A Path of Complete Description of Hadron Spectrum

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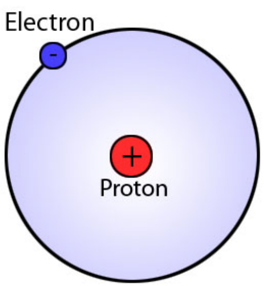
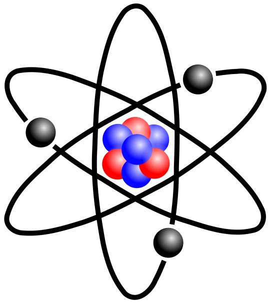
Fundamental Forces versus **Bound States:**

QED



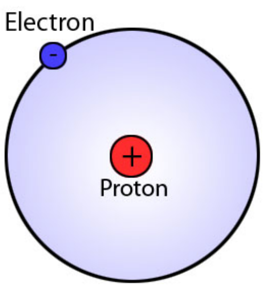


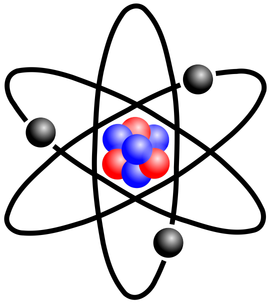
hydrogen

Fundamental Forces versus **Bound States:**

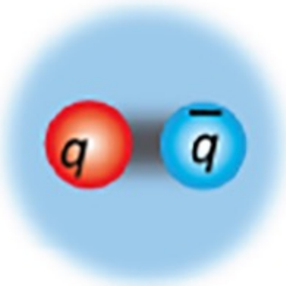
QED

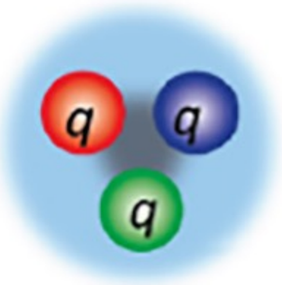
QCD





hydrogen

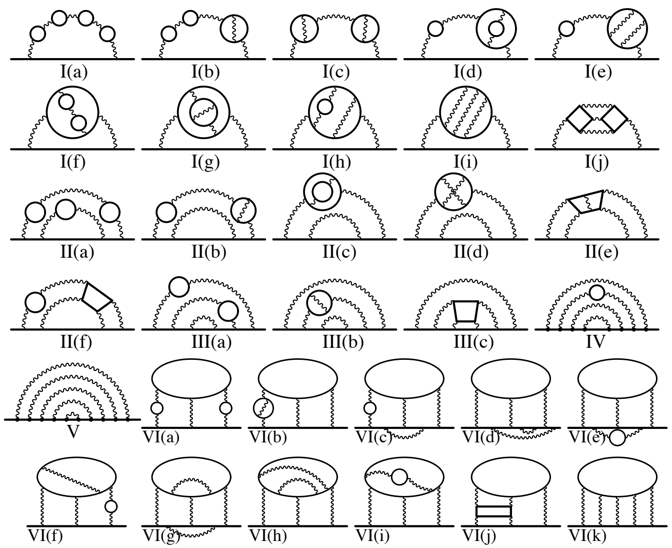




meson

Fundamental Forces versus Bound States:

Perturbative



$$\alpha^{-1} = 137.035\ 999\ 174\ (35)$$

QED fine-structure constant

Fundamental Forces versus Bound States:

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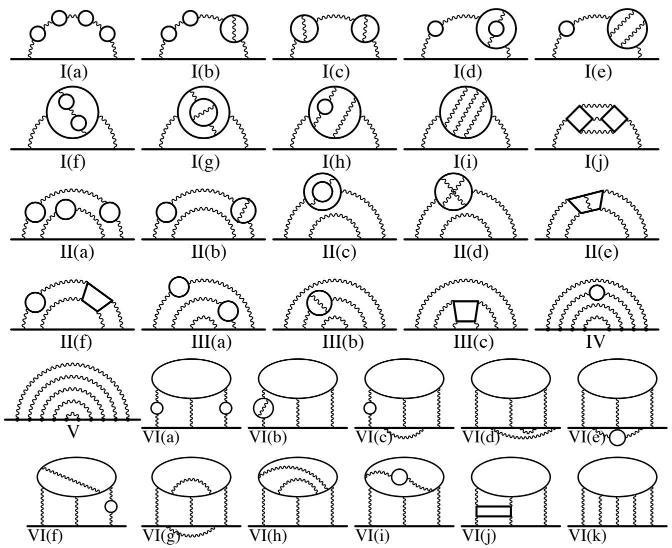

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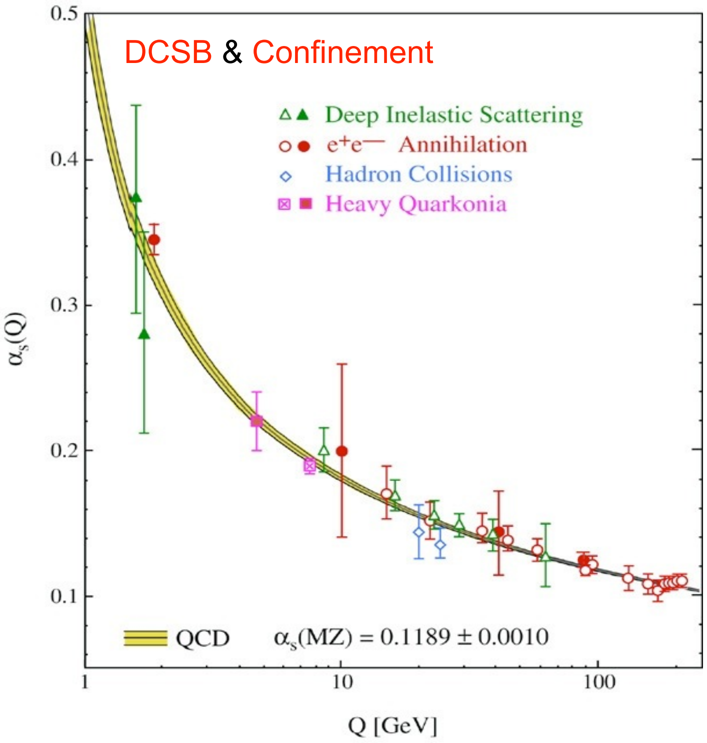
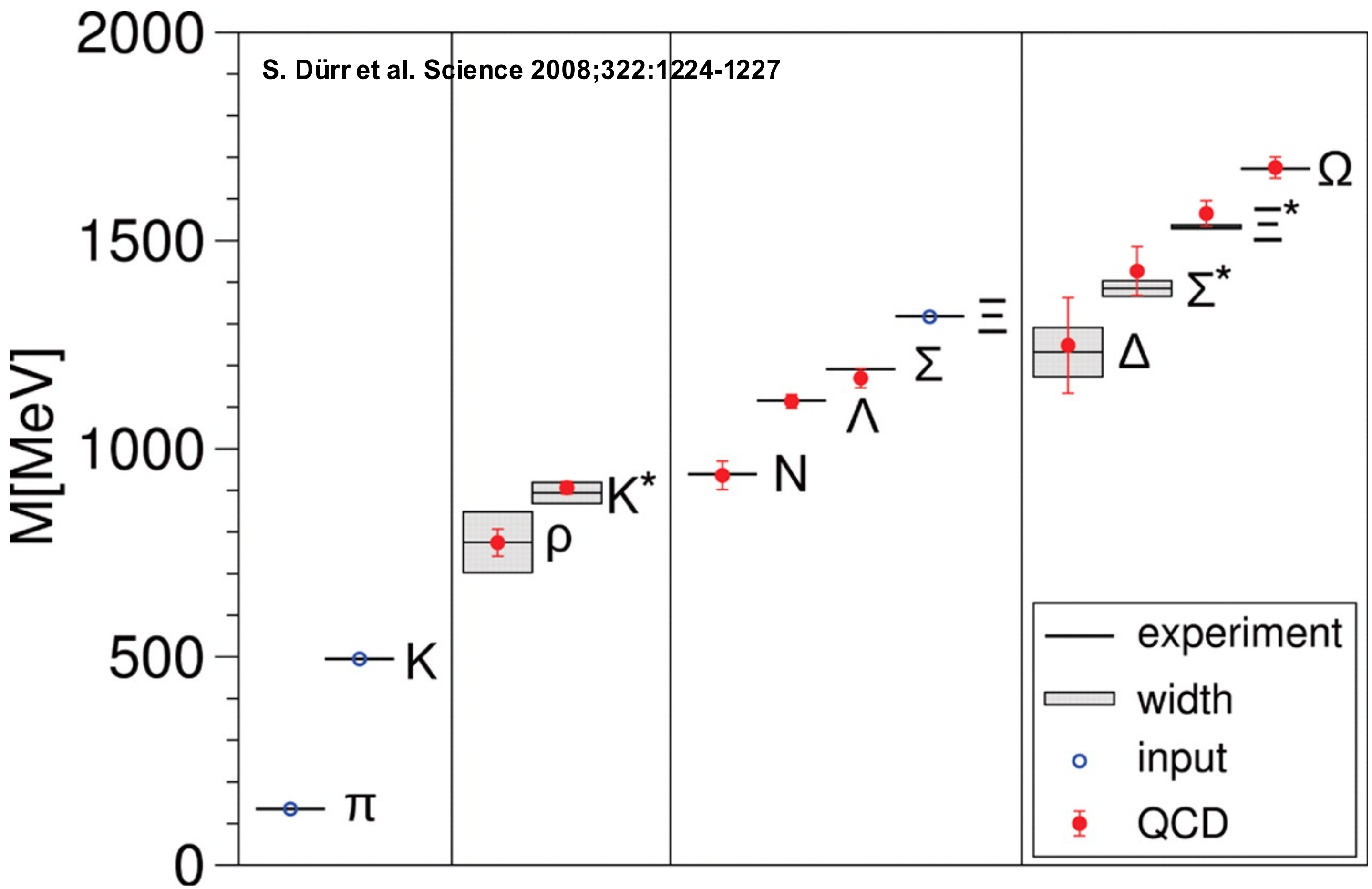
QED fine-structure constant

Non-perturbative



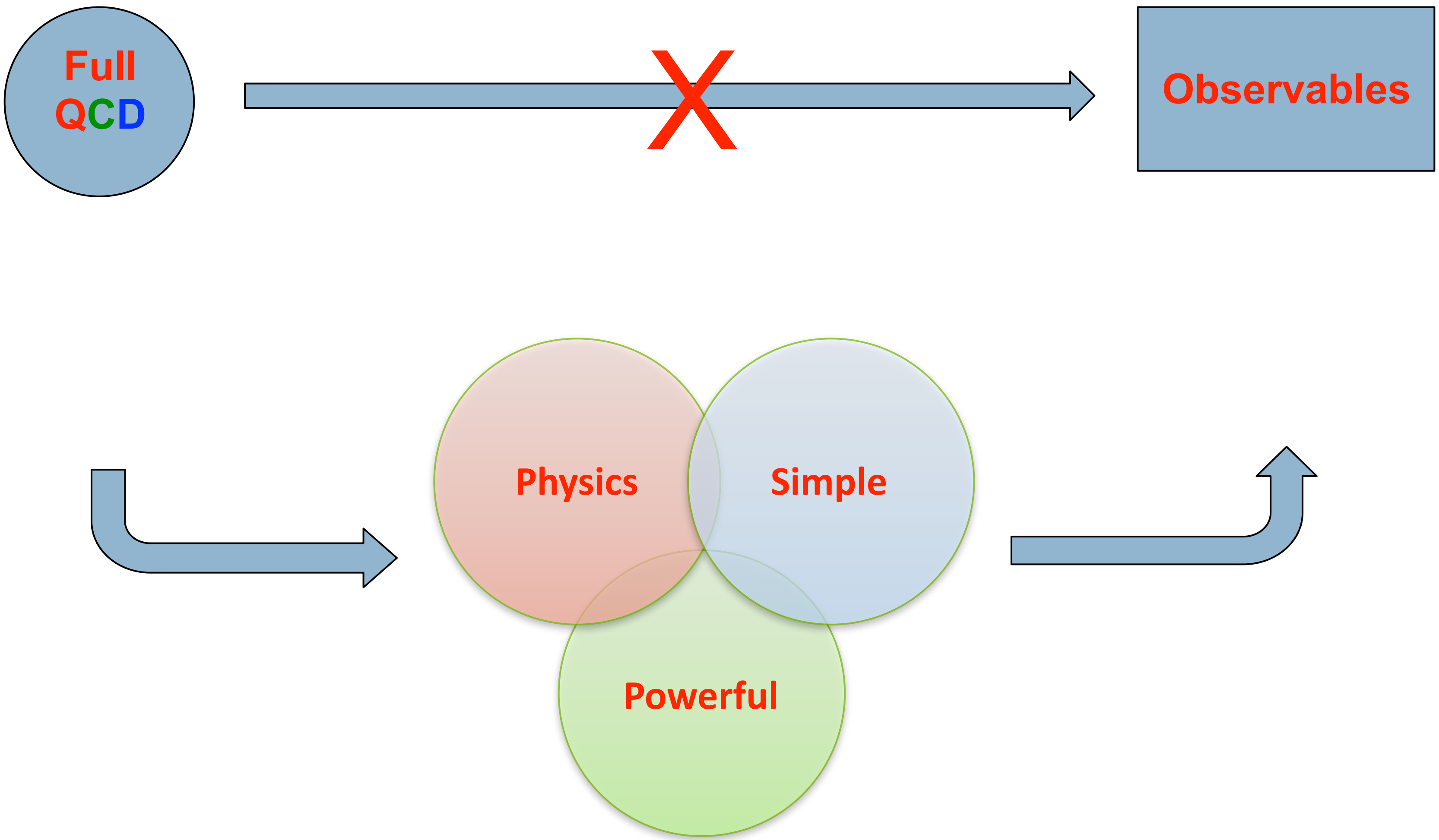
QCD running coupling constant

Fundamental Forces versus **Bound States:** Lattice QCD



The light hadron spectrum of QCD

Fundamental Forces versus Bound States: QCD approaches



Dyson-Schwinger Equations: EoM of Green functions

Classical Mechanics | Quantum Field Theory

Principle of Least Action

$$\frac{\delta S[q]}{\delta q} = 0$$

$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

Equations of Motion (EoM)

Euler-Lagrange Equation | Dyson-Schwinger Equation

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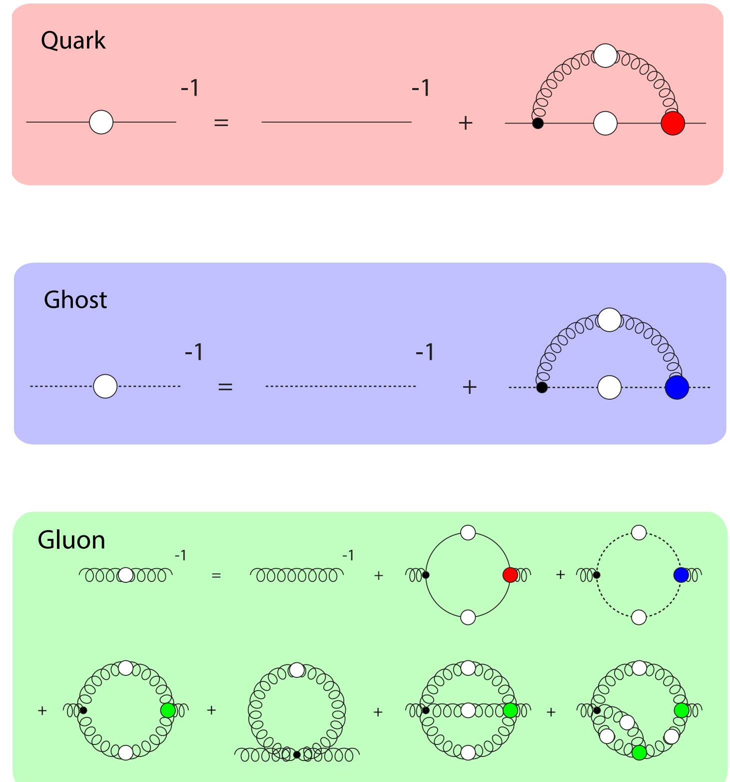
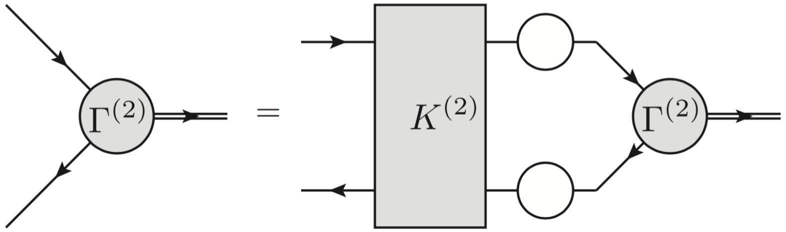
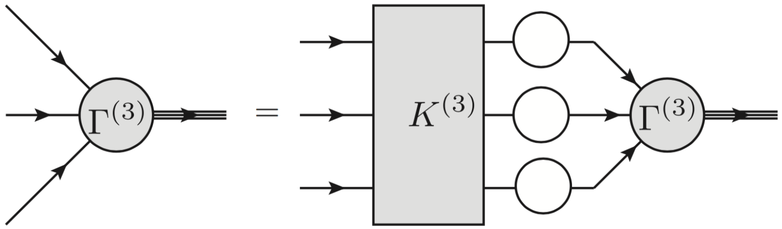


- ✦ Complicated integral equations
- ✦ Coupled tower of all equations

Dyson-Schwinger Equations: Equations for hadrons





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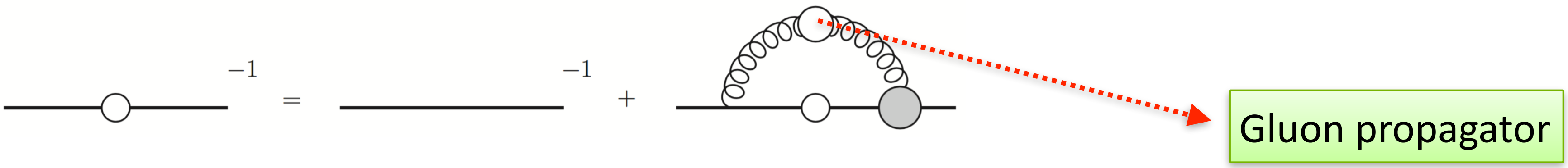

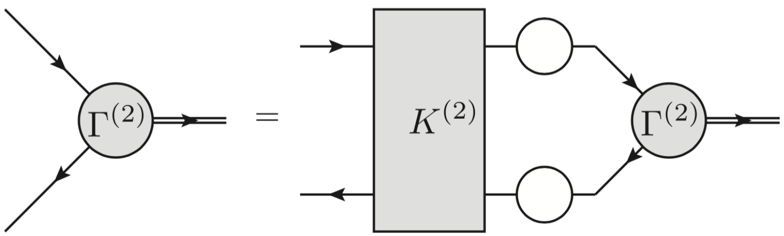

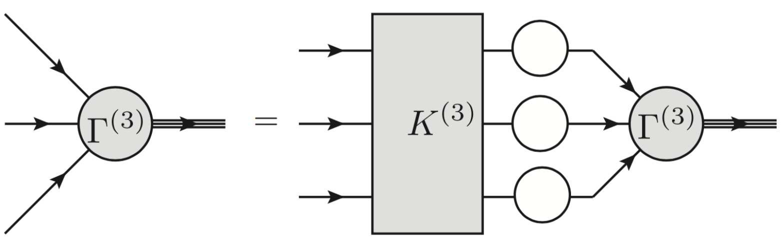

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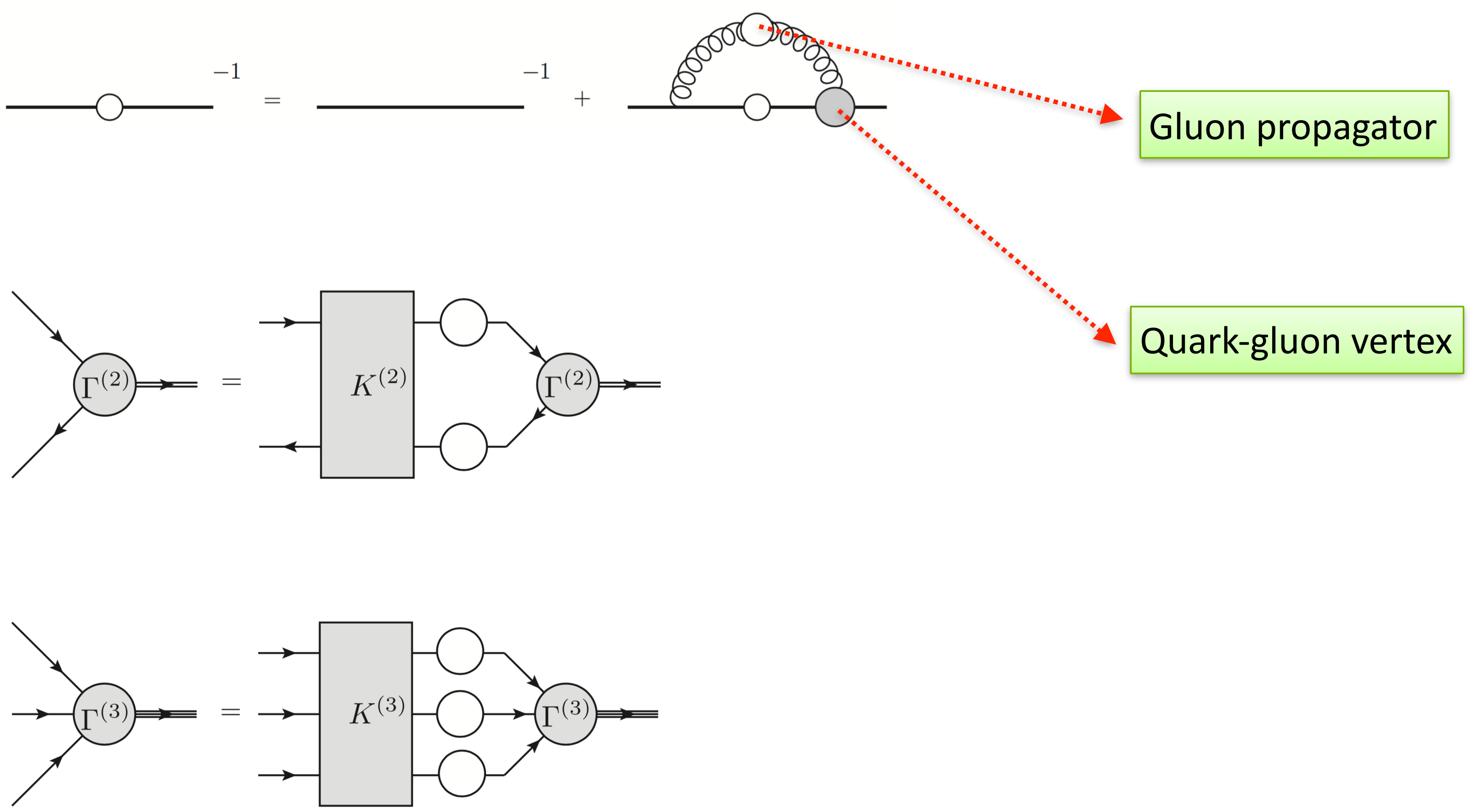

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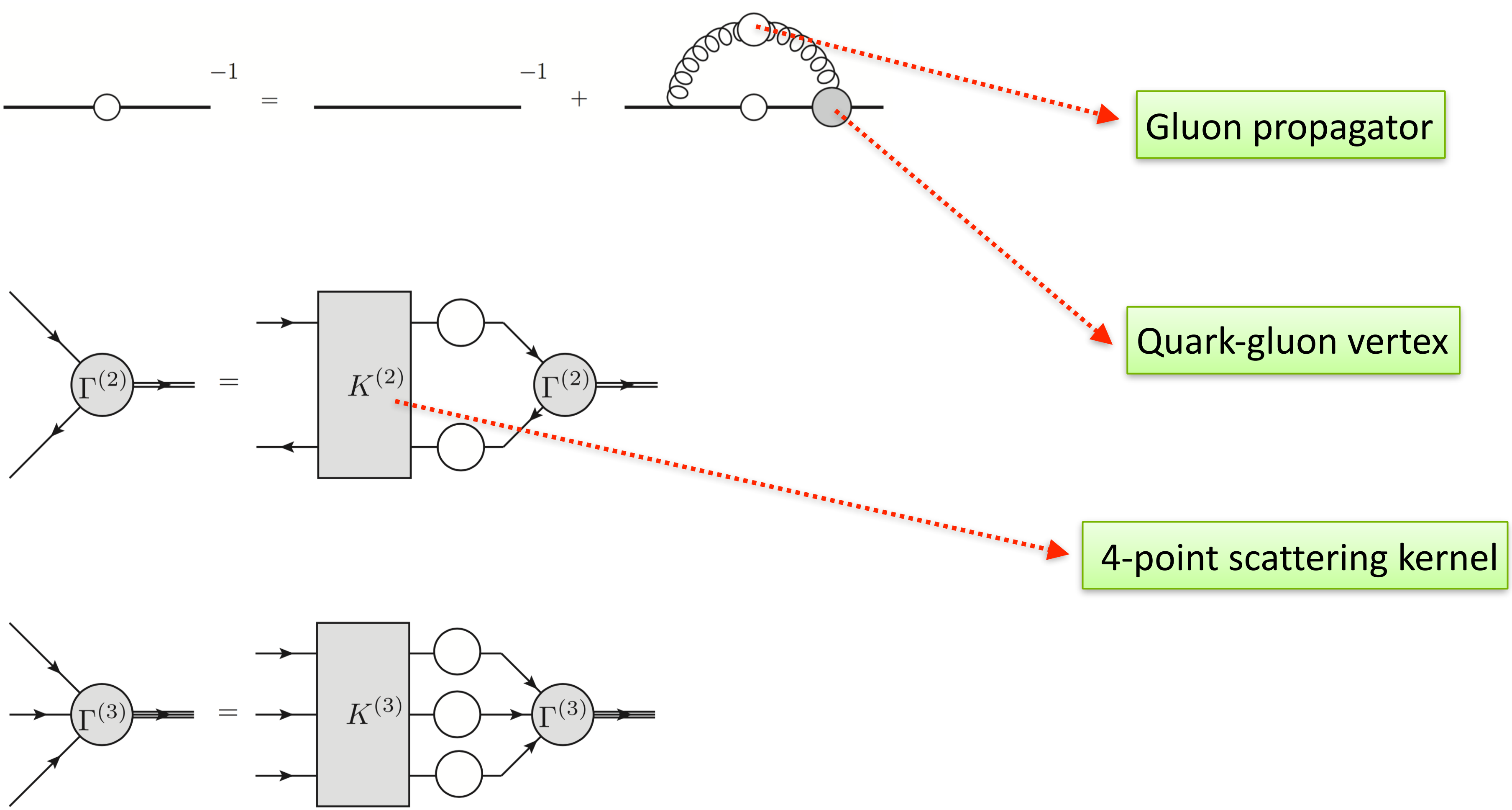

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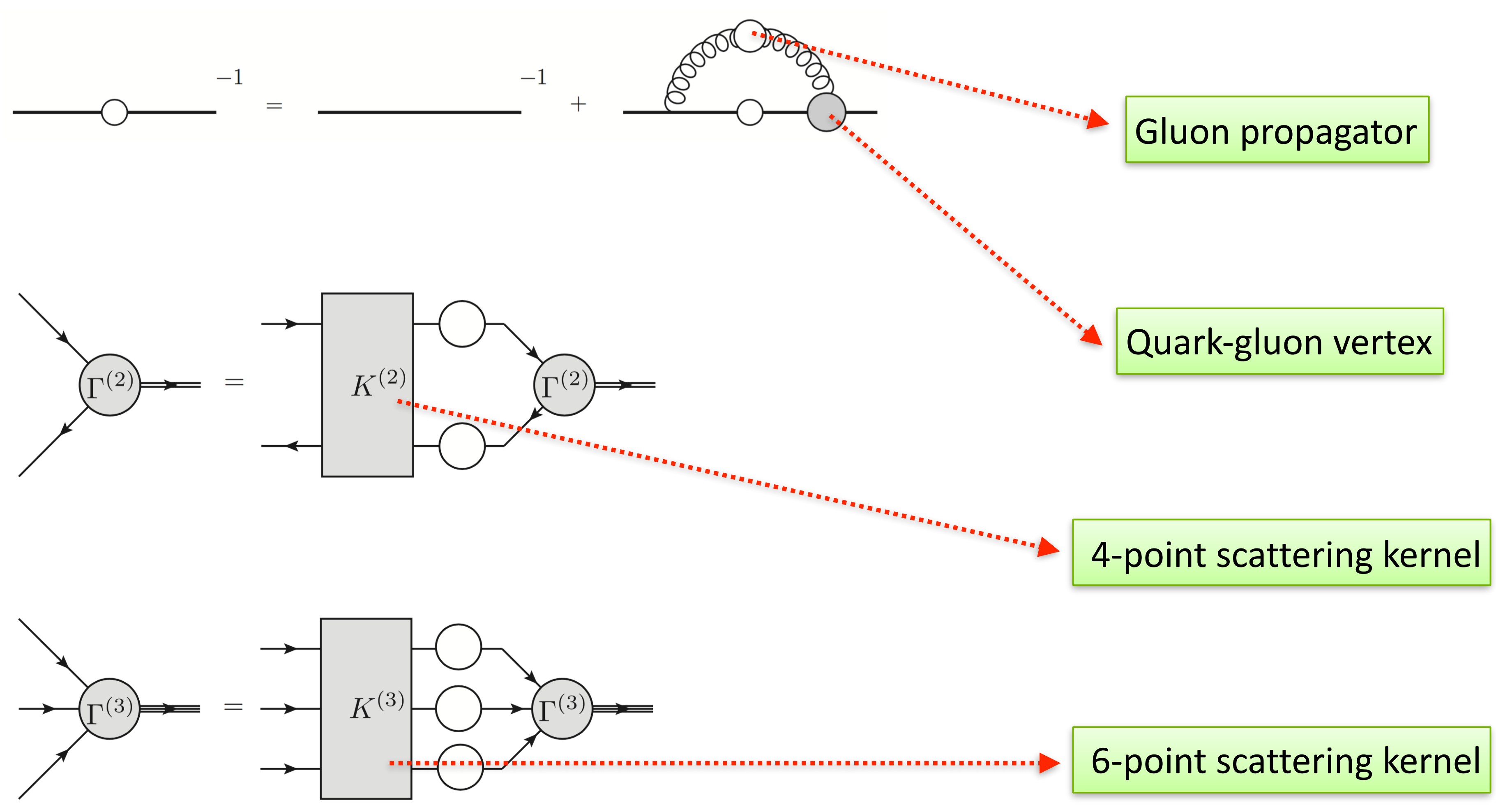


Dyson-Schwinger Equations: The simplest (RL) approximation

I. Gluon propagator

II. Quark-gluon vertex

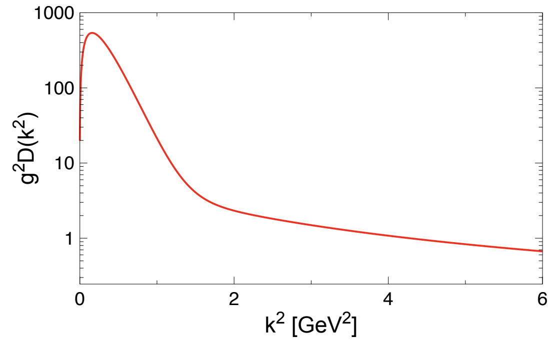
III. Scattering kernels

Dyson-Schwinger Equations: The simplest (RL) approximation

I. Gluon propagator

Maris-Tandy model

$$g^2 D_{\mu\nu}^{ab}(k) = \delta_{ab} D_{\mu\nu}^{\text{free}}(k) \mathcal{G}(k^2)$$



II. Quark-gluon vertex

III. Scattering kernels

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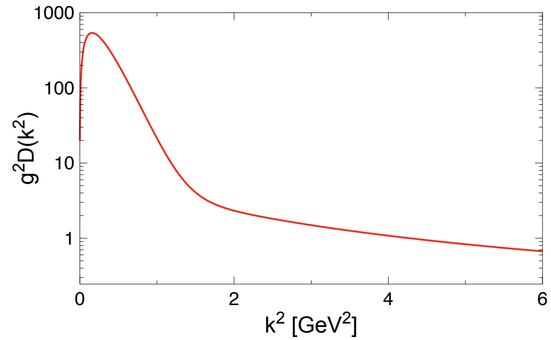
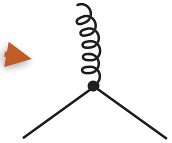


II. Quark-gluon vertex

rainbow approximation

$$\Gamma_\nu^a(k,p) = \frac{\lambda^a}{2}\gamma_\nu$$



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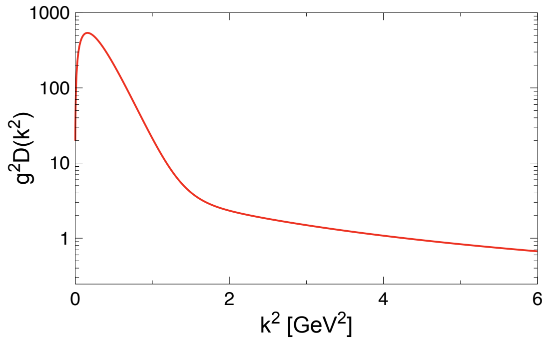

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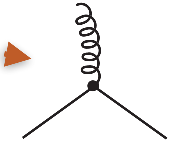
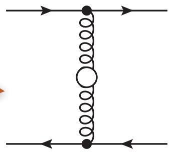


III. Scattering kernels

ladder approximation

$$\mathcal{K}^{ab}_{\mu\nu}(k,q,P) = g^2 D^{ab}_{\mu\nu}(k) \left[\frac{\lambda^a}{2}\gamma_\mu\right] \left[\frac{\lambda^b}{2}\gamma_\nu\right]$$



Dyson-Schwinger Equations: The simplest (RL) approximation

I. Gluon propagator — Maris-Tandy model

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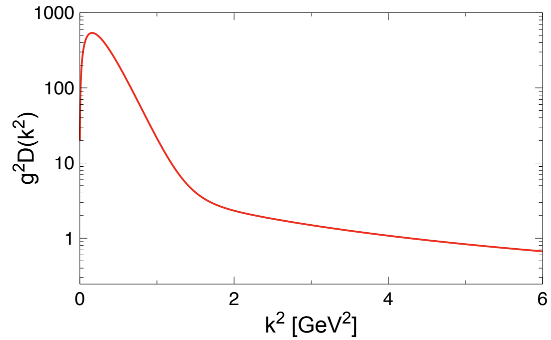
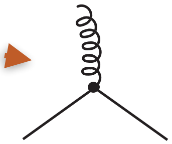


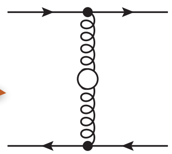
II. Quark-gluon vertex — rainbow approximation

$$\Gamma^a_\nu(k,p) = \frac{\lambda^a}{2}\gamma_\nu$$

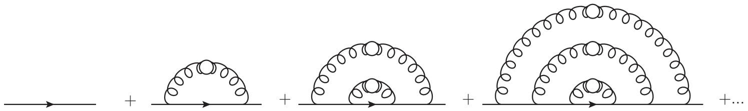


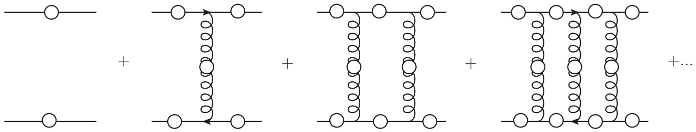
III. Scattering kernels — ladder approximation

$$\mathcal{K}^{ab}_{\mu\nu}(k,q,P) = g^2 D^{ab}_{\mu\nu}(k)\left[\frac{\lambda^a}{2}\gamma_\mu\right]\left[\frac{\lambda^b}{2}\gamma_\nu\right]$$



For example, $S(p)$ and $G^{(4)}(k,q;P)$:





Rainbow-Ladder truncation: T = 0

✦ Global properties: mass spectra, decay constants, radii, and etc.

Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle^0_\mu$	(0.236 GeV)^3	(0.241†)^3
m_π	0.1385 GeV	0.138†
f_π	0.0924 GeV	0.093†
m_K	0.496 GeV	0.497†
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6\ s^{-1}$	7.38
$\Gamma(K_{\mu 3})$	$5.2 \cdot 10^6\ s^{-1}$	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
$m_{K^\star}$	0.892 GeV	0.936
$f_{K^\star}$	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^\star K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^\star K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^\star K\gamma}/m_K)^0$	1.28	1.19

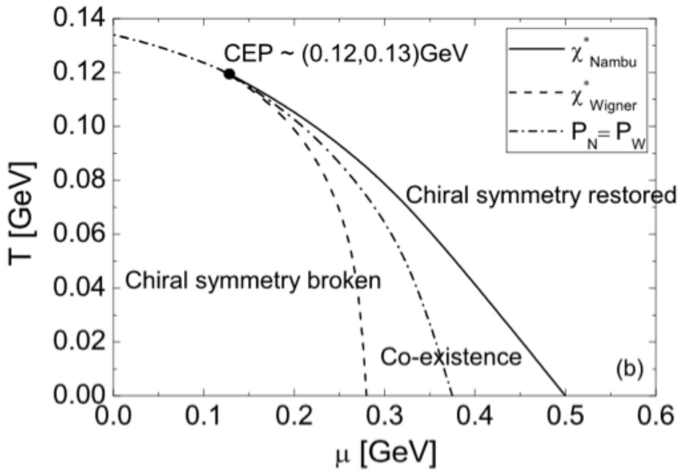
Scattering length (PM, Cotanch, PRD66, 116010)

a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

Tandy @ Beijing Lectures 2010

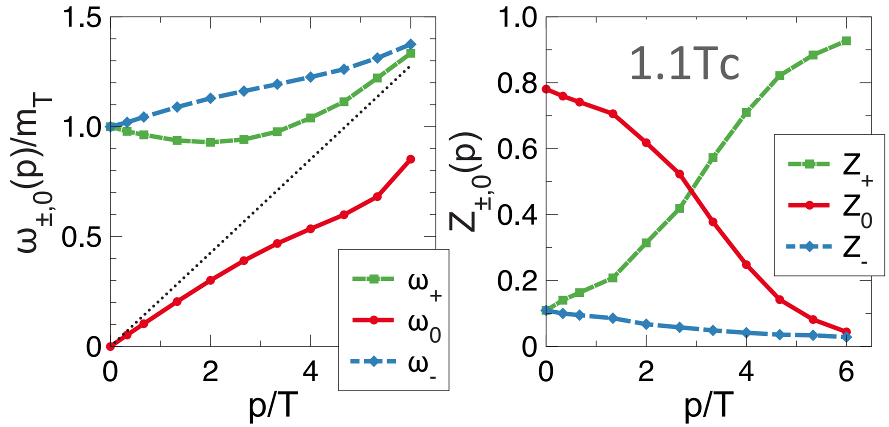
Rainbow-Ladder truncation: T > 0

✦ QCD phase diagram



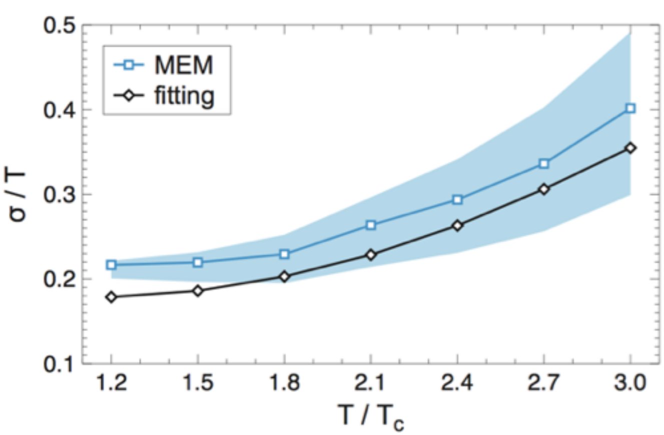
Qin et. al., PRL 106, 172301 (2011)

✦ sQGP collective excitations



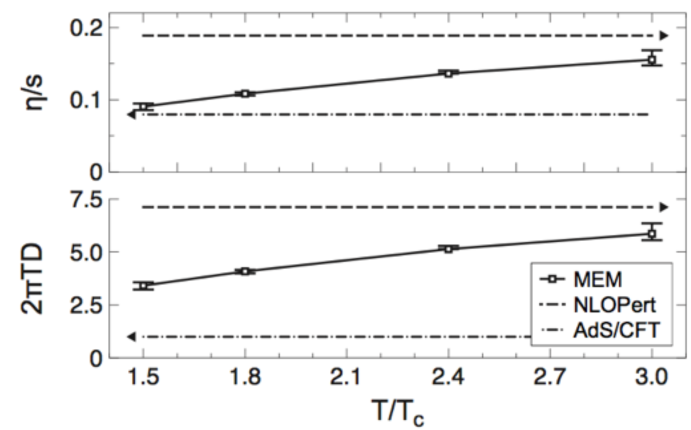
Qin et. al., PRD 84, 014017 (2011)

✦ QGP electrical conductivity



Qin, PLB 742, 358 (2015)

✦ QGP viscosity



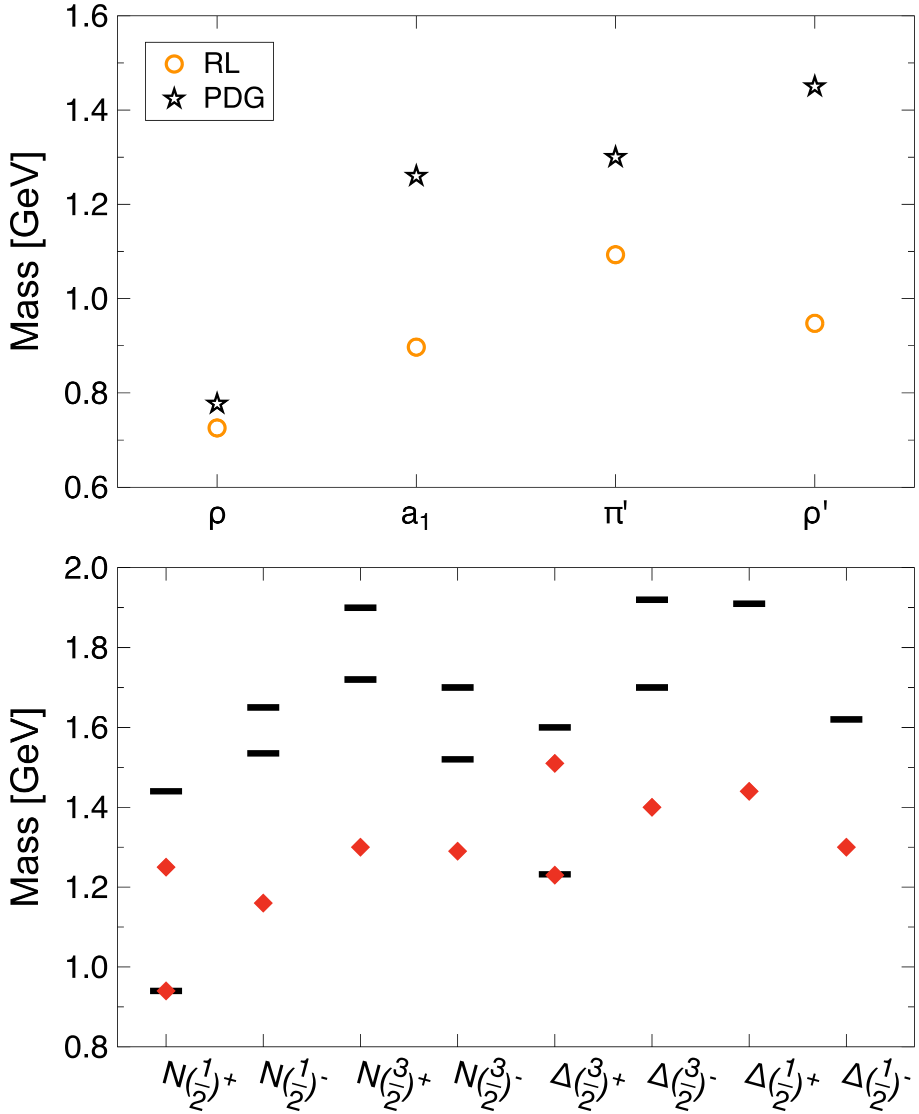
Qin et. al., PLB 734, 157 (2014)

Rainbow-Ladder truncation: Failures

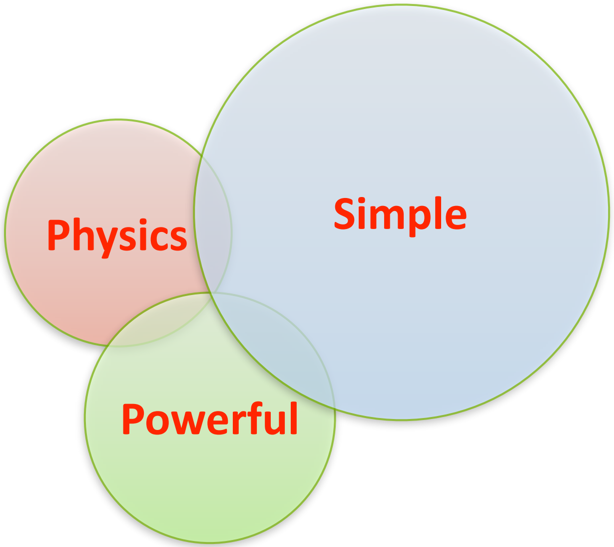
✦ Heavy ground states: light, e.g., rho-a_1 mass splitting;

✦ Radial excitation states: light, e.g., pion', rho', excited baryons;

✦ Hadron spectrum: systematically wrong ordering and magnitudes.



Rainbow-Ladder



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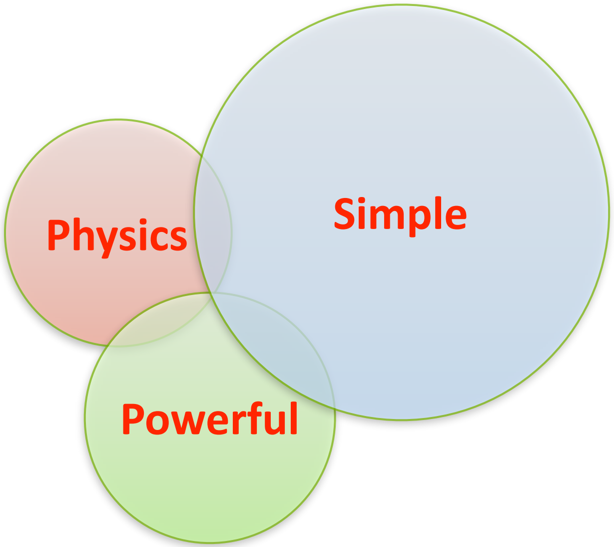
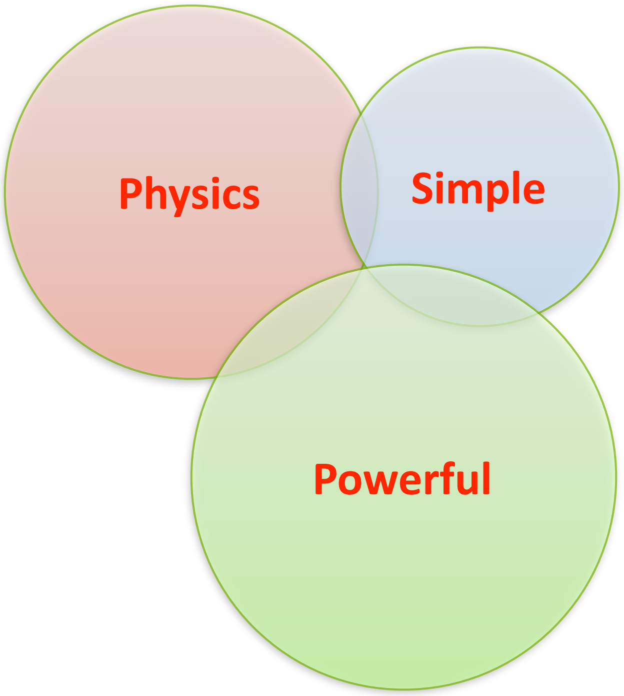


Beyond Rainbow-Ladder



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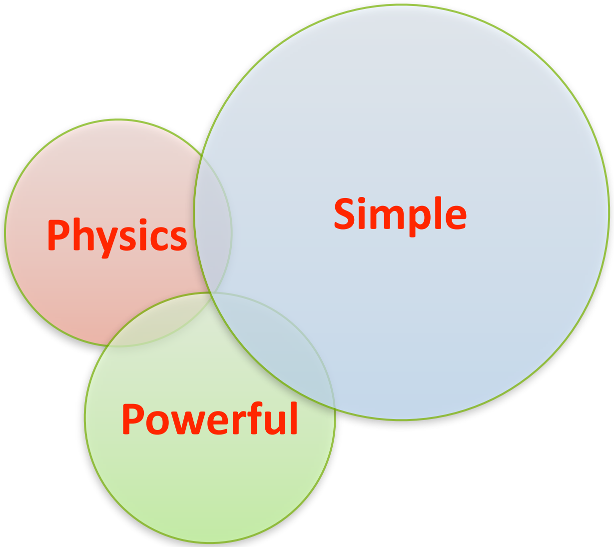

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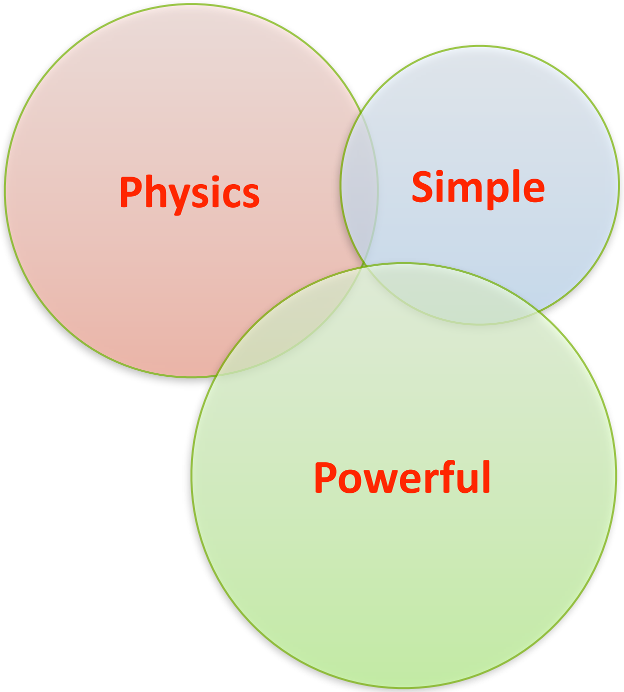


I. Dynamically massive gluon

II. DCSB in quark-gluon vertex

III. Symmetries of the kernels

IV. Meson cloud and diquark

I. Dynamically massive gluon: Lattice QCD

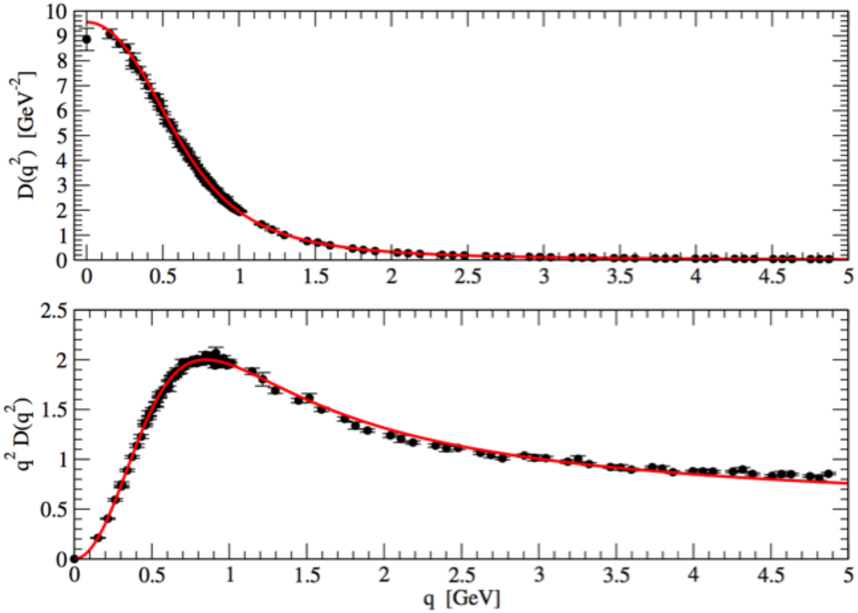
✦ In Landau gauge (a fixed point of the renormalization group):

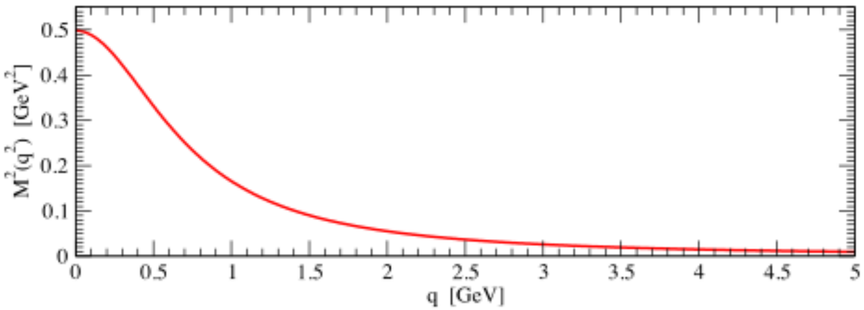
$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})$$

✦ Modeling the dress function:
gluon mass scale + effective running coupling constant

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)},$$

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2},$$





O. Oliveira et. al., J.Phys. G38, 045003 (2011)

I. Dynamically massive gluon: Phenomenological model

Qin et. al., PRC 84, 042202R (2011)

- Model the gluon propagator as two parts: Infrared + Ultraviolet, i.e., an expansion of delta function + a form of one-loop perturbative calculation.

$$\delta^4(k) \overset{\omega\sim 0}{\approx} \frac{1}{\pi^2}\frac{1}{\omega^4}e^{-k^2/\omega^2} \qquad \mathcal{G}(s) = \frac{8\pi^2}{\omega^4}De^{-s/\omega^2} + \frac{8\pi^2\gamma_m\mathcal{F}(s)}{\ln\left[\tau + \left(1 + s/\Lambda_{QCD}^2\right)^2\right]}$$

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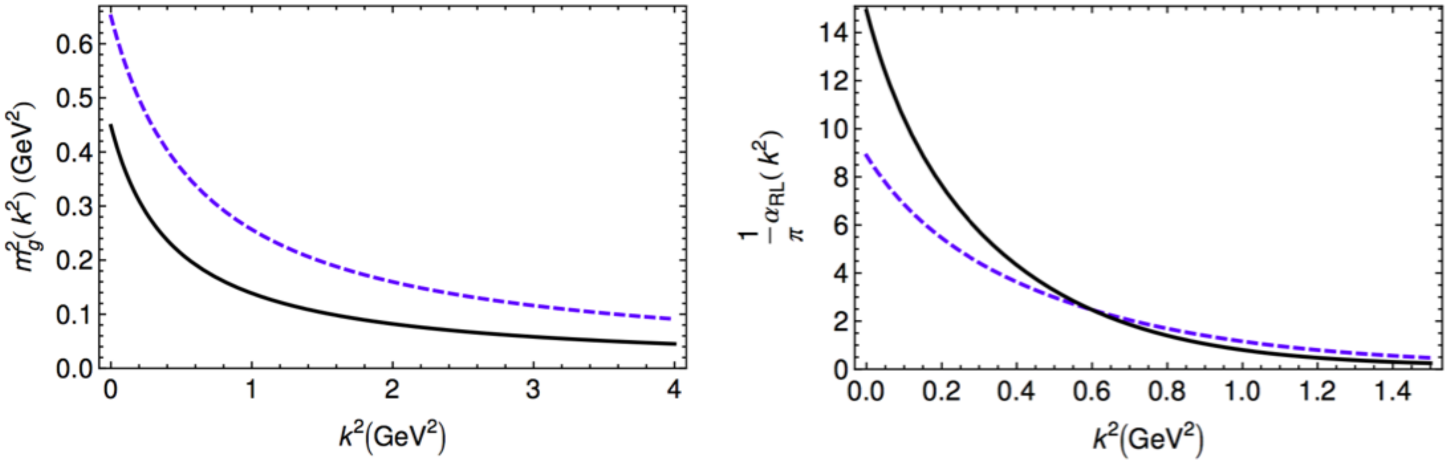
- The gluon mass scale is *typical values of lattice QCD* in our parameter range: *Mg* in [0.6, 0.8] GeV.
- The gluon mass scale is inversely proportional to the confinement length.



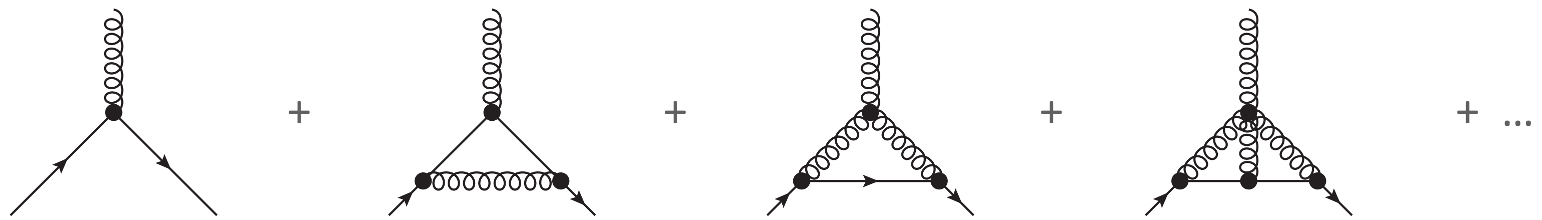
ω = 0.5 GeV (solid curve) and ω = 0.6 GeV (dashed curve)

I. Dynamically massive gluon: Summary

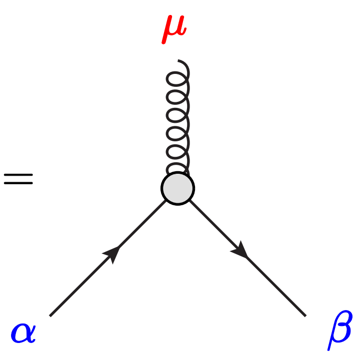
- Two parameters, i.e., coupling strength and width, shapes the interaction in the infrared region, and a perturbation tail dominates that in the ultraviolet region.

- The realistic interaction model includes: gluon mass scale and monotonically decreasing coupling constant.

II. DCSB in quark-gluon vertex: General structure



$$[\Gamma_\mu(p,q)]_{\alpha\beta} =$$



$$\{\gamma_\mu, p_\mu, q_\mu\} \times \{\mathbf{1},\ \gamma\cdot p,\ \gamma\cdot q,\ \sigma_{p,q}\}$$

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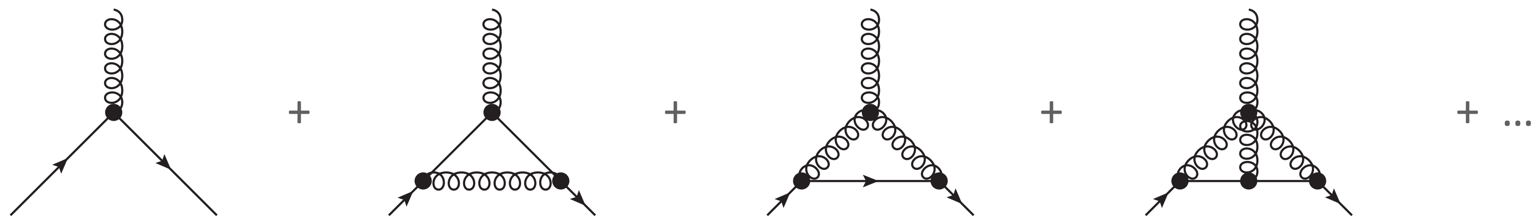

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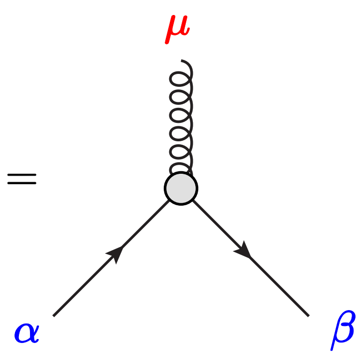
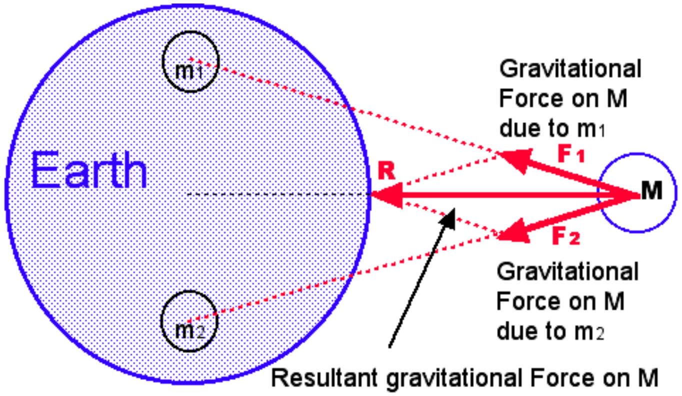


$$\{\gamma_\mu, p_\mu, q_\mu\} \times \{\mathbf{1},\ \gamma\cdot p,\ \gamma\cdot q,\ \sigma_{p,q}\}$$

✦ The vertex has **3 x 4 = 12** independent Lorentz structures.
✦ The appearance may be **modified** in nonperturbative QCD.

II. DCSB in quark-gluon vertex: Ward-Green-Takahashi Identities



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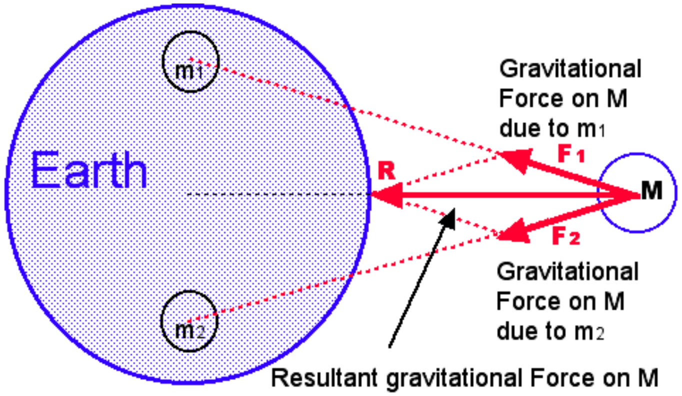

- **Gauge symmetry:** vector WGTI

$$iq_\mu \Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p)$$

- **Chiral symmetry:** axial-vector WGTI

$$q_\mu \Gamma_\mu^A(k,p) = S^{-1}(k) i\gamma_5 + i\gamma_5 S^{-1}(p) - 2im\Gamma_5(k,p)$$

- **Lorentz symmetry + :** transverse WGTIs

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) = {} & S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ & + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \varepsilon_{\lambda\mu\nu\rho} \Gamma_\rho^A(k,p) \\ & + A_{\mu\nu}^V(k,p), \\ q_\mu \Gamma_\nu^A(k,p) - q_\nu \Gamma_\mu^A(k,p) = {} & S^{-1}(p)\sigma_{\mu\nu}^5 - \sigma_{\mu\nu}^5 S^{-1}(k) \\ & + t_\lambda \varepsilon_{\lambda\mu\nu\rho} \Gamma_\rho(k,p) \\ & + V_{\mu\nu}^A(k,p), \qquad \sigma_{\mu\nu}^5 = \gamma_5 \sigma_{\mu\nu} \end{aligned}$$

He, PRD, 80, 016004 (2009)

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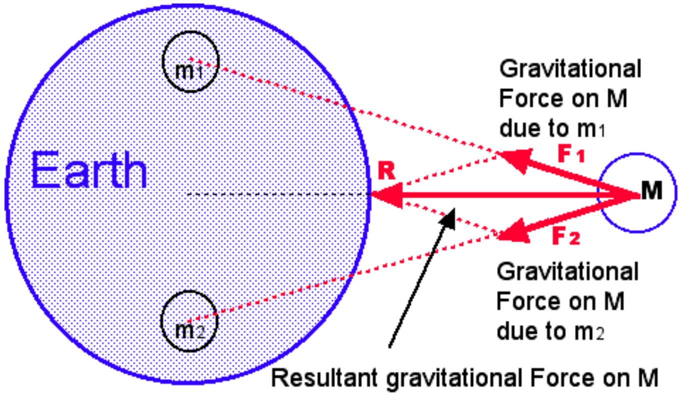

- ✦ The WGTIs express the curls and divergences of the vertices.
- ✦ The WGTIs of the vertices in different channels couple together.
- ✦ The WGTIs involve contributions from high-order Green functions.

❑ **Gauge symmetry:** vector WGTI

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$\nabla \cdot \Phi$

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II. DCSB in quark-gluon vertex: Solution of WGTIs

Qin et. al., PLB 722, 384 (2013)

- Defining proper projection tensors and contract them with the transverse WGTIs, one can decouple the WGTIs and obtain a group of equations for the vector vertex:

$$T^1_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta} t_\alpha q_\beta \mathbf{I}_D, \qquad T^2_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta}\gamma_\alpha q_\beta.$$

$$q_\mu i\Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p),$$

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- It is a group of full-determinant linear equations and a unique solution:

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$$\Gamma^{\rm Full}_\mu(k,p) = \Gamma^{\rm BC}_\mu(k,p) + \Gamma^{\rm T}_\mu(k,p) + \Gamma^{\rm FP}_\mu(k,p).$$

- The quark propagator contributes to the longitudinal and transverse parts. The **DCSB** terms are highlighted.

$$\Gamma^{\rm BC}_\mu(k,p) = \gamma_\mu\Sigma_A + t_\mu \not{t}\frac{\Delta_A}{2} - it_\mu\Delta_B,$$

$$\Gamma^{\rm T}_\mu(k,p) = -\sigma_{\mu\nu}q_\nu\Delta_B + \gamma^T_\mu q^2\frac{\Delta_A}{2} - \left(\gamma^T_\mu[\not{q},\not{t}] - 2t^T_\mu\not{q}\right)\frac{\Delta_A}{4}.$$

$$S(p) = \frac{1}{i\gamma\cdot p\,A(p^2) + B(p^2)}$$

$$\Sigma_\phi(x,y) = \frac{1}{2}[\phi(x) + \phi(y)],$$

$$\Delta_\phi(x,y) = \frac{\phi(x)-\phi(y)}{x-y}.$$

$$X^T_\mu = X_\mu - \frac{q\cdot X q_\mu}{q^2}$$

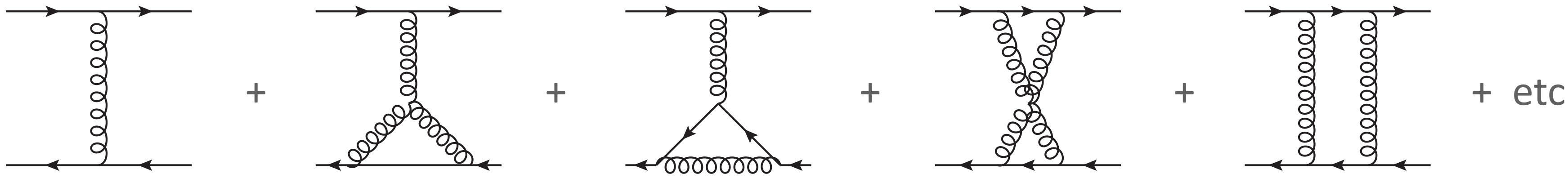
- The unknown high-order terms contribute to the transverse part, i.e., the longitudinal part has been completely determined by the quark propagator.

II. DCSB in quark-gluon vertex: Summary

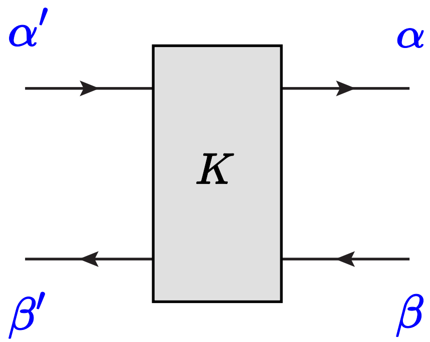
◆ The Lagrangian symmetries are able to constrain structures of the fermion—gauge-boson vertex, and determine some structures uniquely.

◆ DCSB reshapes the appearance of the vertex, dramatically. This must result in remarkable consequences in observables.

III. Symmetries of the kernels: General structure



$$K(p_f, k_f; q_i, k_i)_{\alpha\alpha',\beta'\beta} =$$



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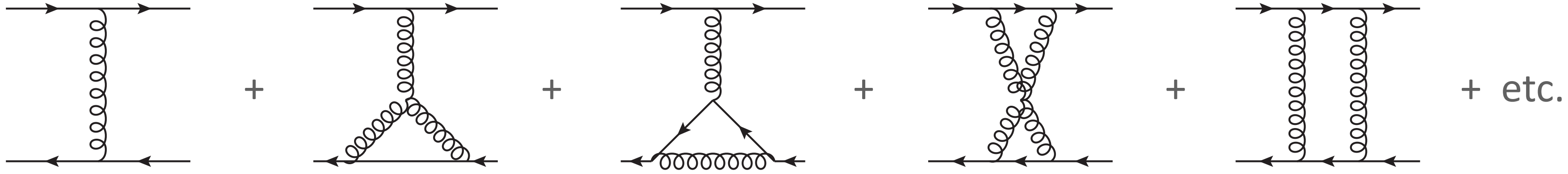

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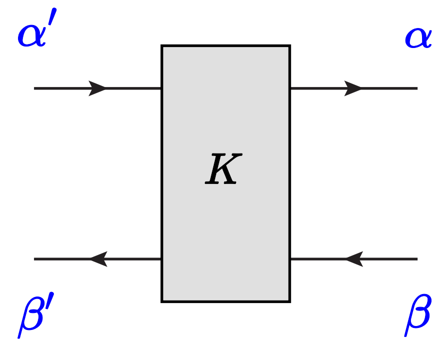
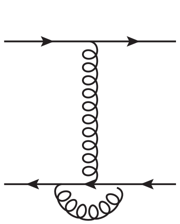
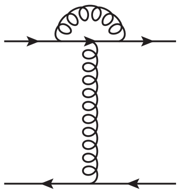


$$K^{L}_{\alpha\alpha'} \otimes K^{R}_{\beta\beta'}$$

- ✦ The kernel has **4 x 4 x 4 x 4 = 256** independent Lorentz structures.
- ✦ It is extremely complicated and must be constrained by symmetries.

III. Symmetries of the kernels: Discrete symmetries

✦ Permutation: $\mathscr{P}\mathcal{K}(q_\pm, k_\pm) = \mathcal{K}^*(q_\pm, k_\pm) = K_R^\mu(k_\mp, q_\mp) \otimes K_L^\mu(k_\mp, q_\mp)$

$\frac{1}{2!}$  +  + etc.

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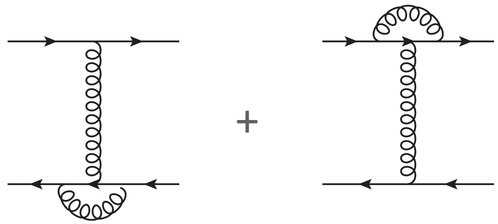
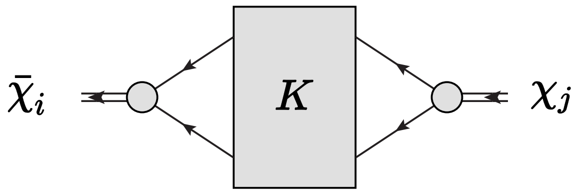
$\frac{1}{2!}$  + etc.

✦ Charge-conjugation: $\mathsf{C}\,\mathcal{K}(q_\pm, k_\pm) = \overline{\mathcal{K}}(q_\pm, k_\pm) = C\,K_L^\mu(-k_\pm, -q_\pm)^T\,C^{-1} \otimes C\,K_R^\mu(-k_\pm, -q_\pm)^T\,C^{-1}$

$\langle \chi_i | K | \chi_j \rangle =$  $= \delta_{ij}$

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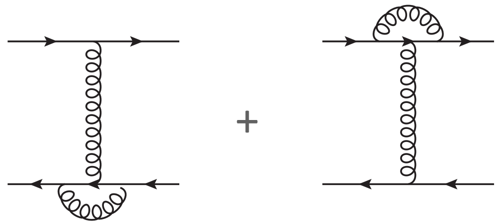
$\frac{1}{2!}$  + etc.

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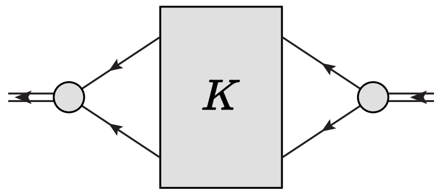
$\langle\chi_i|K|\chi_j\rangle =$ 

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$$K = \mathbf{1}\otimes\mathbf{1} \;+\; \gamma_5\otimes\gamma_5 \;+\; \mathbf{1}\otimes\gamma_5 \;+\; \gamma_5\otimes\mathbf{1}$$

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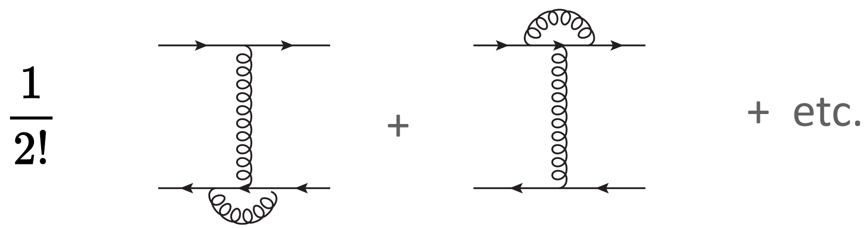

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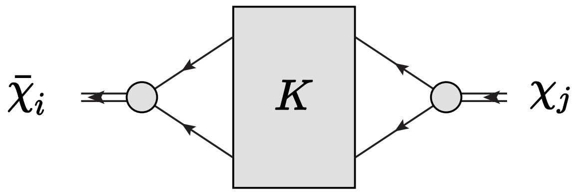

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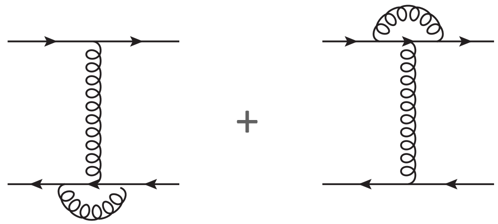
$\frac{1}{2!}$  + etc.

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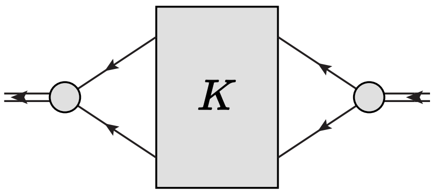
$\langle\chi_i|K|\chi_j\rangle =$ $\bar{\chi}_i$  χ_j $= \delta_{ij}$

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$$K = \mathbf{1}\otimes\mathbf{1} + \gamma_5\otimes\gamma_5 + \cancel{\mathbf{1}\otimes\gamma_5 + \gamma_5\otimes\mathbf{1}}$$

Lorentz covariance guarantees CPT-symmetry; T-symmetry is obtained for free.

III. Symmetries of the kernels: Continuous symmetries

In the chiral limit, the color-singlet axial-vector WGTI (**chiral symmetry**) is written as

$$P_\mu \Gamma_{5\mu}(k,P) = S^{-1}\left(k+\frac{P}{2}\right) i\gamma_5 + i\gamma_5 S^{-1}\left(k-\frac{P}{2}\right)$$

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The axial-vector vertex must involve a pseudo scalar pole (Goldstone theorem)

$$\Gamma_{5\mu}(k,0) \sim \frac{2i\gamma_5 f_\pi E_\pi(k^2) P_\mu}{P^2} \propto \frac{P_\mu}{P^2} \qquad f_\pi E_\pi(k^2) = B(k^2)$$

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Assuming there is a radially excited pion, its decay constant vanishes

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DCSB means **much more** than massless pseudo-scalar meson.

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$$\Gamma^H_{\alpha\beta}(k,P) = \gamma^H_{\alpha\beta} + \int_q \mathcal{K}(k_\pm, q_\pm)_{\alpha\alpha',\beta'\beta}[S(q_+)\Gamma^H(q,P)S(q_-)]_{\alpha'\beta'},$$

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The kernel satisfies the following WGTIs: quark propagator + quark-gluon vertex

$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+) - S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+) - S(q_-)\Gamma_\nu(q_-,k_-)],$$

$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+)\gamma_5 + \gamma_5 S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+)\gamma_5 - \gamma_5 S(q_-)\Gamma_\nu(q_-,k_-)].$$

III. Symmetries of the kernels: Continuous symmetries

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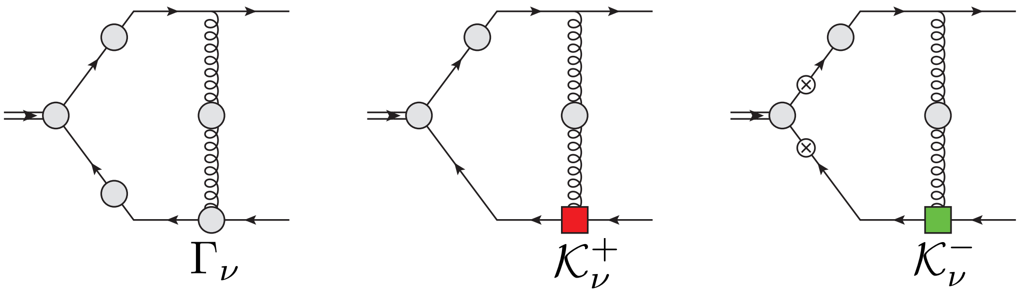

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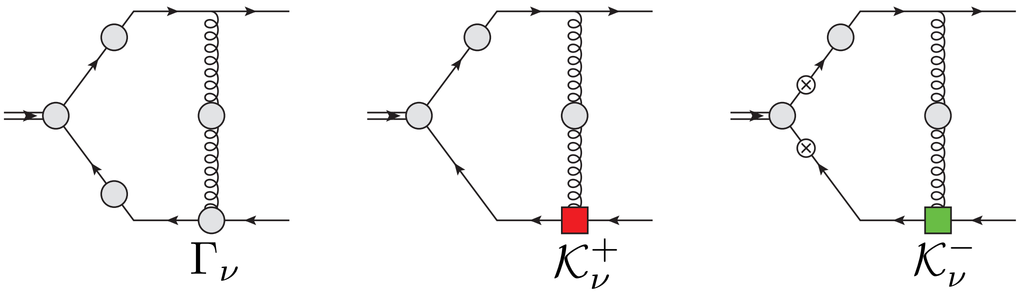

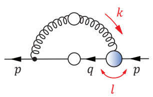

Ladder-like term

Symmetry-rescuing term

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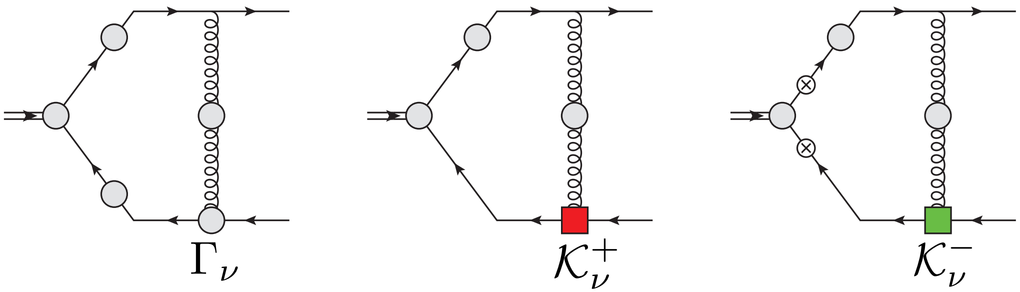

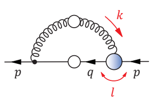

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Inserting the ansatz for the kernel into its WGTIs, we have

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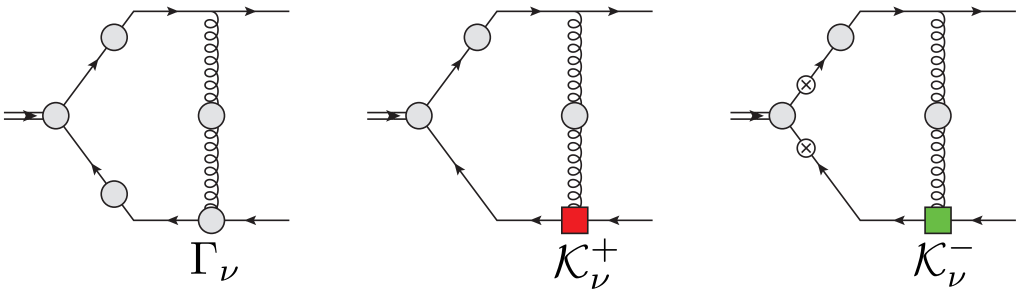

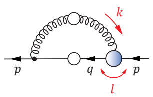

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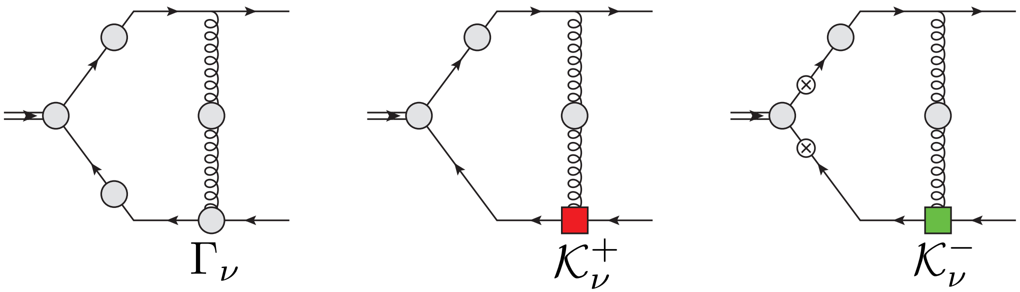
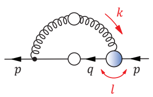


$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

$$\Gamma_\nu^\Sigma = \Gamma_\nu^+ + \gamma_5 \Gamma_\nu^+ \gamma_5 \qquad \Gamma_\nu^\Delta = \Gamma_\nu^+ - \Gamma_\nu^-$$

$$B_\Sigma = 2B_+ \qquad B_\Delta = B_+ - B_-$$

$$A_\Delta = i(\gamma \cdot q_+)A_+ - i(\gamma \cdot q_-)A_-$$



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Eventually, the solution is straightforward:

$$\mathcal{K}_\nu^\pm = (2B_\Sigma A_\Delta)^{-1}[(A_\Delta \mp B_\Delta)\Gamma_\nu^\Sigma \pm B_\Sigma \Gamma_\nu^\Delta].$$

- ✦ The form of scattering kernel is simple.
- ✦ The kernel has no kinetic singularities.
- ✦ All channels share the same kernel.

III. Symmetries of the kernels: Summary

◆ The quark—anti-quark scattering kernel can be constrained by discrete symmetries, aka, CPT-symmetries.

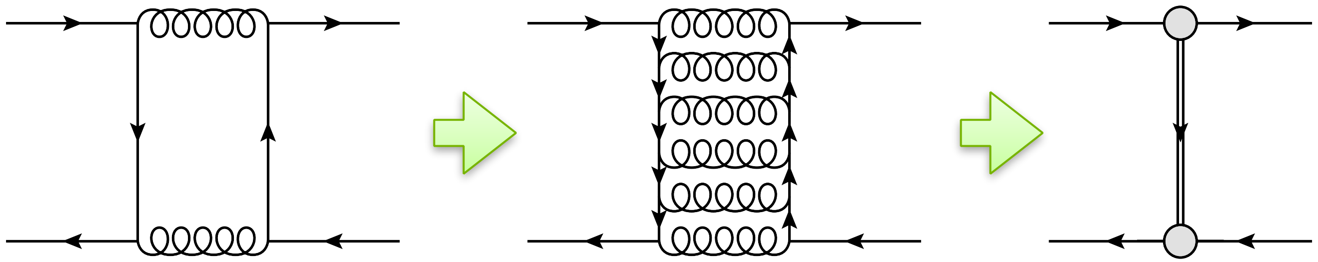
◆ The quark—anti-quark scattering kernel can be constrained by continuous symmetries, aka, vector and axial-vector WGTIs.

◆ The kernel can be constructed systematically and self-consistently.

IV. Meson cloud and diquark: Physics and challenges

In Quantum Field theory (infinitely many degrees of freedom), high-order Green functions cannot completely truncated by low-order ones (unclosed).

For example, meson cloud, e.g., pion cloud, goes into the scattering kernel:



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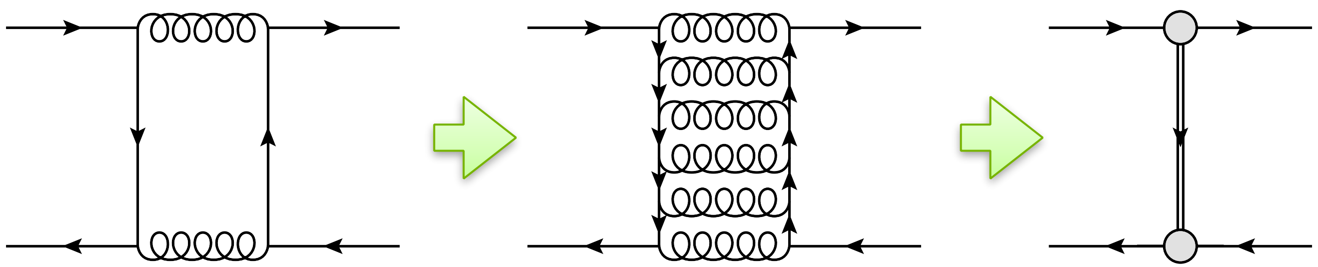
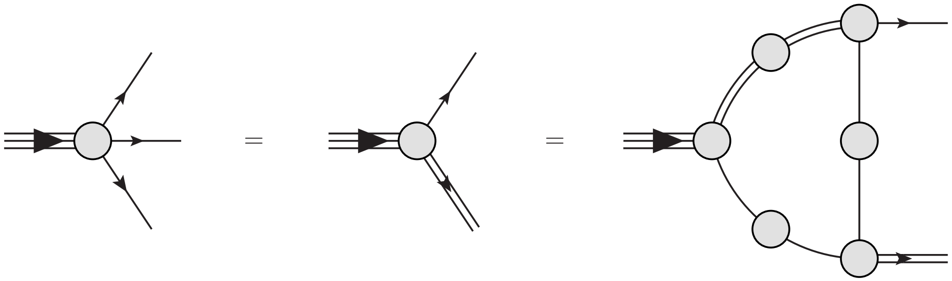


In baryons, two quarks tend to bind together to form a particle-like soft object:



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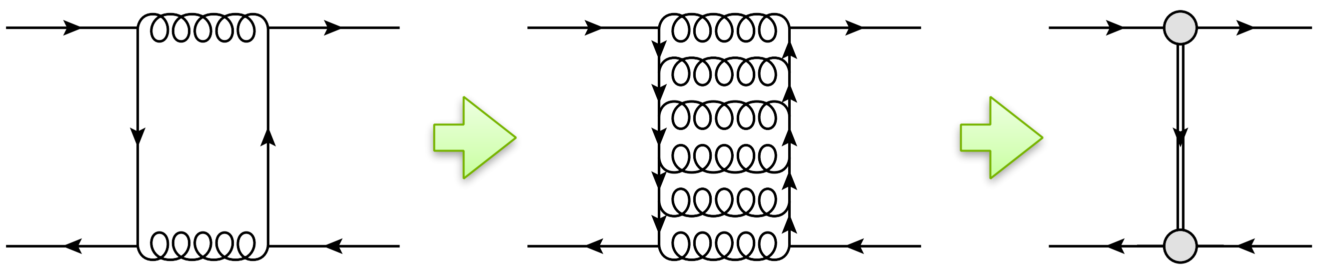

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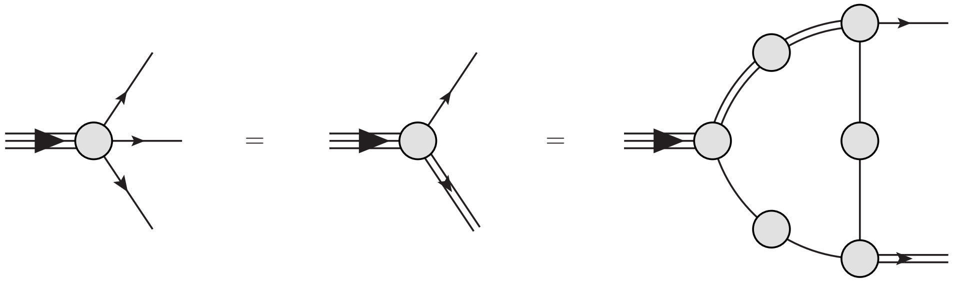
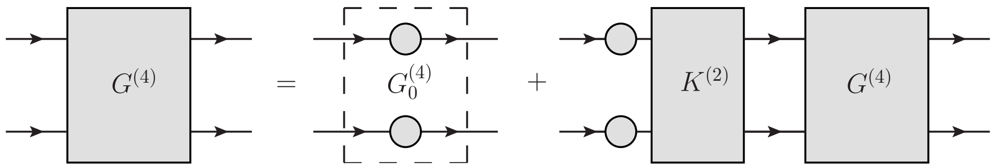


- ✦ What is the **off-shell** meson and diquark?
- ✦ How to make the system **self-consistent**?

IV. Meson cloud and diquark: Off-shell correlation

In QFT, Meson cloud and diquark are encoded in the four-point Green function:



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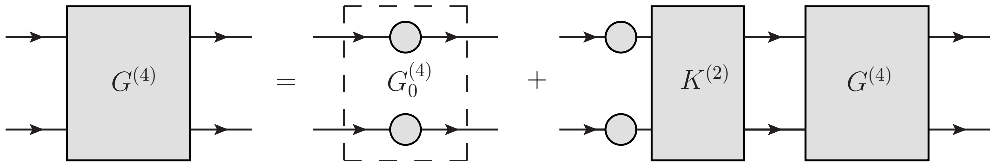

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The kernel can be decomposed by its orthogonal eigenbasis:

$$G_0^{(4)}|\Gamma_i\rangle = \lambda_i\, G_0^{(4)} \cdot K^{(2)} \cdot G_0^{(4)}|\Gamma_i\rangle \qquad \langle\Gamma_i|G_0^{(4)}|\Gamma_j\rangle = \delta_{ij} \qquad K^{(2)} = \sum_i \lambda_i\, |\Gamma_i\rangle\langle\Gamma_i|$$

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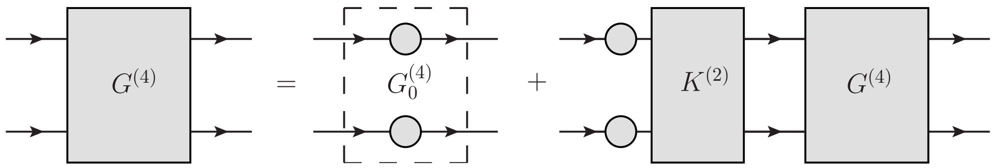

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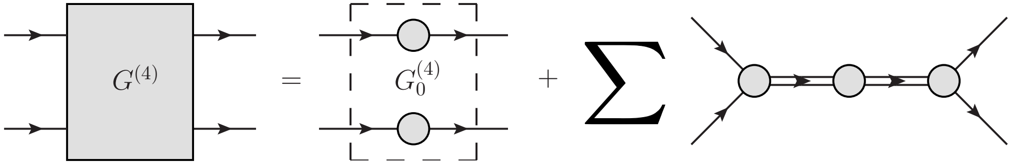
Accordingly, the four-point Green function can be decomposed:



$$G^{(4)} = G_0^{(4)} + \sum_i |\chi_i\rangle \frac{\lambda_i(P^2)}{1-\lambda_i(P^2)} \langle\chi_i|$$

- ✦ The basis is classified by J^P quantum number, and radial quantum number n_r.
- ✦ Meson cloud and diquark correspond to components with quantum numbers.

IV. Meson cloud and diquark: Self-consistency with WGTIs

The start point is the Bethe-Salpeter equation with meson cloud

$$\Gamma^H_{\alpha\beta}(k,P) = \gamma^H_{\alpha\beta} + \int_q \mathcal{K}(k_\pm, q_\pm)_{\alpha\alpha',\beta'\beta}[S(q_+)\Gamma^H(q,P)S(q_-)]_{\alpha'\beta'}.$$

The color-singlet axial-vector and vector WGTIs (|P| = 0) are written as

$$i\hat{P}_\mu \Gamma_\mu(k,0) = \hat{P}_\mu \frac{\partial S^{-1}(k)}{\partial k_\mu},$$
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The Bethe-Salpeter kernel can modify the quark propagator as

$$\left[\hat{P}_\mu \frac{\partial S^{-1}(k)}{\partial k_\mu}\right]_{\alpha\beta} = [i\hat{\not{P}}]_{\alpha\beta} - \int_q \mathcal{K}(k,q)_{\alpha\alpha',\beta'\beta}\left[\hat{P}_\mu \frac{\partial S(q)}{\partial q_\mu}\right]_{\alpha'\beta'},$$

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Using the quark dress functions, the new quark gap equation reads

$$\begin{cases} \dfrac{\partial |k| A(k^2)}{\partial |k|} = 1 + \dfrac{1}{4}\displaystyle\int_q \left[k^{\parallel}_\mu\right]_{\beta\alpha} \mathcal{K}_{\alpha\alpha',\beta'\beta}\left[\frac{\partial S(q)}{\partial q_\mu}\right]_{\alpha'\beta'}, \\ B(k^2) = m + \dfrac{1}{4}\displaystyle\int_q \left[\gamma_5\right]_{\beta\alpha} \mathcal{K}_{\alpha\alpha',\beta'\beta}\left[\gamma_5\sigma_B(q^2)\right]_{\alpha'\beta'}, \end{cases}$$

IV. Meson cloud and diquark: Summary

◆ The meson cloud and diquark can be expressed as components of four-point Green function with corresponding quantum numbers.

◆ The self-consistency can be restored by WGTIs. The quark self-energy and BS kernel can be expressed as the core part plus the meson cloud part.

V. Application: ground and radially excited mesons

Let the quark-gluon vertex includes both longitudinal and transverse parts:

$$\Gamma_\mu(p,q) = \Gamma_\mu^{\mathrm{BC}}(p,q) + \eta\,\Gamma_\mu^{\mathrm{T}}(p,q) \qquad \Gamma_\mu^{\mathrm{T}}(p,q) = \Delta_B \tau_\mu^8 + \Delta_A \tau_\mu^4$$

$$\begin{aligned} \tau_\mu^4 &= 4 l_\mu^{\mathrm{T}} \gamma\cdot k + 4i\gamma_\mu^{\mathrm{T}} \sigma_{\nu\rho} l_\nu k_\rho, \\ \tau_\mu^8 &= 3\, l_\mu^{\mathrm{T}} \sigma_{\nu\rho} l_\nu k_\rho / (l^{\mathrm{T}}\cdot l^{\mathrm{T}}). \end{aligned}$$

- ✦ The longitudinal part is the Ball-Chiu vertex—an exact piece from symmetries.
- ✦ The transverse part is the Anomalous Chromomagnetic Moment (ACM) vertex.

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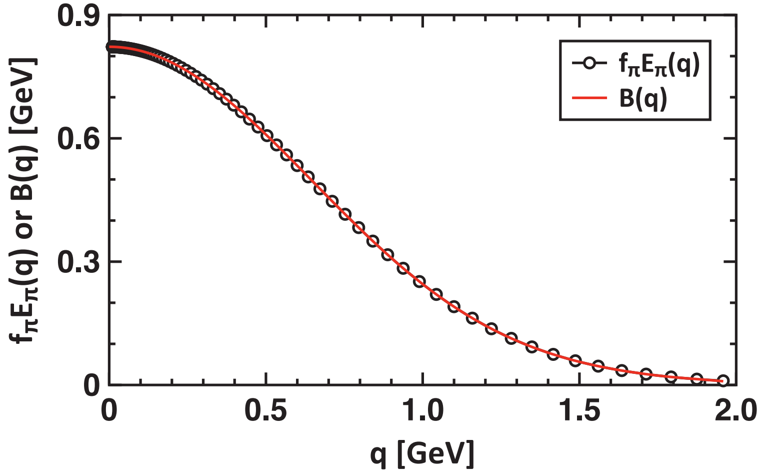

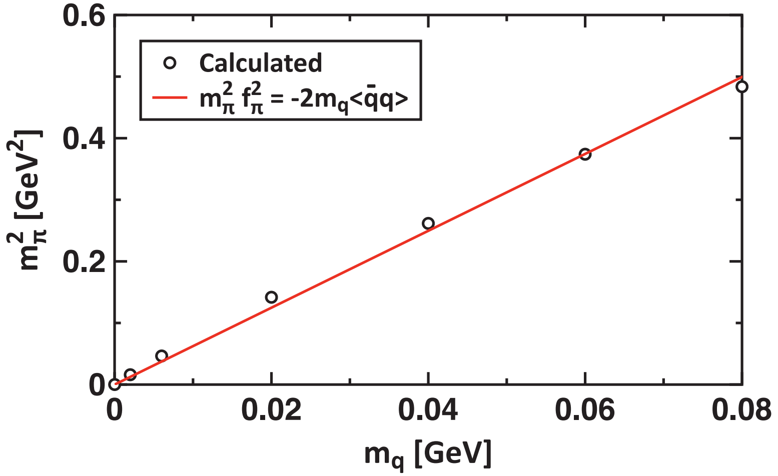

The **DCSB feedback** in the vertex is significant to generate the **quark mass scale** which is comparable to that of LQCD; The **symmetries**, i.e., **WGTIs** are respected.

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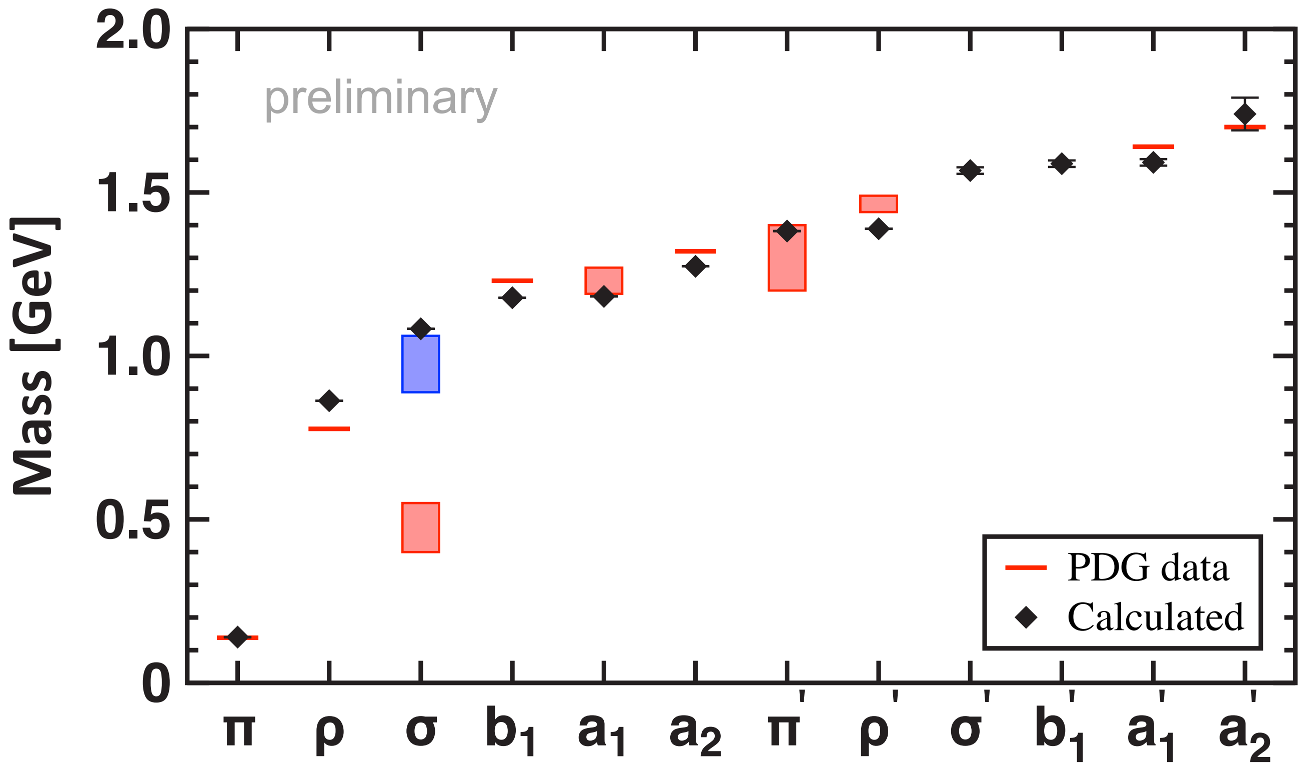

	$-\langle\bar{q}q\rangle_0^{1/3}$	$\rho_\pi^{1/2}$	f_π	m_π	m_ρ	m_σ	m_{b_1}	m_{a_1}	m_{a_2}	$m_{\pi'}$	$m_{\rho'}$	$m_{\sigma'}$	$m_{b_1'}$	$m_{a_1'}$	$m_{a_2'}$
this work	0.291	0.526	0.089	0.14	0.86	1.08	1.17	1.18	1.27	1.38	1.39	1.56 ± 0.01	1.57 ± 0.01	1.58 ± 0.01	1.74 ± 0.05
PDG	-	-	0.092	0.14	0.78	0.50	1.24	1.26	1.32	1.30	1.45	-	-	1.64	1.70

TABLE I: The meson spectrum (Full vertex, $(D\omega)^{1/3} = 0.64$ GeV, $\omega = 0.60$ GeV, $\eta = 0.95$ and $m_q = 2.5$ MeV).

Summary

✦ Based on LQCD and WGTIs, a **systematic** and **self-consistent** method to construct **the gluon propagator, the quark-gluon vertex,** and **the scattering kernels**, beyond the simplest Rainbow-Ladder approximation, is proposed;

✦ A **demonstration** applying the method to light meson spectroscopy, including **ground** and **radially excited mesons**, is presented.

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Outlook

A **path** from **theory** to **experiments** is drawn on the map; it needs to be **paved** in person.