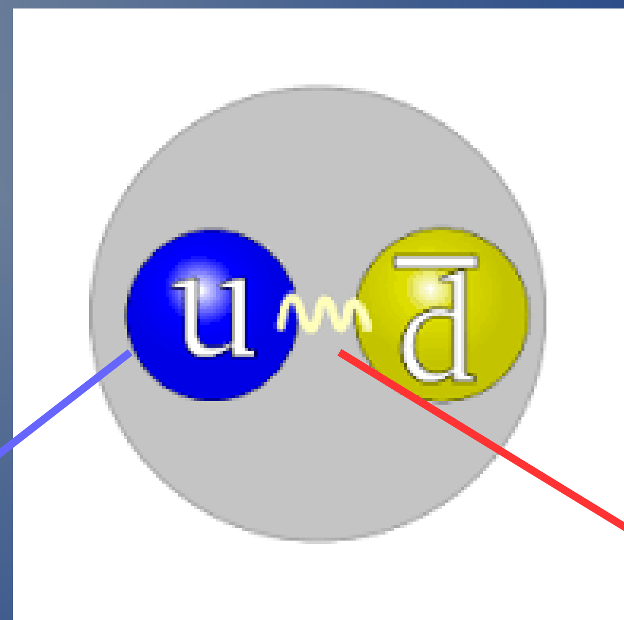
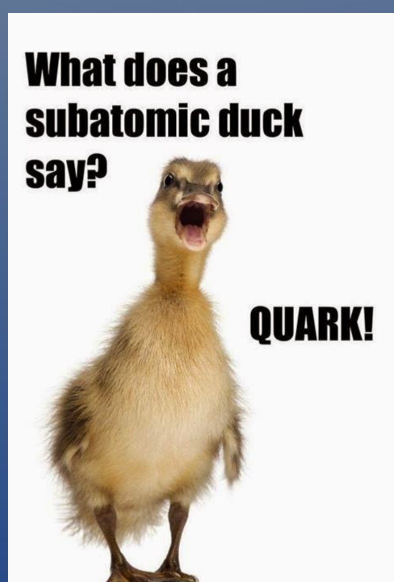
Gluons and Pions





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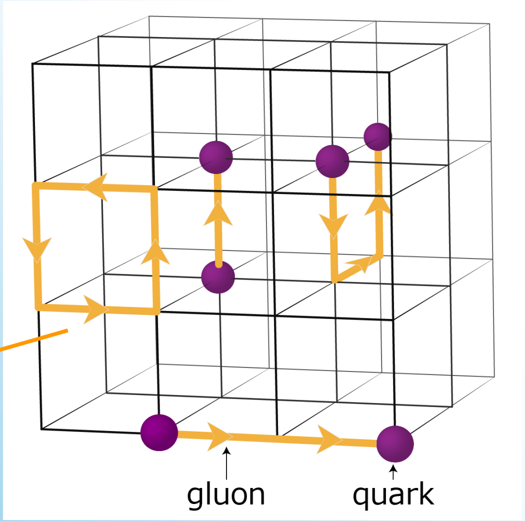
Gluon Green's functions

The vertex and the three-gluon Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

$$\widetilde{A}^a_\mu(q) = \frac{1}{2}\,\mathrm{Tr}\sum_x A_\mu(x+\hat{\mu}/2)\exp[iq\cdot(x+\hat{\mu}/2)]\lambda^a$$



$$A_\mu(x+\hat{\mu}/2) = \frac{U_\mu(x) - U^\dagger_\mu(x)}{2iag_0} - \frac{1}{3}\,\mathrm{Tr}\,\frac{U_\mu(x) - U^\dagger_\mu(x)}{2iag_0}$$

Tree-level Symanzik gauge action

$$S_g = \frac{\beta}{3}\sum_x\left\{b_0\sum_{\substack{\mu,\nu=1\\1\leq\mu<\nu}}^{4}\left[1-\mathrm{Re\,Tr}\,(U^{1\times1}_{x,\mu,\nu})\right] + b_1\sum_{\substack{\mu,\nu=1\\\mu\neq\nu}}^{4}\left[1-\mathrm{Re\,Tr}(U^{1\times2}_{x,\mu,\nu})\right]\right\}$$

The gauge fields are to be nonperturbatively obtained from lattice QCD simulations and applied then to get the gluon Green's functions

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Symmetric configuration:

$q^2 = r^2 = p^2$ and $q\cdot r = q\cdot p = r\cdot p = -q^2/2$;

$$\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\Gamma_{\alpha'\mu'\nu'}(q,r,p)\Delta_{\alpha'\alpha}(q)\Delta_{\mu'\mu}(r)\Delta_{\nu'\nu}(p),$$

$$G_{\alpha\mu\nu}(q,r,p) = T^{sym}(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + S^{sym}(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

$$\Gamma_{\alpha\mu\nu}(q,r,p) = \Gamma^{sym}_{T}(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + \Gamma^{sym}_{S}(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

$$\lambda^{\rm tree}_{\alpha\mu\nu}(q,r,p) = \Gamma^{(0)}_{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p).$$

$$\lambda^{S}_{\alpha\mu\nu}(q,r,p) = (r-p)_\alpha(p-q)_\mu(q-r)_\nu/r^2.$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q)A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

In Landau gauge and for particular kinematical configurations, transversality and Bose symmetry make possible a simple tensorial decomposition of the gluon Green's function

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$$T^{\rm sym}(q^2) = g\,\Gamma^{\rm sym}_T(q^2)\,\Delta^3(q^2),$$
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$$W_{\alpha\mu\nu} = \lambda^{tree}_{\alpha\mu\nu} + \lambda^{S}_{\alpha\mu\nu}/2$$

$$T^{\rm sym}(q^2) = g\,\Gamma^{\rm sym}_T(q^2)\,\Delta^3(q^2),$$
$$S^{\rm sym}(q^2) = g\,\Gamma^{\rm sym}_S(q^2)\,\Delta^3(q^2).$$

$$T^{\rm sym}(q^2) = \left.\frac{W_{\alpha\mu\nu}(q,r,p)\,\mathcal{G}_{\alpha\mu\nu}(q,r,p)}{W_{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,r,p)}\right|_{\rm sym},$$

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Asymmetric configuration:

$$q \to 0\,;\; r^2 = p^2 = -p\cdot r$$

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$$W_{\alpha\mu\nu} = \lambda^{tree}_{\alpha\mu\nu} + \lambda^{S}_{\alpha\mu\nu}/2$$

$$T^{\text{asym}}(r^2) = g\,\Gamma^{\text{asym}}_T(r^2)\,\Delta(0)\,\Delta^2(r^2),$$

$$T^{\text{asym}}(r^2) = \left.\frac{W_{\alpha\mu\nu}(q,r,p)\,\mathcal{G}_{\alpha\mu\nu}(q,r,p)}{W_{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,r,p)}\right|_{\text{asym}}$$

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Symmetric configuration:

$q^2 = r^2 = p^2$ and $q\cdot r = q\cdot p = r\cdot p = -q^2/2$;

$$\Delta_R(q^2;\mu^2) = Z_A^{-1}(\mu^2)\,\Delta(q^2),$$
$$T_R^{\rm sym}(q^2;\mu^2) = Z_A^{-3/2}(\mu^2) T^{\rm sym}(q^2),$$

$$g^{\rm sym}(q^2) = q^3 \frac{T^{\rm sym}(q^2)}{[\Delta(q^2)]^{3/2}} = q^3 \frac{T_R^{\rm sym}(q^2;\mu^2)}{[\Delta_R(q^2;\mu^2)]^{3/2}}.$$

MOM renormalization prescription:

$$\Delta_R(q^2;q^2) = Z_A^{-1}(q^2)\,\Delta(q^2) = 1/q^2,$$
$$T_R^{\rm sym}(q^2;q^2) = Z_A^{-3/2}(q^2)\,T^{\rm sym}(q^2) = g_R^{\rm sym}(q^2)/q^6.$$

$$T^{\rm sym}(q^2) = g\,\Gamma_T^{\rm sym}(q^2)\,\Delta^3(q^2),$$

$$g^{sym}(\mu^2)\,\Gamma^{sym}_{T,R}(q^2;\mu^2) = \frac{g^{sym}(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2) P_{\mu\nu}(q),$$

$$T^{\rm sym}(q^2) = \left.\frac{W_{\alpha\mu\nu}(q,r,p)\,\mathcal{G}_{\alpha\mu\nu}(q,r,p)}{W_{\alpha\mu\nu}(q,r,p) W_{\alpha\mu\nu}(q,r,p)}\right|_{\rm sym},$$

After the required projection and the appropriate renormalization, one can define a QCD coupling from the Green's functions, and relate it to the 1PI vertex form factor, in both symmetric...

The vertex and the three-gluon Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

Asymmetric configuration:

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$$T_R^{\rm sym}(q^2;\mu^2) = Z_A^{-3/2}(\mu^2) T^{\rm sym}(q^2),$$

$$g^{\rm asym}(r^2) = r^3 \frac{T^{\rm asym}(r^2)}{[\Delta(r^2)]^{1/2}\Delta(0)} = r^3 \frac{T_R^{\rm asym}(r^2;\mu^2)}{[\Delta_R(r^2;\mu^2)]^{1/2}\Delta_R(0;\mu^2)}$$

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$$g^{asym}(\mu^2)\,\Gamma^{asym}_{T,R}(q^2;\mu^2) = \frac{g^{asym}(q^2)}{\left[q^2\,\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

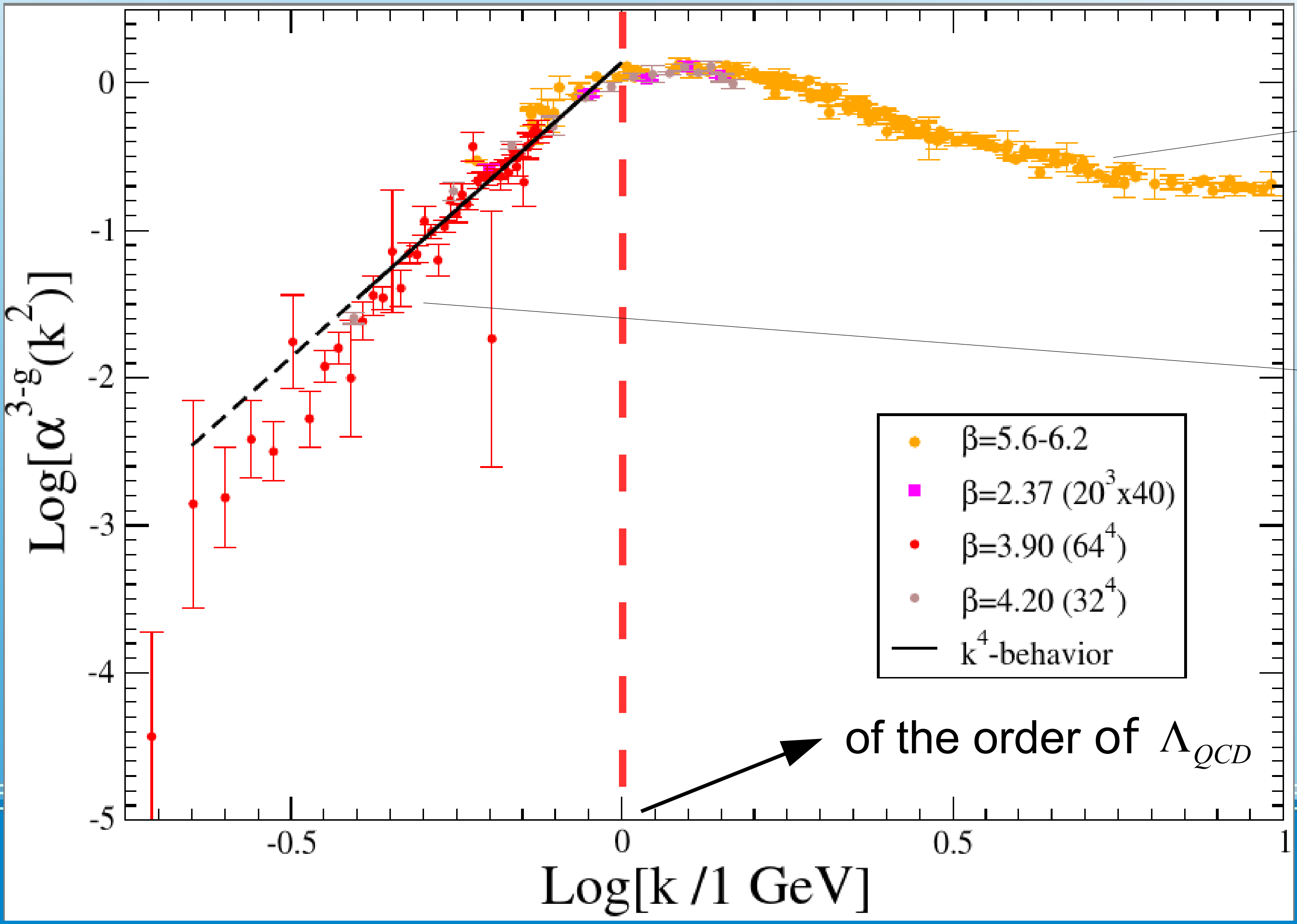
$$T^{\rm asym}(r^2) = \left.\frac{W_{\alpha\mu\nu}(q,r,p)\,\mathcal{G}_{\alpha\mu\nu}(q,r,p)}{W_{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,r,p)}\right|_{\rm asym}$$

After the required projection and the appropriate renormalization, one can define a QCD coupling from the Green's functions, and relate it to the 1PI vertex form factor, in both symmetric and asymmetric kinematical configurations.

The vertex and the three-gluon Green's function

Let's focus on the symmetric coupling:

$$\alpha^{sym}(q^2) = \frac{\left(g^{sym}(q^2)\right)^2}{4\pi} = \frac{q^6}{4\pi} \frac{\left[T^{sym}(q^2)\right]^2}{\left[\Delta(q^2)\right]^3}$$



Logarithmic running accounted for by perturbation theory

A k^4 power law clearly appears to rise up from data within the IR domain

Can we somehow interpret this feature?

Two domains, wherein very different running behaviors appear to dominate each, lie separated by a momentum scale of the order of Λ_{QCD}

Multi-instanton background

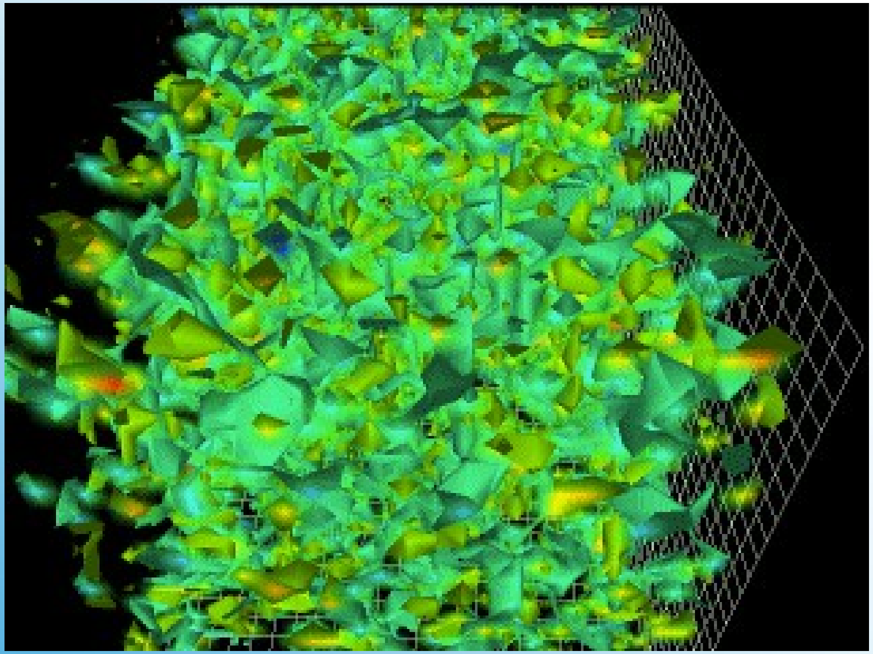
The classical gauge field solution from a multi-instanton ensemble can be cast as the so-called *ratio ansatz* [E.V. Shuryak; Nucl.Phys.B302(1988)574]

$$g_0 B^a_\mu(x) = \frac{2 \sum_{i=I,A} R^{a\alpha}_{(i)} \bar{\eta}^\alpha_{\mu\nu} \frac{y^\nu_i}{y^2_i} \rho^2_i \frac{f(|y_i|)}{y^2_i}}{1 + \sum_{i=I,A} \rho^2_i \frac{f(|y_i|)}{y^2_i}},$$

$$y_i = x - z_i$$

$\bar{\eta}_{\mu\nu}, R^{a\alpha}_{(i)}$ `t Hooft symbols and color rotation matrices

ρ_i instanton radius



http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/

"Visualizations of QCD" by Derek B. Leinweber

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$y_i = x - z_i$

$$\to \begin{cases} \sim 2 \sum_{i=I,A} R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \rho_i^2 \frac{f(|y_i|)}{y_i^2} & y_i >> \rho_i \text{ for all } i, \\ \sim 2 \sum_{i=I,A} R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \frac{f(|y_i|)}{f(|y_i|) + \frac{y_i^2}{\rho_i^2}} & y_j << \rho_j, \; y_i >> \rho_i \text{ for any } i \neq j, \end{cases}$$

$\overline{\eta}_{\mu\nu}, R^{a\alpha}_{(i)}$ `t Hooft symbols and color rotation matrices

ρ_i instanton radius

f(z) is a shape function [f(0)=1] that might be eventually obtained by minimization of the action per particle for some statistical ensemble of instantons (classical background).

Then:

$$g_0 B^a_\mu(\mathbf{x}) = 2 \sum_i R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \phi_{\rho_i}\left(\frac{|y_i|}{\rho_i}\right)$$

D. Diakonov, V. Petrov; Nucl.Phys.B45386(1992)236
Boucaud et al.; Phys.Rev.D70(2004)114503

$$\phi_\rho(z) = \begin{cases} \frac{f(\rho z)}{f(\rho z) + z^2} \simeq \frac{1}{1+z^2} & z \ll 1 \\ \frac{f(\rho z)}{z^2} & z \gg 1 \end{cases}$$

The classical gauge field can be effectively accounted for by an independent pseudo-particule sum ansatz approach in both large- and low-distance regimes.

Multi-instanton background

$$g_0^m G^{(m)}(k^2) = \frac{1}{N} W^{\mu_1 \dots \mu_m}_{a_1 \dots a_m} \langle g_0 A^{a_1}_{\mu_1}(k_1) \dots g_0 A^{a_m}_{\mu_m}(k_m) \rangle$$

$$G^{(2)}(k^2) = \Delta(k^2);\; G^{(3)}(k^2) = T^{sym}(k^2)$$

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Multi-instanton background

Instanton density

$$g_0^m G^{(m)}(k^2) = \frac{1}{N} W^{\mu_1\ldots\mu_m}_{a_1\ldots a_m} \langle g_0 A^{a_1}_{\mu_1}(k_1)\ldots g_0 A^{a_m}_{\mu_m}(k_m)\rangle = \frac{k^{2-m}}{m4^{m-1}}\, n \,\langle\, \rho^{3m} I^m(k\rho)\,\rangle$$

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$$I(s) = \frac{8\pi^2}{s}\int_0^\infty z\,dz\, J_2(sz)\phi(z)$$

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$$\alpha^{sym}(k^2) = \frac{k^6}{4\pi} \frac{\left[G^{(3)}(k^2)\right]^2}{\left[G^{(2)}(k^2)\right]^3} = \frac{k^4}{18\pi n} \frac{\langle \rho^9 I^3(k\rho) \rangle^2}{\langle \rho^6 I^2(k\rho) \rangle^3}$$

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$$\phi_\rho(z) = \begin{cases} [illegible] & z \ll 1 \\ \dfrac{f(\rho z)}{z^2} & z \gg 1 \end{cases}$$

$$\Phi_\rho(0) = 1$$

$$\alpha^{sym}(k^2) = \frac{k^6}{4\pi} \frac{\left[G^{(3)}(k^2)\right]^2}{\left[G^{(2)}(k^2)\right]^3} = \frac{k^4}{18\pi n} \frac{\langle \rho^9 I^3(k\rho)\rangle^2}{\langle \rho^6 I^2(k\rho)\rangle^3}$$

$$1 + \mathcal{O}\left(\frac{\delta\rho^2}{k^2 \bar{\rho}^4}\right)$$

where $\bar{\rho} = \sqrt{\langle \rho^2 \rangle}$ and $\delta\rho^2 = \langle (\rho - \bar{\rho})^2 \rangle$

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$$I(s) = \frac{8\pi^2}{s} \int_0^\infty z dz \, J_2(sz) \phi(z)$$

$$\phi_\rho(z) = \begin{cases} \frac{f(\dots)}{f(\rho z) \dots} & z \ll 1 \\ \frac{f(\rho z \dots)}{z^2} & z \gg 1 \end{cases}$$

$$\Phi_\rho(0) = 1$$

$$o(z^\infty)$$

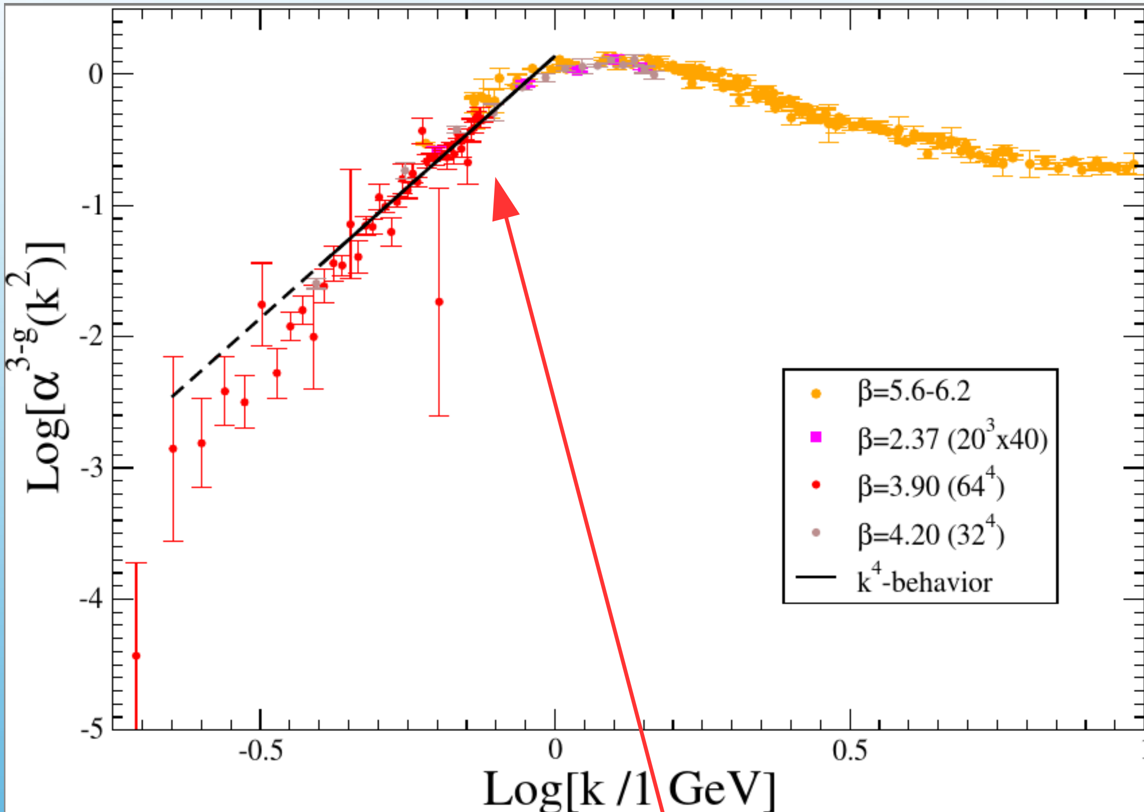
$$\alpha^{sym}(k^2) = \frac{k^6}{4\pi} \frac{\left[G^{(3)}(k^2)\right]^2}{\left[G^{(2)}(k^2)\right]^3} = \frac{k^4}{18\pi n} \frac{\langle \rho^9 I^3(k\rho) \rangle^2}{\langle \rho^6 I^2(k\rho) \rangle^3}$$

$$\begin{cases} 1 + \mathcal{O}\left(\frac{\delta\rho^2}{k^2 \bar{\rho}^4}\right) \\ 1 + 48 \frac{\delta\rho^2}{\bar{\rho}^2} + \mathcal{O}\left(k^2 \delta\rho^2, \frac{\delta\rho^4}{\bar{\rho}^4}\right) \end{cases}$$

where $\bar{\rho} = \sqrt{\langle \rho^2 \rangle}$ and $\delta\rho^2 = \langle (\rho - \bar{\rho})^2 \rangle$

The asymptotic behavior at both the large- and low-momentum limits appears to be driven by the fourth power of the momentum, the result relying on a very general ground, irrespective of the details of the profile and its breaking of the scale independence.

Multi-instanton background



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This is consistent with the low-momentum behavior obtained from the lattice gluon correlation functions!!!

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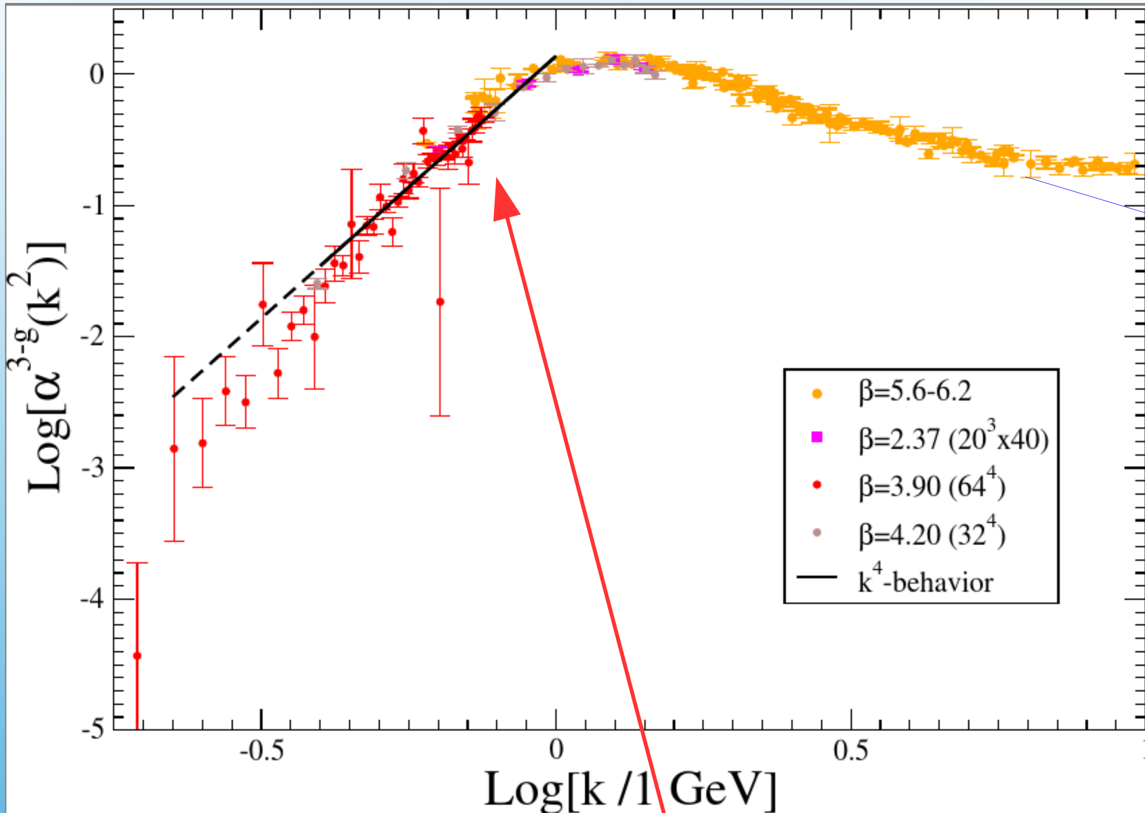

The large-momentum limit in the field of a multi-instanton solution appears here hidden by the quantum UV fluctuations!!!

$$\alpha^{sym}(k^2) = \frac{k^6}{4\pi}\frac{\left[G^{(3)}(k^2)\right]^2}{\left[G^{(2)}(k^2)\right]^3} = \frac{k^4}{18\pi n}\frac{\langle \rho^9 I^3(k\rho)\rangle^2}{\langle \rho^6 I^2(k\rho)\rangle^3}$$

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This is consistent with the low-momentum behavior obtained from the lattice gluon correlation functions!!!

The Wilson flow smoothing procedure

The Wilson flow $B_\mu(t,x)$ of an SU(N) gauge field is defined by [M. Luescher; JHEP02(2010)071]

$$\partial_t B_\mu = D_\nu G_{\nu\mu}$$

where $t = a^2 \tau$ is the so-called flow time and

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$
$$D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

with the initial condition $B_\mu(0,x) = A_\mu(x)$.

Then, the expansion in terms of $A_\mu(x)$ gives at tree-level:

$$B_\mu(t,x) = \int d^4 y\, K(t; x-y)\, A_\mu(x)$$

$$K(t;x) = \frac{e^{-x^2/4t}}{(4\pi t)^2}$$

The Wilson flow has been proven to be an useful tool to deprive the lattice gauge fields from their short-distance (UV) quantum fluctuations.

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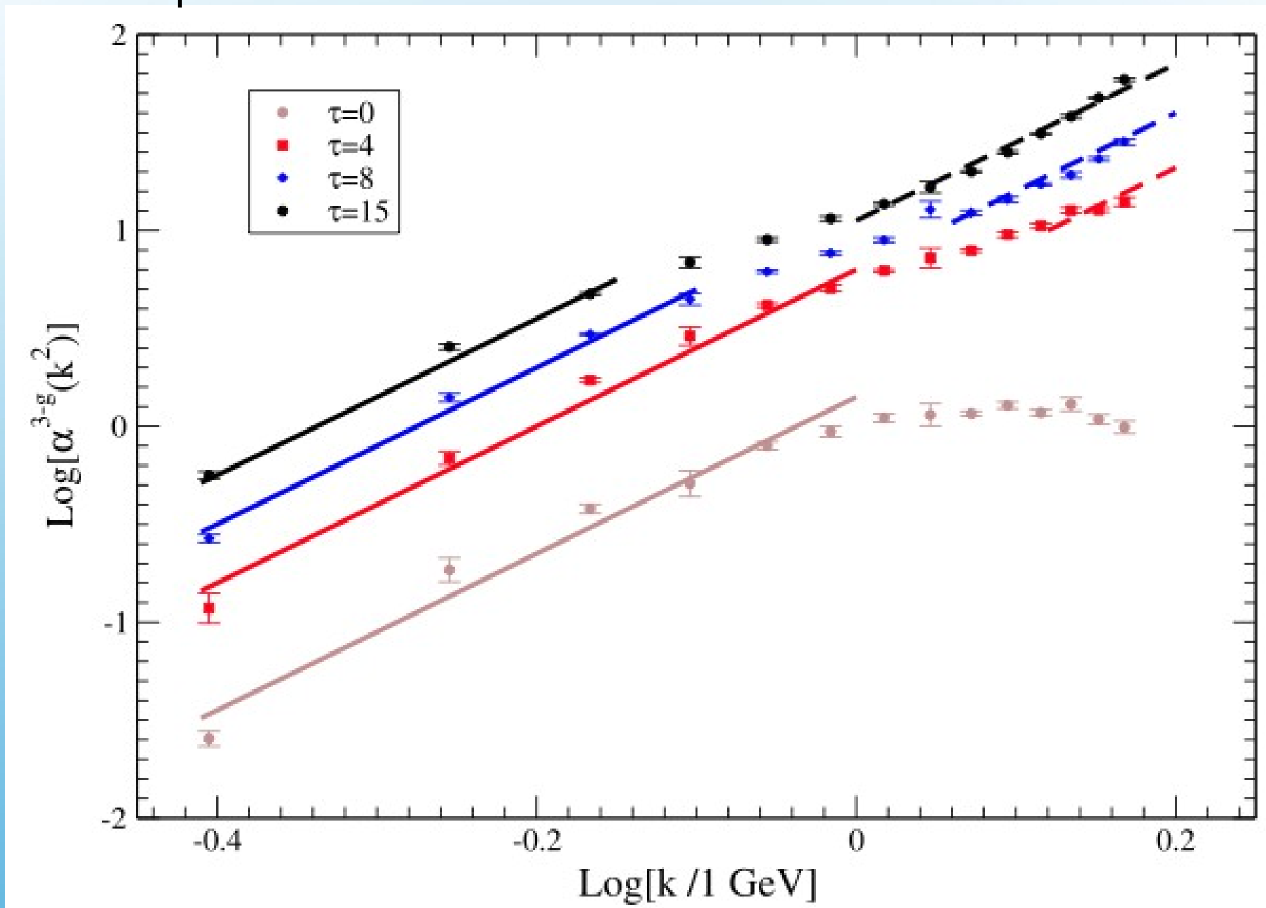
Table 1
Estimates for the densities, obtained as explained in the text, for the different flow times, also expressed in physical units. For this to be done, according to [27], we have defined $\sqrt{8t_0} = 0.3$ fm, whence $t_0 = a^2\tau_0 = 0.0113$ fm^2 and $t = \frac{\tau}{\tau_0}t_0$. At $\tau = 4$, in the unquenched case, the characteristic diffusion length is so small that quantum fluctuations have not been properly removed yet.

	τ	t/t_0	n (fm^{-4})
Quenched	4	6.84	
	8	13.7	
	15	25.6	
Unquenched	4	2.34	
	8	4.70	
	15	8.84	

The Wilson flow has been proven to be an useful tool to deprive the lattice gauge fields from their short-distance (UV) quantum fluctuations.

The Wilson flow smoothing procedure

$\beta = 4.20$



$$\alpha(k^2) = \frac{k^4}{18\pi n} \times \begin{cases} 1 + \mathcal{O}\left(\frac{\delta\rho^2}{k^2\bar{\rho}^4}\right) \\ 1 + 48\frac{\delta\rho^2}{\bar{\rho}^2} + \mathcal{O}\left(k^2\delta\rho^2, \frac{\delta\rho^4}{\bar{\rho}^4}\right) \end{cases}$$

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	τ	t/t_0	n (fm^{-4})
Quenched	4	6.84	3.5(1)
	8	13.7	1.75(4)
	15	25.6	0.98(5)
Unquenched	4	2.34	
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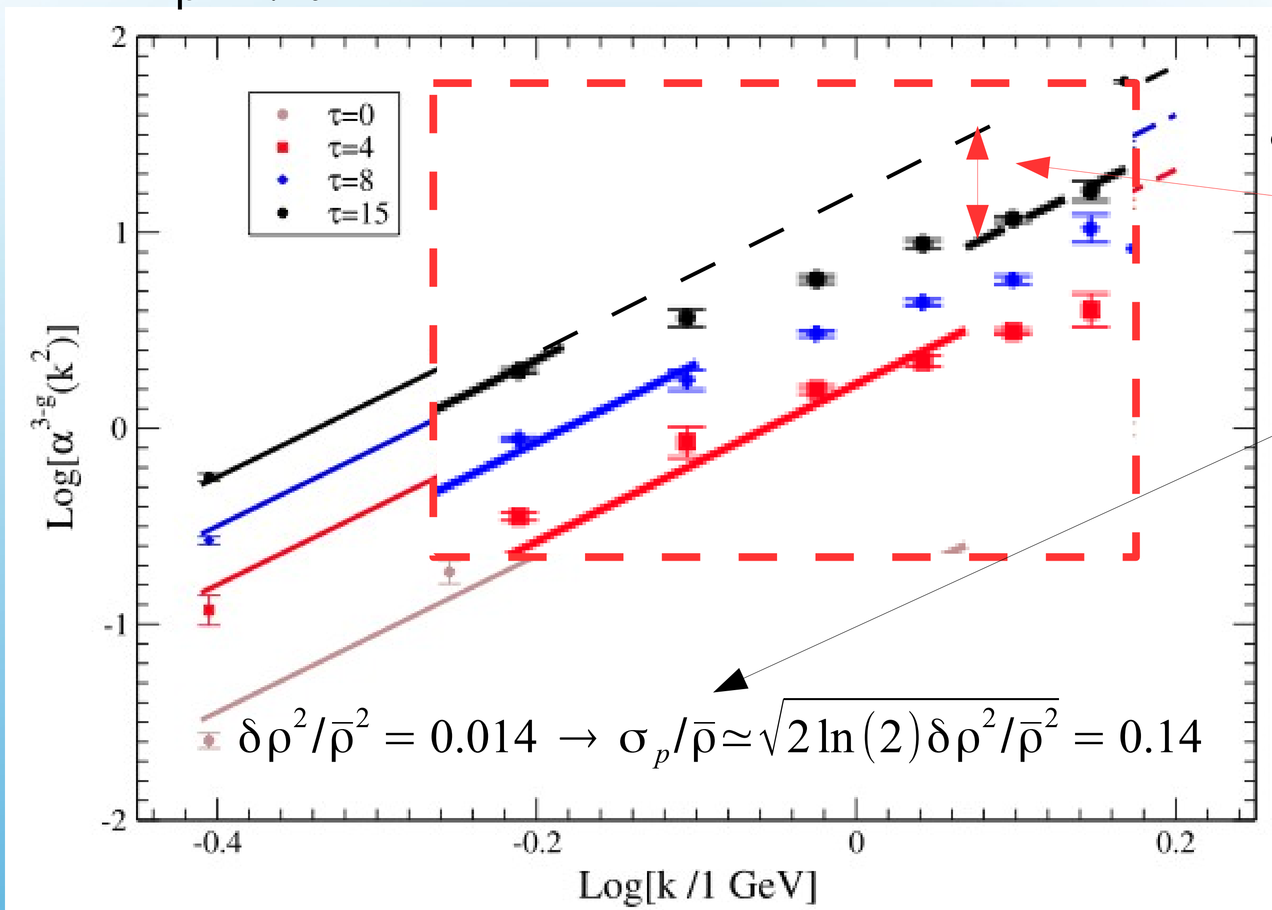

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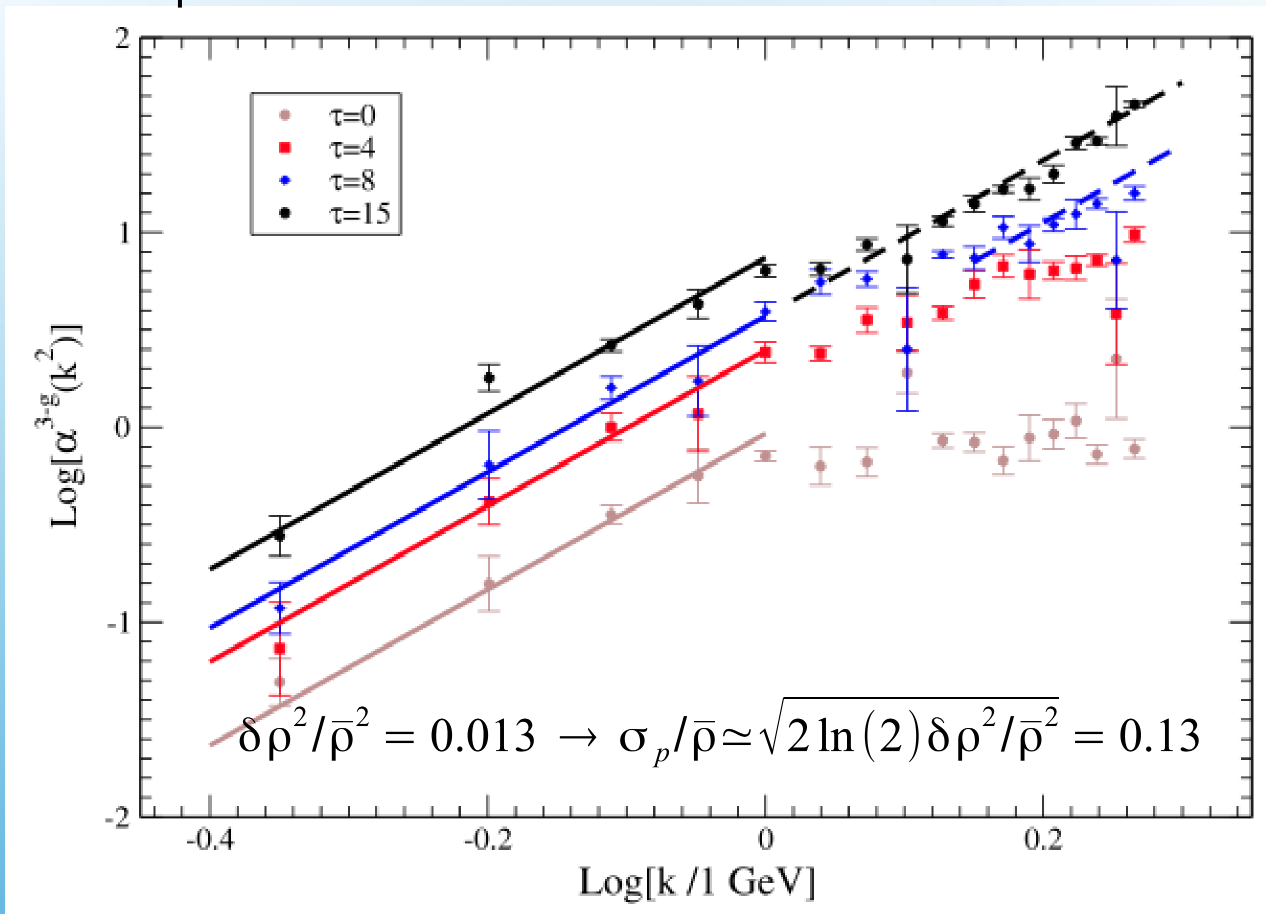
Fairly consistent with lattice estimates made by applying direct instanton detection: $\sigma_p/\bar{\rho} \sim 0.17 - 0.22$ [D.A. Smith, M.J. Teper; PRD58(1998)014505]

The Wilson flow has been proven to be an useful tool to deprive the lattice gauge fields from their short-distance (UV) quantum fluctuations.
The main features observed in the gluon correlations obtained with lattice flown gauge fields can be well described within the multi-instanton approach framework.

The Wilson flow smoothing procedure

$\beta = 1.95$



$$\alpha(k^2) = \frac{k^4}{18\pi n} \times \begin{cases} 1 + \mathcal{O}\left(\frac{\delta\rho^2}{k^2\bar{\rho}^4}\right) \\ 1 + 48\frac{\delta\rho^2}{\bar{\rho}^2} + \mathcal{O}\left(k^2\delta\rho^2, \frac{\delta\rho^4}{\bar{\rho}^4}\right) \end{cases}$$

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Unquenched	4	2.34	–
	8	4.70	6.8(5)
	15	8.84	3.0(2)

The Wilson flow has been proven to be an useful tool to deprive the lattice gauge fields from their short-distance (UV) quantum fluctuations.
The main features observed in the gluon correlations obtained with lattice flown gauge fields can be well described within the multi-instanton approach framework.

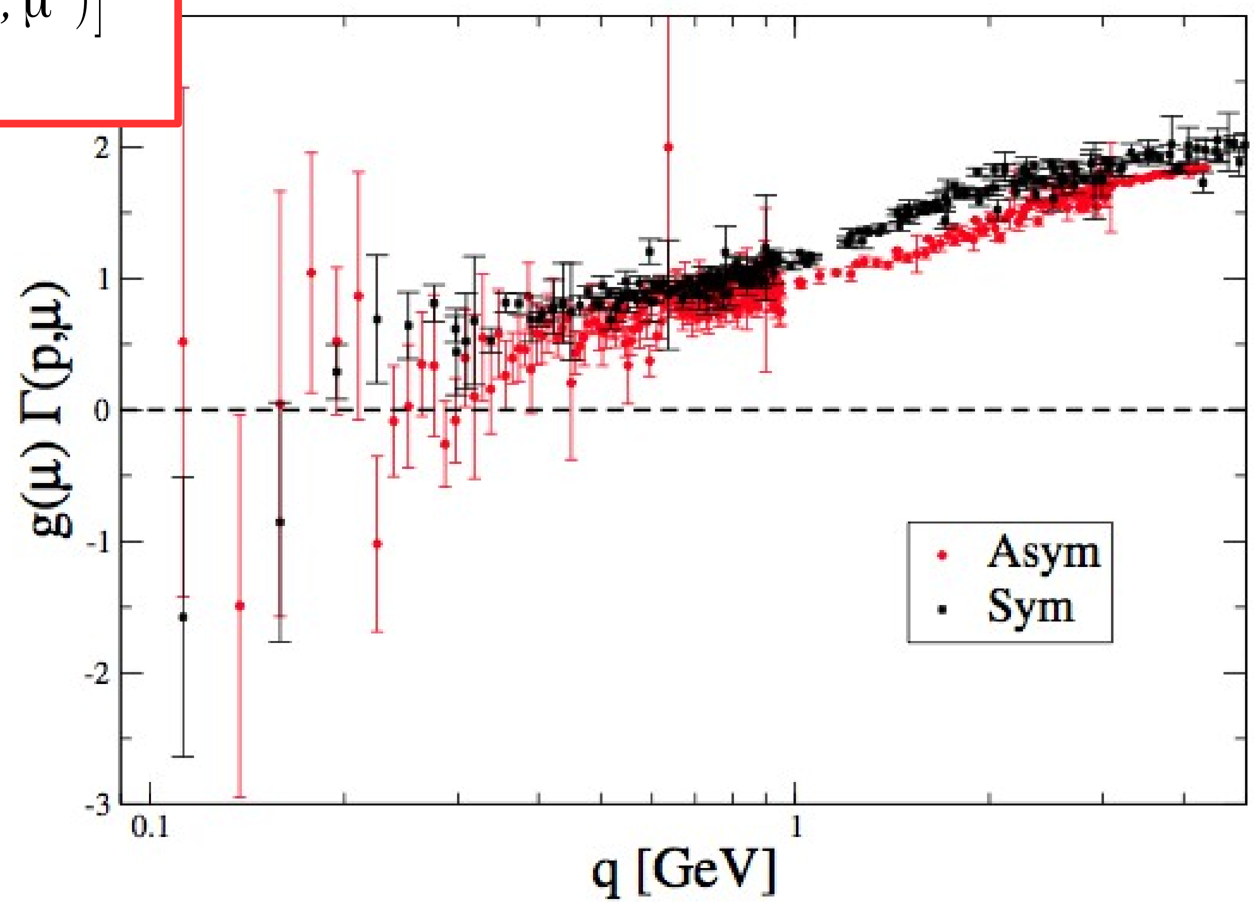
The zero-crossing of the three-gluon vertex

$$g^i(\mu^2)\,\Gamma^i_{T,R}(q^2;\mu^2) = \frac{g^i(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$i = sym\,,\, asym.$

$$g^{sym}(q^2) = q^3\frac{T^{sym}(q^2)}{[\Delta(q^2)]^{3/2}}$$

$$g^{asym}(q^2) = q^3\frac{T^{asym}(q^2)}{\Delta(0)[\Delta(q^2)]^{1/2}}$$



The form factor for the tree-level tensor structure of the 1PI three-gluon vertex appear to show similar IR behavior in both symmetric and asymmetric kinematic configurations of momenta. The asymmetric case is however noisier than the symmetric one!

The zero-crossing of the three-gluon vertex

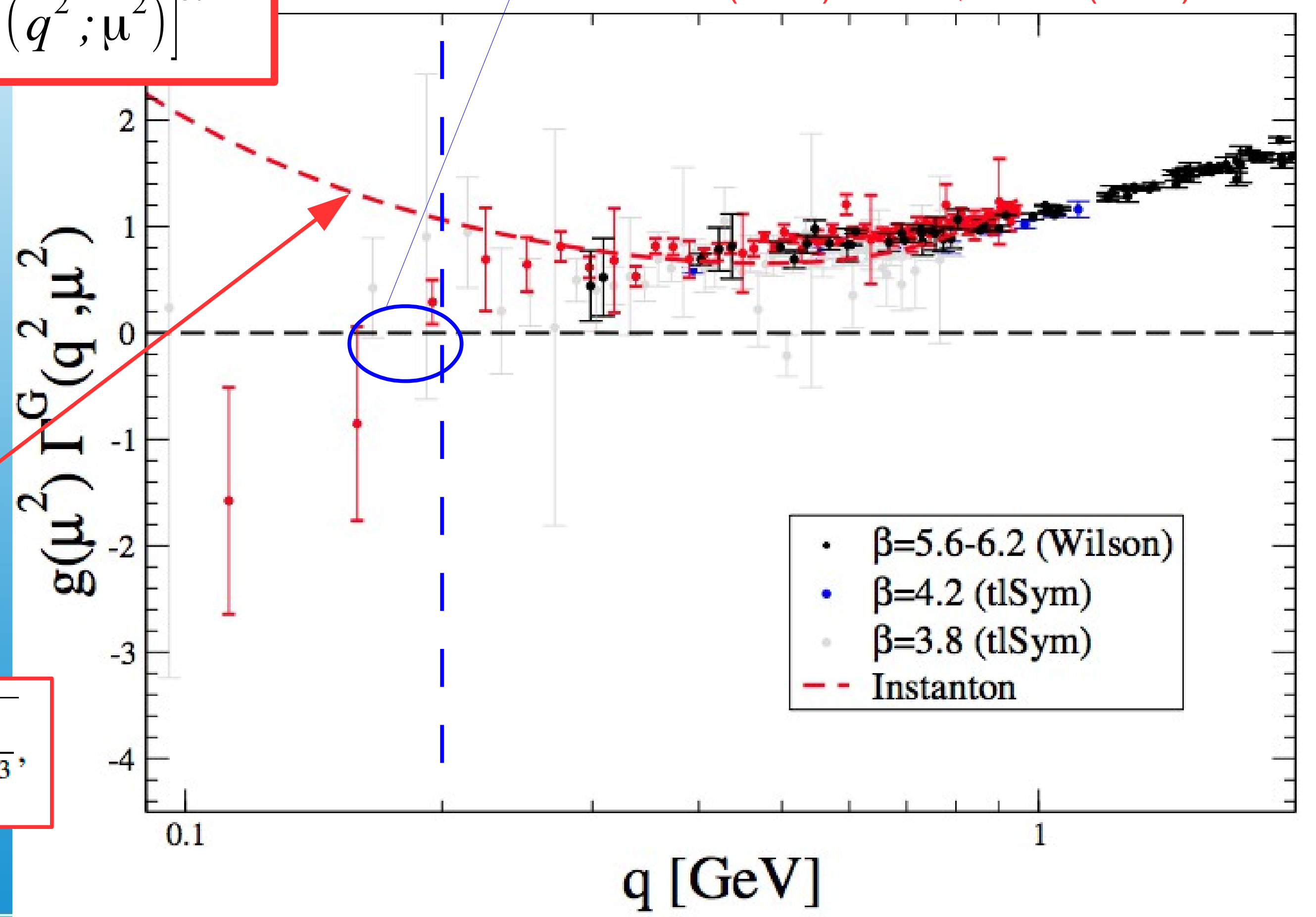
A.C Aguilar et al.; PRD89(2014)05008
A. Blum et al.; PRD89(2014)061703
G. Eichmann et al.; PRD89(2014)105014
A.K. Cyrol et al.; arXiv:1605.01856[hep-ph]
A. Cucchieri, A. Maas, T. Mendes;
PRD74(2006)014503;PRD77(2008)094510

$$g^{sym}(\mu^2)\Gamma^{sym}_{T,R}(q^2;\mu^2) = \frac{g^{sym}(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$$g^{sym}(q^2) = q^3\frac{T^{sym}(q^2)}{[\Delta(q^2)]^{3/2}}$$

$$g^{\rm sym}(\mu^2)\Gamma^{\rm sym}_{T,R}(q^2;\mu^2) \simeq \sqrt{\frac{2}{9np^2\,[\Delta(p^2;\mu^2)]^3}},$$

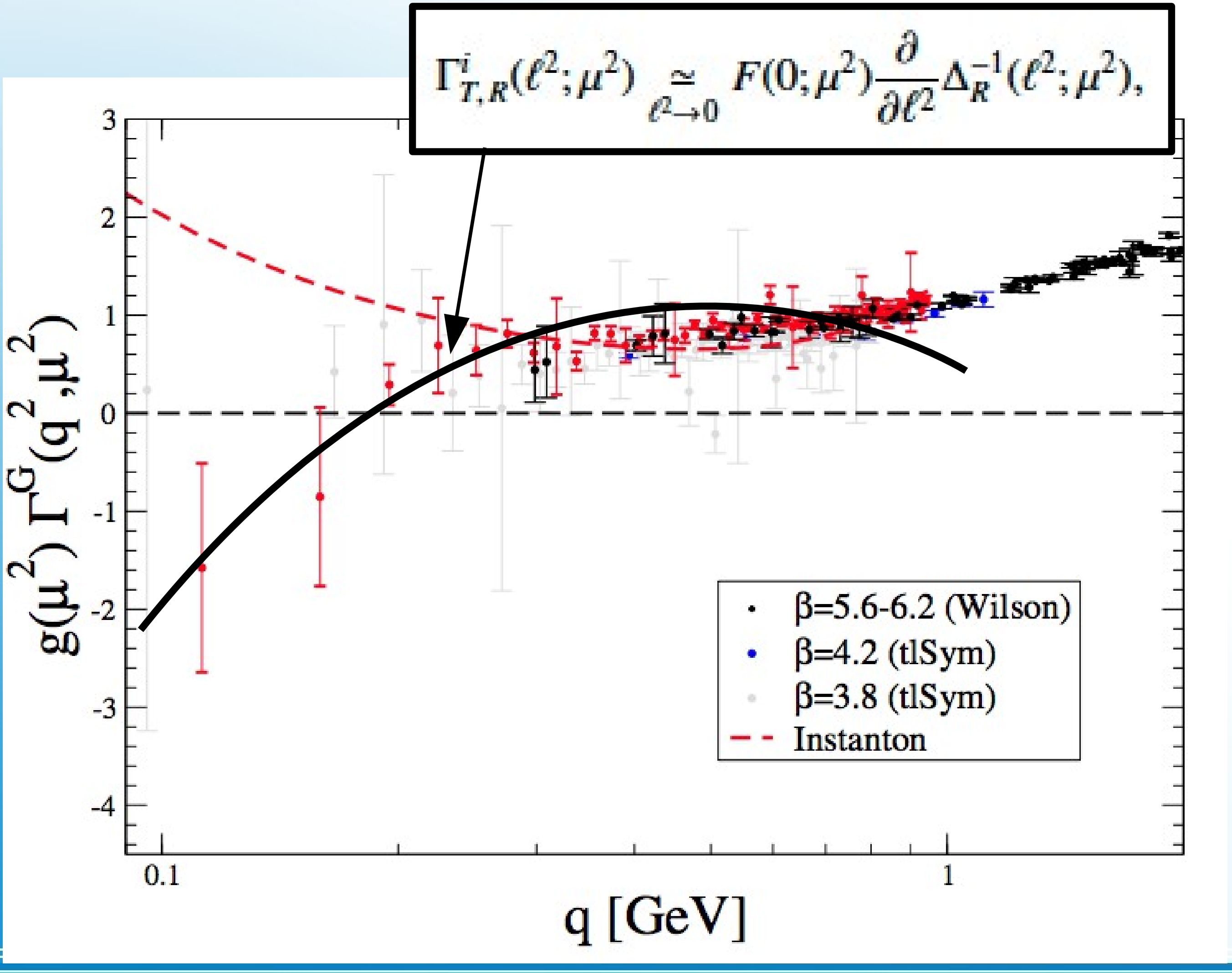
zero-crossing



Let's then focus (again) on the symmetric case: the form factor appears to change its sign at very deep IR momenta and show then a zero-crossing. This feature, happening below ~0.2 GeV, is not accounted for by the semiclassical instanton picture.

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008



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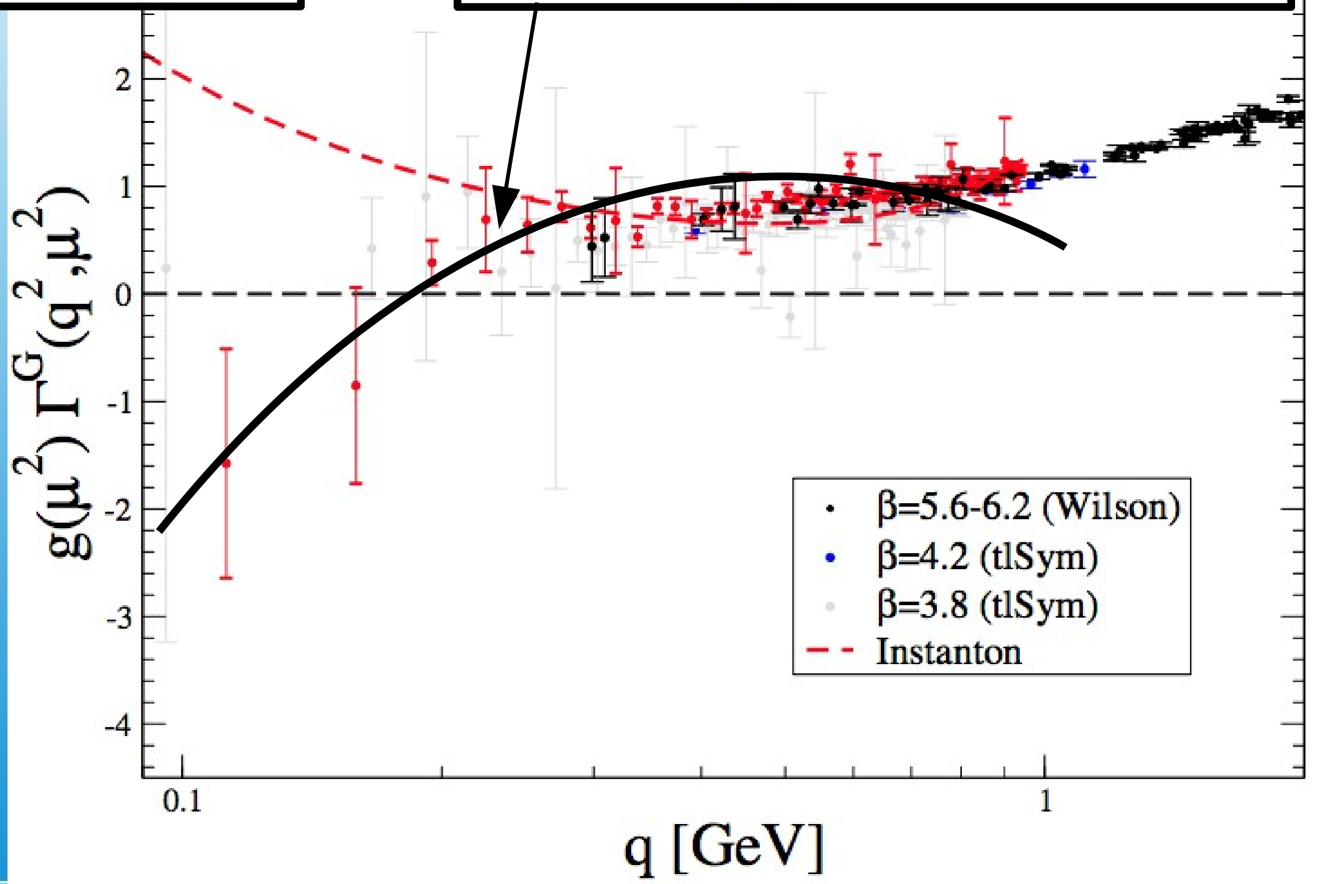
The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008

$$\Delta_R^{-1}(q^2;\mu^2) \underset{q^2\to 0}{=} q^2\left[a + b\log\frac{q^2+m^2}{\mu^2} + c\log\frac{q^2}{\mu^2}\right] + m^2,$$

$$\Gamma^i_{T,R}(\ell^2;\mu^2) \underset{\ell^2\to 0}{\simeq} F(0;\mu^2)\frac{\partial}{\partial \ell^2}\Delta_R^{-1}(\ell^2;\mu^2),$$



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The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008
M.Tissier, N.Wschebor; PRD84(2011)045018

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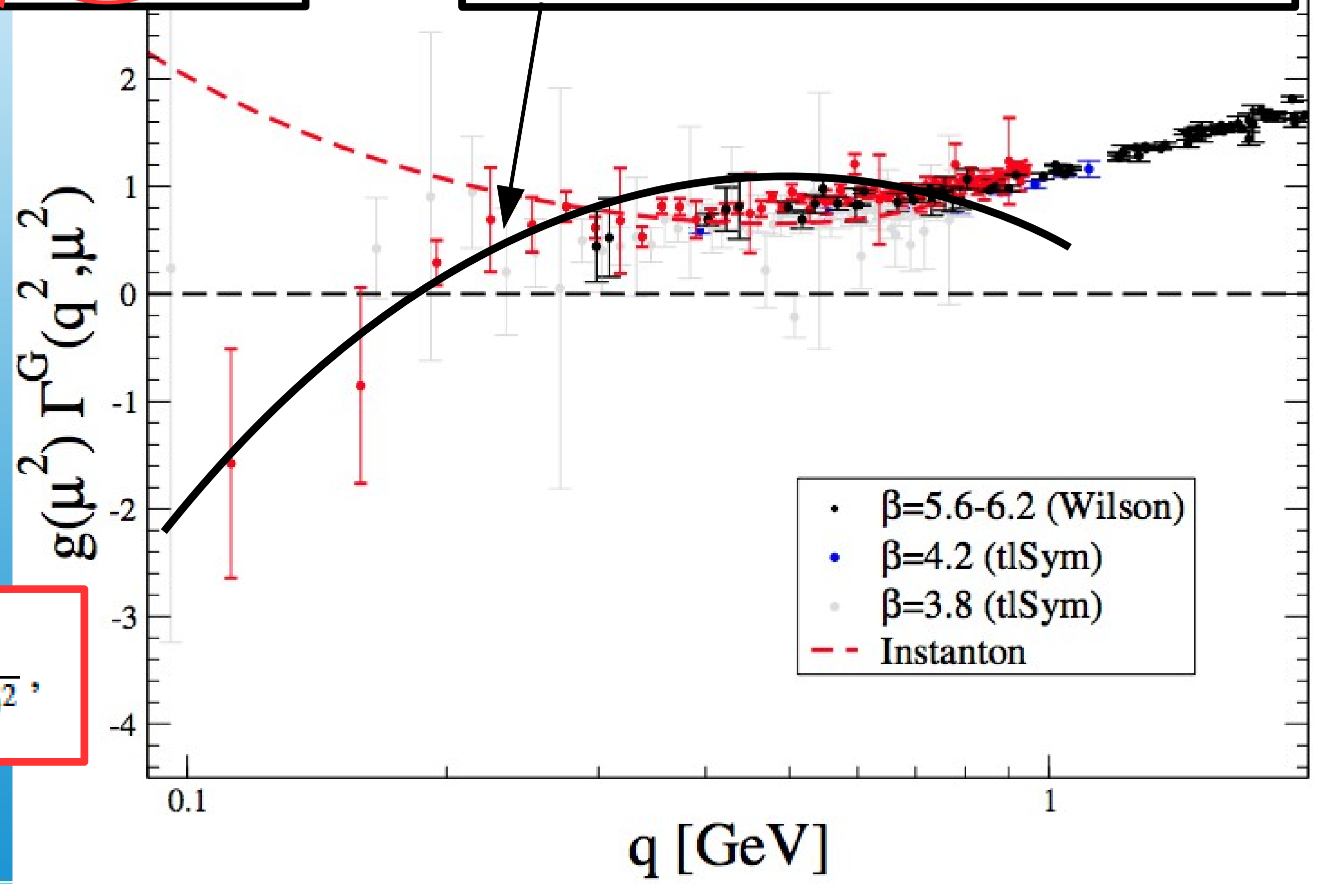

$$\Pi_c(q^2) = \frac{g^2 C_A}{6} q^2 F(q^2) \int_k \frac{F(k^2)}{k^2(k+q)^2},$$

Ghost loop contribution!!!

Let's then focus (again) on the symmetric case: the form factor appears to change its sign at very deep IR momenta and show then a zero-crossing. This feature, happening below ~0.2 GeV, is not accounted for by the semiclassical instanton picture. It's a soft quantum effect!!!

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008
M.Tissier, N.Wschebor; PRD84(2011)045018

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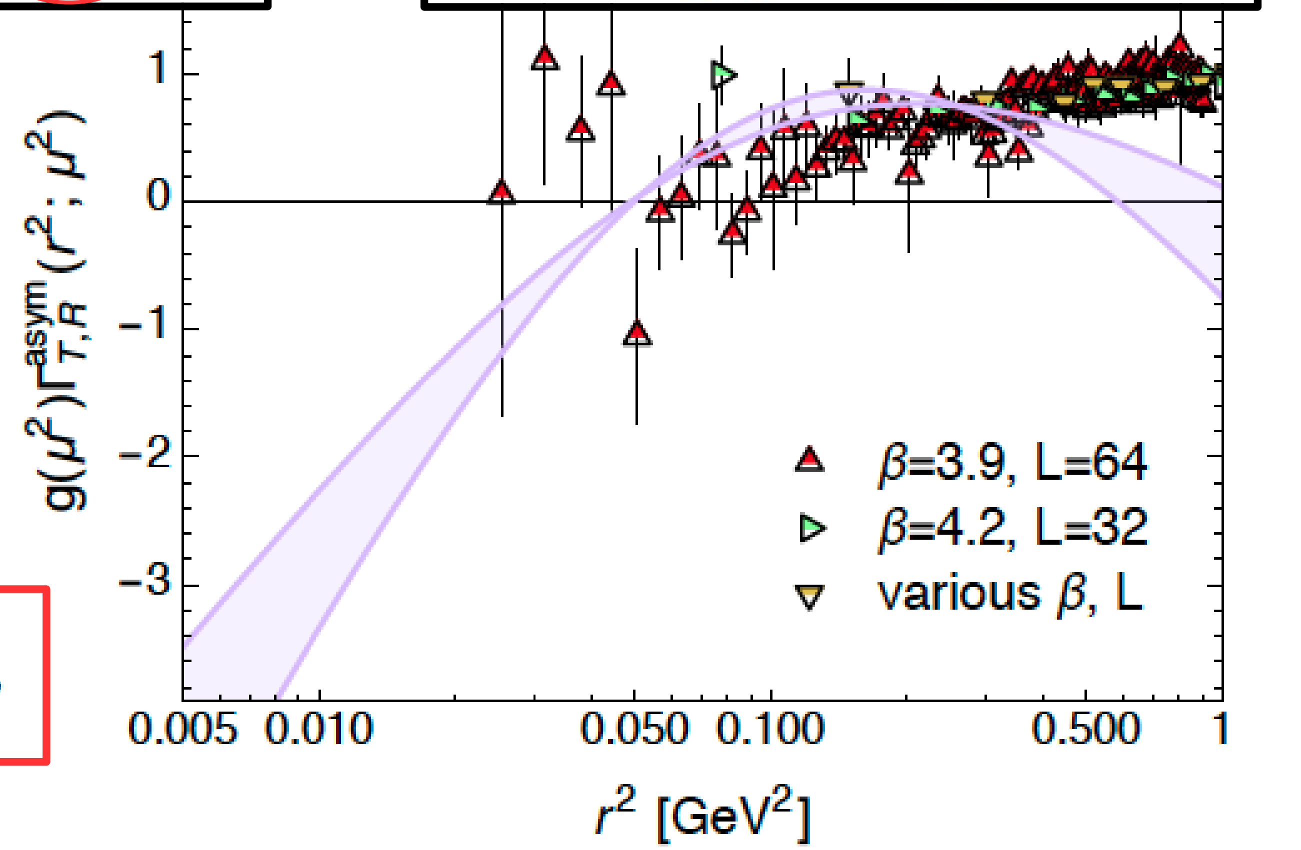

The data for the asymmetric case display a behavior much noisier... but compatible with the predicted one on the basis of the soft quantum effect that comes out from the ghost sector.

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008
M.Tissier, N.Wschebor; PRD84(2011)045018

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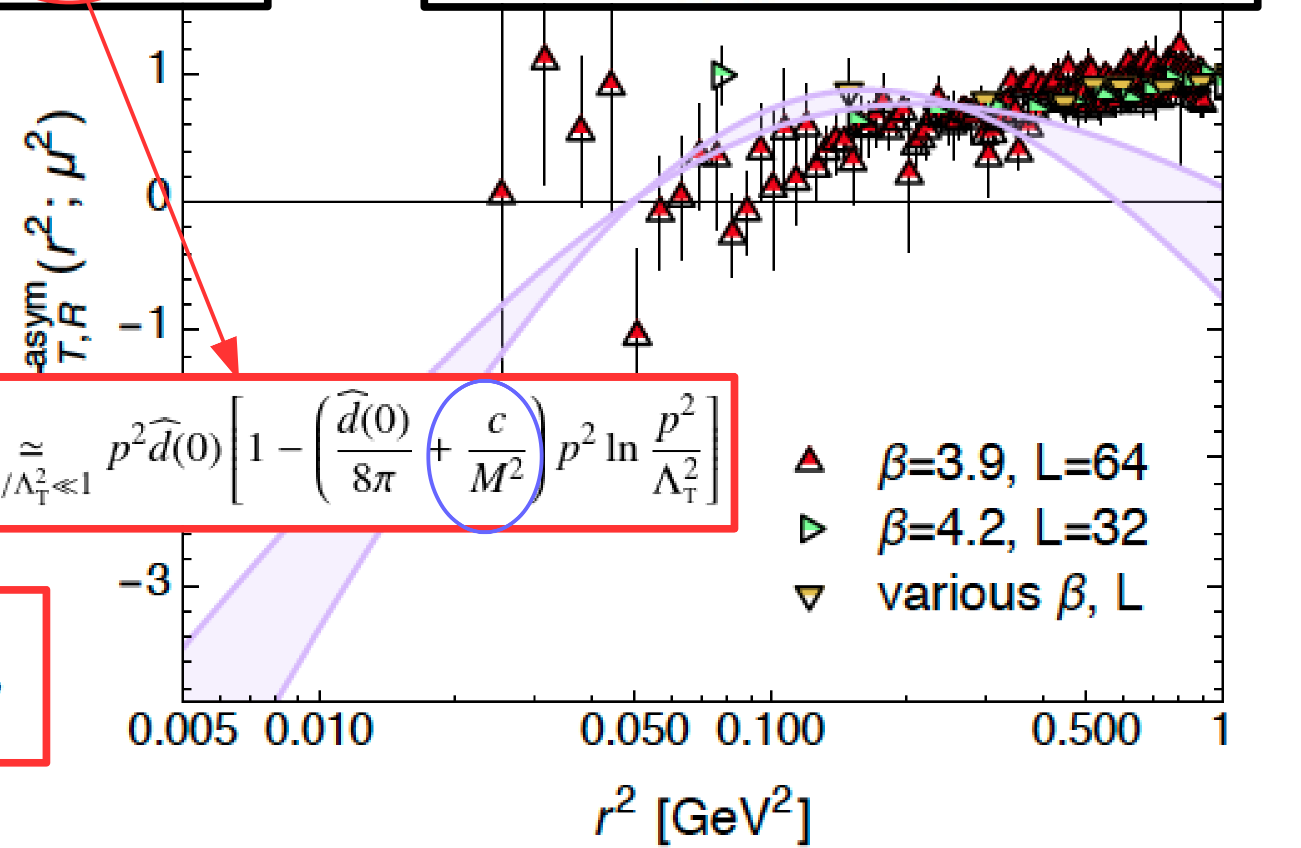


$$\mathcal{I}(p^2) \underset{p^2/\Lambda_T^2\ll 1}{\simeq} p^2\widehat{d}(0)\left[1-\left(\frac{\widehat{d}(0)}{8\pi}+\frac{c}{M^2}\right)p^2\ln\frac{p^2}{\Lambda_T^2}\right]$$

$$\Pi_c(q^2) = \frac{g^2 C_A}{6}q^2 F(q^2)\int_k \frac{F(k^2)}{k^2(k+q)^2},$$

The ghost-loop contribution responsible for the zero-crossing can be also related to the running interaction for the quark-gap equation (Daniele's talk).

Conclusions:

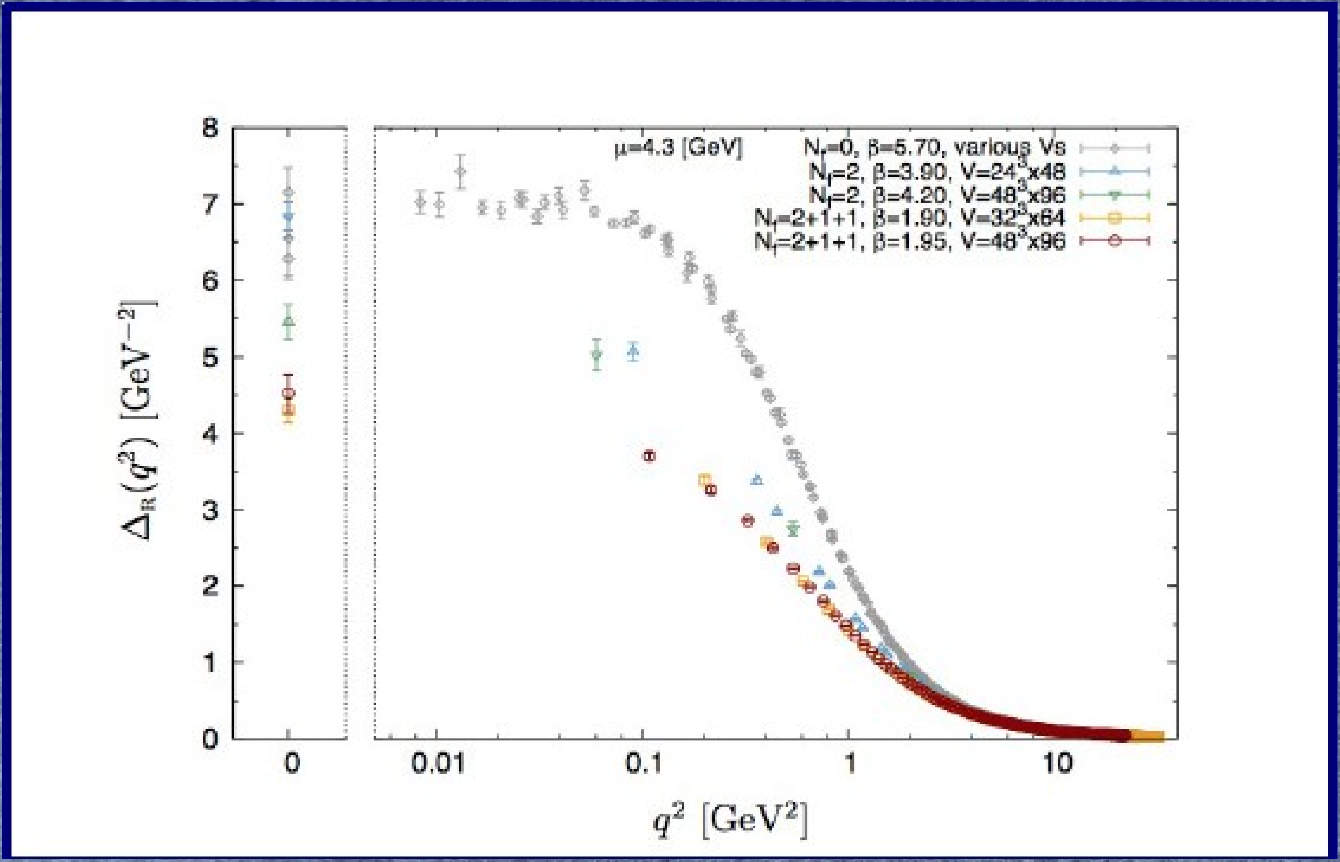
- 2- and 3-gluon Green functions have been deprived from the UV quantum fluctuations by applying the Wilson flow and then shown to be well described as correlations in the field of a multi-instanton ensemble.

- The Wilson flow smoothing procedure leaves the low-momentum domain of these Green functions essentially unmodified; and gets rid of the fundamental QCD scale Λ_{QCD} (which indicates where the mechanism driving the transition from asymptotically free to confinement regimes take place).

- Nevertheless, the three-gluon Green function shows a feature at very low-momentum not fitting in the multi-instanton picture: the zero-crossing which can be explained as a soft quantum effect induced by the contribution of unprotected (by a mass) ghost-loops.

A few words about gluons and confinement

A. Ayala, A. Bashir, D. Binosi, M Cristoforetti, J. R-Q,
Phys. Rev. D86 (2012) 074512

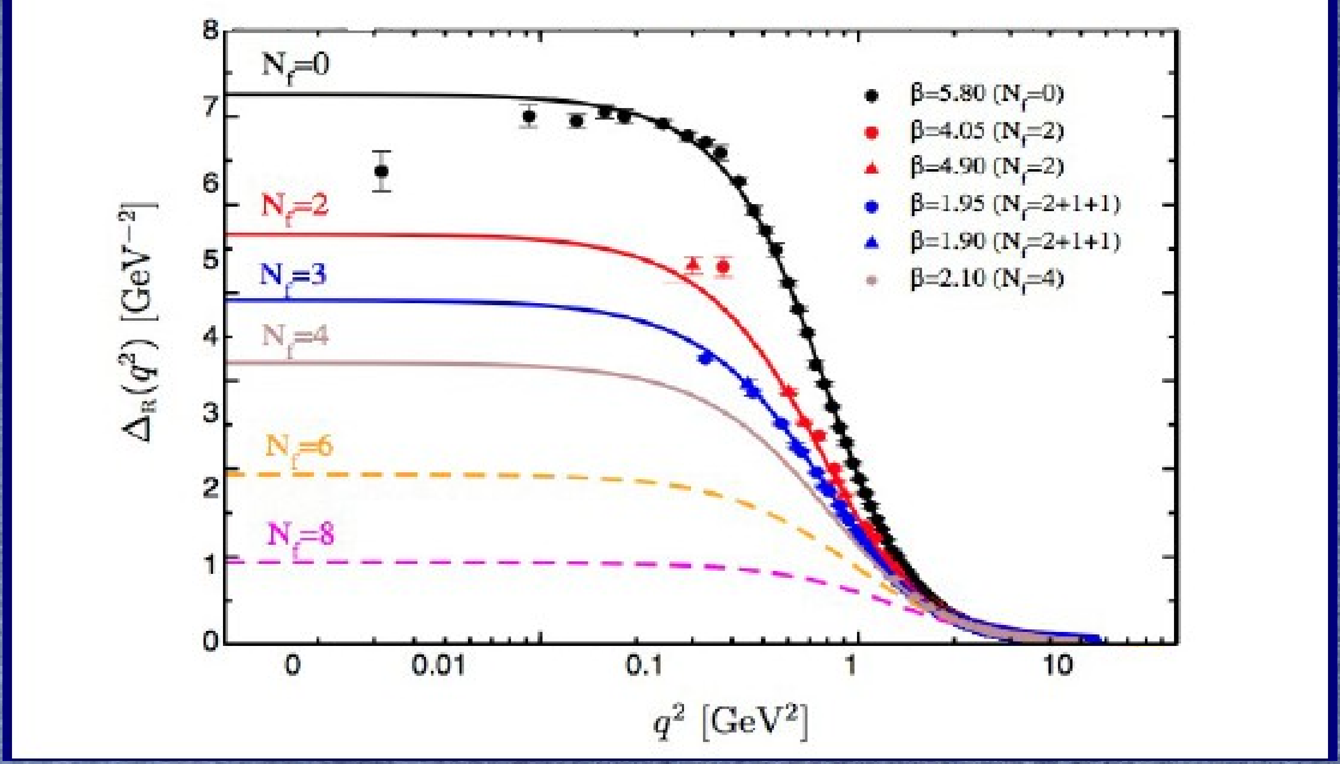


A few words about gluons and confinement

A. Raya, A. Bashir, J. R-Q,
Phys. Rev. D88 (2013) 054003

Refined Gribov-Zwanziger:

$$D(q^2) = \frac{z(\mu^2)\, q^2(q^2+M^2)}{q^4 + q^2\left(M^2 - 13g^2\langle A^2\rangle/24\right) + M^2 m_0^2}$$



A few words about gluons and confinement

A. Raya, A. Bashir, J. R-Q,
Phys. Rev. D58 (2013) 054003c

Quark SDE:

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m) + \Sigma(p)$$

$$S^{-1}(p) = \frac{i\gamma \cdot p + M(p^2)}{Z(p^2, \mu^2)}$$

$$\Sigma(p) = Z_1 \int \frac{d^4q}{(2\pi)^4} g^2 \Delta_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma^a_\nu(q,p)$$

Mass function

$$Z_1 g^2 \Delta_{\mu\nu}(p-q) \Gamma_\nu(p,q) \rightarrow g^2_{\rm eff}(q^2)\, \Delta^N_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\nu$$

P. Maris, C.D. Roberts, P. Tandy; Phys.Lett.B420(2998)267

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Phys. Rev. D58 (2013) 054003

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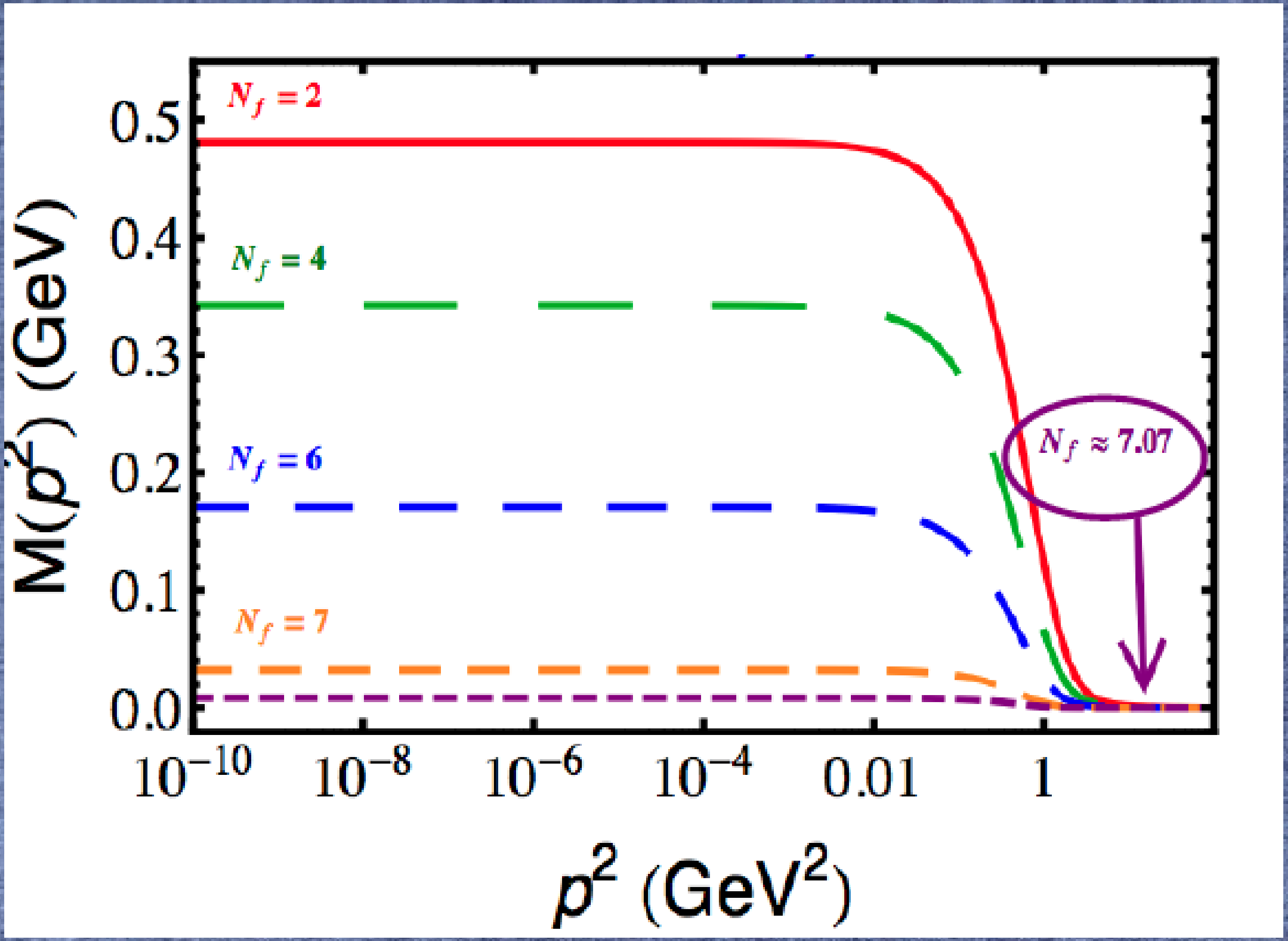
P. Maris, C.D. Roberts, P. Tandy; Phys.Lett.B420(2998)267 at Nf= 2 flavours

A few words about gluons and confinement

A. Raya, A. Bashir, J. R-Q,
Phys. Rev. D58 (2013) 054003

The mass function:



A few words about gluons and confinement

A. Raya, A. Bashir, J. R-Q,
Phys. Rev. D58 (2013) 054003

P. Maris; **Phys. Rev. D 52 6087 (1995)**
A. Bashir, A. Raya, S. Sánchez-Madrigal,
C.D. Roberts, **Few Body Sys. 46, 229 (2009)**

$$\Delta(t) = \int d^3x \int \frac{d^4p}{(2\pi)^4}\, e^{i(p_4 t + \mathbf{p}\cdot\mathbf{x})} \sigma_s(p^2)$$

$$Z(p^2,\mu^2)M(p^2)/(p^2 + M(p^2))$$

Asymptotic free state of
mass m (deconfined phase)

$$\Delta(t) \sim e^{-mt}$$

$$\Delta(t) \sim e^{-at}\cos(bt+\delta)$$

Complex mass poles
(confined phase)

A few words about gluons and confinement

A. Raya, A. Bashir, J. R-Q,
Phys. Rev. D58 (2013) 054003

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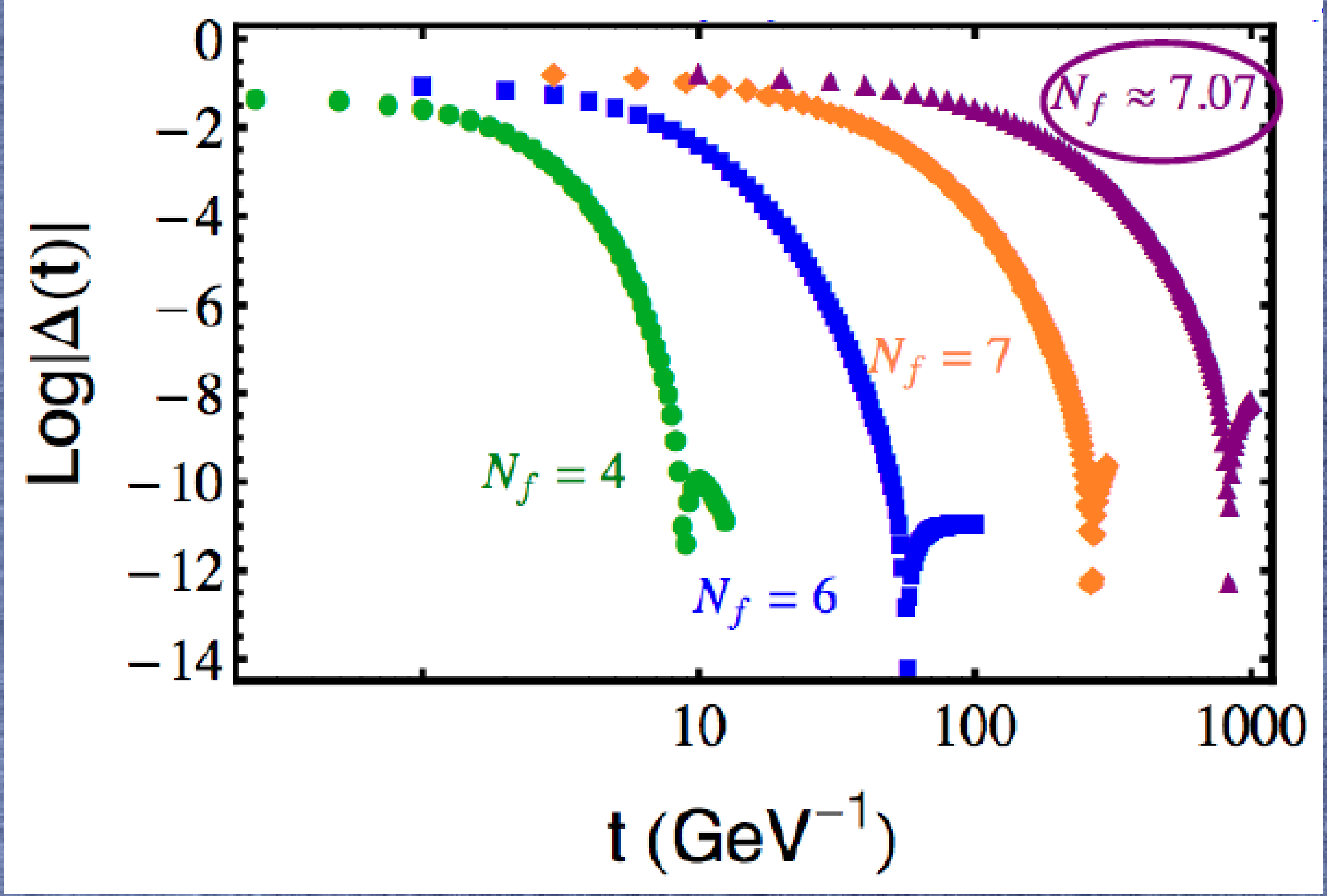

A few words about gluons and confinement

$$\Delta(t) \;=\; \int d^3x \int \frac{d^4p}{(2\pi)^4}\, \mathrm{e}^{i(p_4 t+\mathbf{p}\cdot\mathbf{x})}\sigma_s(p^2)$$

$$\Delta(t) \;=\; \int d^3\vec{x} \int \frac{d^4p}{(2\pi)^4} e^{i(p_4 t+\vec{p}\cdot\vec{x})} D(p^2)$$

A few words about gluons and confinement

$$\Delta(t) = \int d^3x \int \frac{d^4p}{(2\pi)^4}\, e^{i(p_4 t + \mathbf{p}\cdot\mathbf{x})} \sigma_s(p^2)$$

$$\Delta(t) = \int \frac{d^4p}{(2\pi)^4} e^{ip_4 t} D(p^2) \underbrace{\int d^3\vec{x} e^{i\vec{p}\cdot\vec{x}}}_{(2\pi)^3\delta(\vec{p})}$$

A few words about gluons and confinement

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$$z_0 \frac{p^2 + M^2}{p^4 + (m^2 + M^2)p^2 + \lambda^4}$$

First important remark: multiplicative renormalizability implies that M, m and lambda need to be RGI

A few words about gluons and confinement

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$$z = \lambda\, e^{\pm i(\frac{\pi}{2} \pm \frac{\varphi_0}{2})}$$

$$\cos\frac{\varphi_0}{2} = \left(\frac{1}{2} + \frac{m^2+M^2}{4\lambda^2}\right)^{1/2}$$

$$z_0 \quad \frac{p^2+M^2}{\left(p^2+\underbrace{\frac{m^2+M^2}{2}}_{\lambda_1^2}\right)^2 + \underbrace{\lambda^4 - \frac{(m^2+M^2)^2}{4}}_{\lambda_2^4}}$$

A few words about gluons and confinement

$$\Delta(t) = \int d^3x \int \frac{d^4p}{(2\pi)^4} \, \mathrm{e}^{i(p_4 t + \mathbf{p}\cdot\mathbf{x})} \sigma_s(p^2)$$

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$$\Delta(t) = \frac{e^{-\lambda \cos\frac{\varphi_0}{2} t}}{2\lambda \sin\varphi_0} \left[\left(1 + \frac{M^2}{\lambda^2}\right) \sin\frac{\varphi_0}{2} \cos\left(\lambda \sin\frac{\varphi_0}{2} t\right) - \left(1 - \frac{M^2}{\lambda^2}\right) \cos\frac{\varphi_0}{2} \sin\left(\lambda \sin\frac{\varphi_0}{2} t\right) \right]$$

A few words about gluons and confinement

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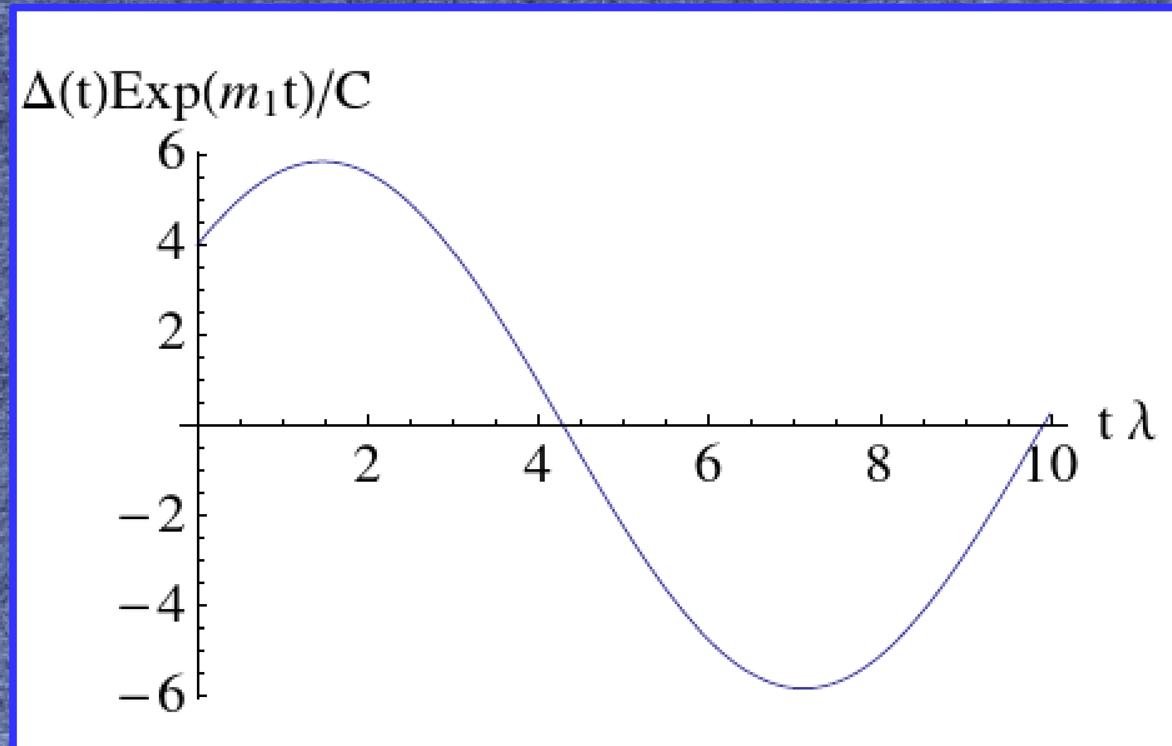
$$\sim e^{-mt}$$

A low-t partonic behaviour?

A few words about gluons and confinement

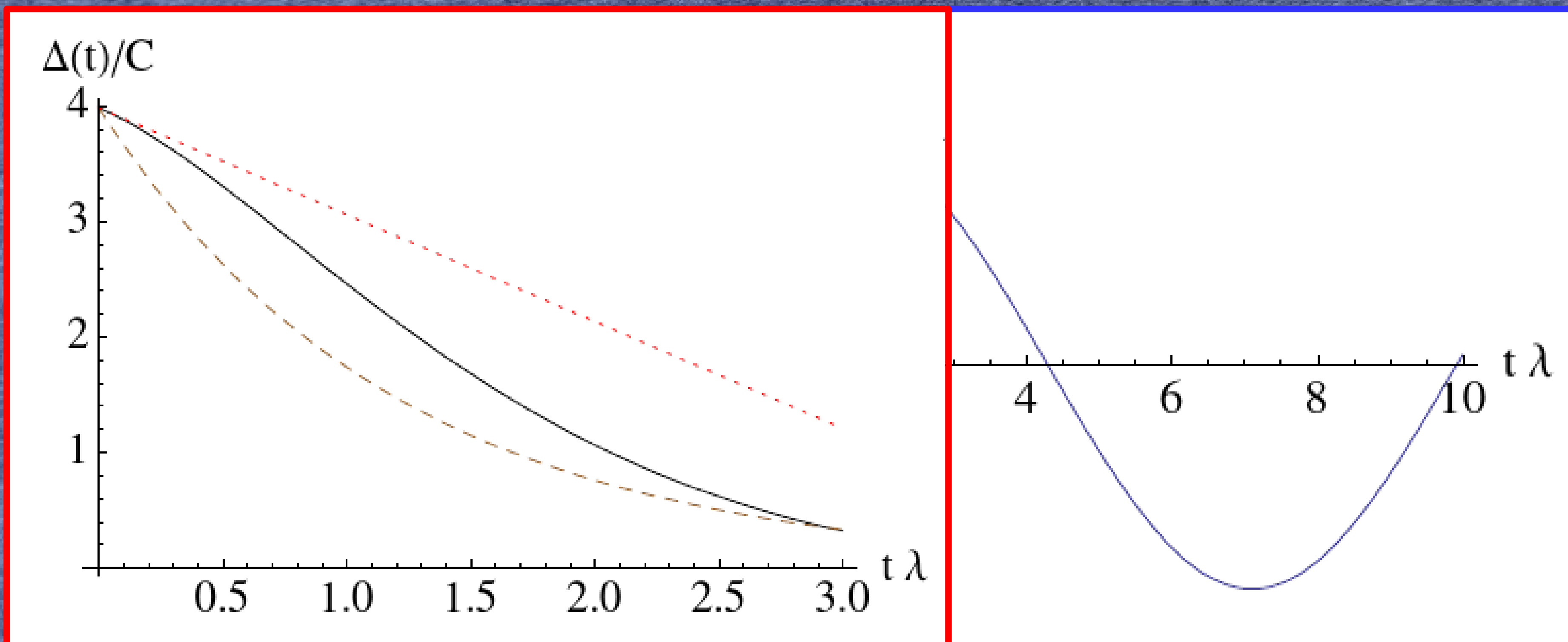


$$\Delta(t) = \frac{e^{-\lambda \cos\frac{\varphi_0}{2} t}}{2\lambda \sin\varphi_0}\left[\left(1+\frac{M^2}{\lambda^2}\right)\sin\frac{\varphi_0}{2}\cos\left(\lambda \sin\frac{\varphi_0}{2} t\right) - \left(1-\frac{M^2}{\lambda^2}\right)\cos\frac{\varphi_0}{2}\sin\left(\lambda\sin\frac{\varphi_0}{2} t\right)\right]$$

Using fitted parameters from the lattice gluon with no additional constraints!!!

A few words about gluons and confinement

No partonic behaviour at low t for the gluon...

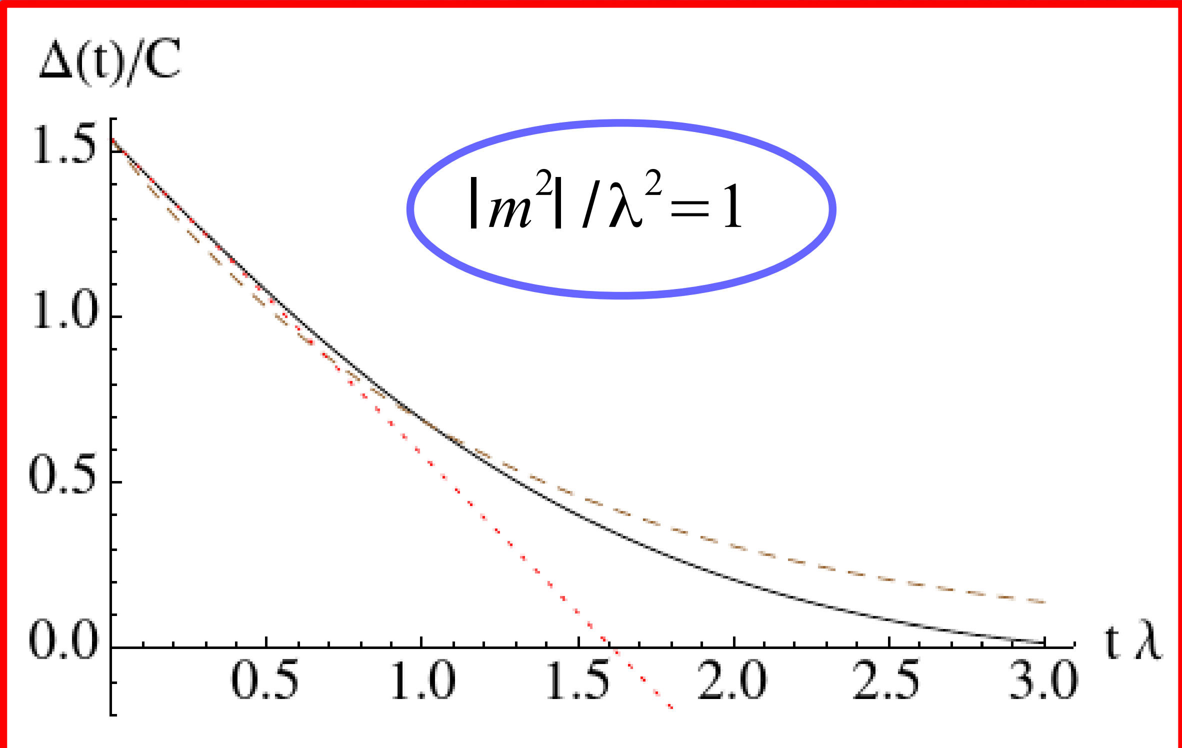


$$\Delta(t)=\frac{e^{-\lambda\cos\frac{\varphi_0}{2}t}}{2\lambda\sin\varphi_0}\left[\left(1+\frac{M^2}{\lambda^2}\right)\sin\frac{\varphi_0}{2}\cos\left(\lambda\sin\frac{\varphi_0}{2}t\right)-\left(1-\frac{M^2}{\lambda^2}\right)\cos\frac{\varphi_0}{2}\sin\left(\lambda\sin\frac{\varphi_0}{2}t\right)\right]$$

Using fitted parameters from the lattice gluon with no additional constraints!!!

A few words about gluons and confinement

No partonic behaviour at low t for the gluon... but can be imposed by means of a very simple condition relating the RGZ parameters in a non-trivial way!!!



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Fitting the lattice gluon data with a “partonic” constraint: concavity of SF!!!

$|m^2| / \lambda^2 \leqslant 1$

A few words about gluons and confinement

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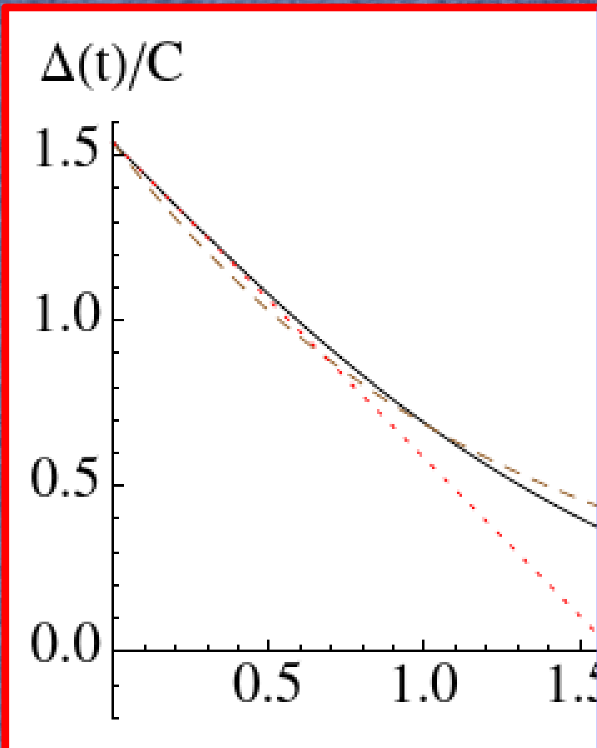
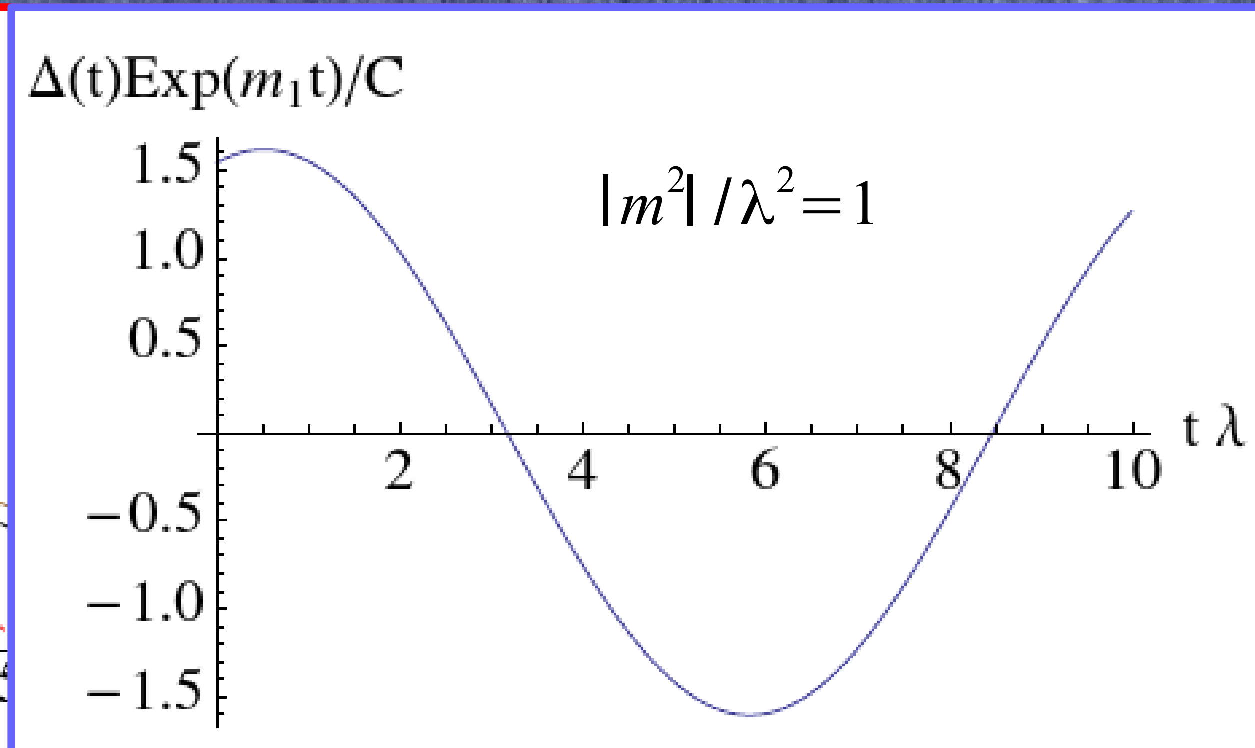




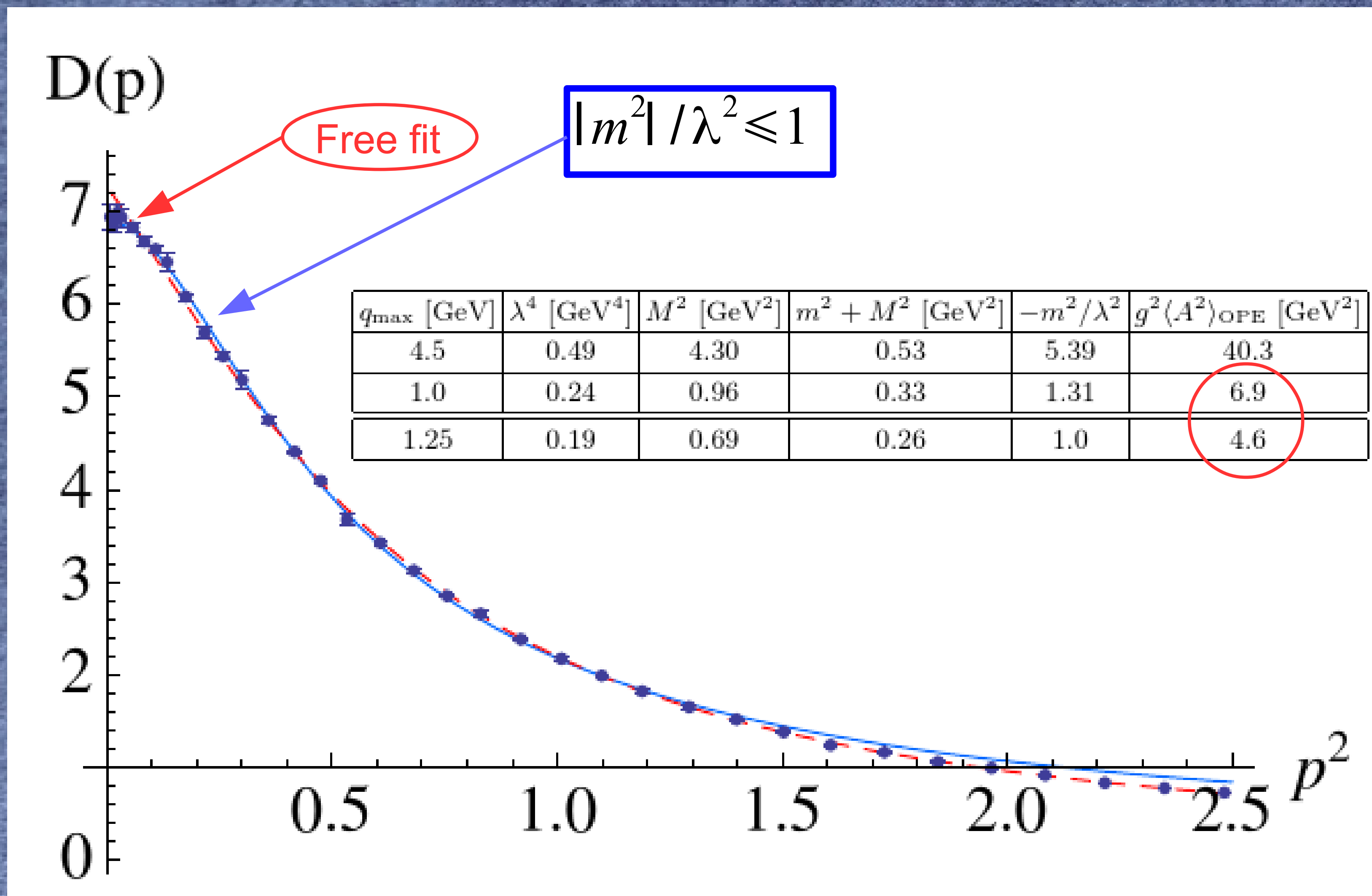
$$\Delta(t)=\frac{e^{-\lambda\cos\frac{\varphi_0}{2}t}}{2\lambda\sin\varphi_0}\left[\left(1+\frac{M^2}{\lambda^2}\right)\sin\frac{\varphi_0}{2}\cos\left(\lambda\sin\frac{\varphi_0}{2}t\right)-\left(1-\frac{M^2}{\lambda^2}\right)\cos\frac{\varphi_0}{2}\sin\left(\lambda\sin\frac{\varphi_0}{2}t\right)\right]$$

Fitting the lattice gluon data with a "partonic" constraint: concavity of SF!!!

$$|m^2|/\lambda^2\leqslant 1$$

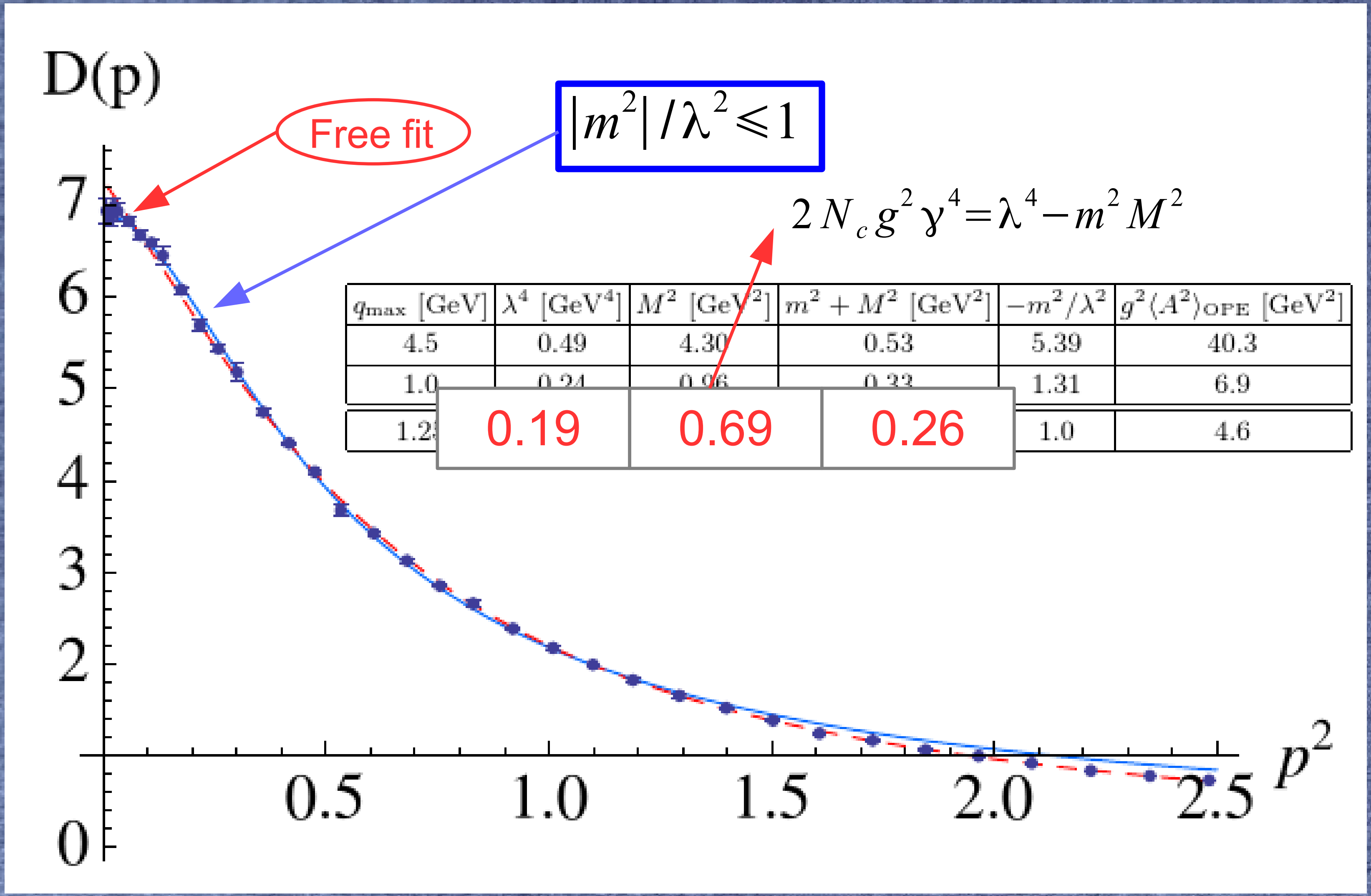
A few words about gluons and confinement

After imposing partonic behaviour, consistency with direct OPE analysis of the gluon propagator appears to be restored!!!



$q_{\rm max}$ [GeV]	λ^4 [GeV^4]	M^2 [GeV^2]	m^2+M^2 [GeV^2]	$-m^2/\lambda^2$	$g^2\langle A^2\rangle_{\rm OPE}$ [GeV^2]
4.5	0.49	4.30	0.53	5.39	40.3
1.0	0.24	0.96	0.33	1.31	6.9
1.25	0.19	0.69	0.26	1.0	4.6

A few words about gluons and confinement



q_{max} [GeV]	λ^4 [GeV^4]	M^2 [GeV^2]	$m^2 + M^2$ [GeV^2]	$-m^2/\lambda^2$	$g^2\langle A^2\rangle_{OPE}$ [GeV^2]
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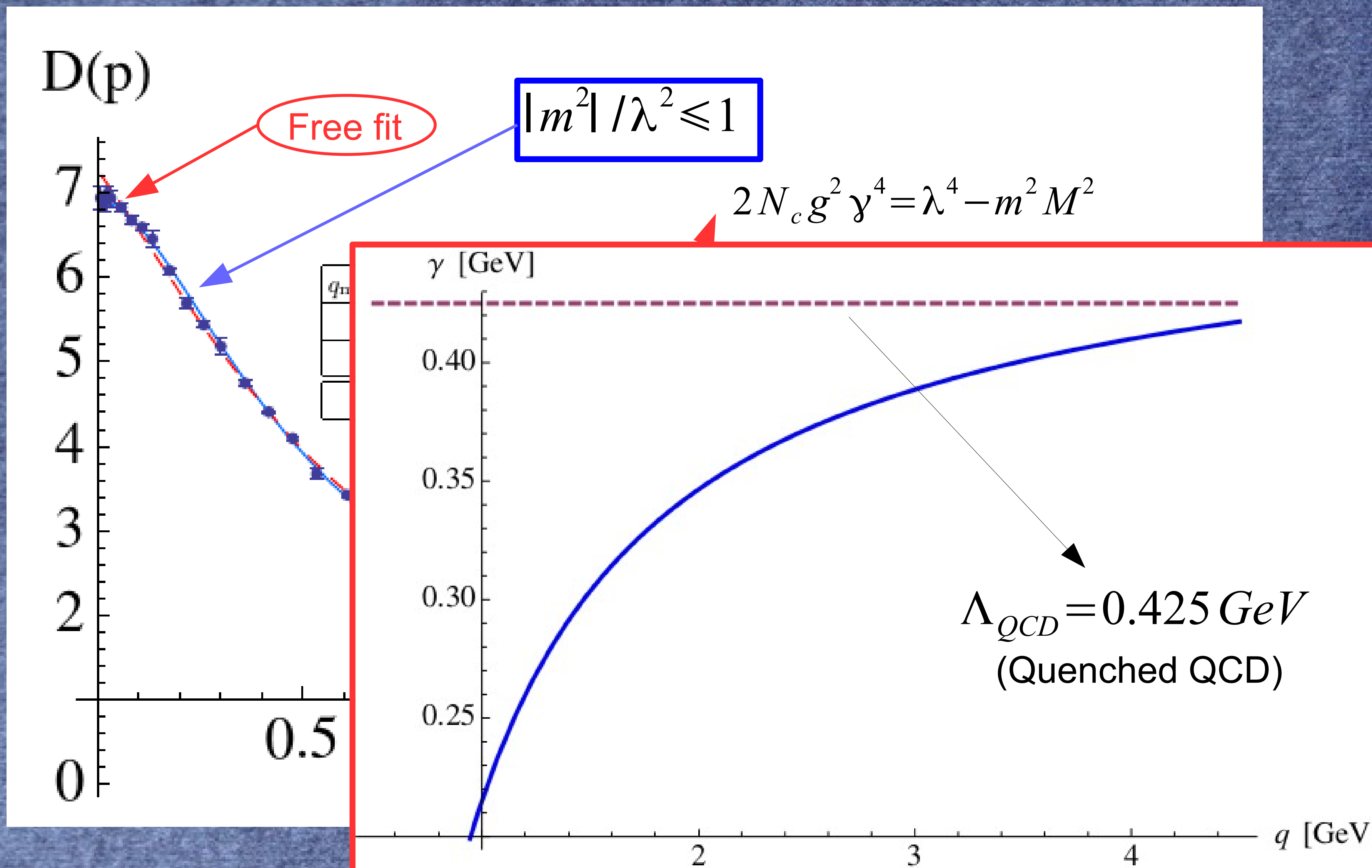
A few words about gluons and confinement

- The running of the effective Gribov parameter depends on the coupling we take: Taylor coupling (see Daniele's talk)



Bringing remarks and concluding:

- Multiplicative renormalizability implies that the RGZ parameters M, m and lambda need to be effectively defined as RGI quantities.
- A partonic behaviour at low t for the gluon can be only imposed by means of a very simple condition relating the RGZ parameters in a non-trivial way. It is no coming for free within the RGZ formalism!
- After imposing partonic behaviour, consistency with direct OPE analysis of the gluon propagator appears to be restored!!!
- The running of the effective Gribov parameter is obtained by assuming a phenomenological behavior of the running for the coupling; and it happens then, in quenched QCD, to be far below LambdaQCD (far outside the scale distance for quark confinement).

Pion 

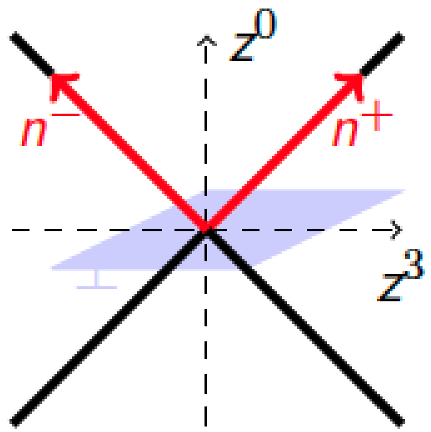
in the BSE and DSE approach

Pion GPD

Definition, constraints and symmetry properties:

$$H_\pi^q(x,\xi,t) \quad = \quad \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P-\frac{\Delta}{2} \right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- From **isospin symmetry**, all the information about pion GPD is encoded in $H_{\pi^+}^u$ and $H_{\pi^+}^d$.
- Further constraint from **charge conjugation**:
  $H_{\pi^+}^u(x,\xi,t) = -H_{\pi^+}^d(-x,\xi,t)$.

Pion GPD

Definition, constraints and symmetry properties:

- PDF **forward limit**
- Form factor **sum rule**
- **Polynomiality** — Lorentz invariance
- **Positivity** — Positivity of Hilbert space norm
- H^q is an **even function** of ξ from time-reversal invariance.
- H^q is **real** from hermiticity and time-reversal invariance.
- H^q has support $x \in [-1, +1]$. — Relativistic Quantum mechanics
- **Soft pion theorem** (pion target) — Dinamical CSB

Numerous theoretical constraints on GPDs.

- There is no known GPD parameterization **relying only on first principles**.
- Modeling becomes a key issue.

Focus here on polynomiality and positivity!

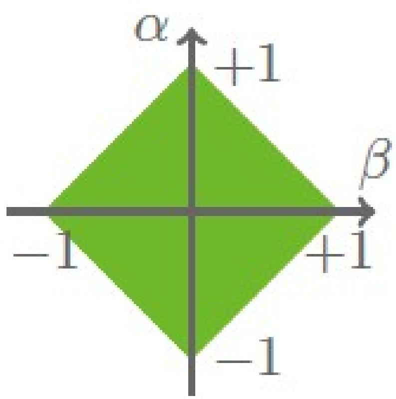
Double Distributions

A well fitted tool to encode GPD properties

- Define Double Distributions F^q and G^q as matrix elements of **twist-2 quark operators**:

$$\left\langle P+\frac{\Delta}{2}\right|\bar{q}(0)\gamma^{\{\mu}i\overleftrightarrow{D}^{\mu_1}\dots i\overleftrightarrow{D}^{\mu_m\}}q(0)\left|P-\frac{\Delta}{2}\right\rangle = \sum_{k=0}^{m}\binom{m}{k}$$

$$[F^q_{mk}(t)2P^{\{\mu}-G^q_{mk}(t)\Delta^{\{\mu}]P^{\mu_1}\dots P^{\mu_{m-k}}\left(-\frac{\Delta}{2}\right)^{\mu_{m-k+1}}\dots\left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$



with

$$F^q_{mk} = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\,\alpha^k\beta^{m-k}F^q(\beta,\alpha)$$

$$G^q_{mk} = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\,\alpha^k\beta^{m-k}G^q(\beta,\alpha)$$

[Muller et al., Fortschr.Phys. 42 (1994)101
[Radyshkin, Phys.Rev.D59(1999)014030;Phys.Lett.B499(1999)81

Double Distributions

Relation to Generalized Parton Distributions

- Representation of GPD:

$$H^q(x,\xi,t) = \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, \delta(x-\beta-\alpha\xi)\big(F^q(\beta,\alpha,t)+\xi G^q(\beta,\alpha,t)\big)$$

See Hervé's talk for more details!!!

- Support property: $x \in [-1,+1]$.
- Discrete symmetries: F^q is α-even and G^q is α-odd.
- **Gauge**: any representation (F^q, G^q) can be recast in one representation with a single DD f^q:

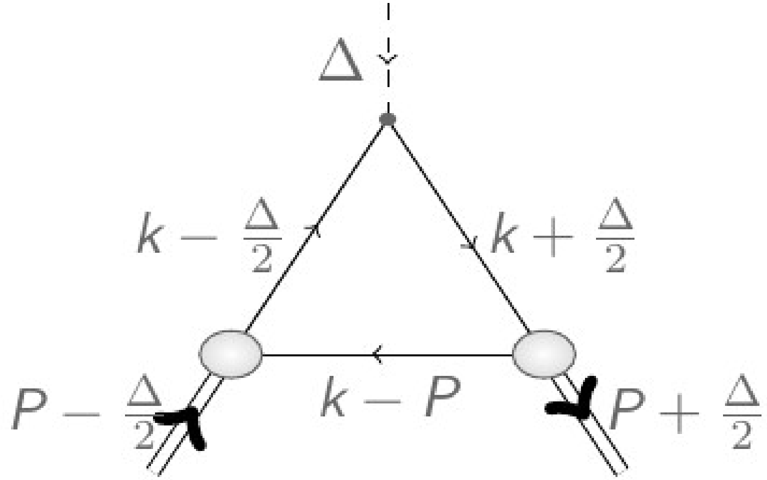
$$H^q(x,\xi,t) = x\int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, f^q_{\mathrm{BMKS}}(\beta,\alpha,t)\delta(x-\beta-\alpha\xi)$$

[Belitsky et al., Phys.Rev.D64 (2001)110062]

GPD in the DSE-BSE approach

Evaluation *via* the triangle diagram approximation:

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$



- Compute **Mellin moments** of the pion GPD H.

GPD in the DSE-BSE approach

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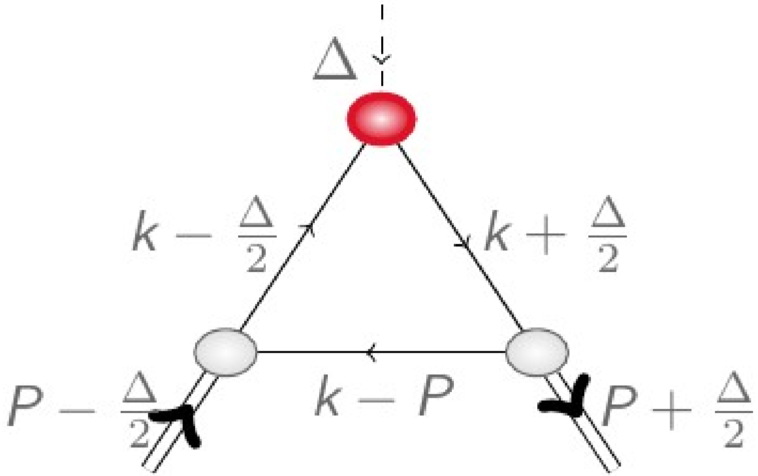

- Compute **Mellin moments** of the pion GPD *H*.
- Triangle diagram approx.

GPD in the DSE-BSE approach

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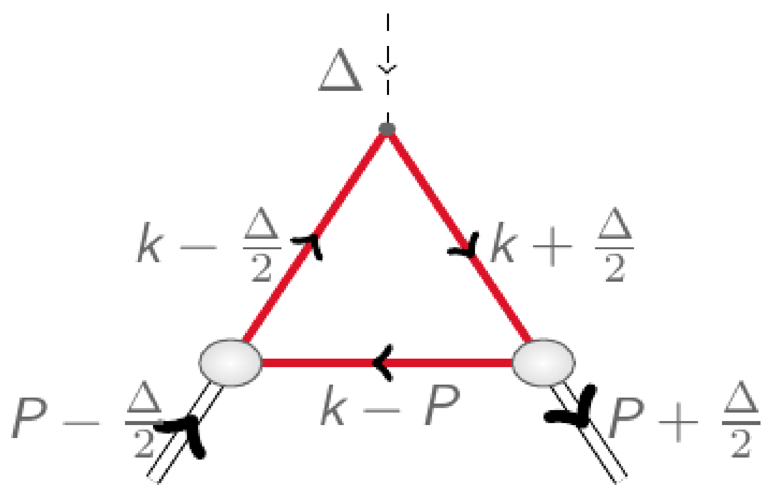
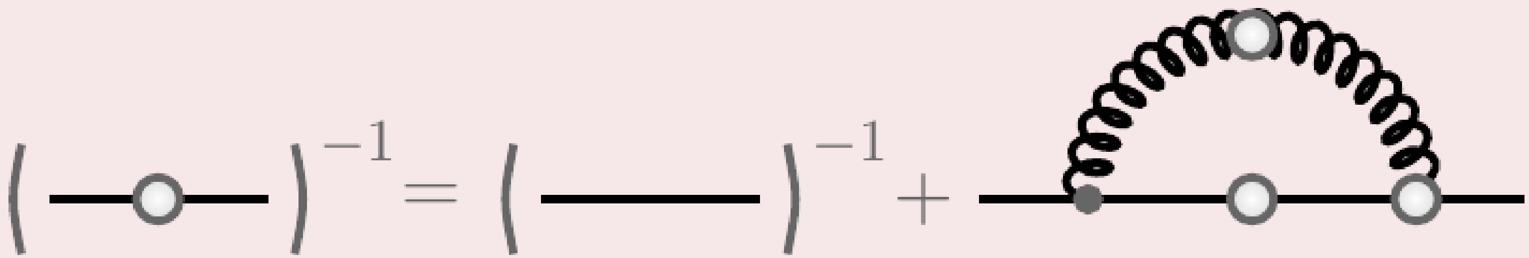


- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.

Dyson - Schwinger equation



GPD in the DSE-BSE approach

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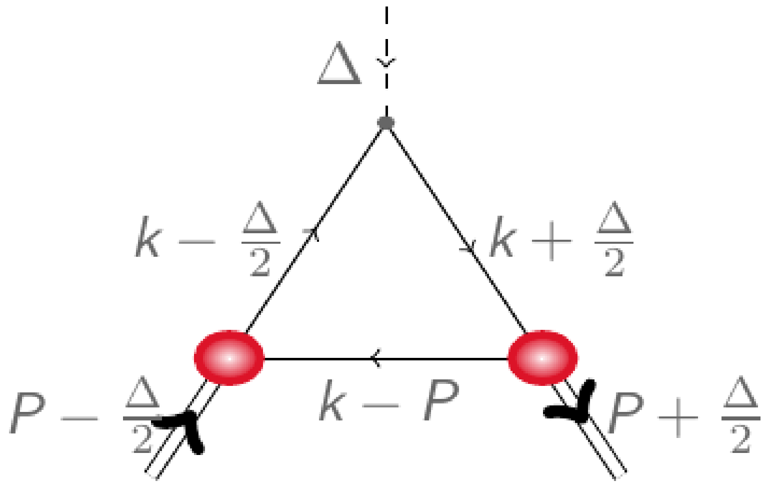

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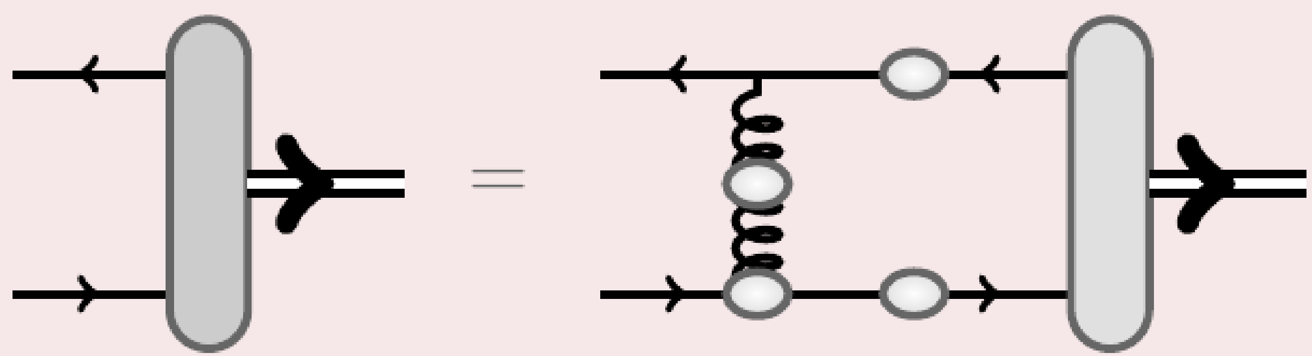
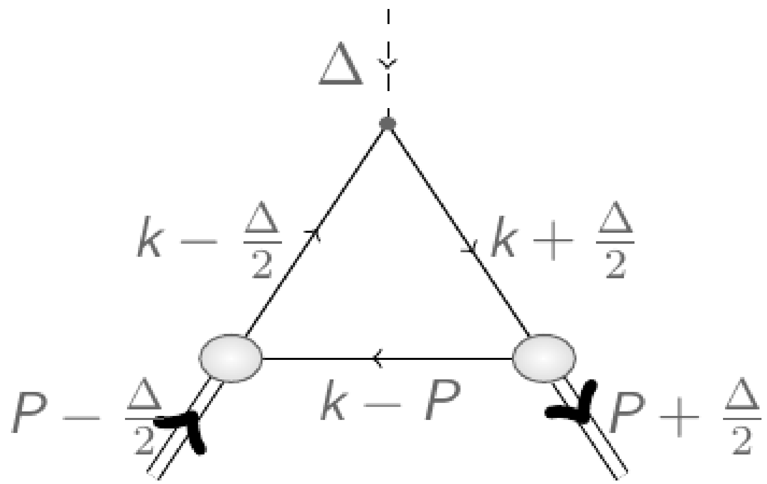
Bethe - Salpeter equation



GPD in the DSE-BSE approach

Evaluation *via* the triangle diagram approximation:

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$



- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.
- **Nonperturbative** modeling.

GPD in the DSE-BSE approach

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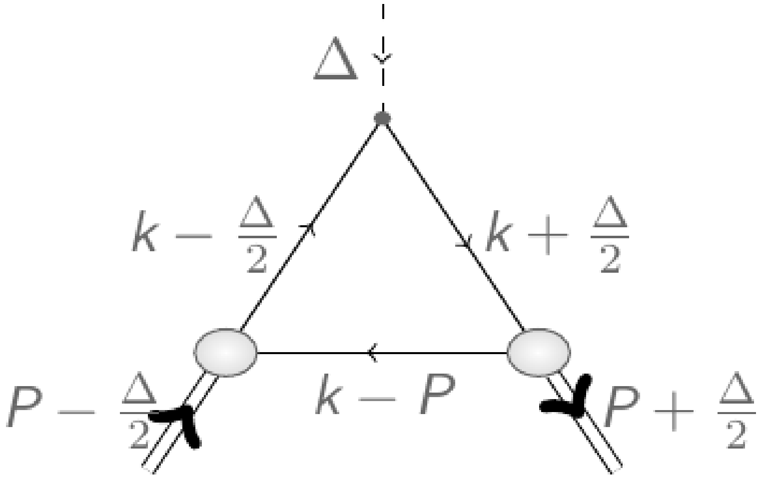


- Compute **Mellin moments** of the pion GPD *H*.
- Triangle diagram approx.
- Resum **infinitely many** contributions.
- **Nonperturbative** modeling.

- Most GPD properties **satisfied by construction**.
- Also compute crossed triangle diagram.

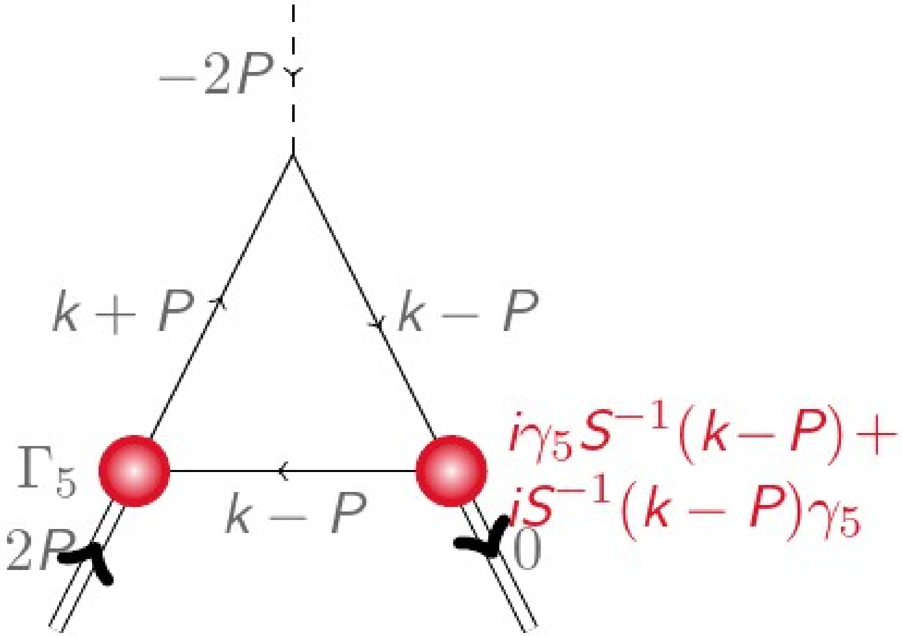
Mezrag *et al.*, arXiv:1406.7425 [hep-ph]
and Phys. Lett. **B741**, 190 (2015)

GPD in the DSE-BSE approach

Most of the properties made sure by construction:

- **Polynomiality** from Poincaré covariance.
- **Soft pion theorem** from **symmetry-preserving** truncation of Bethe-Salpeter and gap equations.

Mezrag *et al.*, Phys. Lett. **B741**, 190 (2015)



- Mellin moments.
- Soft pion kinematics.
- Axial and axial vector vertices Γ_5, Γ_5^μ in chiral limit.
- Axial-vector Ward identity.
- Recover pion DA Mellin moments.

Algebraic DSE-BSE inspired GPD model

Have to deal with DSEs and BSEs solutions:

- Numerical resolution of gap and Bethe-Salpeter equations in Euclidean space.
- Analytic continuation to Minkowskian space required.
- **Ill-posed** problem in the sense of Hadamard.
- Parameterize solutions and fit to numerical solution:

Gap Complex-conjugate pole representation:

$$S(k) = \sum_{i=0}^{N} \left[\frac{z_i}{i\not{k} + m_i} + \frac{z_i^*}{i\not{k} + m_i^*} \right]$$

Bethe-Salpeter Nakanishi representation of amplitude $\mathcal{F}_\pi$:

$$\mathcal{F}_\pi(q^2, q \cdot P) = \int_{-1}^{+1} \mathrm{d}\alpha \int_0^\infty \mathrm{d}\lambda \frac{\rho(\alpha, \lambda)}{(q^2 + \alpha q \cdot P + \lambda^2)^n}$$

Algebraic DSE-BSE inspired GPD model

A first intermediate step before dealing with numerical solutions:

- Expressions for vertices and propagators:

$$
\begin{aligned}
S(p) &= \big[-i\gamma\cdot p + M\big]\Delta_M(p^2) \\
\Delta_M(s) &= \frac{1}{s+M^2} \\
\Gamma_\pi(k,p) &= i\gamma_5\frac{M}{f_\pi}M^{2\nu}\int_{-1}^{+1}\mathrm{d}z\,\rho_\nu(z)\ \left[\Delta_M(k_{+z}^2)\right]^\nu \\
\rho_\nu(z) &= R_\nu(1-z^2)^\nu
\end{aligned}
$$

  with R_ν a normalization factor and $k_{+z} = k - p(1-z)/2$.

  Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

- Only two parameters:

Algebraic DSE-BSE inspired GPD model

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- Only two parameters:
  - Dimensionful parameter M.

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  - Dimensionless parameter ν

Algebraic DSE-BSE inspired GPD model

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\end{aligned}
$$

  with R_ν a normalization factor and $k_{+z} = k - p(1-z)/2$.

  Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

- Only two parameters:
  - Dimensionful parameter M.
  - Dimensionless parameter ν. **Fixed to 1** to recover asymptotic pion DA.

Results for the pion GPD

Verification of the theoretical constraints:

- **Analytic expression** in the DGLAP region.

$$
\begin{aligned}
H^{u}_{x\geq\xi}(x,\xi,0) \quad = \quad & \frac{48}{5}\Bigg\{ \frac{3\left(-2(x-1)^4\left(2x^2-5\xi^2+3\right)\log(1-x)\right)}{20\left(\xi^2-1\right)^3} \\
& \frac{3\left(+4\xi\left(15x^2(x+3)+(19x+29)\xi^4+5(x(x(x+11)+21)+3)\xi^2\right)\tanh^{-1}\left(\frac{(x-1)}{x-\xi^2}\right.\right.}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(x^3(x(2(x-4)x+15)-30)-15(2x(x+5)+5)\xi^4\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(-5x(x(x(x+2)+36)+18)\xi^2-15\xi^6\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(2(x-1)\left((23x+58)\xi^4+(x(x(x+67)+112)+6)\xi^2+x(x((5-2x)x+15)+3\right.\right.}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(\left(15(2x(x+5)+5)\xi^4+10x(3x(x+5)+11)\xi^2\right)\log\left(1-\xi^2\right)\right)}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(2x(5x(x+2)-6)+15\xi^6-5\xi^2+3\right)\log\left(1-\xi^2\right)}{20\left(\xi^2-1\right)^3}\Bigg\}
\end{aligned}
$$

Results for the pion GPD

Verification of the theoretical constraints:

- **Analytic expression** in the DGLAP region.
- Similar expression in the ERBL region.

Results for the pion GPD

Verification of the theoretical constraints:

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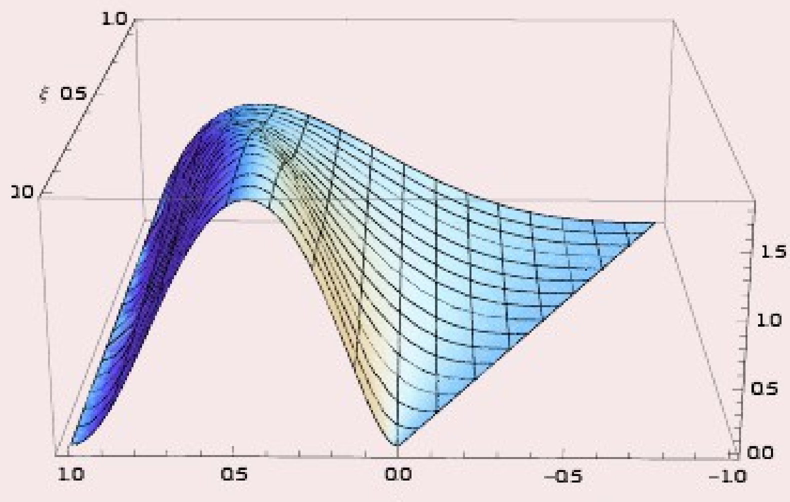
- Also direct verification using Mellin moments of H.

Valence $H^u(x, \xi, t)$ as a function of x and ξ at vanishing t.



Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

Results for the pion GPD

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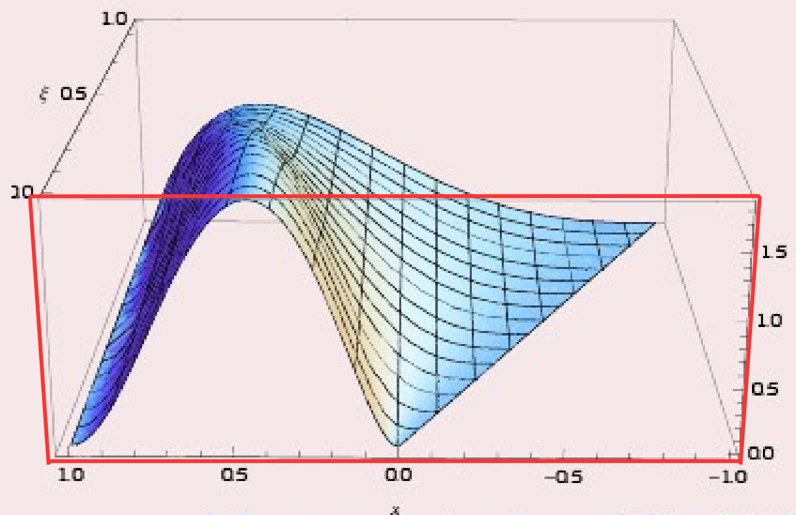
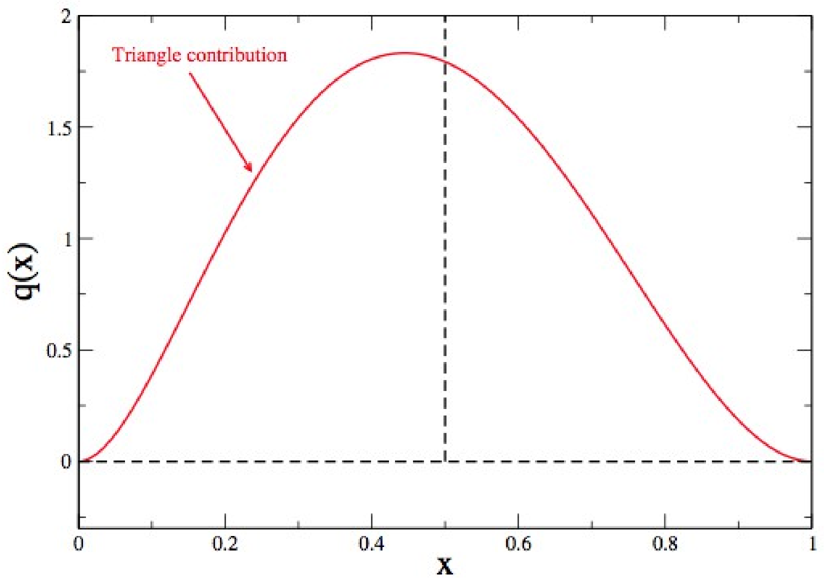


Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

Results for the pion GPD

The two-body problem:



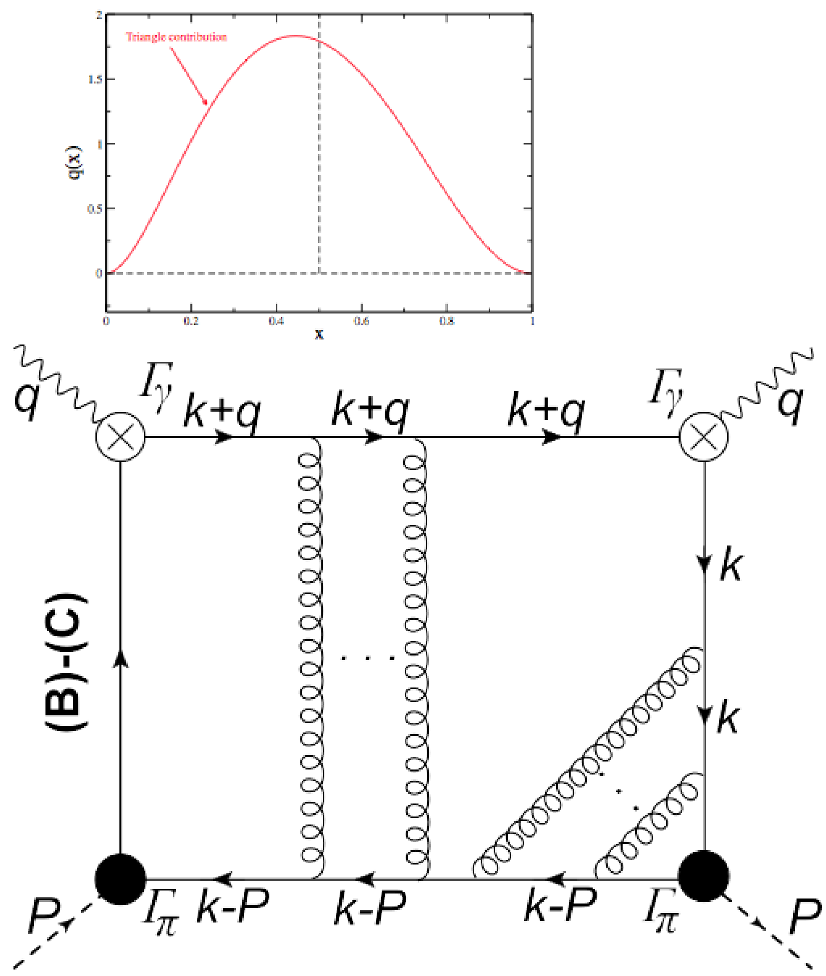
$$q_A^\pi(x) = n_q \Big[x^3(x[-2(x-4)x-15]+30)\ln(x) + (2x^2+3) \times (x-1)^4 \ln(1-x) + x[x(x[2x-5]-15)-3](x-1) \Big],$$

- The PDF appears not to be symmetric around $x = \frac{1}{2}$.

Craig's talk on Monday!!!

Results for the pion GPD

The two-body problem:



- The PDF appears not to be symmetric around $x = \frac{1}{2}$.
- Part of the gluons contribution is neglected in the triangle diagram approach.

Craig's talk on Monday!!!

Results for the pion GPD

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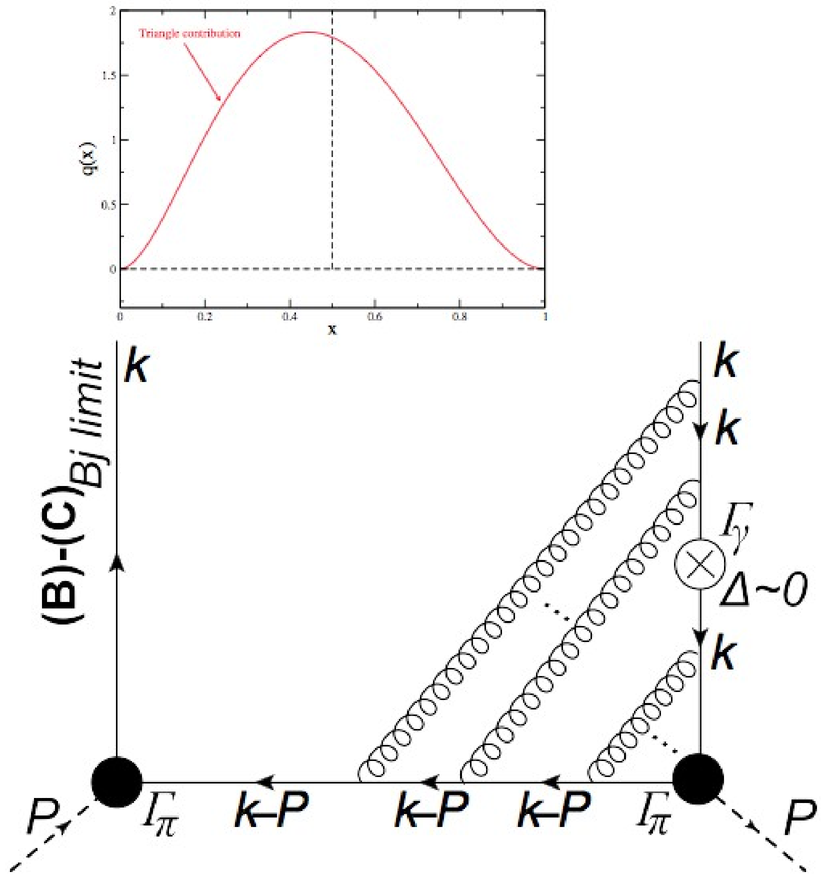

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Results for the pion GPD

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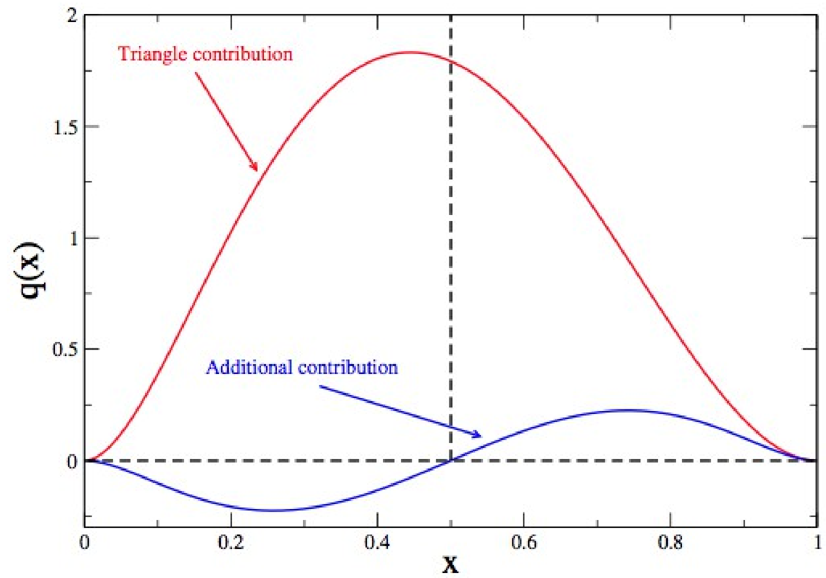


$$q_A^\pi(x) = n_q\Big[x^3(x[-2(x-4)x-15]+30)\ln(x) + (2x^2+3) \times(x-1)^4\ln(1-x) + x[x(x[2x-5]-15)-3](x-1)\Big],$$

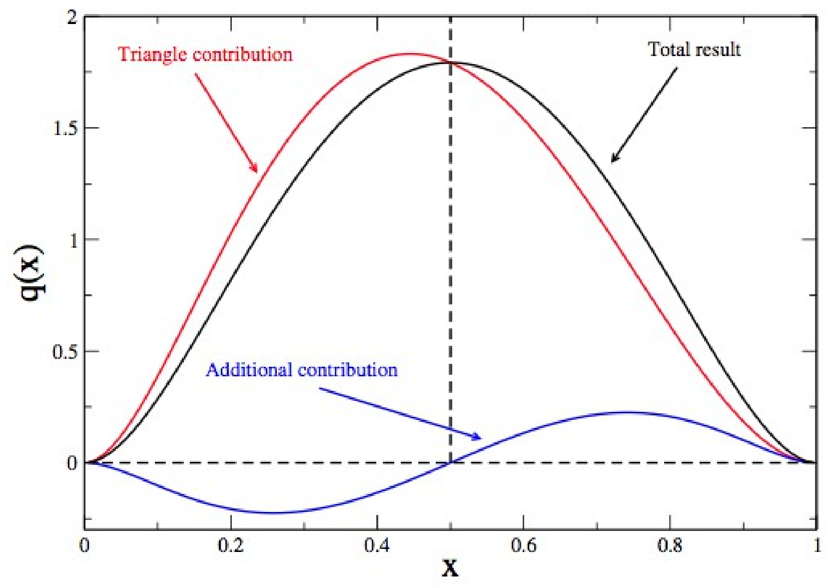
$$q_{BC}^\pi(x) = n_q\Big[x^3(2x([x-3]x+5)-15)\ln(x) - (2x^3+4x+9) \times(x-1)^3\ln(1-x) - x(2x-1)([x-1]x-9)(x-1)\Big]. \quad (13)$$

- The PDF appears not to be symmetric around $x = \frac{1}{2}$.
- Part of the gluons contribution is neglected in the triangle diagram approach.

Craig's talk on Monday!!!

Results for the pion GPD

The two-body problem:



$$q_L^\pi(x) = \frac{72}{25}\Big[x^3(x[2x-5]+15)\ln(x) + (x[2x+1]+12)$$
$$\times(1-x)^3\ln(1-x) + 2x(6-[1-x]x)(1-x)\Big]\,.$$

- The PDF appears not to be symmetric around $x = \frac{1}{2}$.
- Part of the gluons contribution is neglected in the triangle diagram approach.
- Adding this contribution allows us to recover a symmetric PDF [L. Chang *et al.*, Phys.Lett.B737(2014)2329].

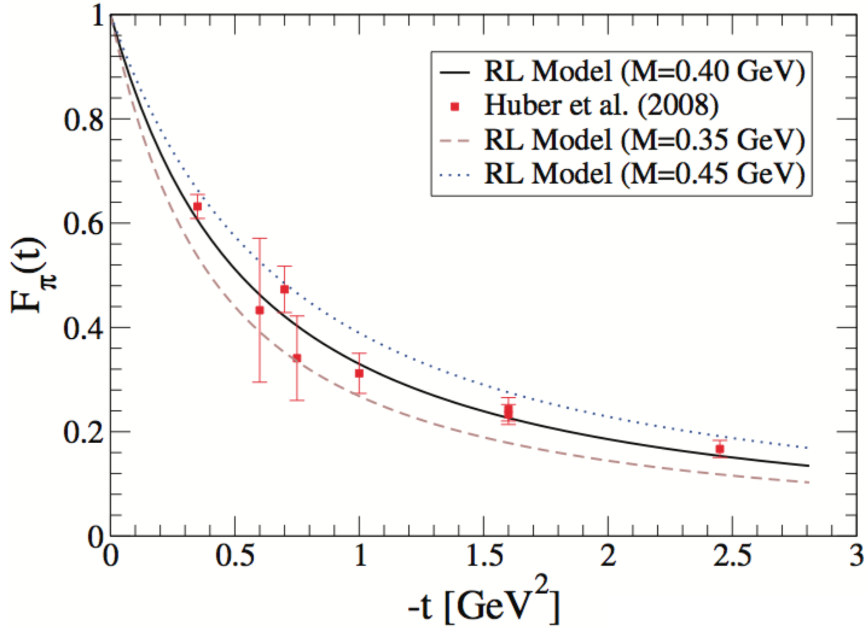
Results for the pion GPD

The form factor and the dimensionful parameter:

- Pion form factor obtained from isovector GPD:

$$\int_{-1}^{+1} \mathrm{d}x \, H^{I=1}(x, \xi, t) = 2F_\pi(t)$$

- Single dimensionful parameter $M \simeq 400$ MeV.



C. Mezrag et al., Phys. Lett. B190 (2015) 741

Results for the pion GPD

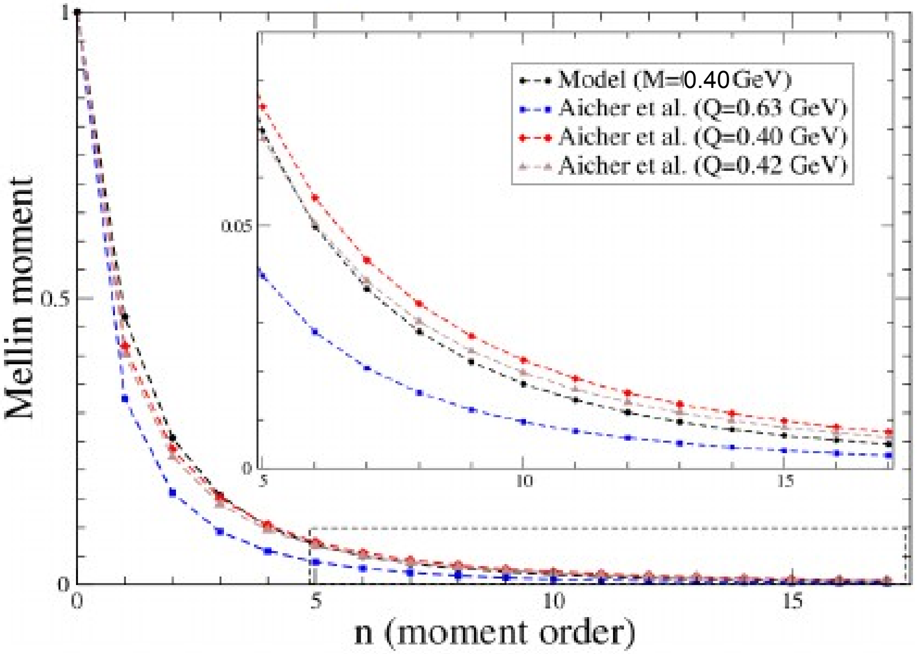
The parton distribution function:

- Pion PDF obtained from forward limit of GPD:

$$q(x) = H^q(x, 0, 0)$$

- Use LO DGLAP equation and compare to PDF extraction.

Aicher *et al.*, Phys. Rev. Lett. **105**, 252003 (2010)



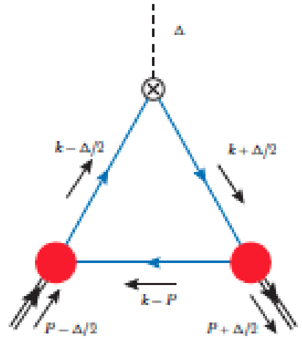
Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

- Find model initial scale $\mu \simeq 400$ MeV.

Results for the pion GPD

The off-forward (non-skewed) GPD:

The model:



$$2(P\cdot n)^{m+1}\left\langle x^m\right\rangle^u = \mathrm{tr}_{CFD}\int\frac{\mathrm{d}^4k}{(2\pi)^4}(k\cdot n)^m\,\tau_+ i\Gamma_\pi\left(\eta(k-P)+(1-\eta)\left(k-\frac{\Delta}{2}\right),P-\frac{\Delta}{2}\right)$$
$$S(k-\frac{\Delta}{2})\; i\gamma\cdot n\; S(k+\frac{\Delta}{2})$$
$$\tau_- i\bar{\Gamma}_\pi\left((1-\eta)\left(k+\frac{\Delta}{2}\right)+\eta(k-P),P+\frac{\Delta}{2}\right)S(k-P),$$

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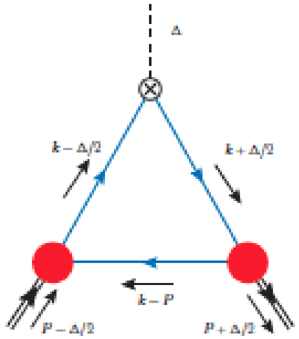
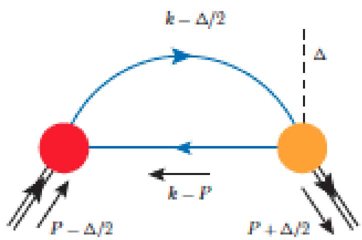
Results for the pion GPD

The off-forward (non-skewed) GPD:

The full model:





$$
\begin{aligned}
2(P\cdot n)^{m+1}\langle x^m\rangle^u \;=\;& \mathrm{tr}_{CFD}\int \frac{\mathrm{d}^4k}{(2\pi)^4}(k\cdot n)^m\,\tau_+ i\Gamma_\pi\left(\eta(k-P)+(1-\eta)\left(k-\frac{\Delta}{2}\right),P-\frac{\Delta}{2}\right)\\
& S(k-\frac{\Delta}{2})\; i\gamma\cdot n\; S(k+\frac{\Delta}{2})\\
& \tau_- i\bar{\Gamma}_\pi\left((1-\eta)\left(k+\frac{\Delta}{2}\right)+\eta(k-P),P+\frac{\Delta}{2}\right)S(k-P),
\end{aligned}
$$

$$
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& S(k-\frac{\Delta}{2})\tau_-\;\frac{\partial}{\partial k}\bar{\Gamma}_\pi\left((1-\eta)\left(k+\frac{\Delta}{2}\right)+\eta(k-P),P+\frac{\Delta}{2}\right)S(k-P)
\end{aligned}
$$

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Results for the pion GPD

The off-forward (non-skewed) GPD:

$$2(P\cdot n)^{m+1}\langle x^m\rangle^u = \mathrm{tr}_{CFD}\int \frac{\mathrm{d}^4k}{(2\pi)^4}(k\cdot n)^m \tau_+ i\Gamma_\pi\left(\eta(k-P)+(1-\eta)\left(k-\frac{\Delta}{2}\right), P-\frac{\Delta}{2}\right)$$

$$S\left(k-\frac{\Delta}{2}\right)\tau_- \frac{\partial}{\partial k}\bar{\Gamma}_\pi\left((1-\eta)\left(k+\frac{\Delta}{2}\right)+\eta(k-P), P+\frac{\Delta}{2}\right)S(k-P)$$

$$F^{BC}(\beta,\alpha,t),\ G^{BC}(\beta,\alpha,t)$$

$$H^{BC}(x,\xi,t) = \int_{-1}^{1}\mathrm{d}\beta\int_{-1+|\beta|}^{1-|\beta|}\mathrm{d}\alpha\left(F^{BC}(\beta,\alpha,t)+\xi\, G^{BC}(\beta,\alpha,t)\right)\delta(x-\beta-\alpha\xi)$$

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Results for the pion GPD

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$$H^{BC}(x,0,0) = \int_{-1+|x|}^{1-|x|}\mathrm{d}\alpha\,F^{BC}(x,\alpha,0) \equiv q^{\pi}_{BC}(x)$$

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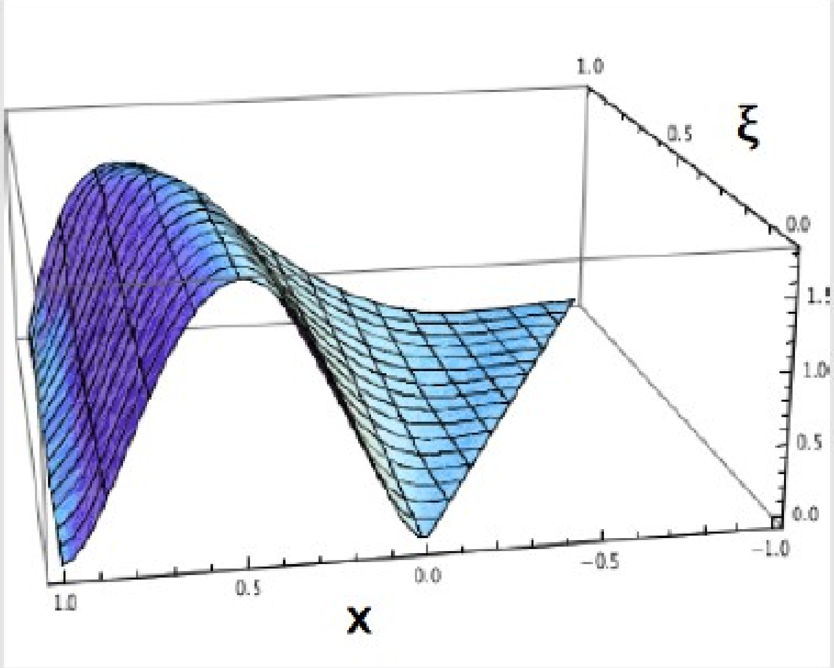
Results for the pion GPD

The off-forward (non-skewed) GPD:

$$H(x,\xi,0) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \left(F(\beta,\alpha,0) + \xi G(\beta,\alpha,0)\right) \delta(x-\beta-\alpha\xi)$$

GPD 3D-plot (t=0)



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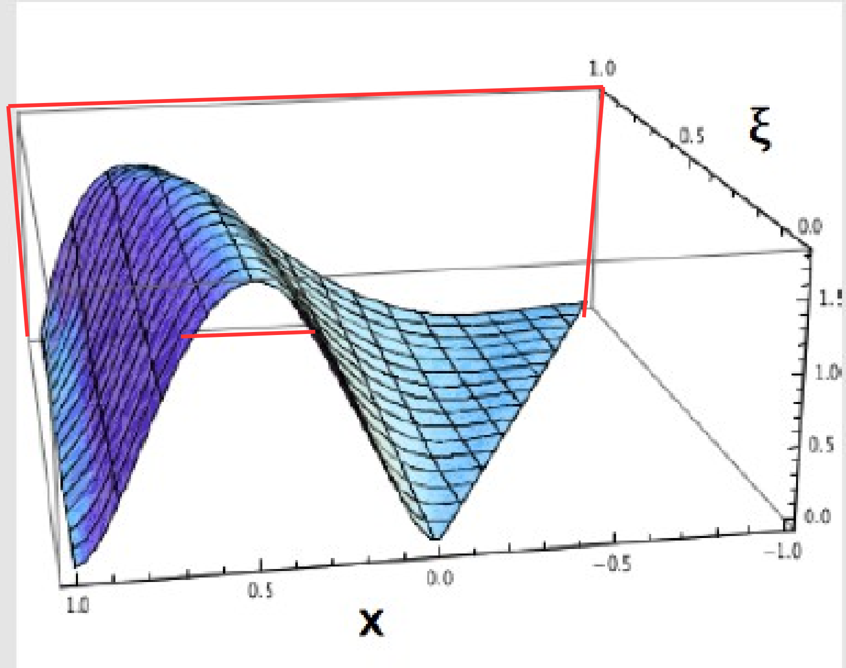

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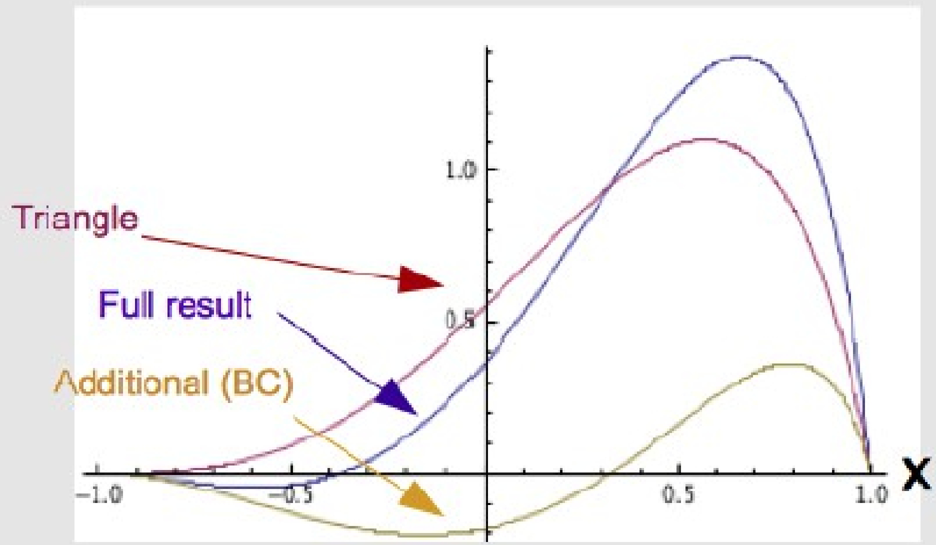
GPD $(t=0, \xi = 1)$



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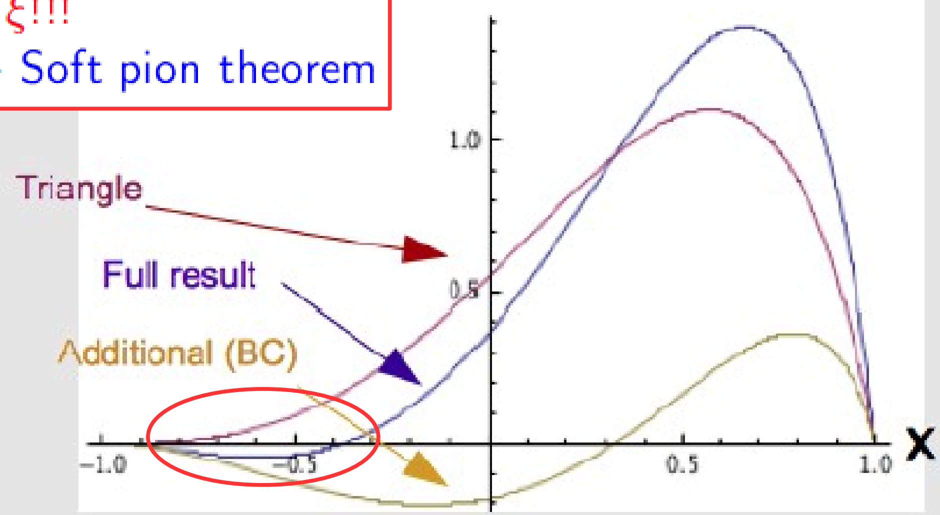
Results for the pion GPD

The off-forward (non-skewed) GPD:

$$H(x,\xi,0) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \left(F(\beta,\alpha,0) + \xi G(\beta,\alpha,0)\right) \delta(x-\beta-\alpha\xi)$$

GPD (t=0, $\xi = 1$)

Problems at large ξ!!!
AVWT Identity $\Rightarrow$ Soft pion theorem



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Results for the pion GPD

The off-forward (non-skewed) GPD:

The pion GPD

$$H^q(x,0,t) = \int_{-1+|x|}^{1-|x|} \mathrm{d}\alpha \left(F^0(x,\alpha,t) + F^{BC}(x,\alpha,t)\right)$$

$$H(x,0,t) = H(x,0,0)\mathcal{N}(t)C_\pi(x,t)F_\pi(t)\,,$$
$$1 = \mathcal{N}(t)\int_{-1}^{1} dx\; H(x,0,0)C_\pi(x,t)\,.$$

$$F(\beta,\alpha,t) = \frac{1}{\left(1+\frac{t}{4M^2}(1-\beta+\alpha)(1-\beta+\alpha)\right)^2} \times \left(F_S(\beta,\alpha) + t\,[\cdots]\right)$$

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Results for the pion GPD

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$$F(\beta,\alpha,t) = \frac{1}{\left(1+\frac{t}{4M^2}(1-\beta)(1-\beta)\right)^2} \times F_S(\beta,\alpha)$$

Simplified analytical model:

$$C(x,t) = \frac{1}{\left(1+\frac{t}{4M^2}(1-x)^2\right)^2}$$

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Results for the pion GPD

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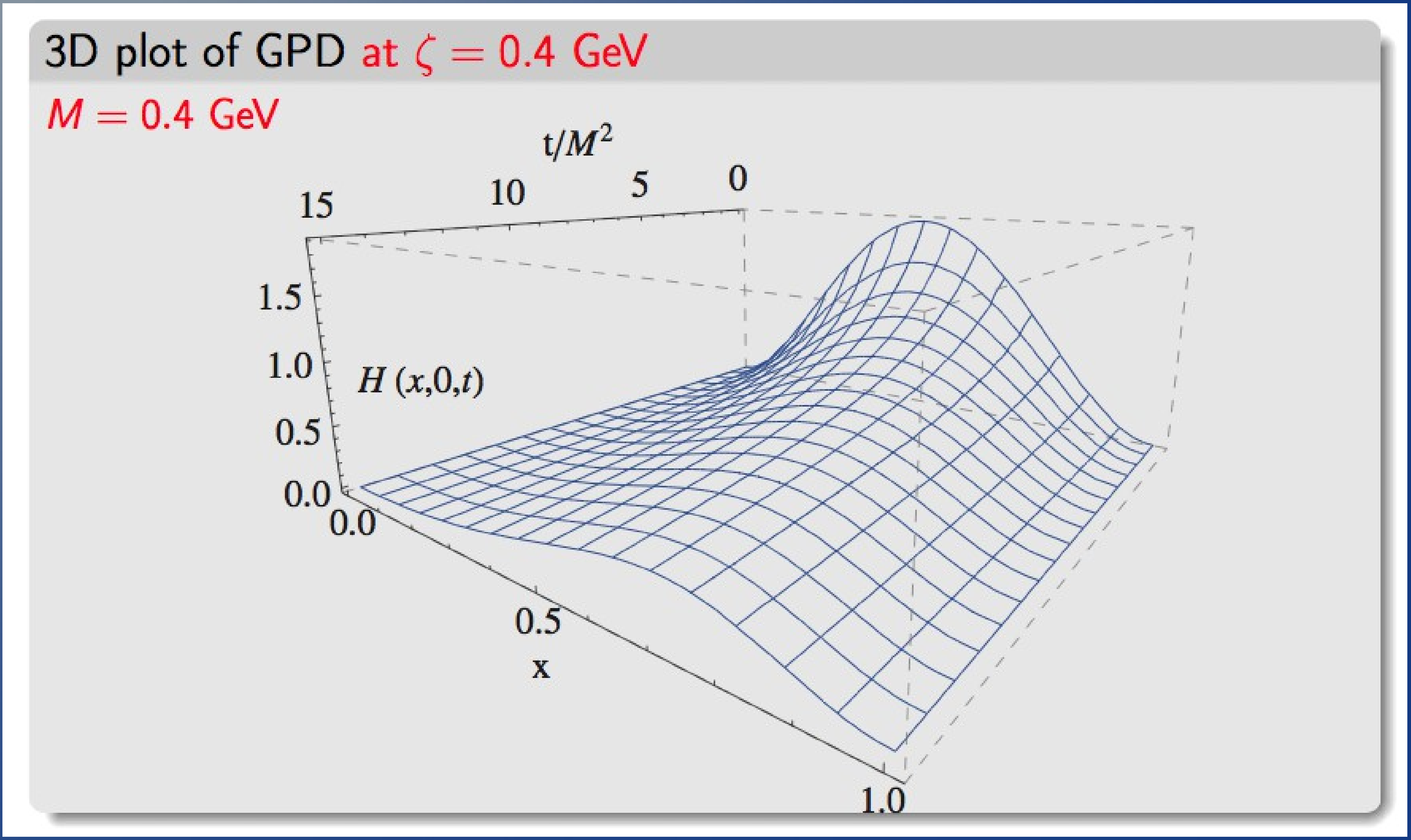

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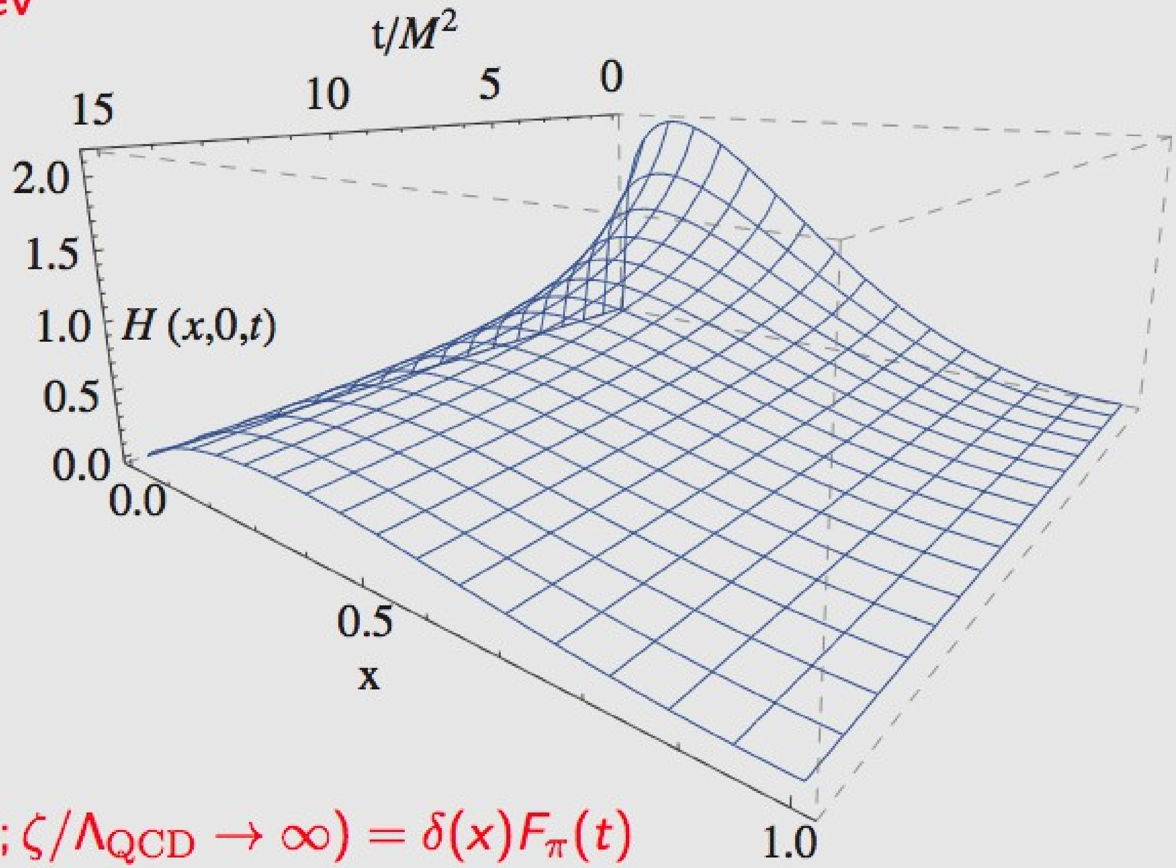
Results for the pion GPD

The off-forward (non-skewed) GPD:

3D plot of GPD at $\zeta = 2$ GeV (DGLAP running; $x > \xi$)

$M = 0.4$ GeV



$$H(x, 0, t; \zeta/\Lambda_{QCD} \to \infty) = \delta(x) F_\pi(t)$$

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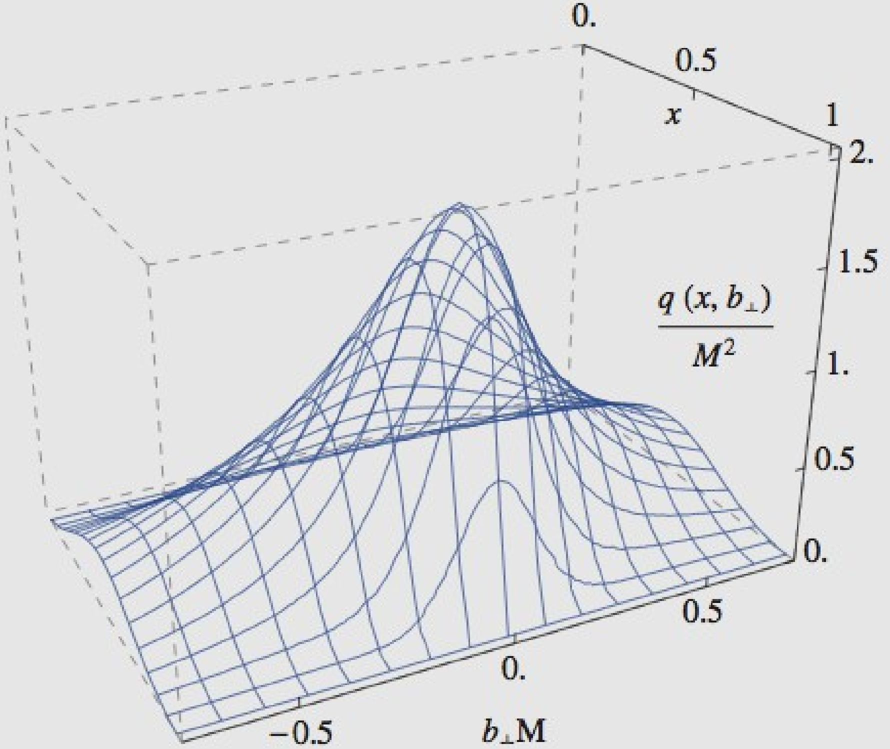
Results for the pion GPD

The off-forward (non-skewed) GPD:

$$q(x,|\vec{b}|) = \int \frac{d|\vec{\Delta}_\perp|}{2\pi}|\vec{\Delta}_\perp| J_0(|\vec{b}_\perp||\vec{\Delta}_\perp|) H(x,0,-\Delta_\perp^2)$$

Impact parameter space GPD at $\zeta = 0.4$ GeV

$M = 0.4$ GeV



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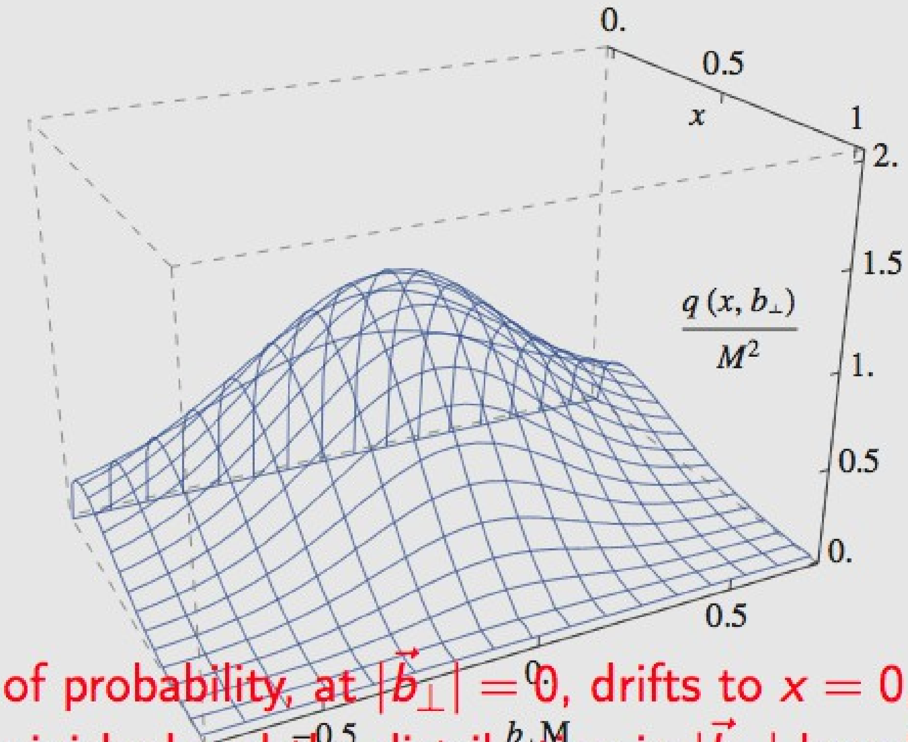
Results for the pion GPD

The off-forward (non-skewed) GPD:

$$q(x, |\vec{b}|) = \int \frac{d|\vec{\Delta}_\perp|}{2\pi} |\vec{\Delta}_\perp| J_0(|\vec{b}_\perp||\vec{\Delta}_\perp|) H(x, 0, -\Delta_\perp^2)$$

Impact parameter space GPD at $\zeta = 2$ GeV

$M = 0.4$ GeV



The peak of probability, at $|\vec{b}_\perp| = 0$, drifts to $x = 0$, its height is diminished and the distribution in $|\vec{b}_\perp|$ broadens.

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Results for the pion GPD

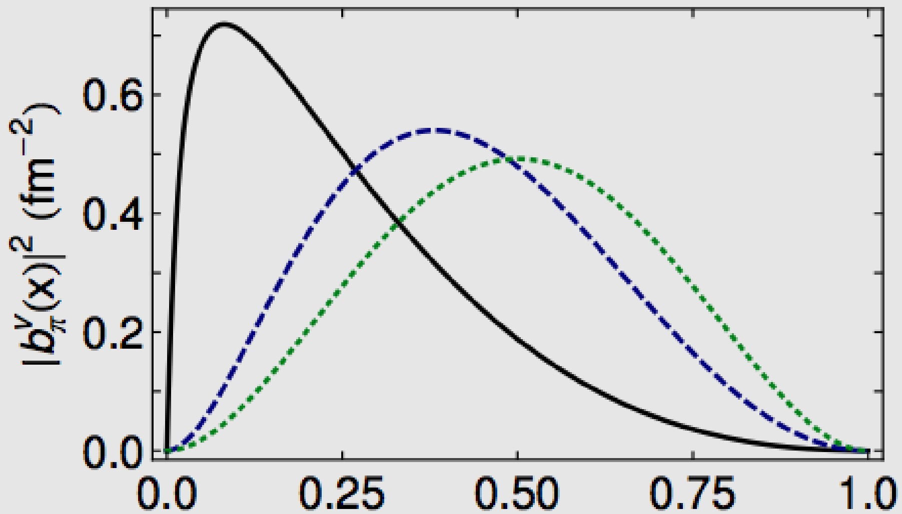
The off-forward (non-skewed) GPD:

$$q(x,|\vec{b}|) = \int \frac{d|\vec{\Delta}_\perp|}{2\pi} |\vec{\Delta}_\perp| J_0(|\vec{b}_\perp||\vec{\Delta}_\perp|) H(x,0,-\Delta_\perp^2)$$

$$\langle|\vec{b}_\perp|^2\rangle = \int_{-1}^{1} dx \, \langle|\vec{b}_\perp(x;\zeta)|^2\rangle = \int_{-1}^{1} dx \int_0^\infty d|\vec{b}_\perp| |\vec{b}_\perp|^3 \int_0^\infty d\Delta \Delta J_0(\vec{b}_\perp|\Delta) F_\pi(\Delta^2)$$

Impact parameter space GPD

$r_\pi = \sqrt{3/2\langle|\vec{b}_\perp|^2\rangle} = 0.674$ fm $\Longleftrightarrow$ $r_\pi = 0.672(8)$ fm [PRD86(2012)010001]



$\zeta = 2$ GeV; $\zeta = 0.4$ GeV; $\zeta = 0.4$GeV [c(x,t)=1].

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Extension: The overlap approach

A first-principle connection with Light-Front Wave Function:

- Decompose an hadronic state $|H; P, \lambda\rangle$ in a Fock basis:

$$|H; P, \lambda\rangle = \sum_{N,\beta} \int [\mathrm{d}x \mathrm{d}\mathbf{k}_\perp]_N \psi_N^{(\beta,\lambda)}(x_1, \mathbf{k}_{\perp 1}, \ldots, x_N, \mathbf{k}_{\perp N}) \, |\beta, k_1, \ldots, k_N\rangle$$

- Derive an expression for the pion GPD in the DGLAP region $\xi \leq x \leq 1$:

$$H^q(x, \xi, t) \propto \sum_{\beta,j} \int [\mathrm{d}\bar{x} \mathrm{d}\bar{\mathbf{k}}_\perp]_N \delta_{j,q} \delta(x - \bar{x}_j) \psi_N^{(\beta,\lambda)*}(\hat{x}', \hat{\mathbf{k}}'_\perp) \psi_N^{(\beta,\lambda)}(\tilde{x}, \tilde{\mathbf{k}}_\perp)$$

  with $\tilde{x}, \tilde{\mathbf{k}}_\perp$ (resp. $\hat{x}', \hat{\mathbf{k}}'_\perp$) generically denoting incoming (resp. outgoing) parton kinematics.

  Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)

- Similar expression in the ERBL region $-\xi \leq x \leq \xi$, but with overlap of N- and $(N+2)$-body LFWF.

Extension: The overlap approach

A first-principle connection with Light-Front Wave Function:

- Decompose an hadronic state $|H; P, \lambda\rangle$ in a Fock basis:

$$|H; P, \lambda\rangle = \sum_{N,\beta} \int [\mathrm{d}x\mathrm{d}\mathbf{k}_\perp]_N \psi_N^{(\beta,\lambda)}(x_1, \mathbf{k}_{\perp 1}, \ldots, x_N, \mathbf{k}_{\perp N}) \, |\beta, k_1, \ldots, k_N\rangle$$

- Derive an expression for the pion GPD in the DGLAP region $\xi \leq x \leq 1$:

$$H_\pi^q(x,\xi,t)_{\xi \leq x \leq 1} = C^q \int d^2\mathbf{k}_\perp^2 \Psi^*\left(\frac{x-\xi}{1-\xi}, \mathbf{k}_\perp + \frac{1-x}{1-\xi}\frac{\Delta_\perp}{2}; P_-\right)\Psi\left(\frac{x+\xi}{1+\xi}, \mathbf{k}_\perp - \frac{1-x}{1+\xi}\frac{\Delta_\perp}{2}; P_+\right)$$

with $\tilde{x}, \tilde{\mathbf{k}}_\perp$ (resp. $\hat{x}', \hat{\mathbf{k}}'_\perp$) generically denoting incoming (resp. outgoing) parton kinematics.

Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)

- Similar expression in the ERBL region $-\xi \leq x \leq \xi$, but with overlap of N- and $(N+2)$-body LFWF.

Extension: The overlap approach

First step: DGLAP GPD from Light Front Wave Functions

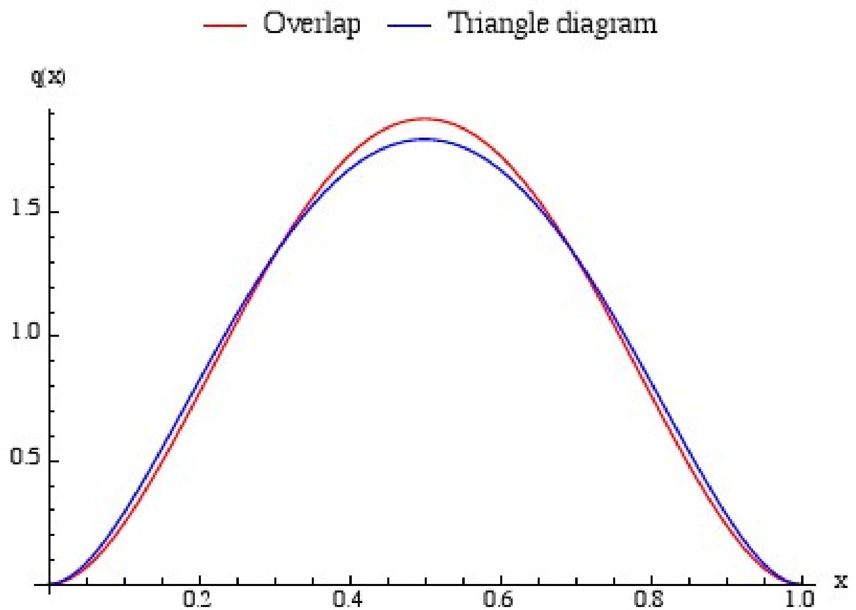
Illustration:

- Evaluate LFWF in algebraic model:

$$\psi(x, \mathbf{k}_\perp) \propto \frac{x(1-x)}{[(\mathbf{k}_\perp - x\mathbf{P}_\perp)^2 + M^2]^2}$$

- Expression for the GPD at $t = 0$:

$$H(x, \xi, 0) \propto \frac{(1-x)^2(x^2-\xi^2)}{(1-\xi^2)^2}$$



- Manifest 2-body symmetry.
- Expression for the PDF:

$$q(x) = 30x^2(1-x)^2$$

Extension: The overlap approach

First step: DGLAP GPD from Light Front Wave Functions

Illustration:

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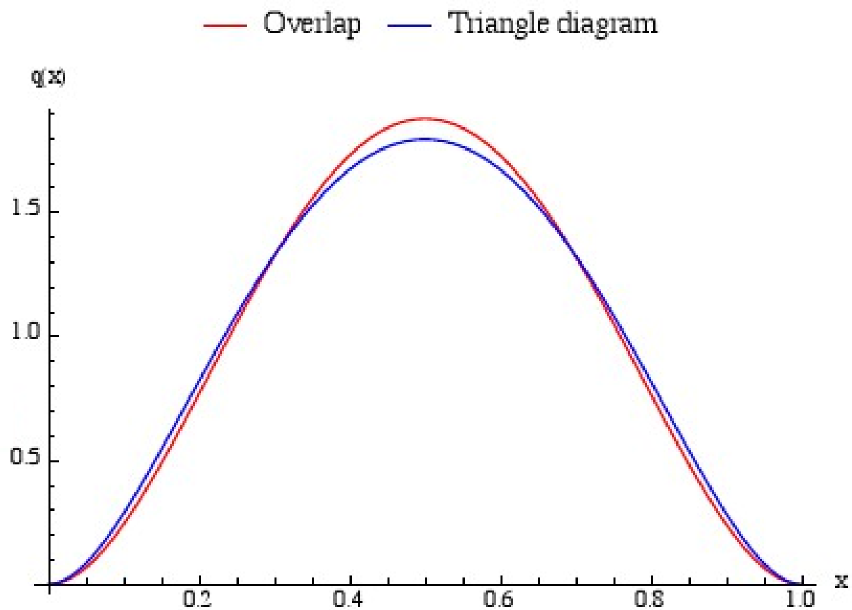

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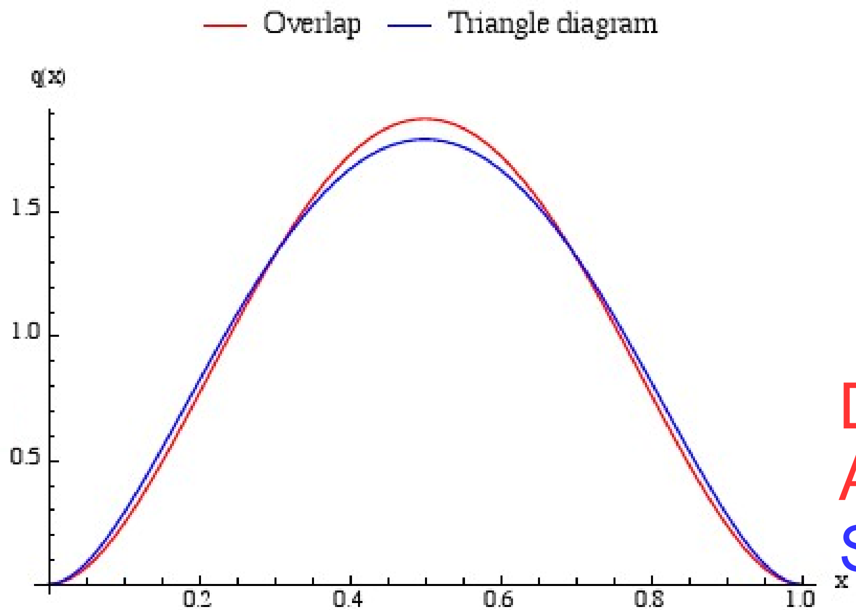

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DGLAP → ERBL?

A systematic procedure?

See this afternoon Nabil's talk!!!

Conclusions:

Just made a few modest steps in a very long way!!!

- **Nonperturbative** computation of GPDs, DDs, LFWFs,...from Dyson-Schwinger equations.
- **Explicit check** of several theoretical constraints, including polynomiality, support property and soft pion theorem.
- **Systematic** procedure to construct GPD models from any "reasonable" Ansatz of LFWFs.

(Don't miss Nabil's talk this afternoon!!!)

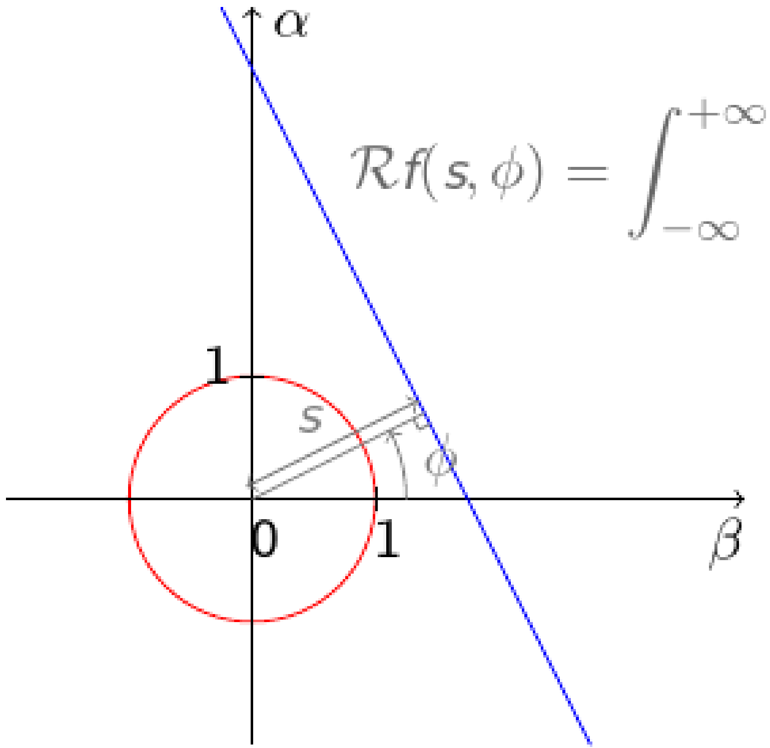
...much work in progress and to do!!!

Thank you.

Backslides:

Radon transform

Definition and properties



For $s > 0$ and $\phi \in [0, 2\pi]$:

$$\mathcal{R}f(s,\phi) = \int_{-\infty}^{+\infty} \mathrm{d}\beta \mathrm{d}\alpha\, f(\beta,\alpha)\delta(s - \beta\cos\phi - \alpha\sin\phi)$$

and:

$$\mathcal{R}f(-s,\phi) = \mathcal{R}f(s,\phi \pm \pi)$$

Relation to GPDs:

$$x = \frac{s}{\cos\phi} \text{ and } \xi = \tan\phi$$

Relation between GPD and DD in Belistky *et al.* gauge

$$\frac{\sqrt{1+\xi^2}}{x} H(x,\xi) = \mathcal{R}f_{\mathrm{BMKS}}(s,\phi)$$

Radon transform

Polinomiality and Ludwig-Helgason condition

- The Mellin moments of a Radon transform are **homogeneous polynomials** in $\omega = (\sin\phi, \cos\phi)$.
- The converse is also true:

Theorem (Hertle, 1983)

Let $g(s, \omega)$ an even compactly-supported distribution. Then g is itself the Radon transform of a compactly-supported distribution if and only if the ***Ludwig-Helgason consistency condition*** *hold:*

(i) *g is C^∞ in ω,*

(ii) *$\int \mathrm{d}s\, s^m g(s, \omega)$ is a homogeneous polynomial of degree m for all integer $m \geq 0$.*

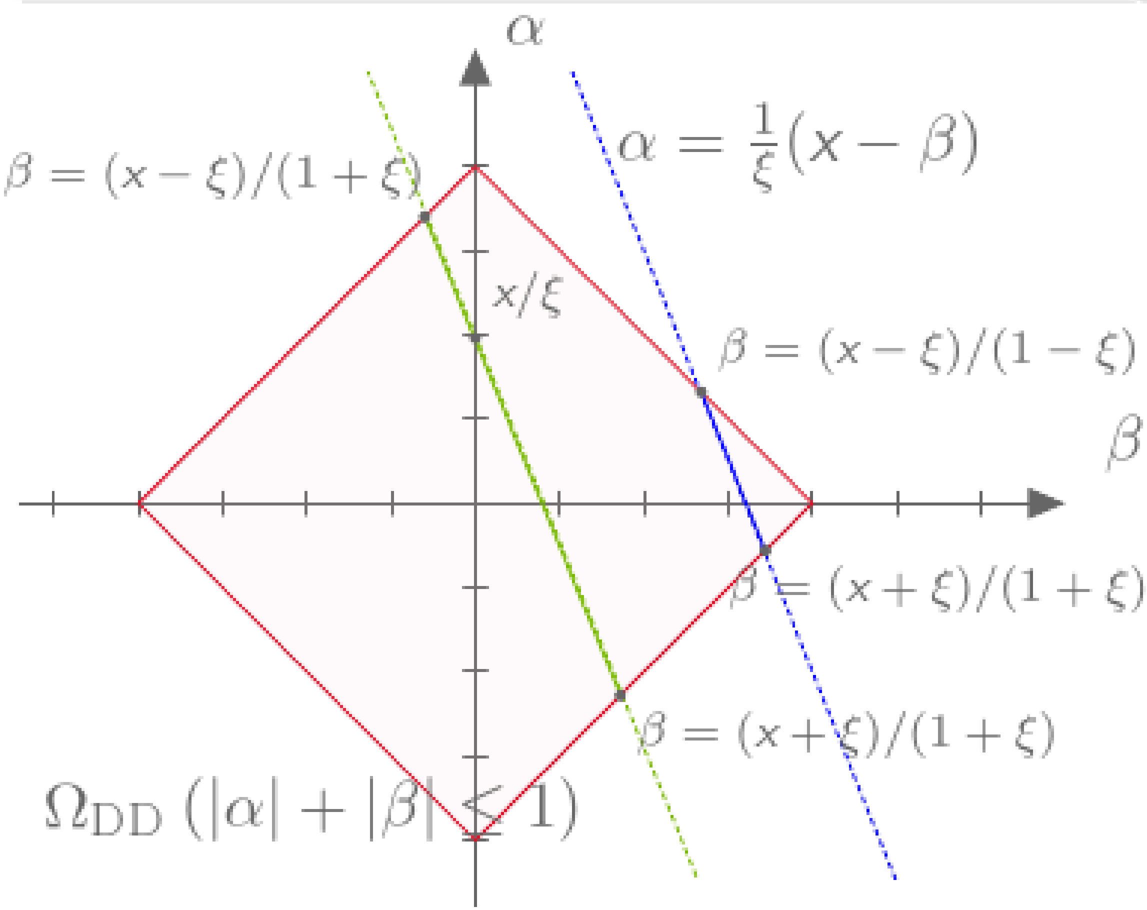
- Double Distributions and the Radon transform are the **natural solution** of the polynomiality condition.

Implementing Lorentz covariance

From GPD DGLAP to whole GPD domain

DGLAP and ERBL regions

$$(x,\xi) \in \text{DGLAP} \quad \Leftrightarrow \quad |s| \geq |\sin\phi| \, ,$$
$$(x,\xi) \in \text{ERBL} \quad \Leftrightarrow \quad |s| \leq |\sin\phi| \, .$$



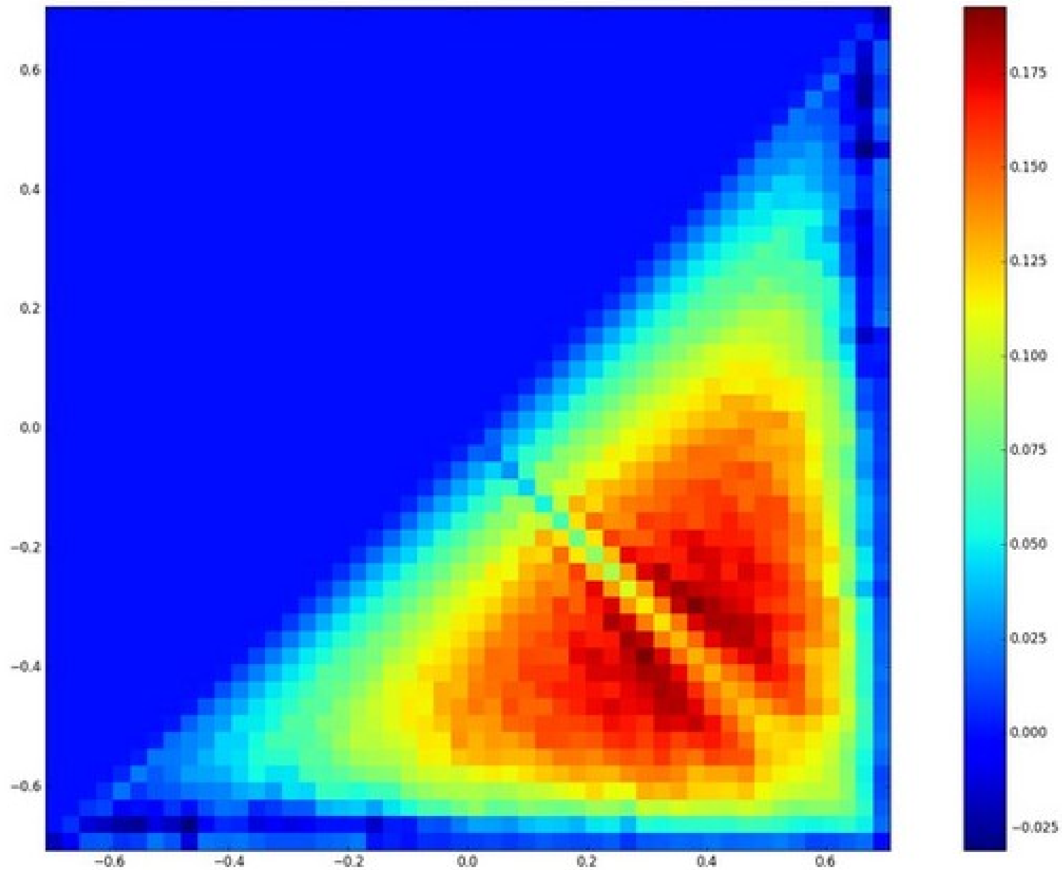
- Each point (β,α) with $\beta \neq 0$ contributes to **both** DGLAP and ERBL regions.
- Expressed in **support theorem**.

Inverse Radon transform

Preliminary results

Illustration of inverse Radon results with a Radyushkin DD ansatz

$$f(\beta, \alpha; t) = \Re^{-1} H_{RDDA}(\beta, \alpha; t)$$



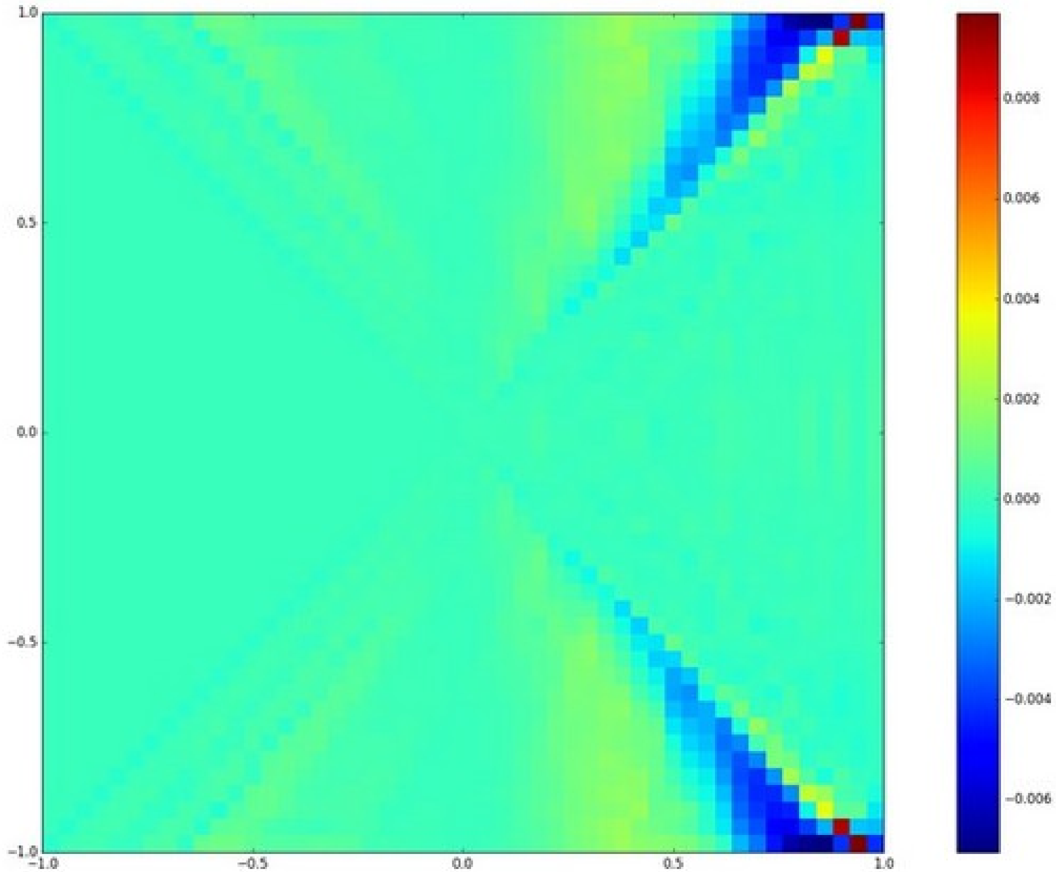
Nabil Chouika's preliminary results!!!

Inverse Radon transform

Preliminary results

Illustration of inverse Radon results with a Radyushkin DD ansatz

$$\Delta(x,\xi;t) = \left[H_{RDDA} - \Re \Re^{-1} H_{RDDA} \right](x,\xi;t)$$



Nabil Chouika's preliminary results!!!