ON THE SATISFACTION OF THE FUNCTIONAL EQUATION $A(x, N(x)) = c$

Ana Pradera

Dept. de Ciencias de la Computación
Universidad Rey Juan Carlos, Móstoles, Madrid
ana.pradera@urjc.es

Abstract

With the aim of analyzing the behavior of aggregation functions when receiving contradictory information, this paper studies the satisfaction of the functional equation $A(x, N(x)) = c$, where A is an aggregation function on $[0, 1]$, N is a strong negation and c is a constant value belonging to $[0, 1]$. Special attention is paid to the values $c \in \{0, 1, x_N, e, a\}$, where x_N is the fixed point of the negation and e/a denote, respectively, neutral/absorbing elements.

Keywords: Aggregation functions, strong negations, contradictory information.

1 INTRODUCTION

Due to the key role they play in many practical applications, the interest in *aggregation functions*, which perform the combination of several inputs into a single output, is unceasingly growing (see e.g. the recent monographs on the topic [4, 12, 2, 6]). Large families of aggregation functions, as well as different construction methods, are available, so an important -and difficult- issue is how to choose the most appropriate function for a given application. Several criteria may help in making this choice, such as the satisfaction of some specific properties, the fitting to some empirical data or the fulfillment of a given optimization condition.

In some contexts, the behavior of the aggregation function when receiving *contradictory information* could be a useful criterion. For instance, depending on the application, one may be expecting a low, a high or a medium score when aggregating couples of contradictory information, thus seeking an aggregation function presenting, respectively, an intolerant, a tolerant or a compensative attitude towards contradictory inputs.

This paper proposes a preliminary approach to the above problem, representing contradictory information by means of couples $(x, N(x))$, where N is a negation function, and then studying the satisfaction of the functional equation $A(x, N(x)) = c$ for some distinguished values $c \in [0, 1]$. It is organized as follows. Section 2 briefly recalls the main issues regarding aggregation and negation functions that are needed later on. Section 3 includes the main results of the paper. It first provides some general statements about the fulfillment of the equation $A(x, N(x)) = c$, and then addresses the following distinguished cases: $c = 0$ and $c = 1$, accounting, respectively, for strict intolerant and tolerant behaviors; the case $c = x_N$, where x_N is the negation's fixed point, presenting a compensative behavior; and then the neutral and absorbing behaviors, represented, respectively, by $c = e$ and $c = a$, where e and a stand for neutral and absorbing elements. The paper ends with a summary of the obtained results.

2 PRELIMINARIES

Although *aggregation functions* are defined for inputs of any size, in this paper these terms will be used to refer to *bivariate* aggregation functions, i.e., non-decreasing functions $A : [0, 1]^2 \to [0, 1]$ verifying the boundary conditions $A(0, 0) = 0$ and $A(1, 1) = 1$. Aggregation functions may be compared pointwise as follows: given two functions A_1 and A_2, it is said that A_1 is *weaker* than A_2 (or A_2 is *stronger* than A_1), and it is denoted $A_1 \leq A_2$, when it is $A_1(x, y) \leq A_2(x, y)$ for any $x, y \in [0, 1]$. The relation $\leq$, along with the distinguished functions Min (minimum) and Max (maximum), allows one to classify aggregation functions into the following four categories:

- *Conjunctive* functions, which are those verifying $A \leq Min$. This class includes the well-known *triangular norms* (*t-norms*) as well as *copulas*.
- *Disjunctive* functions, verifying $Max \leq A$, such as *triangular conorms (t-conorms)* and *dual copulas*.
- *Averaging* functions, which verify $Min \leq A \leq Max$, and include, in particular, different functions based on the arithmetic mean, such as *quasi-linear means* or *OWA operators*, as well as integral-based aggregations.
- Finally, the class of *mixed* aggregation functions contains all the operators that do not belong to any of the three previous categories, such as *uninorms*, *nullnorms*, *compensatory T-S operators* or *symmetric sums*.

Recall also that an aggregation function A has a *neutral element* $e \in [0,1]$ whenever $A(x,e) = A(e,x) = x$ for any $x \in [0,1]$, and an *absorbing element* $a \in [0,1]$ when $A(a,x) = A(x,a) = a$ for any $x \in [0,1]$.

On the other hand, contradictory information will be represented with the help of *strong negations* ([13]). A strong negation is a non-increasing function $N : [0,1] \to [0,1]$ which is involutive, that is, verifies $N(N(x)) = x$ for any $x \in [0,1]$. Recall that a function $N : [0,1] \to [0,1]$ is a strong negation if and only if there exists a strictly increasing bijection $\varphi : [0,1] \to [0,1]$ such that $N = N_\varphi$, where $N_\varphi(x) = \varphi^{-1}(1 - \varphi(x))$ for any $x \in [0,1]$. Due to their definition, strong negations are continuous and strictly decreasing functions, they satisfy the boundary conditions $N(0) = 1$ and $N(1) = 0$, and they have a unique fixed point $x_N = \varphi^{-1}(\frac{1}{2}) \in]0,1[$, verifying $N(x_N) = x_N$. The set made of all the strong negations defined on $[0,1]$ will be denoted with $\mathcal{N}_{[0,1]}$.

3 MAIN RESULTS

As already stated in the Introduction, the aim of this paper is to study which aggregation functions satisfy the equation

$$\forall x \in]0,1[, \quad A(x, N(x)) = c \qquad (1)$$

for a given strong negation N and a constant value $c \in [0,1]$ (the points $(0,1)$ and $(1,0)$ are excluded because of their singularity). Note first of all that the weakest and the strongest aggregation functions satisfying (1) are, respectively,

$$A(x,y) = \begin{cases} 0 & \text{if } y < N(x), \\ 1 & \text{if } x = y = 1, \\ c & \text{otherwise.} \end{cases}$$

and

$$A(x,y) = \begin{cases} 0 & \text{if } x = y = 0, \\ c & \text{if } y \leq N(x), \\ 1 & \text{otherwise.} \end{cases}$$

It is also easy to prove that given a strong negation N, the class made of all the aggregation functions satisfying equation (1) is closed under composition by means of any aggregation function having c as idempotent element, i.e., if A_1, A_2 are two aggregation functions satisfying equation (1), then $A(A_1, A_2)$, defined as $A(A_1, A_2)(x,y) = A(A_1(x,y), A_2(x,y))$ for any $x, y \in [0,1]$, does also satisfy the equation as long as $A(c,c) = c$. When $c \in \{0,1\}$ any aggregation function may be used as the outer function A; otherwise, A may be chosen, for example, among averaging functions.

Observe on the other hand that equation (1) refers to one specific strong negation, whereas situations may arise in which one is interested in aggregation functions satisfying the equation for *any* strong negation, i.e., independently of the particular negation that is used to represent contradictory information. This consideration suggests the following stronger version of (1):

$$\forall x \in]0,1[, \quad \forall N \in \mathcal{N}_{[0,1]}, \quad A(x, N(x)) = c \qquad (2)$$

The above equation is evidently much more restrictive that the former one. Averaging functions are to be discarded, and the weakest and the strongest aggregation functions satisfying equation (2) are, respectively

$$A(x,y) = \begin{cases} 0 & \text{if } x = 0 \text{ or } y = 0, \\ 1 & \text{if } x = 1 \text{ and } y = 1, \\ c & \text{otherwise.} \end{cases}$$

and

$$A(x,y) = \begin{cases} 0 & \text{if } x = 0 \text{ and } y = 0, \\ 1 & \text{if } x = 1 \text{ or } y = 1, \\ c & \text{otherwise.} \end{cases}$$

The next subsections are devoted to the following particular cases:

- $c = 0$ and $c = 1$, which reflect, respectively, strict intolerance or large tolerance with respect to contradictory information.
- $c = x_N$, where x_N is the negation's fixed point. Since it is always $Min(x, N(x)) \leq x_N \leq Max(x, N(x))$, this case provides a compensative behavior.
- $c = e$ and $c = a$, when dealing, respectively, with aggregation functions with a neutral element $e \in]0,1[$ or an absorbing element $a \in]0,1[$.

3.1 THE CASES $c = 0$ and $c = 1$

Equations $A(x, N(x)) = 0$ and $A(x, N(x)) = 1$ were analyzed in [11] while studying the satisfaction of the logical Non-Contradiction and Excluded-Middle principles. The main results regarding the former (the approach to the latter is, thanks to duality, similar) are summarized below:

Proposition 1 [11] *Equation* (1) *with* $c = 0$ *is satisfied if and only if for any* $x, y \in [0, 1]$ *it is:*

$$A(x,y) = \begin{cases} 0, & \text{if } y \leq N(x), \{x,y\} \neq \{0,1\} \\ B(x,y), & \text{otherwise} \end{cases}$$

where B *is any bivariate non-decreasing function verifying* $B(1, 1) = 1$.

Clearly the above characterization provides either conjunctive aggregation functions (when B is chosen such that $B \leq Min$) or mixed ones (otherwise). Among the former, one may find different triangular norms, such as the drastic t-norm, the nilpotent minimum or some t-norms isomorphic to the Łukasiewicz t-norm. Regarding mixed functions satisfying equation (1) with $c = 0$, some instances may be found, for example, within the class of compensatory T-S functions (see [11] for details).

Concerning equation (2) with $c = 0$, the following characterization is available:

Proposition 2 [11] *Equation* (2) *with* $c = 0$ *is satisfied if and only if for any* $x, y \in [0, 1]$ *it is:*

$$A(x,y) = \begin{cases} 0, & \text{if } (x,y) \in [0,1[^2 \\ B(x,y), & \text{otherwise} \end{cases}$$

where B *is any bivariate non-decreasing function verifying* $B(1, 1) = 1$.

Aggregation functions characterized in Proposition 2 may only be conjunctive (such as the drastic t-norm, which is the unique t-norm satisfying (2) with $c = 0$) or mixed (e.g. the exponential convex combination of the drastic t-norm and any t-conorm, see [11] for details).

3.2 THE CASE $c = x_N$

The main results associated to the case $c = x_N$, which accounts for a compensative behavior, are the following:

Proposition 3 *Let* A *be an aggregation function and let* N *be a strong negation with fixed point* x_N.

1. *If* A *is either conjunctive or disjunctive, then it does not satisfy equation* (1) *with* $c = x_N$.
2. *If* A *has neutral element* $e \in]0, 1[$, *then* A *satisfies* (1) *with* $c = x_N$ *if and only if it satisfies* (1) *with* $c = e$ *(and in such a case it must be* $x_N = e$*).*
3. *If* A *has absorbing element* $a \in]0, 1[$, *then* A *satisfies* (1) *with* $c = x_N$ *if and only if* $x_N = a$.
4. *If* A *is commutative and* N*-self-dual (i.e.,* $A = N \circ A \circ N \times N$*) then* A *satisfies* (1) *with* $c = x_N$.
5. A *does not satisfy equation* (2) *with* $c = x_N$.

Proof. The first statement comes from the inequality $Min(x, N(x)) \leq x_N \leq Max(x, N(x))$ for all $x \in [0, 1]$. The second one is obtained choosing $x = e$ and taking into account that N is an involution with a unique fixed point. To prove the third one it suffices to choose $x = a$ and to notice that, because of monotonicity, any aggregation function with absorbing element a is such that $A(x, y) = a$ whenever $(x, y) \in [0, a] \times [a, 1] \cup [a, 1] \times [0, a]$. The involutive character of N provides the fourth statement. Finally, to prove the last statement take $(u, v) \in]0, 1[^2$ such that $u \neq v$ and choose two strong negations N_1 and N_2 such that $x_{N_1} \neq x_{N_2}$ and $v = N_1(u) = N_2(u)$; then the fulfillment of equation (2) with $c = x_N$ would entail the contradiction $x_{N_1} = x_{N_2}$. ∎

The particular cases where A has a neutral or an absorbing element will be treated in the next subsections. For the general case, the condition given by the fourth item of Proposition 3 is very useful for finding both averaging and mixed aggregation functions satisfying $A(x, N(x)) = x_N$ for any $x \in]0, 1[$, since different families and construction methods for N-self-dual aggregation functions are available (see e.g. [9]). For instance:

Proposition 4 *Given a strong negation* $N = N_\varphi$, *the* quasi-arithmetic mean *generated by* φ, *given by* $M_\varphi(x, y) = \varphi^{-1}(\frac{\varphi(x)+\varphi(y)}{2})$, *is an averaging function satisfying* $A(x, N(x)) = x_N$ *for any* $x \in [0, 1]$.

Proposition 5 *Given a strong negation* $N = N_\varphi$ *and an arbitrary commutative aggregation function* B, *the two following constructions provide aggregation functions verifying* $A(x, N(x)) = x_N$ *for any* $x \in [0, 1]$ *(the first one with convention* $\frac{0}{0} = \frac{1}{2}$*):*

$$A(x,y) = \varphi^{-1}\left(\frac{B(x,y)}{B(x,y) + B(N(x),N(y))}\right)$$

$$A(x,y) = \varphi^{-1}\left(\frac{B(x,y) + 1 - B(N(x),N(y))}{2}\right)$$

3.3 THE CASE $c = e$, $e \neq 0, 1$

As proved in Proposition 3, when dealing with aggregation functions with a neutral element $e \in]0,1[$ it appears that $A(x, N(x)) = e$ for all $x \in]0,1[$ is equivalent to $A(x, N(x)) = x_N$ for all $x \in]0,1[$. Hence, the results given in Proposition 3 also apply here: only averaging and mixed aggregation functions with neutral element $e = x_N$ can be considered, and a wide family of functions in this class satisfying $A(x, N(x)) = e$ for all $x \in]0,1[$ is given by those which are commutative and N-self-dual.

Remark 6 *Note that the satisfaction of $A(x, N(x)) = e$ for any $x \in]0,1[$ is equivalent to the fact that all the tuples $(x, N(x))$ are* neutral tuples *(see [1]) for the aggregation function A.*

Within the class of averaging functions, the two following functions are, respectively, the weakest and the strongest satisfying $A(x, N(x)) = e$ for any $x \in]0,1[$:

$$A(x,y) = \begin{cases} Min(x,y) & \text{if } y < N(x), \\ Max(x,y) & \text{if } x, y \geq e, \\ e & \text{otherwise.} \end{cases}$$

and

$$A(x,y) = \begin{cases} Min(x,y) & \text{if } x, y \leq e, \\ Max(x,y) & \text{if } y > N(x), \\ e & \text{otherwise.} \end{cases}$$

The next proposition provides a means for building commutative N-self-dual aggregation functions with a neutral element $e = x_N$ that, as stated in Proposition 3, satisfy $A(x, N(x)) = e$:

Proposition 7 *Given a strong negation $N = N_\varphi$ and a commutative aggregation function B with neutral element x_N, the following construction provides aggregation functions with neutral element x_N satisfying $A(x, N(x)) = x_N$ for any $x \in [0,1]$:*

$$A(x,y) = \varphi^{-1}\left(\frac{\varphi(B(x,y)) + 1 - \varphi(B(N(x), N(y)))}{2}\right)$$

The most popular aggregation functions with a neutral element $e \in]0,1[$ are those which are associative and commutative, known as *uninorms* ([14]). Are there uninorms satisfying $A(x, N(x)) = e$ for all $x \in]0,1[$? The next proposition shows that, among the most important classes of uninorms, only one satisfies the above equation.

Proposition 8 *Let $U : [0,1]^2 \to [0,1]$ be a uninorm and let N be a strong negation.*

1. *If U is locally internal on $[0,e] \times [e,1] \cup [e,1] \times [0,e]$ (i.e., it is $U(x,y) \in \{x,y\}$ for any (x,y) in that region), then it does not satisfy $U(x, N(x)) = e$ for all $x \in]0,1[$.*
2. *If U is continuous on $]0,1[^2$, then it satisfies $U(x, N(x)) = e$ for all $x \in]0,1[$ if and only if U is a representable uninorm with generating function u and $N = N_u$, where $N_u = u^{-1} \circ (-u)$.*

Proof. Note first that $e = x_N$ is a necessary condition for the satisfaction of $U(x, N(x)) = e$ for all $x \in]0,1[$ (choosing $x = e$ the equation provides $N(e) = e$, which entails $e = x_N$ due to the uniqueness of the strong negation's fixed point). This means that any couple $(x, N(x))$ belongs to the region $[0,e] \times [e,1] \cup [e,1] \times [0,e]$. Then:

1. If U is locally internal, $U(x, N(x)) \in \{x, N(x)\}$, and hence $U(x, N(x)) \neq e$ for any $x \neq e$.
2. The structure of $]0,1[$-continuous uninorms given by Hu and Li in [7] proves that, unless U is a representable uninorm, there is always a value $0 < k < e$ such that the output of U on the region $[0,k] \times [k,1] \cup [k,1] \times [0,k]$ coincides either with the minimum or with the maximum, and is, hence, different from e. Now, if U is a representable uninorm, then it is well-known that it may be written (see e.g. [5]) as $U(x,y) = u^{-1}(u(x) + u(y))$ for any $(x,y) \in [0,1]^2 \setminus \{(0,1),(1,0)\}$, where $u : [0,1] \to [-\infty, +\infty]$ is a strictly increasing bijection such that $u(e) = 0$. Then the equation $U(x, N(x)) = e$ is clearly equivalent to $N = N_u$.

∎

Remark 9 *Note that the class described in the first item of Proposition 8 includes well-known uninorms, such as* idempotent uninorms *or uninorms in the classes $\mathcal{U}_{\min}$ and $\mathcal{U}_{\max}$ (those obtained when choosing, respectively, Min and Max in the region $[0,e] \times [e,1] \cup [e,1] \times [0,e]$). Observe in addition that the result regarding representable uninorms (Proposition 8, item 2) could have been obtained, alternatively, from the fourth item in Proposition 3, since representable uninorms are self-dual, except at the points $(0,1)$ and $(1,0)$, with respect to their associated negation N_u (see e.g. [5]).*

3.4 THE CASE $c = a$, $a \neq 0, 1$

When dealing with aggregation functions possessing an absorbing element $a \in]0,1[$, both conjunctive and disjunctive functions are to be discarded (since they have absorbing elements 0 and 1, respectively) and the following may be stated:

Proposition 10 *Let A be an aggregation function with absorbing element $a \in]0,1[$ and let N be a strong negation with fixed point x_N.*

1. *If $a = x_N$, A satisfies equation* (1) *with $c = a$.*
2. *A satisfies equation* (2) *with $c = a$ if and only if $A(x,y) = a \quad \forall (x,y) \in]0,1[^2$.*

Proof. The first item is a consequence of item 3 in Proposition 3; the second one is a matter of calculation. ∎

Let us focuss on the class of *nullnorms* ([10, 3]), the most important aggregation functions with an absorbing element. Recall that nullnorms are aggregation functions $V : [0,1]^2 \to [0,1]$ which are associative, commutative and possess an element $a \in]0,1[$ such that for any $x \in [0,1]$, it is $V(x,0) = x$ if $x \leq a$ and $V(x,1) = x$ if $x \geq a$. Nullnorms have absorbing element a, they behave as a t-conorm in $[0,a]^2$ and as a t-norm in $[a,1]^2$, and verify $V(x,y) = a$ otherwise. They have the following structure (where T is a t-norm and S is a t-conorm):

$$V_{a,T,S}(x,y) = \begin{cases} a \cdot S(\frac{x}{a}, \frac{y}{a}), & \text{if } x,y \leq a \\ a + (1-a) \cdot T(\frac{x-a}{1-a}, \frac{y-a}{1-a}), & \text{if } x,y \geq a \\ a, & \text{otherwise} \end{cases}$$

Proposition 11 *Let $V_{a,T,S}$ be a nullnorm and let N be a strong negation with fixed point x_N.*

1. *$V_{a,T,S}$ satisfies equation* (1) *with $c = a$ if and only if one of the following conditions holds:*
   (a) $x_N = a$
   (b) $x_N < a$ and $S(u,v) = 1 \; \forall u \geq \frac{N(a)}{a}, \forall v \geq \frac{N(ua)}{a}$.
   (c) $x_N > a$ and $T(u,v) = 0 \; \forall u \leq \frac{N(a)-a}{1-a}, \forall v \leq \frac{N(a+(1-a)u)-a}{1-a}$.
2. *$V_{a,T,S}$ satisfies equation* (2) *with $c = a$ if and only if T and S are, respectively, the drastic t-norm and the drastic t-conorm.*

Remark 12 *Note that cases (b) and (c) in Proposition 11 pose the problem of the N_α-annihilation of t-conorms and t-norms, a generalization of the N-annihilation problem (see e.g. [8]). Indeed, focussing on (c) (the problem for t-conorms is dual), and denoting $\alpha = \frac{N(a)-a}{1-a}$ and $N_\alpha(u) = \frac{N(a+(1-a)u)-a}{1-a}$, condition (c) may be equivalently written as*

$$x_N > a \text{ and } T(u,v) = 0 \; \forall u \leq \alpha, \forall v \leq N_\alpha(u)$$

where $N_\alpha : [0,\alpha] \to [0,\alpha]$ is a strong negation on $[0,\alpha]$.

4 CONCLUSIONS

This paper has proposed a simple preliminary approach for studying the behavior of bivariate aggregation functions when receiving contradictory information. The existence of aggregation functions satisfying equations (1) and (2) has been investigated, taking into account the four different categories in which aggregation functions may be classified, as well as the most important distinguished values for the constant c (see tables 1 and 2 for a summary). Whenever possible, concrete examples of aggregation functions satisfying equations (1) and (2) have been provided.

Table 1: Existence of aggregation functions satisfying $A(x,N(x)) = c \;\; \forall x \in]0,1[$

	Conj.	Disj.	Aver.	Mix.
$c = 0$	✓	×	×	✓
$c = 1$	×	✓	×	✓
$c = x_N$	×	×	✓ (any if $x_N = a$)	
$c = e \neq 0,1$	×	×	✓ (only if $e = x_N$)	
$c = a \neq 0,1$	×	×	✓ (any if $a = x_N$)	

Table 2: Existence of aggregation functions satisfying $A(x,N(x)) = c \;\; \forall x \in]0,1[, \forall N \in \mathcal{N}_{[0,1]}$

	Conj.	Disj.	Aver.	Mix.
$c = 0$	✓	×	×	✓
$c = 1$	×	✓	×	✓
$c = x_N$	×	×	×	×
$c = e \neq 0,1$	×	×	×	×
$c = a \neq 0,1$	×	×	×	✓

References

[1] G. Beliakov, T. Calvo, and A. Pradera. Handling of neutral information by aggregation operators. *Fuzzy Sets and Systems*, 158(8):861–880, 2007.

[2] G. Beliakov, A. Pradera, and T. Calvo. *Aggregation Functions: A Guide for Practitioners.* Springer, 2007.

[3] T. Calvo, B. De Baets, and J. Fodor. The functional equations of Frank and Alsina for uninorms and nullnorms. *Fuzzy Sets and Systems*, 120:385–394, 2001.

[4] T. Calvo, A. Kolesárová, M. Komorníková, and R. Mesiar. Aggregation operators: properties, classes and construction methods. In T. Calvo, G. Mayor, and R. Mesiar, editors, *Aggregation Operators. New Trends and Applications*, pages 3–104. Physica-Verlag, Heidelberg, New York, 2002.

[5] J. Fodor, R. Yager, and A. Rybalov. Structure of uninorms. *Int. J. of Uncertainty, Fuzziness and Knowledge-Based Systems*, 5:411–427, 1997.

[6] M. Grabisch, J.L. Marichal, R. Mesiar, and E. Pap. *Aggregation Functions*. Cambridge University Press, 2009.

[7] S.k. Hu and Z.f. Li. The structure of continuous uni-norms. *Fuzzy Sets and Systems*, 124(1):43–52, 2001.

[8] S. Jenei. How to construct left-continuous triangular norms-state of the art. *Fuzzy Sets and Systems*, 143:27–45, 2004.

[9] K. C. Maes, S. Saminger, and B. De Baets. Representation and construction of self-dual aggregation operators. *Europ. J. Oper. Res.*, 177:472–487, 2007.

[10] M. Mas, G. Mayor, and J. Torrens. T-operators. *Int. J. of Uncertainty, Fuzziness and Knowledge-Based Systems*, 7(1):31–50, 1999.

[11] A. Pradera and E. Trillas. Aggregation, Non-Contradiction and Excluded-Middle. *Mathware & Soft Computing*, XIII:189–201, 2006.

[12] V. Torra and Y. Narukawa. *Modeling Decisions: Information Fusion and Aggregation Operators*. Springer, 2007.

[13] E. Trillas. Sobre funciones de negación en la teoría de los subconjuntos difusos. *Stochastica*, III:47–59 (in Spanish) Reprinted (English version) in Advances of Fuzzy Logic, S. Barro et al. (eds), Universidad de Santiago de Compostela 31–43, 1998., 1979.

[14] R. Yager and A. Rybalov. Uninorm aggregation operators. *Fuzzy Sets and Systems*, 80:111–120, 1996.