OPTIMIZATION OF ROUTE DISTRIBUTION PROBLEMS WITH FUZZY OBJECTIVES

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Abstract

Logistics planning is an area which offers many opportunities to study optimization problems such as route planning. These problems can be modeled according to the standard forms seen in vehicle route problems. One of the most common objectives in these kinds of problems is to minimize the time to cover the vehicle routes. In many real life situations the values for time can not be obtained in a precise way. In this article we propose a methodological approximation to obtain solutions to the vehicle route problem when the objective function is expressed in terms of fuzzy times.

Keywords: Vehicle route problems, fuzzy Optimization, Metaheuristics.

1 INTRODUCTION

Route distribution planning problems play a key role in supply chain management and especially in logistical systems. Proper planning can help improve the effectiveness and efficiency in these systems. Route planning problems in the present day context of the global economy have an important associated impact beside the obvious transport costs. They also affect the efficient use of resources, added value in service benefits and client satisfaction.

A traditional objective of these product distribution problems from a depot to a set of geographically disperse nodes is to determine the set of routes for the available vehicles that minimize total operating costs of the fleet, satisfying some, but not all, constraints. These problems have been studied under varied approaches and many different results have been obtained through the formulation of vehicle route problems (VRP). The VRP is concerned with finding an optimal set of routes that begin and end in a depot for a specified fleet of vehicles that satisfy client demand (see Cordeau et al. [4]). Client, depot and vehicle characteristics, in addition to different route operating restrictions, offer different variations to the problem.

The two lines of research we wish to highlight provide solutions to the models that increasingly account for those characteristics observed in real cases and those that specifically seek more effective procedures to solve these problems.

Apart from the clearly practical aspects of this problem, the formal, academic study of the combinatorial optimization aspects of the problem are noteworthy. They are, for the most part, NP hard problems and it is not possible to find exact procedures and algorithms that solve large instances of the problem in polynomial time. It is there where the relevance of the utilized heuristic methods is acquired, specifically in general search strategies that carry out a limited or intensive study of the solution space and where reasonable quality solutions are produced with modest computation times.

In many typical vehicle route planning problems it is not always possible to have access to all of the necessary information, consequently leaving a problem characterized by incomplete or imprecise information of the problem parameters and variables. One of the current approaches in the modeling and resolution of problems containing uncertainty is found in an area known as Soft Computing and in the Theory of Fuzzy Sets (Zadeh, 1965). These approaches are used to construct computation systems that solve decision and optimization problems and where the modeling is difficult to define with precision, managing vagueness and the imprecision of available information, in addition to the formulation of preferences, constraints and objectives expressed by the decision makers[1].

In this paper we deal with the problem in which travel times required by a vehicle to go from one demand node to another is imprecise. Traffic and road conditions cause difficulties in the calculation of exact travel times. A great amount of uncertainty is present in the calculation of the travel times, for instance the day of the week or the time of day that a vehicle travels or other specific circumstances, such as traffic density and the speed at which the vehicle moves in roads and highways can be more or less distinct. On the other hand, in route planning, the travel time of the routes is usually one of the criteria to measure costs of the available fleet of vehicles and the efficient use of that fleet. Thus, in order to model the vehicle route problem, find solutions and evaluate the best ones, we use an objective function that is expressed in terms of duration times of the routes.

In this situation we consider the classic model of the VRP, known as the capacitated vehicle route problem (CVRP), where the coefficients of the objective function are imprecise, and the set of constraints are known exactly, even though the capacity of the vehicles is homogenous. To model this kind of imprecision in the coefficients we can use fuzzy quantities, namely triangular fuzzy numbers that can be expressed in terms of known values, using the knowledge at hand, derived from the expertise of the drivers of the vehicles or using the historic information provided by positioning tools in the vehicles.

The objective of this paper is to formulate the problem based on the factors mentioned earlier, and find a procedure that will allow optimal solutions to be found in an efficient way. After introducing the problem and its context in this section, the organization of the paper is as follows. In section 2 we formulate the CVRP by taking into account the conditions of uncertainty in the travel times. In Section 3 we explain how we apply some models proposed in the literature to obtain a solution to the combinatorial optimization problem where there are fuzzy coefficients in the objective function. In section 4 we solve this model with an example, using different heuristics, which generate satisfactory solutions.

2 CAPACITATED VEHICLE ROUTING PROBLEMS

The standard VRP (usually called capacitated VRP; CVRP) calls for the determination of a set of m routes whose total travel length is minimized such that: (a) each customer is visited exactly once by one route, (b) each route starts and ends at a single depot, and (c) the total demand of the customers served by a route does not exceed a given vehicle capacity C_k where q_j is the demand of a node j. Travel times and travel costs are considered equivalent. If each vehicle i is assigned to a route R_i, a feasible solution for VRP is made up of a partition from nodes set V into m routes $R_1, R_2, ..., R_m$ and the corresponding permutations of R_i that specify client order along the routes. A formulation of the VRP as Integer Linear Programming problem can be described [6], and assumed that the depot is the node 0 and n+1, n is the number of customers to be served by m vehicles and the decision variables are $x_{ij}^k \in \{0,1\}, i = 0,1,...,n, j = 1,2,...,n+1, k = 1,2,...,m$, where $x_{ij}^k = 1$ if vehicle k travels from customer i to j and 0 otherwise. We also consider the continuous variables r_i^k , representing the load of vehicle k after visiting node i. If the vehicle k goes from customer i to customer j ($x_{ij}^k = 1$) then $r_j^k = r_i^k - q_i$.

In particular, objective function may be formulated as follows:

$$\min \sum_{k=1}^{m}\sum_{i=0}^{n}\sum_{j=1}^{n+1} c_{ij}^k x_{ij}^k \tag{1}$$

where c_{ij}^k is the *cost of travelling* from customer i to customer j by vehicle k. The total travel cost is an objective function to be minimized and is expressed in term of total time, then

$$\min \left(\sum_{k=1}^{m}\sum_{i=0}^{n}\sum_{j=1}^{n+1} t_{ij}^k x_{ij}^k + \sum_{k=1}^{m}\sum_{i=0}^{n}\sum_{j=1}^{n+1} u_i^k x_{ij}^k \right) \tag{2}$$

where t_{ij}^k denotes the time needed to go from customer i to customer j and u_i^k is the required time by vehicle k to unload demand to customer i. The objective function is to minimize the total travel time.

The CVRP has the follow constraints:

- Constraints ensuring that each customer is served exactly once.

$$\sum_{k=1}^{m}\sum_{i=0}^{n} x_{ij}^k = 1, \quad j \in [1..n]$$

$$\sum_{k=1}^{m}\sum_{i=1}^{n+1} x_{ji}^k = 1, \quad j \in [1..n]$$

- Constraints ensuring that each vehicle is used no more than once.

$$\sum_{j=1}^{n} x_{0j}^k = 1, \quad k \in [1..m]$$

$$\sum_{i=1}^{n} x_{in+1}^k = 1, \quad k \in [1..m]$$

- Constraints ensuring route continuity.

$$\sum_{i=0}^{n} x_{ij}^{k} - \sum_{i=1}^{n+1} x_{ji}^{k} = 0,\ j \in [1..n], k \in [1..m]$$

- Constraints ensuring that the load of each vehicle is not greater that its capacity.

$$r_i^k + q_j - r_j^k \leq C_k(1 - x_{ij}^k)$$
$$i \in [1..n], j \in [1..n], k \in [1..m]$$

$$q_i \leq r_i^k \leq C_k,\ \ i \in [0..n], k \in [1..m]$$

These constraints are an alternative set of constraints equivalent to the standard constraints of capacity cuts and ensure, in addition to the restriction of capacities, the connectivity of the routes and the elimination of subtours.

- In addition the conditions for the variables:

$$x_{ij}^k \in \{0,1\}, r_i^k \geq 0,$$
$$i \in [0..n], j \in [1..n], k \in [1..m]$$

This formalization assumes that the decision-maker has access to specific information on the components that define the problem; that is, on objective functions and constraints. However in a logistics context the information of time travel is actually imprecise or incomplete.

Thus, if we suppose that all parameters of the problems are crisp: demand q_i , capacity C_k, unload time u_i^k, and available load r_i^k, travel times t_{ij}^k that can be fuzzy, the traditional model becomes a Fuzzy optimization problem. Intuitively, when any of these quantities are fuzzy numbers, the objective functions become fuzzy as well. If these parameters are approximately known, they can be represented by the fuzzy numbers $\tilde{t}_{ij}^k$, with their corresponding membership functions. Then, for instance, the objective function (1) and (2) can be expressed as:

$$\min \sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=1}^{n+1} \tilde{c}_{ij}^k x_{ij}^k$$

and

$$\min \left(\sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=1}^{n+1} \tilde{t}_{ij}^k x_{ij}^k + \sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=1}^{n+1} u_i^k x_{ij}^k \right) \qquad (3)$$

The summation symbol Σ in the objective functions and constraints refers to an addition of fuzzy numbers. Hence there is a need to seek appropriate procedures for its solution.

3 FUZZY OPTIMIZATION PROBLEMS

An *optimization problem* can be described as the search for the value of specific *decision variables* so that identified *objective functions* attain their optimum values. The value of the variables is subject to stated *constraints*. In these problems the objective functions are defined on a set of solutions that we will denote by X. The objective function is not subject to any condition or property nor is the definition of the set X. Typically the number of elements of X is very high, essentially eliminating the possibility of a complete evaluation of all its solutions while determining the optimal solution.

Optimization problems in their most general form involve finding an optimal solution according to stated criteria. In practice, however, many situations lack the exact information that is needed in the problem, including its constraints, or in other cases, where it is unreasonable to access such specific constraints or clearly defined objective functions. In these situations it is advantageous to model and solve the problem using soft computing and fuzzy techniques.

Among all the optimization problems, the models that have received the most attention and have offered the most useful applications in different areas are Linear Programming (LP) models, which is the single objective linear case with linear constraints. The classic problem of LP is to find the maximum or minimum values of a linear function subject to constraints that are represented by linear equations or inequalities. The most general formulation of the LP problem is:

$$\begin{array}{ll} \text{min} & Z = cx \\ \text{subject to} & Ax \leq b \\ & x \geq 0 \end{array}$$

The vector $x = (x_1, x_2, ..., x_n) \in \Re^n$ represents the decision variables. The objective function is denoted by z, the numbers c_j are coefficients and the vector $c = (c_1, c_2, ..., c_n) \in \Re^n$ is known as the cost vector. The matrix $A = [a_{ij}] \in \Re^{n \times m}$ is called the constraint or technological matrix and the vector $b = (b_1, b_2, ..., b_m) \in \Re^m$ represents the independent terms or right-hand-side of the constraints.

In many real situations not all the constraints and objective functions can be valued in a precise way. In these situations we are dealing with the general problem form of Fuzzy Linear Programming (FLP). FLP is characterized as follows: a_{ij}, b_j and c_i can be expressed as *fuzzy numbers*, x_i as variables whose states are fuzzy numbers, addition and multiplication operates with fuzzy numbers, and the inequalities are among fuzzy numbers.

Different FLP models can be considered according to the elements that contain imprecise information that is taken as a basis for the classification proposed in [2], [5]. In our case use a *model with fuzzy costs as objective function coefficients*.

These models are those whose costs are not fully known (with imprecision). Therefore, they are represented by an m-dimensional fuzzy vector $c^f = (c_1^f, c_2^f, ..., c_n^f)$, and the following model:

$$\begin{array}{ll} \min & Z = c^f x \\ \text{subject to} & Ax \leq b \\ & x \geq 0 \end{array}$$

Obviously, z is also a fuzzy number, but x can be a vector of fuzzy or non-fuzzy numbers, and each fuzzy cost is described by its corresponding membership function $\mu_i(x)$.

This model can be transformed into a simpler auxiliary model following the approach proposed at [3], [7]. This method proposed the use of an ordering function g that allows the comparison between fuzzy numbers, which facilitates minimizing the objective function. If $\tilde{u}$ and $\tilde{v}$ are two fuzzy numbers, $\tilde{u} \leq \tilde{v}$ if $g(\tilde{u}) \leq g(\tilde{v})$. In our problem we use the Yager third index as an ordering function for triangular fuzzy numbers $\tilde{u} = Tr(u_1, u_2, u_3)$ is $g(\tilde{u}) = u_1 + 2u_2 + u_3$. Therefore the objective function (3) can be replaced by:

$$\min \left(\sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=1}^{n+1} g(\tilde{t}_{ij}^k) x_{ij}^k + \sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=1}^{n+1} g(u_i^k) x_{ij}^k \right)$$

and using triangular fuzzy numbers and the Yager third index, we obtain the following objective function:

$$\min \quad \left(\sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=1}^{n+1} (t_{ij}^{1k} x_{ij}^k + 2t_{ij}^{2k} x_{ij}^k + t_{ij}^{3k} x_{ij}^k) + \sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=1}^{n+1} 4u_i^k x_{ij}^k \right)$$

where $\tilde{t}_{ij}^k$ is a triangular fuzzy number $Tr(t_{ij}^{1k}, t_{ij}^{2k}, t_{ij}^{3k})$ and u_i^k is a crisp number equivalent to a triangular fuzzy number $Tr(u_i^k, u_i^k, u_i^k)$.

We use this fuzzy optimization approximation to solve the problem formulation described earlier. In the next section we provide specific examples.

Table 4: Results obtained

Heuristic	Total Time	route number
GRASP	1840	3
GRASP-LS	1840	3
VNS	1820	3
GRASP-VNS	1740	3

3.1 SOLUTIONS AND EXPERIMENTATION

An evaluation of the formulation and the proposed method is carried out using an example from Zheng and Liu [8]. The proposed instances with triangular travel times in their paper include 18 customers, labelled "1", "2",. . . , "18", a depot, labelled '0' and vehicles with homogeneous capacity.

The amount of demand at each customer and the distances between customers (and the depot) are given in Table 1. The travel times are triangular fuzzy numbers shown in Tables 2 and 3, with the time windows of each customer. The unloading time at each location is 15 minutes and the capacity of each of the four available vehicles is 1000.

We have found solutions for the corresponding optimization problem using four heuristics: Greedy Randomize Adaptive Search Procedure (GRASP), GRASP with Local Search, a Variable Neighborhood Search (VNS) and a GRASP-VNS Hybrid procedure. The results were performed on a PC Intel CORE 2 Duo (2.26GHz) processors and 4GB RAM.

Table 4 presents the values of the best solutions obtained in run time 60s for the objective function.

The best solutions was obtained using VNS and GRASP-VNS hybrid heuristic in near 10min and consist in the following three routes, respectively:

$$\begin{array}{ll} R1: & 0 \to 2 \to 1 \to 3 \to 6 \to 4 \to 7 \to 10 \to 0 \\ R2: & 0 \to 8 \to 11 \to 5 \to 9 \to 14 \to 12 \to 16 \to 13 \to 0 \\ R3: & 0 \to 15 \to 18 \to 17 \to 0 \end{array}$$

The total time is T = 1700 and the loads of the vehicles are 1000, 985 and 500.

$$\begin{array}{ll} R1: & 0 \to 2 \to 3 \to 1 \to 18 \to 14 \to 15 \to 17 \to 0 \\ R2: & 0 \to 10 \to 11 \to 7 \to 9 \to 6 \to 5 \to 13 \to 16 \to 0 \\ R3: & 0 \to 4 \to 8 \to 12 \to 0 \end{array}$$

The total time is T = 1700 and the loads of the vehicles are 1000, 990 and 495.

Table 1: Demands (q) and distances (d) between customers.

d	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	q
1	19.0																		200
2	17.5	6.0																	100
3	28.0	11.0	10.5																140
4	24.0	21.0	15.0	20.0															160
5	24.5	32.0	26.0	34.0	15.5														200
6	31.2	44.5	39.5	49.0	31.0	16.0													60
7	31.0	48.5	45.0	55.5	41.5	28.5	16.0												200
8	21.0	37.5	33.5	44.0	30.0	18.5	13.0	11.5											135
9	18.0	36.0	33.0	44.0	38.0	24.0	20.0	13.5	7.0										120
10	21.5	40.0	39.0	49.5	43.0	36.5	32.5	21.0	20.0	13.0									140
11	36.5	55.0	54.0	65.0	56.0	46.0	37.0	21.0	28.0	23.0	15.5								100
12	31.5	46.5	48.0	57.0	55.5	51.0	48.5	36.0	36.0	29.0	16.0	21.5							200
13	23.0	38.5	39.0	48.5	47.0	44.0	43.0	32.5	30.0	23.0	12.0	23.0	9.0						80
14	28.0	38.5	41.5	49.0	52.0	51.0	52.5	43.5	40.0	33.0	22.5	33.0	13.0	11.0					60
15	34.5	40.0	44.0	50.0	56.5	58.5	62.0	54.0	50.0	43.0	34.0	44.5	24.0	22.0	11.5				200
16	30.0	29.5	34.5	38.0	48.5	54.0	60.5	56.0	49.0	43.5	38.0	51.5	33.0	28.0	20.5	14.0			90
17	18.5	16.5	21.3	26.5	35.0	41.0	50.0	48.0	39.0	35.0	33.0	48.0	34.0	27.0	24.0	23.5	13.5		200
18	24.0	14.0	20.0	22.0	35.0	44.0	54.5	55.0	45.0	41.5	41.0	56.5	43.0	35.0	32.0	36.0	17.0	8.5	100

Table 2: fuzzy travel times between customers (part I).

T	0	1	2	3	4	5	6	7	8
1	(25,50,75)								
2	(5,10,15)	(20,40,60)							
3	(25,50,75)	(5,10,15)	(20,40,60)						
4	(7,15,23)	(25,50,75)	(7,15,23)	(22,45,68)					
5	(25,50,75)	(17,35,53)	(17,35,53)	(15,30,45)	(17,35,53)				
6	(25,50,75)	(7,15,23)	(20,40,60)	(2,5,8)	(22,45,68)	(15,30,45)			
7	(12,25,38)	(20,40,60)	(15,30,45)	(17,35,53)	(7,15,23)	(12,25,38)	(17,35,53)		
8	(7,15,23)	(20,40,60)	(5,10,15)	(22,45,68)	(10,20,30)	(17,35,53)	(20,40,60)	(17,35,53)	
9	(25,50,75)	(7,15,23)	(22,45,68)	(5,10,15)	(22,45,68)	(15,30,45)	(5,10,15)	(20,40,60)	(20,40,60)
10	(10,20,30)	(22,45,68)	(12,25,38)	(22,45,68)	(7,15,23)	(15,30,45)	(20,40,60)	(5,10,15)	(12,25,38)
11	(25,50,75)	(5,10,15)	(17,35,53)	(15,30,45)	(17,35,53)	(5,10,15)	(15,30,45)	(5,10,15)	(17,35,53)
12	(27,55,83)	(17,35,53)	(17,35,53)	(15,30,45)	(17,35,53)	(2,5,8)	(15,30,45)	(7,15,23)	(17,35,53)
13	(5,10,15)	(20,40,60)	(5,10,15)	(20,40,60)	(7,15,23)	(15,30,45)	(17,35,53)	(17,35,53)	(5,10,15)
14	(25,50,75)	(5,10,15)	(20,40,60)	(2,5,8)	(22,45,68)	(15,30,45)	(2,5,8)	(17,35,53)	(17,35,53)
15	(22,45,68)	(5,10,15)	(20,40,60)	(5,10,15)	(22,45,68)	(15,30,45)	(5,10,15)	(17,35,53)	(17,35,53)
16	(7,15,23)	(22,45,68)	(7,15,23)	(22,45,68)	(10,20,30)	(15,30,45)	(22,45,68)	(17,35,53)	(10,20,30)
17	(15,30,45)	(20,40,60)	(12,25,38)	(20,40,60)	(10,20,30)	(12,25,38)	(17,35,53)	(2,5,8)	(12,25,38)
18	(25,50,75)	(5,10,15)	(22,45,68)	(5,10,15)	(25,50,75)	(15,30,45)	(7,15,23)	(17,35,53)	(20,40,60)

4 CONCLUSIONS

Uncertainty needs to be managed in real logistic and transportation systems. Fuzzy Logic systems have been used to model the uncertainty. We consider the CVRP with uncertainty in the travel times as triangular fuzzy numbers. We propose a simple approximation to obtain optimal solutions for these problems when fuzzy times are objective function coefficients. We offer an approximation and an example in which several heuristics are used to obtain solutions in short periods of time. The best solution is obtained with the GRASP-VNS hybrid procedure. These results indicate new avenues of possible future research.

Acknowledgements

The first three authors have been supported by project TIN2008-06873-C04-01 (70% FEDER) from the Spanish Government and PI2007/019 from the Canary Islands Government. J.L. Verdegay has been supported by projects TIN2008-06872-C04-04/TIN and TIC-02970-JA

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Table 3: fuzzy travel times between customers (part II).

T	9	10	11	12	13	14	15	16	17
10	(22,45,68)								
11	(17,35,53)	(15,30,45)							
12	(17,35,53)	(12,25,38)	(7,15,23)						
13	(20,40,60)	(20,40,60)	(17,35,53)	(17,35,53)					
14	(5,10,15)	(20,40,60)	(15,30,45)	(15,30,45)	(20,40,60)				
15	(2,5,8)	(20,40,60)	(15,30,45)	(20,40,60)	(20,40,60)	(2,5,8)			
16	(22,45,68)	(15,30,45)	(17,35,53)	(7,15,23)	(7,15,23)	(22,45,68)	(22,45,68)		
17	(20,40,60)	(5,10,15)	(12,25,38)	(12,25,38)	(12,25,38)	(17,35,53)	(17,35,53)	(12,25,38)	
18	(7,15,23)	(20,40,60)	(15,30,45)	(20,40,60)	(20,40,60)	(7,15,23)	(7,15,23)	(20,40,60)	(20,40,60)

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