Distribution Amplitudes in a Diquark-Quark approach

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Argonne National Laboratory

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In collaboration with:
C.D. Roberts

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$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

$$|P,N\rangle \propto \sum_\beta \Psi_\beta^{qqq}|qqq\rangle + \sum_\beta \Psi_\beta^{qqq,q\bar{q}}|qqq,q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the N-particles Lightcone-Wave Functions (LCWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LCWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

Nucleon Distribution Amplitudes

- 3 bodies matrix element:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle$$

- 3 bodies matrix element expanded at leading twist:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle = \frac{1}{4}\Big[\left(\not{p}C\right)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma V(z_i^-)$$
$$+\left(\not{p}\gamma_5 C\right)_{\alpha\beta}\left(N^+\right)_\gamma A(z_i^-) - \left(ip^\mu\sigma_{\mu\nu}C\right)_{\alpha\beta}\left(\gamma^\nu\gamma_5 N^+\right)_\gamma T(z_i^-)\Big]$$

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- Isospin symmetry:

$$2T(x_1,x_2,x_3) = \varphi(x_1,x_3,x_2) + \varphi(x_2,x_3,x_1)$$

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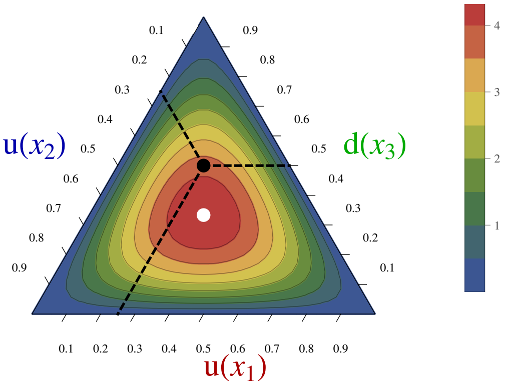
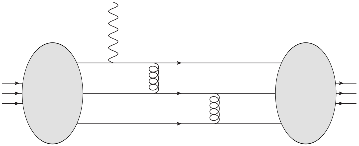
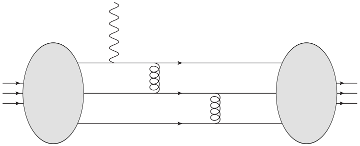
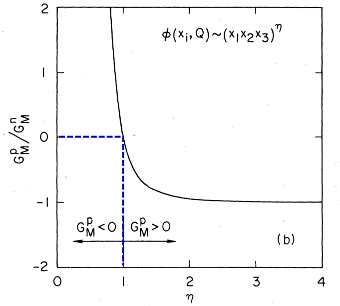
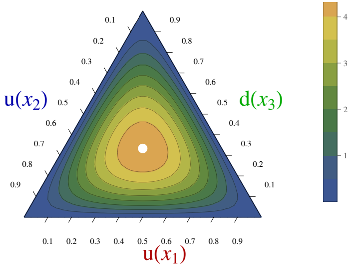






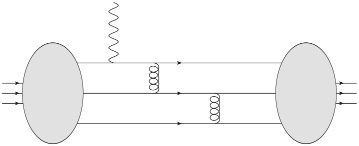


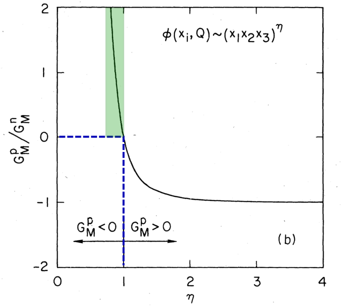


$\eta = 1$

S. Brodsky and G. Lepage, PRD 22, (1980)

Form Factors: Nucleon case





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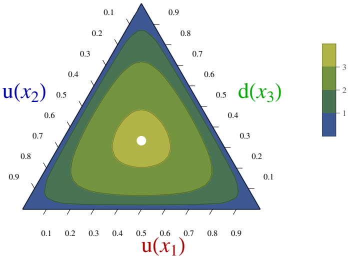

$\eta = 0.5$

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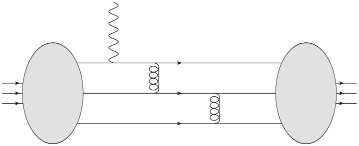
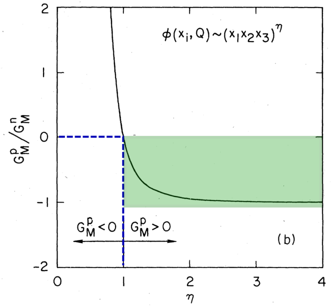
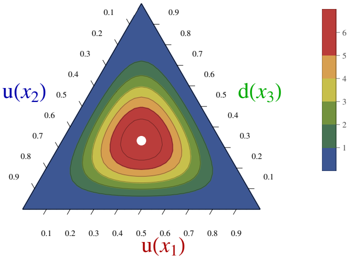






$\eta = 2$

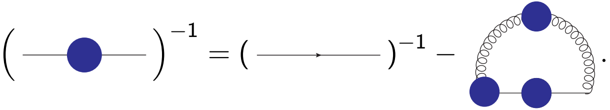
S. Brodsky and G. Lepage, PRD 22, (1980)

Some previous studies of DA

- QCD Sum Rules
  - V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246 (1984)
- Relativistic quark model
  - Z. Dziembowski, PRD 37 (1988)
- Scalar diquark clustering
  - Z. Dziembowski and J. Franklin, PRD 42 (1990)
- Phenomenological fit
  - J. Bolz and P. Kroll, Z. Phys. A 356 (1996)
- Lightcone quark model
  - B. Pasquini *et al.*, PRD 80 (2009)
- Lightcone sum rules
  - I. Anikin *et al.*, PRD 88 (2013)
- Lattice Mellin moment computation
  - G. Bali *et al.*, JHEP 2016 02

Dyson-Schwinger equations

- The gap equation for the quark propagator $S(q)$:



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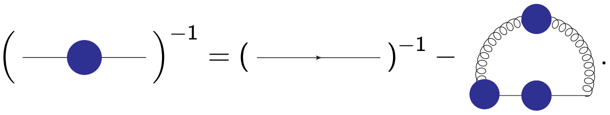
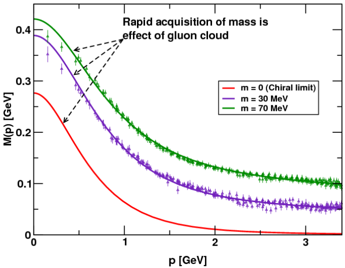


- It has successfully described the quark mass behaviour:



- $S(q)^{-1} = i\not{q}A(q^2) + B(q^2)$
- Non-perturbative description of the quark mass
- Dynamical mass generation
- Figure from Bashir *et al.* (2012)

Faddeev Equation

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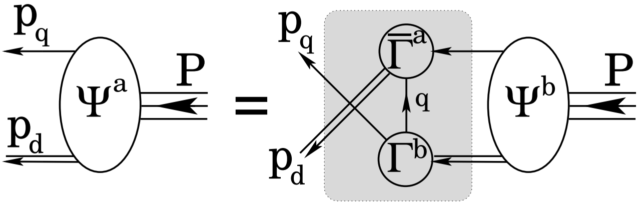
- It predicts the existence of strong diquarks correlations inside the nucleon.



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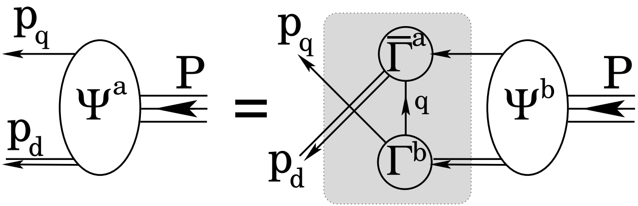

- Mostly two types of diquark are dynamically generated by the Faddeev equation:
  - Scalar diquarks, with mass roughly 2/3 of the nucleon mass,
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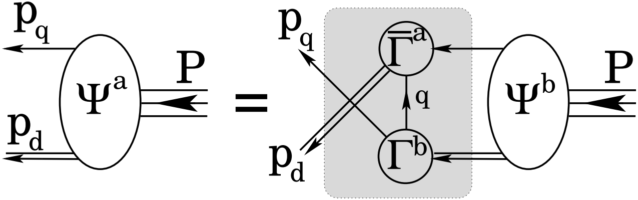

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- Can we understand the nucleon DA in terms of quark-diquarks correlations?

The Nakanishi Representation

Propagator:

- Complexe conjugate poles:

$$S(p) = \sum_j \frac{\alpha_j}{-i\not{p} + m_j} + \mathrm{c.c.}$$

Amplitude:

- Nakanishi representation:

$$F(p,K) \propto \sum_j \int \frac{\mathrm{d}z \;\rho_{\nu_j}(z)}{\left[\left(p - \frac{1-z}{2}K\right)^2 + \Lambda_j^2\right]^{\nu_j}}$$

- The parameters m_j, α_j, ν_j and Λ_j are fitted on the numerical solution of the DSEs.
- Both have been successfully used in the case of the mesons DSE/Bethe-Salpeter equation.
- The Nakanishi representation will be at the center of the following work.

Nucleon DA and DSE

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)C\not{n}u^j_\downarrow(z_2)\right)\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to \varphi(x_i) \to O_\varphi,$$

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)Ci\sigma_{\perp\nu}n^\nu u^j_\uparrow(z_2)\right)\gamma^\perp\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to T(x_i) \to O_T,$$

Braun *et al.*, Nucl.Phys. B589 (2000)

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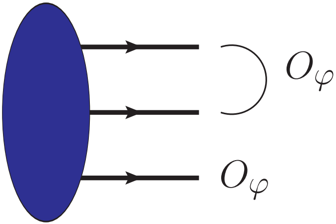

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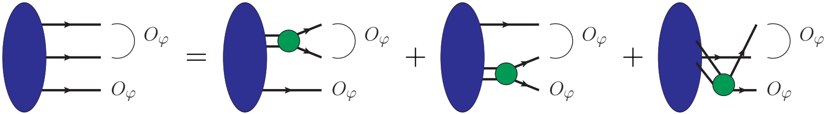

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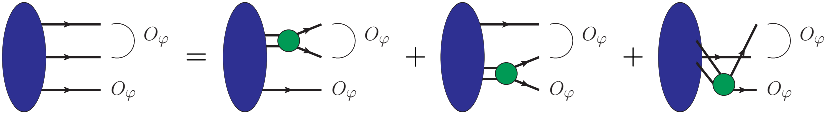
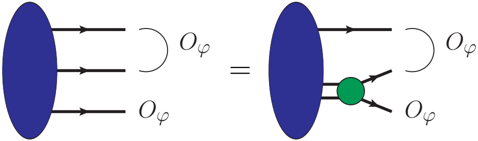


- The operator then selects the relevant component of the wave function.

Scalar diquark contribution

- In the scalar diquark case, only one contribution remains (φ case):



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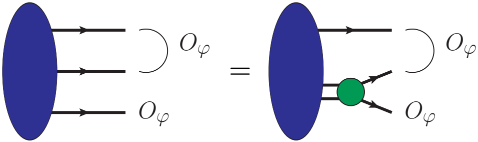

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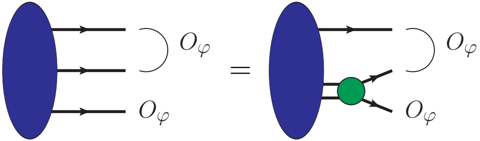

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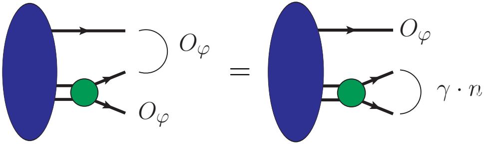

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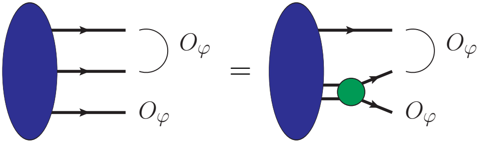

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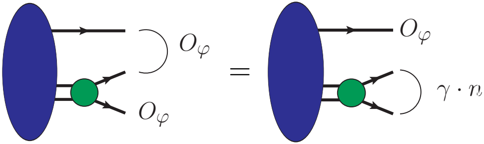
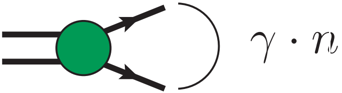
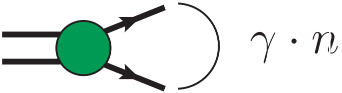


We recognise the leading twist DA of a scalar diquark



- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$



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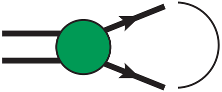
- Bethe-Salpeter amplitude (1 out of 4 structures):

$$\Gamma^{0+}_{\mathrm{PL}}(q, K) = i\gamma_5 C\mathcal{N}^{0+}$$



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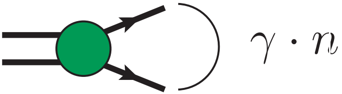
- This point-like case leads to a flat DA:

$$\phi_{\mathrm{PL}}(x) = 1$$



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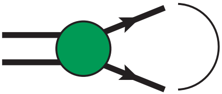

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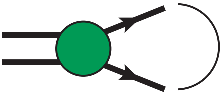
Scalar diquark II: the Nakanishi case



$$\gamma \cdot n$$

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- The Nakanishi case leads to a non trivial DA:

$$\phi(x) = 1 - \frac{M^2}{K^2} \frac{\ln\left[1 + \frac{K^2}{M^2} x(1-x)\right]}{x(1-x)}$$

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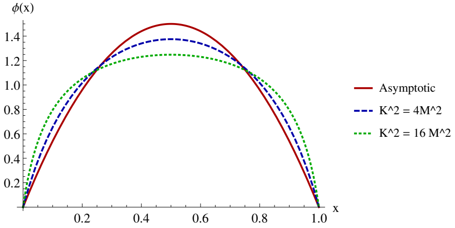
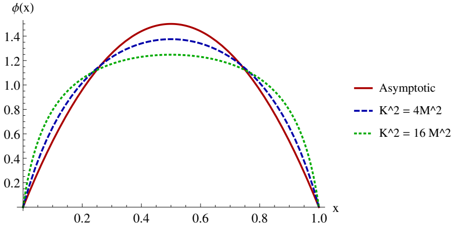


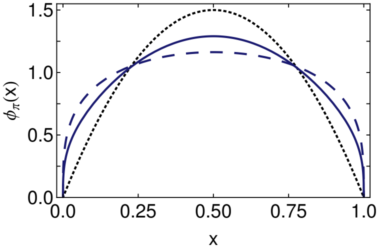
Scalar DA behaviour

$$\phi(x) = 1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}$$

Scalar diquark



Pion

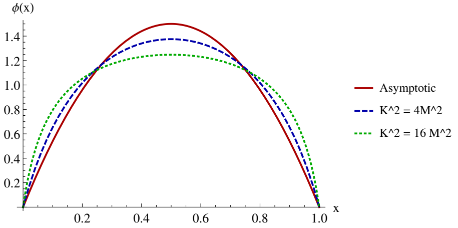


Pion figure from L. Chang *et al.*, PRL 110 (2013)

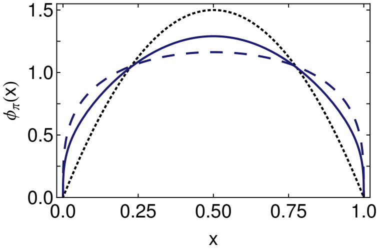
Scalar DA behaviour

$$\phi(x) = 1 - \frac{M^2}{K^2} \frac{\ln\left[1 + \frac{K^2}{M^2} x(1-x)\right]}{x(1-x)}$$

Scalar diquark

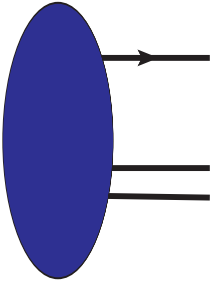


Pion



Pion figure from L. Chang *et al.*, PRL 110 (2013)

This extended version of the DA seems promising!



- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

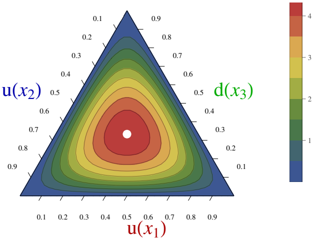
- Diquark propagator:

$$\Delta(K) = \frac{1}{K^2 + \widetilde{M}^2}$$

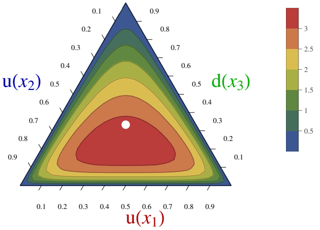
- Faddeev Amplitude:

$$s_1(K,P) = \mathcal{N}_1 \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]}$$

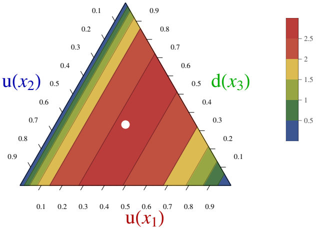
Results in the scalar channel



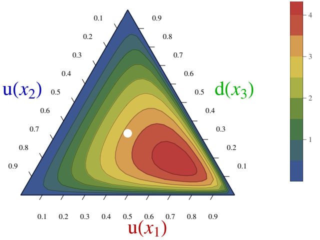
Asymptotic DA



Extended case: T



Point-like case: φ



Extended case: φ

Comparison with lattice

$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$

	Pointlike	Extended	Lattice
$< x_1 >_\varphi$	0.354	0.427	0.358
$< x_2 >_\varphi$	0.323	0.286	0.319
$< x_3 >_\varphi$	0.323	0.286	0.323
$< x_1 >_T$	0.338	0.357	0.34
$< x_2 >_T$	0.338	0.357	0.34
$< x_3 >_T$	0.323	0.286	0.319

Lattice data from G. Bali *et al.*, JHEP 2016 02

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Lattice data from G. Bali *et al.*, JHEP 2016 02

How are these results modified by the AV component?

- Leading twist chiral-even DA:

$$\langle 0|\bar{u}(0)\gamma^\mu d(z)|\rho^+(P,\lambda)\rangle = P^\mu \frac{\epsilon^{(\lambda)}\cdot n}{P\cdot n} f_\rho m_\rho \int_0^1 \mathrm{d}u \phi_{||}(u) e^{-iuP\cdot n}$$

- Leading twist chiral-odd DA:

$$\langle 0|\bar{u}(0)\sigma_{\mu\nu} d(z)|\rho^+(P,\lambda)\rangle =$$
$$i\left(\epsilon^{(\lambda)}_{\perp\mu} p_\nu - \epsilon^{(\lambda)}_{\perp\nu} p_\mu\right) f_\rho^\perp \int_0^1 \mathrm{d}u \phi_\perp(u) e^{-iuP\cdot n}$$

P.Ball and V. Braun,Nucl.Phys. B543 (1999)

Vector Meson DA

- Leading twist chiral-even DA:

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P.Ball and V. Braun,Nucl.Phys. B543 (1999)

Same structures appear in the Axial-Vector diquark.

Selecting the AV components

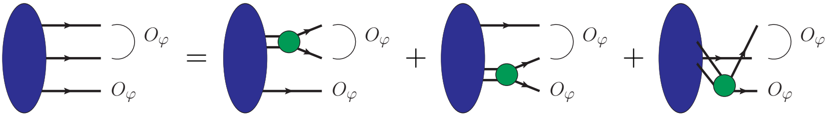
$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)C\not{n}u^j_\downarrow(z_2)\right)\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to \varphi(x_i) \to O_\varphi,$$

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)Ci\sigma_{\perp\nu}n^\nu u^j_\uparrow(z_2)\right)\gamma^\perp\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to T(x_i) \to O_T,$$

Selecting the AV components

$$\langle 0|\epsilon^{ijk}\left(u_{\uparrow}^{i}(z_1)C\not{n}u_{\downarrow}^{j}(z_2)\right)\not{n}d_{\uparrow}^{k}(z_3)|P,\lambda\rangle \to \varphi(x_i) \to O_\varphi,$$

$$\langle 0|\epsilon^{ijk}\left(u_{\uparrow}^{i}(z_1)Ci\sigma_{\perp\nu}n^{\nu}u_{\uparrow}^{j}(z_2)\right)\gamma^{\perp}\not{n}d_{\uparrow}^{k}(z_3)|P,\lambda\rangle \to T(x_i) \to O_T,$$



Selecting the AV components

$$\langle 0|\epsilon^{ijk}\left(u_\uparrow^i(z_1)C\not{n}u_\downarrow^j(z_2)\right)\not{n}d_\uparrow^k(z_3)|P,\lambda\rangle \to \varphi(x_i) \to O_\varphi,$$

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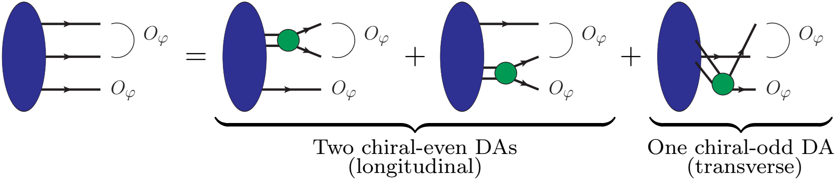

Selecting the AV components

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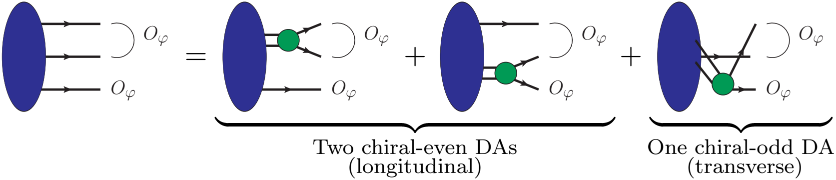
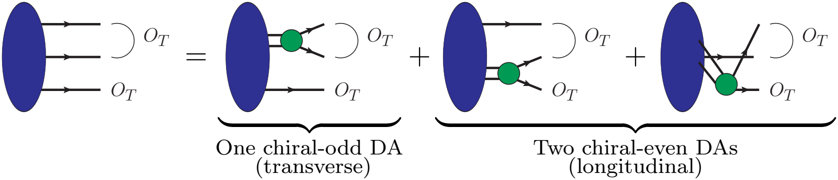
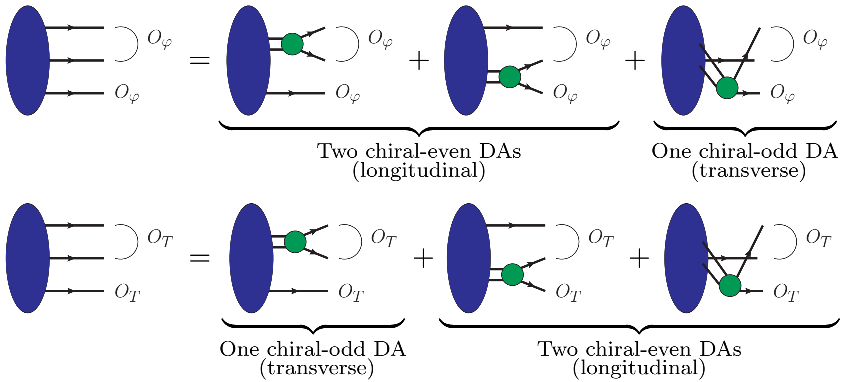




Selecting the AV components

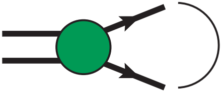
$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)C\not{n}u^j_\downarrow(z_2)\right)\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to \varphi(x_i) \to O_\varphi,$$

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)Ci\sigma_{\perp\nu}n^\nu u^j_\uparrow(z_2)\right)\gamma^\perp\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to T(x_i) \to O_T,$$

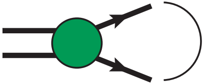


$$2T(x_1,x_2,x_3) = \varphi(x_1,x_3,x_2) + \varphi(x_2,x_3,x_1)$$

AV diquark DA



$\gamma \cdot n$

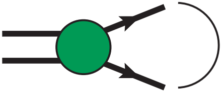
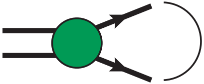


$\sigma^{\mu\nu} n_\nu$

- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

AV diquark DA

 $\gamma \cdot n$  $\sigma^{\mu\nu} n_\nu$

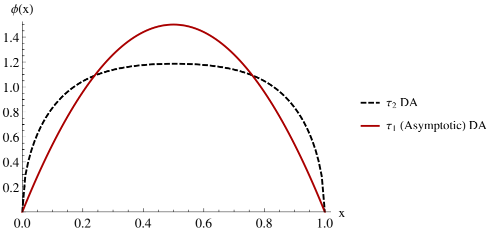
- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

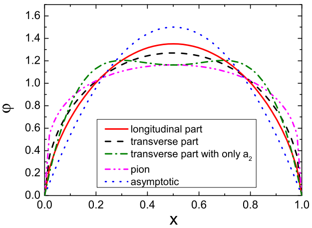
- Bethe-Salpeter amplitude (2 out of 8 structures):

$$\begin{aligned}
\Gamma^\mu_{\mathrm{PL}}(q,K) &= (\mathcal{N}_1 \tau_1^\mu + \mathcal{N}_2 \tau_2^\mu)\, C \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(q - \frac{1-z}{2}K\right)^2 + \Lambda_q^2\right]} \\
\tau_1^\mu &= i\left(\gamma^\mu - K^\mu \frac{\not{K}}{K^2}\right) \rightarrow \text{Chiral even} \\
\tau_2^\mu &= \frac{K \cdot q}{\sqrt{q^2 (K-q)^2}\sqrt{K^2}} \left(-i\tau_1^\mu \not{q} + i\not{q}\tau_1^\mu\right) \rightarrow \text{Chiral odd}
\end{aligned}$$

AV diquark

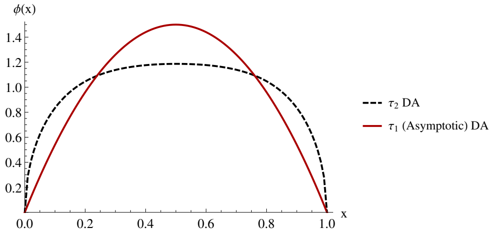


ρ meson

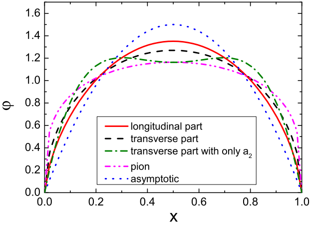


ρ figure from F. Gao *et al.*, PRD 90 (2014)

AV diquark



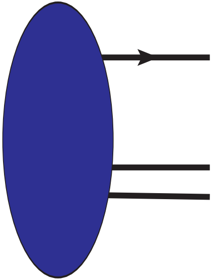
ρ meson



ρ figure from F. Gao *et al.*, PRD 90 (2014)

- Same "shape ordering" $\to$ $\phi_\perp$ is flatter in both cases.
- Farther apart compared to the ρ meson case.

Faddeev Amplitude for the AV case



- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

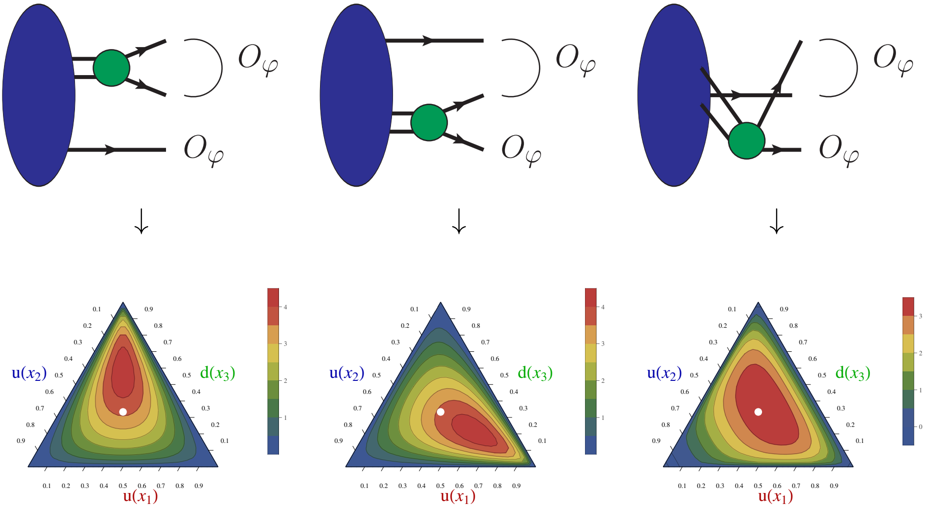
- Diquark propagator:

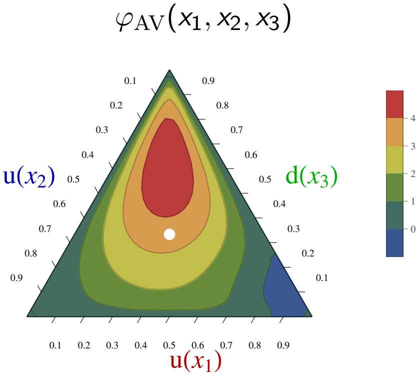
$$\Delta^{\mu\nu}(K) = \frac{\delta^{\mu\nu} + \frac{K^\mu K^\nu}{K^2}}{K^2 + \widetilde{M}^2}$$

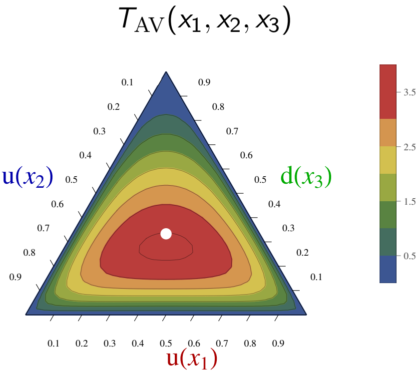
- AV Faddeev amplitude (2 out of 6 structures):

$$A^\mu(K,P) = \left(\gamma_5\gamma^\mu - i\gamma_5\hat{P}^\mu\right)\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]}$$

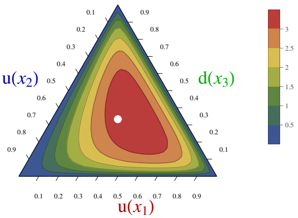
Preliminary results for the AV contributions I

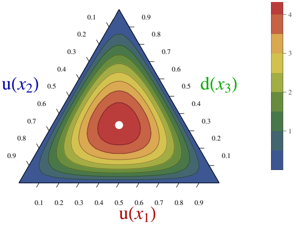



$\varphi_{\rm AV}(x_1, x_2, x_3)$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
u(x_2)
d(x_3)
u(x_1)
4
3
2
1
0


$T_{\rm AV}(x_1, x_2, x_3)$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
u(x_2)
d(x_3)
u(x_1)
3.5
2.5
1.5
0.5

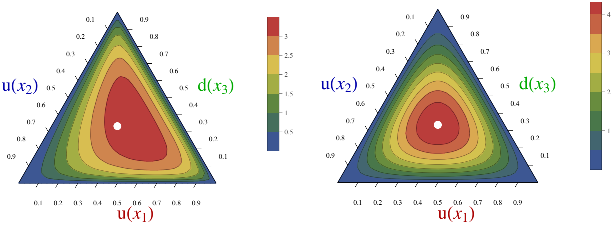
- We use the prediction from the Faddeev equation to weight the scalar and AV contributions 62/38:





Complete results for φ

- We use the prediction from the Faddeev equation to weight the scalar and AV contributions 62/38:



- which can be compared with LCSR results:

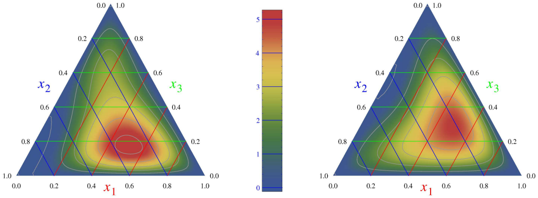
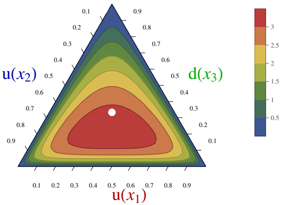


figure from I

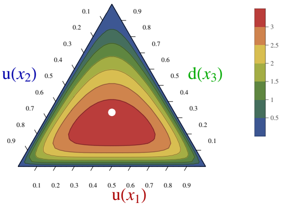
Anikin *et al.*, PRD

88 (2013)

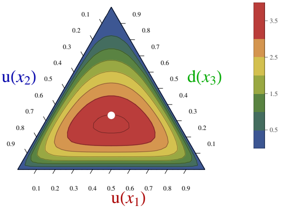
Complete results for T



100% Scalar



62% Scalar



100% AV

- Like ϕ , T received both contributions from the scalar and AV diquarks.
- Both contributions are similar and thus contrary to ϕ, T is hardly modified.

Comparison with lattice

	Scalar	S+AV (62/38)	Lattice	Relative Discrepancy
$< x_1 >_\varphi$	0.427	0.368	0.358	$\approx 3\%$
$< x_2 >_\varphi$	0.286	0.294	0.319	$\approx 8\%$
$< x_3 >_\varphi$	0.286	0.339	0.323	$\approx 5\%$
$< x_1 >_T$	0.357	0.353	0.34	$\approx 4\%$
$< x_2 >_T$	0.357	0.353	0.34	$\approx 4\%$
$< x_3 >_T$	0.286	0.294	0.319	$\approx 8\%$

Lattice data from Bali *et al.*, JHEP 1602, 2016

Comparison with lattice

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$< x_3 >_\varphi$	0.286	0.339	0.323	$\approx 5\%$
$< x_1 >_T$	0.357	0.353	0.34	$\approx 4\%$
$< x_2 >_T$	0.357	0.353	0.34	$\approx 4\%$
$< x_3 >_T$	0.286	0.294	0.319	$\approx 8\%$

Lattice data from Bali *et al.*, JHEP 1602, 2016

Quark-diquark correlation seems able to explain the nucleon DA.

Extension to the Roper

- Everything done before can actually be extended to the Roper case.

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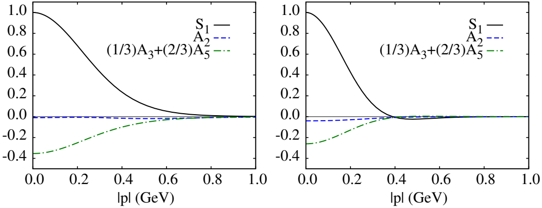

figures from J. Segovia *et al.*,PRL 115 (2015)

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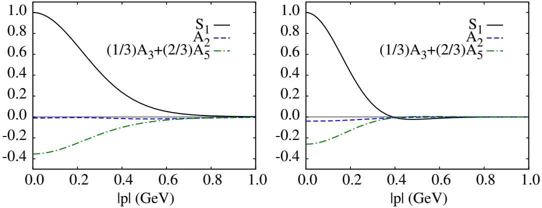
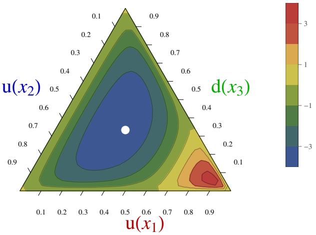


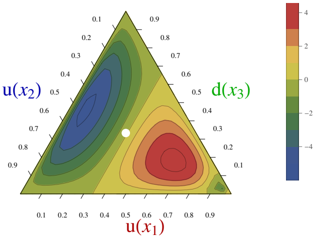
figures from J. Segovia *et al.*,PRL 115 (2015)

- This behaviour can be obtained by adding a zero in the Faddeev amplitude through:

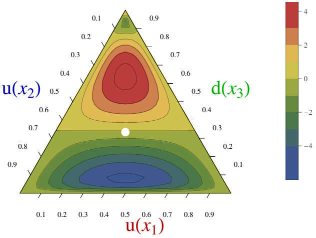
$$\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]} \rightarrow \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)(z-\kappa)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_R^2\right]}$$



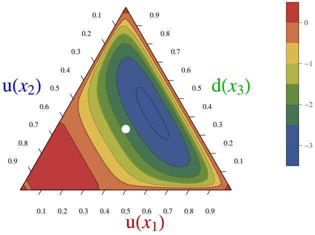
Scalar



AV Long.

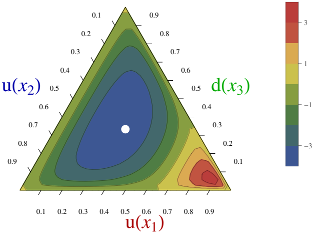


AV Long.

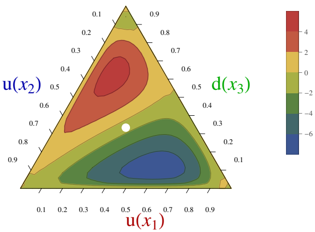


AV Trans.

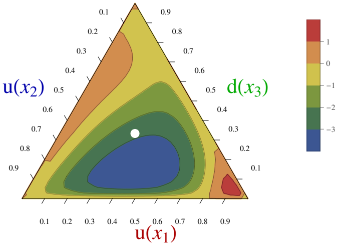
Complete results for φ_R



100 % Scalar

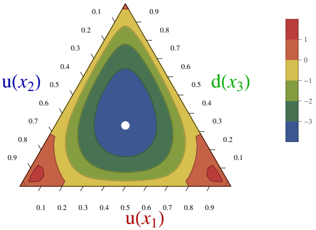


100% AV

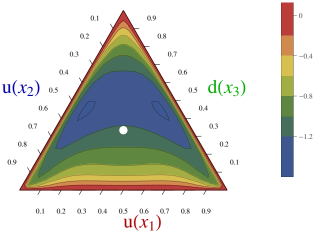


62% Scalar

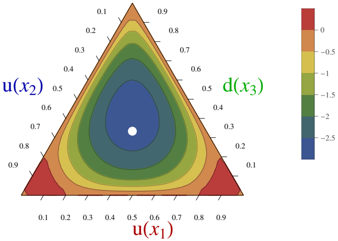
Complete results for T_R



100 % Scalar



100% AV



62% Scalar

Summary

- Both nucleon DAs φ and T can be described using a quark-diquark approximation.
- We show how the diquark types and diquarks polarisations were selected.
- The comparison with lattice computation explains how the different diquarks contribute to the total DAs, and the respective sensitivity of the latter to the AV-diquarks.
- The comparison with the lattice data is encouraging.
- It is possible to extend the work on the nucleon to the Roper case.
- In the Roper case, the results of individual diquarks contributions seems to be consistent with a $n = 1$ excited state.

Outlooks

- Adding the full Dirac structure may help to correct the discrepancy.
- The use of the numerical solutions of the DSE-Faddeev Equation will certainly modify the previous results, and improve our understanding of the physics of the nucleon.
- From Distribution Amplitudes to observables: the computation of the form factor at large momentum transfer within this framework is one of the future goals.
- On a longer time scale, this work also opens the door to the computations of nucleon PDFs, GPDs and TMDs within the DSE framework.

Thank you for your attention...

Thank you for your attention...

... and if you have a position available in 2017, I am looking for a job!