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Chasing the Veneziano ghost (in linear covariant gauges)

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Overview

QCD with 3 light flavours (u, d, s)

- Theory enjoys an (almost) $U_V(3) \times U_A(3)$ symmetry.
- DχSB for the chiral (axial) $U(3)$ ($\leftrightarrow$ dynamical quark mass generation)
- Expected nonet of (almost) massless Goldstone modes ($=$ 3 pions, 4 kaons, η, η')

Problem: the η' mass

- The η' particle is way too massive to be called the ninth "almost Goldstone" boson as $m'_\eta \approx 958$ MeV
- Underlying reason: the corresponding $U(1)$ chiral anomaly

$$\partial_\mu J^5_\mu = -\frac{g^2}{32\pi^2} N_f \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \equiv -\frac{g^2}{16\pi^2} N_f F_{\mu\nu} \tilde{F}_{\mu\nu} \equiv -N_f \mathcal{Q}$$

  so no a priori reason to talk about a Goldstone boson. Nonetheless, as

$$\mathcal{Q} = \partial_\mu \mathcal{K}^\mu \,, \mathcal{K}^\mu = \frac{g^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} A^a_\nu \left(\partial_\rho A^a_\sigma + \frac{g}{3} f^{abc} A^b_\rho A^c_\sigma \right)$$

  is a total derivative, still possible to define a conserved (albeit gauge variant) current and associated chiral Ward identities. $\Rightarrow$ still leading to (almost) Goldstonic poles etc. (see Coleman, "Uses of Instantons")
- More is needed!

Instanton solution ('t Hooft)

- Instantons give zero modes for the Dirac operator.
  (Ward identities, nontrivial windings, index theorema, etc.)
- One gets DχSB, but the Goldstone poles can only appear in gauge variant correlation functions, not in gauge invariant ones ("strengthening/cancelling Kogut-Susskind dipoles"). Based on semiclassical tools, inherent problem of (divergent) instanton sampling in the *IR*.
- Where does $m_{\eta'}$ comes from?
  Can be connected to $U(1)$ violating higher-quark operators ('t Hooft determinant). η' should become massless (in the chiral limit) when "instantons are turned off" (whatever that means).

Topological susceptibility

- Consider the topological charge density

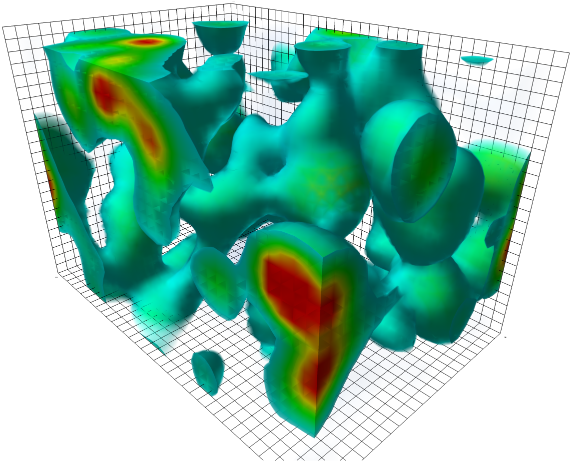
$$\mathcal{Q} = \frac{g^2}{16\pi^2} F_{\mu\nu}\tilde{F}_{\mu\nu}$$

(topological because $\int d^4x\mathcal{Q} \neq 0$ with $\mathcal{Q} = d\mathcal{K}$ necessitates nontrivial boundary conditions at infinity $\sim$ topological content)

- $$\text{Topological susceptibility } \chi^4 = \lim_{p^2\to 0} \langle \mathcal{Q}\mathcal{Q} \rangle$$

Highly nontrivial to get $\chi^4 \neq 0$. Can be (approximately) computed in lattice gauge theory at small/large N (e.g. Del Debbio, Giusti, Pica, PRL94 (2005); Cè, García Vera, Giusti, Schaefer, PLB762 (2016)). Characterizes the topological nature of the QCD (YM) vacuum.

The QCD vacuum



Leinweber et al (Adelaide group)

Topological susceptibility via θ-vacuum

- $\mathcal{Q}$ can be coupled to action via

$$S_{QCD} \to S_{QCD} + \int d^4 x i\theta\mathcal{Q}$$

- If there are massless fermions, $\theta \to 0$ by an anomalous chiral $U(1)$ rotation
-

$$\chi^4 = \frac{\partial^2 E_{vac}}{\partial\theta^2}$$

- Thus also $\chi^4 = 0$ in presence of massless flavour(s)

Large N solution

- Veneziano and Witten, using large N formulation, provided

$$m^2_{\eta'} \approx \frac{4N_f}{f^2_\pi}\chi^{4,N_f=0}_{\theta=0} \sim \frac{1}{N}$$

- Very intricate formula, since lhs refers to QCD (with flavours), rhs to pure gauge theory.

  $\chi^4 \neq 0$ can explain size of $m_{\eta'}$ and thus explain there is no $U(1)$ problem.

  Unfortunately, no recipe to compute χ^4. But relationship can be tested, with satisfactory comparison, via lattice, see e.g. Del Debbio, Giusti, Pica, PRL94 (2005). It can also be incorporated in DSE meson studies, e.g. Bhagwat, Chang, Liu, Roberts, Tandy, PRC76 (2007) 045203.

Instantons vs. large *N* resolution

- No instantons in Veneziano-Witten picture. In 't Hooft picture, most important ingredient?
- Naive instanton extrapolation (at large N)

$$m_{\eta'}^2 \sim e^{-N} \quad \text{vs.} \quad m_{\eta'}^2 \sim \frac{1}{N}$$

So sometimes stated that instantons are irrelevant for large N dynamics.

Though, instanton calculus plagued by *IR* problems, so extrapolation not reliable. In cases where it is (finite volume), large N and instanton result do agree. Marino, "Instantons and Large N : An Introduction to Non-Perturbative Methods in Quantum Field Theory".

A subtlety

- $$\chi^4 = \lim_{p^2\to 0} \langle \mathcal{Q}\mathcal{Q}\rangle_p = \int d^4x \, \langle \mathcal{Q}(x)\mathcal{Q}(0)\rangle$$

  makes no sense as it stands, as $\langle \mathcal{Q}\mathcal{Q}\rangle$ needs additive subtractions ($\mathcal{Q}$ itself is RG invariant in pure gauge theory)

  $$\langle \mathcal{Q}\mathcal{Q}\rangle_p = a_0 + a_1 p^2 + a_2 p^4 + p^6 \int \frac{\rho(t)dt}{(t+p^2)t^3}$$

- Freedom of subtraction constants in absence of low energy theorems $\Rightarrow$ freedom of χ^4? (see later).
- Seiler, PLB525 (2002) questions the derivation of Witten-Veneziano (not the result). Rather, subtraction constant a_0 is linked to (assumed) dominance of) η', then $\chi^4 = a_0$.
  Subtraction constants correspond to contact terms in x-space, so accessible via continuum extrapolation of lattice data? (see Horvath et al, PLBB617 (2005) for some evidence.)

The Veneziano ghost

- By definition $\chi^4 \geq 0$
- Since $\mathcal{Q}$ is t-odd (thanks to Levi-Civita), reflection positivity implies Vicari, Panagopoulos, Phys. Rept. 470 (2009)

$$\mathcal{F}(x) = \langle \mathcal{Q}(x)\mathcal{Q}(0)\rangle < 0\,, \qquad \text{for } |x| > 0,$$

(this corresponds to negative spectral function)

(Strong) positive contact term is indispensable to get $\chi^4 > 0$

$$\langle \mathcal{Q}(x)\mathcal{Q}(0)\rangle = \mathcal{F}(x) + C\delta(x)\,, \qquad \text{for } |x| \geq 0,$$

- How to control C (to some extent at least)?

The Veneziano ghost

We also have

$$\lim_{p^2\to 0} p_\mu p_\nu \langle \mathcal{K}_\mu \mathcal{K}_\nu \rangle = -\chi^4 < 0$$

$\Rightarrow$ The (gauge variant!) correlator $\langle \mathcal{K}\mathcal{K} \rangle$ must have a zero mass pole with wrong sign ($=$ Veneziano ghost).

Veneziano ghost $\to$ topological susceptibility

If we can describe the Veneziano ghost, we can describe the topological susceptibility.
The ghost is not direcly accessible to a “standard” lattice gauge theorist (gauge fixing).

The Veneziano ghost: pro

Adopting now as *definition*

$$\chi^4 = -\lim_{p^2 \to 0} p^2 \langle \mathcal{K}\mathcal{K} \rangle$$

- This χ^4 is the quantity appearing in anomalous chiral Ward identities (Crewther, Riv. Nuovo Cim. 2N8 (1979))
- It coincides with the precise θ-definition of Witten (Meggiolaro, PRD58 (1998) 085002)
- $\langle \mathcal{K}\mathcal{K} \rangle$ also requires a (double) subtraction, but $p^2 \to 0$ in front will wipe out that freedom.

The Veneziano ghost

Research plans

- Emerging project with Oliveira, Silva (UCoimbra) and new PhD student to probe the Veneziano ghost using lattice Landau gauge configurations. (Costly) fermions are not needed to access the relevant χ^4.
  A little more general, we also develop tools to probe the spectral representation of such "unphysical" correlation function.
- Using non-perturbative gluon propagators, see if we can set up a kind of moment extrapolation to get from (non-perturbative) *UV* approximation to $\langle \mathcal{K}\mathcal{K} \rangle \to$ "*IR* extrapolation" with $\lim_{p^2\to 0} \langle \mathcal{K}\mathcal{K} \rangle \neq 0$? ($\sim$ OPE + spectral sum rules inspiration)

Overview

Topological susceptibility and the Veneziano ghost

The linear covariant gauge: some preliminaries

Veneziano ghost and Gribov copies?

Dealing with Gribov copies in linear covariant gauges

Outlook

The (perturbative) linear covariant gauge

The (Faddeev-Popov) gauge fixed action reads

$$
\begin{aligned}
S_{FP} &= S_{YM} + s\int d^4x \left(\bar{c}^a\partial_\mu A^a_\mu - i\frac{\alpha}{2}\bar{c}^a b^a\right) \\
&= S_{YM} + \int d^4x \left(ib^a\partial_\mu A^a_\mu + \frac{\alpha}{2}b^a b^a + \bar{c}^a\partial_\mu D^{ab}_\mu c^b\right)
\end{aligned}
$$

it implements the linear covariant gauge

$$\partial A = -i\alpha b$$

with nilpotent BRST symmetry s

$$sA^a_\mu = -D^{ab}_\mu c^b\ ,\ sc^a = \frac{g}{2}f^{abc}c^b c^c\ ,\ s\bar{c}^a = ib^a\ ,\ sb^a = 0\ ,\ s^2 = 0$$

The (perturbative) linear covariant gauge

The gluon propagator reads

$$\langle A^a_\mu A^b_\nu \rangle_p = \left[\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) D(p^2) + \frac{p_\mu p_\nu}{p^2} \frac{\alpha}{p^2} \right] \delta^{ab}$$

- It holds to all orders that the longitudinal piece remains $\frac{\alpha}{p^2}$. Consequence of the Slavnov-Taylor identity (BRST) combined with another Ward identity.
  Intuitive understanding: gluon self energy ($1PI$ sector) is transverse due to BRST, so no coupling possible to longitudinal (connected) sector of the propagator. (see also talk of Papavassiliou)
- Feynman gauge $\alpha = 1$ gives simple propagator $\sim \delta_{\mu\nu}$, at least at tree level ($\alpha = 1$ is no RG fixed point).

The (non-perturbative) linear covariant gauge

- Faddeev-Popov procedure actually fails, since it tacitly assumed there is a single solution to the gauge condition (to implement the δ(gauge condition) in the path integral).
  But $A \to A + D\omega$ (infinitesimal gauge transform) gives gauge (Gribov) copies iff
  $$-\partial D\omega = 0$$
  i.e. if the FP operator has zero modes.
  One must deal with these copies, just as in the Landau or Coulomb gauge, in particular on lattice. In continuum: see later.
- Dyson-Schwinger approach, see Aguilar, Papavassiliou, PRD77 (2008); Aguilar, Binosi, Papavassiliou, PRD91 (2015); Huber, PRD91 (2015).
- Lattice implementation, see Cucchieri, Mendes, Santos, PRL103 (2009); Bicudo, Binosi, Cardoso, Oliveira, Silva, PRD92 (2015).

The (non-perturbative) linear covariant gauge

Why conceptually interesting?

Most studies so far are in Landau gauge. Interesting to search for "common features" independent of α (or features specific to the Landau gauge $\alpha = 0$).

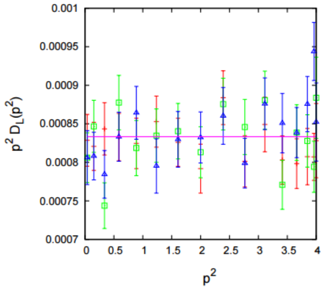
Studies of *gauge invariant physical quantities* as QCD phase diagram (Polyakov loop), the spectrum, DχSB, etc always necessitate at some point simplifying assumptions and/or modelling.

Not so easy to fully appreciate gauge invariance if only one gauge is used. Interesting opportunity to explicitly verify (almost) α-independence of QCD observables.

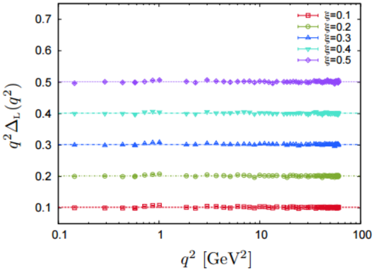
The (non-perturbative) linear covariant gauge

One thing everybody agrees upon

The longitudinal gluon propagator receives no quantum corrections.



Cucchieri et al, $SU(2)$



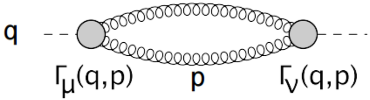
Bicudo et al, $SU(3)$

Overview

Veneziano ghost from modified perturbation theory

Setup of Kharzeev, Levin, PRL114 (2015): Feynman gauge with tree level gluon propagator $D_{\mu\nu}(p^2) = \frac{\delta_{\mu\nu}}{p^2}$

- Assume a new non-perturbative coupling Γ_μ between Veneziano ghost and gluons, such that



- For

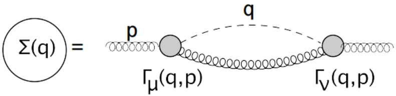
$$\Gamma_\mu(q,p)\Gamma_\nu(q,p)_{q\leq p} \propto \frac{-\chi^4}{p^2}\delta_{\mu\nu}$$

one gets

$$\langle K_\mu K_\nu \rangle_{q\to 0} = \int \frac{d^4p}{(2\pi)^4}\Gamma_\mu(q,p)\frac{1}{p^2}\frac{1}{(q-p)^2}\Gamma_\nu(q,p) = -\frac{\chi^4}{q^2}\delta_{\mu\nu}$$

Veneziano ghost from modified perturbation theory

- Resumming the corresponding self-energy corrections *at*



small momenta leads to

$$D_{\mu\nu}(p^2) = \frac{p^2}{p^4 + \chi^4}\delta_{\mu\nu}$$

Conclusion of Kharzeev-Levin

This is the Gribov propagator that cures the Gribov problem. So, taking into account the QCD vacuum topology (χ^4) automatically also solves the Gribov gauge fixing ambiguity, linking topology to confinement (violation of positivity by complex conjugate (cc) poles of Gribov propagator). *Interesting idea, but. . .*

Intermezzo: positivity violation

- A physical (observable) particle's propagator ought to obey Källén-Lehmann (KL) spectral representation

$$D(p^2) = \int_0^{+\infty} dx \, \frac{\rho(x)}{x+p^2} \,, \qquad \rho(x) \geq 0$$

- Violation can be detected from (Schwinger) function

$$C(t) = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} e^{ipt} D(p^2) = \int_0^{+\infty} dy \rho(y^2) e^{-ty}$$

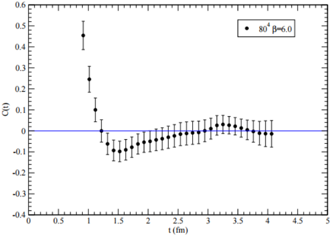
  $C(t) \geq 0$ (reflection positivity) $\Leftarrow \rho(x) \geq 0$
  (or from inflection point in $D(p^2)$)
- In presence of cc poles (see e.g. Rodriguez-Quintero's talk)

$$C(t) \not\geq 0$$

  so no physical particle interpretation.
- Of course, KL itself is not possible for cc poles.

Intermezzo: positivity violation

- cc poles are sufficient (but not necessary) for positivity violation.
- Lattice confirmation of positivity violation (independent of favourite model)



Silva, Oliveira, Dudal, Bicudo, Cardoso, PoS QCD-TNT-III (2013).

Intermezzo: positivity violation

- Ongoing challenge: probe the gluon (and ghost, quark) in the complex momentum plane.

  See also Frederico's and Rodriguiz-Quintero's talk.
- Even the "father" of massive gluons (Cornwall, PRD26(1982)) has a complicated complex structure.

Veneziano ghost from modified perturbation theory

but. . . Dudal, Guimaraes, PRD93 (2016)

- Wrong behaviour of Kharzeev-Levin longitudinal propagator

$$\text{Feyman gauge:} \quad D_{\mu\nu}(p^2) = \frac{p^2}{p^4 + \chi^4}\delta_{\mu\nu}$$

$\rightarrow$ violation of (perturbative) BRST!
Perhaps longitudinal behaviour no more valid beyond perturbation theory? $\rightarrow$ lattice data immediately rules this out! See also DSE work.

- It looks like the Gribov propagator, but in Landau gauge (Gribov, NPB139 (1978); Zwanziger, NPB323 (1989). Not so clear why this would be correct for general linear covariant gauge?

  We shall soon see this is indeed *not* correct.

Veneziano ghost from modified perturbation theory

But first, a counterexample to "vacuum topology $\leftrightarrow$ Gribov copies/confinement" in Landau gauge (Dudal, Guimaraes, PRD93 (2016)).

- Assuming another effective vertex

$$\frac{1}{(2\pi)^4}\int \mathrm{d}^4 p \Gamma_\mu^{\alpha\beta}\Gamma_\nu^{\rho\sigma}\frac{P_{\alpha\rho}(p)}{p^2}\frac{P_{\beta\sigma}(p-q)}{(p-q)^2} = -\frac{\chi^2}{q^2}\frac{q_\mu q_\nu}{q^2}$$

with

$$\Gamma_\mu^{\alpha\beta}(q,p) \propto X q_\mu (p-q)^\alpha q^\beta \; ; \quad q \leq p$$

$$X^2 = \frac{-\chi^2}{p^2 q^2}\frac{1}{(p-q)_\alpha (p-q)_\rho q_\beta q_\sigma P^{\alpha\rho}(q) P^{\beta\sigma}(p-q)}$$

gives

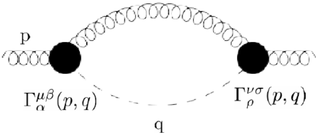
$$\langle K_\mu K_\nu \rangle_{q\to 0} = -\frac{\chi^4}{q^2}\frac{q_\mu q_\nu}{q^2}$$

also sufficient to get the appropriate χ^4.

Veneziano ghost from modified perturbation theory

- The effective *IR* gluon self-energy then becomes

$$\Sigma_{\mu\nu}(p) \propto p_\mu p_\nu$$



and this vanishes when contracted with the transverse Landau gauge gluon propagator, which would thus be not affected by the QCD vacuum topology. Nonetheless, the Gribov problem is very much there in Landau gauge.

Future strategy: (partial) lattice study of possible $\langle \mathcal{K} A A \rangle$ vertex in Landau gauge. Would allow to translate non-trivial vacuum topology to non-trivial interaction in a more controllable setup.

Veneziano ghost from modified perturbation theory

- Interesting constraint on gauge variant correlator $\langle \mathcal{K}_\mu \mathcal{K}_\nu \rangle$

$$\langle \mathcal{K}_\mu \mathcal{K}_\nu \rangle_p = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \mathcal{K}_\perp(p^2) + \frac{p_\mu p_\nu}{p^2} \mathcal{K}_\parallel(p^2)$$

from connection with gauge invariant correlator ($\mathcal{Q} = d\mathcal{K}$)

$$\langle \mathcal{Q}_\mu \mathcal{Q}_\nu \rangle_p = p^2 \mathcal{K}_\parallel(p^2)$$

So the longitudinal projection ought to be gauge invariant, thus powerful internal check when working with general α (also for future lattice analysis).

Overview

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Outlook

Treating the copy problem in Landau gauge: a short survey

From Faddeev-Popov (FP) to Gribov-Zwanziger (GZ)

- A class of copies was related to zero modes of Faddeev-Popov operator $\mathcal{M} = -\partial D$
- Let us restrict path integral to region Ω where $\partial A = 0$ and $\mathcal{M} > 0$.
- Ω corresponds to local minima of the functional $\int d^4 x A_\mu^2$!
- $\Rightarrow$ This is already an improvement of Faddeev-Popov!
- Compare with lattice where one seeks for (in theory) global minima of $\int d^4 x A_\mu^2$
- How to implement restriction to Ω in continuum??? Work of Gribov (leading order) and Zwanziger (all orders).

The Gribov-Zwanziger action

- The Gribov-Zwanziger action is given by

$$S_h = S_{YM} + S_{gf} + \gamma^4 \int \mathrm{d}^4 x\, h(x)$$

with the (non-local) horizon function

$$h(x) = g^2 f^{abc} A_\mu^b \left(\mathcal{M}^{-1}\right)^{ad} f^{dec} A_\mu^e$$

with horizon condition (= gap equation to obtain $\gamma^2 \propto \Lambda_{\text{QCD}}^2$)

$$\langle h(x) \rangle = d(N^2 - 1)$$

- For $\gamma = 0$, everything reduces to Faddeev-Popov.

The Gribov-Zwanziger action

- We replace the action with a local (equivalent) action

$$S_{GZ} = S_{YM+GF} + S_h$$

with

$$\begin{aligned} S_h = & \int d^4x \Big(\overline{\varphi}_\mu^{ac} \partial_\nu \left(\partial_\nu \varphi_\mu^{ac} + g f^{abm} A_\nu^b \varphi_\mu^{mc} \right) - \overline{\omega}_\mu^{ac} \partial_\nu \left(\partial_\nu \omega_\mu^{ac} + g f^{abm} A_\nu^b \omega_\mu^{mc} \right) - g \left(\partial_\nu \overline{\omega}_\mu^{ac} \right) f^{abm} \left(D_\nu c \right)^b \varphi_\mu^{mc} \\ & - \gamma^2 g \left(f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \overline{\varphi}_\mu^{bc} + \frac{4}{g} \left(N^2 - 1 \right) \gamma^2 \right) \Big) \end{aligned}$$

- horizon condition (= gap equation)

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0 \Leftrightarrow \underbrace{\left\langle g f^{abc} A_\mu^a (\varphi + \overline{\varphi})_\mu^{bc} \right\rangle = 2d(N^2 - 1)\gamma^2}_{d=2 \text{ condensate!!}}$$

The Gribov-Zwanziger action

Gribov-Zwanziger quantization

- The GZ formalism is a geometrically inspired path-integral construction with good quantum properties (renormalizable etc) that improves upon the standard FP quantization.
- **Nice property:** closely related to lattice formulation, as in both cases minimization of $\int A^2$ along the gauge orbit is used to define the (a) non-perturbative Landau gauge.

The Refined Gribov-Zwanziger action

- We included extra dynamical effects due to non-perturbative $d = 2$ condensates (Dudal, Gracey, Sorella, Vandersickel, Verschelde, PRD78 (2008)).
- Ghost propagator $G(p^2) \sim \frac{1}{p^2}$ for $p^2 \sim 0$.
- Gluon propagator

$$D(p^2) \;=\; \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + \lambda^4}$$

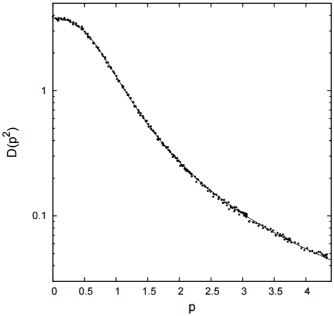
m^2 and M^2 are mass scales corresponding to condensates

$$m^2 \sim \langle A^2 \rangle \;, \qquad M^2 \sim \langle \overline{\varphi}\varphi - \overline{\omega}\omega \rangle$$

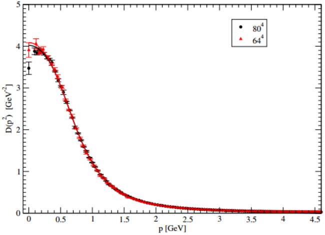
- Works pretty well to describe lattice data, with indeed 2 cc poles. Corresponds to a massive gluon propagator, with running mass (Dudal, Oliveira, Rodriguez-Quintero, PRD86 (2012))

$$D(p^2) = \frac{1}{p^2 + \mathcal{M}^2(p^2)} \;, \quad \mathcal{M}^2(p^2) = \frac{\lambda^4 + m^2 M^2}{p^2 + M^2} + \frac{m^2 p^2}{p^2 + M^2}$$

RGZ fits to gluon lattice data



Cucchieri, Dudal, Mendes, Vandersickel, PRD85 (2012) ($SU(2)$)



Dudal, Oliveira, Rodriguez-Quintero, PRD86 (2012) ($SU(3)$)

BRST symmetry in Gribov-Zwanziger formalism

- The standard BRST symmetry s is clearly broken when using (R)GZ:

$$sS_{GZ} = g\gamma^2 \int d^4x \left(f^{abc} A^a_\mu \omega^{bc}_\mu - (D^{am}_\mu c^m)(\overline{\varphi}^{bc}_\mu + \varphi^{bc}_\mu) \right)$$

- Without BRST, it is hard to "connect" different gauges (also with Gribov problem) while maintaining gauge invariance (or better said: gauge parameter independence).
- based on Dudal, Guimaraes, Sorella et al, PRD92 (2015); PRD93 (2016); PRD94 (2016) and work in progress.

Preliminaries

- Consider A^2-functional

$$\mathrm{Tr}\int d^4x\, A^u_\mu A^u_\mu = \mathrm{Tr}\int d^4x \left(u^\dagger A_\mu u + \frac{i}{g}u^\dagger \partial_\mu u\right)\left(u^\dagger A_\mu u + \frac{i}{g}u^\dagger \partial_\mu u\right)$$

and set $v = he^{ig\omega}$.

- Variation to identify minima:

$$2\mathrm{Tr}\int d^4x \left(\omega\partial_\mu A^h_\mu\right) - \mathrm{Tr}\int d^4x\, \omega\partial_\mu D_\mu(A^h)\omega + O(\omega^3)$$

$\Rightarrow$ $\partial_\mu A^h_\mu = 0 \quad \& \quad -\partial_\mu D_\mu[A^h] > 0$

We recognize the Landau gauge and defining condition of the Gribov region (positive FP operator).

Preliminaries

- The “minimum configuration” can be solved for

$$A^h_\mu = A_\mu - \frac{1}{\partial^2}\partial_\mu\partial A - ig\frac{\partial_\mu}{\partial^2}\left[A_\nu, \partial_\nu\frac{\partial A}{\partial^2}\right] - i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\partial A, \frac{1}{\partial^2}\partial A\right] + ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] + i\frac{g}{2}\left[\frac{1}{\partial^2}\partial A, \frac{\partial_\mu}{\partial^2}\partial A\right] + O(A^3)$$

  A^h is transverse and gauge invariant order by order.
- Observation: if $\partial A = 0$, $A = A^h$. More precisely

$$A = A^h + \text{non-local power series in } (A, \partial A)$$

Rewriting GZ action

- $$A = A^h + \text{non-local power series in } (A, \partial A)$$
- Consider GZ action with non-local horizon action $H(A) = g^2 \int d^4x d^4y \; f^{abc} A_\mu^b(x) \left[\mathcal{M}^{-1}(x,y)\right]^{ad} f^{dec} A_\mu^e(y)$

$$\begin{aligned} S_{GZ} &= S_{YM} + \int d^4x \left(b\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) + \gamma^4 H(A) \\ &= S_{YM} + \int d^4x \left(b\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) + \gamma^4 H(A^h) - \gamma^4 R(A)(\partial A) \\ &= S_{YM} + \int d^4x \left(b^h\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) + \gamma^4 H(A^h) \end{aligned}$$

with a new field b^h

$$b^h = b - \gamma^4 R(A)$$

Identification of new BRST

- Introduce auxiliary fields to obtain

$$
\begin{aligned}
S_{GZ} &= S_{YM} + \int d^4x \left(b^h \partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c \right) \\
&\quad + \int d^4x \, \left(\bar{\phi}\mathcal{M}(A^h)\phi - \bar{\omega}\mathcal{M}(A^h)\omega + \gamma^2 A^h(\bar{\phi}+\phi) \right)
\end{aligned}
$$

- This new (equivalent) GZ action in the Landau gauge enjoys a nilpotent (unbroken) BRST symmetry

$$
s_{\gamma^2} = s + \delta_{\gamma^2} \,, \qquad s_{\gamma^2} S_{GZ} = 0
$$

with

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b \,, \; sc^a = \frac{g}{2} f^{abc} c^b c^c \,, \; s\bar{c}^a = b^a \,, \; sb^a = 0 \,, \\
s\phi^{ab}_\mu &= \omega^{ab}_\mu \,, \; s\omega^{ab}_\mu = 0 \,, \; s\bar{\omega}^{ab}_\mu = \bar{\phi}^{ab}_\mu \,, \; s\bar{\phi}^{ab}_\mu = 0 \\
\delta_{\gamma^2}\bar{c}^a &= -\gamma^4 R^a(A) \,, \quad \delta_{\gamma^2} b^a = \gamma^4 sR^a(A) \,, \\
\delta_{\gamma^2}\bar{\omega}^{ac}_\mu &= \gamma^2 g f^{kbc} A^{h,k}_\mu \left[\mathcal{M}^{-1}(A^h) \right]^{ba} \,, \; \delta_{\gamma^2}(\text{rest}) = 0
\end{aligned}
$$

Identification of new BRST

- Operator algebra

$$\{s, \delta_{\gamma^2}\} = s^2 = \delta_{\gamma^2}^2 = s_{\gamma^2}^2 = 0$$

  and clearly, for $\gamma^2 \to 0$ (GZ$\to$FP) we have $s_{\gamma^2} \to s$.
- γ^2 is a physical parameter, as it does not couple to s_{γ^2}-exact piece.
- Pretty non-local formulation. How to work with this?

Generalization to linear covariant gauge (LCG)

We propose for the LCG the (by construction BRST invariant) action

$$
\begin{aligned}
S_{FP} &= S_{YM} + s_{\gamma^2} \int d^4x \left(\bar{c}^a \partial_\mu A^a_\mu - i\frac{\alpha}{2} \bar{c}^a b^a \right) \\
&+ \int d^4x \, \left(-\bar{\varphi}^{ac}_\mu \mathcal{M}^{ab}(A^h) \varphi^{bc}_\mu + \bar{\omega}^{ac}_\mu \mathcal{M}^{ab}(A^h) \omega^{bc}_\mu + g\gamma^2 f^{abc} A^{h,a}_\mu (\bar{\varphi} + \varphi)^{bc}_\mu \right) \\
&= S_{YM} + \int d^4x \left(ib^a \partial_\mu A^a_\mu + \frac{\alpha}{2} b^a b^a + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) \\
&+ \int d^4x \, \left(-\bar{\varphi}^{ac}_\mu \mathcal{M}^{ab}(A^h) \varphi^{bc}_\mu + \bar{\omega}^{ac}_\mu \mathcal{M}^{ab}(A^h) \omega^{bc}_\mu + g\gamma^2 f^{abc} A^{h,a}_\mu (\bar{\varphi} + \varphi)^{bc}_\mu \right)
\end{aligned}
$$

which corresponds to a restriction to

$$
\Omega^h = \{ \, A_\mu | \partial_\mu A^a_\mu = \alpha b^a, \; \partial_\mu A^h_\mu = 0, \; \mathcal{M}^{ab}(A^h) > 0 \, \}
$$

Comments about Ω^h

The (non-Hermitian) FP operator for general α reads

$$\mathcal{M}^{ab}(A) = -\partial_\mu D_\mu^{ab} = -\partial_\mu(\delta^{ab}\partial_\mu - gf^{abc}A_\mu^c) \\ = -\delta^{ab}\partial^2 + \alpha g f^{abc} b^c + g f^{abc} A_\mu^c \partial_\mu$$

Infinitesimal Gribov copies will appear whenever

$$\mathcal{M}^{ab}(A)\zeta^b = 0$$

with ζ^a a normalizable zero mode.

$$\mathcal{M}^{ab}(A^h) = -\partial_\mu(\delta^{ab}\partial_\mu - gf^{abc}A_\mu^{h,c})$$

is Hermitian, it thus makes sense to define the region Ω^h. It is convex and bounded in all directions.

Restriction to $\Omega^h \Rightarrow$ elimination of copies?

Why does imposing $\mathcal{M}^{ab}(A^h) > 0$ removes zero modes of $\mathcal{M}^{ab}(A)$?

- Assume ζ^a is a zero mode of $\mathcal{M}^{ab}(A)$ with a Taylor expansion in α,

$$\zeta^a = \sum_{n=0}^{\infty} \alpha^n \zeta_n^a$$

- Decompose the gauge field A_μ^a via

$$A_\mu = A_\mu^h + \tau_\mu \;, \qquad \partial_\mu \tau_\mu = \alpha b$$

with $\tau_\mu = \sum_{n=0}^{\infty} \alpha^{n+1} \tau_\mu^n = \alpha \hat{\tau}_\mu$ since $A = A^h$ for $\alpha \to 0$

$$\begin{aligned} A_\mu \in \Omega^h \Rightarrow \zeta^a &= -g \left[\mathcal{M}(A^h)^{-1}\right]^{ad} f^{dbc} \partial_\mu \left(\tau_\mu^b \zeta^c\right) \\ &= -g\alpha \left[\mathcal{M}(A^h)^{-1}\right]^{ad} f^{dbc} \partial_\mu \left(\hat{\tau}_\mu^b \zeta^c\right) \end{aligned}$$

Restriction to $\Omega^h \Rightarrow$ elimination of copies?

- Expand in powers of α

$$\sum_n \alpha^n \zeta_n^a = -\sum_n g\alpha^{n+1} \left[\mathcal{M}(A^h)^{-1}\right]^{ad} f^{dbc}\partial_\mu \left(\zeta_n^c \hat{\tau}_\mu^b\right)$$

- Matching orders of α

$$\zeta_n^a \propto \zeta_{n-1}^a \propto \ldots \zeta_0^a \equiv 0 \Rightarrow \zeta \equiv 0$$

All zero modes that possess a Taylor expansion around $\alpha = 0$ are automatically vanishing if $A^h \in \Omega^h$. The restriction to Ω^h thus at least excludes these from the game.

Localization of the action

- Reconsider the action (with non-local A^h)

$$S_{GZ}^{LCG} = S_{YM} + \int d^4x \left(\alpha\frac{b^a b^a}{2} + ib^a\partial_\mu A_\mu^a + \bar{c}^a\partial_\mu D_\mu^{ab}(A)c^b\right)$$
$$+\int d^4x \left(-\bar{\varphi}_\mu^{ac}\mathcal{M}^{ab}(A^h)\varphi_\mu^{bc} + \bar{\omega}_\mu^{ac}\mathcal{M}^{ab}(A^h)\omega_\mu^{bc} + g\gamma^2 f^{abc}(A^h)_\mu^a(\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc})\right)$$

- Introduce an auxiliary Stueckelberg field h via

$$A_\mu^h = (A^h)_\mu^a T^a = h^\dagger A_\mu^a T^a h + \frac{i}{g} h^\dagger \partial_\mu h\,, \quad h = e^{ig\xi^a t^a}$$

A^h is indeed gauge invariant as

$$A_\mu \to V^\dagger A_\mu V + \frac{i}{g} V^\dagger \partial_\mu V\,, \qquad h \to V^\dagger h\,, \qquad h^\dagger \to h^\dagger V$$

Localization of the action

- A_μ^h is now a local field and can be expanded in ξ^a

$$(A^h)_\mu^a = A_\mu^a - D_\mu^{ab}\xi^b - \frac{g}{2} f^{abc}\xi^b D_\mu^{cd}\xi^d + \mathcal{O}(\xi^3)$$

- Two Lagrange multipliers b^a and τ^a enforcing two constraints.
  - $b^a \to \partial_\mu A_\mu^a = i\alpha\, b^a$ (linear covariant gauge)
  - $\tau^a \to \partial_\mu (A^h)_\mu^a = 0$ which is a *(crucial)* constraint on the Stueckelberg field h (or ξ).
    If ξ is eliminated through the transversality constraint $\partial A^h = 0$, we go back to the non-local expression for the field A^h and by further integrating over the auxiliary fields $(\bar{\varphi}, \varphi)$ and $(\bar{\omega}, \omega)$, one goes back to the original non-local GZ action.

Localization of the BRST

- Albeit the action is local, the BRST transformation itself still contains non-localities
- In PRD94 (2016), we showed that a fully local formulation can be obtained, so all tools of local *QFT* become available.
- At the end, one arrives at

$$
\begin{aligned}
S_{GZ}^{LCG} &= S_{YM} + s_{loc} \int d^4x \left(-i\frac{\alpha}{2}\bar{c}^a b^a + \bar{c}^a \partial_\mu A_\mu^a - \frac{1}{\sqrt{2}} \bar{\omega}_\mu^{ac} \mathcal{M}(A^h)^{ab} \kappa_\mu^{bc} \right) + \int d^4x\, \tau^a \partial_\mu (A^h)_\mu^a. \\
&\quad + \int d^4x \left(-\bar{\lambda}_\mu^{ac} \mathcal{M}(A^h)^{ab} \lambda_\mu^{bc} + \bar{\zeta}_\mu^{ac} \mathcal{M}(A^h)^{ab} \zeta_\mu^{bc} + g\gamma^2 f^{abc} (A^h)_\mu^a (\lambda_\mu^{bc} + \bar{\lambda}_\mu^{bc}) \right) \\
&= S_{YM} + \int d^4x \left(\alpha \frac{b^a b^a}{2} + ib^a \partial_\mu A_\mu^a + \bar{c}^a (\partial_\mu D_\mu)^{ab} c^b \right) + \int d^4x\, \tau^a \partial_\mu (A^h)_\mu^a. \\
&\quad + \int d^4x \left(-\bar{\kappa}_\mu^{ac} \mathcal{M}(A^h)^{ab} \kappa_\mu^{bc} + \bar{\omega}_\mu^{ac} \mathcal{M}(A^h)^{ab} \omega_\mu^{bc} \right) \\
&\quad + \int d^4x \left(-\bar{\lambda}_\mu^{ac} \mathcal{M}(A^h)^{ab} \lambda_\mu^{bc} + \bar{\zeta}_\mu^{ac} \mathcal{M}(A^h)^{ab} \zeta_\mu^{bc} + g\gamma^2 f^{abc} (A^h)_\mu^a (\lambda_\mu^{bc} + \bar{\lambda}_\mu^{bc}) \right)
\end{aligned}
$$

- The fields $(\kappa, \bar{\kappa}, \omega, \bar{\omega})$ do not only constitute a BRST quartet, but even a trivial unity (so they can be integrated out).

Localization of the BRST

- Local BRST variations

$$
\begin{aligned}
s_{loc}A_\mu^a &= -D_\mu^{ab}c^b\,, \quad s_{loc}c^a = \frac{g}{2}f^{abc}c^bc^c\,, \\
s_{loc}\bar{c}^a &= ib^a\,, \quad s_{loc}b^a = 0\,. \\
s_{loc}h^{ij} &= -igc^a(T^a)^{ik}h^{kj}\,, \quad s_{loc}(A^h)_\mu^a = 0\,, \\
s_{loc}\kappa_\mu^{ab} &= \sqrt{2}\omega_\mu^{ab}\,, \quad s_{loc}\omega_\mu^{ab} = 0\,, \\
s_{loc}\bar{\omega}_\mu^{ab} &= \sqrt{2}\bar{\kappa}_\mu^{ab}\,, \quad s_{loc}\bar{\kappa}_\mu^{ab} = 0\,, \\
s_{loc}\tau^a &= 0\,, \\
s_{loc}\bar{\lambda}_\mu^{ab} &= 0 \quad s_{loc}\lambda_\mu^{ab} = 0 \\
s_{loc}\zeta_\mu^{ab} &= 0 \quad s_{loc}\bar{\zeta}_\mu^{ab} = 0\,,
\end{aligned}
$$

in addition to

$$
s_{loc}\xi^a = -c^a + \frac{g}{2}f^{abc}c^b\xi^c - \frac{g^2}{12}f^{amr}f^{mpq}c^p\xi^q\xi^r + O(g^3) \Rightarrow s_{loc}A_\mu^{h,a} = 0
$$

Localization of the action

- The action is now local, albeit non-polynomial. This is not unknown in literature ($\mathcal{N} = 1$ SUSY YM, chiral Wess-Zumino models, etc).
- The theory (in particular its renormalizability) can still be controlled by the Quantum Action Principle (algebraic formalism). Dragon, Hürth, Van Nieuwenhuizen, Nucl. Phys. Proc. Suppl.56B (1997).
- Renormalizability for $\gamma^2 = 0$ was proven in Sorella et al, PRD94 (2016). As expected, non-linear renormalization of ξ.
- For $\gamma^2 \neq 0$ work in progress, but no problems expected, given that γ^2 is a "soft" change. From

$$\frac{k^2}{k^4 + \gamma^4} = \frac{1}{k^2} - \frac{\gamma^4}{k^2(k^4 + \gamma^4)}$$

and more general identities, a decent *UV* behaviour of GZ propagators can be appreciated (Sorella et al, EPJC76 (2016)).

Localization of the action

- Thanks to crucial constraint $\partial A^h = 0$, $\langle \xi\xi \rangle \sim \frac{1}{p^4}$ consistent with power counting renormalizability, despite Stueckelberg formalism.

  If $\cancel{\int d^4x \tau \partial A^h}$ in the action, we would get $\langle \xi\xi \rangle \sim \frac{1}{p^2}$ which would destroy renormalizability (cf. non-renormalizability of the standard Stueckelberg action.)

  Similar to renormalizability of

$$S_{YM} + \int d^4x \left(\bar{c}\partial Dc + b\partial A + \frac{m^2}{2} A^2 \right)$$

  vs. non-renormalizability of

$$S_{YM} + \int d^4x \left(\frac{m^2}{2} A^2 \right)$$

Applications of the local BRST invariant formulation

- Exact proof that gluon propagator still looks like

$$D(p^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D(p^2) + \frac{\alpha}{p^2}\frac{p_\mu p_\nu}{p^2}$$

  Only (non-)perturbative corrections to the transverse form factor.

- Renormalizability and gauge invariance Sorella et al, PRD94 (2016) of $\langle A^h A^h \rangle_p$ propagator and associated $d = 2$ condensate $\langle A^2 \rangle_{\text{min}} = \langle A^h A^h \rangle$

  (important phenomenological applications, see e.g. Chetyrkin, Narison, Zakharov, NPB550 (1999); Gubarev, Zakharov, PLB501 (2001); Megias, Ruiz Arriola, Salcedo, JHEP 0601 (2006)).

- Nielsen identities.

Nielsen identities

- We extend the BRST transformation with (Piguet, Sibold, NPB253 (1985))

$$s_{loc}\alpha = \chi\ , \qquad s_{loc}\chi = 0$$

- The full action (including sources) becomes

$$\begin{aligned} S &= S_{YM} + \int d^4x \left(\alpha \frac{b^a b^a}{2} + i b^a \partial_\mu A^a_\mu - i\frac{\chi}{2}\bar{c}^a b^a + \bar{c}^a \partial_\mu D^{ab}_\mu(A) c^b \right) + \int d^4x\, \tau^a \partial_\mu (A^h)^a_\mu \\ &\quad + \int d^4x \left(-\bar{\varphi}^{ac}_\mu \mathcal{M}(A^h)^{ab} \varphi^{bc}_\mu + \bar{\omega}^{ac}_\mu \mathcal{M}(A^h)^{ab} \omega^{bc}_\mu + g\gamma^2 f^{abc} (A^h)^a_\mu (\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu) \right) \\ &\quad + \int d^4x\, \mathcal{J}^a_\mu A^{ha}_\mu + \int d^4x \left(\Omega^a_\mu (s_{loc} A^a_\mu) + L^a (s_{loc} c^a) + K^a (s_{loc} \xi^a) \right) \end{aligned}$$

- Extended Slavnov-Taylor identity for $1PI$ generator Γ

$$\text{STI} = \int d^4x \left(\frac{\delta\Gamma}{\delta\Omega^a_\mu}\frac{\delta\Gamma}{\delta A^a_\mu} + \frac{\delta\Gamma}{\delta L^a}\frac{\delta\Gamma}{\delta c^a} + \frac{\delta\Gamma}{\delta K^a}\frac{\delta\Gamma}{\delta \xi^a} + i b^a \frac{\delta\Gamma}{\delta \bar{c}^a} \right) + \chi \frac{\partial\Gamma}{\partial\alpha} = 0$$

Nielsen identities

- Acting with test operators $\frac{\delta^2}{\delta\Phi(x)\Phi(y)}\Big|_{\text{sources}=\text{fields}=\chi=0}$ on STI

$$\partial_\alpha \langle\Phi(x)\Phi(y)\rangle^{1PI} = \text{certain } 1PI \text{ diagrams}$$

- . . . (nice technical analysis, to appear)
- $$\partial_\alpha(\text{pole masses}) = 0$$

  despite that propagators itself are α-dependent.

  Thus, the cc pole masses of the gluon propagator attain a "gauge invariant" meaning (also: RG invariant).
- Moreover

$$\text{poles of } \langle AA\rangle = \text{poles of } \langle A^h A^h\rangle$$

Overview

Things to do

- Study α-independence for explicitly gauge invariant quantities? (e.g. Polyakov loop to probe the phase transition).
- Variational probing of self-consistent values for the dynamical mass scales? Nielsen identities important to ensure gauge parameter independence.
- Gauge invariant signal of confinement in (positivity violation of) $\langle A^h A^h \rangle$ propagator? What happens when Higgs fields are added? Positivity violation averted?
- What happens with $\langle A^h A^h \rangle$ at finite temperature?
- "Inverse logic" of Kharzeev-Levin: we now have a GZ for LCG $\rightarrow$ some way to access topological susceptibility, "sourced" by gluon mass scales?
- Test the non-perturbative LCG in a lattice formulation?
- What happens with the ghost?
- ...

Lattice formulation of LCG

- We proposed (inspired by Cucchieri, Mendes, PRL103 (2009))

$$\mathcal{R}(A,B,U,V) \equiv \mathrm{Tr}\int d^4x \left(A^U_\mu A^U_\mu + \frac{2}{g}\mathrm{Re}(iU\Lambda)\right) + \mathrm{Tr}\int d^4x \left(B^V_\mu B^V_\mu\right)$$
$$+\mathrm{Tr}\int d^4x \left(B^V_\mu - P_{\mu\nu}A^U_\nu\right)^2$$

with $P_{\mu\nu} = \delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}$.

- Gauging $\to$ minima of $\mathcal{R}(A,B,U,V)$, for a fixed function $\Lambda(x) = \Lambda^a(x)t^a$ in function of variable U, V.
- (Local) minimization leads to

$$\begin{aligned} B &= \text{transverse part of } A \\ \partial B &= 0\,, \qquad \partial A = \Lambda \text{ (LCG)} \\ \mathcal{M}(B) &> 0 \end{aligned}$$

Non-perturbative B corresponds to A^h!

The ghost propagator in LCG

- For $\alpha \neq 0$, quite different behaviour for ghost expected (*IR* suppression), predicted by DSE work Aguilar, Papavassiliou, PRD77 (2008); Aguilar, Binosi, Papavassiliou, PRD91 (2015); Huber, PRD91 (2015).
- At one-loop, we get

$$G(k^2) = \frac{1}{k^2}\frac{1}{1-\omega(k^2)}$$

where

$$\omega(k^2) = \omega^T(k^2) + \omega^L(k^2)$$

$\omega^{T,L}(k^2)$ corresponding to transverse/longitudinal gluon propagator. In particular

$$\omega^L(k^2) = \alpha\frac{Ng^2}{64\pi^2}\log\frac{k^2}{\bar{\mu}^2}$$

which suggests *IR* suppression because of $\ln(k^2)$ ($\omega^T(0) =$ number).

The ghost propagator in LCG

- No ghost propagator data yet for $\alpha \neq 0$.
- As $\mathcal{M}^{ab}(A)$ is not Hermitian for $\alpha \neq 0$, neither will its inverse be (= the ghost propagator).

  To have Hermitian action for $\alpha \neq 0$, the FP ghost c and anti-ghost $\bar{c}$ are to be chosen to be independent and real, resp. purely imaginary (see f.i. Alkofer, Von Smekal, Phys. Rept. **353** (2001)). The relevant (Hermitian) operator becomes matrix valued

$$\begin{pmatrix} 0 & \partial D \\ -\partial D & 0 \end{pmatrix}$$

- Lattice ghost propagator usually computed via (real) eigenvalues of (supposedly Hermitian) FP matrix $\Rightarrow$ the matrix propagator is what need to be studied.

The End.

Thanks!