PROBABILISTIC DECISION MAKING WITH THE OWA OPERATOR AND THE 2-TUPLE LINGUISTIC APPROACH

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Abstract

We present a new probabilistic decision making model by using the 2-tuple linguistic representation approach. We introduce the 2-tuple linguistic probabilistic ordered weighted averaging (2T-LPOWA) operator. We study some of its particular cases including the 2-tuple linguistic probabilistic aggregation and the 2-tuple linguistic arithmetic OWA operator. We show an application of the new approach in a decision making problem about the selection of investments.

Keywords: Decision making, Probability, OWA operator, 2-tuple linguistic approach.

1 INTRODUCTION

Decision making problems are very common in a lot of disciplines [3-11,15-19]. A very useful tool to accomplish aggregation process in decision making is the ordered weighted averaging (OWA) operator [15]. Such an aggregation operator provides a method for representing the attitudinal attitude of the decision maker in the aggregation process. Since its appearance, the OWA operator has been studied and applied by many authors [1,3-19].

Recently, Merigó [9] has suggested a new model that unifies the probability with the OWA operator in the same formulation and considering the degree of importance that each concept has in the aggregation. It was called the probabilistic ordered weighted averaging (POWA) operator. Note that in the literature there are other aggregation operators analyzing the use of probabilities and the OWA in the same formulation such as the immediate probability. Note that it is also possible to extend other approaches that use OWAs and weighted averages (WAs) in the same formulation to the probabilistic framework [12,14].

When using the POWA operator, it is assumed that the available information is numerical. However, this may not be the real situation in decision making problems. Sometimes, the available information is vague or imprecise and it is not possible to analyze it with numerical values and this uncertainty does not have a probability character because they are related to imprecision and vagueness of meaning. Then, a better approach may be the use of linguistic assessments [20]. In the literature, we find a wide range of methods for dealing with linguistic information such as [2,4-6,8,10,13,20]. In this paper, we will use the linguistic 2-tuple representation model [5-6,8,10,13] in order to accomplish processes of computing with words without loss of information.

The aim of this paper is to present a new model for decision making when the available information can be represented with probabilities, OWA operators and linguistic variables. Thus, we are able to deal with decision making problems under risk and under uncertain environments in the same formulation. Moreover, we can assess the uncertain information with linguistic variables obtaining a more complete representation of the decision problem. For doing so, we introduce the 2-tuple linguistic probabilistic ordered weighted averaging (2T-LPOWA) operator. It is a new aggregation operator that includes a wide range of particular cases such as the simple linguistic probabilistic aggregation, the 2-tuple OWA, the 2-tuple linguistic probabilistic arithmetic mean and a lot of other cases. We study some of its main properties and provide an illustrative example in a decision making problem about investment selection.

This paper is organized as follows. In Section 2, we briefly review some basic concepts. Section 3 presents the 2T-LPOWA operator and Section 4 gives an illustrative example. In Section 5 we end the paper summarizing the conclusions.

2 PRELIMINARIES

2.1. 2-TUPLE LINGUISTIC APPROACH

In [5], Herrera and Martínez developed a fuzzy linguistic representation model, which represents the linguistic information with a pair of values called 2-tuple, (s, α), where s is a linguistic label and α is a numerical value that represents the value of the symbolic translation. With this model, it is possible to accomplish CW processes without loss of information, solving one of the main limitations of the previous linguistic computational models [2,4,20].

Definition 1. *Let β be the result of an aggregation of the indexes of a set of labels assessed in the linguistic label set $S = \{s_0, s_1, \dots, s_g\}$, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g + 1$ the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-0.5, 0.5)$, then α is called a symbolic translation.*

Note that the 2-tuple (s_i, α) that expresses the equivalent information to β is obtained with the following function:

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5),$$

$$\Delta(\beta) = \begin{cases} s_i & i = round(\beta), \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5). \end{cases} \quad (1)$$

where round is the usual round operation, s_i has the closest index label to β and α is the value of the symbolic translation.

Proposition 1. *Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and (s_i, α_i) be a linguistic 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g]$.*

Proof. *It is trivial, we consider the following function:*

$$\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

For further information on the 2-tuple linguistic representation model, see [5-6,8,10,13].

2.2. THE OWA OPERATOR

The OWA operator was introduced by Yager in [15] and it provides a parameterized family of aggregation operators that include the arithmetic mean, the maximum and the minimum. It can be defined as follows.

Definition 2. An OWA operator of dimension n is a mapping OWA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, then:

$$OWA\ (a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j b_j \quad (2)$$

where b_j is the jth largest of the a_i.

The OWA operator is commutative, monotonic, bounded and idempotent. For further information about the OWA and its applications, see, [1,3-19].

2.3. THE PROBABILISTIC OWA OPERATOR

The probabilistic ordered weighted averaging (POWA) operator is an aggregation operator that unifies the probability and the OWA operator in the same formulation considering the degree that each concept has in the analysis [8-9]. With this approach, we can either under estimate or over estimate the probabilities according to the attitudinal attitude of the decision maker. It can be defined as follows.

Definition 3. A POWA operator of dimension n is a mapping POWA: $R^n \rightarrow R$ that has associated two weighting vector W and V of dimension n such that w_j *and* $v_i \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$ and $\sum_{i=1}^{n} v_i = 1$, according to the following formula:

$$POWA\ (a_1, \dots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j \quad (3)$$

where b_j is the jth largest of the a_i, each argument a_i has an associated weight (probability) v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, $\hat{v}_j = \delta w_j + (1-\delta) v_j$ with $\delta \in [0, 1]$ and v_j is the weight (probability) v_i ordered according to b_j, that is, according to the jth largest of the a_i.

Note that it is possible to obtain a wide range of particular types of POWA operators [8-9]. Especially, when $\delta = 0$, we get the probabilistic approach, and if $\delta = 1$, we get the OWA operator. Other interesting cases are found when $w_j = 1/n$, for all a_i, because then, we get the arithmetic probability (AP). And if $v_i = 1/n$, for all a_i, we get the arithmetic OWA operator. Note that inside the arithmetic OWA we find the arithmetic maximum and the arithmetic minimum, and so on.

3 LINGUISTIC POWA OPERATOR

The 2-tuple linguistic probabilistic ordered weighted averaging (2T-LPOWA) operator is an extension of the POWA operator for situations where the available information is uncertain but can be assessed with the 2-tuple linguistic representation model. It can also be seen as a unification between linguistic decision making problems under uncertainty (with linguistic OWA operators) and under risk (with probabilities with linguistic information). This approach seems to be complete, at least as an initial real unification between OWA operators and probabilities under a linguistic environment.

However, note that some previous models already considered the possibility of using OWA operators and probabilities in the same formulation. The main model is the concept of immediate probability explained in Section 2 [3,7-8,17-19].

Definition 4. Let S be the set of the 2-tuples. An IP-LOWA operator of dimension n is a mapping IP-LOWA: $S^n \to S$ that has associated two weighting vector W and V of dimension n such that w_j *and* $v_i \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$ and $\sum_{i=1}^{n} v_i = 1$, according to the following formula:

$$IP\text{-}LOWA((s_1, \alpha_1), \ldots, (s_n, \alpha_n)) = \Delta(\sum_{j=1}^{n} \hat{v}_j \Delta^{-1}(s_j, \alpha_j)) \quad (4)$$

where $\Delta^{-1}(s_j, \alpha_j)$ is the jth largest of the $\Delta^{-1}(s_i, \alpha_i)$ values, each $\Delta^{-1}(s_i, \alpha_i)$ has associated a probability v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$ and v_j is the probability v_i ordered according to $\Delta^{-1}(s_j, \alpha_j)$, that is, according to the jth largest of the $\Delta^{-1}(s_i, \alpha_i)$ values.

Although it seems to be a good approach it is not so complete than the POWA because it can unify OWAs and probabilities in the same model but it can not take in consideration the degree of importance of each case in the aggregation process. Note that the IP-OWA (or IP-LOWA) could be formulated in other ways in order to obtain the immediate weights. For example, we could use $\hat{v}_j = ((w_j + v_j) / \sum_{j=1}^{n} (w_j + v_j))$, and the results would be also similar and consistent because they would accomplish the main aggregation properties, $\hat{v}_j = ((\alpha w_j + (1-\alpha) v_j) / \sum_{j=1}^{n} (\alpha w_j + (1-\alpha) v_j))$, and so on.

Other methods that could be considered are the hybrid averaging (HA) operator [14] and the weighted OWA (WOWA) operator [12]. These methods are focused on the weighted average (WA) but it is easy to extend them to probabilities because sometimes the WA is used as a subjective probability. Note that in this case, we could be talking about the 2-tuple WOWA or the 2-tuple HA or its probabilistic version. As said before, these an other approaches are useful for some particular situations but they does not seem to be so complete than the POWA because they can unify OWAs with probabilities (or with WAs) but they can not unify them giving different degrees of importance to each case. Note that in future research we will also prove that these models can be seen as a special case of a general POWA operator (or its respective model with WAs) that uses quasi-arithmetic means. Obviously, it is possible to develop more complex models of the IP-OWA, the HA and the WOWA that takes into account the degree of importance of the OWAs and the probabilities (or WAs) in the model but they seem to be artificial and not a natural unification as it will be shown below.

In the following, we are going to analyze the 2T-LPOWA (or LPOWA) operator. Note that we use the 2-tuple linguistic representation model but it would be possible to use a lot of other linguistic approaches [8]. It can be defined as follows.

Definition 5. Let S be the set of the 2-tuples. A 2T-LPOWA operator of dimension n is a mapping 2T-PLOWA: $S^n \to S$ that has associated two weighting vector W and V of dimension n such that w_j *and* $v_i \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$ and $\sum_{i=1}^{n} v_i = 1$, according to the following formula:

$$2T\text{-}LPOWA((s_1, \alpha_1), \ldots, (s_n, \alpha_n)) = \Delta(\sum_{j=1}^{n} \hat{v}_j \Delta^{-1}(s_j, \alpha_j)) \quad (5)$$

where $\Delta^{-1}(s_j, \alpha_j)$ is the jth largest of the $\Delta^{-1}(s_i, \alpha_i)$ values, each $\Delta^{-1}(s_i, \alpha_i)$ has associated a probability v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, $\hat{v}_j = \delta w_j + (1-\delta) v_j$ with $\delta \in [0, 1]$ and v_j is the probability v_i ordered according to $\Delta^{-1}(s_j, \alpha_j)$, that is, according to the jth largest of the $\Delta^{-1}(s_i, \alpha_i)$ values.

Note that it is also possible to formulate the 2T-LPOWA operator separating the part that strictly affects the OWA operator and the part that affects the probabilities. This representation is useful to see both models in the same formulation but it does not seem to be as a unique equation that unifies both models.

Remark 1. Let S be the set of the 2-tuples. A 2T-LPOWA operator of dimension n is a mapping 2T-PLOWA: $S^n \to S$ that has an associated weighting vector W, with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$ and a probabilistic vector V, with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, such that:

2T-LPOWA $((s_1, \alpha_1), \ldots, (s_n, \alpha_n)) =$

$$= \Delta\left(\delta \sum_{j=1}^{n} w_j \Delta^{-1}(s_j, \alpha_j) + (1-\delta) \sum_{i=1}^{n} v_i \Delta^{-1}(s_i, \alpha_i)\right) \quad (6)$$

where $\Delta^{-1}(s_j, \alpha_j)$ is the jth largest of the $\Delta^{-1}(s_i, \alpha_i)$ values and $\delta \in [0, 1]$.

Note that we can distinguish between the descending 2T-LPOWA (2T-LDPOWA) and the ascending 2T-LPOWA (2T-LAPOWA) operator by using $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the 2T-LDPOWA and w^*_{n-j+1} the jth weight of the 2T-LAPOWA operator.

If B is a vector corresponding to the ordered arguments $\Delta^{-1}(s_j, \alpha_j)$, we shall call this the ordered argument vector and W^T is the transpose of the weighting vector, then, the 2T-LPOWA operator can be expressed as:

$$\text{2T-LPOWA } ((s_1, \alpha_1), \ldots, (s_n, \alpha_n)) = \Delta\left(\hat{V}^T B\right) \quad (7)$$

Remark 2. Note that in some situations we may find that the weighting vector is not normalized, i.e., $\hat{V} = \sum_{j=1}^{n} \hat{v}_j \neq 1$, then, the 2T-LPOWA operator can be reformulated as follows:

$$\textit{2T-LPOWA } ((s_1, \alpha_1), \ldots, (s_n, \alpha_n)) = \Delta\left(\frac{1}{\hat{V}} \sum_{j=1}^{n} \hat{v}_j \beta_j^*\right) \quad (8)$$

The 2T-LPOWA is monotonic, bounded and idempotent. It is not commutative because the probabilistic aggregations (with WAs) are not commutative.

- It is monotonic because if $(s_i, \alpha_i) \geq (s_i^*, \chi_i)$, for all i, then, 2T-LPOWA $((s_1, \alpha_1), \ldots, (s_n, \alpha_n)) \geq$ 2T-LPOWA $((s_1^*, \chi_1), \ldots, (s_n^*, \chi_n))$.

- It is bounded because the 2T-LPOWA aggregation is delimitated by the linguistic minimum and the linguistic maximum. That is, Min$\{(s_i, \alpha_i)\} \leq$ 2T-LPOWA $((s_1, \alpha_1), \ldots, (s_n, \alpha_n)) \leq$ Max$\{(s_i, \alpha_i)\}$.

- It is idempotent because if $(s_i, \alpha_i) = (s, \alpha)$, for all (s_i, α_i), then, 2T-LPOWA $((s_1, \alpha_1), \ldots, (s_n, \alpha_n)) = (s, \alpha)$.

Another interesting issue to analyze are the measures for characterizing the weighting vector W. Following a similar methodology as it has been developed for the OWA operator [8,15,19] we can formulate the attitudinal character, the entropy of dispersion, the divergence of W and the balance operator. Note that these measures affect the weighting vector W but not the probabilities because they are given as some kind of objective information. For example, the attitudinal character can be formulated as follows.

$$\alpha(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1}\right) \quad (9)$$

Another interesting issue to analyze is the different particular cases of the 2T-LPOWA operator that can be obtained by studying the weighting vector W and the parameter δ. For example, we can consider the following cases.

- If $\delta = 0$, we get the 2-tuple linguistic probabilistic aggregation.
- If $\delta = 1$, we get the 2-tuple linguistic OWA operator.
- The 2-tuple linguistic arithmetic probabilistic aggregation (if $w_j = 1/n$, for all j).
- The 2-tuple linguistic arithmetic OWA operator (if $v_i = 1/n$, for all i).
- The 2-tuple linguistic average (if $v_i = 1/n$, for all i, and $w_j = 1/n$, for all j).
- The 2-tuple linguistic maximum probabilistic aggregation ($w_1 = 1$ and $w_j = 0$, for all $j \neq 1$).
- The 2-tuple linguistic minimum probabilistic aggregation ($w_n = 1$ and $w_j = 0$, for all $j \neq n$).
- The 2-tuple linguistic Hurwicz probabilistic criteria ($w_1 = \alpha$, $w_n = 1 - \alpha$ and $w_j = 0$, for all $j \neq 1, n$).
- The step-2T-LPOWA ($w_k = 1$ and $w_j = 0$, for all $j \neq k$).
- The olympic-2T-LPOWA operator ($w_1 = w_n = 0$, and $w_j = 1/(n-2)$ for all others).
- The general olympic-2T-LPOWA operator ($w_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$; and for all others $w_{j^*} = 1/(n-2k)$, where $k < n/2$).

- The general probabilistic olympic-2T-LOWA operator ($\hat{v}_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$; and for all others $\hat{v}_{j*} = 1/(n-2k)$, where $k < n/2$).
- The S-2T-LPOWA ($w_1 = (1/n)(1 - (\alpha + \beta) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta)$ for $j = 2$ to $n - 1$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$).
- The centered-2T-LPOWA (if it is symmetric, strongly decaying from the center to the maximum and the minimum, and inclusive).
- Etc.

Note that other families of 2T-LPOWA operators may be used following a similar methodology as it has been developed for the OWA operator and its extensions [1,7-19]. Moreover, we could extend this analysis to other types of linguistic approaches [2,4,13,20].

4 ILLUSTRATIVE EXAMPLE

The 2T-LPOWA operator is applicable in a wide range of situations where it is possible to use probabilities, linguistic information and OWA operators. Therefore, we see that the applicability is incredibly broad because all the previous models and theories that use the probability can be extended by using the 2T-LPOWA operator. The reason is that all the problems with probabilities deal with uncertainty. Therefore, in a lot of situations the information is not clear and needs to be assessed with linguistic variables. Moreover by using the 2T-LPOWA operator we can underestimate or overestimate the results according to a degree of orness that we want to have in the aggregation.

Summarizing some of the main fields where it is possible to apply the 2T-LPOWA operator, we can mention: Statistics (especially in probability theory), Mathematics, Economics, Decision theory, Engineering, Physics, etc.

In this paper, we focus on an application in decision making about selection of investments. The main reason for using the 2T-LPOWA operator is that we are able to assess the decision making problem considering linguistic information, probabilities and the attitudinal character of the decision maker. Thus, we get a more complete representation of the decision problem.

We analyze a company that operates in Europe and North America that wants to invest some money in a new market. They consider five alternatives.

- A_1 = Invest in the Asian market.
- A_2 = Invest in the South American market.
- A_3 = Invest in the African market.
- A_4 = Invest in all three markets.
- A_5 = Do not invest money in any market.

In order to evaluate these investments, the investor has brought together a group of experts. This group considers that the key factor is the economic situation of the world economy for the next period. They consider 5 possible states of nature that could happen in the future:

- N_1 = Very bad economic situation.
- N_2 = Bad economic situation.
- N_3 = Regular economic situation.
- N_4 = Good economic situation.
- N_5 = Very good economic situation.

The results of the available investments, depending on the state of nature N_i and the alternative A_k that the decision maker chooses, are shown in Table 1. Note that we assume a set of seven linguistic terms $S = \{s_1 = None, s_2 = Very\ Low, s_3 = Low, s_4 = Medium, s_5 = High, s_6 = Very\ High, s_7 = Perfect\}$.

Table 1: Available investment alternatives

	N_1	N_2	N_3	N_4	N_5
A_1	$(s_3, 0)$	$(s_5, 0)$	$(s_6, 0)$	$(s_2, 0)$	$(s_4, 0)$
A_2	$(s_7, 0)$	$(s_6, 0)$	$(s_2, 0)$	$(s_2, 0)$	$(s_2, 0)$
A_3	$(s_4, 0)$	$(s_2, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_5, 0)$
A_4	$(s_6, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_5, 0)$	$(s_2, 0)$
A_5	$(s_2, 0)$	$(s_3, 0)$	$(s_7, 0)$	$(s_4, 0)$	$(s_2, 0)$

In this problem, the experts assume the following weighting vector: $W = (0.1, 0.2, 0.2, 0.2, 0.3)$. They assume that the probability that each state of nature will happen is: $P = (0.3, 0.3, 0.2, 0.1, 0.1)$. Note that the OWA operator has an importance of 30% and the probabilistic information an importance of 70%. With this information, we can aggregate the expected results in order to make a decision. In Table 2, we present different results obtained by using different types of 2T-LPOWA operators.

Table 2: Aggregated results

	2T-Prob.	2TOWA	2T-PAM	POWA
A_1	$(S_4, 0.2)$	$(S_4, -0.4)$	$(S_4, 0.14)$	$(S_4, 0.02)$
A_2	$(S_5, -0.3)$	$(S_3, 0.3)$	$(S_4, 0.43)$	$(S_4, 0.28)$
A_3	$(S_3, 0.5)$	$(S_3, 0.5)$	$(S_4, -0.41)$	$(S_3, 0.5)$
A_4	$(S_4, 0.5)$	$(S_4, -0.2)$	$(S_4, 0.41)$	$(S_4, 0.29)$
A_5	$(S_3, 0.5)$	$(S_3, 0.1)$	$(S_4, -0.47)$	$(S_3, 0.38)$

If we establish an ordering of the investments, a typical situation if we want to consider more than one alternative, then, we get the results shown in Table 3. Note that the first alternative in each ordering is the optimal choice.

Table 3: Ordering of the investments

	Ordering
2-tuple probabilistic	$A_2 \succ A_4 \succ A_1 \succ A_3 = A_5$
2T-OWA	$A_4 \succ A_1 \succ A_3 \succ A_2 \succ A_5$
2T-PAM	$A_2 \succ A_4 \succ A_1 \succ A_3 \succ A_5$
2T-POWA	$A_4 \succ A_2 \succ A_1 \succ A_3 \succ A_5$

As we can see, depending on the aggregation operator used, the ordering of the investments may be different. Sometimes, it is clear which the best alternative is because one alternative is clearly better than the others. But sometimes, it is not clear because the alternatives give similar results and depending on the state of nature that happens in the future the optimal alternative may be different. Therefore, depending on the particular type of 2T-LPOWA operator used, the optimal choice may change because the 2T-LPOWA provides a parameterized family of aggregation operators between the maximum and the minimum. Thus, when looking to the maximum and similar results, we may find an optimal alternative that is different than looking to the minimum.

5 CONCLUSIONS

We have presented the 2T-LPOWA operator. It is a new aggregation operator that unifies the probability and the OWA operator in the same formulation and considering the degree of importance that each concept has in the aggregation. Moreover, it also deals with uncertain environments where the available information can be assessed with linguistic variables. Thus we get a more complete representation of the uncertain information and we are able to compute with words in the aggregation process. We have analyzed some of its main particular cases such as the 2-tuple linguistic probabilistic aggregation, the 2-tuple linguistic OWA operator, the 2-tuple linguistic arithmetic mean, the 2-tuple linguistic arithmetic OWA operator and the 2-tuple linguistic probabilistic arithmetic mean.

We have also studied the applicability of the new approach and we have seen that it is very broad because all the studies that use the probability can be revised and extended with this new model. We have focussed on a decision making problem about the selection of investments. We have seen that depending on the particular type of 2T-LPOWA operator used, the results may lead to different decisions.

In future research we expect to develop further developments of this approach by using other types of linguistic information and adding new characteristics and generalizations such as the use of order-inducing variables, distance measures and quasi-arithmetic means. We will also consider other business applications and we will extend it to group decision making.

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References

[1] G. Beliakov, A. Pradera and T. Calvo, *Aggregation Functions: A Guide for Practitioners*. Berlin: Springer-Verlag, 2007.

[2] H. P. Bonissone, "A Fuzzy Sets Based Linguistic Approach: Theory and Applications", in: M.M. Gupta and E. Sanchez, Eds., *Approximate Reasoning in Decision Analysis*, North-Holland, pp. 329-339, 1982.

[3] K.J. Engemann, D.P. Filev and R.R. Yager, Modelling decision making using immediate probabilities. *International Journal of General Systems*, 24:281-294, 1996.

[4] F. Herrera, E. Herrera-Viedma and J.L. Verdegay, "A Sequential Selection Process in Group Decision Making with a Linguistic Assessment Approach", *Information Sciences*, 85, 1995, pp. 223-239.

[5] F. Herrera and L. Martínez, "A 2-tuple Fuzzy Linguistic Representation Model for Computing with Words", *IEEE Transactions on Fuzzy Systems*, 8, 2000, pp. 746-752.

[6] F. Herrera and L. Martínez, "The 2-tuple Linguistic Computational Model. Advantages of its linguistic description, accuracy and consistency", *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 9, 2001, pp. 33-48.

[7] J.M. Merigó, Using immediate probabilities in fuzzy decision making. In: *Proceedings of the 22nd ASEPELT Conference*, Barcelona, pp. 1650-1664, 2008.

[8] J.M. Merigó, *New extensions to the OWA operators and its application in decision making* (In Spanish). PhD Thesis, Department of Business Administration, University of Barcelona, 2008.

[9] J.M. Merigó, Probabilistic decision making with the OWA operator and its application in investment management. In: *Proceedings of the IFSA-EUSFLAT Conference*, Lisbon, Portugal, pp. 1364-1369, 2009.

[10] J.M. Merigó, M. Casanovas, L. Martínez, A decision making model based on Dempster-Shafer theory and linguistic hybrid aggregation operators. In:

Proceedings of the 8th HIS Conference, Barcelona, pp. 80-85, 2008.

[11] J.M. Merigó and A.M. Gil-Lafuente, The induced generalized OWA operator. *Information Sciences*, 179:729-741, 2009.

[12] V. Torra, The weighted OWA operator. *International Journal of Intelligent Systems*, 12:153-166, 1997.

[13] J.H. Wang and J. Hao, "A new version of 2-tuple fuzzy linguistic representation model for computing with words", *IEEE Transactions on Fuzzy Systems*, 14:435-445, 2006.

[14] Z.S. Xu and Q.L. Da, An overview of operators for aggregating information. *International Journal of Intelligent Systems*, 18:953-968, 2003

[15] R.R. Yager, On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Transactions on Systems, Man and Cybernetics B*, 18:183-190, 1988.

[16] R.R. Yager, Families of OWA operators. *Fuzzy Sets and Systems*, 59:125-148, 1993.

[17] R.R. Yager, Including decision attitude in probabilistic decision making. *International Journal of Approximate Reasoning*, 21:1-21, 1999.

[18] R.R. Yager, K.J. Engemann, D.P. Filev, On the concept of immediate probabilities, *International Journal of Intelligent Systems*, 10:373-397, 1995.

[19] R.R. Yager and J. Kacprzyk, *The Ordered Weighted Averaging Operators: Theory and Applications*. Norwell: Kluwer Academic Publishers, 1997.

[20] L.A. Zadeh, "The Concept of a Linguistic Variable and its application to Approximate Reasoning. Part 1", *Information Sciences*, 8:199-249, 1975; "Part 2", *Information Sciences*, 8:301-357, 1975; "Part 3", *Information Sciences*, 9:43-80, 1975.