Hamiltonian approach to QCD in Coulomb gauge - zero and finite temperature

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G. Burgio, D. Campagnari, J. Heffner, M. Quandt,

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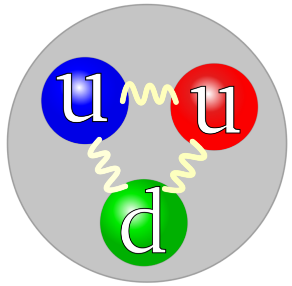
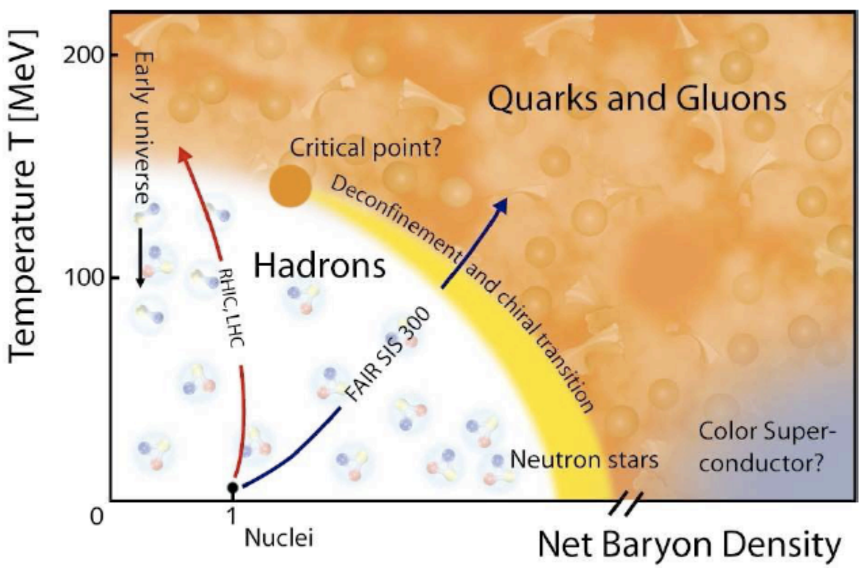
QCD

- *vacuum*
  - confinement
  - SB chiral symmetry



- *phase diagram*
  - deconfinement
  - rest. chiral symm.



- latticeMC–fail at large chemical potential
  continuum approaches required
  Hamiltonian approach in Coulomb gauge

Why Hamiltonian approach?

- *QFT: functional integral approach*
  - *perturbation theory*
  - *lattice gauge theory*
- *QM: solving the Schrödinger equation is much simpler and more efficient than calculating the functional integral,* *see e.g. hydrogen atom*
- *non-perturbative continuum QFT:*
  - *Hamiltonian approach is very efficient!*

Outline

- introduction
- basics of the Hamiltonian approach to QCD in Coulomb gauge
- results for YMT at T=0
- comparison with the lattice
    - gluon & ghost propagators
    - Coulomb string tension
- quark sector of QCD
- finite temperature by compactification of a spatial dimension
    - Polyakov loop
    - chiral & dual quark condensate
- conclusions

Canonical Quantization of Yang-Mills theory

cartesian coordinates $A_\mu^a(x)$

momenta $\Pi_i^a(x) = \delta S / \delta \dot{A}_i^a(x) = E_i^a(x)$

$\Pi_0^a(x) = 0$ Weyl gauge : $A_0^a(x) = 0$

$$H = \tfrac{1}{2}\int d^3x(\Pi^2(x) + B^2(x))$$

quantization: $\Pi_k^a(x) = \delta / i\delta A_k^a(x)$

Gauß law: $D\Pi\Psi = \rho_m \Psi$

residual gauge invariance $\mathrm{U}(\vec{\mathrm{x}})$: $\Psi(A^U) = \Psi(A)$

Coulomb gauge

$$\partial A = 0, \qquad A = A^{\perp}$$

curved space

$$\langle \Psi | \Phi \rangle = \int DA^{\perp} J(A^{\perp}) \Psi^{*}(A^{\perp}) \Phi(A^{\perp})$$

Faddeev-Popov

$$J(A^{\perp}) = Det(-D\partial)$$

$$\Pi = \Pi^{\perp} + \Pi^{\parallel}, \qquad \Pi^{\perp} = \delta / i\delta A^{\perp}$$

Gauß law:

$$D\Pi\Psi = \rho_m \Psi$$

resolution of Gauß´ law

$$\Pi^{\parallel} = -\partial(-D\partial)^{-1}\rho, \qquad \rho = (-\hat{A}^{\perp}\Pi^{\perp} + \rho_m)$$

Hamiltonian approach to YMT in Coulomb gauge $\partial A = 0$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi J \Pi + B^2) + H_C$$

$$\Pi = \delta / i\delta A$$

Christ and Lee

$$J(A^{\perp}) = Det(-D\partial) \qquad D^{ab} = \delta^{ab}\partial + gf^{abc}A^c$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho J(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}\rho$$

Coulomb term

color charge density $\qquad \rho^a = -f^{abc}A^b\Pi^c + \rho_m^a$

$$\langle\phi|\ldots|\psi\rangle = \int DAJ(A)\phi^*(A)\ldots\psi(A)$$

$$H\psi[A] = E\psi[A]$$

Variational approach to YMT

- Gaussian ansatz,

$$\Psi\left(A\right)=\exp\left[-\tfrac{1}{2}\int dx\,dy\,A(x)\,\omega(x,y)\,A(y)\right]$$

D. Schütte 1984

....

A.Szczepaniak & E. Swanson 2002

C. Feuchter & H. R. 2004

-ansatz

-FP determinant

-renormalization

- Greensite, Matevosyan,Olejnik,Quandt, Reinhardt, Szczepaniak,PRD83

Variational approach to YMT

- trial ansatz

C.Feuchter & H. R. PRD70(2004)

$$\Psi(A) = \frac{1}{\sqrt{\mathrm{Det}(-D\partial)}} \exp\left[-\tfrac{1}{2}\int dx\, dy\, A(x)\omega(x,y)A(y)\right]$$

gluon propagator $\langle A(x)A(y)\rangle = (2\omega(x,y))^{-1}$

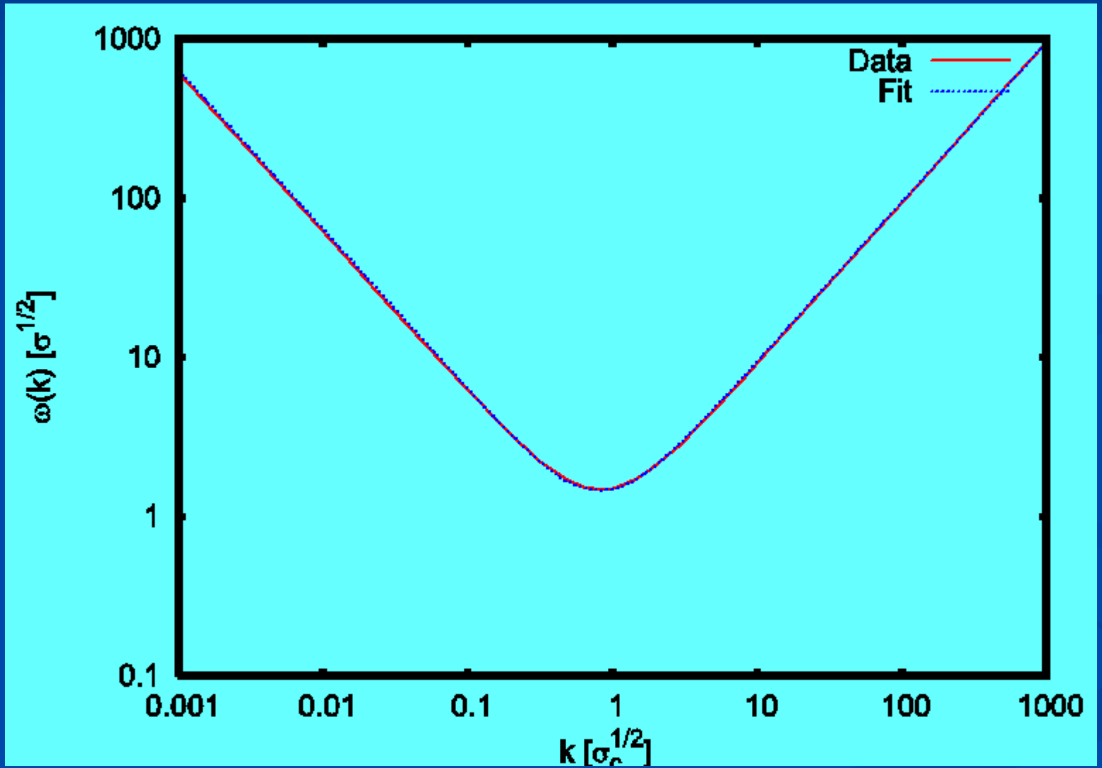
variational kernel $\omega(x,x')$ determined from

$$\boxed{\langle\Psi|H|\Psi\rangle \to \min}$$

Numerical results

gluon energy

D. Epple, H. R., W.Schleifenbaum, PRD 75 (2007)

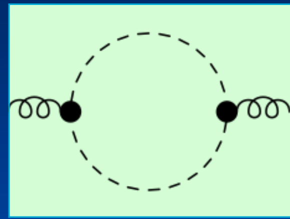


$$IR: \quad \omega(k) \sim 1/k \qquad\qquad UV: \quad \omega(k) \sim k$$

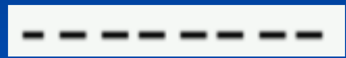
equation of motion

$$\omega^2(k) = k^2 + \chi^2(k) + \cancel{\ldots\ldots\ldots}$$

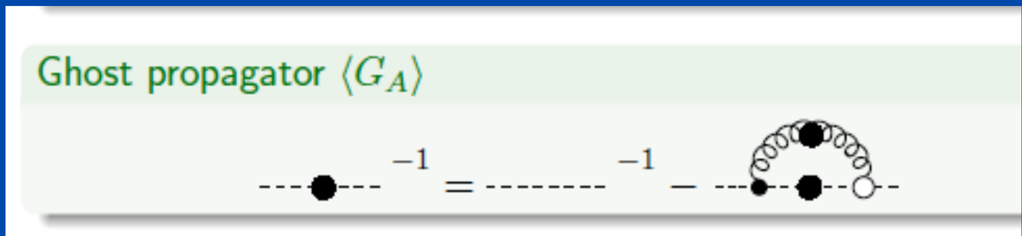
$\chi(k)$



ghost propagator

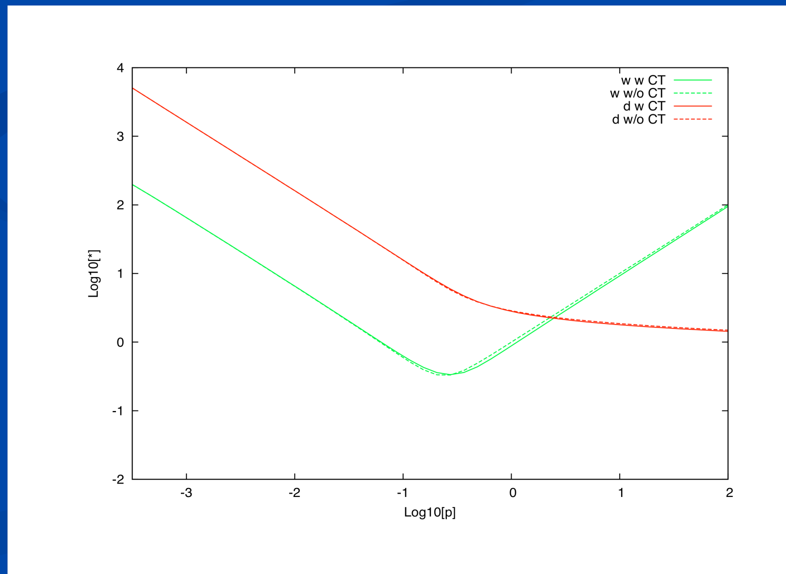


gost loop



$$\left\langle (-D\partial)^{-1} \right\rangle = d \,/\, (-\Delta)$$

horizon condition $\mathrm{d}^{-1}(0) = 0$



The color dielectric function of the QCD vacuum

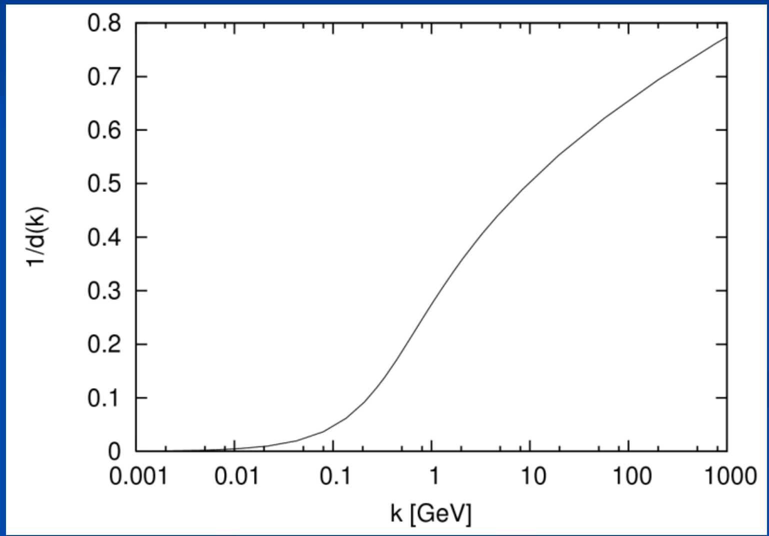
- ghost propagator

$$\left\langle (-D\partial)^{-1} \right\rangle = d\,/\,(-\Delta)$$

- dielectric „constant“

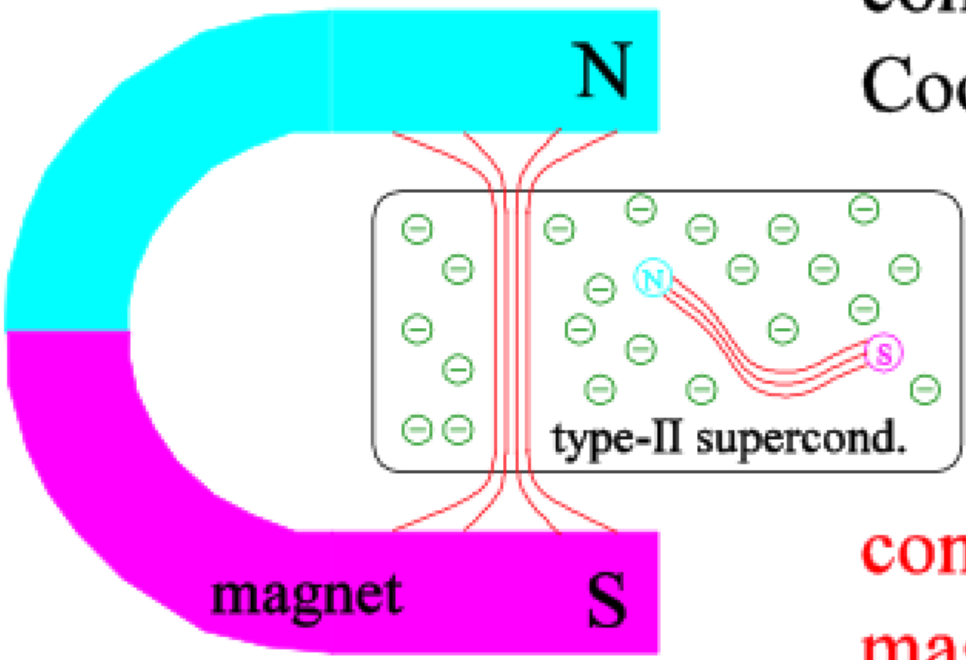
$$\varepsilon = d^{-1}$$

H.R. PRL101 (2008)



- horizon condition:
  - : $d^{-1}(k=0)=0 \qquad \varepsilon(k=0)=0$
- QCD vacuum: perfect color dia-electricum
  - dual superconductor
  - $\varepsilon(k)<1$ anti-screening

superconductor



condensate of
Cooper pairs

confinement of
magn. monopoles

dual superconductor

magnetic field	$\longleftrightarrow$	*electric field*
magnetic charge		*electric charge*

The color dielectric function of the QCD vacuum

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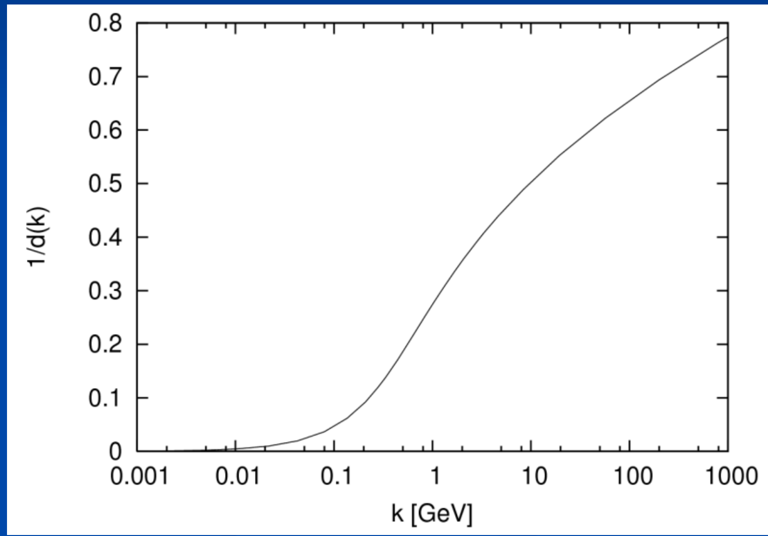

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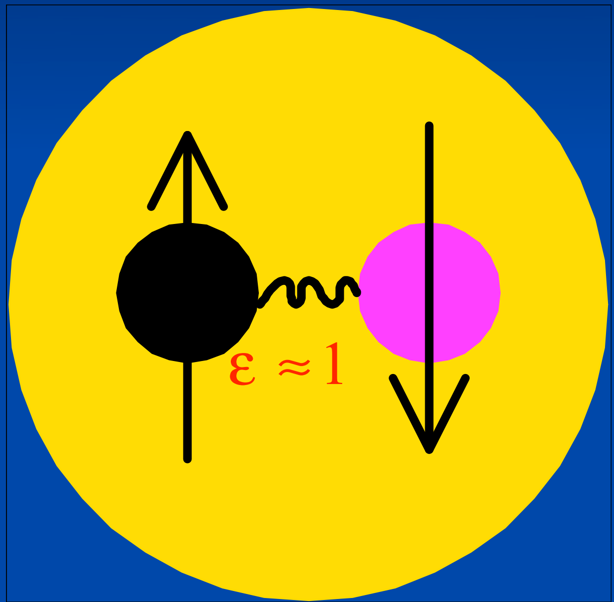
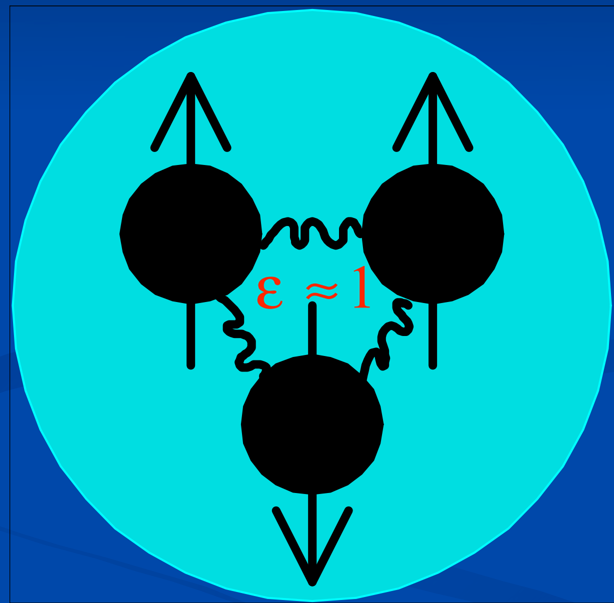
$$D = \varepsilon E \qquad \partial D = \rho_{free}$$



$$\varepsilon = 0$$

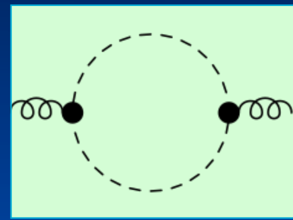


no free color charges in the vacuum: confinement

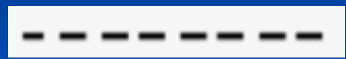
equation of motion

$$\omega^2(k) = k^2 + \chi^2(k) + \cancel{\ldots\ldots\ldots}$$

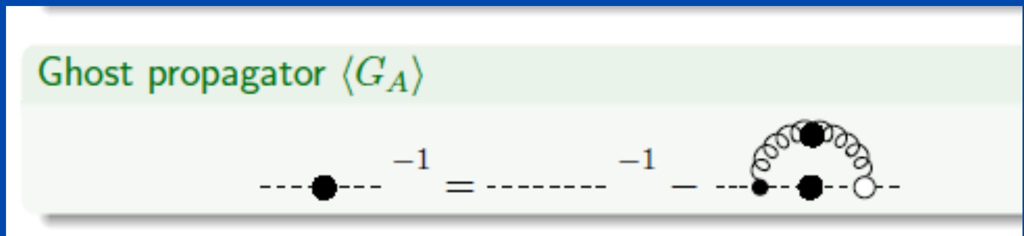
$\chi(k)$

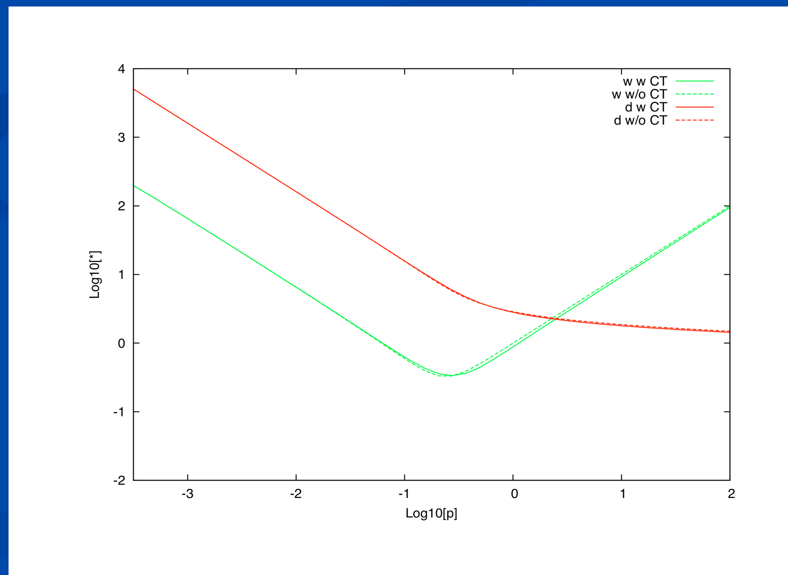


ghost propagator



gost loop





$$\left\langle (-D\partial)^{-1} \right\rangle = d \,/\, (-\Delta)$$

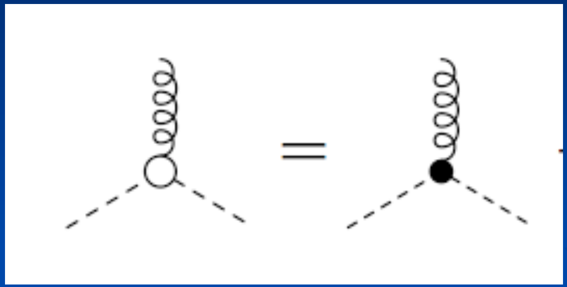
horizon condition $\mathrm{d}^{-1}(0) = 0$

Infrared analysis

Lerche & von Smekal,PRD65
Schleifenbaum,Leder, H.R. PRD73

gluon energy $\omega(p) = A / p^{\alpha}$ ghost form factor $d(p) = B / p^{\beta}$

assumptions: **bare ghost gluon vertex**



horizon condition $d^{-1}(0) = 0$ supported by the lattice

sum rule $\alpha = 2\beta + 2 - d$

$d = 3$ $\alpha = 1\ (0.6)$ $\beta = 1\ (0.8)$

Static gluon propagator in D=3+1

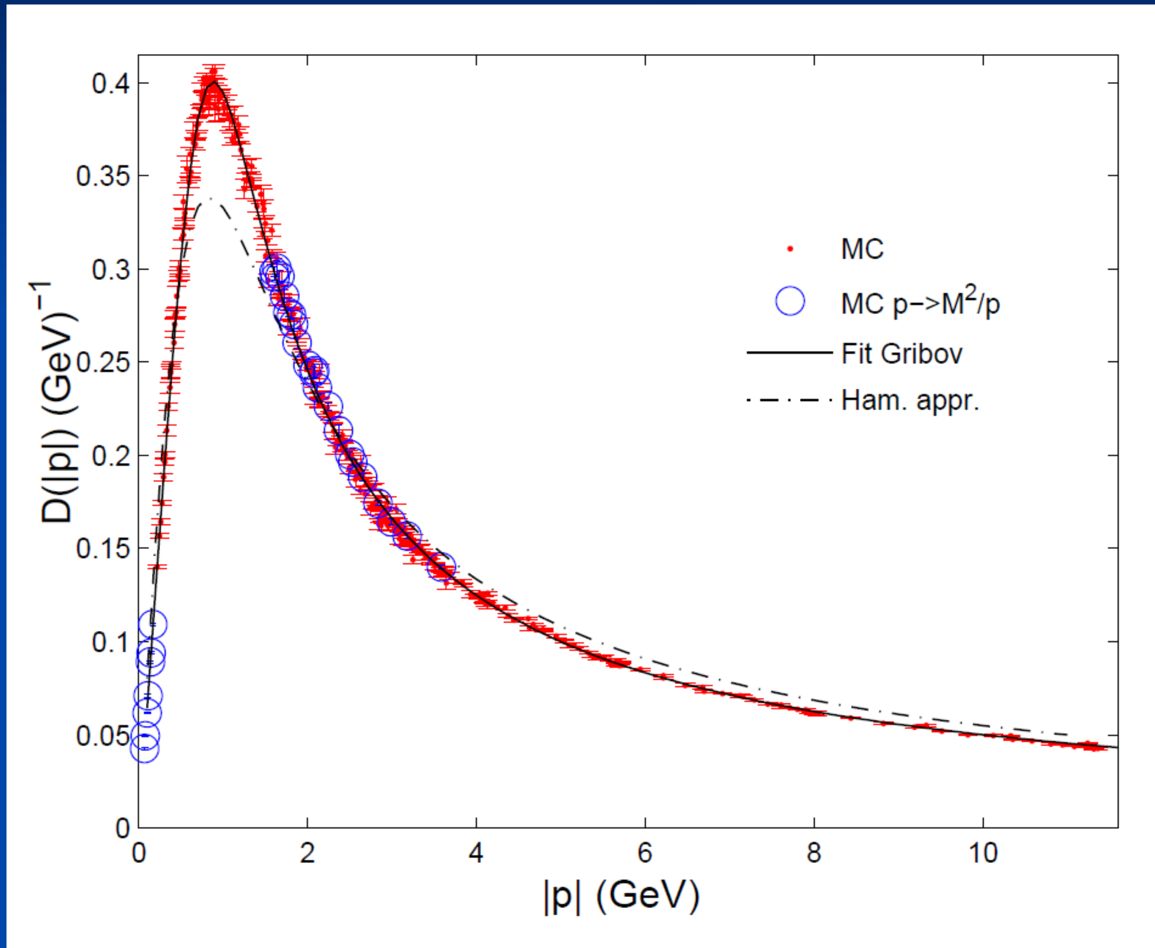
$$D(k) = (2\omega(k))^{-1}$$

Gribov's formula

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

$$M = 0.88 GeV$$

missing strength in
mid momentum regime:
missing gluon loop



G. Burgio, M.Quandt , H.R., **PRL102(2009)**

Variational approach to YMT with non-Gaussian wave functional

D. Campagnari & H.R,
Phys.Rev.D82(2010)
Phys.Rev.D92(2015)

wave functional

$$|\psi[A]|^2 = \exp(-S[A])$$

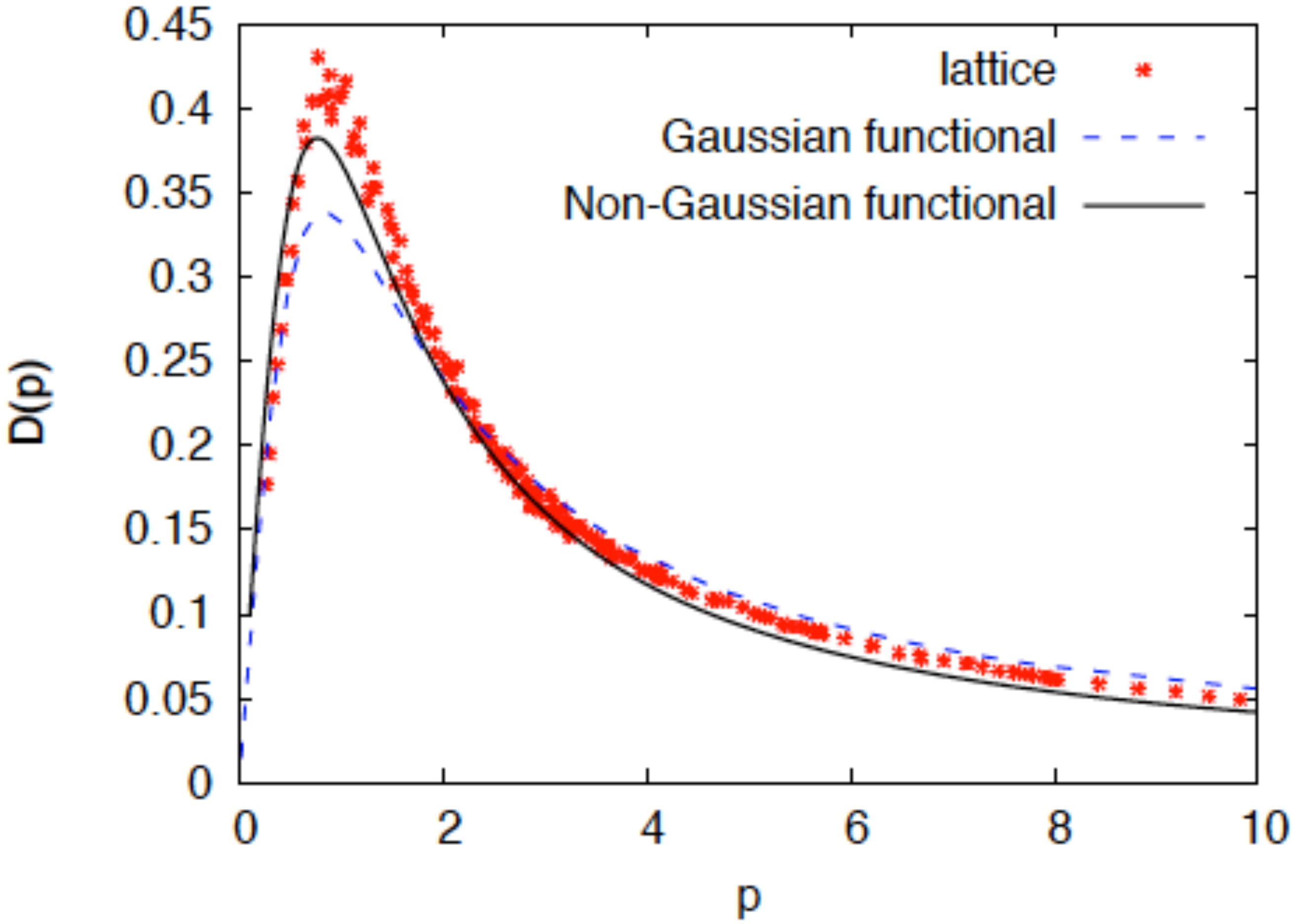
ansatz

$$S[A] = \int \omega A^2 + \tfrac{1}{3!}\int \gamma^{(3)} A^3 + \tfrac{1}{4!}\int \gamma^{(4)} A^4$$

exploit DSE

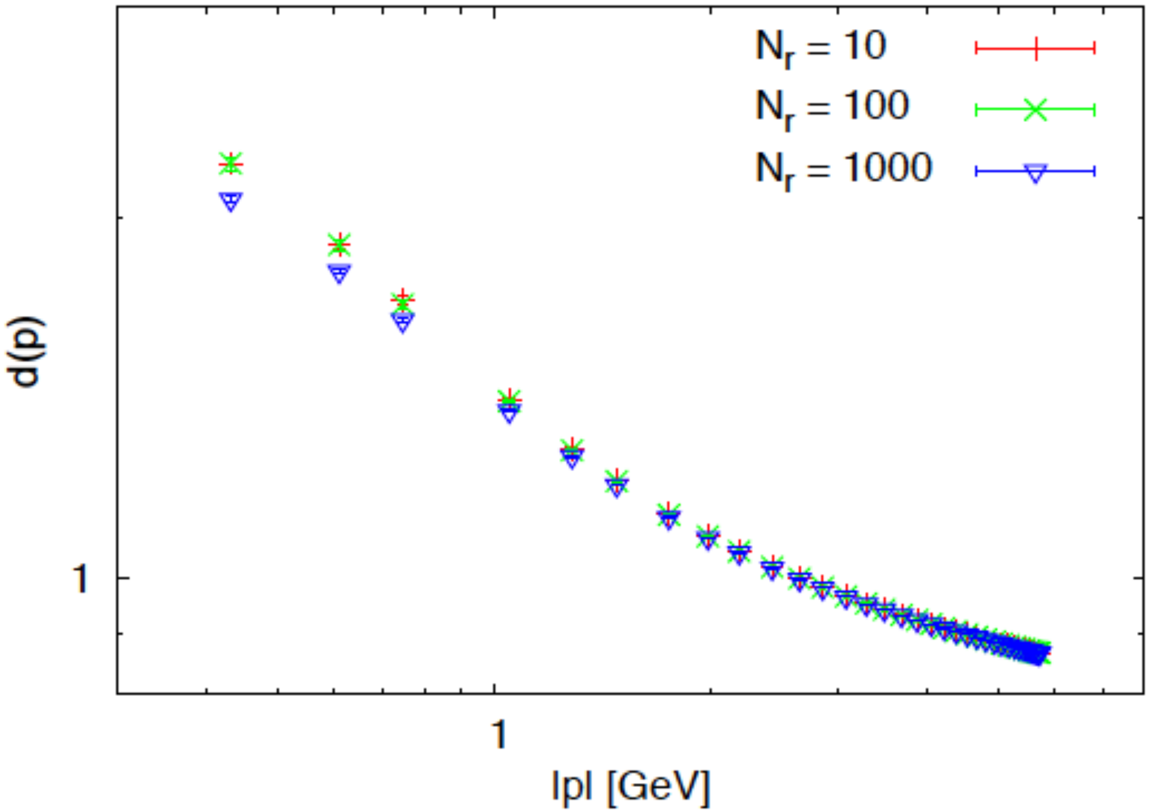
see talk by D. Campagnari

Corrections to the gluon propagator



D. Campagnari & H.R, Phys.Rev.D82(2010)

lattice ghost form factor



IR-exponent fixed at 0.5

G. Burgio, M. Quandt, H. R. & H. Vogt, arXiv:1608.05795

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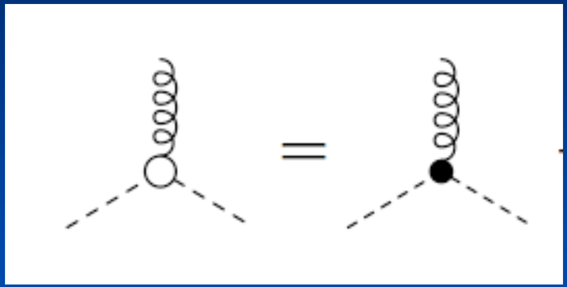

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sum rule $\alpha = 2\beta + 2 - d$

$d = 3$ $\alpha = 1\ (0.6)$ $\beta = 1\ (0.8)$

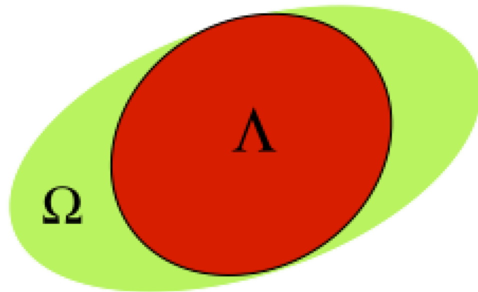
lattice : $\alpha \approx 1$ $\beta \approx 0.5$

lattiice propagators violate the sum rule?

Coulomb gauge fixing on the lattice & Gribov copies

$$F_t^U[g] = \sum_{\vec{x},i} \mathrm{Re}\, tr\left[U_i^g(t,\vec{x})\right] \rightarrow \max$$

continuum: $Tr(\vec{A}^g \vec{A}^g) \rightarrow \min$



local maximum: first Gribov region

global maximum: fundamental modular region

bc: „best" Gribov copy: largest local maximum
= „best representative" of the global maximum!?

counter example: U(1)-LGT on S^2 *de Forcrand & Hetrick*

Sternbeck & Müller-Preußker, Maas,(Landau-gauge)

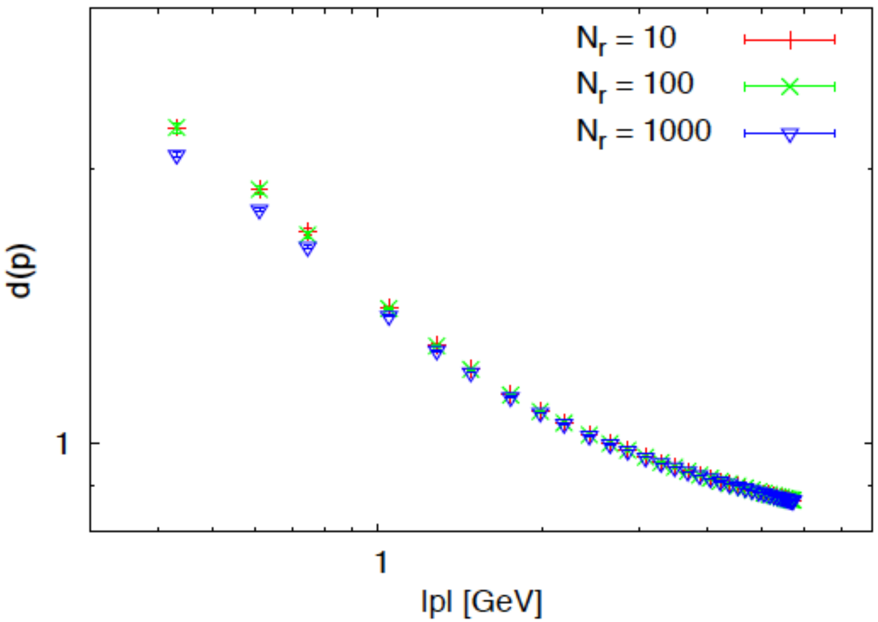
lc: „lowest" Gribov copy: lowest (n.t.)eigenvalue of the FP operator

Cooper & Zwanziger: results closer to the continuum

G. Burgio, M. Quandt, H. R. & H. Vogt, arXiv:1608.05795

Coulomb gauge fixing on the lattice & Gribov copies: ghost form factor

best copy



IR-exponent fixed at 0.5

Coulomb gauge fixing on the lattice & Gribov copies: ghost form factor

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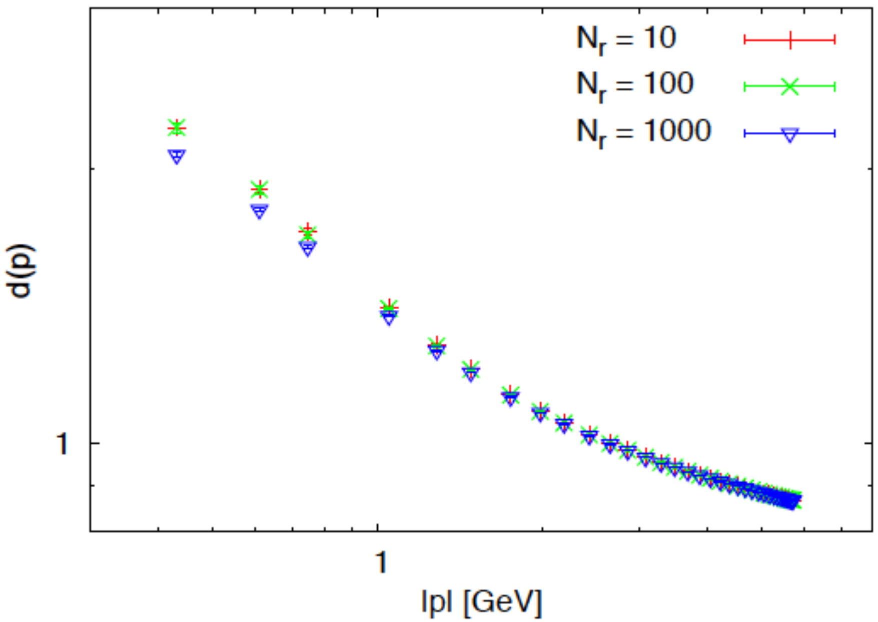
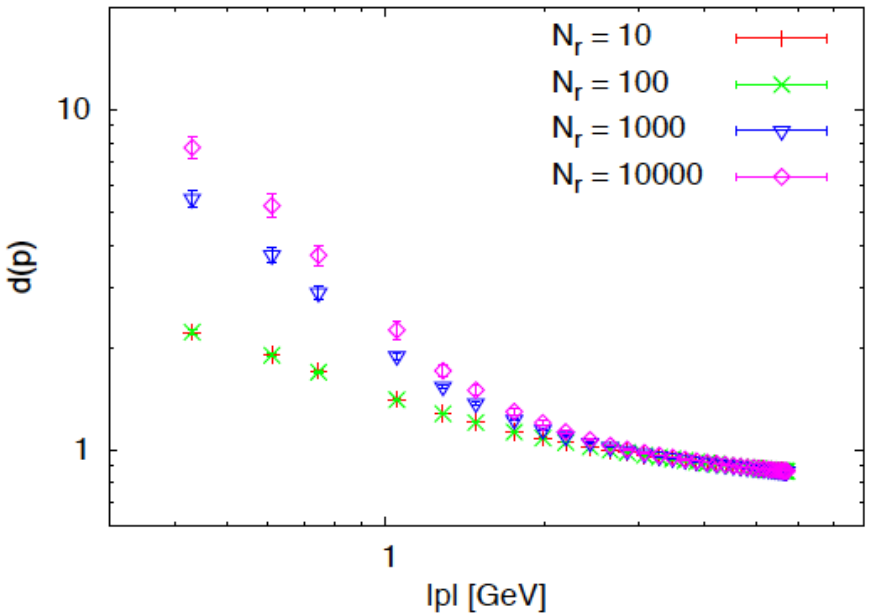


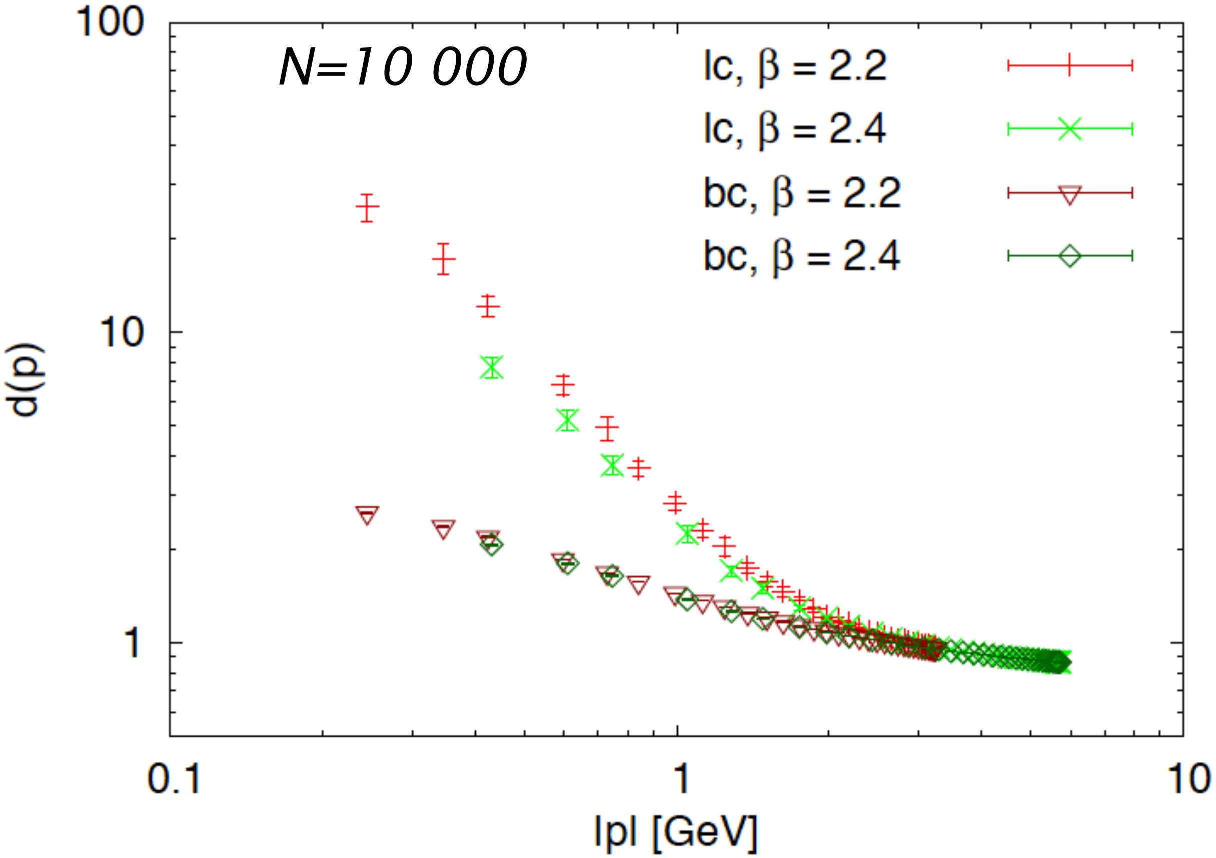
IR-exponent fixed at 0.5

lowest copy



IR-exponent increases and „saturates" near 1

Coulomb gauge fixing on the lattice & Gribov copies: ghost form factor



Coulomb gauge fixing on the lattice & Gribov copies: gluon propagator

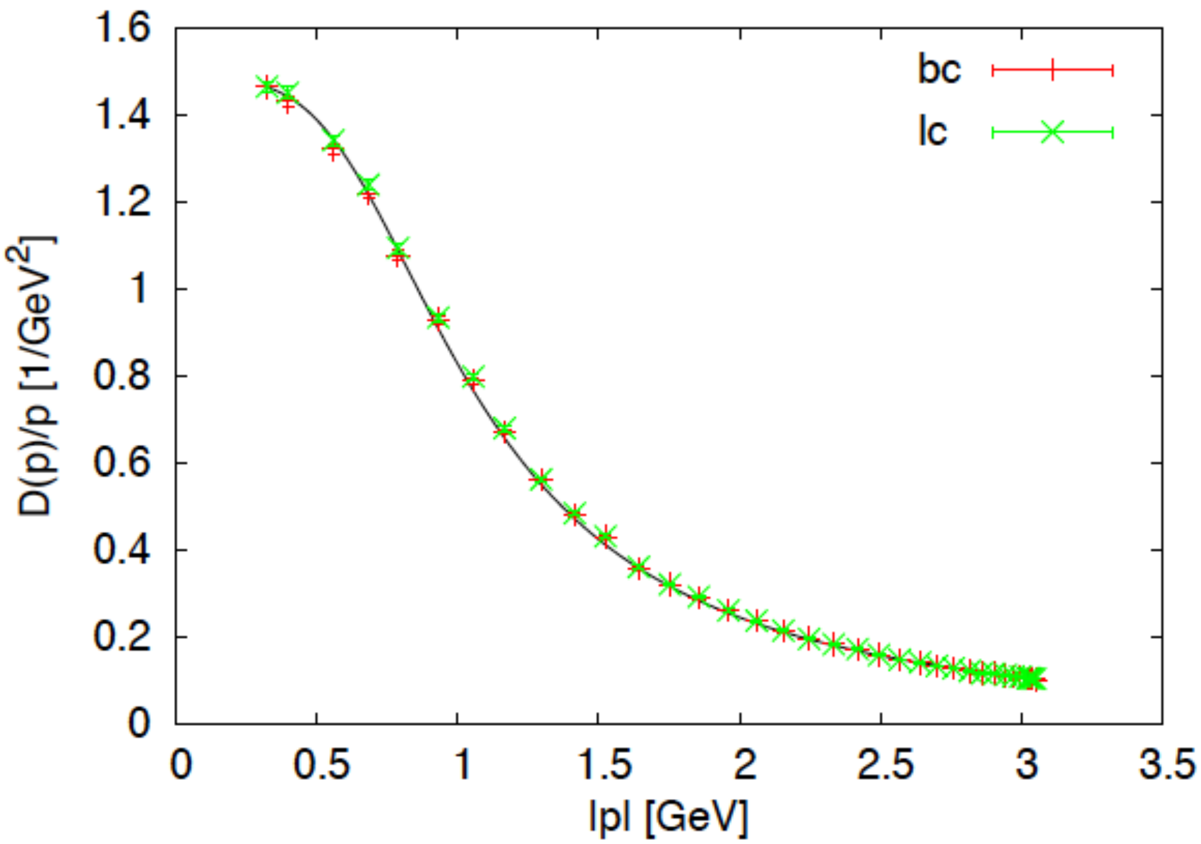


FIG. 6. The gluon propagator for the B1 lattice with the bc and the lc-approach from 1000 trials. The solid line is a fit to the Gribov formula [14, 15]. The choice of Gribov copy apparently makes no visible difference.

C
to
cc
P
m

YM Hamiltonian in $\partial A = 0$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi J\Pi + B^2) + H_C$$

$$\Pi = \delta / i\delta A$$

Christ and Lee

$$J(A^\perp) = Det(-D\partial) \qquad D^{ab} = \delta^{ab}\partial + gf^{abc}A^c$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho J(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}\rho$$

Coulomb term

color charge density $\rho^a = -f^{abc}A^b\Pi^c + \rho^a_m$

YM Hamiltonian in $\partial A = 0$

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Coulomb term

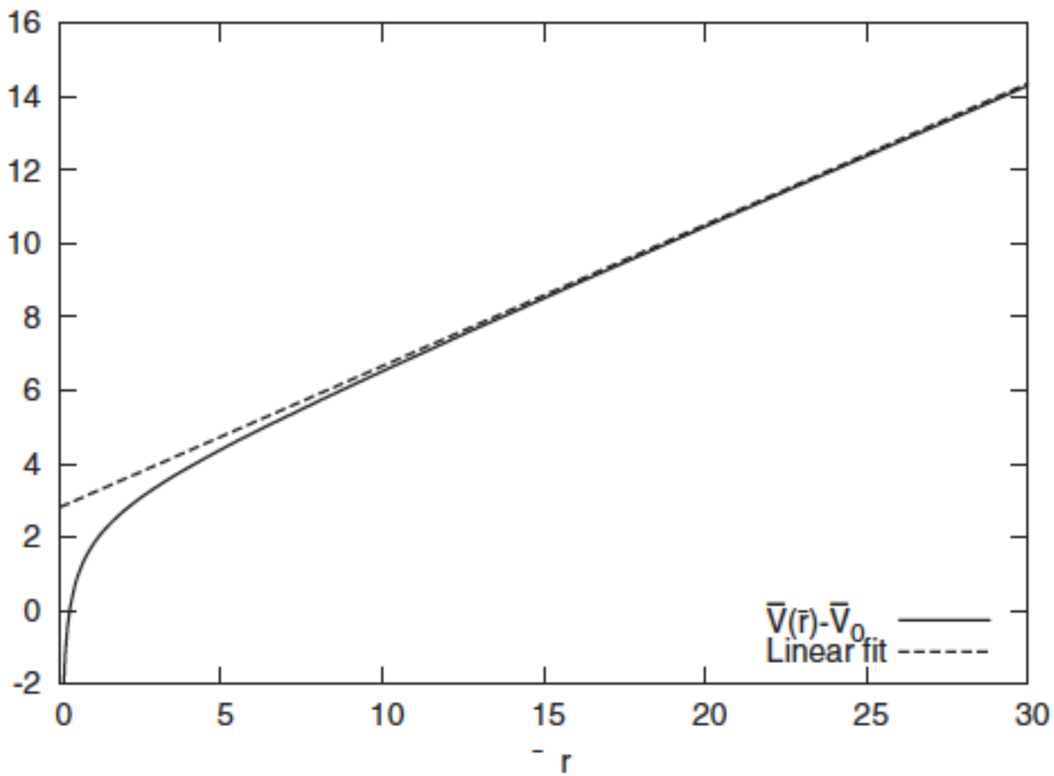
color charge density $\rho^a = -f^{abc}A^b\Pi^c + \rho_m^a$

static quark potential

$$V_C(|\vec{x}-\vec{y}|) = \langle\langle \vec{x}|(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}|\vec{y}\rangle\rangle$$

Non-Abelian Coulomb potential

$$V_C(\vec{x},\vec{y}) = \langle \vec{x} | (-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1} | \vec{y} \rangle$$



D. Epple, H. Reinhardt
W.Schleifenbaum,
PRD 75 (2007)

$$V(r) = \xrightarrow[r\to 0]{} \sim 1/r$$

$$V(r) = \xrightarrow[r\to\infty]{} \sigma_C r, \qquad \text{lattice}: \ \sigma_C = 2...3\sigma_W$$

Gribov´s confinement scenario assumes:

horizon condition $\mathrm{d}^{-1}(0) = 0$

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What are the field configurations
which induce the horizon condition
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What are the field configurations which induce the horizon condition and thus confinement?

-magnetic monopoles

dual superconductor

Gribov´s confinement scenario assumes:

horizon condition $d^{-1}(0) = 0$

What are the field configurations which induce the horizon condition and thus confinement?

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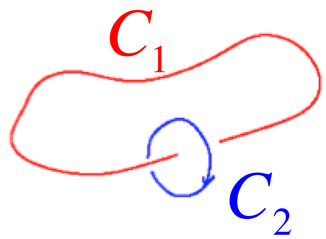
-center vortices

Center Vortices

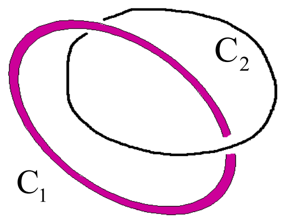
SU(N) **center Z(N)** $$z_k = e^{i2\pi k/N} 1_N, \quad k = 0,1,\ldots,N-1,$$

Wilson loop $$W[A](C) = P\exp(i\oint_C A)$$

center vortex field $$W[A(C_1)](C_2) = z^{L(C_1,C_2)}$$



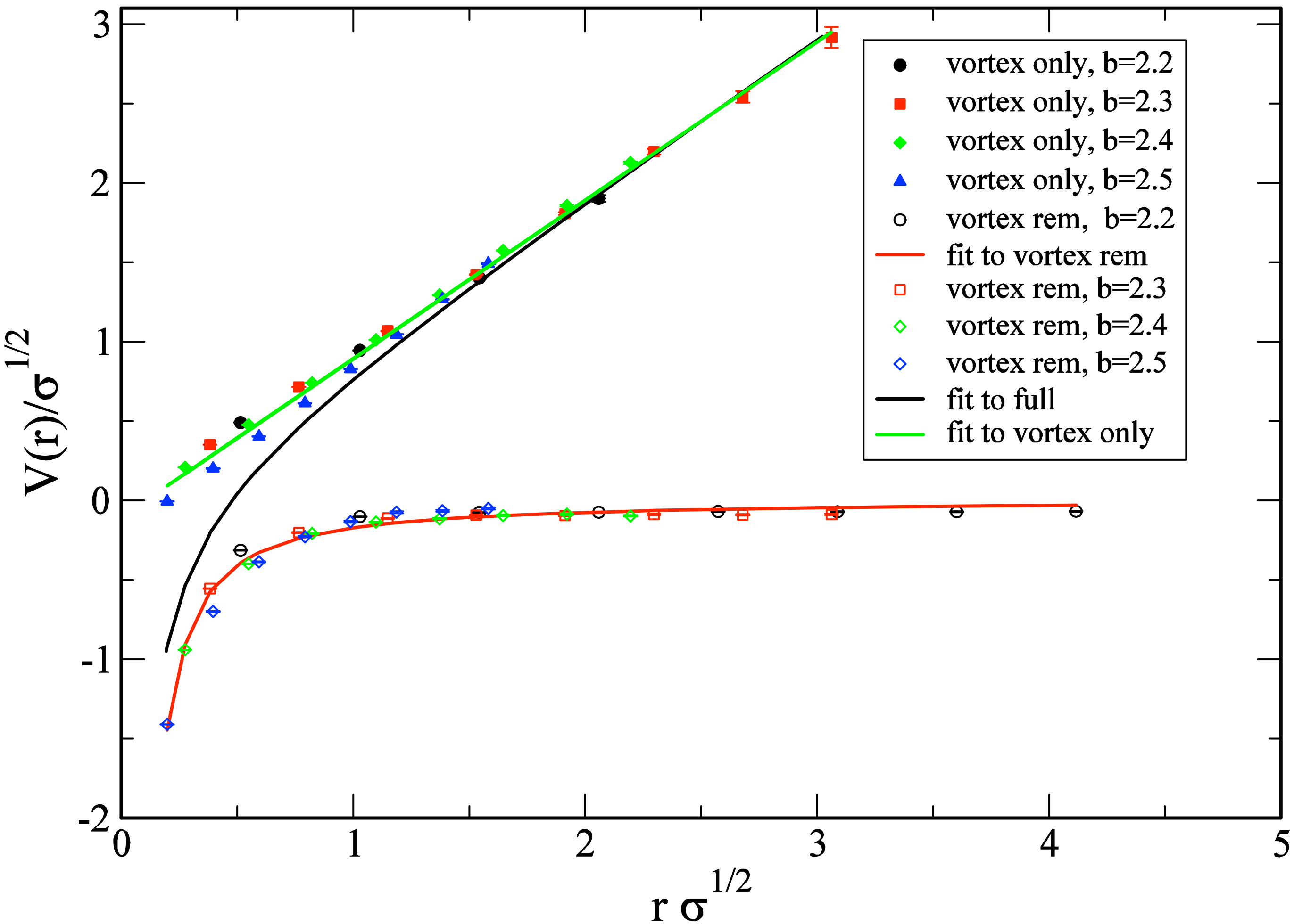
Gauss´ linking number $L(C1,C2)$



non-trivial center element z

SU(2) **center Z(2)** $z = 1,-1 \qquad W[A(C_1)](C_2) = (-1)^{L(C_1,C_2)}$

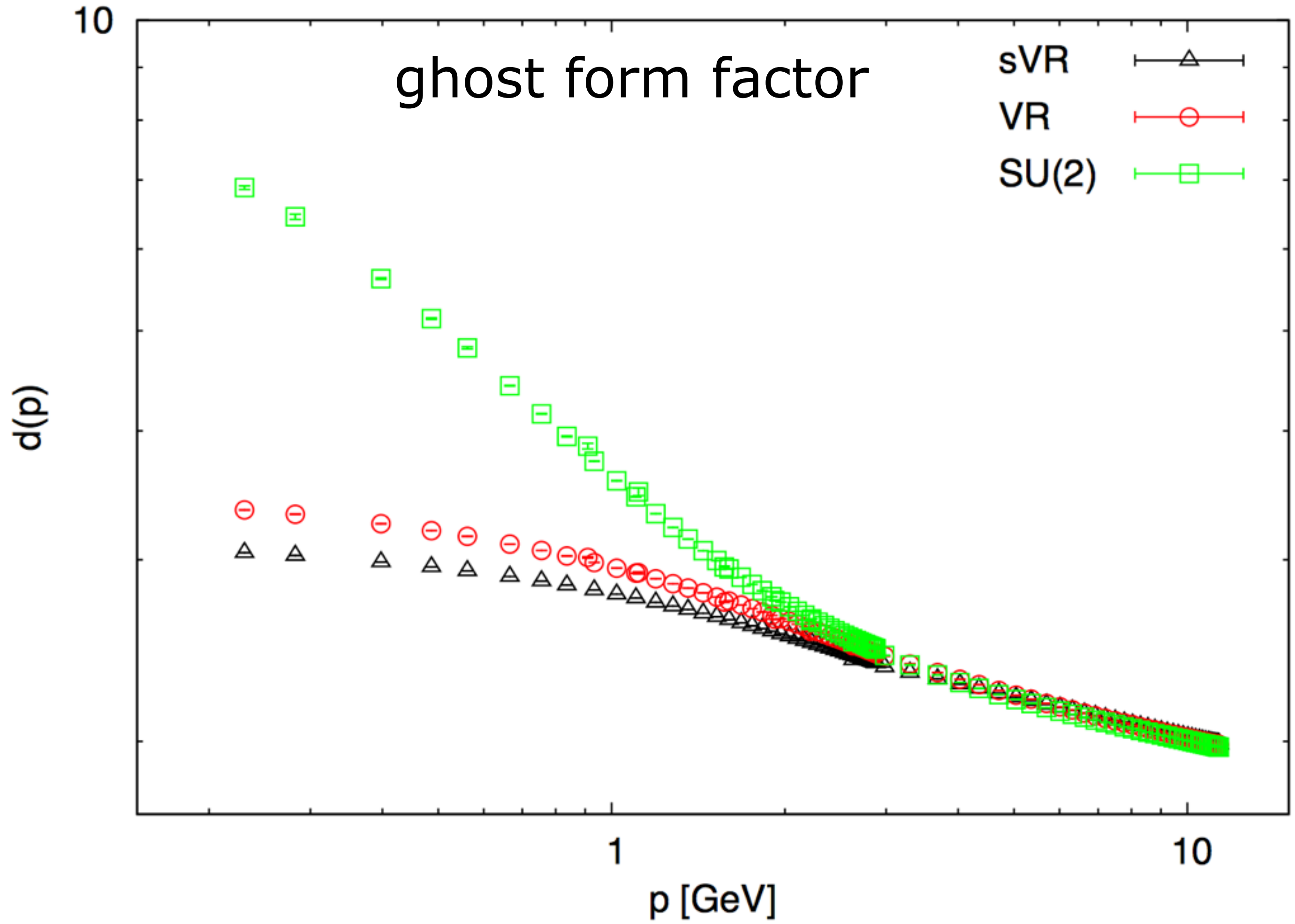
Q-Q-potential: SU(2)



$\sigma_{vortices} \approx \sigma$

K. Langfeld, H. R. & O. Tennert

Gribov scenario & center vortex picture



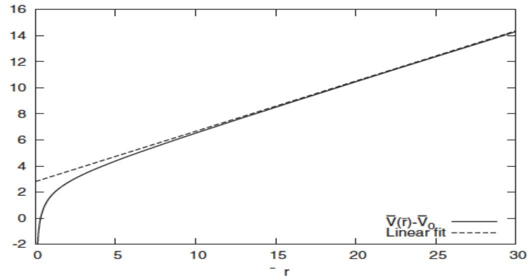
G. Burgio, M. Quandt, H.R. & H.Vogt, Phys. Rev.D92(2015)

- elimination of center vortices: IR enhancement disappears
- horizon condition $d^{-1}(0) = 0$ is lost

Non-Abelian Coulomb potential

$$V_C(\vec{x},\vec{y}) = \langle \vec{x} | (-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1} | \vec{y} \rangle$$

$$\xrightarrow[|\vec{x}-\vec{y}|\to\infty]{} \sigma_C |\vec{x}-\vec{y}|$$



D. Zwanziger $\quad \sigma_C \geq \sigma_W$

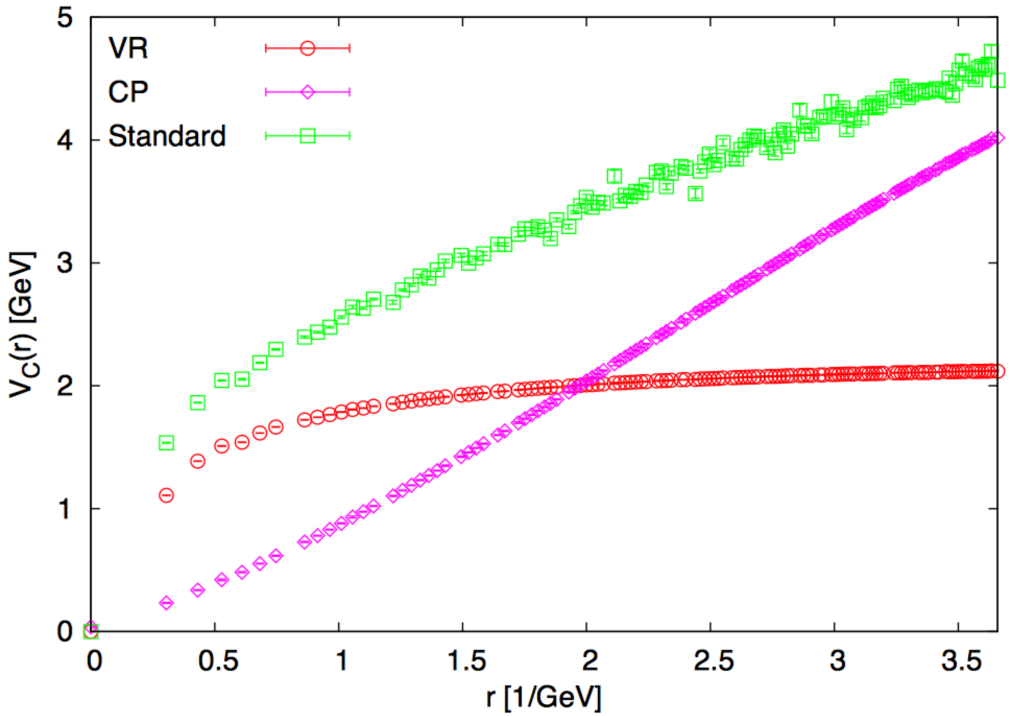
$$\text{lattice}: \ \sigma_C = 2 \ldots 3\sigma_W$$

Gribov scenario & center vortex picture

- *Coulomb potential*

  G. Burgio, M. Quandt, H. R. & H. Vogt, Phys.Rev.D92(2015)

- *Coulomb string tension disappears after elimination of center vortices*



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Non-Abelian Coulomb potential

$$V_C(\vec{x},\vec{y}) = \langle \vec{x} | (-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1} | \vec{y} \rangle$$

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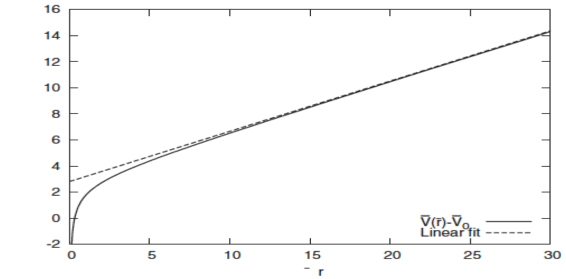
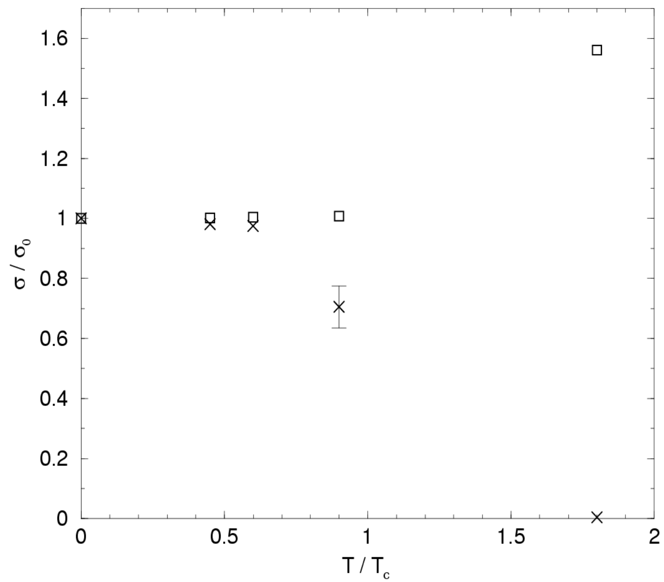


D. Zwanziger $\quad \sigma_C \geq \sigma_W$

$$\text{lattice}: \ \sigma_C = 2...3\sigma_W$$



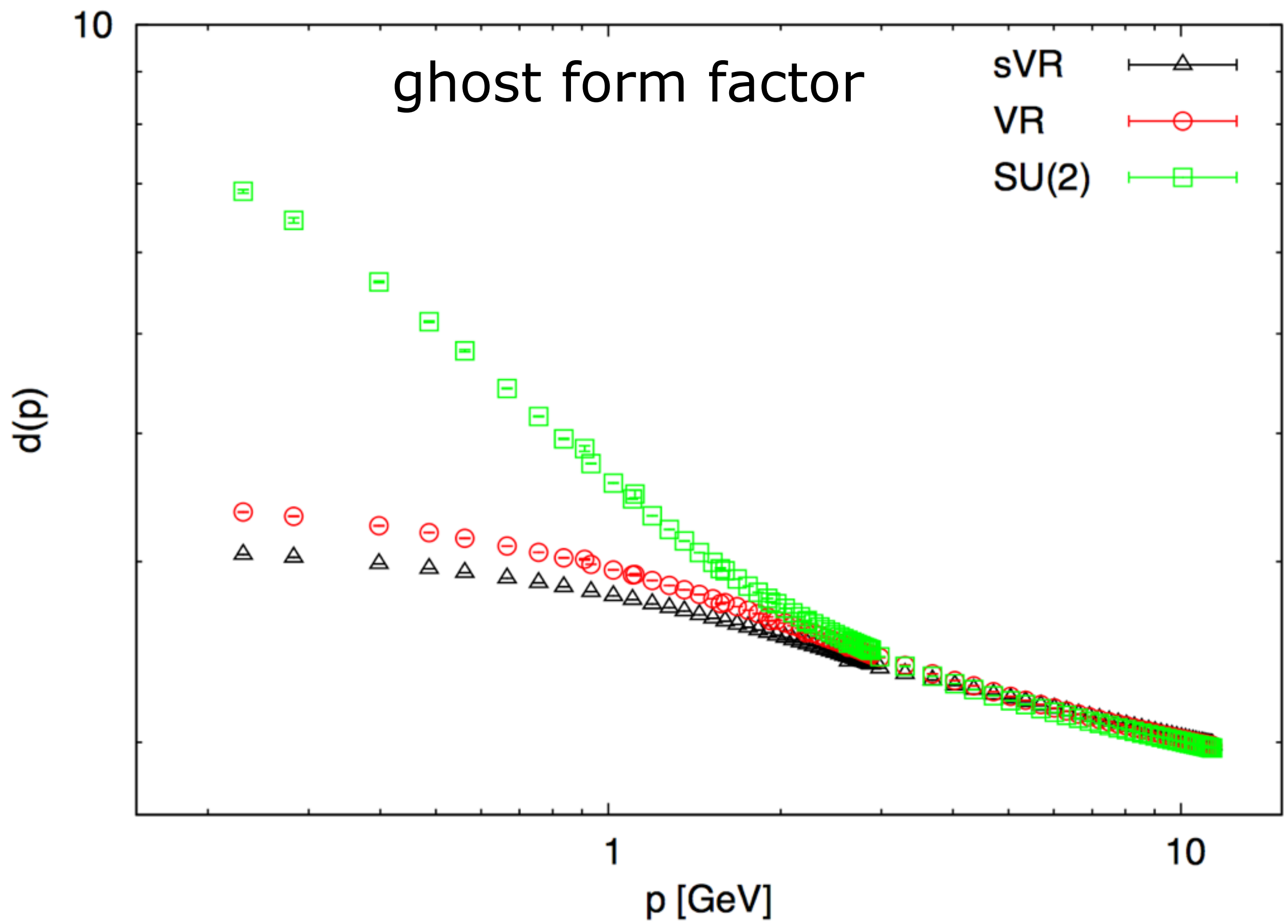
M. Engelhardt & H.R
Nucl.Phys.B585(2000)

elimination of spatial center vortices

>removes spatial string tension

>does not change temporal links
Polyakov loops & temporal string tension
are not affected

Gribov scenario & center vortex picture



G. Burgio, M. Quandt, H.R. & H.Vogt, Phys. Rev.D92(2015)

- elimination of *spatial* center vortices: IR enhancement disappears
- horizon condition $d^{-1}(0)=0$ is lost

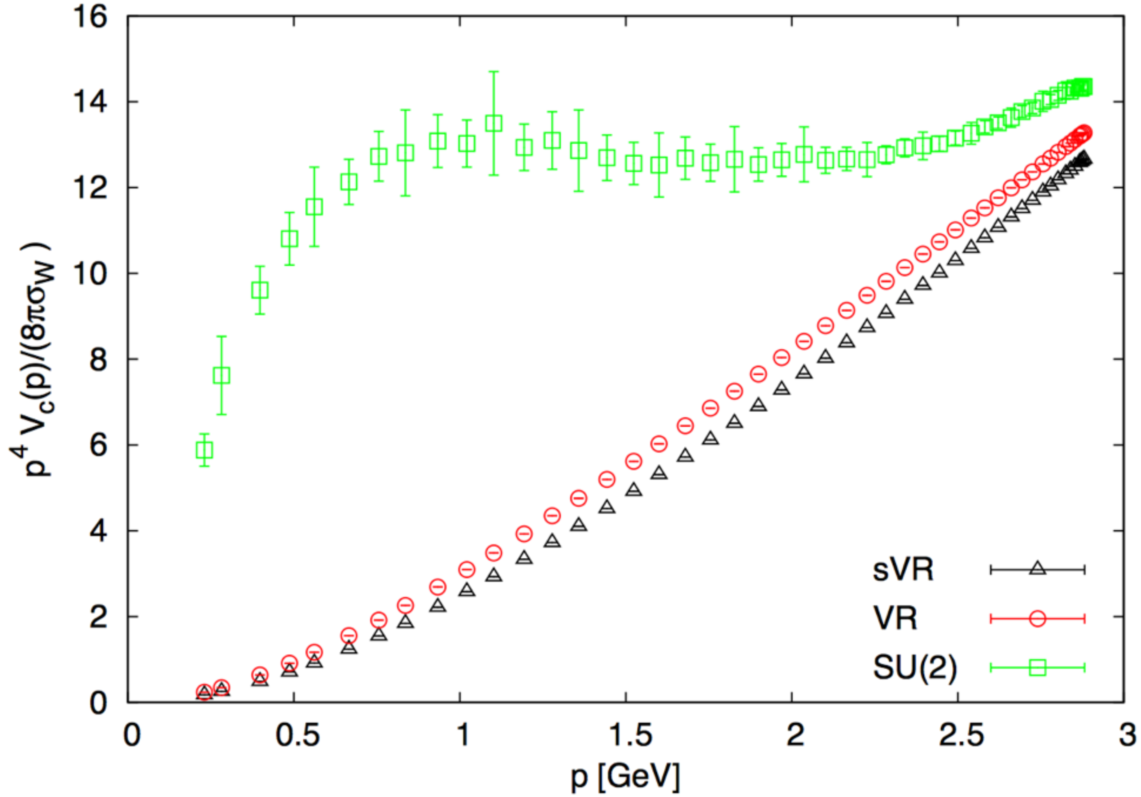
Gribov scenario & center vortex picture

- Coulomb potential

$$V_C(\boldsymbol{p}) = g^2 \mathrm{tr} \left\langle \left(-\hat{\boldsymbol{D}} \cdot \nabla\right)^{-1} (-\nabla^2) \left(-\hat{\boldsymbol{D}} \cdot \nabla\right)^{-1} \right\rangle$$

$$p^4 V_C(p) / 8\pi\sigma_W$$

$$\lim_{p \to 0} p^4 V_C(p) = 8\pi\sigma_C$$

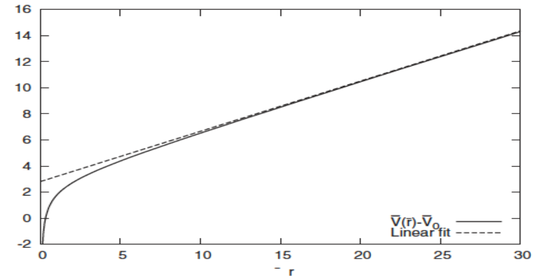


- Coulomb string tension disappears after *spatial* center vortex removal

Non-Abelian Coulomb potential

$$V_C(\vec{x},\vec{y}) = \langle \vec{x} | (-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1} | \vec{y} \rangle$$

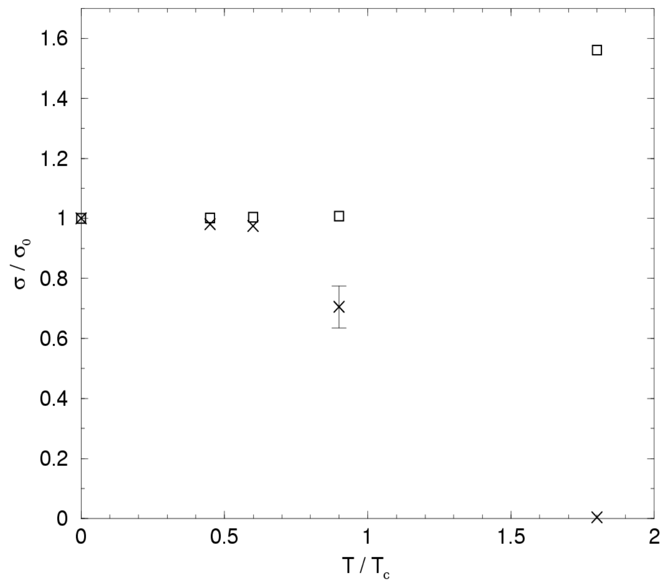
$$\xrightarrow[|\vec{x}-\vec{y}|\to\infty]{} \sigma_C |\vec{x}-\vec{y}|$$



D. Zwanziger $\sigma_C \geq \sigma_W$

lattice : $\sigma_C = 2...3\sigma_W$

G. Burgio, M. Quandt,
H. R. & H. Vogt
Phys.Rev.D92(2015)

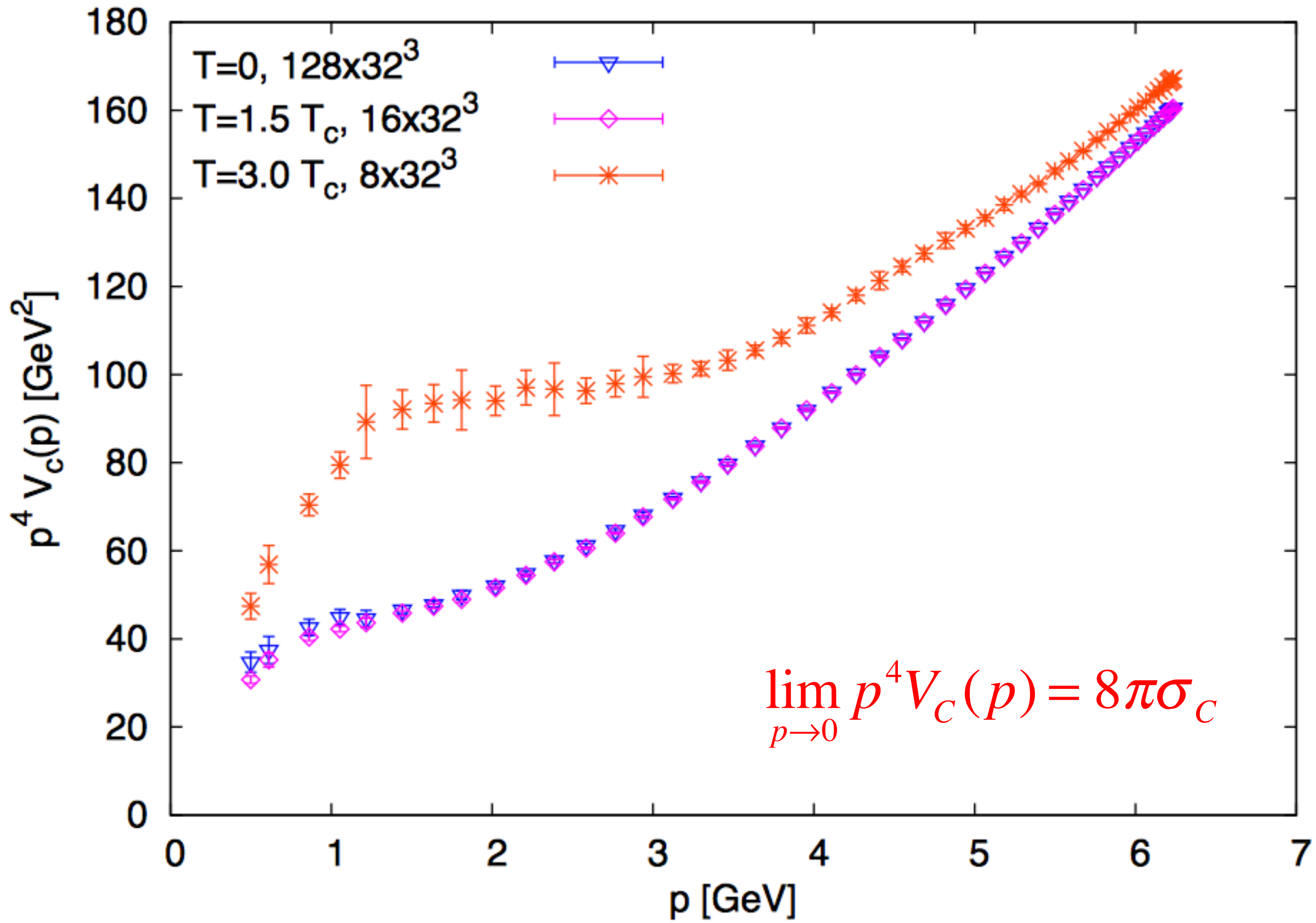


$\sigma_C \not\to \sigma_T \qquad \sigma_C \sim \sigma_S$

explains finite temp behaviour of σ_C

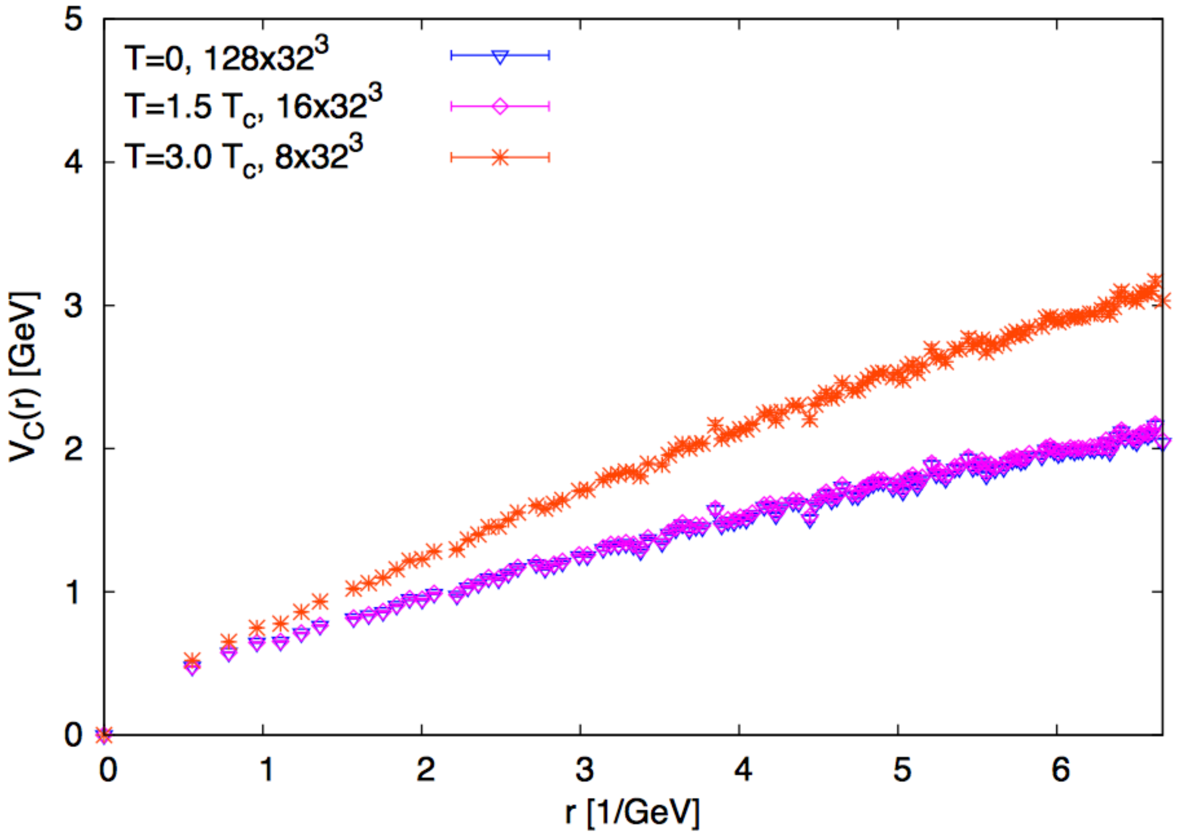
non-Abelian Coulomb potental at finite T

Y. Nakagawa, A. Nakamura, T. Saito, H. Toki, and D. Zwanziger, Phys. Rev., D73:094504, 2006.



G. Burgio, M. Quandt, H. R. & H. Vogt, Phys.Rev.D92(2015)

Coulomb potential



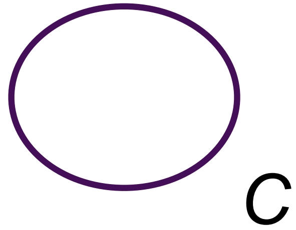
conclusion:

-the Coulomb string tension is tied to the spatial string tension, which is known to decouple from the temporal string tension in the deconfinement phase transition

-explains the temperature behaviour of the Coulomb string tension

Wilson loop

$$W[A](C) = P\exp\left[i\oint_C A \right]$$



order parameter of confinement

$$\langle W[A](C)\rangle \sim \begin{cases} \exp(-\sigma A(C)) & \textit{confinement} \\ \exp(-\kappa P(C)) & \textit{deconfinement} \end{cases}$$

area law = linearly rising potential
between static color acharges

$\langle W[A](C)\rangle$ **difficult to calculate in the continuum theory due to path ordering**

approximate calulation of the Wilson loop

M. Pak & H.R., Phys. Rev 80(2009)

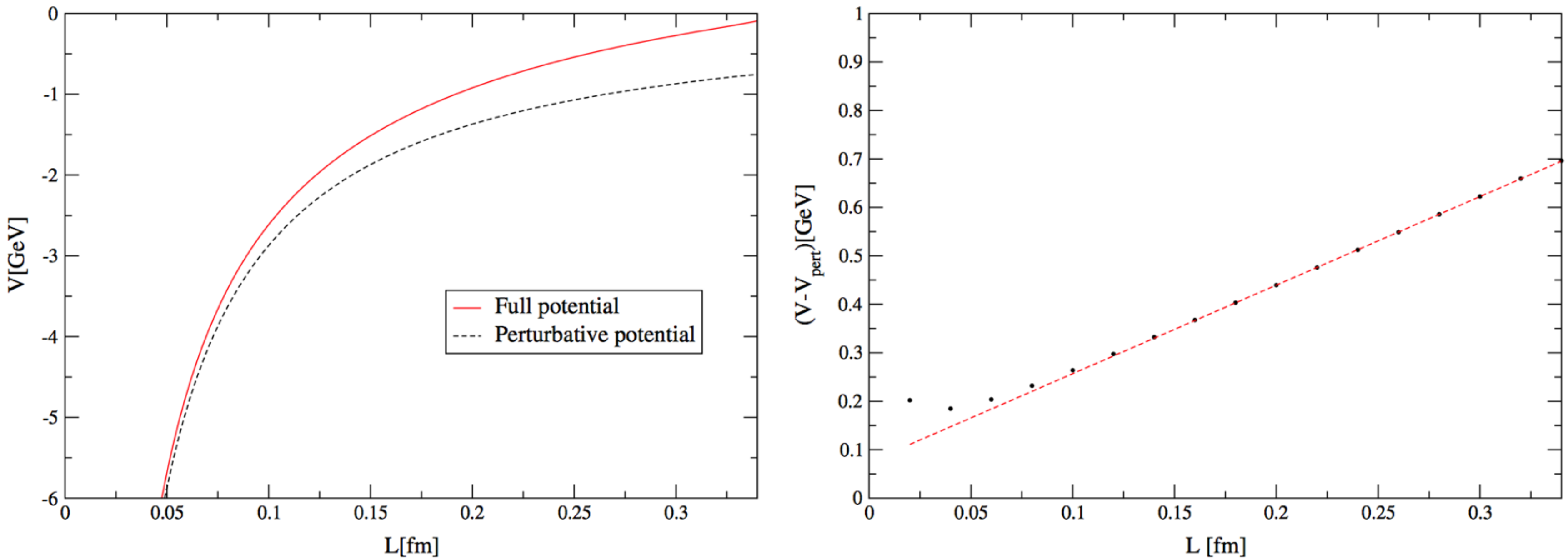


FIG. 7 (color online). Left-hand panel: The full static quark potential $V(L)$ obtained from the full propagator (32) and the perturbative potential $V_{\text{pert}}(L)$ obtained from the perturbative propagator (33). Right-hand panel: The full potential minus its perturbative part.

approximate calulation of the Wilson loop

M. Pak & H.R., Phys. Rev 80(2009)

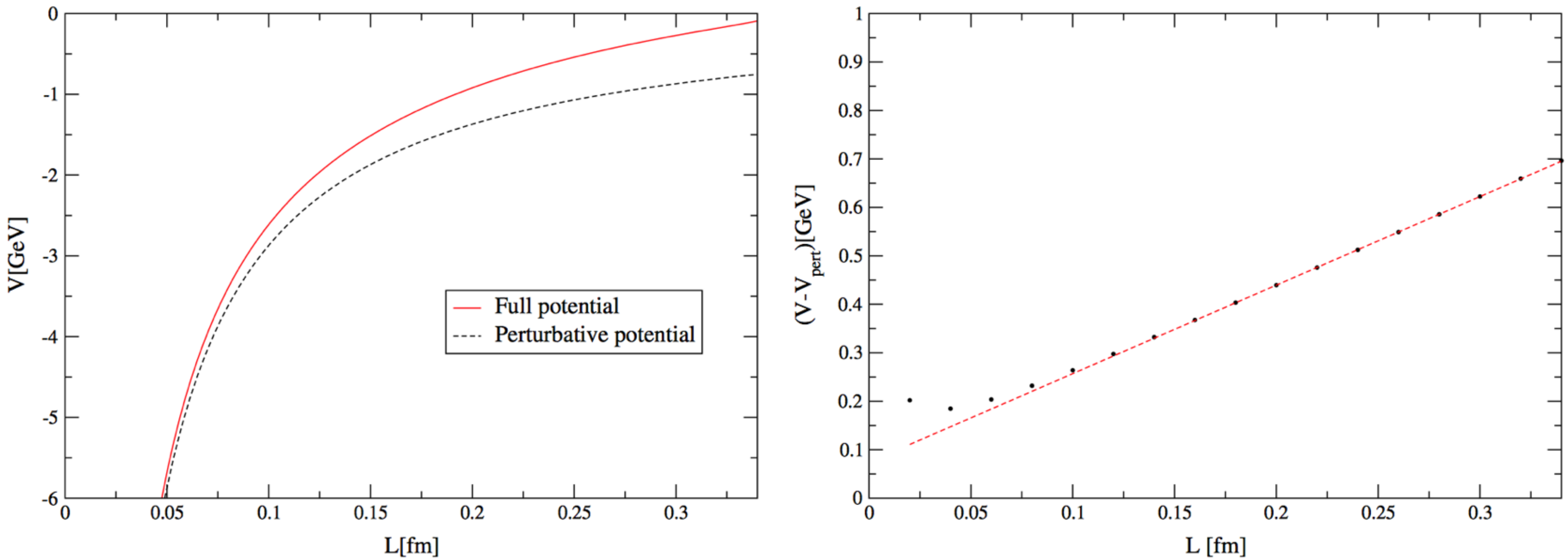
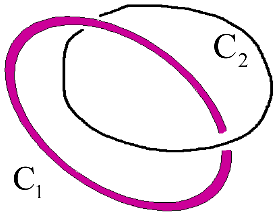


FIG. 7 (color online). Left-hand panel: The full static quark potential $V(L)$ obtained from the full propagator (32) and the perturbative potential $V_{\mathrm{pert}}(L)$ obtained from the perturbative propagator (33). Right-hand panel: The full potential minus its perturbative part.

alternative order parameters:

center of the gauge group

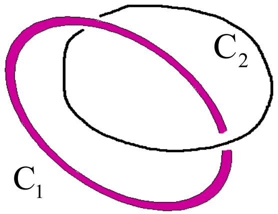
t´Hooft loop



$$\hat{V}(C_1)W(C_2) = z^{L(C_1,C_2)}W(C_2)\hat{V}(C_1)$$

z-center element
L-linking number

t´Hooft loop



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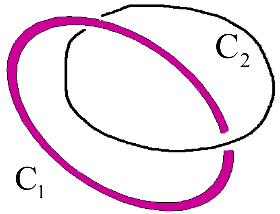
t´Hooft loop



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continuum representation

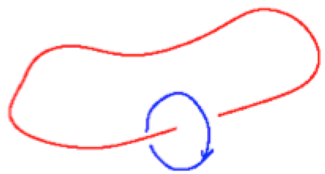
H.R. Phys. Lett. B557(2003)

$$V(C) = \exp\left[i\int_{R^3} A(C)\hat{\Pi} \right]$$

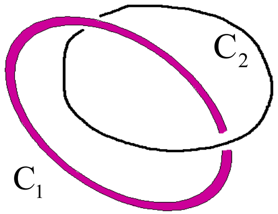
$$\Pi_k^a(x) = \delta / i\delta A_k^a(x)$$

center vortex field

$$W\left[A(C_1)\right](C_2) = z^{L(C_1,C_2)}$$



t´Hooft loop



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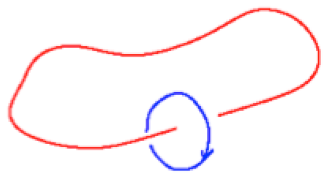
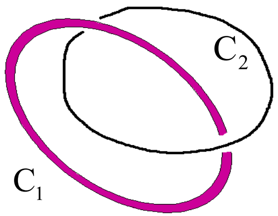


center vortex generator

$$\hat{V}(C)\Psi(A') = \Psi(A' + A(C))$$

t´Hooft loop



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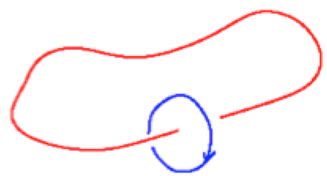


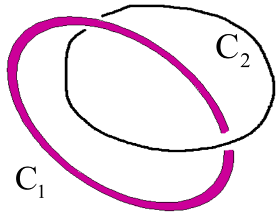
center vortex generator

$$\hat{V}(C)\Psi(A') = \Psi(A' + A(C))$$

QM:

$$\exp(ia\hat{p})\Psi(x) = \Psi(x+a)$$

t´Hooft loop



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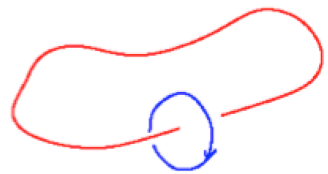

center vortex generator

$$\hat{V}(C)\Psi(A') = \Psi(A' + A(C))$$

Hamiltonian approach to YMT in Coulomb gauge: **perimeter law**

H.R. & D. Epple, Phys. Rev.D76(2007)

The QCD Hamiltonian in Coulomb gauge

$$\boxed{H_{QCD} = H_{YM} + H_C + H_q}$$

gluon part

$$H_{YM} = \tfrac{1}{2}\int (J^{-1}\Pi J \Pi + B^2) \qquad \Pi = -i\delta/\delta A \qquad J(A^{\perp}) = Det(-D\partial)$$

quark part

$$H_q = \int \Psi^{\dagger}(x)[\vec{\alpha}(\vec{p} + g\vec{A}) + \beta m_0]\Psi(x) \qquad \vec{\alpha}, \beta - \textit{Dirac matrices}$$

Coulomb term

$$H_C = \tfrac{1}{2}\int J^{-1}\rho(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}J\rho$$

color charge density

$$\rho^a = -f^{abc}A^b\Pi^c + \Psi^{\dagger}(x)t^a\Psi(x)$$

Hamiltonian approach to QCD in Coulomb gauge

early attempts:

-only Coulomb term
-no coupling of the quarks to the transversal gluons included
-too small quark condensate

Finger & Mandula
Adler & Davis,
Alkofer& Amundsen

with quark-gluon coupling

M. Pak & H.R. Phys.Rev.D88(2013)

P. Vastag, H. R. & D. Campagnari
Phys.Rev.D93(2016)

D. Campagnari , E. Ebadati, H.R. and P: Vastag,
arXiv:1608.06820, Pys.Rev.D, inpress

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger(s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

s, v, w – variational kernels $\quad \vec{\alpha}, \beta$ – Dirac matrices

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

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s,v,w – variational kernels $\quad$ $\vec{\alpha},\beta$ – *Dirac matrices*

$\mathrm{v}=\mathrm{w}=0:$ *BCS – wave function*

Finger & Mandula
Adler & Davis,
Alkofer & Amundsen

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A|\Phi\rangle_q = \exp\Big[\int \Psi_+^\dagger (\mathrm{s}\beta + \mathrm{v}\vec{\alpha}\cdot\vec{A} + \mathrm{w}\beta\vec{\alpha}\cdot\vec{A})\Psi_-\Big]|0\rangle$$

s, v, w – variational kernels $\quad$ $\vec{\alpha}, \beta$ – *Dirac matrices*

$\mathrm{v}=\mathrm{w}=0:$ *BCS – wave function*

Finger & Mandula
Adler & Davis,
Alkofer & Amundsen

$\mathrm{v}\neq 0, \mathrm{w}=0:$ *quark - gluon - coupling*

Pak & Reinhardt,

quark wave functional

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

s, v, w – variational kernels $\quad$ $\vec{\alpha}, \beta$ – *Dirac matrices*

> calculate ⟨H_{QCD}⟩ up to 2 loops

> variation w.r.t. **s, v, w**

$$v(p,q) = f_v[s,\omega] \qquad w(p,q) = f_w[s,\omega]$$

$$\boxed{s(p) = f_s[s,v,w;p]}$$

gap equation

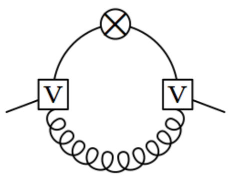
cancelation of all UV-divergencies

cancellation of UV-divergencies

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_- \right]|0\rangle$$

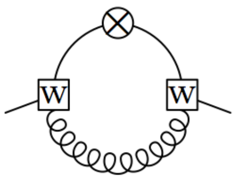
divergent loop contributions to the gap equation

> kernel **V**



$$\frac{C_F}{16\pi^2} g^2 S(k)\left[-2\Lambda + k\ln\frac{\Lambda}{\mu}\left(-\frac{2}{3} + 4P(k)\right)\right]$$

> kernel **W**



$$\frac{C_F}{16\pi^2} g^2 S(k)\left[2\Lambda + k\ln\frac{\Lambda}{\mu}\left(\frac{10}{3} - 4P(k)\right)\right]$$

> Coulomb term V_C

$$-\frac{C_F}{6\pi^2} g^2 k S(k)\ln\frac{\Lambda}{\mu}.$$

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A | \Phi \rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_- \right] | 0 \rangle$$

s, v, w – variational kernels $\quad$ $\vec{\alpha}, \beta$ – *Dirac matrices*

numerical calculation

D. Campagnari , E. Ebadati, H.R. and P: Vastag,
arXiv:1608.06820,PRD, in press

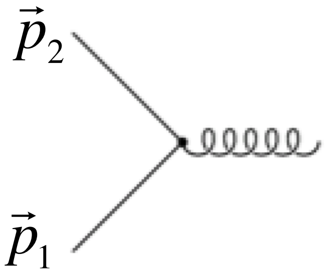
input: $\quad \omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}} \quad M = 0.88 GeV$

lattice: $\sigma_C = 2.5\sigma$

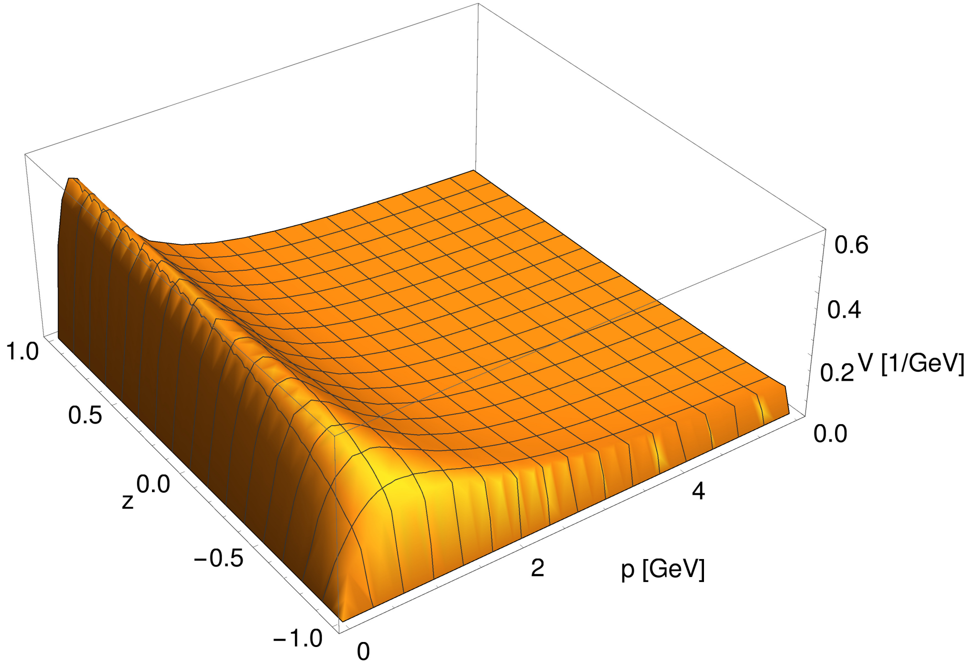
G. Burgio, M.Quandt , H.R.,
PRL102(2009)

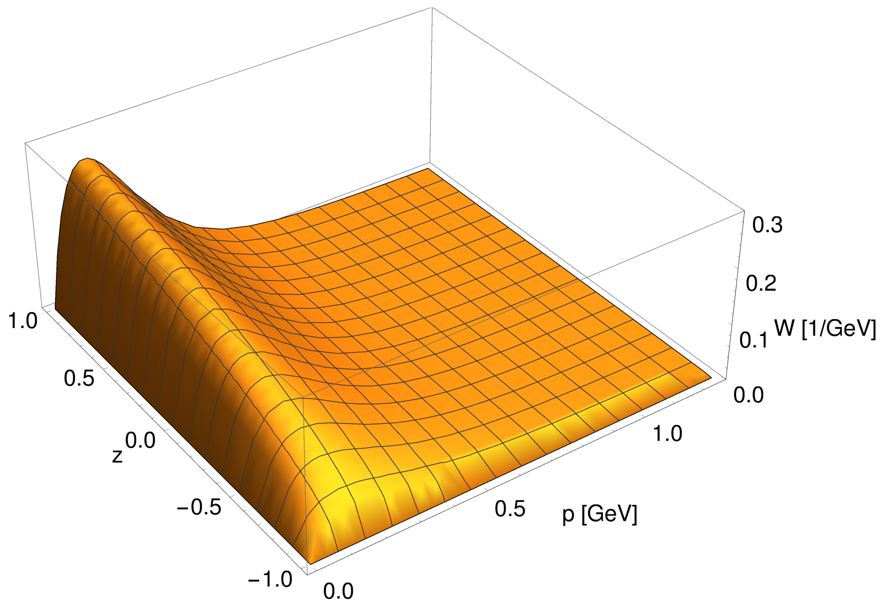
choose g *to reproduce* $\quad \langle \bar{q}q \rangle = (-235 MeV)^3 \quad \Rightarrow g \simeq 2.1$

vector form factors v , w

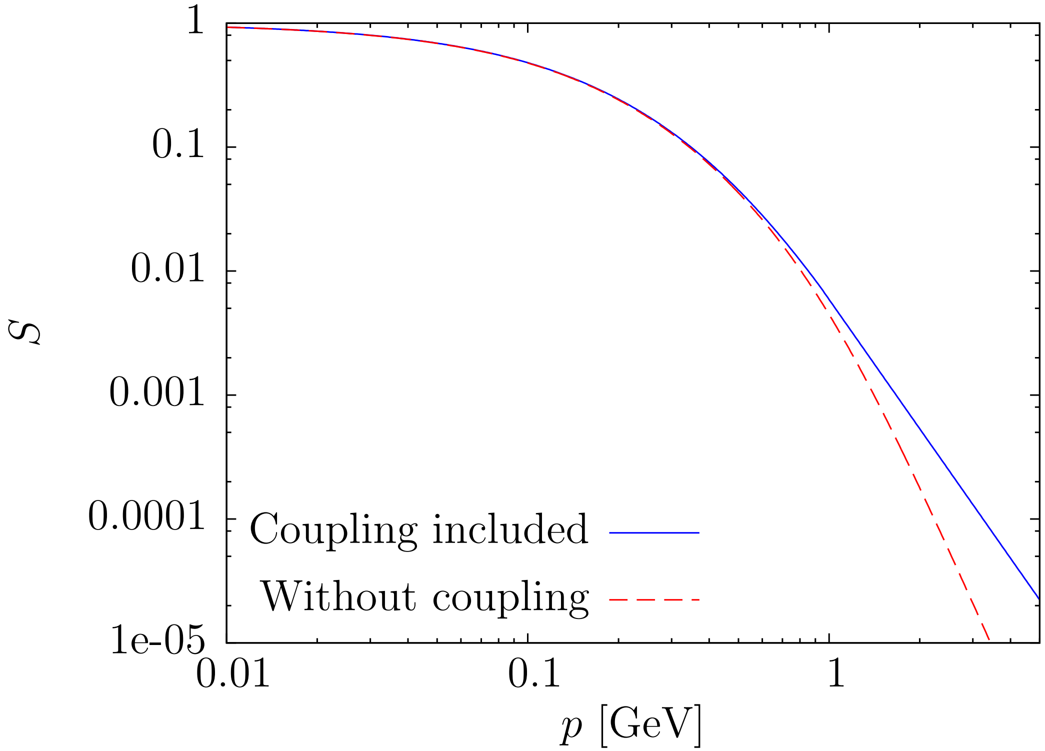


$$v,w(\vec{p}_1,\vec{p}_2): \quad p := |\vec{p}_1| = |\vec{p}_2|, \;\; z = \cos\sphericalangle(\vec{p}_1,\vec{p}_2)$$





scalar form factor



-quark-gluon coupling modifies only the mid- and high-momentum regime

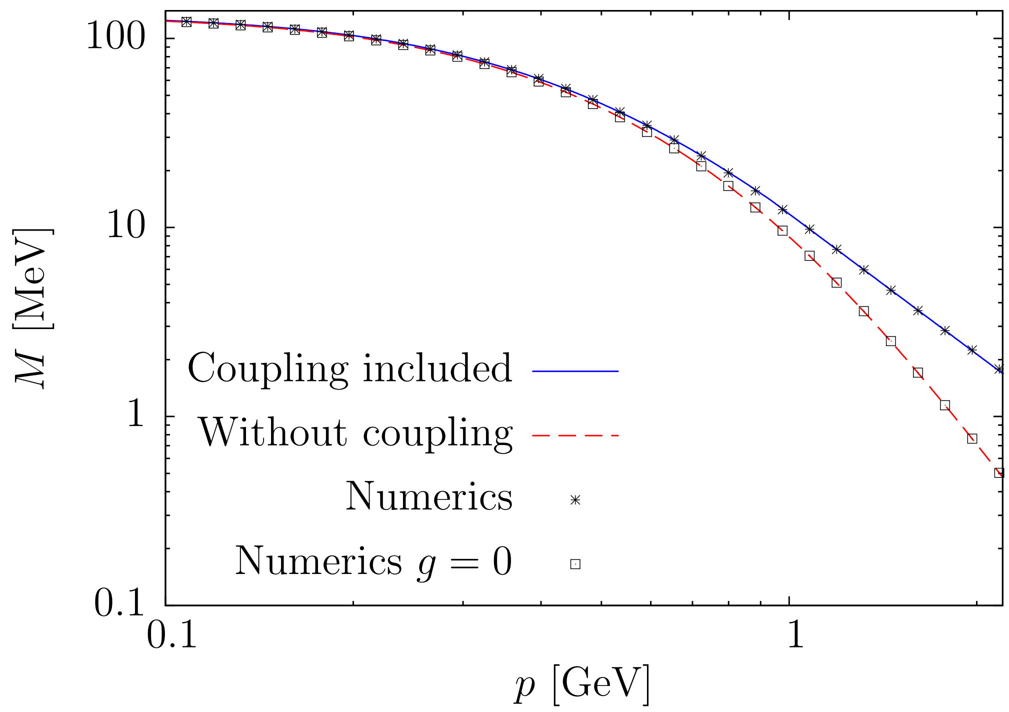
-low-momentum regime is dominated by Coulomb term

effective quark mass

$$M(p) = \frac{2ps(p)}{1-s^2(p)}$$

$$\langle \bar{q}q \rangle_{phen} = (-235\,MeV)^3$$

$$\langle \bar{q}q \rangle_{AD} = (-185\,MeV)^3$$



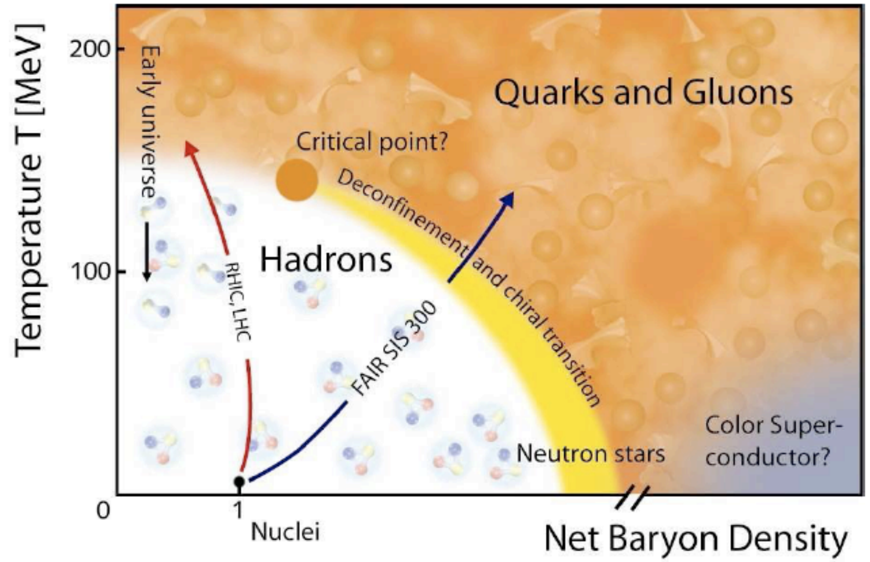
> coupling to tranversal gluons substantially increases chiral symmetry breaking

> no chiral symmetry breaking without the Coulomb term

QCD at finite temperature: grand canonical ensemble

- quasi-particle ansatz for the density operator

- minimization of the thermodynamic potential



YM sector:

H.Reinhardt, D.Campagnari & A. Szczepaniak, Phys.Rev.D84(2011)

J.Heffner, H.Reinhardt & D.Campagnari, Phys.Rev.D85(2012)

Alternative Hamiltonian approach to finite temperature QFT

H. R. arXiv:1604.06273
Phys.Rev.D94(2016)045016

- no ansatz for the density matrix required
- motivation: Polyakov loop

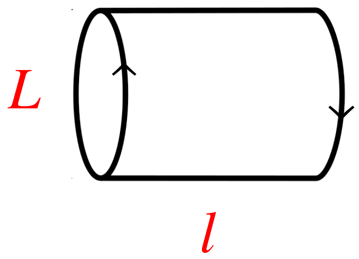
$$P[A_0](\vec{x}) = \frac{1}{d_r} tr P \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

  - $\langle P[A_0] \rangle$ order parameter of confinement
- Hamiltonian approach
  - Weyl gauge $A_0=0$
- How to calculate the Polyakov loop in the Hamiltonian approach?

Finite temperature QFT

- compactification of (Euclidean) time
- bc: $A(x^0 = L/2) = A(x^0 = -L/2)$ Bose fields

  $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$ Fermi fields



$S^1(L) \times \mathbb{R}^3$

- temperature $T = L^{-1}$ $\qquad l \to \infty$

Finite temperature QFT

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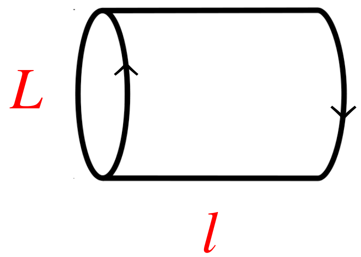
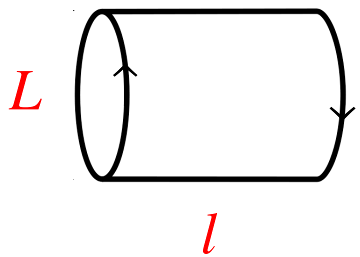


$S^1(L) \times \mathbb{R}^3$

- temperature $T = L^{-1}$ $\quad l \to \infty$
- exploit the O(4)-invariance of the Euclidean Lagrangian
  - O(4)-rotation $x^0 \to x^3 \quad \mathrm{A}^0 \to A^3 \quad \gamma^0 \to \gamma^3$

    $x^1 \to x^0 \quad \mathrm{A}^1 \to A^0 \quad \gamma^1 \to \gamma^0$
  - one compactified spatial dimension
  - bc: $A(x^3 == L/2) = \mathrm{A}(x^3 = -L/2)$ Bose fields

    $\psi(x^3 = L/2) = -\psi(x^3 = -L/2)$ Fermi fields
- spatial manifold: $\mathbb{R}^2 \times S^1(L)$



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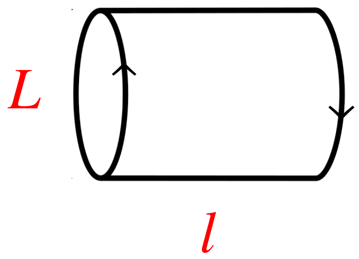
  $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$ Fermi fields

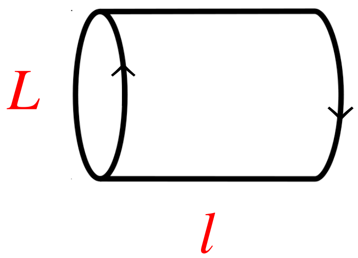


$S^1(L) \times \mathbb{R}^3$

- temperature $T = L^{-1}$ $\quad l \to \infty$
- exploit the O(4)-invariance of the Euclidean Lagrangian
  - O(4)-rotation $x^0 \to x^3 \quad \mathrm{A}^0 \to A^3 \quad \gamma^0 \to \gamma^3$

    $x^1 \to x^0 \quad \mathrm{A}^1 \to A^0 \quad \gamma^1 \to \gamma^0$
  - one compactified spatial dimension
  - bc: $A(x^3 == L/2) = \mathrm{A}(x^3 = -L/2)$ Bose fields

    $\psi(x^3 = L/2) = -\psi(x^3 = -L/2)$ Fermi fields
- spatial manifold: $\mathbb{R}^2 \times S^1(L)$



Hamiltonian approach

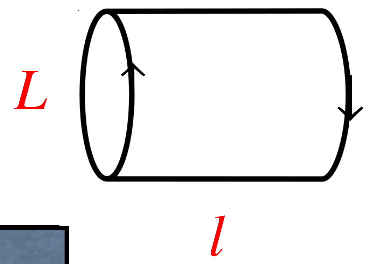
- *temperature is now encoded in one „spatial“ dimension while „time“ has infinite extension independent of the temperature*

Finite temperature QFT

- partition function

$$Z(L) = \lim_{l\to\infty} Tr\exp(-lH(L)) = \lim_{l\to\infty}\sum_n \exp(-lE_n(L)) = \lim_{l\to\infty}\exp(-lE_0(L))$$

- ground state energy $\qquad E_0(L) = l^2 Le(L)$
  - on the spatial manifold: $\qquad \mathbb{R}^2 \times S^1(L)$



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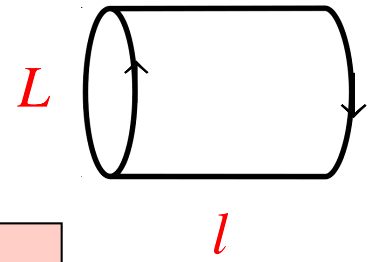

- pressure: $p = -e(L)$
- energy density: $\varepsilon = \partial[Le(L)]/\partial L - \mu\partial e/\partial\mu$
- Dirac fermions with finite chemical potential

$$h = \vec{\alpha}\cdot\vec{p} + \beta m \to h + i\mu\alpha^3$$

Relativistic Bose gas

- grand canonical ensemble $T = L^{-1}$

$$P = \tfrac{2}{3}\int d^3p\,\frac{p^2}{\omega(p)}\,n(p) \qquad n(p) = \frac{1}{e^{L\omega(p)} - 1} \qquad \omega(p) = \sqrt{p^2 + m^2}$$

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- grand canonical ensemble $T = L^{-1}$

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- energy density on $\mathbb{R}^2 \times S^1(L)$

$$e(L) = \frac{1}{2}\int d^2p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty} \sqrt{m^2 + p_\perp^2 + \omega_n^2} \qquad \omega_n = \frac{2\pi n}{L}$$

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- proper-time regularization

$$\sqrt{A} = \frac{1}{\Gamma(-\frac{1}{2})} \lim_{\Lambda\to\infty} \int_{1/\Lambda^2}^{\infty} d\tau \exp(-\tau A)$$

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$$\tfrac{1}{2\pi}\sum_{k=-\infty}^{k=\infty} e^{ikx} = \sum_{n=-\infty}^{n=\infty}\delta(x - 2\pi n)$$

Relativistic Bose gas

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$$P = -e(L) = \frac{1}{2\pi^2}\sum_{n=-\infty}^{\infty}\left(\frac{m}{nL}\right)^2 K_{-2}(nLm)$$

modified Bessel function

Relativistic Bose gas

- grand canonical ensemble $T = L^{-1}$

$$P = \tfrac{2}{3}\int d^3p \frac{p^2}{\omega(p)} n(p) \qquad n(p) = \frac{1}{e^{L\omega(p)} - 1} \qquad \omega(p) = \sqrt{p^2 + m^2}$$

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$$\sqrt{A} = \frac{1}{\Gamma(-\frac{1}{2})} \lim_{\Lambda\to\infty} \int_{1/\Lambda^2}^{\infty} d\tau \exp(-\tau A)$$

  - Poisson resummation

$$\tfrac{1}{2\pi}\sum_{k=-\infty}^{k=\infty} e^{ikx} = \sum_{n=-\infty}^{n=\infty} \delta(x - 2\pi n)$$

$$P = -e(L) = \frac{1}{2\pi^2}\sum_{n=-\infty}^{\infty}\left(\frac{m}{n\beta}\right)^2 K_{-2}(n\beta m)$$

modified Bessel function

- massles bosons: m=0

  Stephan – Boltzmann – law

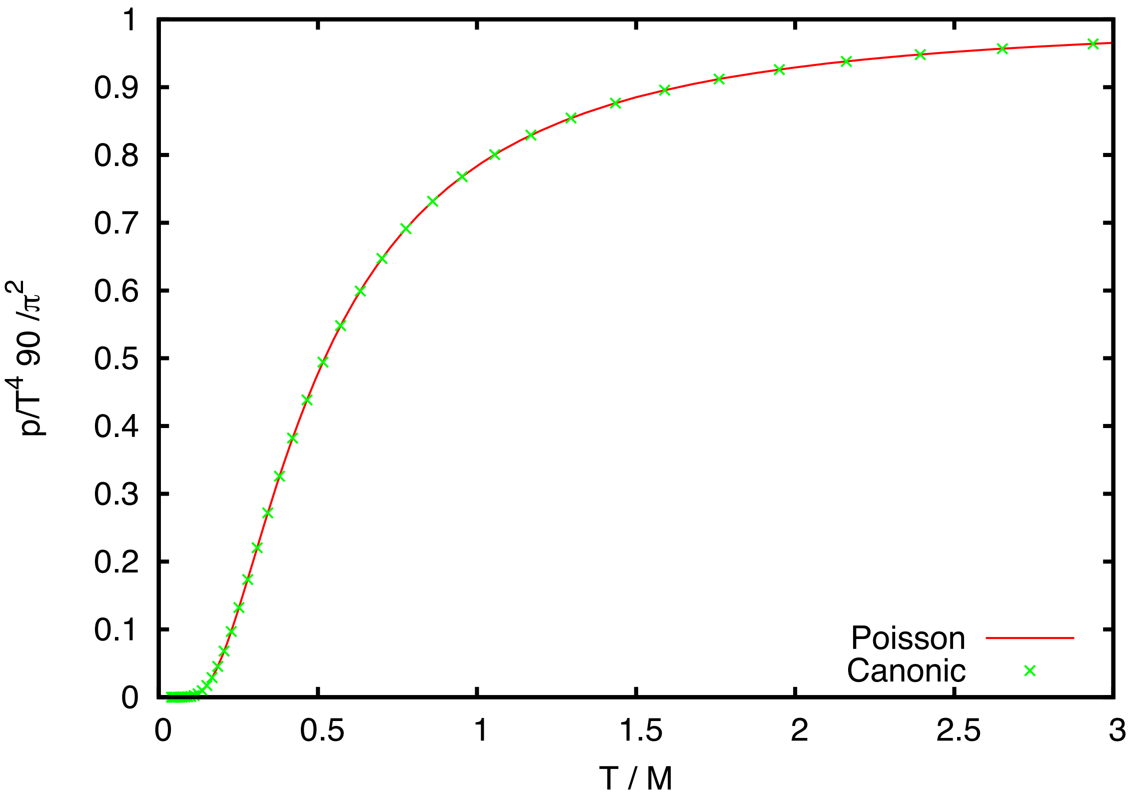
$$\boxed{P = \frac{\zeta(4)}{\pi^2} T^4 = \frac{\pi^2}{90} T^4}$$

massive bosons

$$\omega(p)=\sqrt{p^2+m^2}$$

$$P=\tfrac{2}{3}\int d^3p\,\frac{p^2}{\omega(p)}n(p) \qquad n(p)=\frac{1}{e^{L\omega(p)}-1}$$

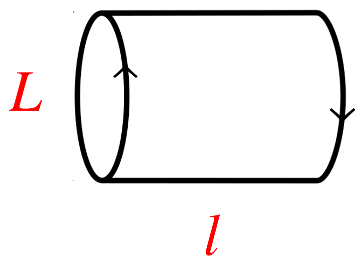
$$P=-e(L)=-\frac{1}{2\pi^2}\sum_{n=-\infty}^{\infty}\left(\frac{m}{n\beta}\right)^2 K_{-2}(n\beta m)$$



pressure of a massive relativistic Bose gas

$$e(L)=\frac{1}{2}\int d^2p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty}\sqrt{m^2+p_\perp^2+\omega_n^2}$$

$$\omega_n=\frac{2\pi n}{L}$$

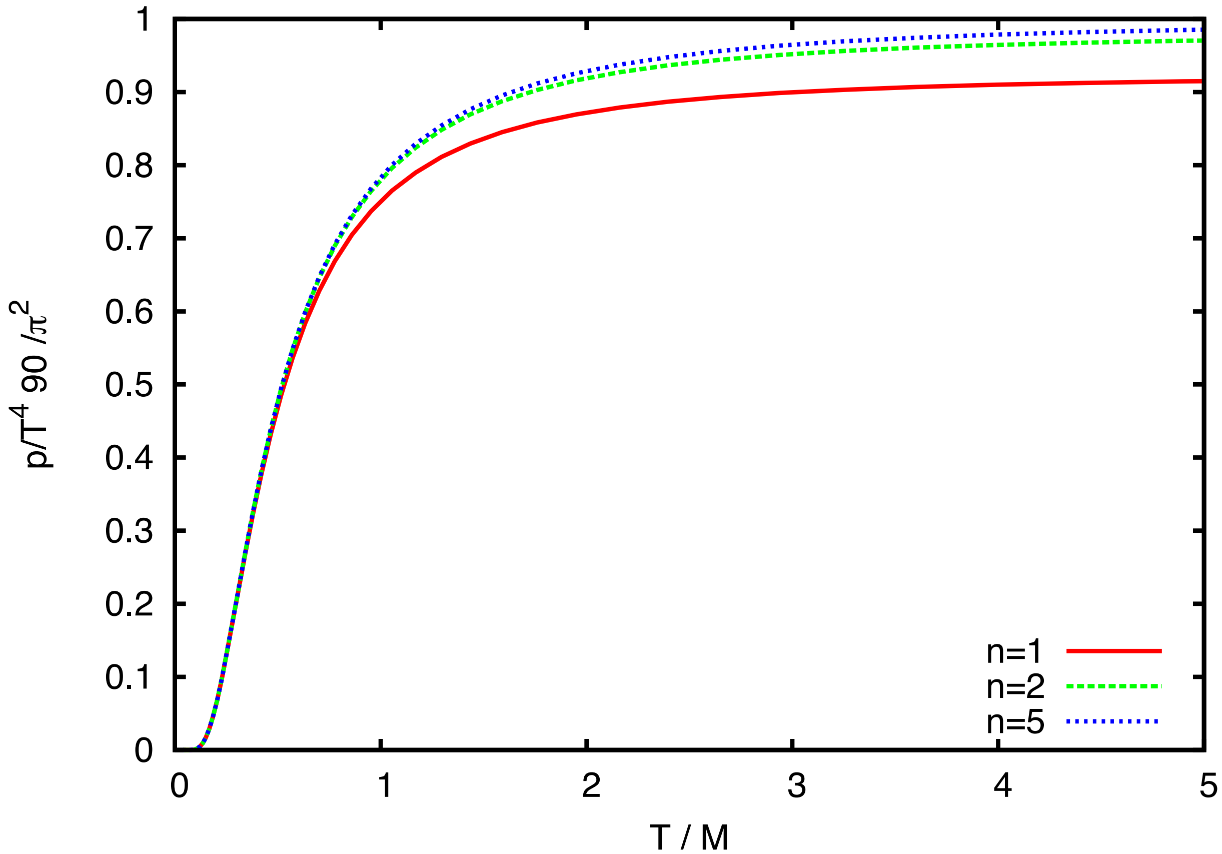


$S^1(L)\times\mathbb{R}^3$

-proper-time regularization
-Poisson resummation
-skip L-independent (div.) const.

$$P=\frac{1}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{m}{n\beta}\right)^2 K_{-2}(nLm)$$

a few terms are sufficient to reproduce the result of the usual grand canonical ensemble



Relativistic Fermi gas

- grand canonical ensemble $T = L^{-1}$

$$P = \tfrac{2}{3}\int d^3p \frac{p^2}{\omega(p)}(n_+(p)+n_-(p)) \qquad n_\pm(p) = \frac{1}{e^{L(\omega(p)\mp\mu)}+1} \qquad \omega(p) = \sqrt{p^2+m^2}$$

- energy density on $\mathbb{R}^2 \times S^1(L)$

$$e(L) = -2\int d^2p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty}\sqrt{m^2+p_\perp^2+(\omega_n+i\mu)^2} \qquad \omega_n = \frac{2n+1}{L}\pi$$

  - proper-time
  - Poisson resummation

$$P = -e(L) = -\frac{2}{\pi^2}\sum_{n=-\infty}^{\infty}\cos\left[nL(\tfrac{\pi}{L}-i\mu)\right]\left(\frac{m}{nL}\right)^2 K_{-2}(nLm)$$

- massles Dirac fermions: m=0
  - analytic continuationfor iμ $\longrightarrow$ x $\quad \sum_{n=1}^{\infty}(-)^n\frac{\cos(nx)}{n^4} = \tfrac{1}{48}\left[-\tfrac{7}{15}\pi^4+2\pi^2x^2-x^4\right]$

$$\boxed{P = \frac{1}{12\pi^2}\left[\frac{7}{15}\pi^4T^4+2\pi^2T^2\mu^2+\mu^4\right]}$$

equivalence of both approaches: analytic continuation

- proper-time regularization $$\sqrt{A} = \frac{1}{\Gamma(-\frac{1}{2})} \lim_{\Lambda\to\infty} \int_{1/\Lambda^2}^{\infty} d\tau \exp(-\tau A)$$

- Poisson resummation $$\sum_{n=-\infty}^{\infty} f(\omega_n) \equiv \int_{-\infty}^{\infty} dz f(z) \sum_{n=-\infty}^{\infty} \exp(inLz)$$

- analytic continuation for μ≠0 $$\sum_{n=1}^{\infty} (-)^n \frac{\cos(inx)}{n^4} = -\tfrac{1}{48}\left[\tfrac{7}{15}\pi^4 + 2\pi^2 x^2 + x^4\right]$$

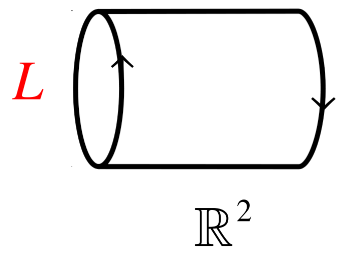
$ix \to x$ $$\sum_{n=1}^{\infty} (-)^n \frac{\cos(nx)}{n^4} = \tfrac{1}{48}\left[-\tfrac{7}{15}\pi^4 + 2\pi^2 x^2 - x^4\right]$$

  - Polylogarithm $$Li_s(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^s} \qquad -Li_s(-e^x) = \frac{1}{\Gamma(s)} \int_0^{\infty} dt \frac{t^{s-1}}{e^{t-x}+1}$$

- $$\boxed{\sum_{n=1}^{\infty} (-)^n \frac{\cos(inx)}{n^4} = -\tfrac{1}{12} \int_0^{\infty} dt\, t^3 \left[\frac{1}{e^{t-x}+1} + \frac{1}{e^{t+x}+1}\right]}$$

QCD at finite T

- Hamiltonian approach in Coulomb gauge on the partially compactified spatial manifold $\mathbb{R}^2 \times S^1(L)$



- variational solution of the Schrödinger equation for the vacuum
- finite temperature QCD is fully encoded in its vacuum

YMT at finite T

J. Heffner & H. R.
Phys.Rev.D91(2015)

Polyakov loop

H. R. & J. Heffner,
Phys.Rev.D88(2013)

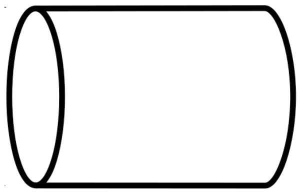
dual quark condensate

H. R. & P. Vastag
arXiv:1605.03740

The Polyakov loop - order parameter of confinement

$$P[A_0](\vec{x}) = \tfrac{1}{d_r} tr P \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

$T^{-1} = L$



Polyakov gauge $\partial_0 A_0 = 0, \quad A_0 = diagonal$

$SU(2):$ $\boxed{P[A_0](\vec{x}) = \cos(\tfrac{1}{2} A_0(\vec{x}) L)}$

$P[A_0] - unique\ function\ of\ A_0$

alternative order parameters of confinement

$$\langle P[A_0](\vec{x}) \rangle \qquad P\left[\langle A_0 \rangle\right](\vec{x}) \qquad \langle A_0(\vec{x}) \rangle$$

- *F.Marhauser and J. M. Pawlowski, arXiv:0812.11144*
- *J. Braun, H. Gies, J. M. Pawlowski, Phys. Lett. B684(2010)262*

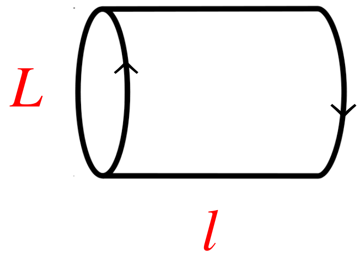
Effective potential of the order parameter for confinement

- background field calculation $a_0 = \langle A_0(\vec{x}) \rangle$ – const, diagonal (Polyakov gauge)
- effective potential $e[a_0] \longrightarrow \min \qquad \Rightarrow a_0 = \bar{a}_0$
- order parameter $\langle P[A_0] \rangle \approx P[\bar{a}_0]$

- ordinary Hamiltonian approach assumes Weyl gauge $A_0 = 0$
  - O(4)-invariance

- compactify (instead of time) one spatial axis to a circle of circumference L and interprete L^{-1} as temperature

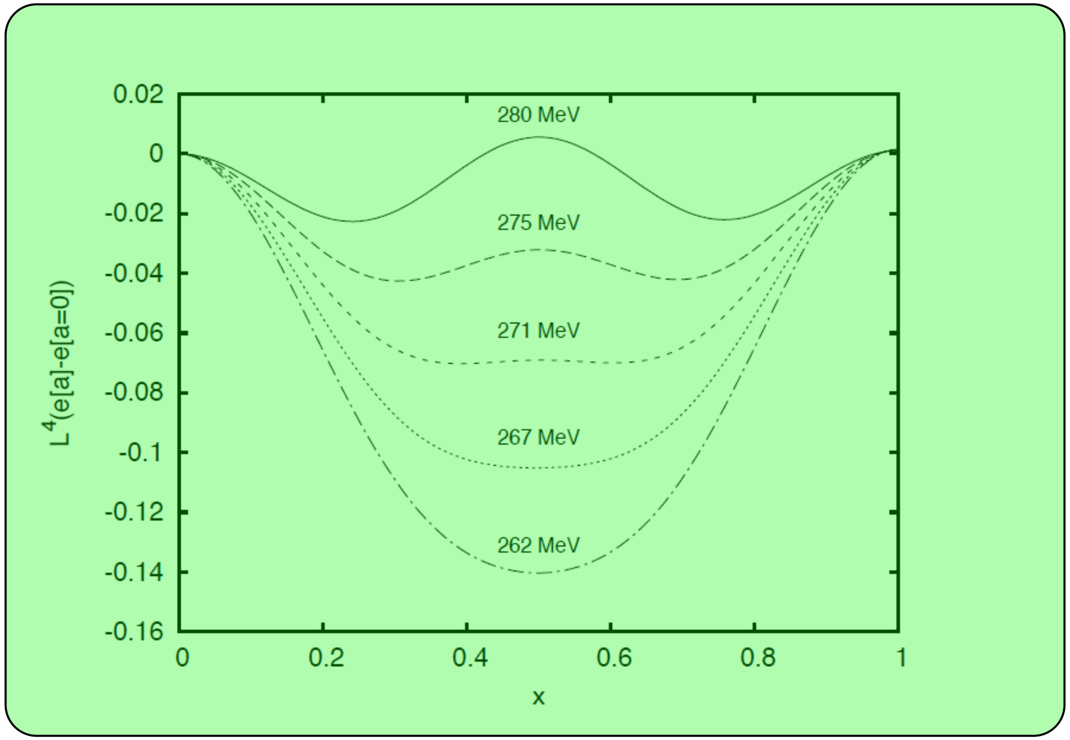
- Hamiltonian approach on $\mathbb{R}^2 \times S^1(L)$



- compactify x_3 – axis $\vec{a} = a\vec{e}_3$
- calculate the effective potential $\boxed{e[a]}$

The gluon effective potential SU(2)

variational calculation in Coulomb gauge



$$x = \frac{aL}{2\pi}$$

second order phase transition:

$$input: \quad M = 880\,MeV \qquad T_C \simeq 269\,MeV$$

H. Reinhardt & J. Heffner, Phys.Rev.D88(2013)

The effective potential for SU(3)

SU(3)-algebra consists of 3 SU(2)-subalgebras characterized by the 3 non-zero positive rooots

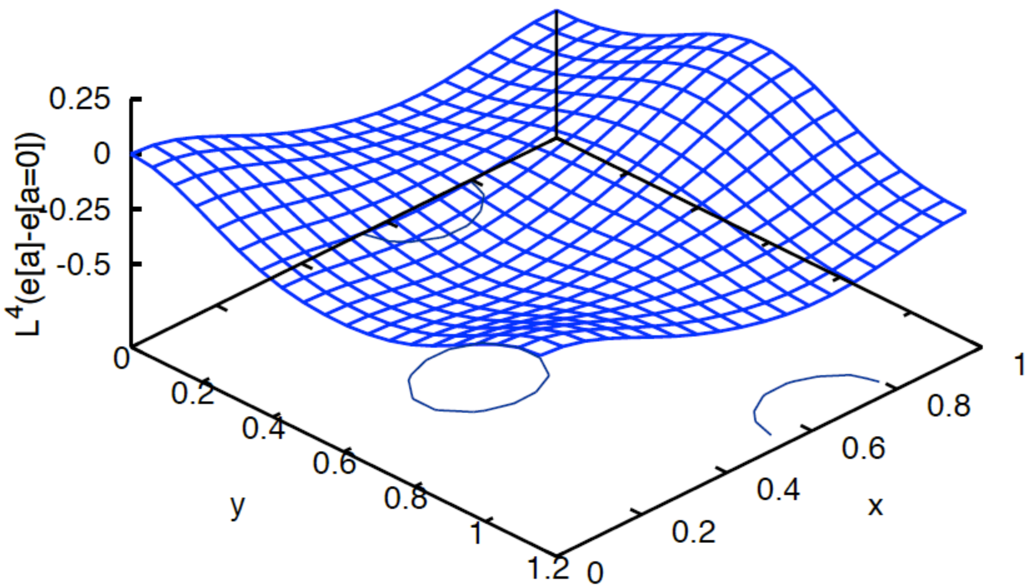
$$\sigma = (1,0), \quad \left(\frac{1}{2}, \frac{1}{2}\sqrt{3}\right), \quad \left(\frac{1}{2}, -\frac{1}{2}\sqrt{3}\right)$$

$$e_{SU(3)}[a] = \sum_{\sigma>0} e_{SU(2)(\sigma)}[a]$$

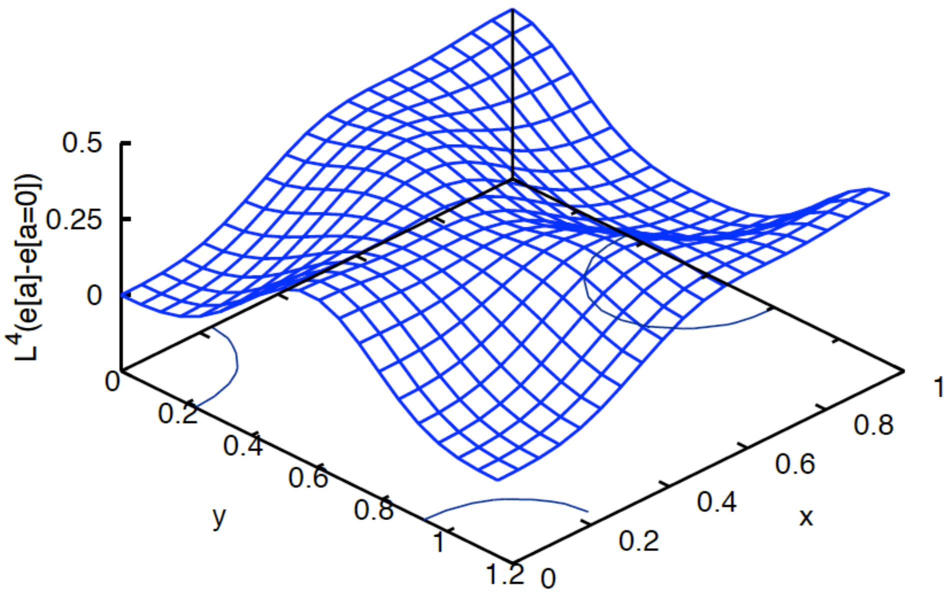
The full effective potential for SU(3)

variational calculation in Coulomb gauge

$T < T_C$



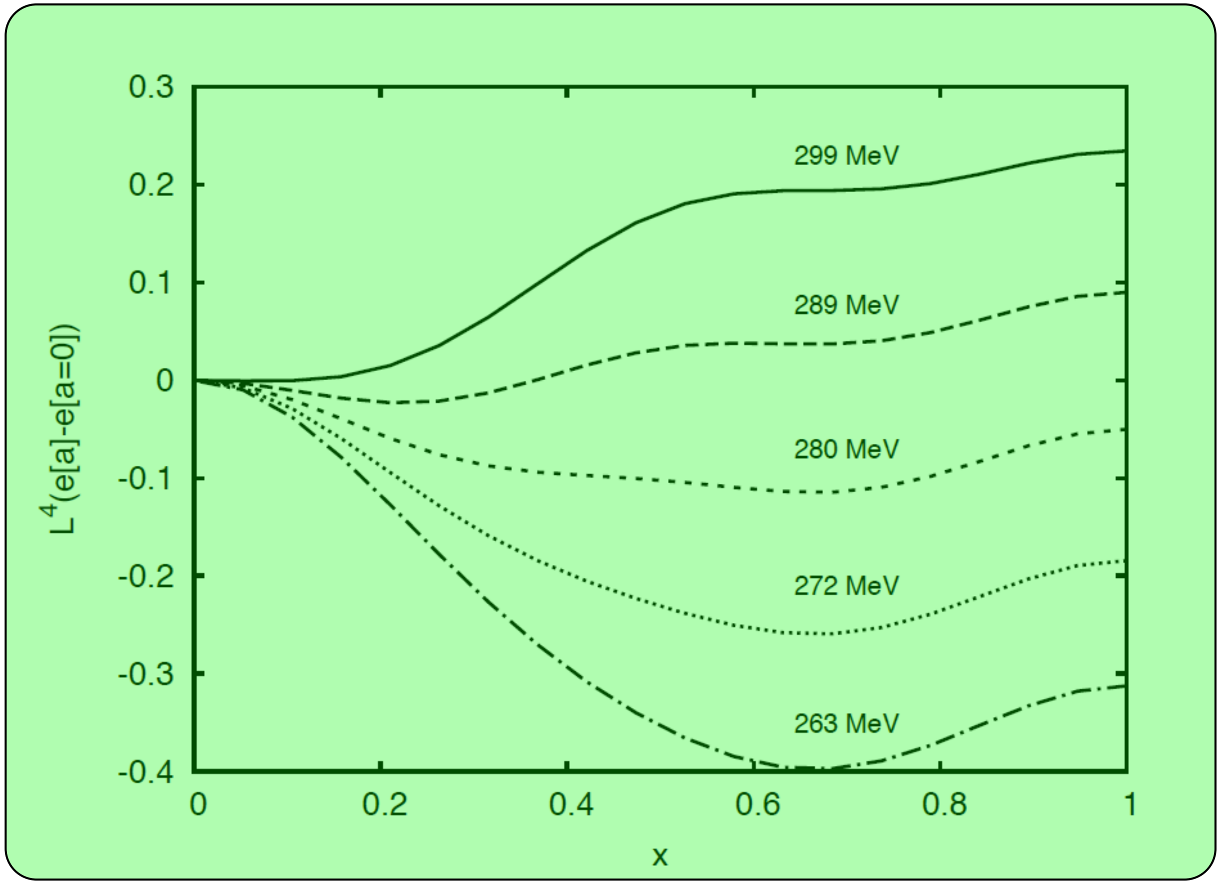
$T > T_C$



$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi}$$

H. Reinhardt & J. Heffner, Phys.Rev.D88(2013)

Polyakov loop potential for SU(3)

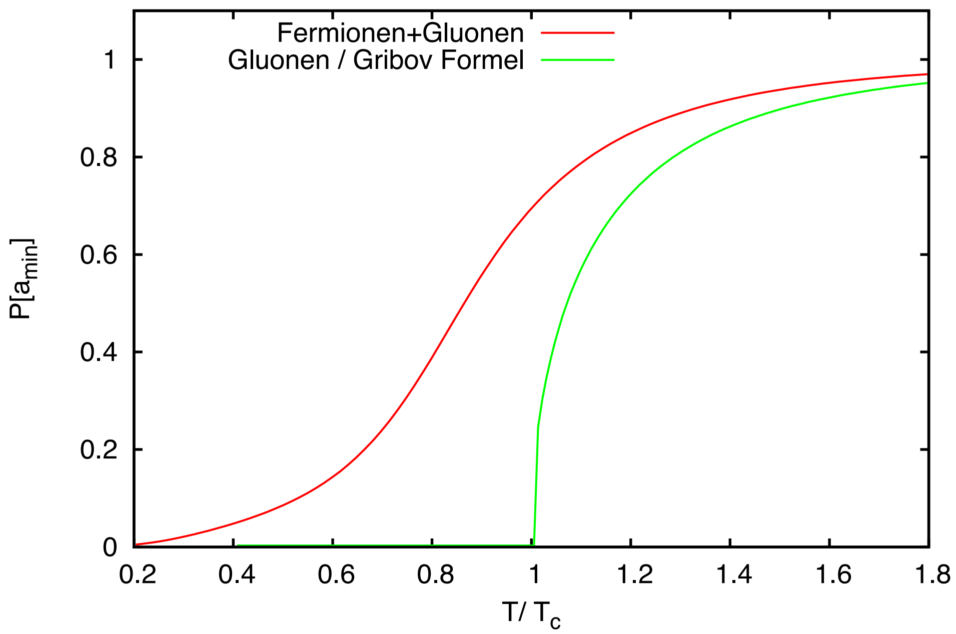


$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi} = 0$$

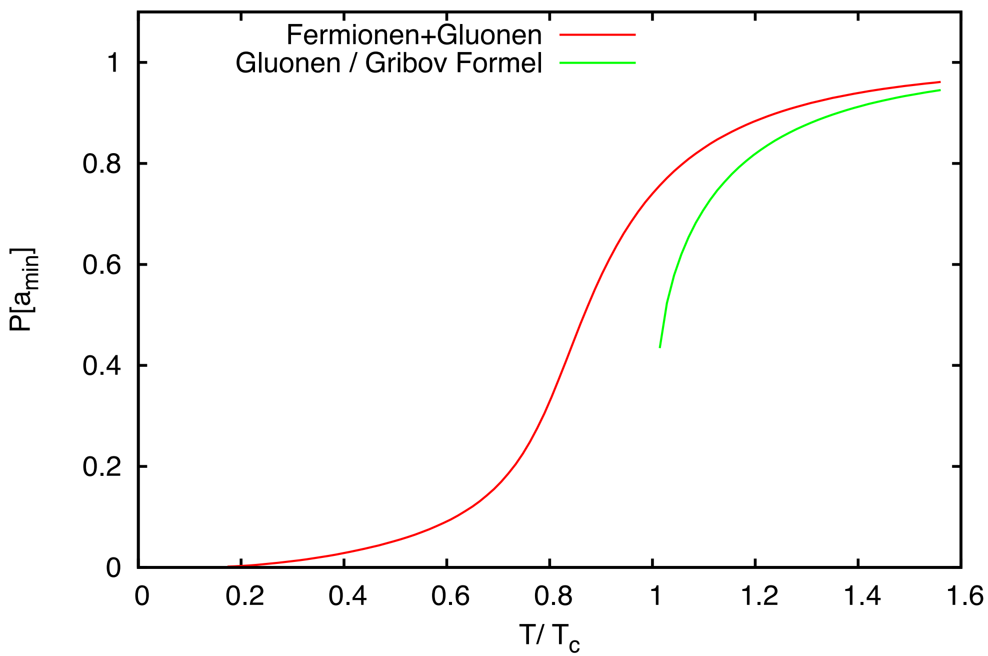
$$input: \ SU(2) - data:$$
$$M = 880 MeV$$

$$T_C = 283 MeV$$

The Polyakov loop



SU(2)



SU(3)

J. Heffner, H. Reinhardt & P. Vastag, to be published

center symmetry

DECONFINEMENT PHASE TRANSION:

confined phase: center symmetry
deconfined phase: center symmetry broken

any observable transforming non-trivially under the center may serve as order parameter for confinement

prototype: Polyakov loop

$$P[A_0](\vec{x}) = \tfrac{1}{d_r} tr P \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

dual quark condensate -dressed Polyakov loop

Gattringer, PRL. 97(2006)
Synatschke, Wipf, Wozar, Phys. Rev. D75(2007)
....

lattice: Bilgici, Bruckmann, Gattringer, Hagen, PR D77(2008)...
FRG: Braun, Haas, Marhauser, Pawlowski, PRL 106(2011)
DSE: Fischer, Maas, Müller, Eur. Phys. J. 68(2010) ...

Hamiltonian approach:
H. R. & P. Vastag, arXiv:1605.03740

dual quark condensate -dressed Polyakov loop

$$\Sigma_n = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-in\varphi} \left\langle (\bar{q}q)_\varphi \right\rangle \qquad q(L) = e^{i\varphi} q(0)$$

Σ_n *loops winding n-times around the compact time axis*

Σ_1 *dressed Polyakov loop*

Gattringer
PRL97(2006)

imaginary chemical potential : $\mu = i\dfrac{\pi - \varphi}{L}$

compactified 3-axis potential : $p_3 = \Omega_n + i\mu = \dfrac{2\pi n + \varphi}{L}$

Dual quark condensate in the Hamiltonian approach in ∂A=0

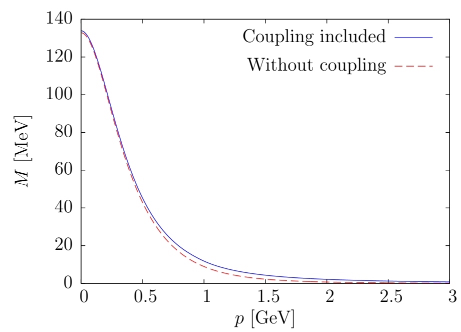
H. R. & P. Vastag, arXiv:1605.03740

$$\Sigma_n = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-in\varphi} \left\langle (\bar{q}q)_\varphi \right\rangle \qquad q(\beta) = e^{i\varphi} q(0)$$

after Poisson resummation

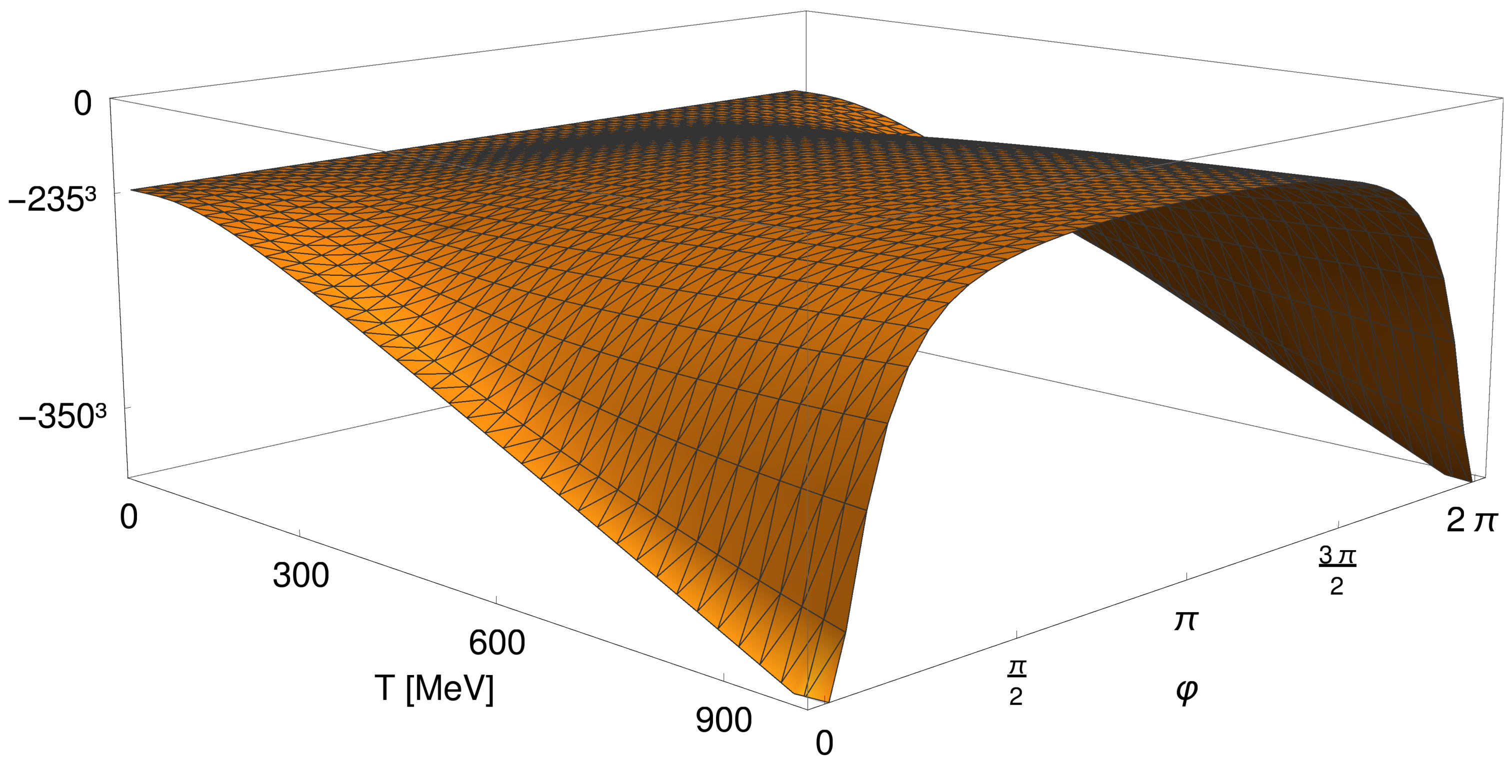
$$\Sigma_n = (-)^{n+1} \frac{2N_C}{2\pi^2} \int_0^\infty dp\, p \frac{\sin(nLp)}{nL} \frac{M(p)}{\sqrt{p^2 + M^2(p)}}$$

effective quark mass $M(p)$

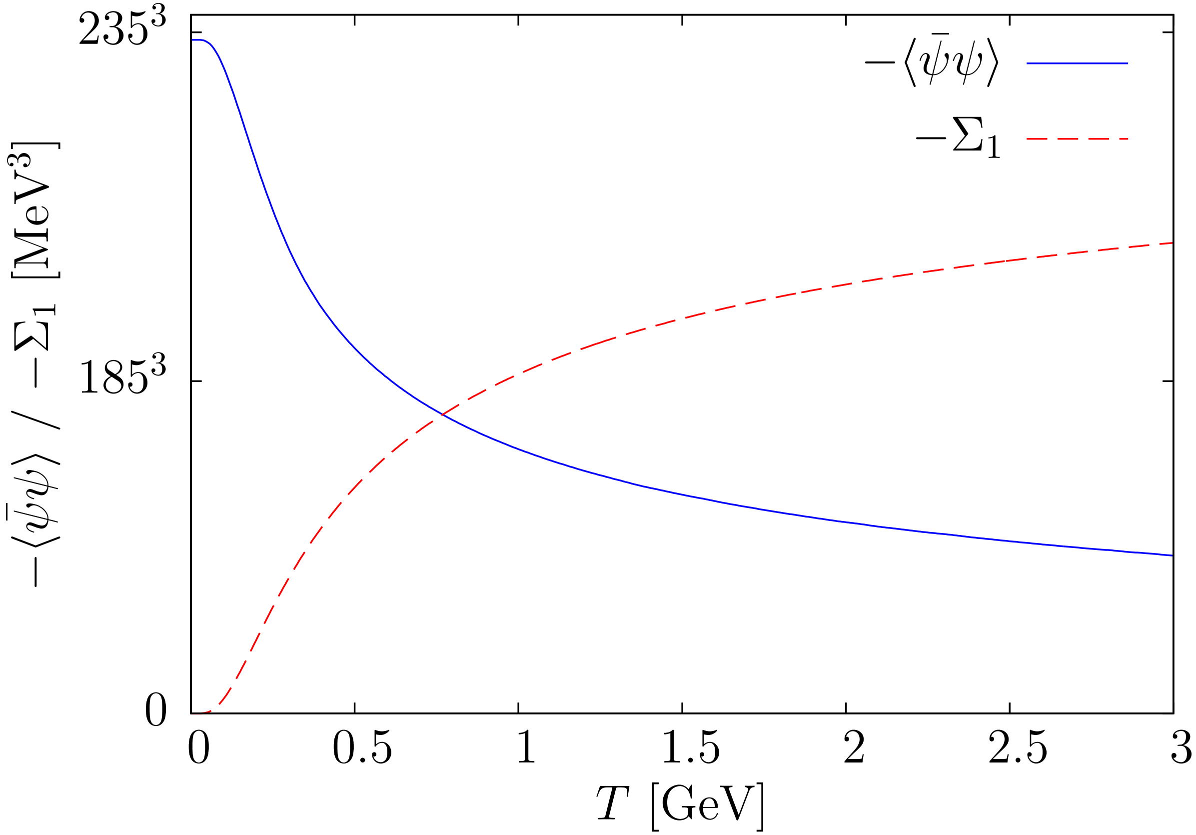


D. Campagnari , E. Ebadati, H.R. and P: Vastag, arXiv:1608.06820

quark condensate $\langle (\bar{q}q)_\varphi \rangle$



chiral & dual condensate



$$\sigma_C = 2.5\sigma \quad (T_{PC})_\chi \simeq 168\,MeV \quad (T_{PC})_C \simeq 196\,MeV$$

Conclusions

- Hamiltonian approach to QCD in Coulomb gauge at T=0
  - Coulomb string tension - spatial string tension
  - no chiral symmetry breaking without Coulomb confinement
- QCD at finite temperature
  - compactification of a spatial dimension $R^2 \times S^1$
    - effective potential of the Polyakov loop
      - deconfinement phase transition in YMT
      - SU(2): 2.order
      - SU(3): 1.order
    - inclusion of quarks:
      - deconfinement phase transition is turned into a crossover
    - pressure & energy density
    - chiral & dual quark condensate

Thanks for your attention

Conclusions

- variational approach to the Hamiltonian formulation of YMT in Coulomb gauge
- Gribov-Zwanziger confinement scenario at work
  - inverse ghost form factor = dielectric function
  - Coulomb string tension - spatial string tension
- connection to the alternative confinement pictures:
  - magnetic monopoles
  - center vortices
- Hamiltonian approach to finite temperature YMT
  - grand canonical ensemble
  - compactification of a spatial dimension $R^2 \times S^1$
    - effective potential of the Polyakov loop
      - deconfinement phase transition
      - SU(2): 2.order
      - SU(3): 1.order
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      - deconfinement phase transition is turned into a crossover
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