The Multidimensional Nucleon Structure

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3DSPIN
MAPPING THE PROTON IN 3D

erc
European Research Council

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Istituto Nazionale di Fisica Nucleare

Principal Investigator: A. Bacchetta

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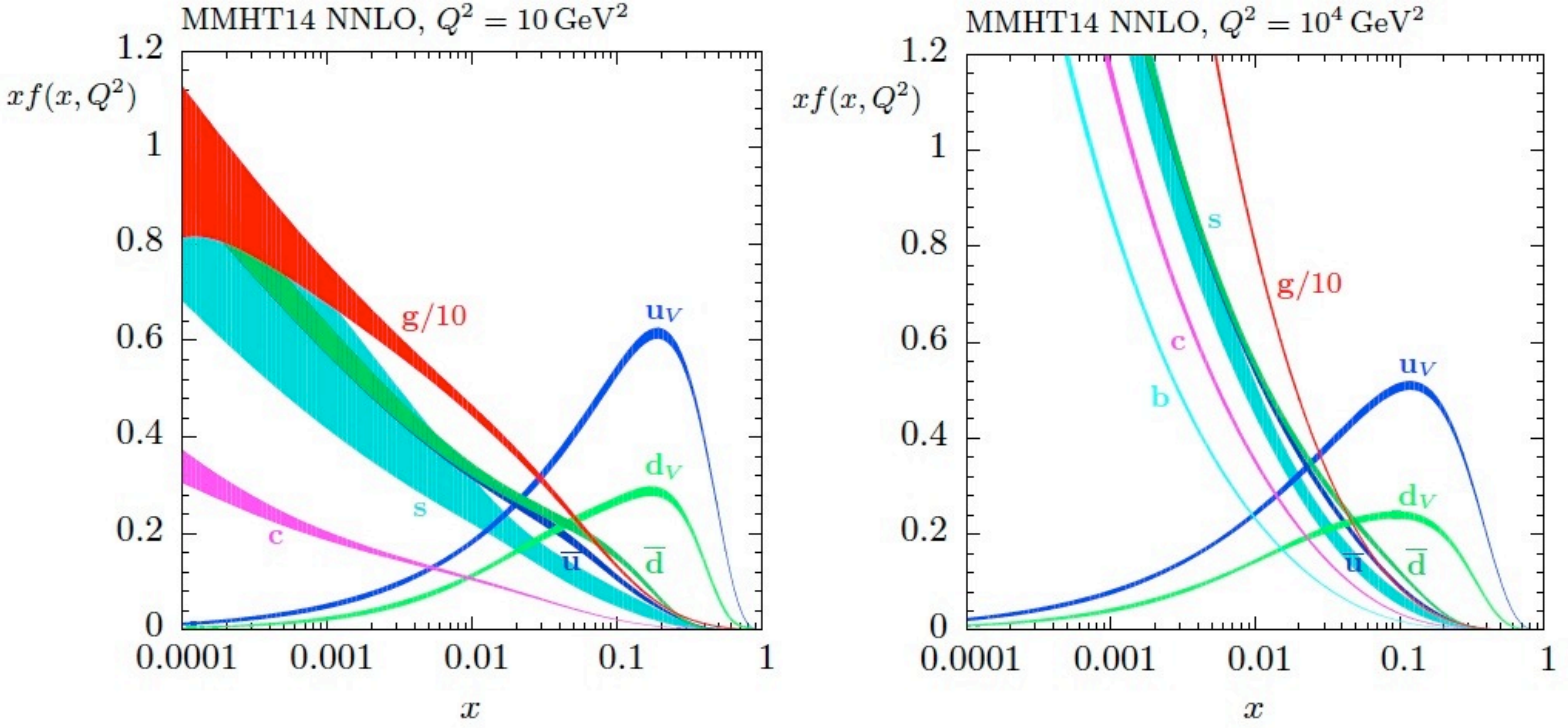
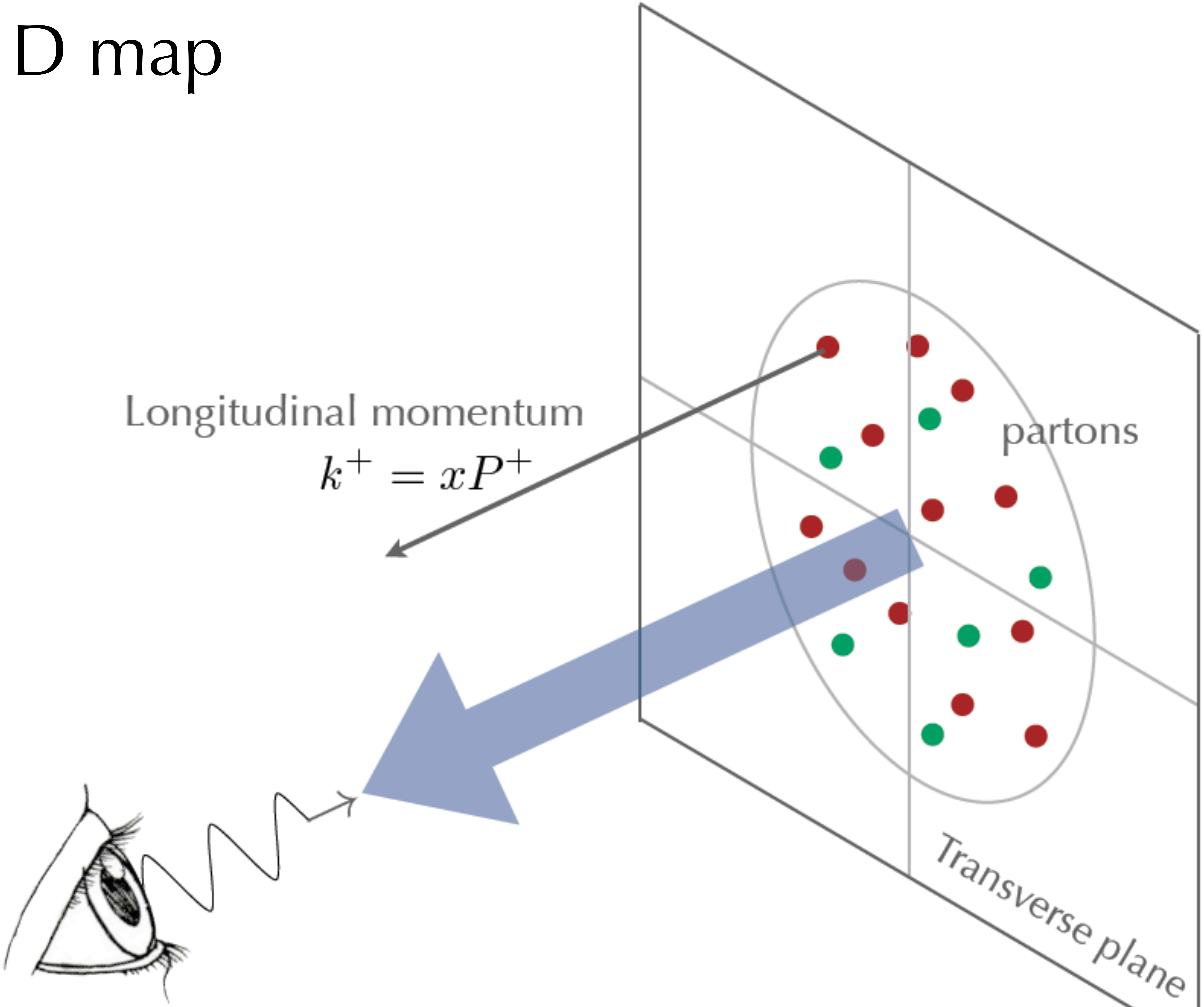
Available Maps: Parton Distribution Functions

monodimensional (in momentum space)



MMHT2014 Eur. Phys. J. C75 (2015) 204

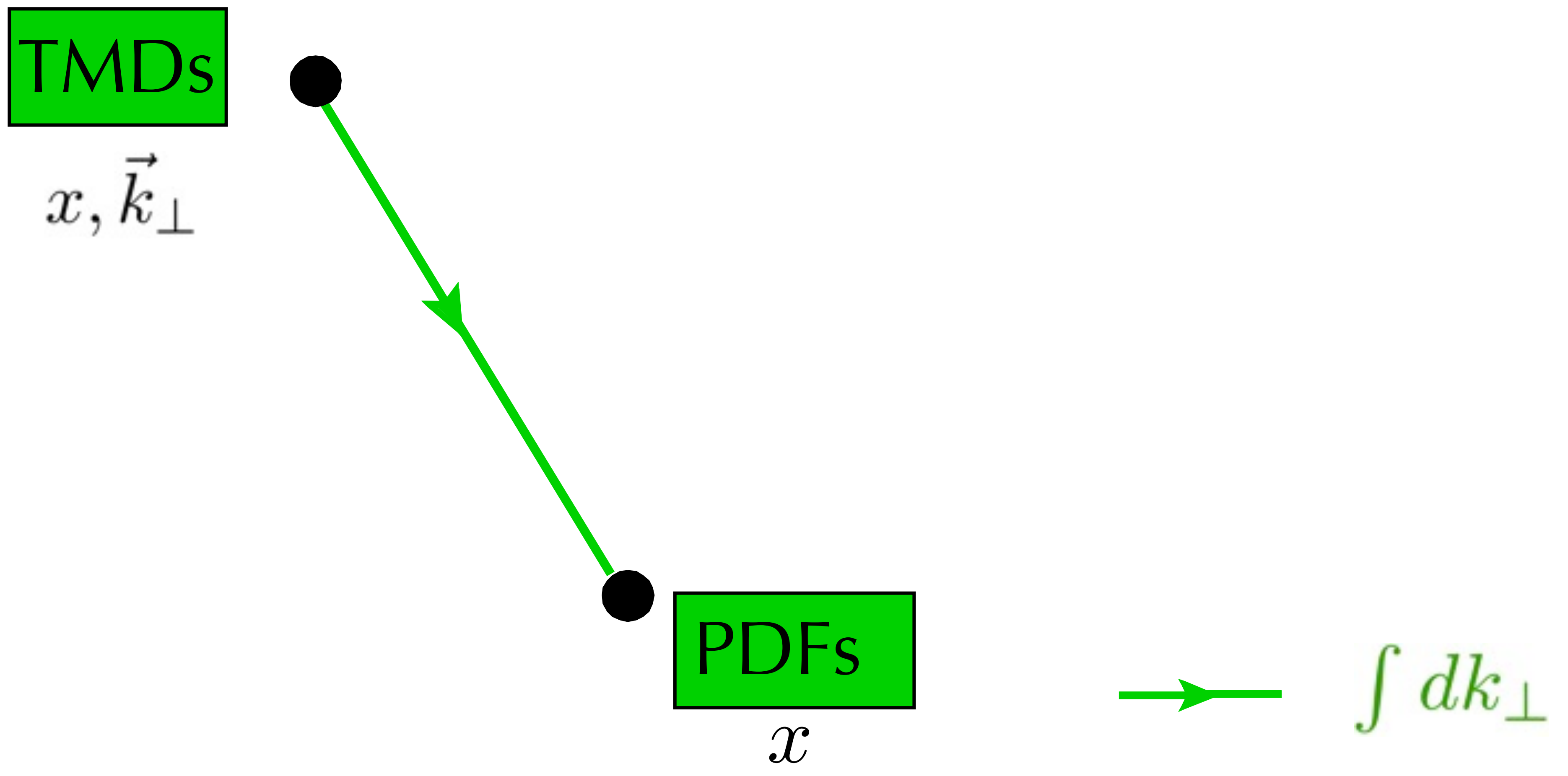
PDFs: 1D map



How can we built up
a multidimensional picture
of the nucleon?

Transverse Momentum PDFs (TMDs)

$$\boxed{\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}^2z_\perp}{(2\pi)^3}e^{ik\cdot z}\langle p^+,\,\vec{0}_\perp,\,\Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+,\,\vec{0}_\perp,\,\Lambda\rangle_{z^+=0}}$$



TMDs

$x, \vec{k}_\perp$

PDFs

x

$\int dk_\perp$

Transverse Momentum PDFs (TMDs)

$$\boxed{\frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2 z_\perp}{(2\pi)^3} e^{ik\cdot z}\langle p^+,\,\vec{0}_\perp,\,\Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+,\,\vec{0}_\perp,\,\Lambda\rangle_{z^+=0}}$$

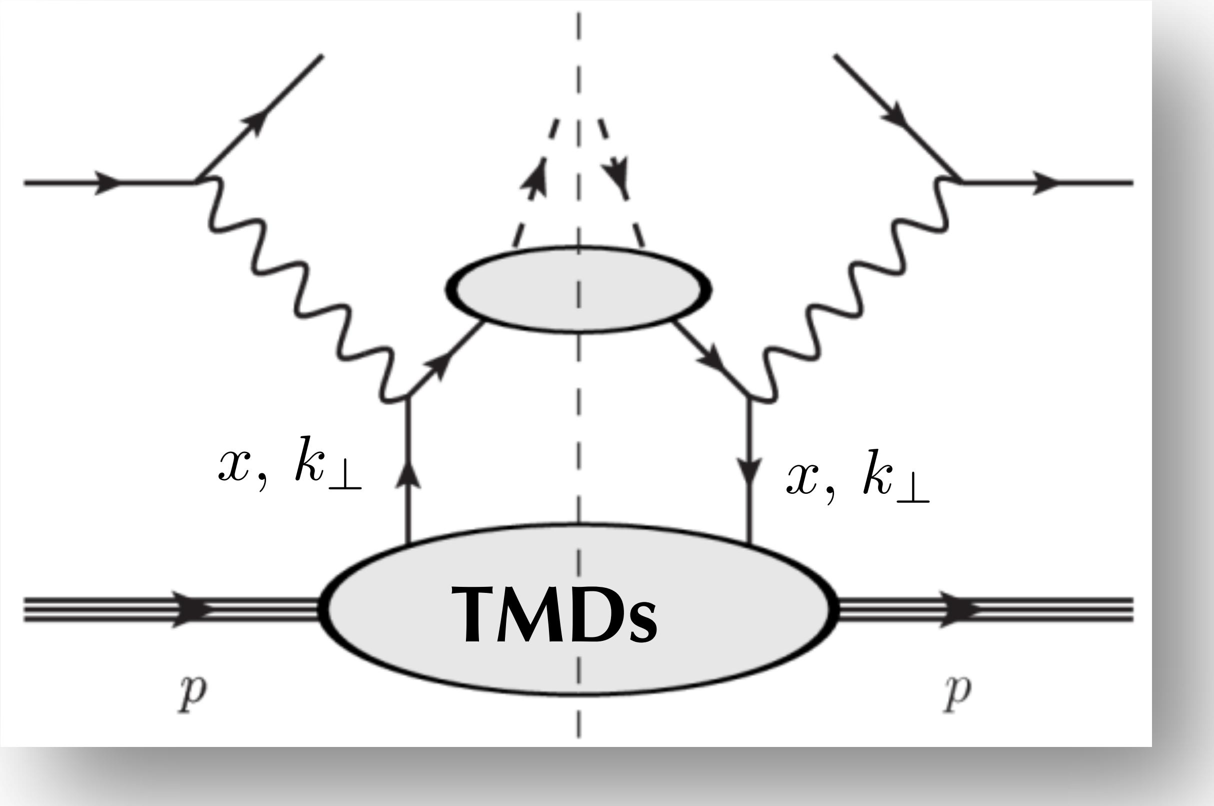
Depend on

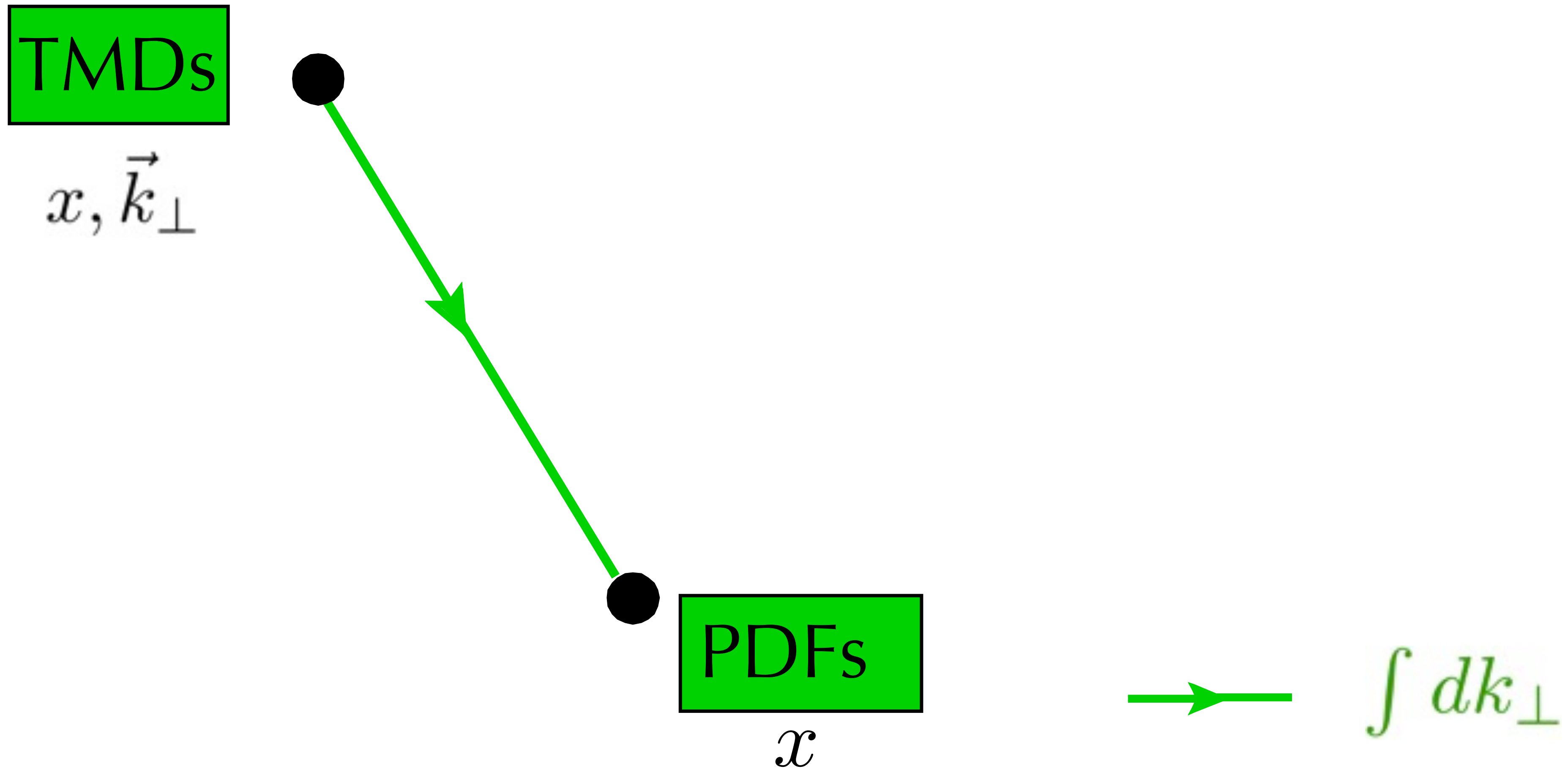
$x = \dfrac{k^+}{P^+}$: longitudinal momentum fraction

$k_\perp$: parton transverse momentum

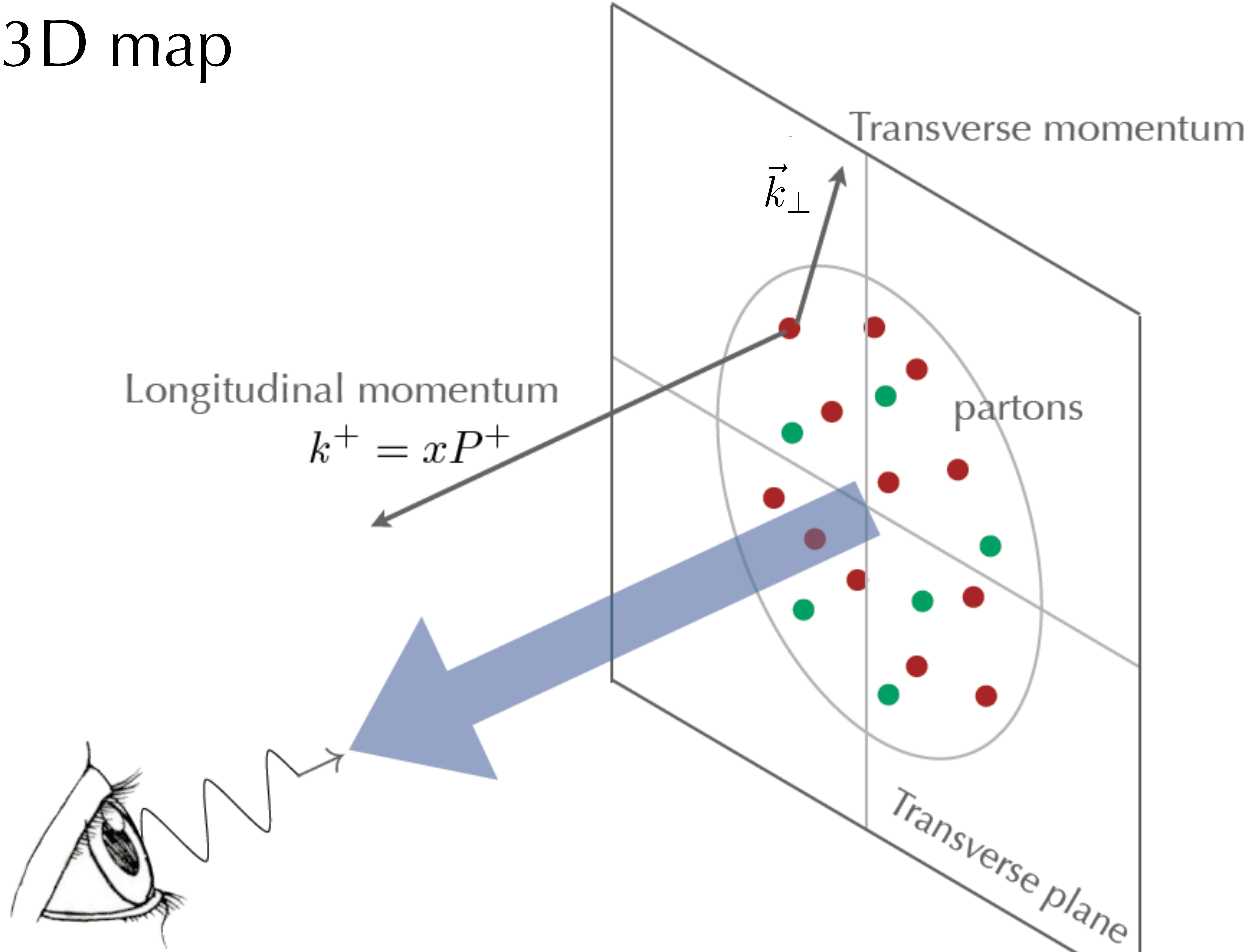
$\Lambda,\ \Lambda',\ \Gamma$: nucleon and quark polarizations

Semi-Inclusive
Deep Inelastic Scattering





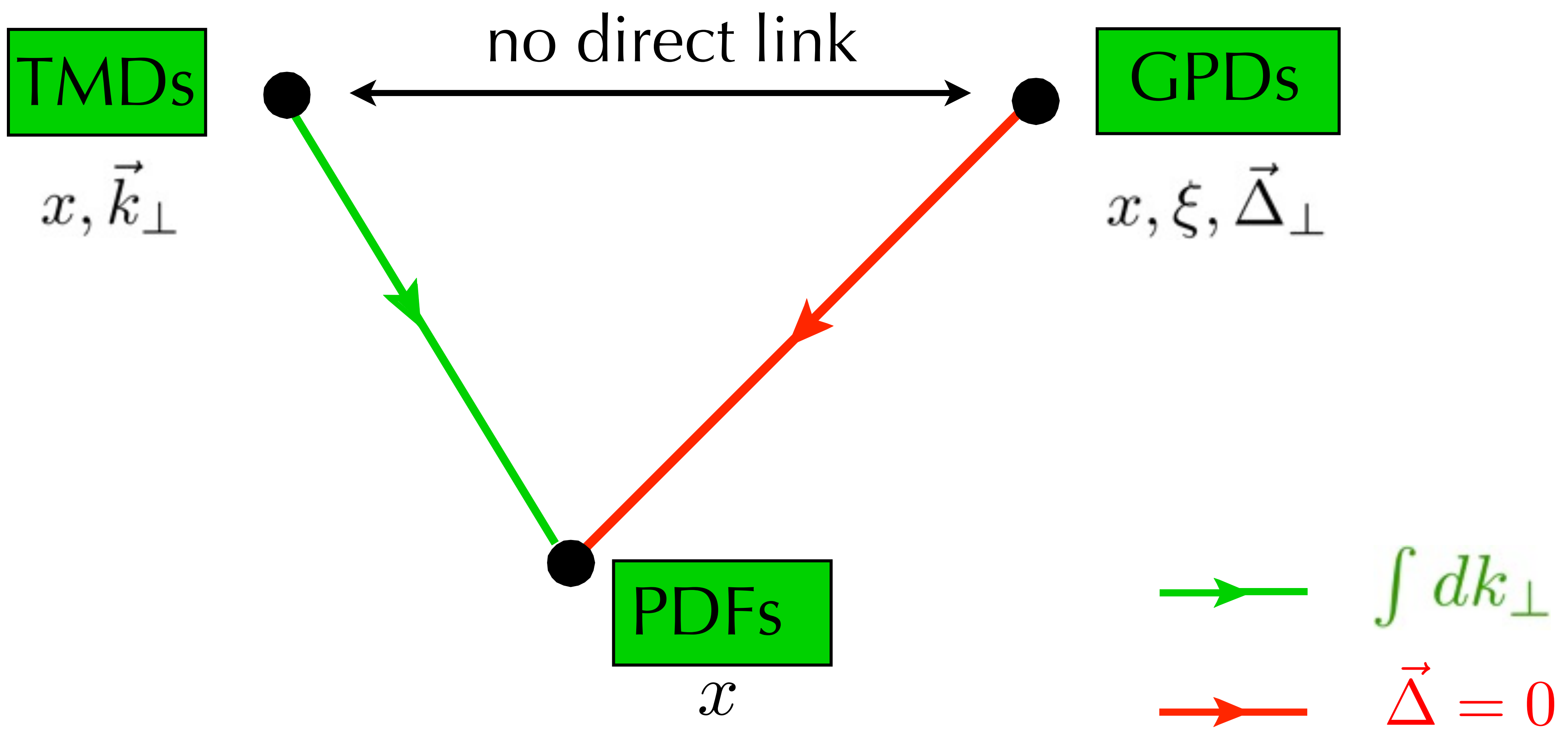
TMDs: 3D map


Transverse momentum
$\vec{k}_\perp$
Longitudinal momentum
$k^+ = xP^+$
partons
Transverse plane

Generalized Parton Distributions (GPDs)

$$\boxed{\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ik^+z^-}\langle p'^+,\, -\frac{\vec{\Delta}_\perp}{2},\, \Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+,\, \frac{\vec{\Delta}_\perp}{2},\, \Lambda\rangle_{z^+=0,z_\perp=0}} \longrightarrow$$

non-diagonal matrix elements



Generalized Parton Distributions (GPDs)

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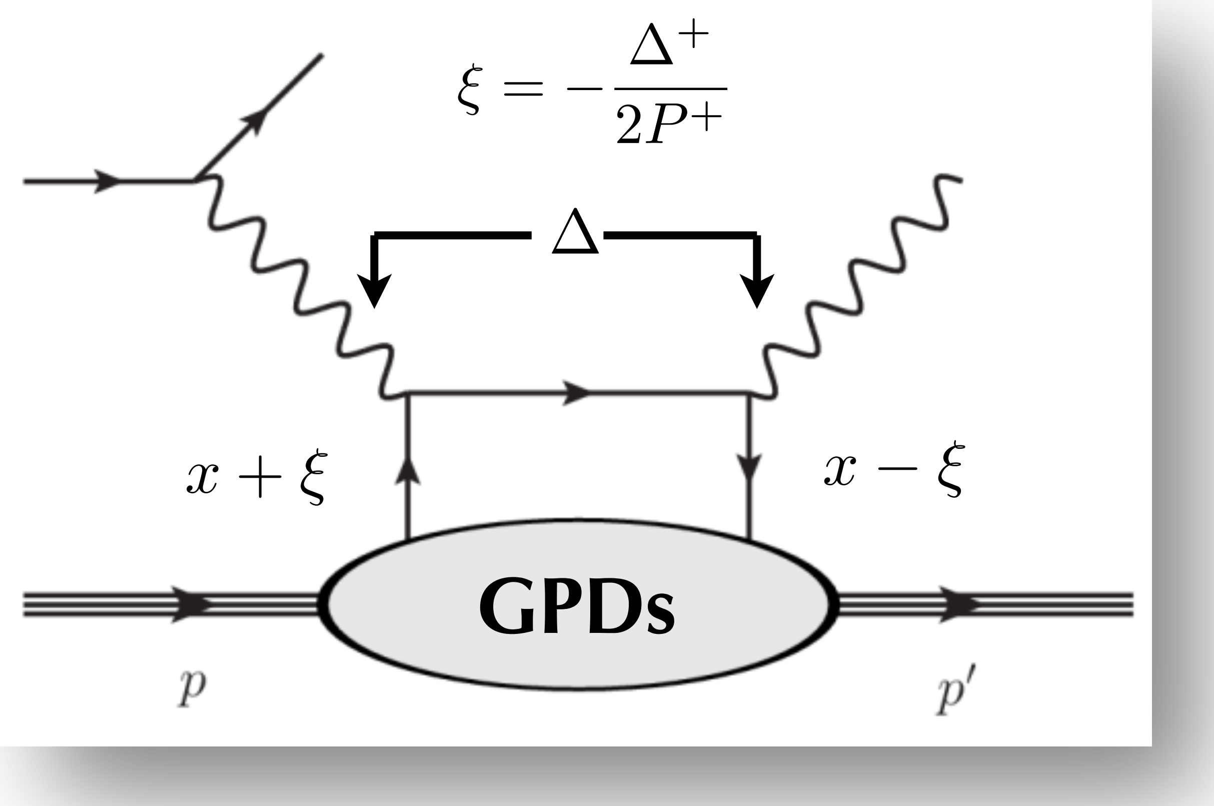
Depend on

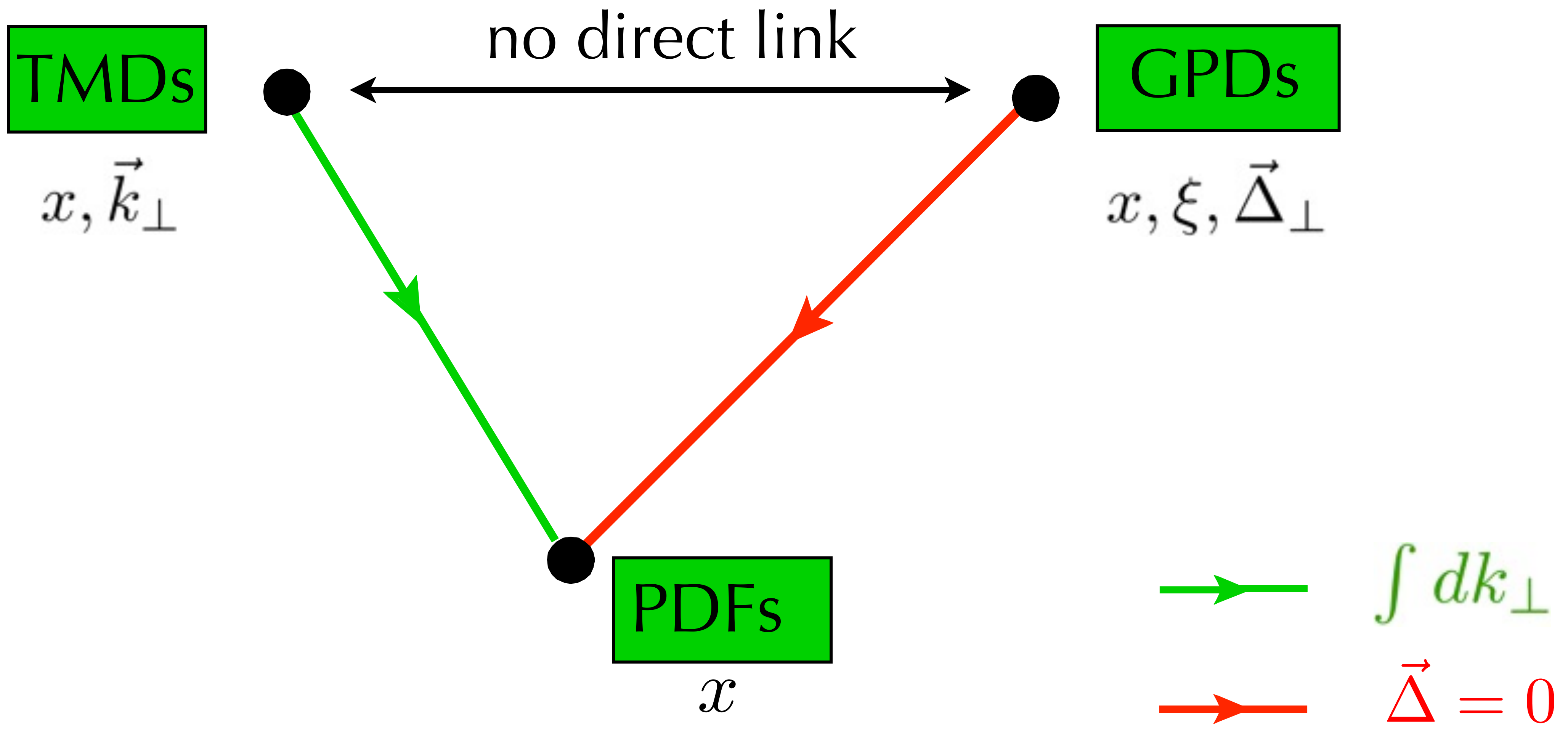
$x = \dfrac{k^+}{P^+}$: longitudinal momentum fraction

Δ : momentum transfer

$\Lambda,\ \Lambda',\ \Gamma$: nucleon and quark polarizations

Deeply Virtual Compton Scattering





TMDs $x, \vec{k}_\perp$

no direct link

GPDs $x, \xi, \vec{\Delta}_\perp$

PDFs x

$\int dk_\perp$

$\vec{\Delta} = 0$

Generalized Parton Distributions (GPDs)

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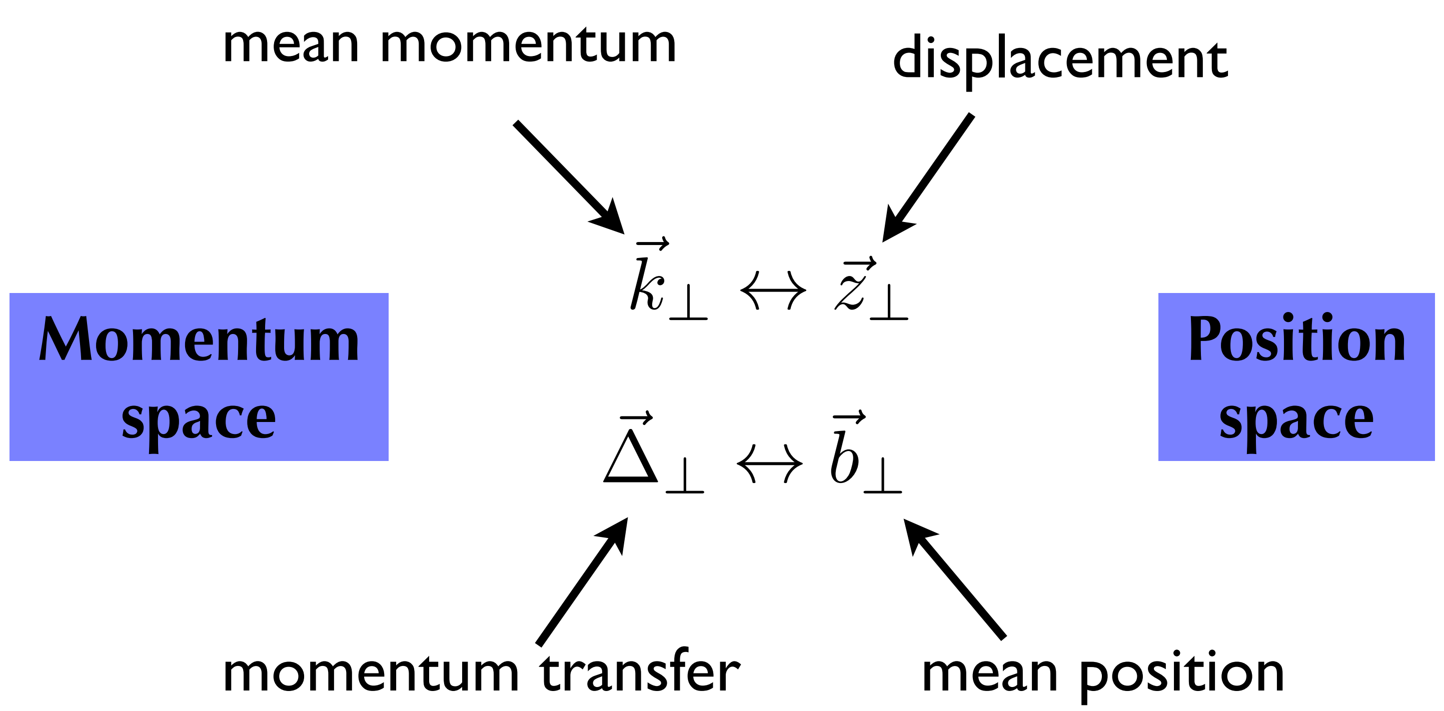
non-diagonal matrix elements

Depend on

$x = \frac{k^+}{P^+}$: longitudinal momentum fraction

Δ : momentum transfer $\quad \vec{\Delta}_\perp \xleftrightarrow{\text{FT}} \vec{b}_\perp$: impact parameter

$\Lambda, \Lambda', \Gamma$: nucleon and quark polarizations



Momentum space

Position space

$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$

$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

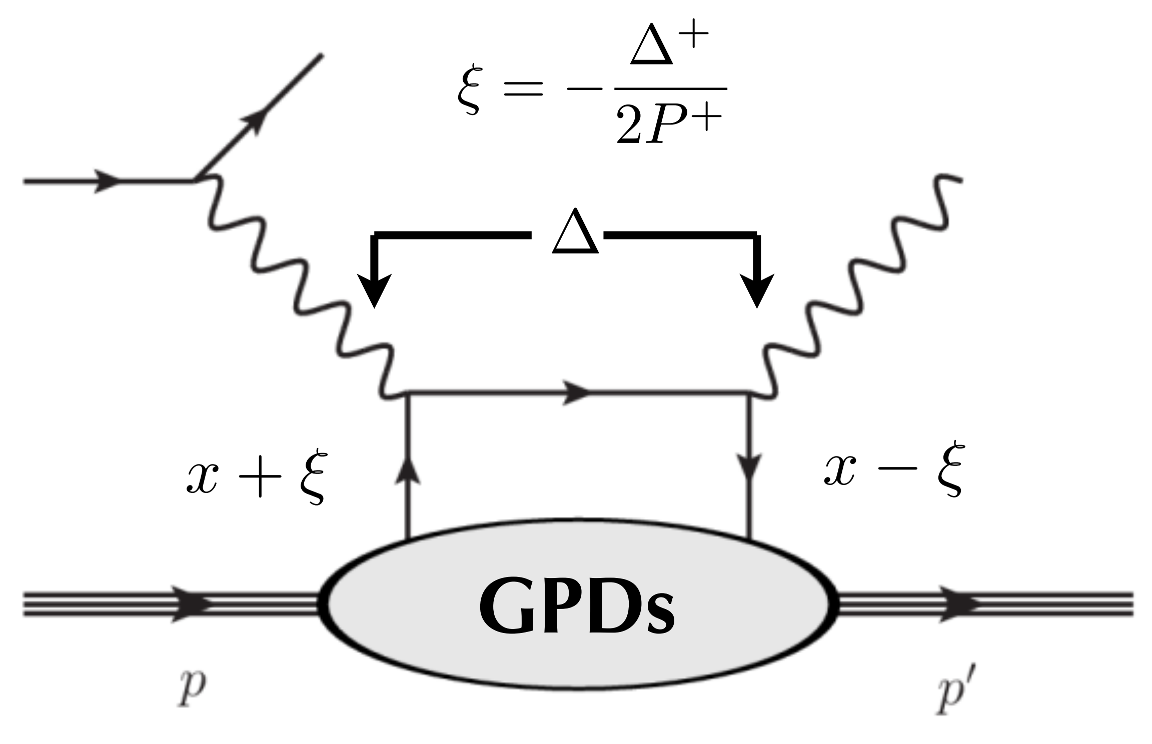
mean momentum

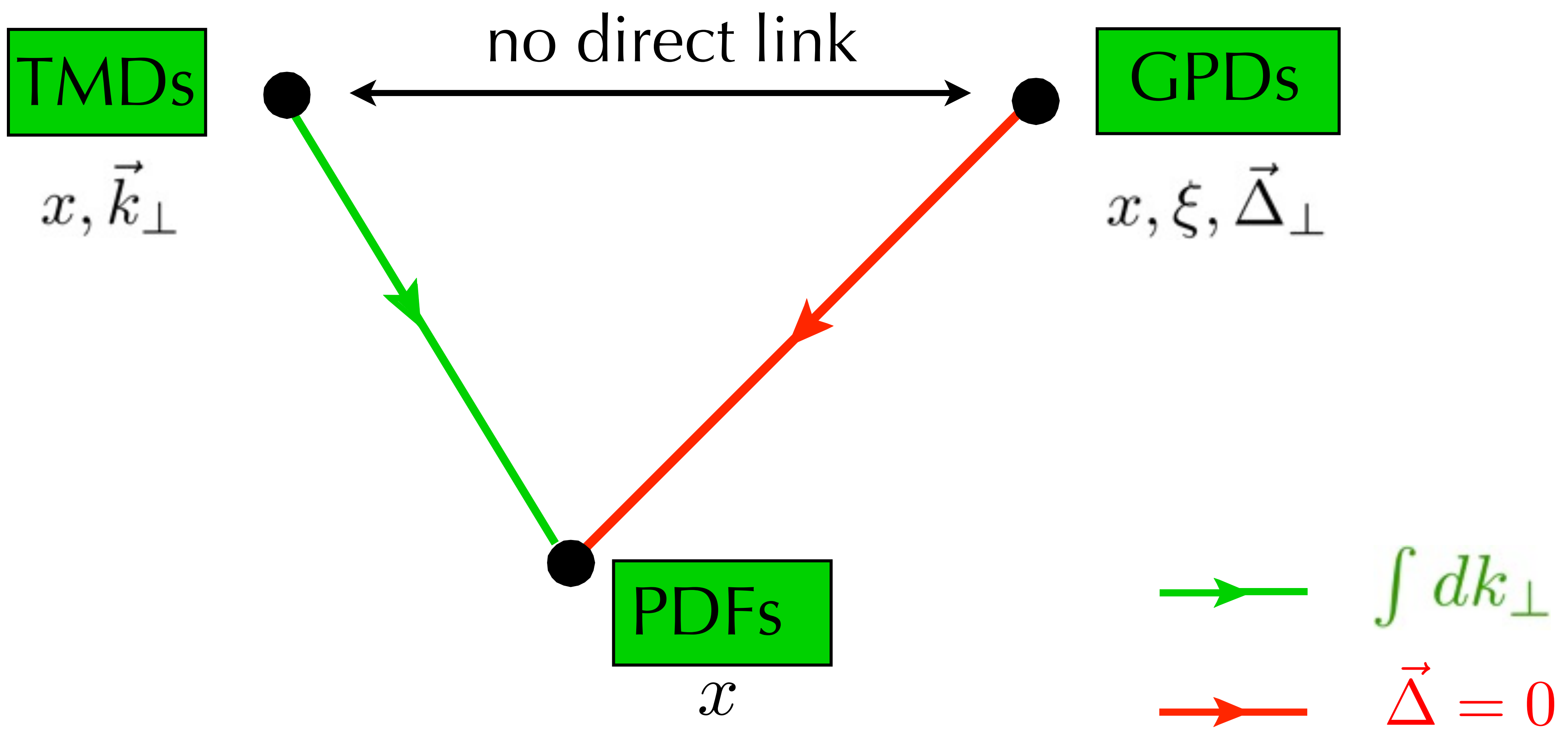
displacement

momentum transfer

mean position

Deeply Virtual Compton Scattering





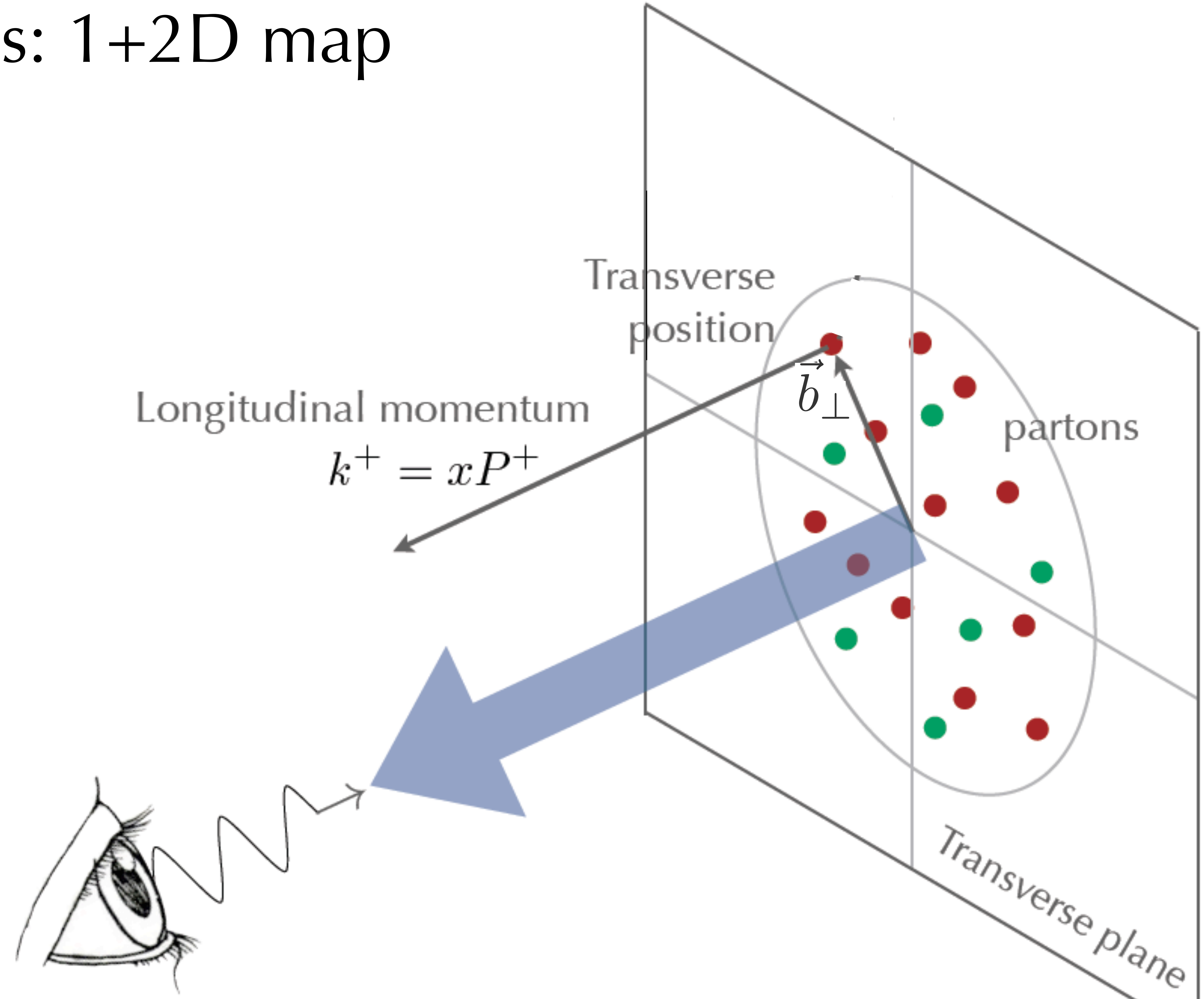
TMDs $x, \vec{k}_\perp$

no direct link

GPDs $x, \xi, \vec{\Delta}_\perp$

PDFs x

GPDs: 1+2D map


Transverse position
Longitudinal momentum
$k^+ = xP^+$
$\vec{b}_\perp$
partons
Transverse plane

Generalized TMDs (GTMDs)

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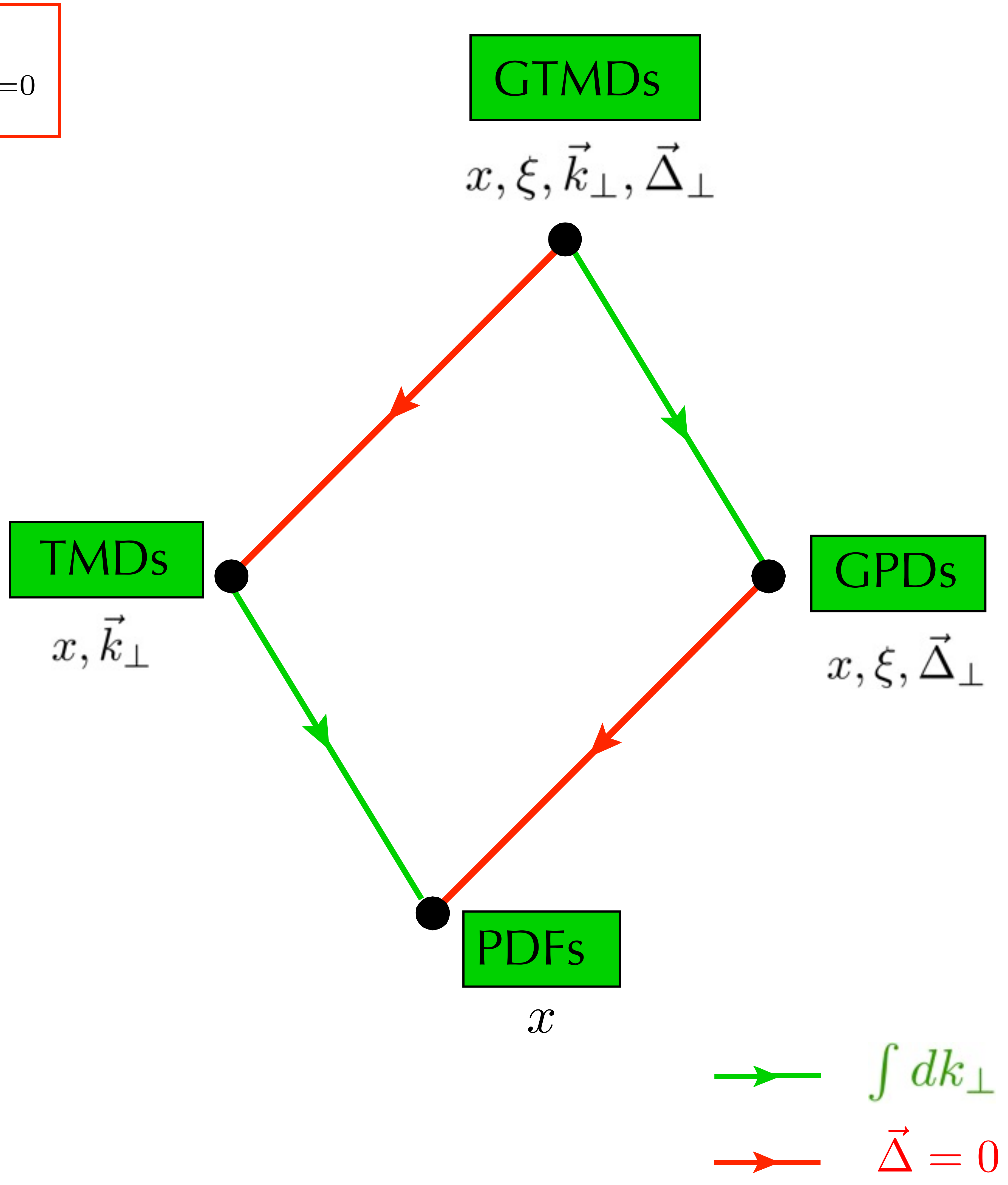

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

PDFs

x

$\int dk_\perp$

$\vec{\Delta} = 0$

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Depend on

$x = \frac{k^+}{P^+}$: longitudinal momentum fraction

Δ : momentum transfer

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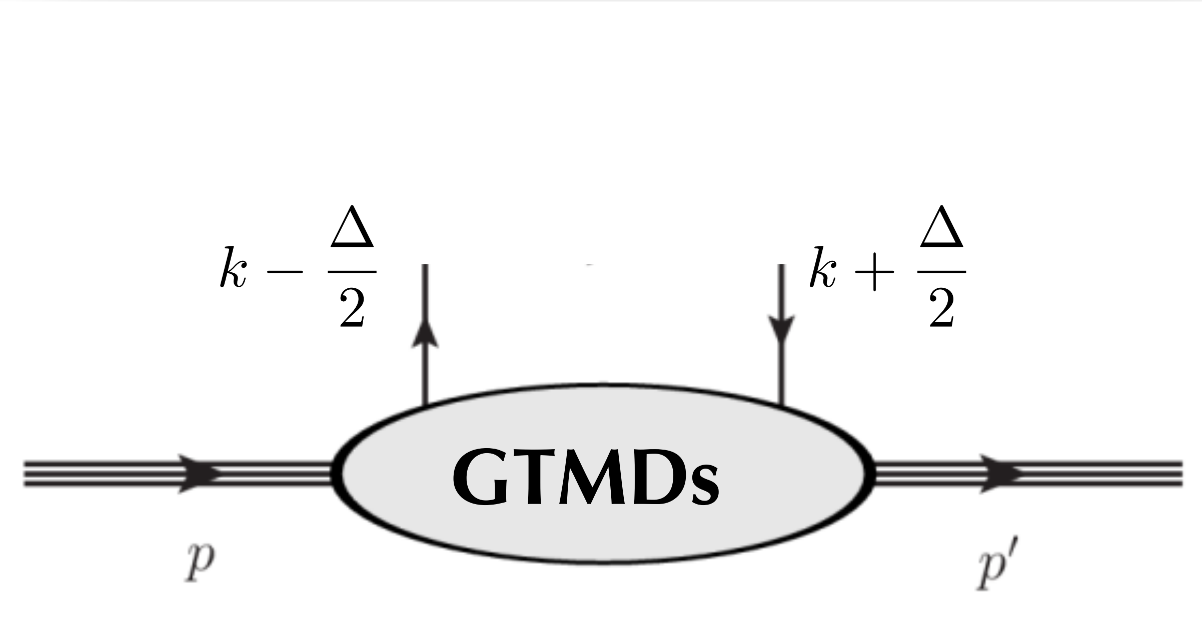
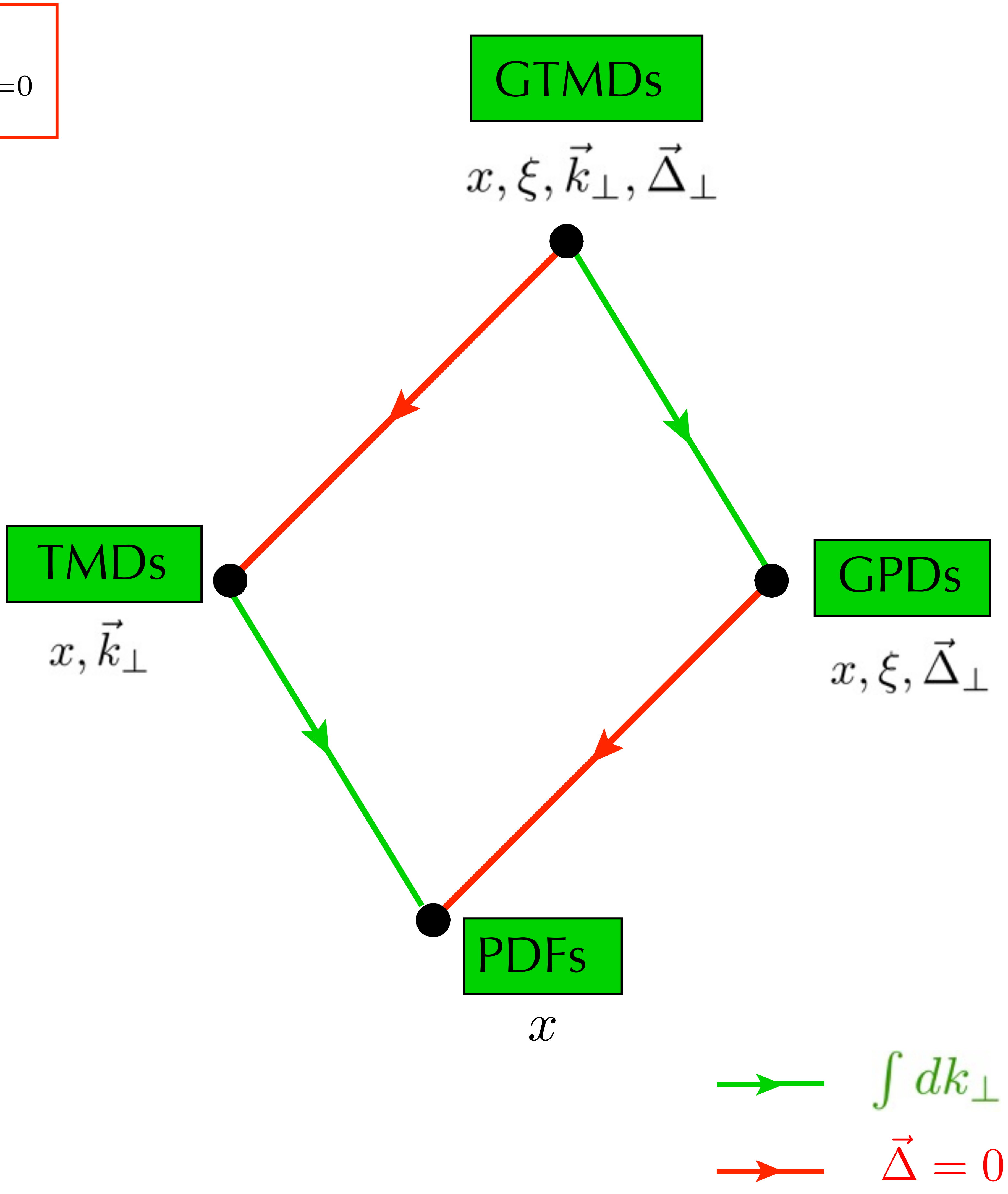
???????



relation of small-x gluon GTMDs to diffractive dijet production in DIS
Hatta, Xiao, Yuan, PRL 116 (2016)



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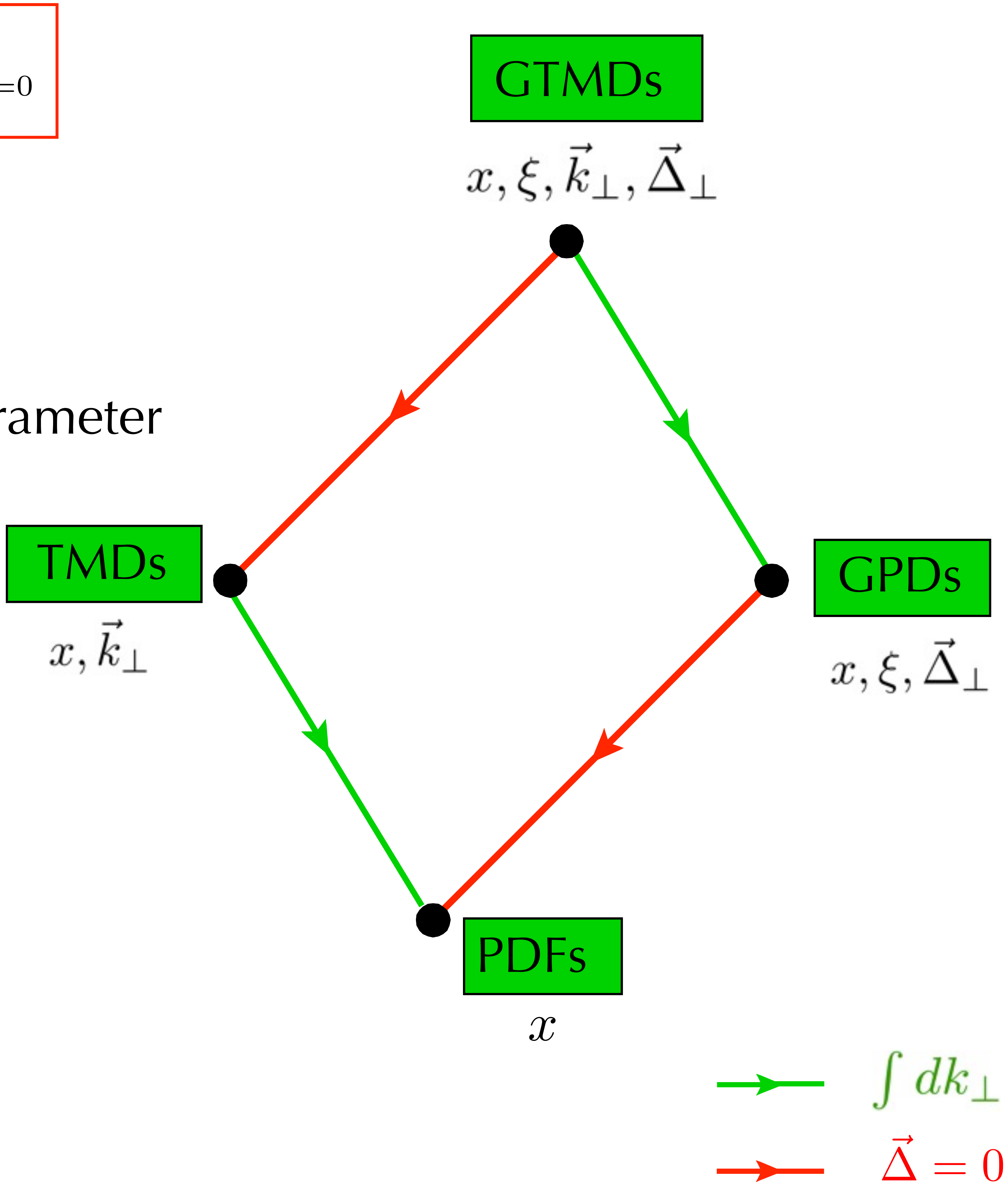


GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

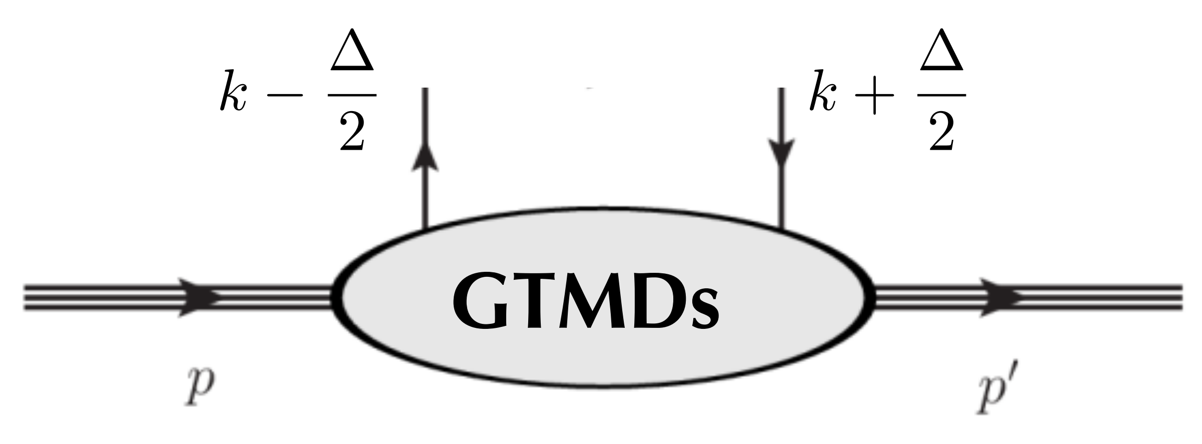
PDFs

x

🟩 $\int dk_\perp$

🟥 $\vec{\Delta} = 0$

???????

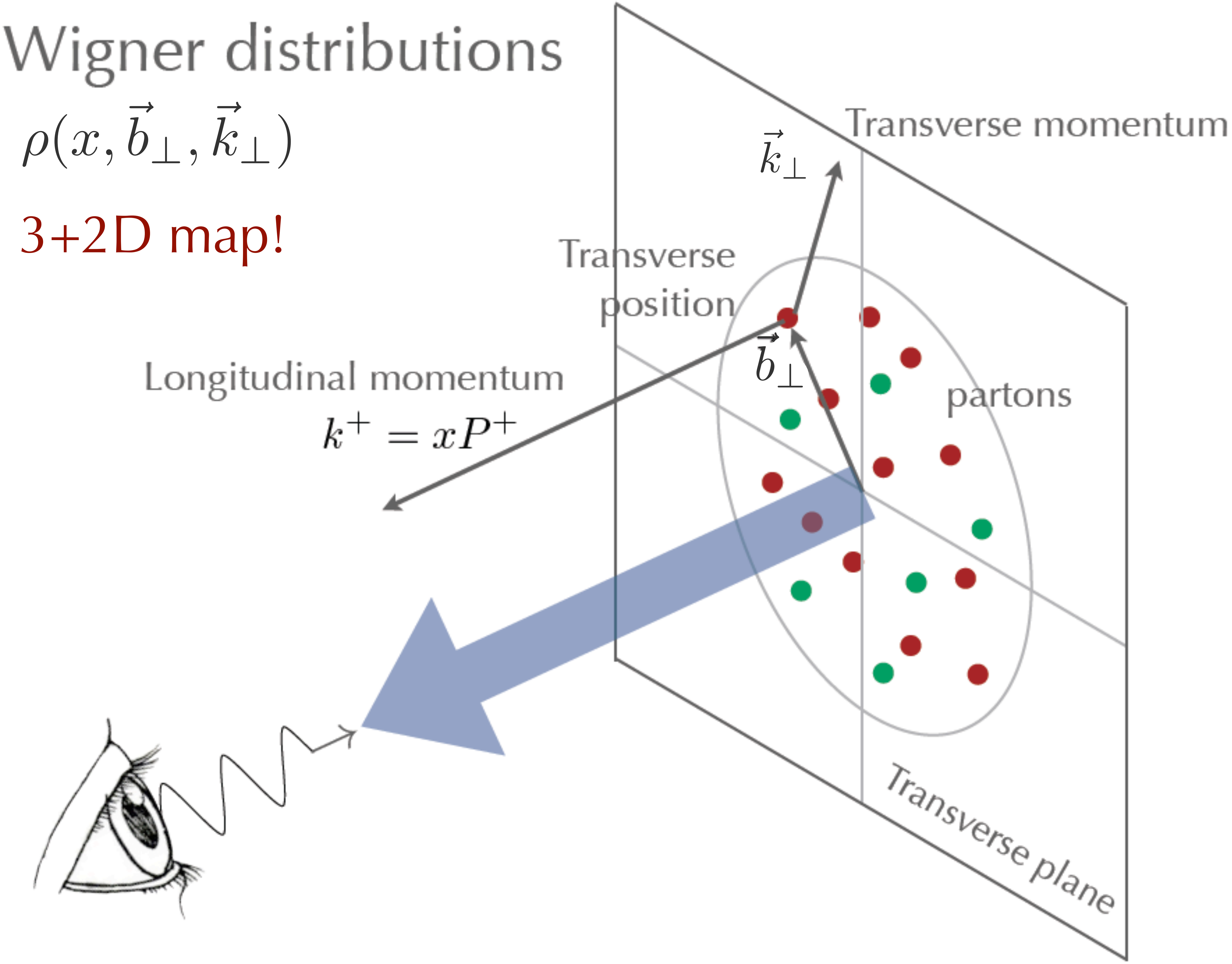


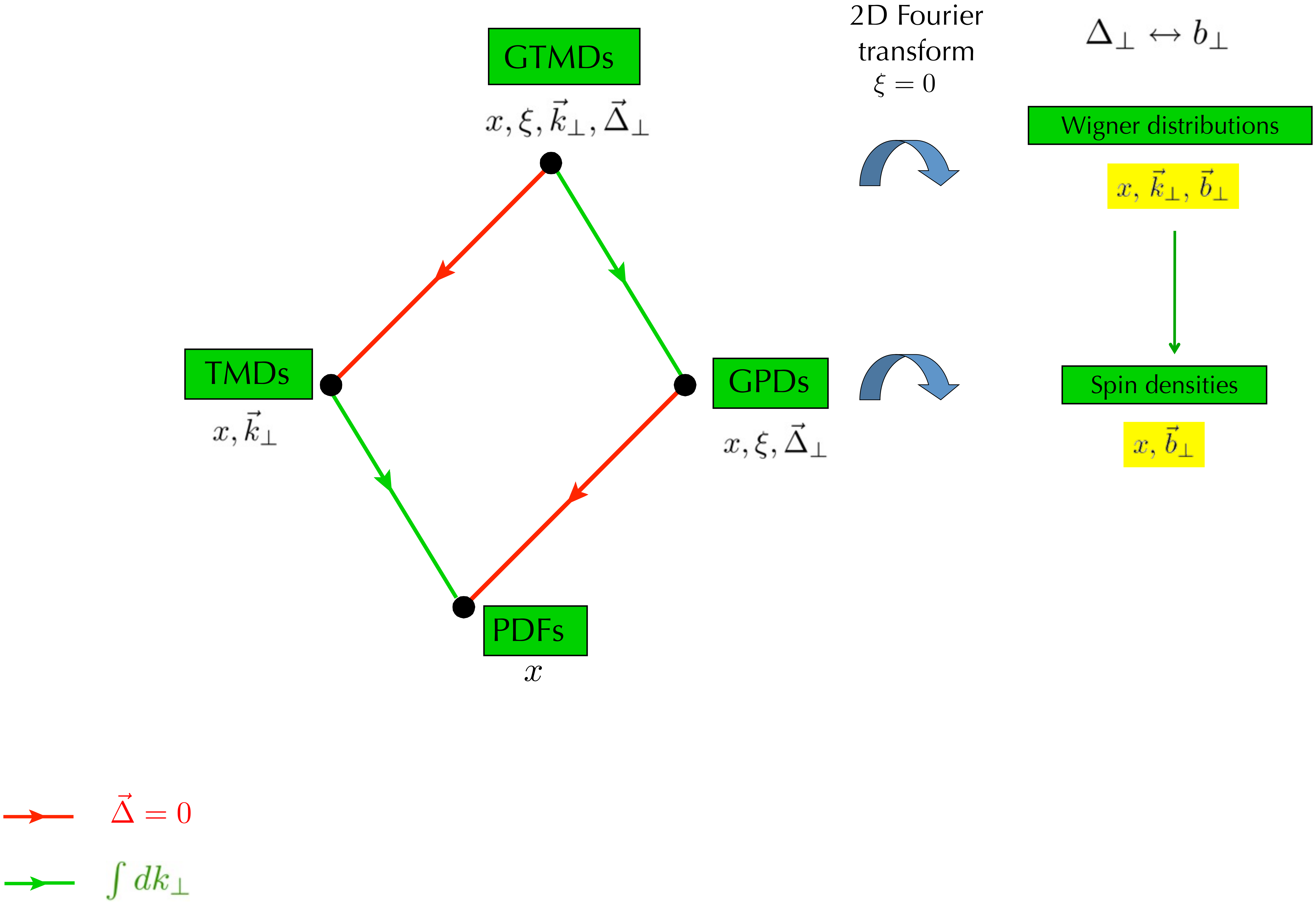
relation of small-x gluon GTMDs to diffractive dijet production in DIS
Hatta, Xiao, Yuan, PRL 116 (2016)

Wigner distributions

$\rho(x, \vec{b}_\perp, \vec{k}_\perp)$

3+2D map!





2D Fourier transform
$\xi = 0$

$\Delta_\perp \leftrightarrow b_\perp$

GTMDs
$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs
$x, \vec{k}_\perp$

GPDs
$x, \xi, \vec{\Delta}_\perp$

PDFs
x

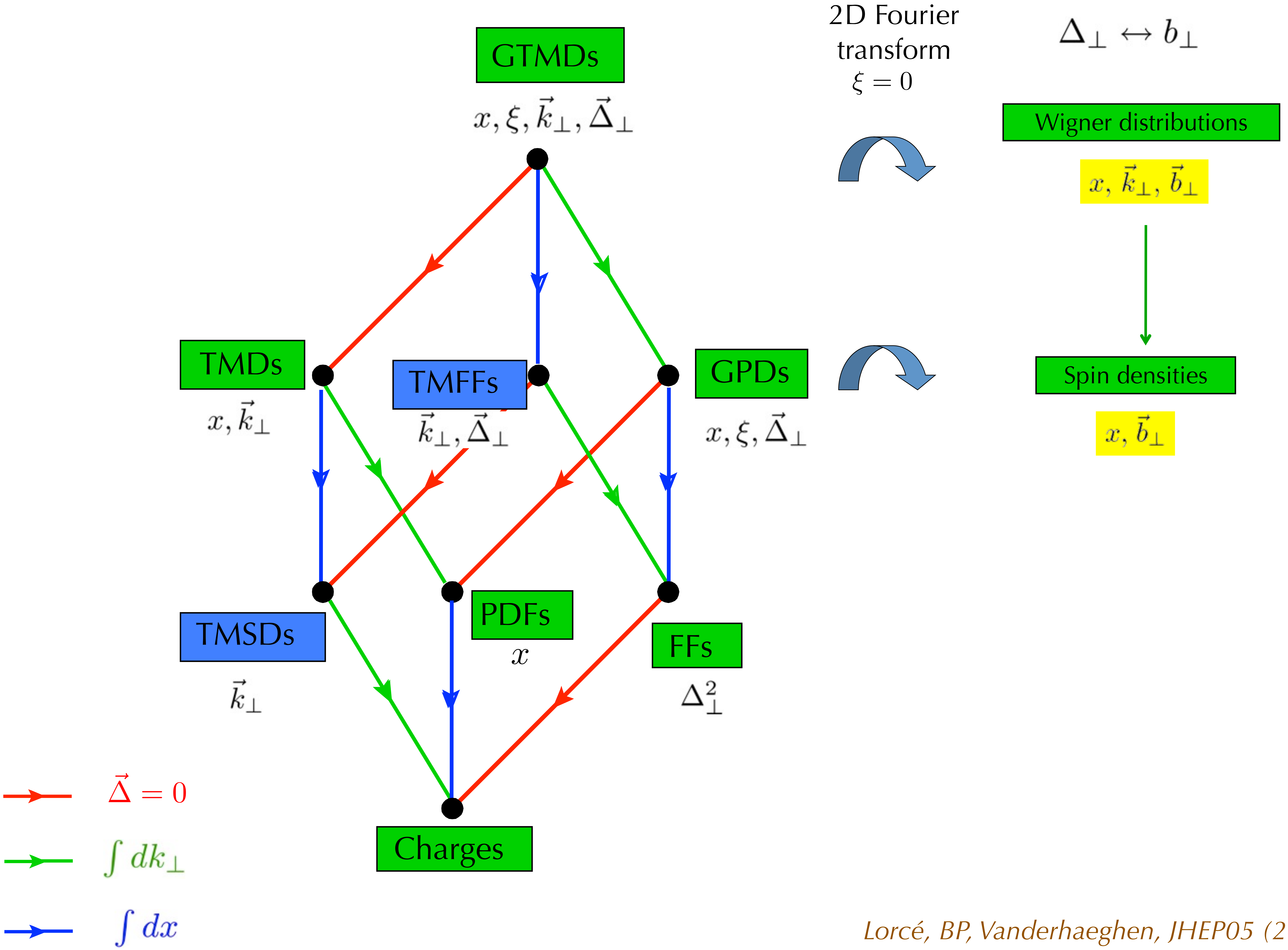
Wigner distributions
$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities
$x, \vec{b}_\perp$

$\vec{\Delta} = 0$

$\int dk_\perp$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

TMSDs

$\vec{k}_\perp$

PDFs

x

FFs

$\Delta_\perp^2$

Charges

2D Fourier transform

$\xi = 0$

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distributions

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

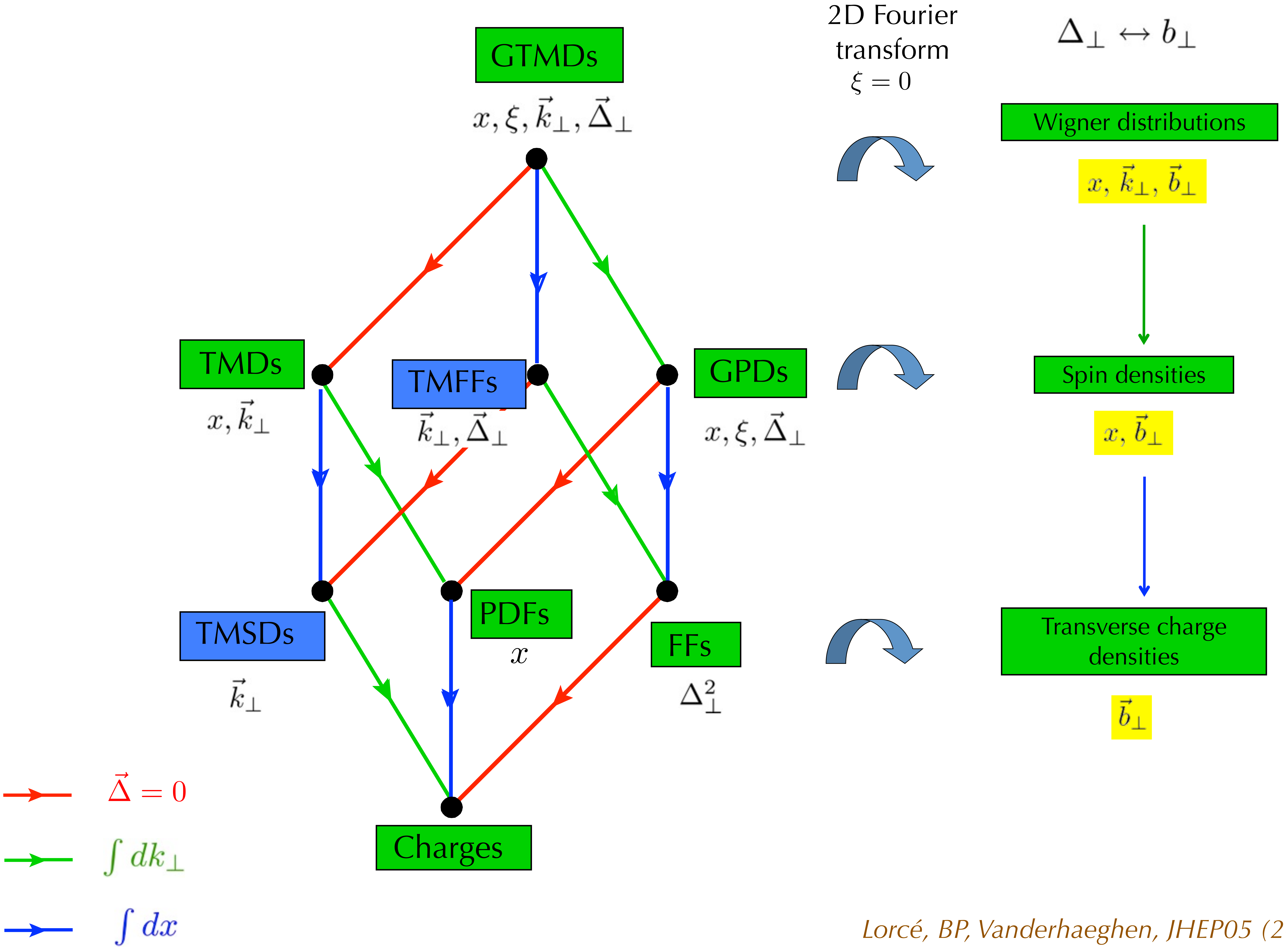
$x, \vec{b}_\perp$

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

TMSDs

$\vec{k}_\perp$

PDFs

x

FFs

$\Delta_\perp^2$

Charges

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

2D Fourier
transform
$\xi = 0$

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distributions

$x, \vec{k}_\perp, \vec{b}_\perp$

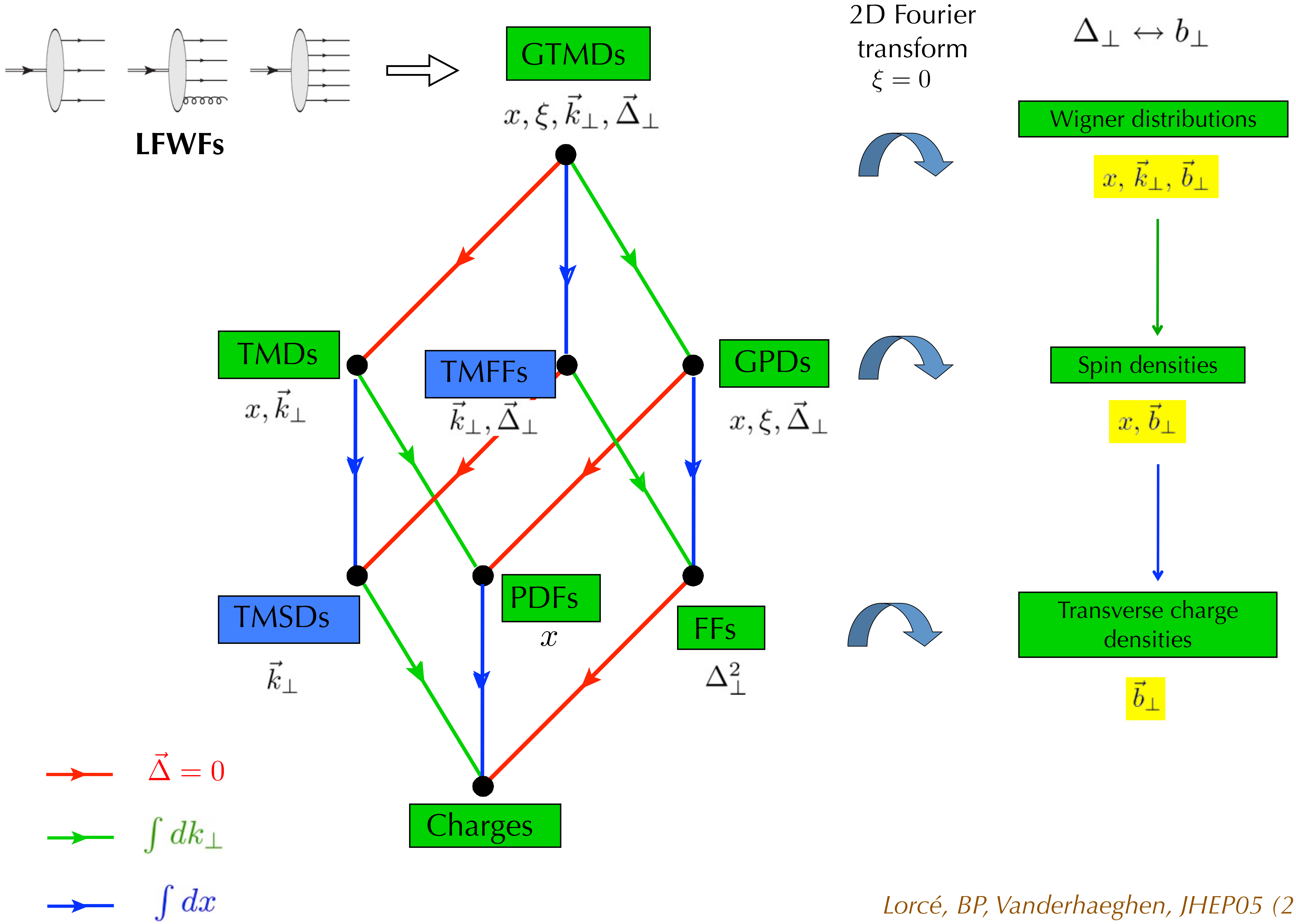
Spin densities

$x, \vec{b}_\perp$

Transverse charge
densities

$\vec{b}_\perp$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



LFWFs

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

TMSDs

$\vec{k}_\perp$

PDFs

x

FFs

$\Delta_\perp^2$

Charges

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

2D Fourier transform

$\xi = 0$

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distributions

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

$x, \vec{b}_\perp$

Transverse charge densities

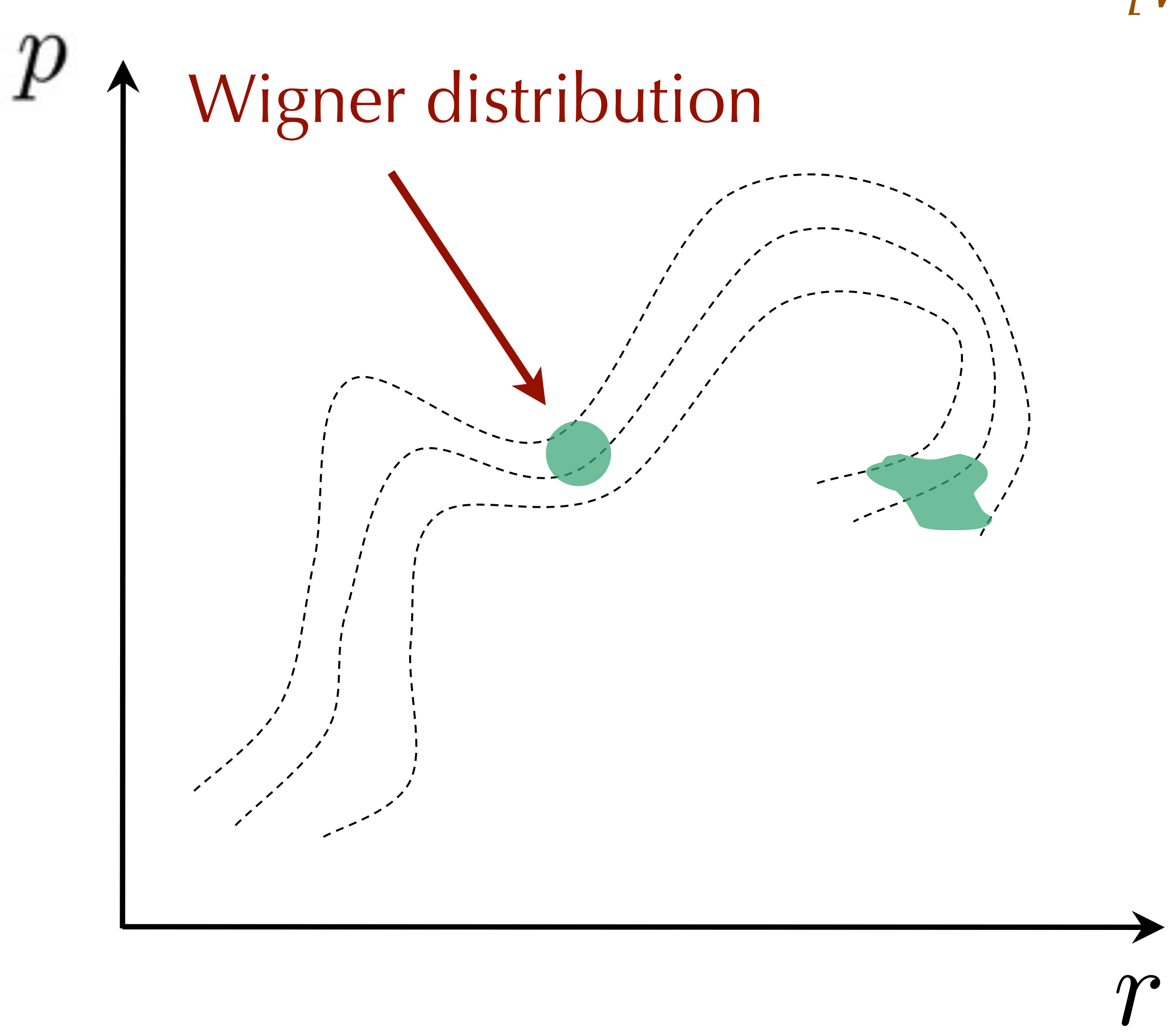
$\vec{b}_\perp$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

Phase-Space Distributions

[Wigner (1932); Moyal (1949)]

$$\rho_W(r,k) = \int \frac{\mathrm{d}z}{2\pi} e^{-ikz}\psi^*(r-\frac{z}{2})\psi(r+\frac{z}{2})$$
$$= \int \frac{\mathrm{d}\Delta}{2\pi} e^{-i\Delta r}\phi^*(k+\frac{\Delta}{2})\phi(k-\frac{\Delta}{2})$$



Position-space density $|\psi(r)|^2 = \int \mathrm{d}k\, \rho_W(r,k)$

Momentum-space density $|\phi(k)|^2 = 2\pi \int \mathrm{d}r\, \rho_W(r,k)$

Phase-Space Distributions

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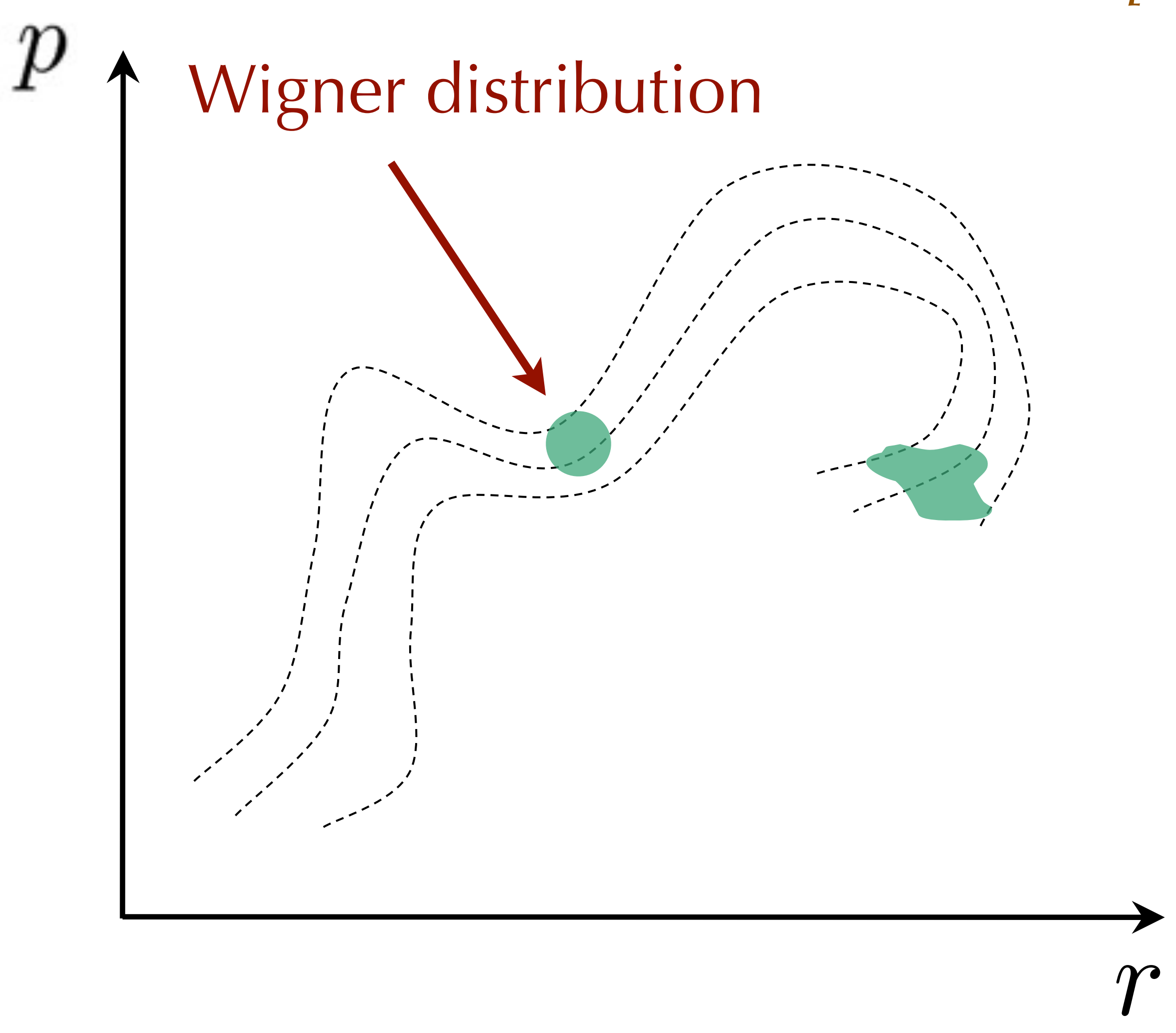

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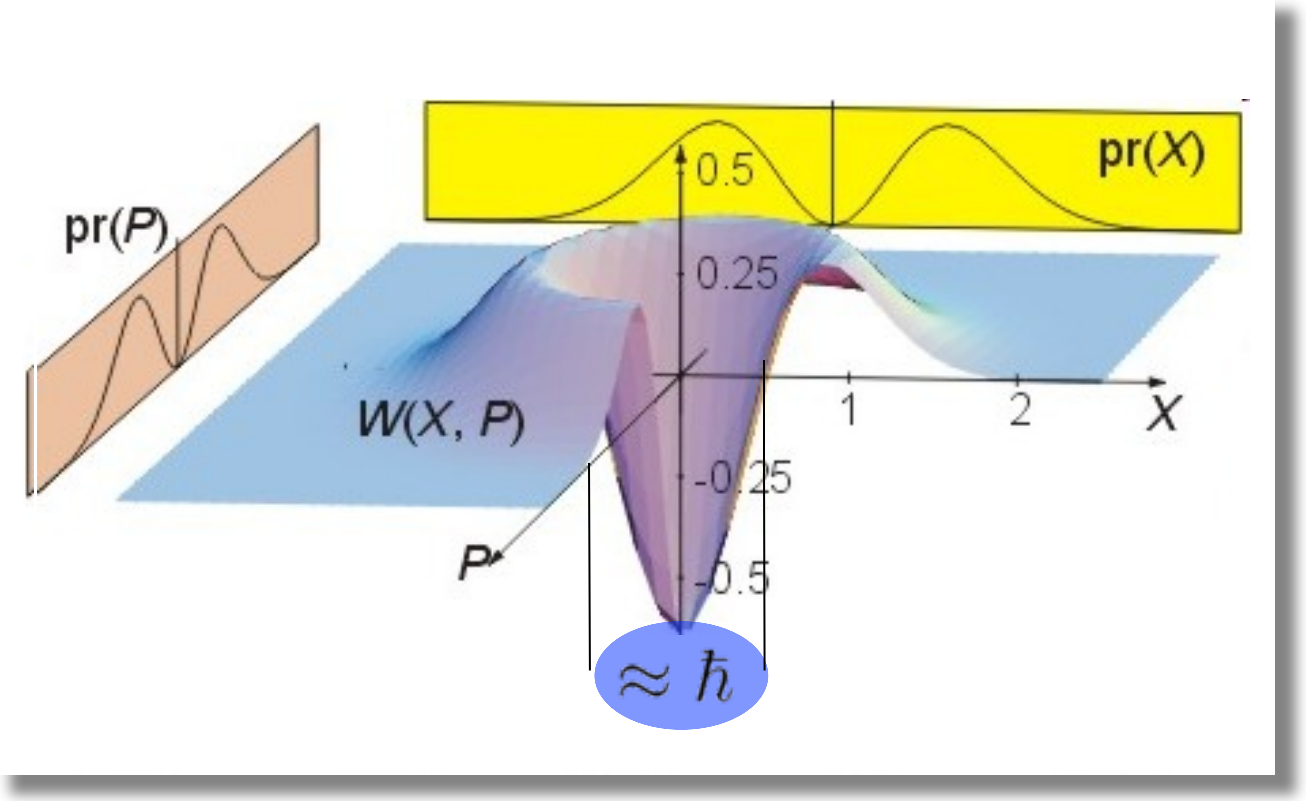
Quasi-probability: ⚠ $\rho(\vec{r},\vec{k})\ngeq 0$

Heisenberg's uncertainty relation



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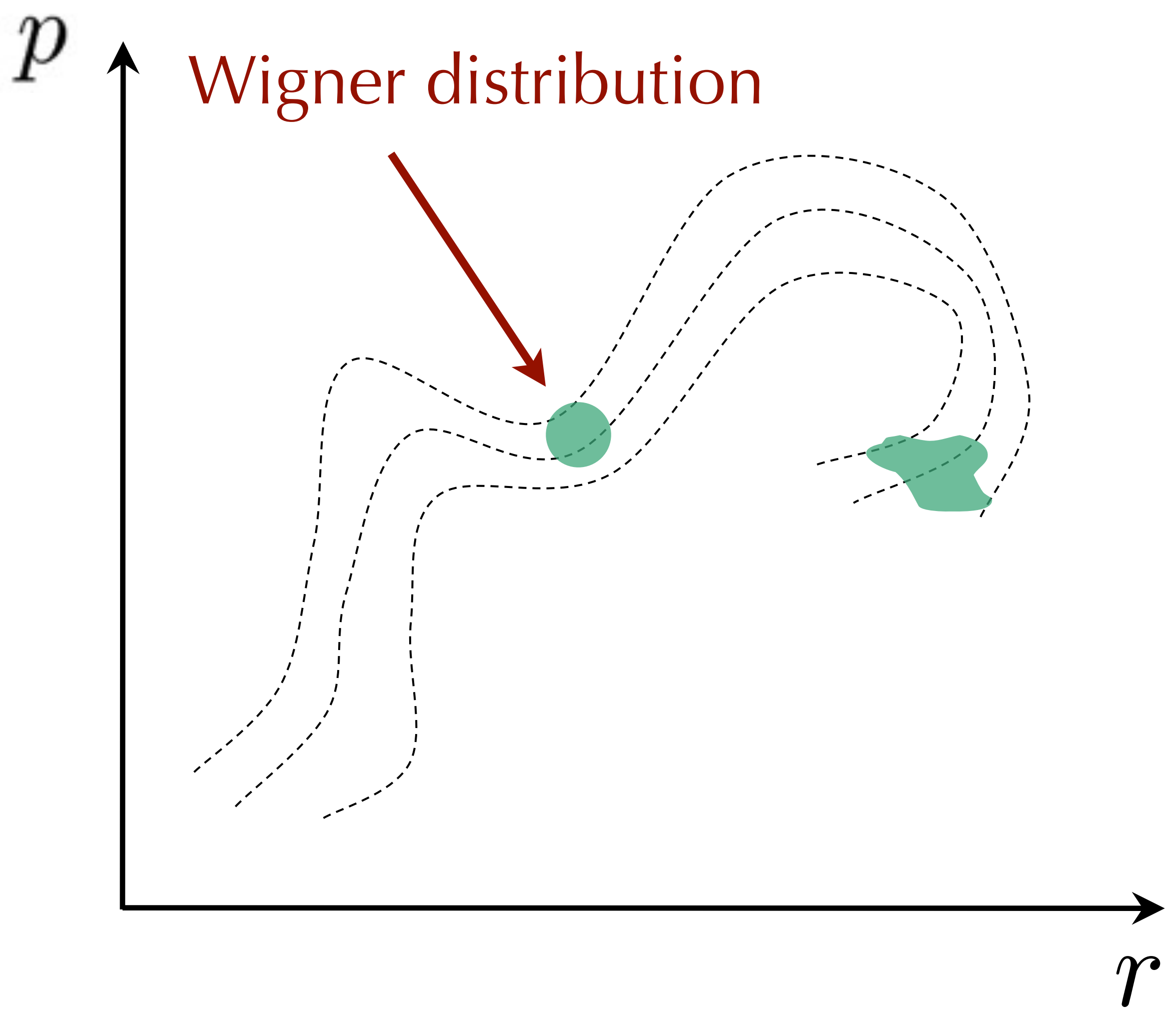

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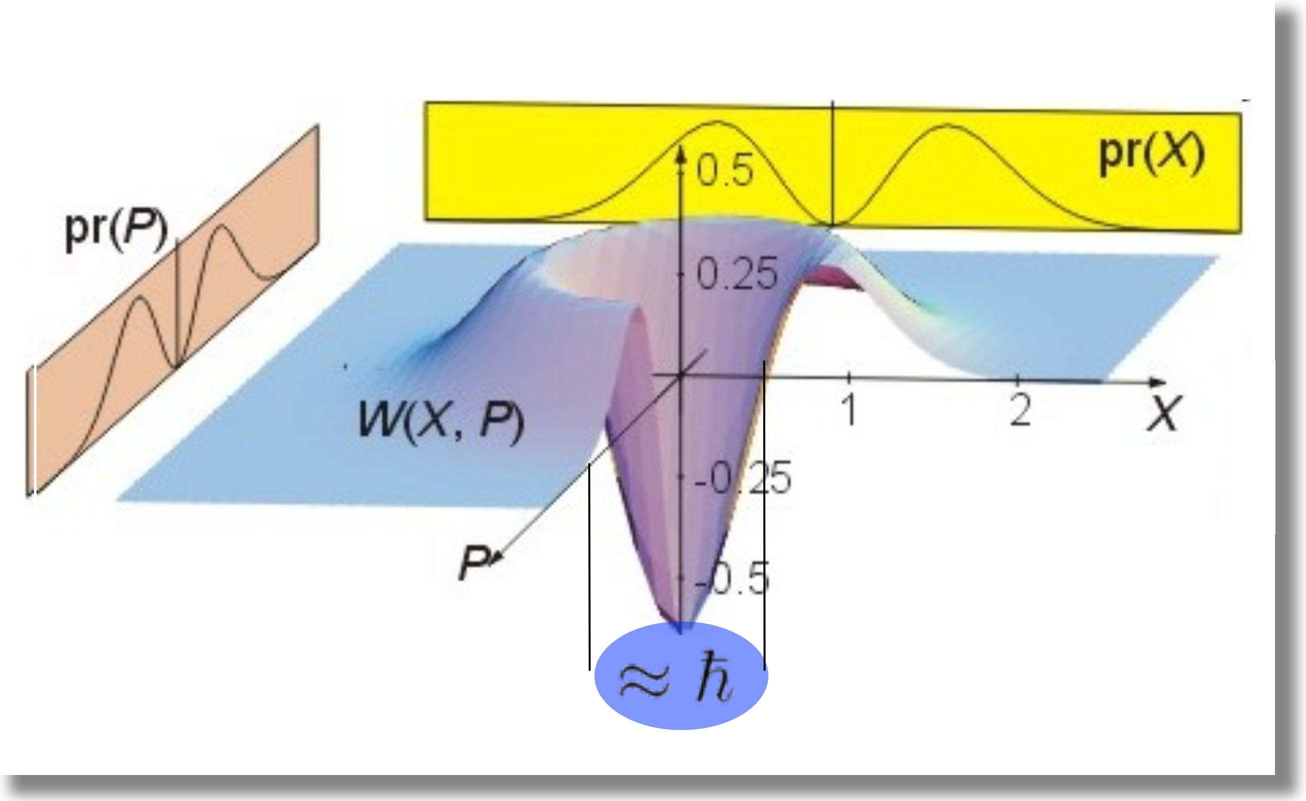

Quantum average

$$\langle \hat{O} \rangle = \int \mathrm{d}r\, \mathrm{d}k\, O(r,k)\, \rho_W(r,k)$$

Wigner Distributions in QFT

Quark Wigner operator

Dirac matrix

$\sim$ quark polarization

$$\widehat{W}^{[\Gamma]}(\vec{r}, k) = \int \frac{\mathrm{d}^4 z}{(2\pi)^4} e^{ik\cdot z} \bar{\psi}(\vec{r} - \frac{z}{2})\, \Gamma\, \mathcal{W}\, \psi(\vec{r} + \frac{z}{2})$$

canonical momentum $k \leftrightarrow i\nabla$

Wilson line

Wigner Distributions in QFT

Dirac matrix

$\sim$ quark polarization

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$$\widehat{W}^{[\Gamma]}(\vec{r}, k) = \int \frac{\mathrm{d}^4 z}{(2\pi)^4} e^{ik\cdot z} \bar{\psi}(\vec{r} - \frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\vec{r} + \frac{z}{2})$$

canonical momentum $k \leftrightarrow i\nabla$

Wilson line

Fixed light-front time

$$z^+ = 0 \quad \Longleftrightarrow \quad \int \mathrm{d}k^-$$

Wigner Distributions in QFT

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Fixed light-front time

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Wigner distributions
in the Breit frame

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{r}, k^+, \vec{k}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \langle \frac{\vec{\Delta}}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|-\frac{\vec{\Delta}}{2},\Lambda\rangle$$

3+3 D

no semi-classical interpretation

Ji (2003)
Belitsky, Ji, Yuan (2004)

Wigner Distributions in QFT

Dirac matrix
$\sim$ quark polarization

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3+3 D

no semi-classical interpretation

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Wigner distributions
in the Drell-Yan frame
$(\Delta^+=0)$

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{b}_\perp,k^+,\vec{k}_\perp)=\frac{1}{2}\int\frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2}e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\langle p^+,\frac{\vec{\Delta}_\perp}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|p^+,-\frac{\vec{\Delta}_\perp}{2},\Lambda\rangle$$

2+3 D

semi-classical interpretation

Lorcé, BP (2011)
Lorcé, BP, Xiong, Yuan (2012)

Wigner Distributions in QFT

Dirac matrix

$\sim$ quark polarization

Quark Wigner operator

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$(\Delta^+=0)$

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{b}_\perp,k^+,\vec{k}_\perp)=\frac{1}{2}\int\frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2}e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\langle p^+,\frac{\vec{\Delta}_\perp}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|p^+,-\frac{\vec{\Delta}_\perp}{2},\Lambda\rangle$$

2+3 D

semi-classical interpretation

Generalized Transverse Momentum Dependent Distributions

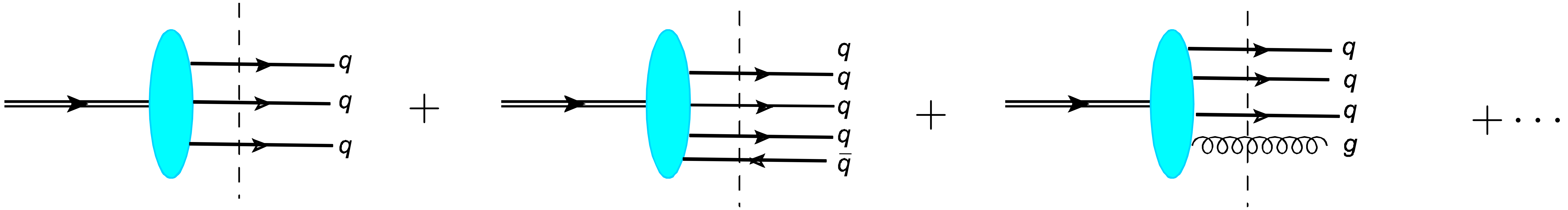
Lorcé, BP (2011)
Lorcé, BP, Xiong, Yuan (2012)

Light-Front Wave Functions (LFWFs)

- Fock expansion of Nucleon state:

$$|N\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3q\,g}|qqqg\rangle + \cdots$$

fixed light-cone time (x^+=0)



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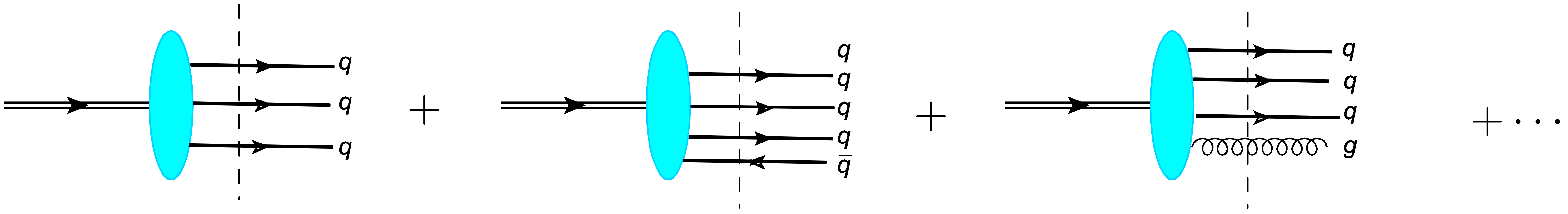
fixed light-cone time (x⁺=0)

- Probability to find N partons in the nucleon $\rho^{\Lambda}_{N,\beta} = \int [dx]_N [d^2k_\perp]_N |\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N}|^2$ normalization $\sum_{N,\beta}\rho^{\Lambda}_{N,\beta} = 1$

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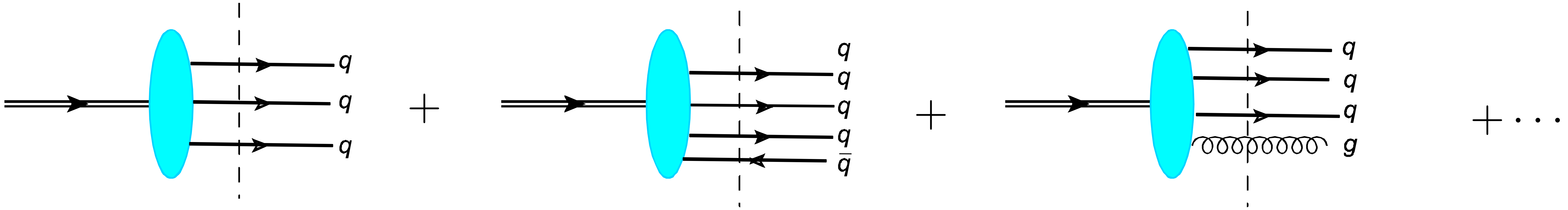

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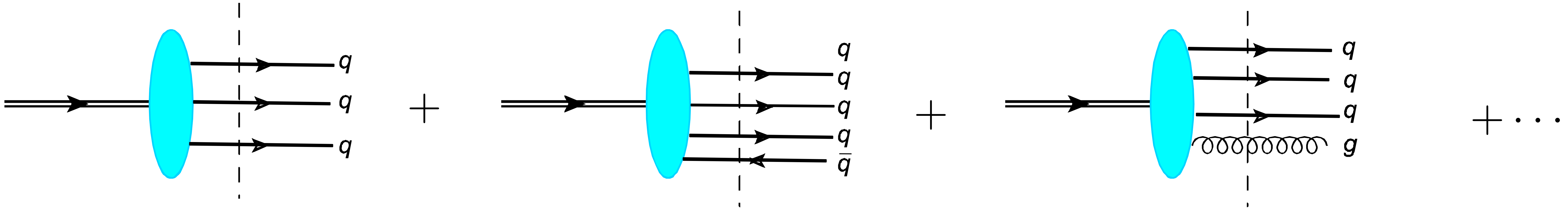

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- Eigenstates of parton light-front helicity $\hat{S}_{iz}\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = \lambda_i\,\Psi^{\Lambda}_{\lambda_1\lambda_2\cdots\lambda_N}$

$$\boxed{\Lambda = \sum_{i=1}^{N} \lambda_i + l_z}$$

- Eigenstates of total orbital angular momentum $\hat{L}_z\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = l_z\,\Psi^{\Lambda}_{\lambda_1\lambda_2\cdots\lambda_N}$

⚠ $A^+ = 0$ gauge

total helicity

$$s_z = \langle \hat{S}_z \rangle = \sum_{N,\beta} \sum_{i=1}^{N} \lambda_i\,\rho^{\Lambda}_{N,\beta}$$

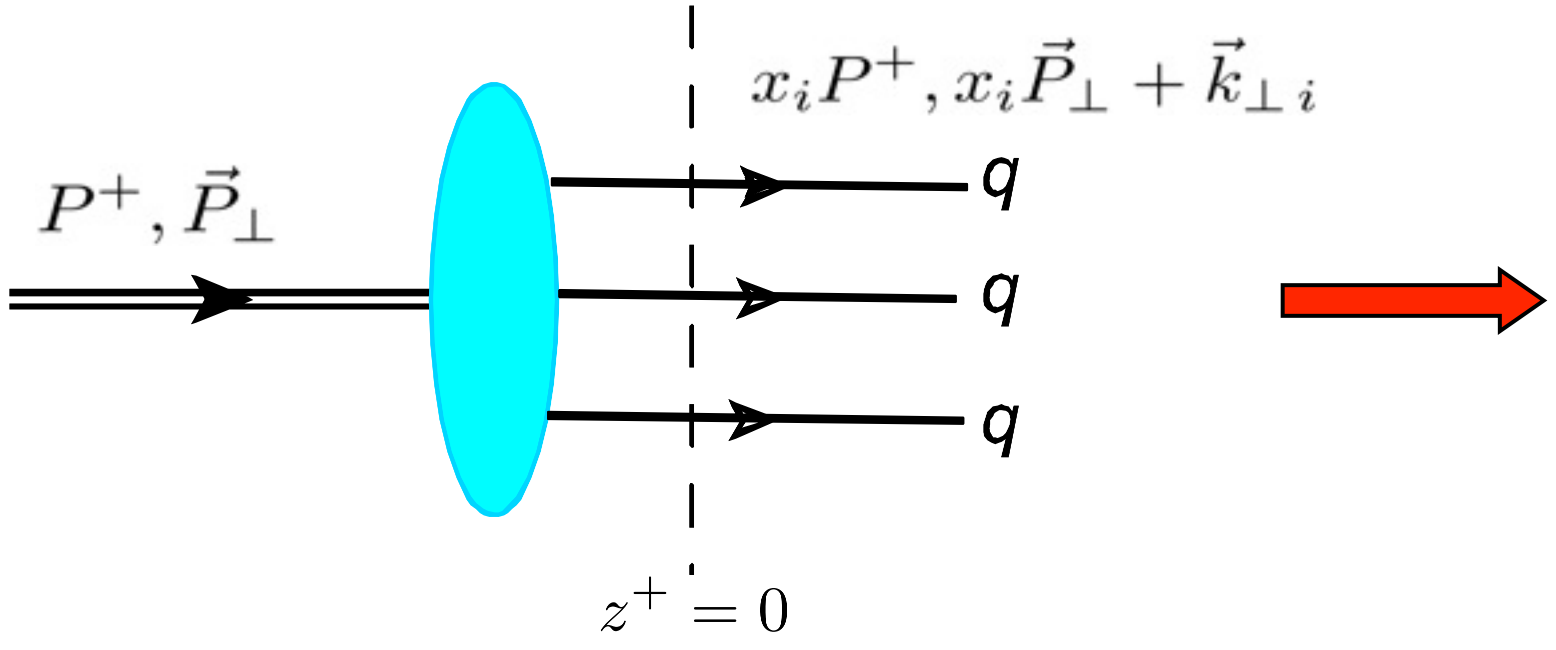
total OAM

$$\ell_z = \langle \hat{L}_z \rangle = \sum_{N,\beta} \sum_{i=1}^{N} l_z\,\rho^{\Lambda}_{N,\beta}$$

nucleon helicity

$$\Lambda = s_z + \ell_z$$

LFWF overlap representation



$\Psi^{\Lambda;q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3}(x_i, \vec{k}_{\perp,i})$

invariant under boost, independent of P^μ

internal variables: $\sum_{i=1}^{3} x_i = 1, \sum_{i=1}^{3} \vec{k}_{\perp\, i} = \vec{0}_\perp$

Brodsky, Pauli, Pinsky, 1998

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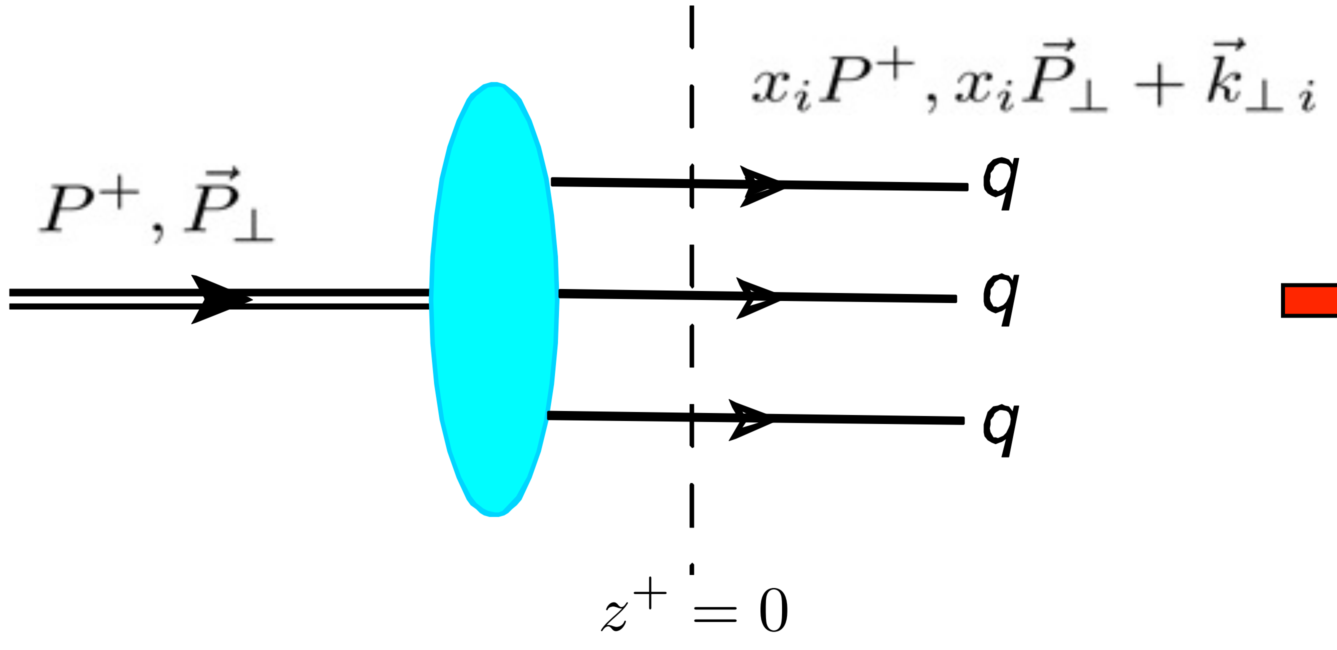

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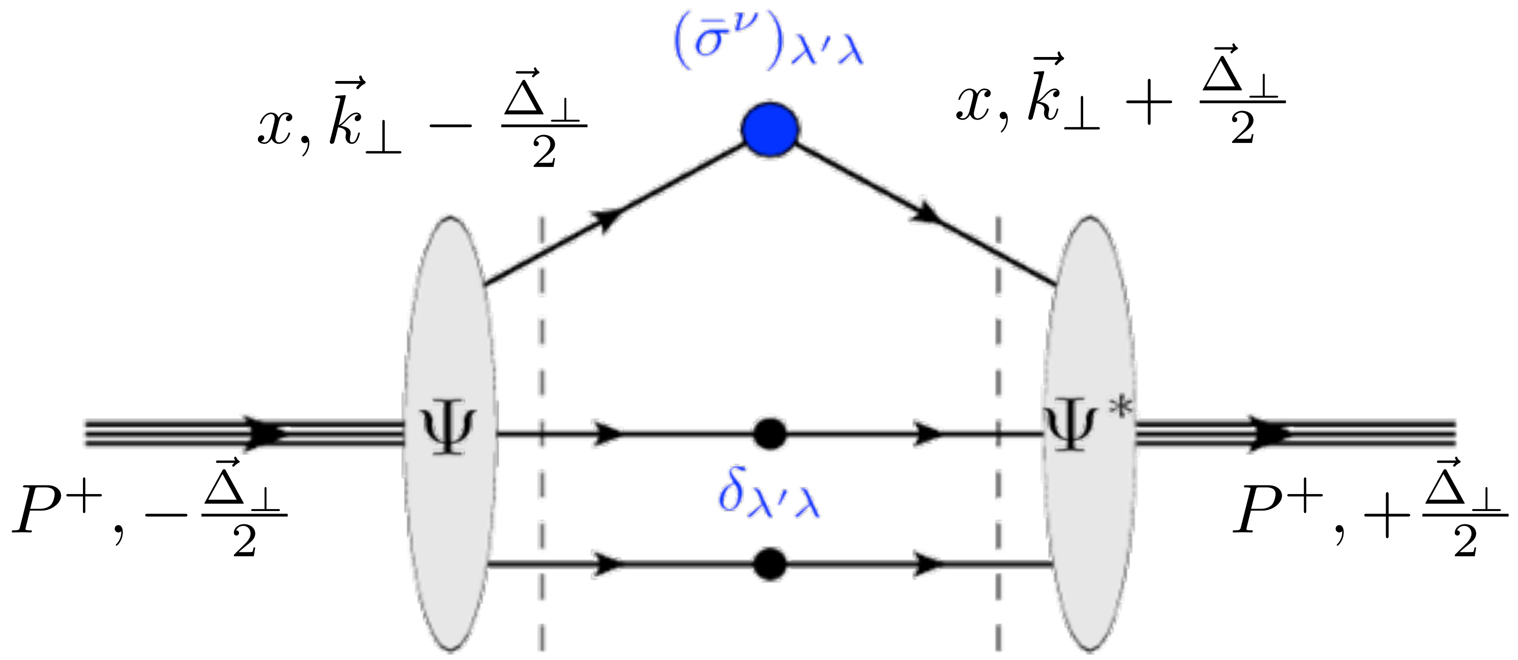
quark-quark correlator

(Δ^+=0)



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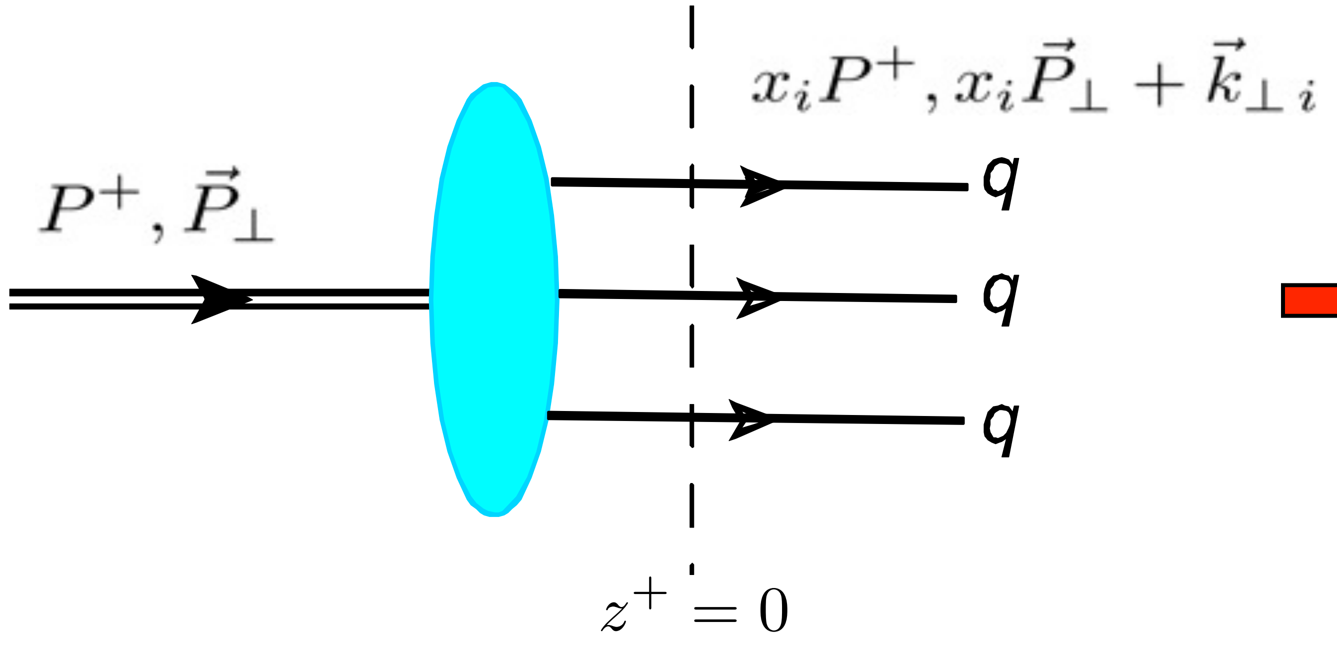

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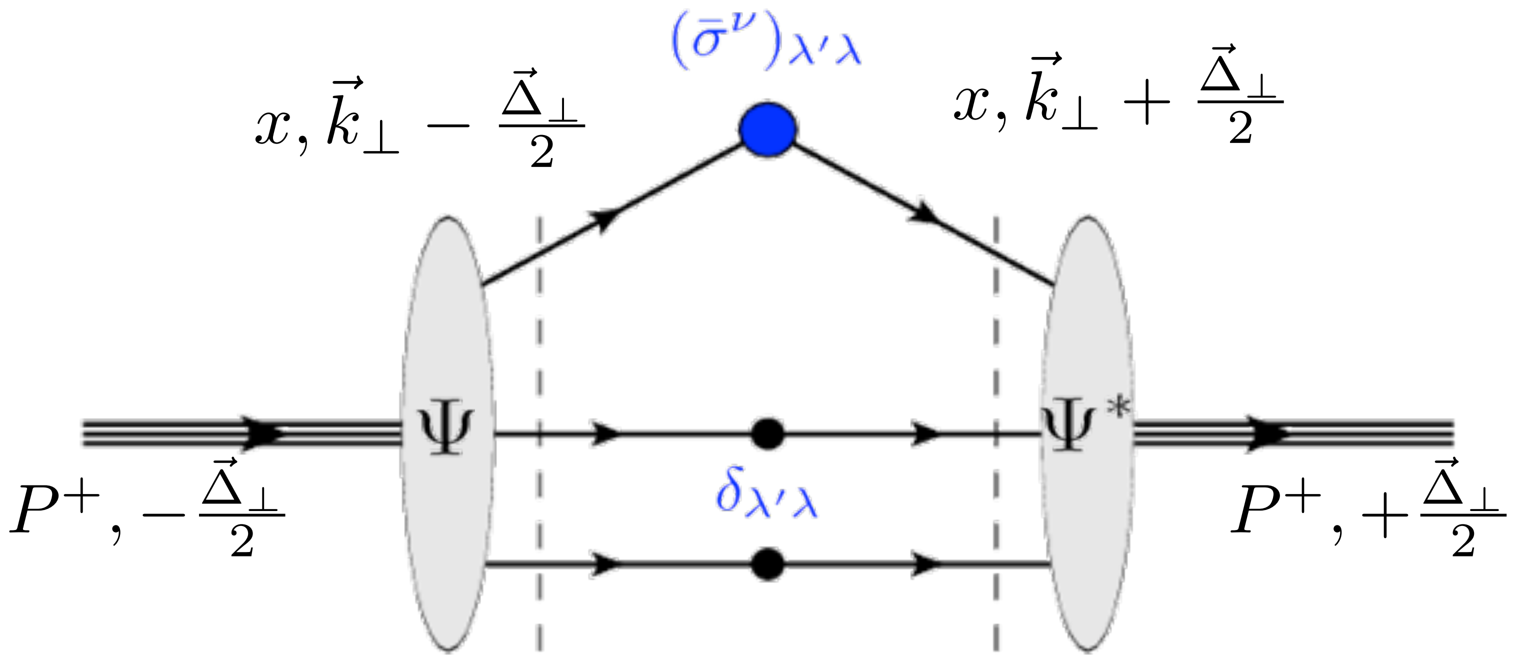

$$\Psi^{\Lambda;q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3}(x_i, \vec{k}_{\perp,i}) = \sum_{s_i} \underbrace{\phi(x_i, \vec{k}_{\perp,i})}_{\text{momentum wf}}\ \underbrace{\Phi^{\Lambda;q_1q_2q_3}_{s_1s_2s_3}}_{\text{spin-flavor wf}} \prod_i \underbrace{D^{1/2*}_{s_i\lambda_i}(R_{cf})}_{\text{rotation from canonical spin to light-cone spin}}$$

General formalism valid for
Bag Model, LFχQSM, LFCQM, Quark-Diquark, Covariant Parton Models

Common assumptions :

- ➢ No gluons
- ➢ Independent quarks

Lorcé, BP, Vanderhaeghen, JHEP05 (2011)

Light-Front Constituent Quark Model

- momentum-space wf

  Schlumpf, Ph.D. Thesis, hep-ph/921155

$$\Psi(k_i) = \frac{N}{(M_0^2 + \beta^2)^\gamma} \qquad M_0 = \sum_i^3 \sqrt{m_i^2 + \vec{k}_i^2}$$

N : normalization constant

$\beta,\ \gamma$ parameters fitted to anomalous magnetic moments of the nucleon

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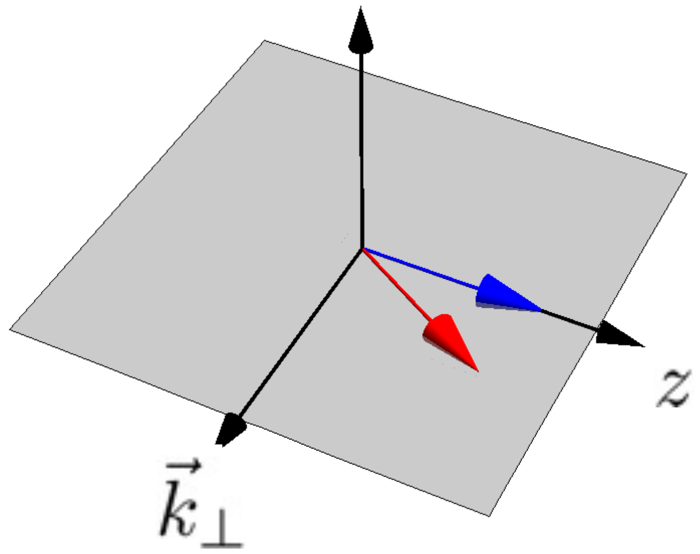
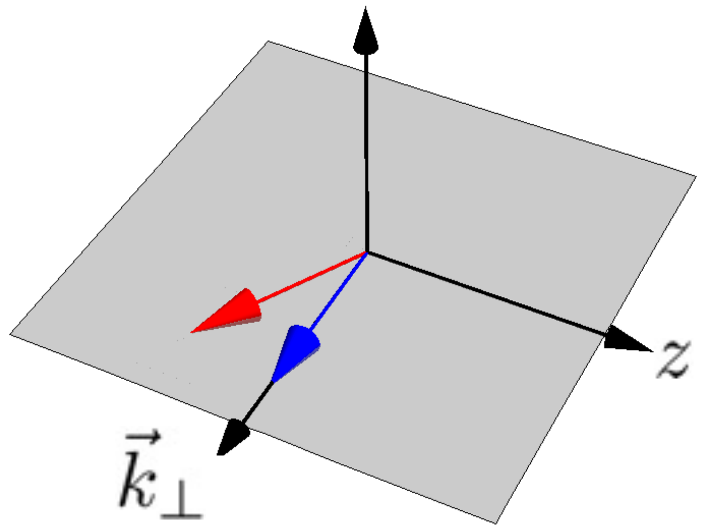
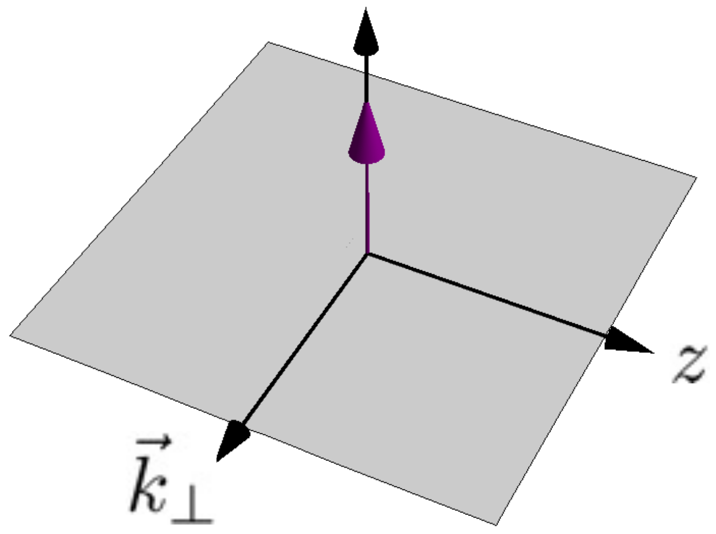
$$q_\lambda^{LC}(k) = D_{\lambda s}^{(1/2)*}\, q_s^C(k) \qquad D_{\lambda s}^{(1/2)*}(k) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$







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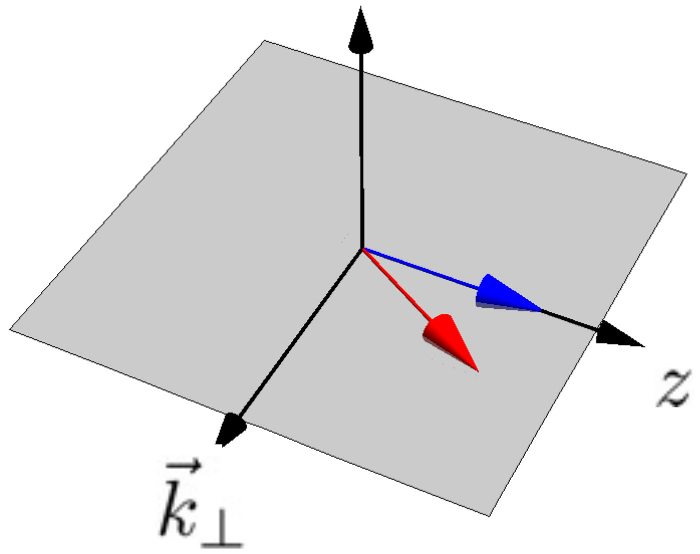
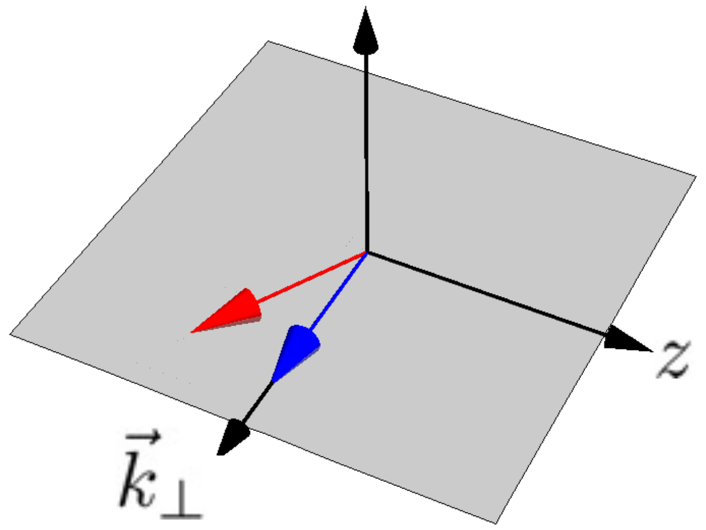
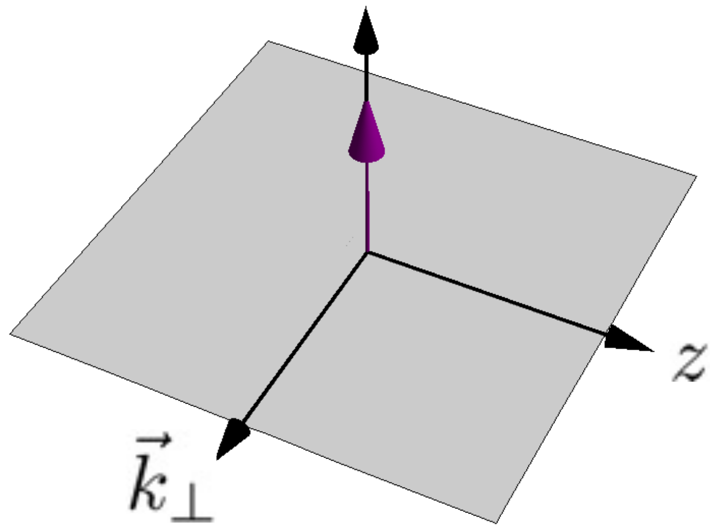
non-interacting quarks → $K_z = m + x\mathcal{M}_0$ $\vec{K}_\perp = \vec{k}_\perp$ (Melosh rotation)







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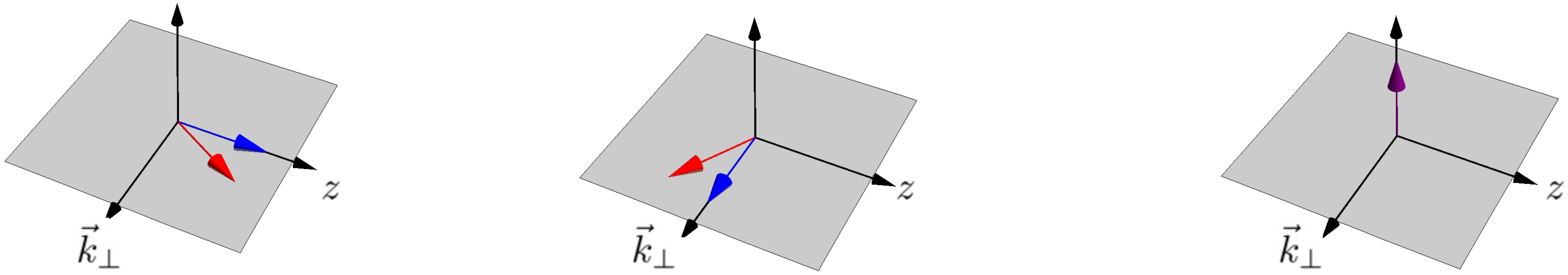

- SU(6) symmetry

Light-Front Constituent Quark Model

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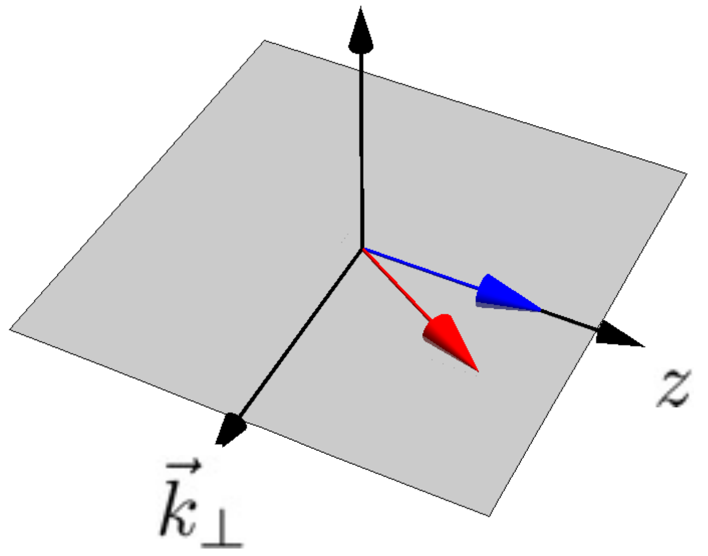
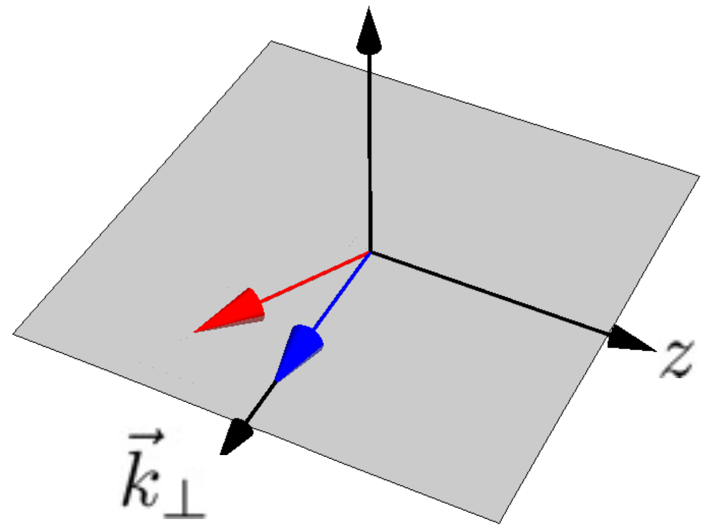
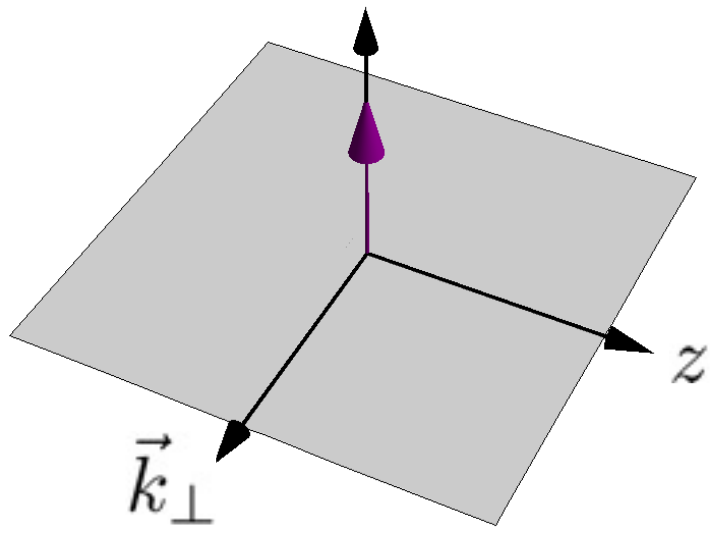






- SU(6) symmetry

Applications of the model to:

GPDs and Form Factors: *BP, Boffi, Traini (2003)-(2005);*
TMDs: *BP, Cazzaniga, Boffi (2008); BP, Yuan (2010);*
Azimuthal Asymmetries: *Schweitzer, BP, Boffi, Efremov (2009)*

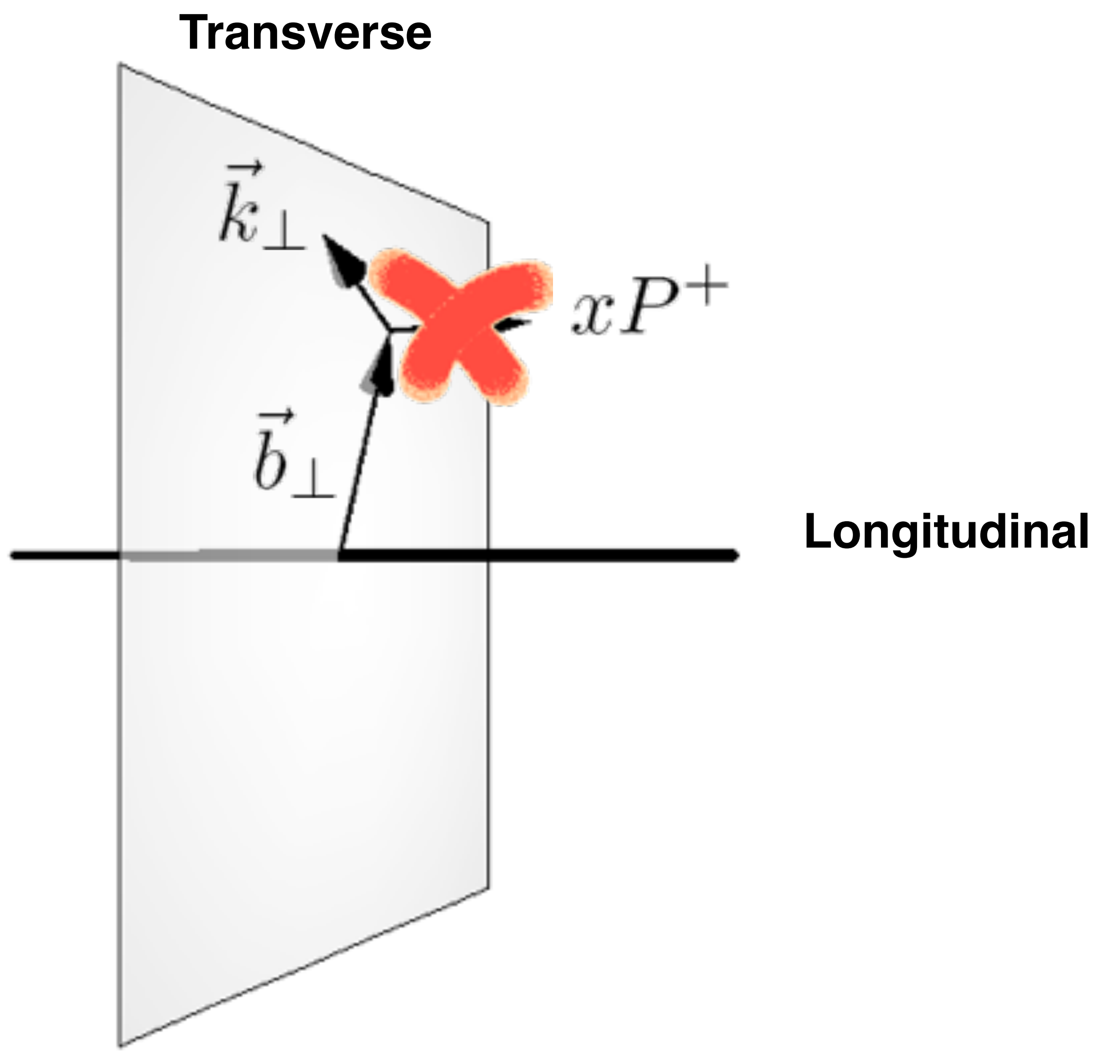
Quark Wigner Distributions

★ Twist-2: $\boxed{\Gamma_{\mathrm{twist}-2} = \gamma^+,\ \gamma^+\gamma_5,\ i\sigma^{j+}\gamma_5}$

quark polarization: **U** **L** **T**

★ Nucleon polarization: **U** **L** **T**

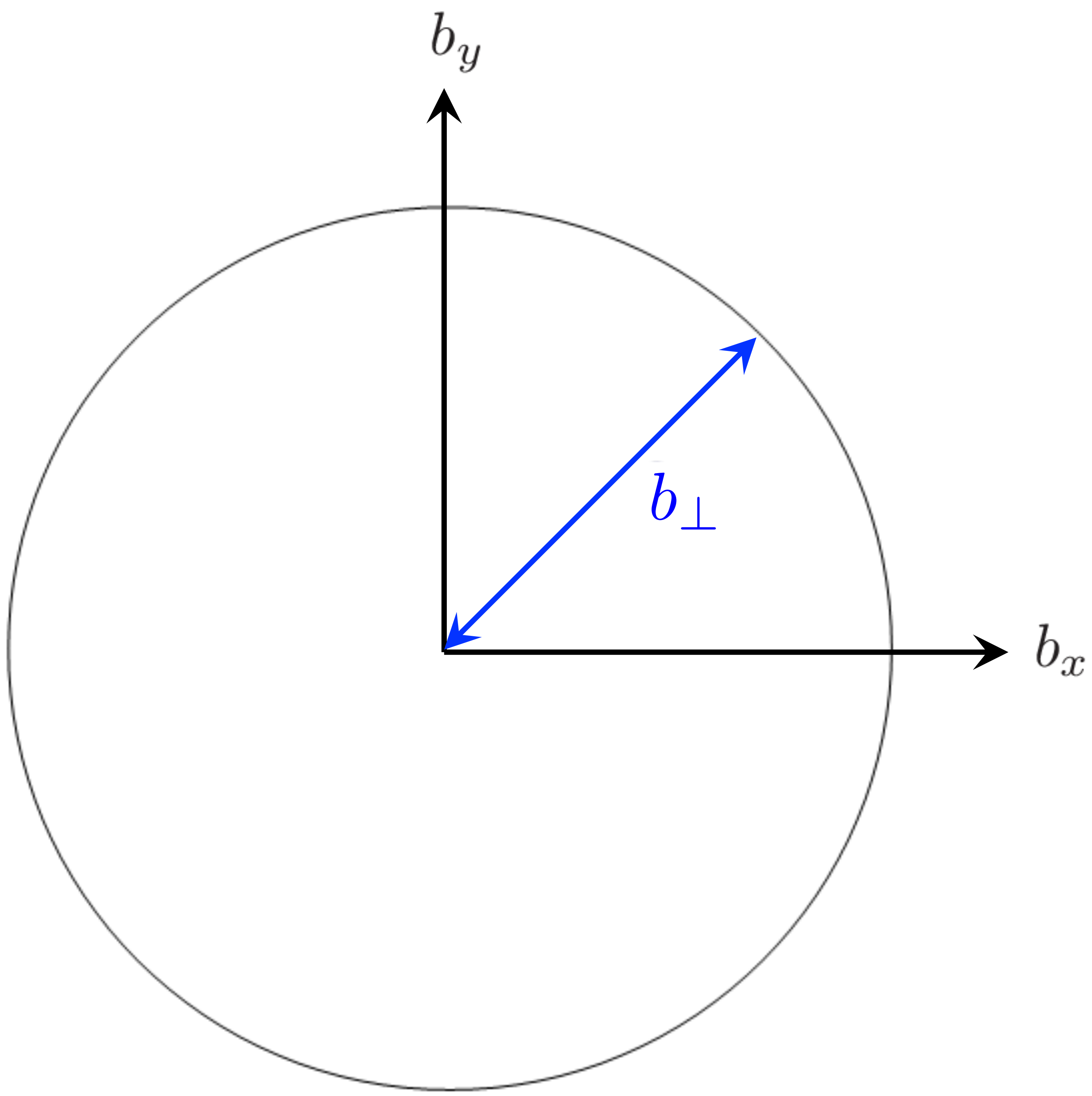
→ 16 complex GTMDs → 32 real Wigner Distributions



Transverse Phase-Space distributions

$$\rho_X(\vec{k}_\perp, \vec{b}_\perp) = \int \mathrm{d}x\, \rho_X(x, \vec{k}_\perp, \vec{b}_\perp) \qquad X = UU,\ UL, UT,\ LU,\ \ldots$$

Phase-Space Transverse Modes



Lorcé, BP, PRD 96, 2016

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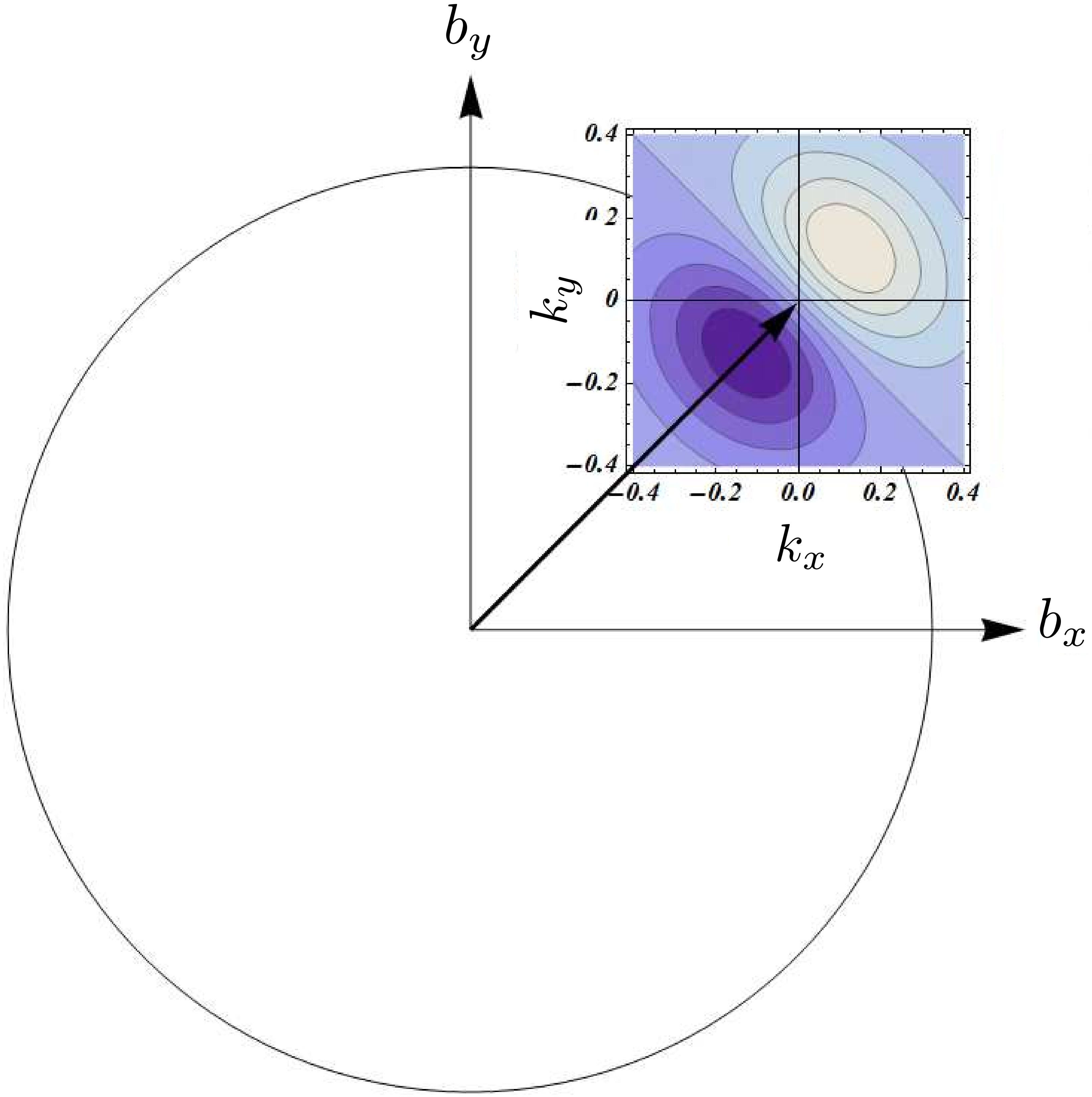
$$\rho_X(\vec{k}_\perp|\,\vec{b}_\perp) = \int \mathrm{d}x\, \rho_X(x, \vec{k}_\perp, \vec{b}_\perp; \hat{P} = \vec{e}_z, \eta = +1)\big|_{\vec{b}_\perp \text{ fixed}}$$



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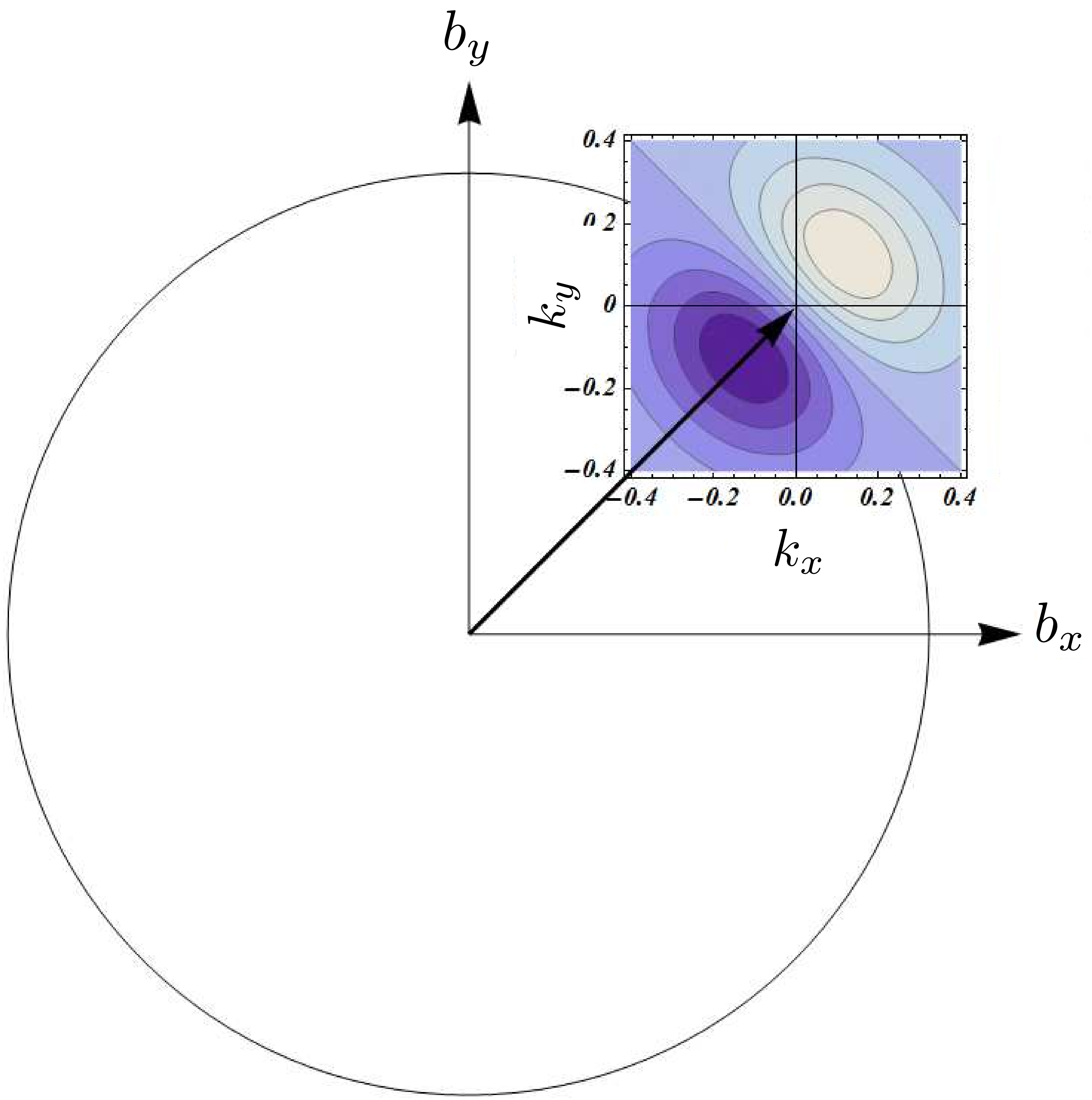


Multipole decomposition

$$\rho_X = \sum_{m_k,m_b} \rho_X^{(m_k,m_b)}$$

from parity and time-reversal properties

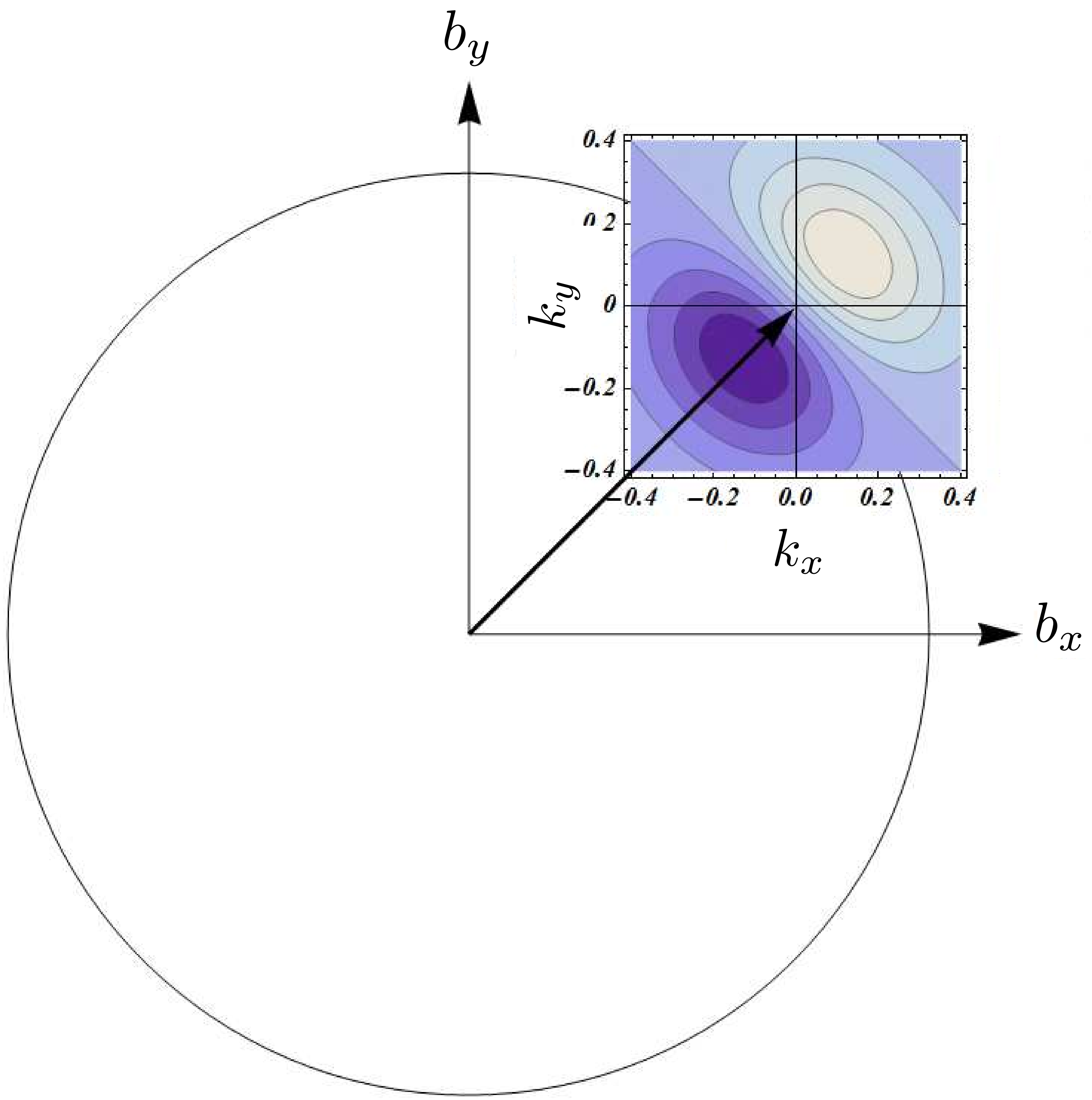
$$\vec{a}_\mathsf{P} = -c_\mathsf{P}\vec{a} \qquad \times_\mathsf{P} = c_\mathsf{P}\times$$
$$\vec{a}_\mathsf{T} = c_\mathsf{T}\vec{a} \qquad \times_\mathsf{T} = c_\mathsf{T}\times$$

	$\vec{b}_\perp$	$\vec{k}_\perp$	$\vec{e}_z \equiv \frac{\vec{P}}{P}$	$\vec{S}$	$\times$
c_P	+	+	+	-	-
c_T	+	-	-	-	+

Lorcé, BP, PRD 96, 2016

Phase-Space Transverse Modes

$$\rho_X(\vec{k}_\perp|\,\vec{b}_\perp) = \int \mathrm{d}x\,\rho_X(x,\vec{k}_\perp,\vec{b}_\perp;\hat{P}=\vec{e}_z,\eta=+1)\big|_{\vec{b}_\perp\text{ fixed}}$$



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c_P	+	+	+	-	-
c_T	+	-	-	-	+

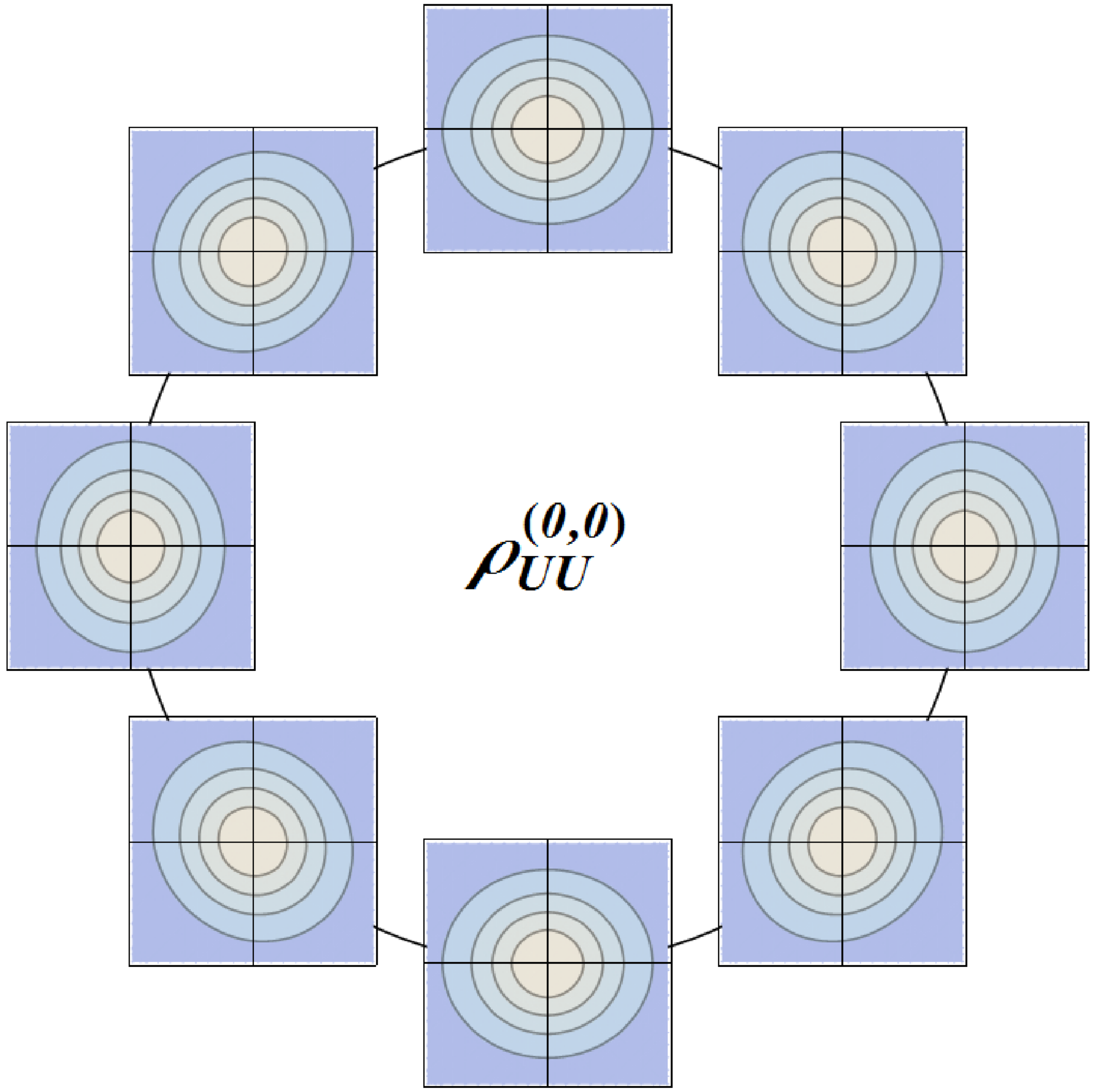
$$\rho_X = \underset{\substack{\big\downarrow \\ \text{T-even}}}{\rho_X^e} + \underset{\substack{\big\downarrow \\ \text{T-odd}}}{\rho_X^o}$$

Lorcé, BP, PRD 96, 2016

UU

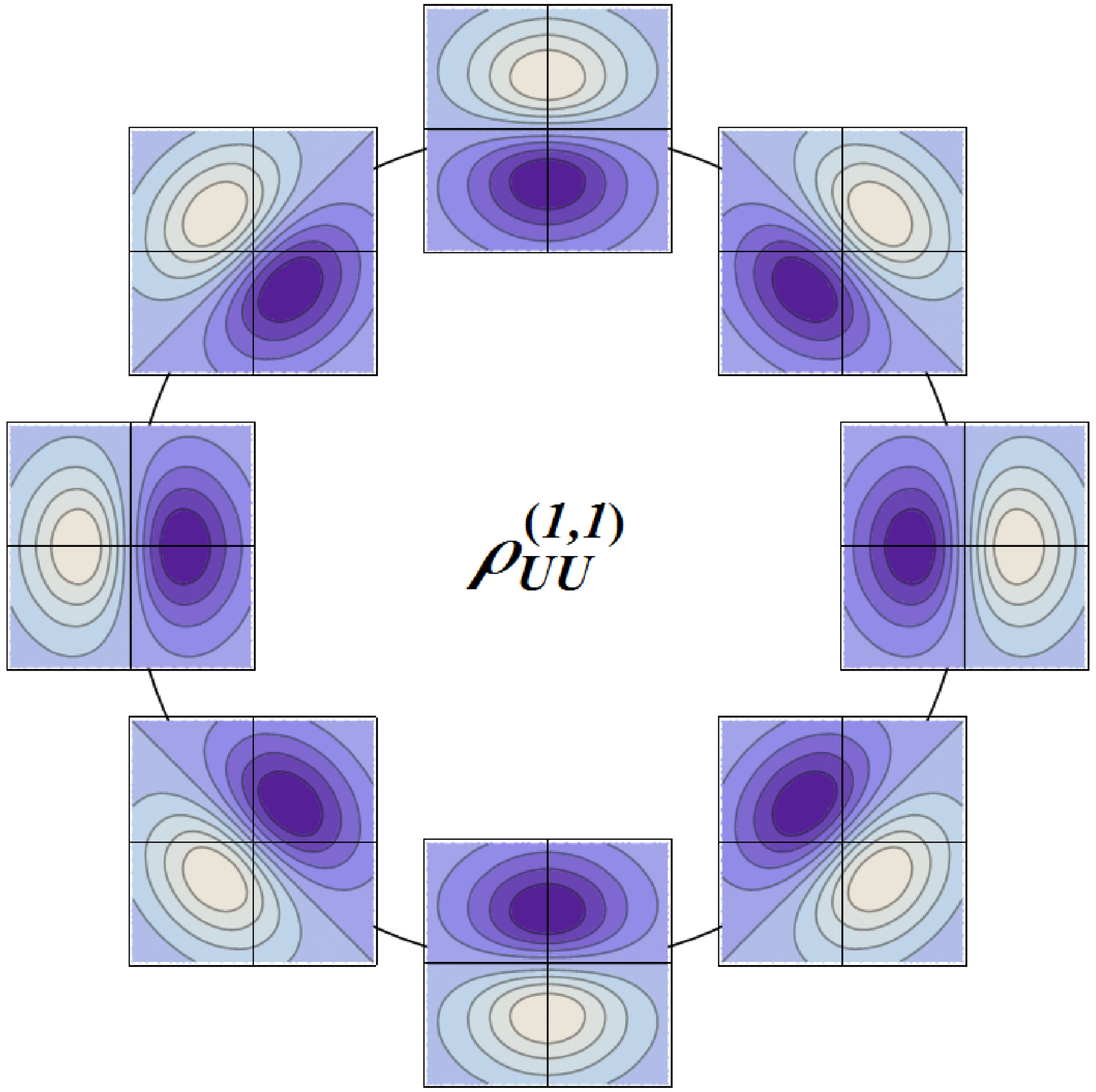
Unpolarized quarks in unpolarized proton

$\Re e[F_{11}] \rightarrow H,\ f_1$



naive time-reversal even

$\Im m[F_{11}]$



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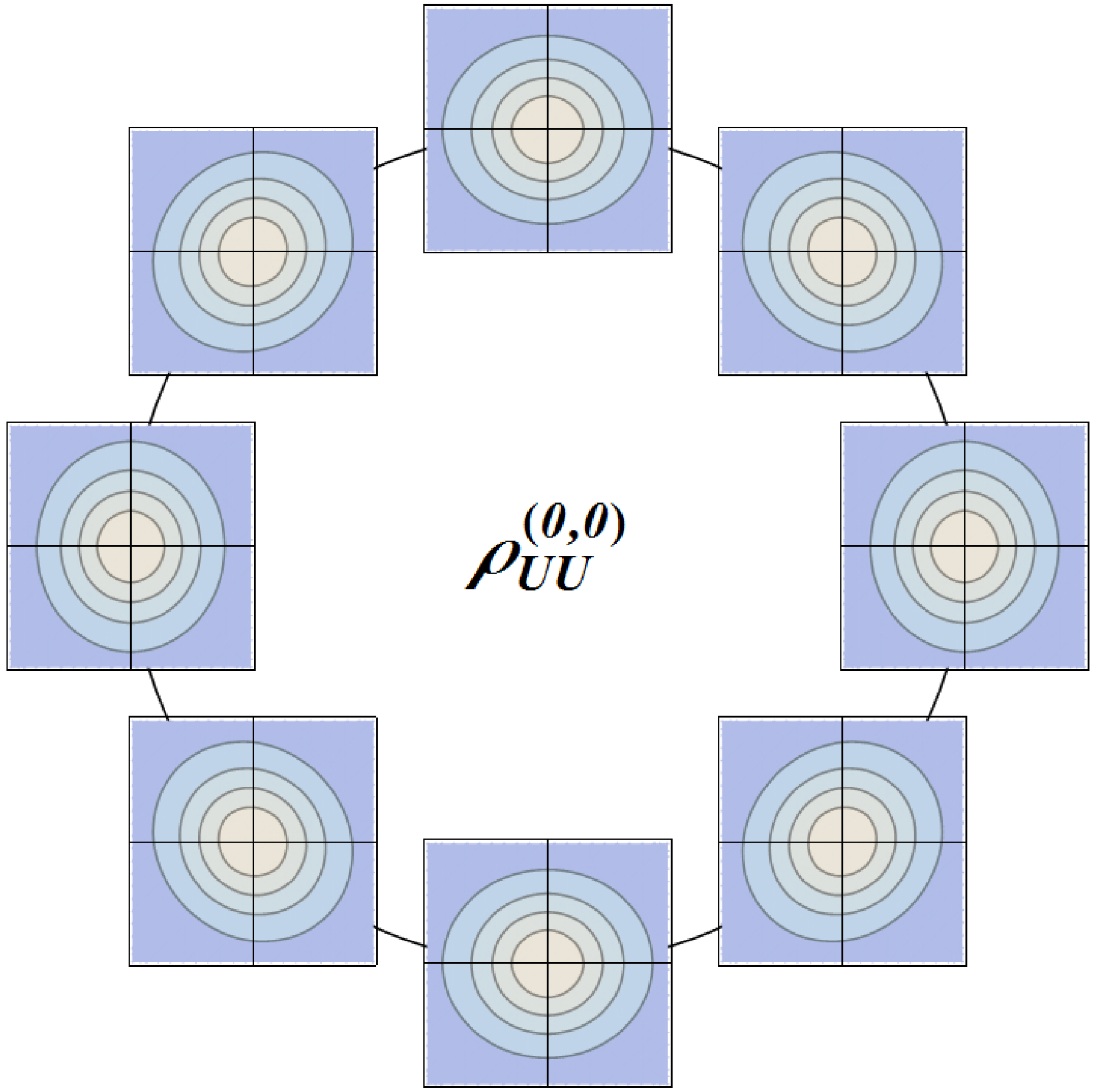

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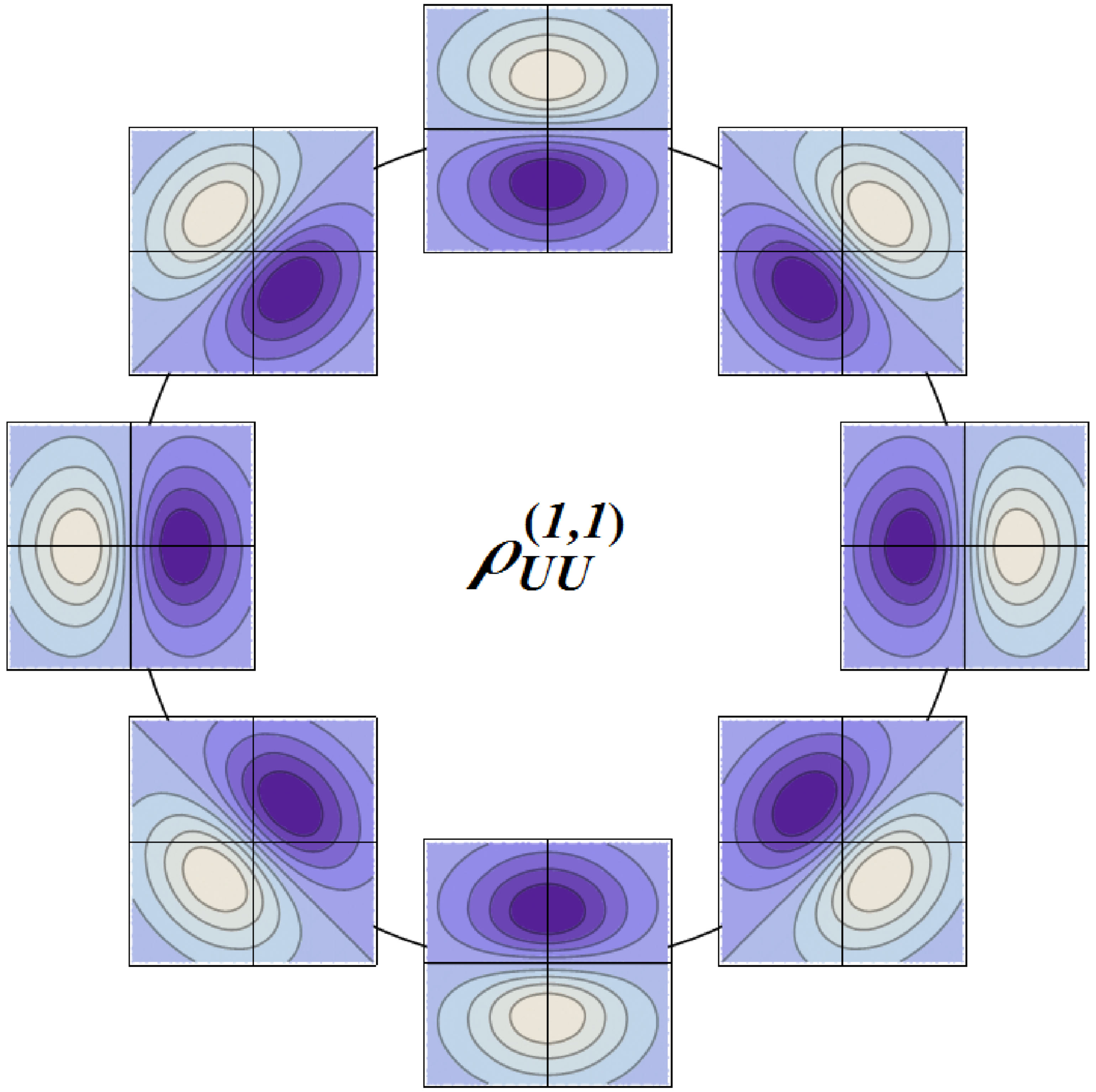


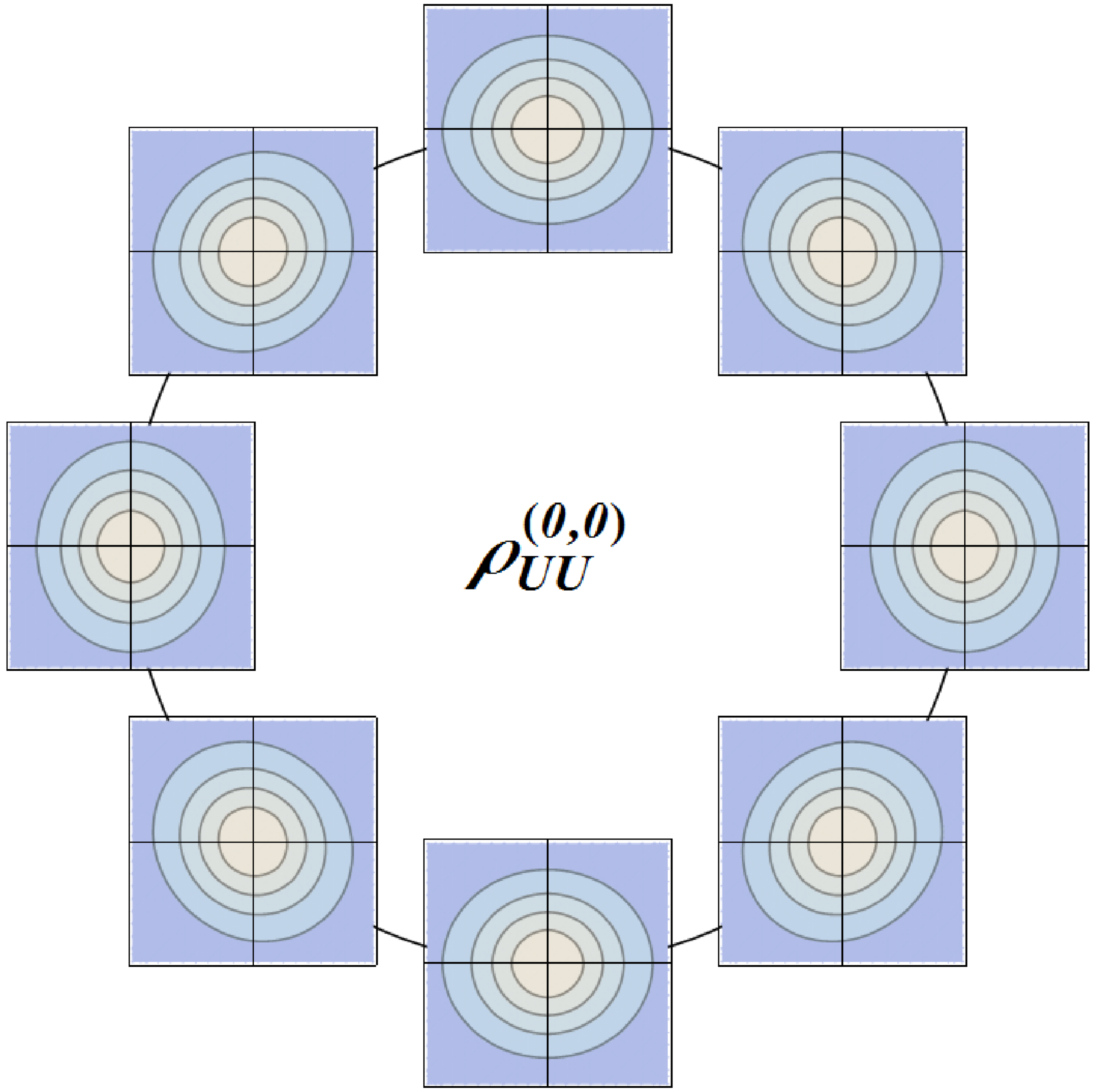
naive time-reversal odd

polar flow $(\vec{k}_\perp \perp \vec{b}_\perp)$ preferred over radial flow $(\vec{k}_\perp \parallel \vec{b}_\perp)$

UU

Unpolarized quarks in unpolarized proton

$\Re e[F_{11}] \rightarrow H,\ f_1$



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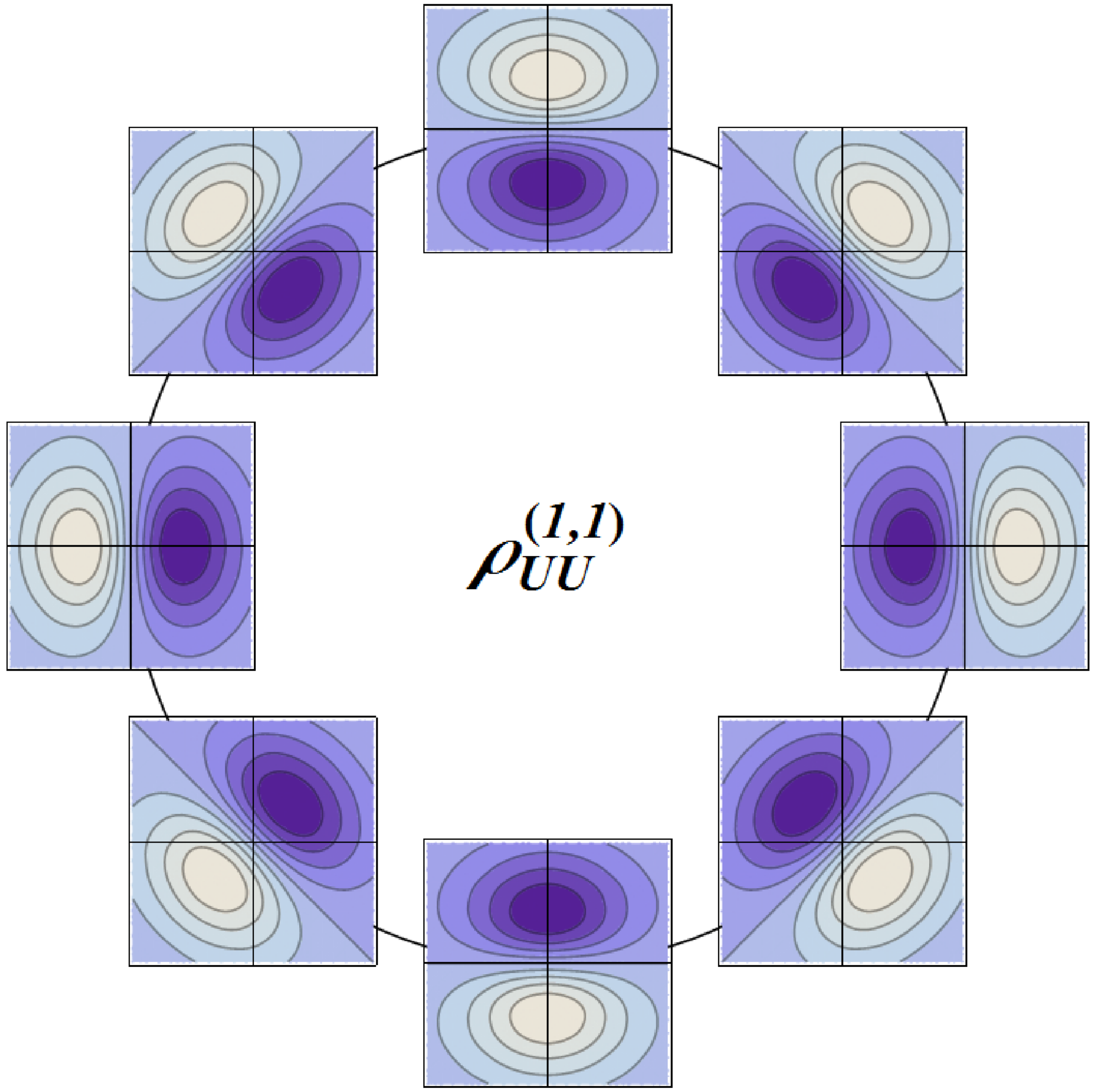

naive time-reversal odd

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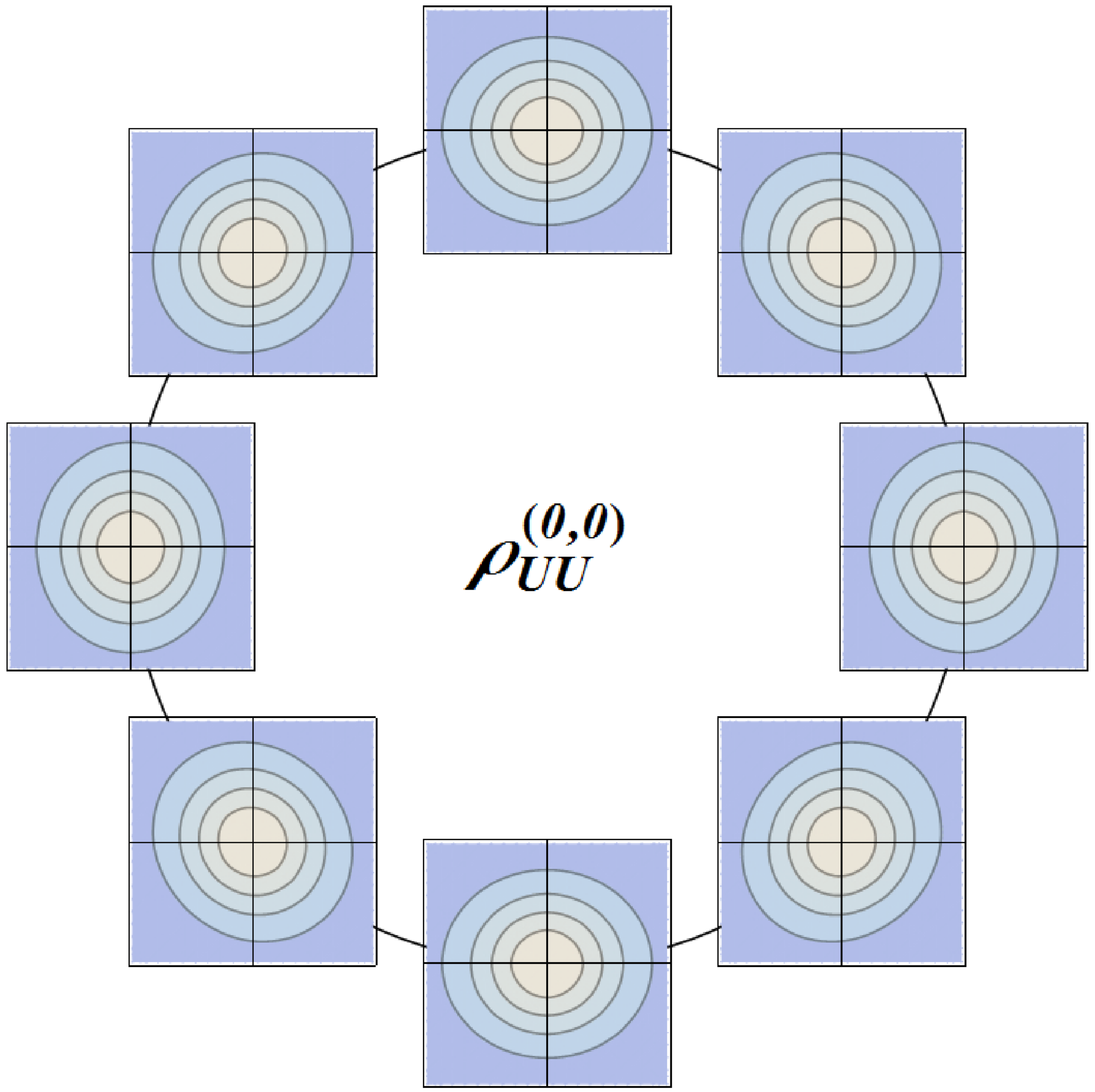
bottom-up symmetry → no net OAM

UU

Unpolarized quarks in unpolarized proton

$\Re e[F_{11}] \rightarrow H,\ f_1$



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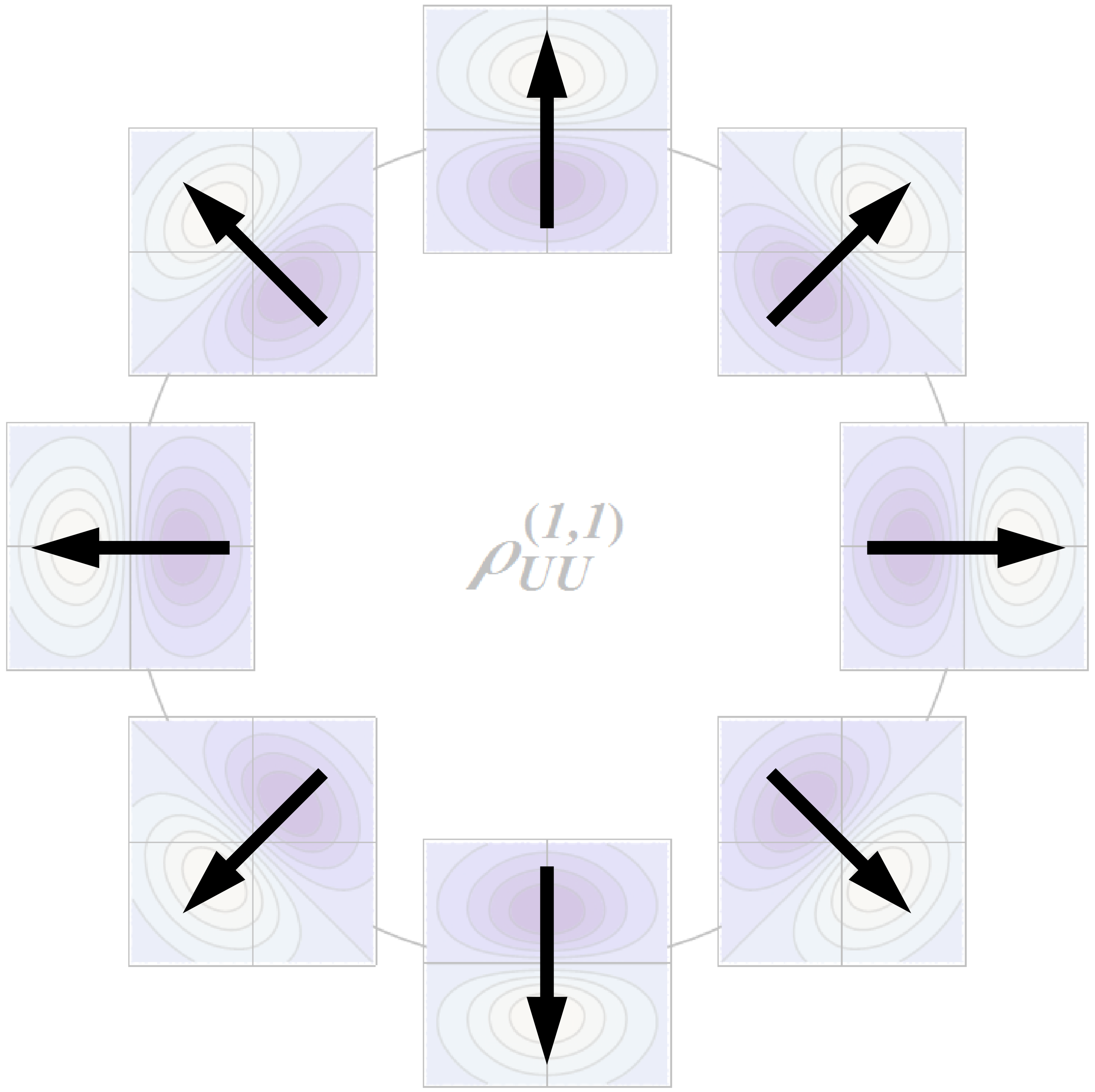


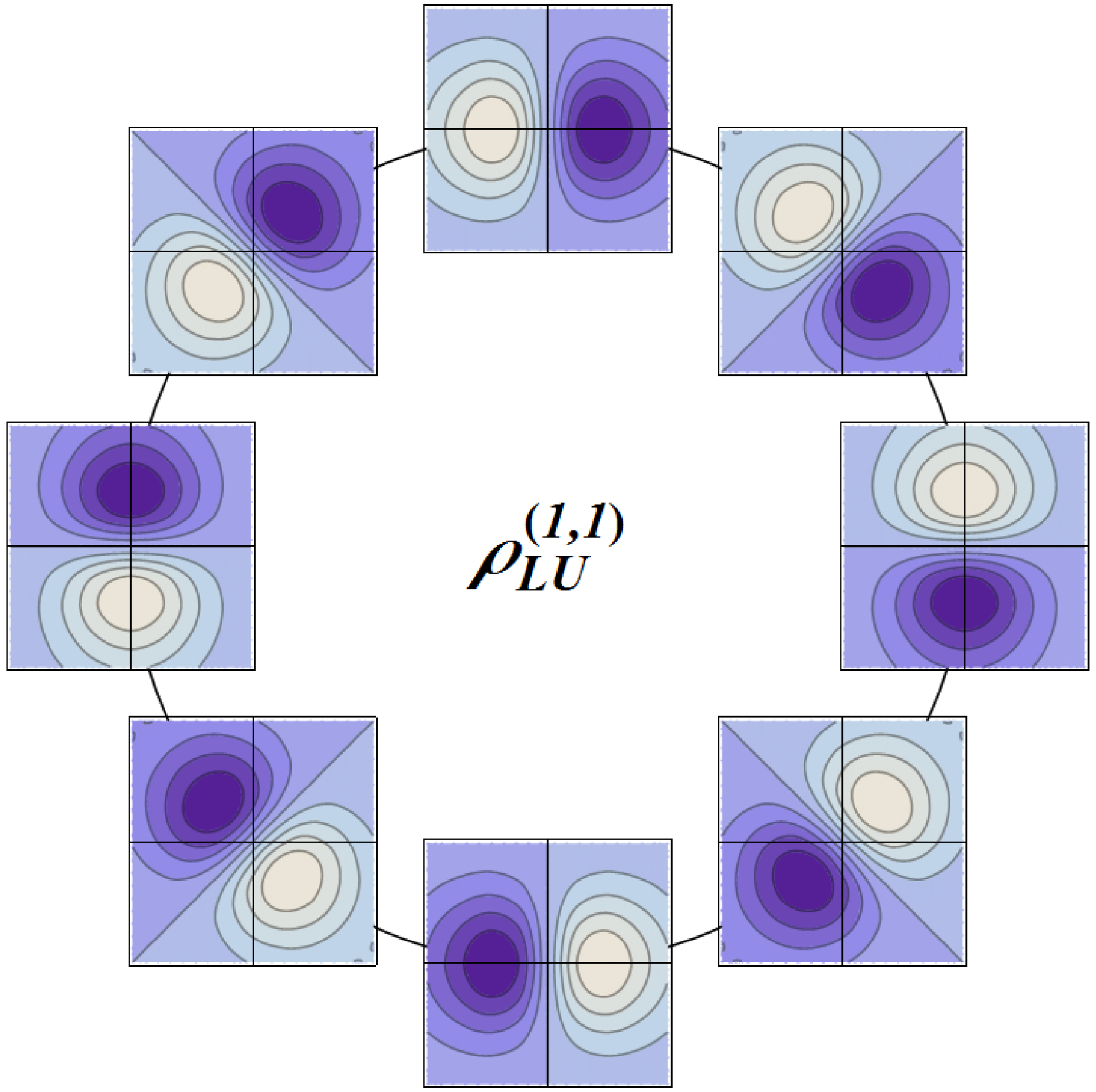
naive time-reversal odd

net radial flow $(\vec{k}_\perp \parallel \vec{b}_\perp)$
due to initial/final state interactions

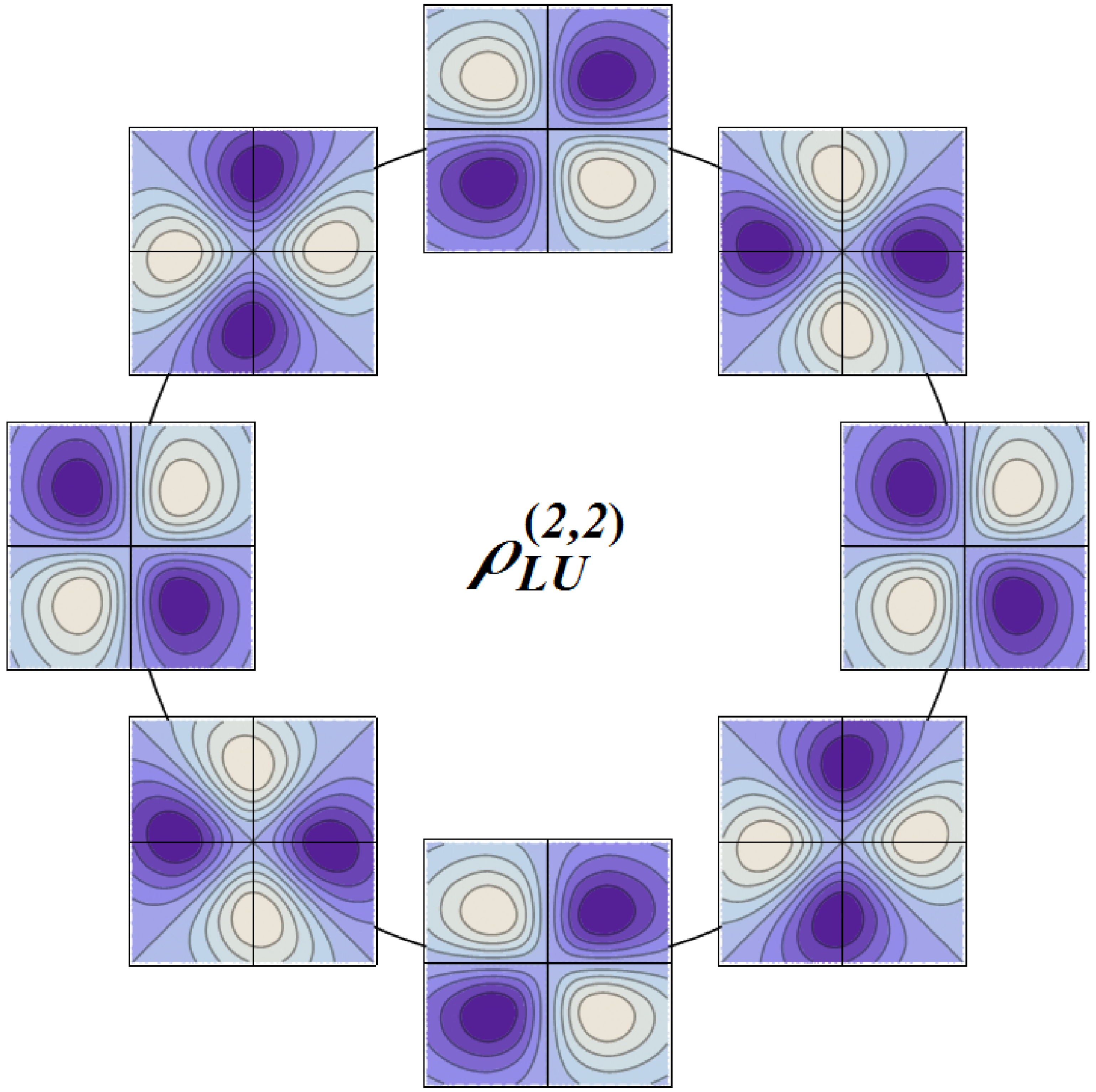
LU

Unpolarized quarks in Longitudinally pol. proton

$\Re e[F_{14}]$ unique information from GTMDs $\Im m[F_{14}]$



naive time-reversal even



naive time-reversal odd

Lorcé, BP, PRD 96, 2016; PRD 84, 2011

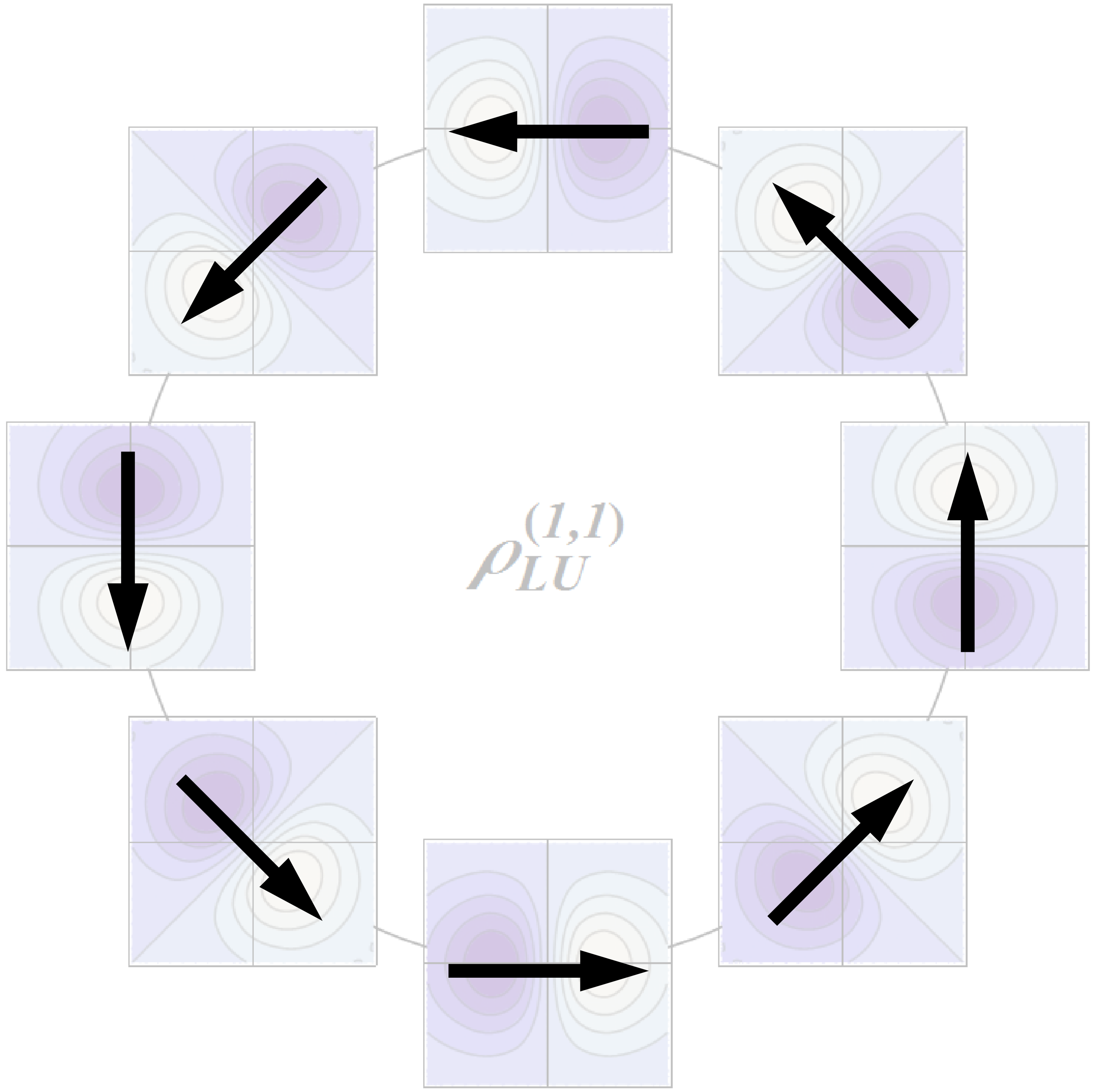
LU

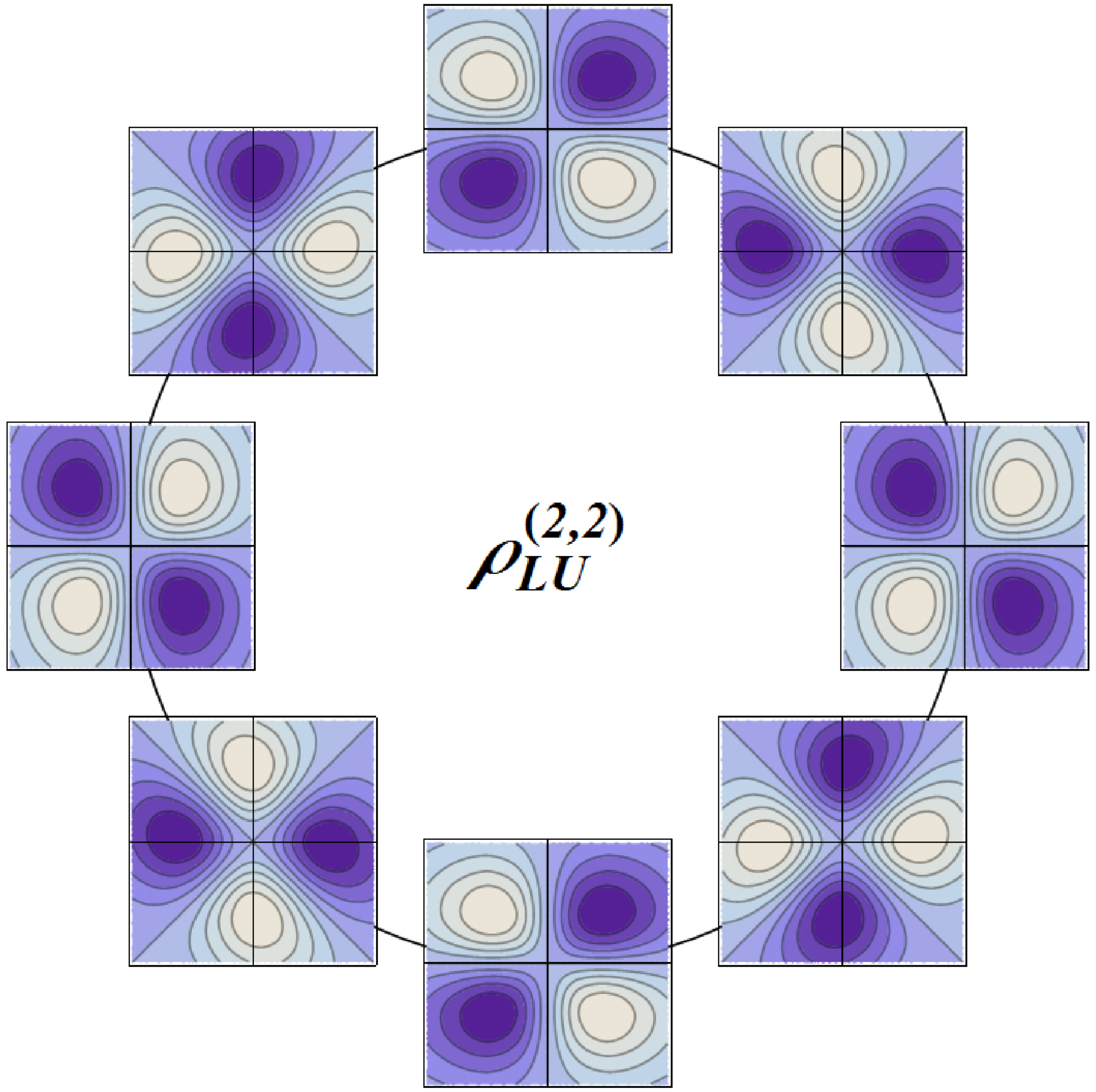
Unpolarized quarks in Longitudinally pol. proton

$\Re e[F_{14}]$

unique information from GTMDs

$\Im m[F_{14}]$





naive time-reversal even

naive time-reversal odd

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z$

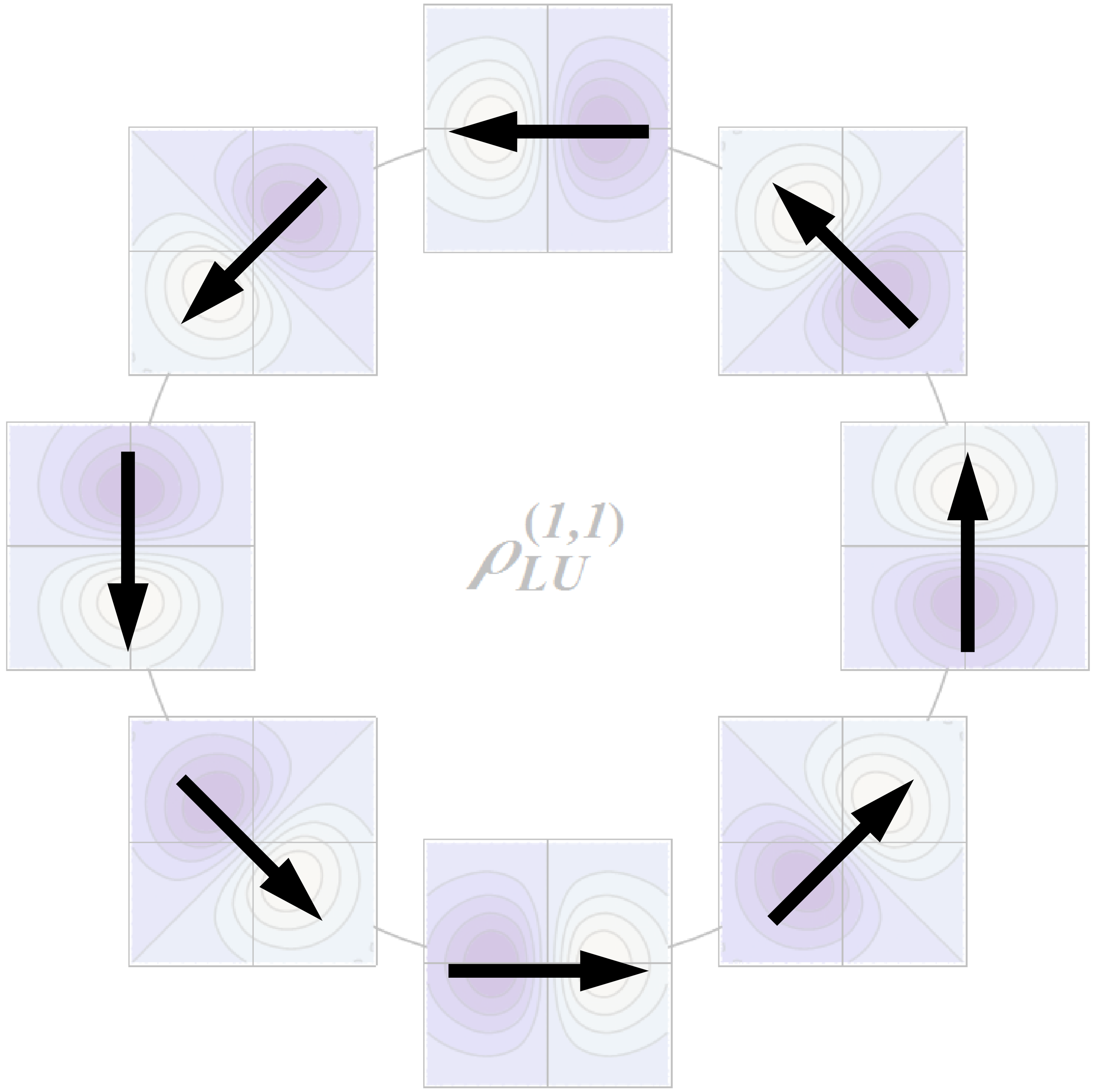
orbital flow → net OAM correlated with S_z

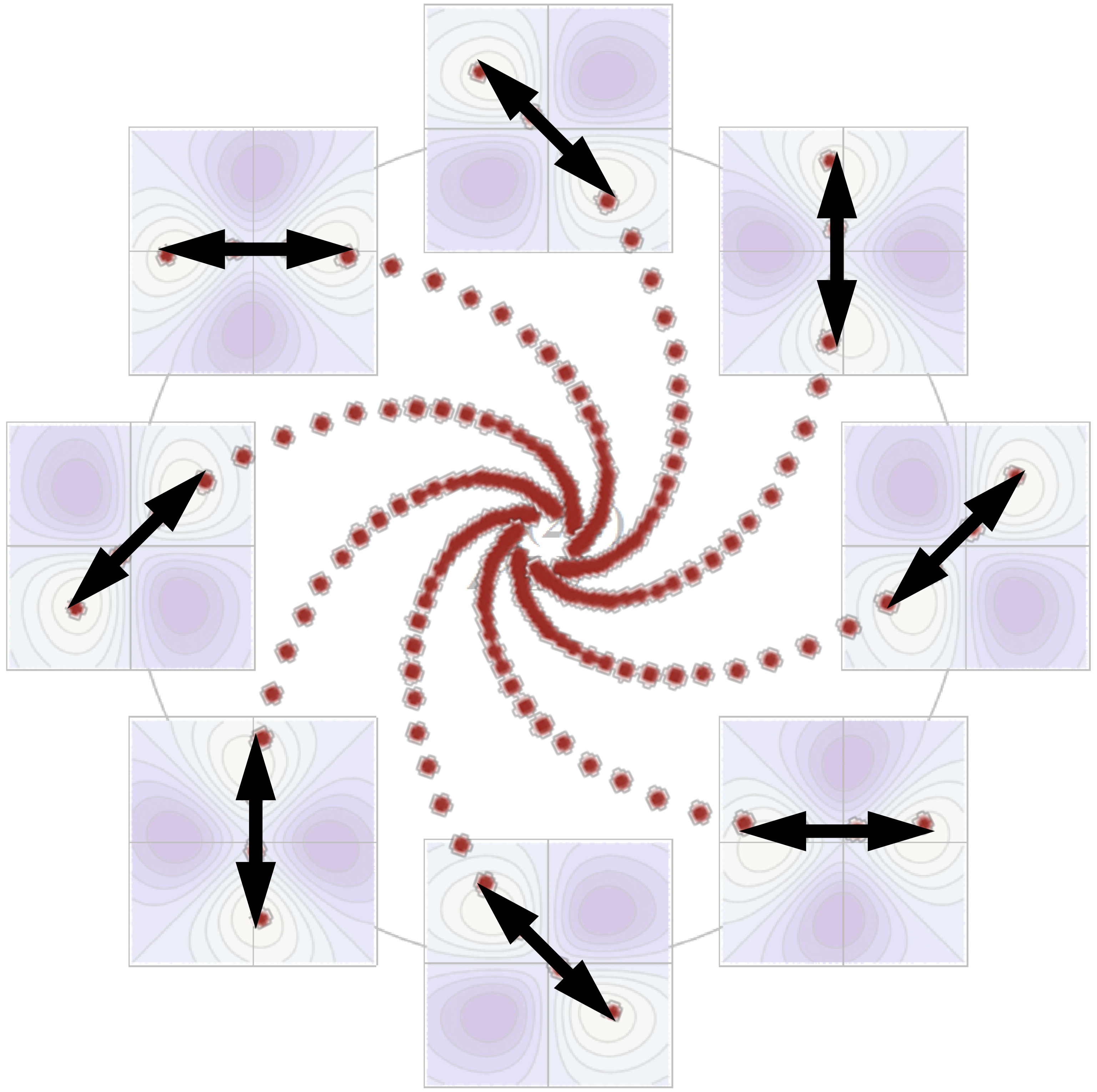
Lorcé, BP, PRD 96, 2016; PRD 84, 2011

Unpolarized quarks in Longitudinally pol. proton

LU

$\Re e[F_{14}]$ unique information from GTMDs $\Im m[F_{14}]$





naive time-reversal even

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z$

orbital flow → net OAM correlated with S_z

naive time-reversal odd

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z\,(\vec{b}_\perp \cdot \vec{k}_\perp)$

spiral flow correlated with S_z

with no-net quark flow

Lorcé, BP, PRD 96, 2016; PRD 84, 2011

Quark Orbital Angular Momentum

$$\ell_z^q = \int \mathrm{d}x\, \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

⬇

Wigner distribution for
Unpolarized quark in a Longitudinally pol. nucleon

Lorcé, BP, PRD **84** (2011) 014015

Lorcé, BP, Xiong, Yuan, PRD **85** (2012) 114006

Quark Orbital Angular Momentum

$$\ell_z^q = \int \mathrm{d}x\, \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

$$= \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}^q_\perp \rangle \longrightarrow \langle \vec{k}^q_\perp \rangle = \int \mathrm{d}x\, \mathrm{d}\vec{k}_\perp\, \vec{k}_\perp \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

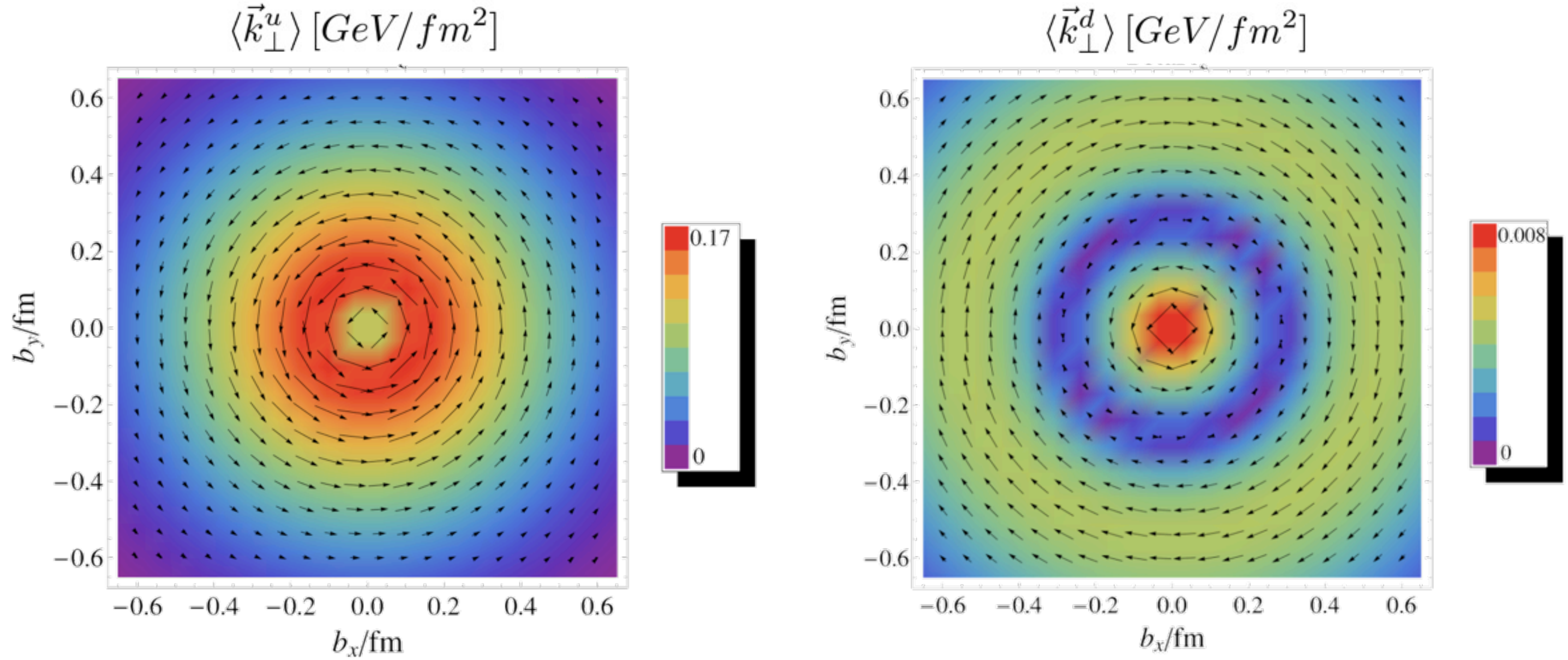
*Lorcé, BP, PRD **84** (2011) 014015*

*Lorcé, BP, Xiong, Yuan, PRD **85** (2012) 114006*

Quark Orbital Angular Momentum

$$\ell_z^q = \int \mathrm{d}x\, \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

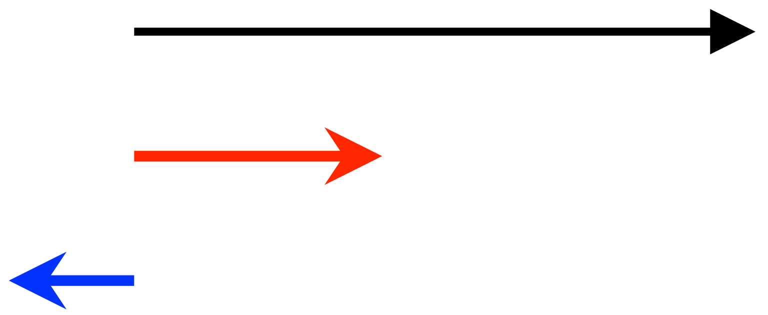
$$= \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}^q_\perp \rangle \longrightarrow \langle \vec{k}^q_\perp \rangle = \int \mathrm{d}x\, \mathrm{d}\vec{k}_\perp\, \vec{k}_\perp \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$



Results in a light-front constituent quark model:

Lorcé, BP, PRD ***84*** *(2011) 014015*

Lorcé, BP, Xiong, Yuan, PRD ***85*** *(2012) 114006*



Proton spin

u-quark OAM

d-quark OAM

$$\ell_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp\mathrm{d}^2\vec{b}_\perp(\vec{b}_\perp \times \vec{k}_\perp)\rho_{LU}(\vec{b}_\perp,\vec{k}_\perp,x)$$

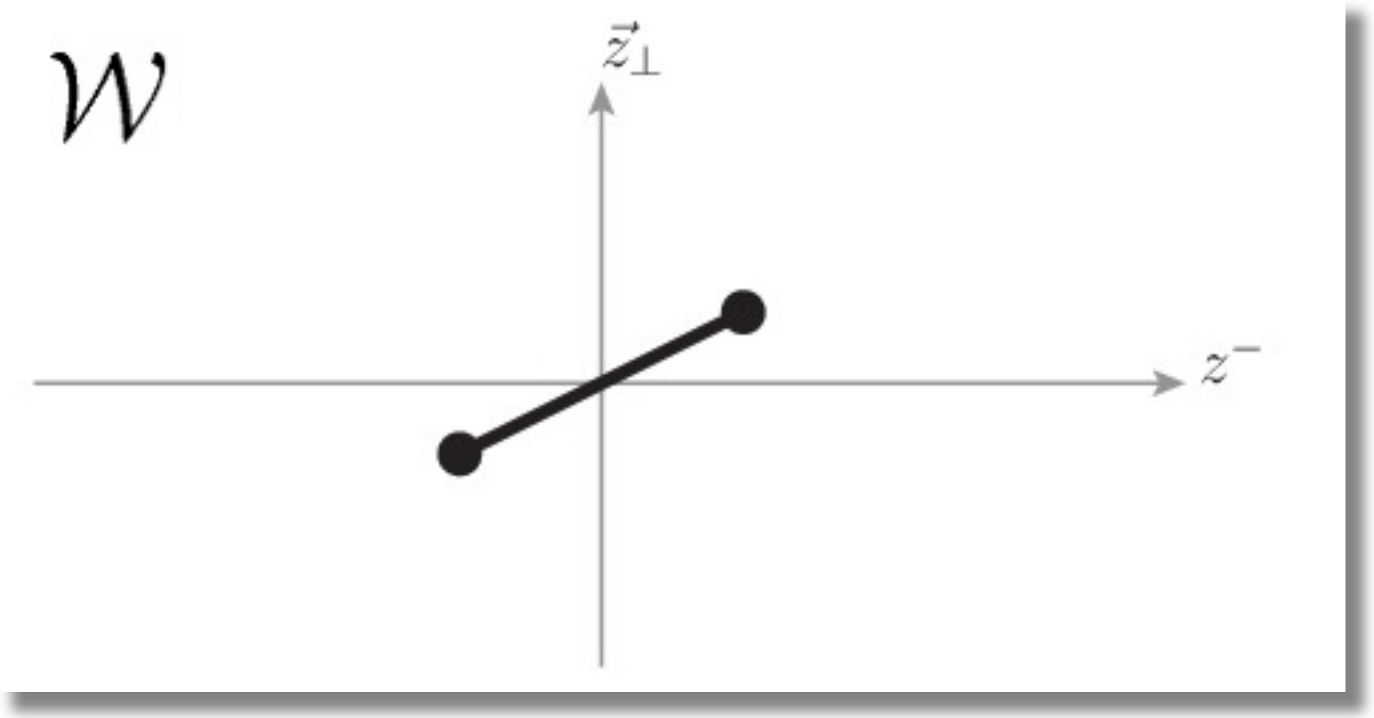
[Lorcé, BP (2011)]
[Lorcé, BP, Xiong, Yuan(2011)]

Light-cone gauge $A^+ = 0$
not gauge invariant, but with simple partonic interpretation

Gauge-invariant extension
$\rho_{LU} \to \rho_{LU}^{\mathcal{W}}$

Kinetic

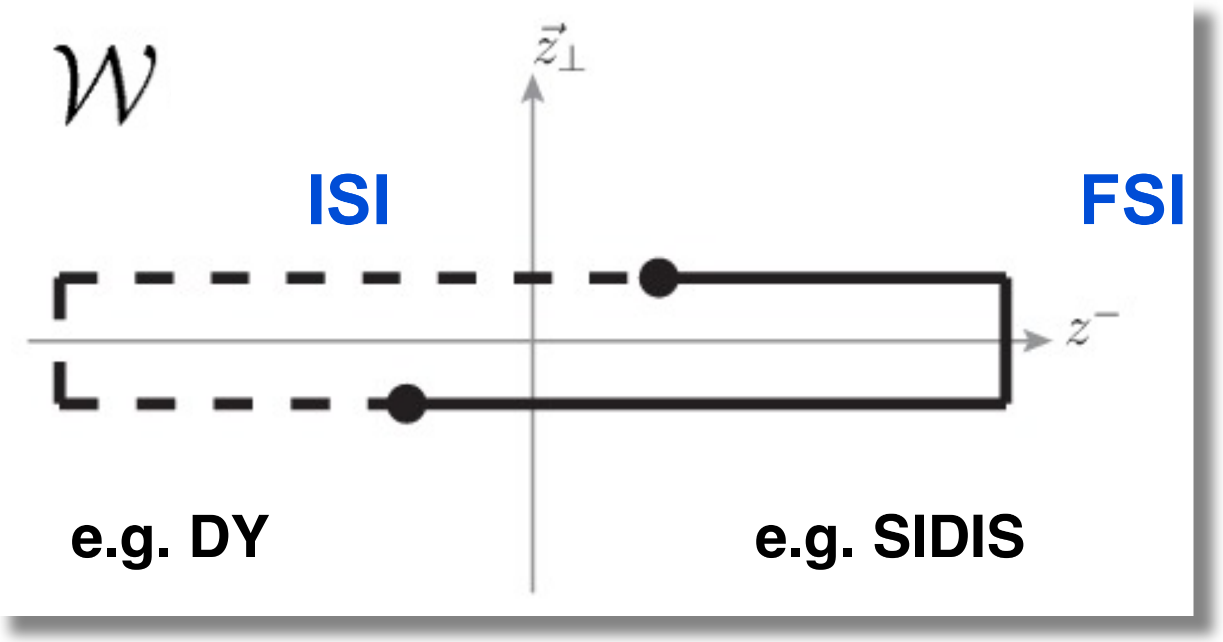
[Ji (1997)]



[Ji, Xiong, Yuan (2012)]
[Burkardt (2012)]

Canonical

[Jaffe, Manohar (1990)]



[Hatta (2012)]

difference between the two definitions can be interpreted as
the change in the quark OAM as the quark leaves the target in a DIS experiment
[M. Burkardt (2013)]

Angular Correlations

$\boxed{\xi = 0}$

quark polarization

nucleon polarization

ρ_X	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S^q_L \ell^q_L \rangle$	$\langle S^q_x \ell^q_x \rangle$	$\langle S^q_y \ell^q_y \rangle$
L	$\langle S_L \ell^q_L \rangle$	$\langle S_L S^q_L \rangle$	$\langle S_L \ell^q_L S^q_x \ell^q_x \rangle$	$\langle S_L \ell^q_L S^q_y \ell^q_y \rangle$
T_x	$\langle S_x \ell^q_x \rangle$	$\langle S_x \ell^q_x S^q_L \ell^q_L \rangle$	$\langle S_x S^q_x \rangle$	$\langle S_x \ell^q_x S^q_y \ell^q_y \rangle$
T_y	$\langle S_y \ell^q_y \rangle$	$\langle S_y \ell^q_y S^q_L \ell^q_L \rangle$	$\langle S_y \ell^q_y S^q_x \ell^q_x \rangle$	$\langle S_y S^q_y \rangle$

Angular Correlations

$\xi = 0$

quark polarization

nucleon polarization

ρ_X	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S_L^q \ell_L^q \rangle$	$\langle S_x^q \ell_x^q \rangle$	$\langle S_y^q \ell_y^q \rangle$
L	$\langle S_L \ell_L^q \rangle$	$\langle S_L S_L^q \rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q \rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q \rangle$
T_x	$\langle S_x \ell_x^q \rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q \rangle$	$\langle S_x S_x^q \rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q \rangle$
T_y	$\langle S_y \ell_y^q \rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q \rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q \rangle$	$\langle S_y S_y^q \rangle$

GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T, \ \tilde{H}_T$

TMD	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, \ h_{1T}^\perp$

the distributions in **red** vanish if there is no quark orbital angular momentum

the distributions in **black** survive in the collinear limit

Angular Correlations

$\boxed{\xi = 0}$

quark polarization

nucleon polarization

ρ_X	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S_L^q \ell_L^q \rangle$	$\langle S_x^q \ell_x^q \rangle$	$\langle S_y^q \ell_y^q \rangle$
L	$\langle S_L \ell_L^q \rangle$	$\langle S_L S_L^q \rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q \rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q \rangle$
T_x	$\langle S_x \ell_x^q \rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q \rangle$	$\langle S_x S_x^q \rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q \rangle$
T_y	$\langle S_y \ell_y^q \rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q \rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q \rangle$	$\langle S_y S_y^q \rangle$

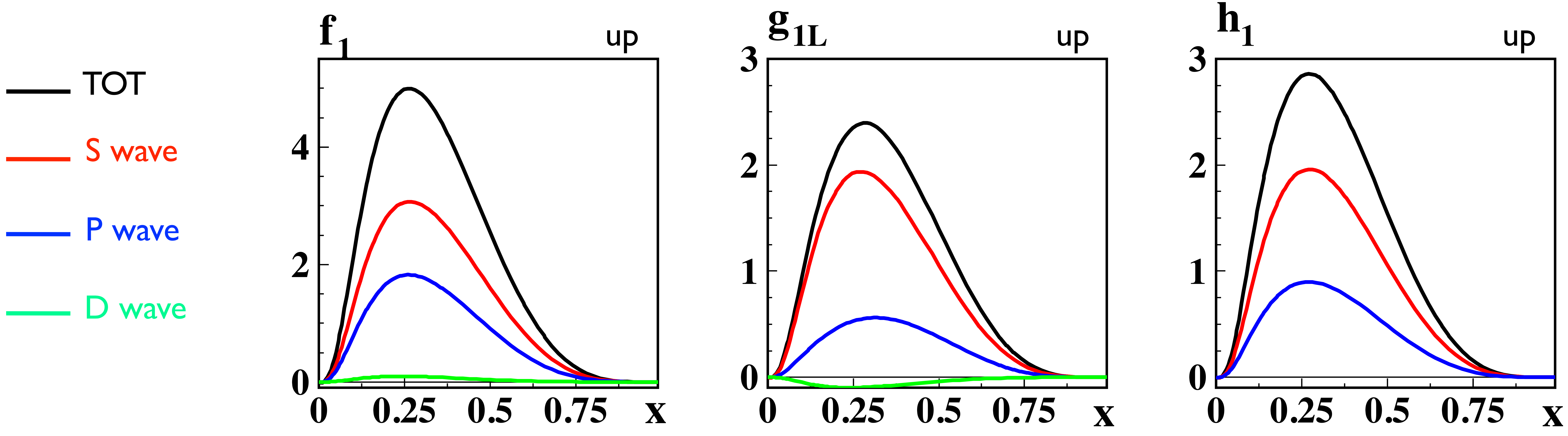
GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\cancel{\tilde{E}_T}$
T	E	$\cancel{\tilde{E}}$	$H_T, \ \tilde{H}_T$

TMD	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_1, \ h_{1T}^{\perp}$

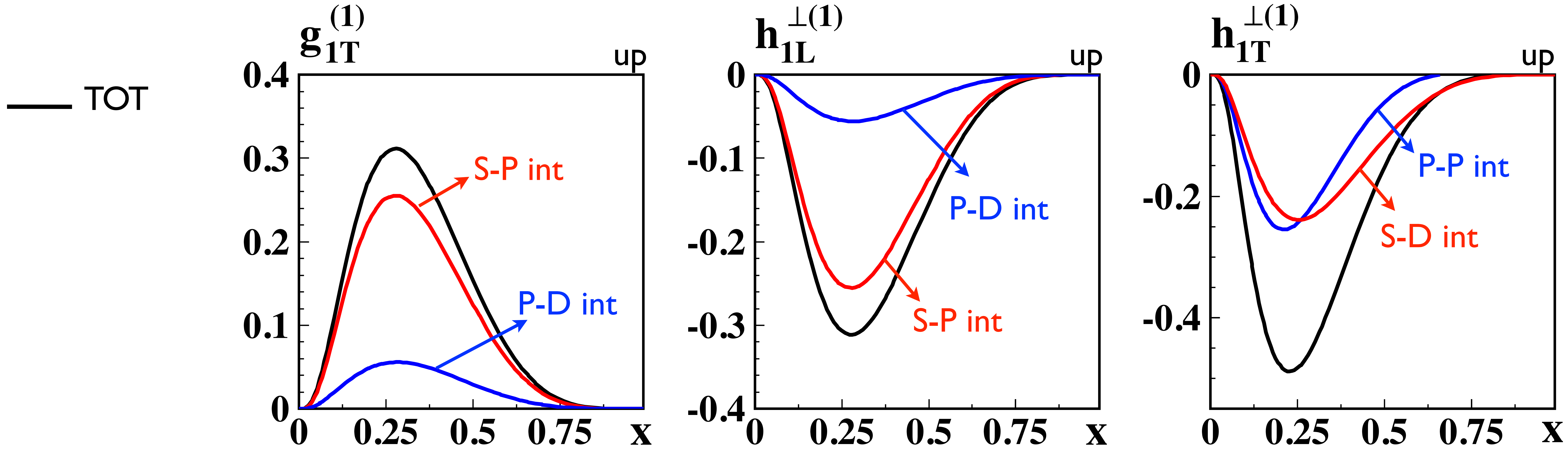
the distributions in **red** vanish if there is no quark orbital angular momentum

the distributions in **black** survive in the collinear limit

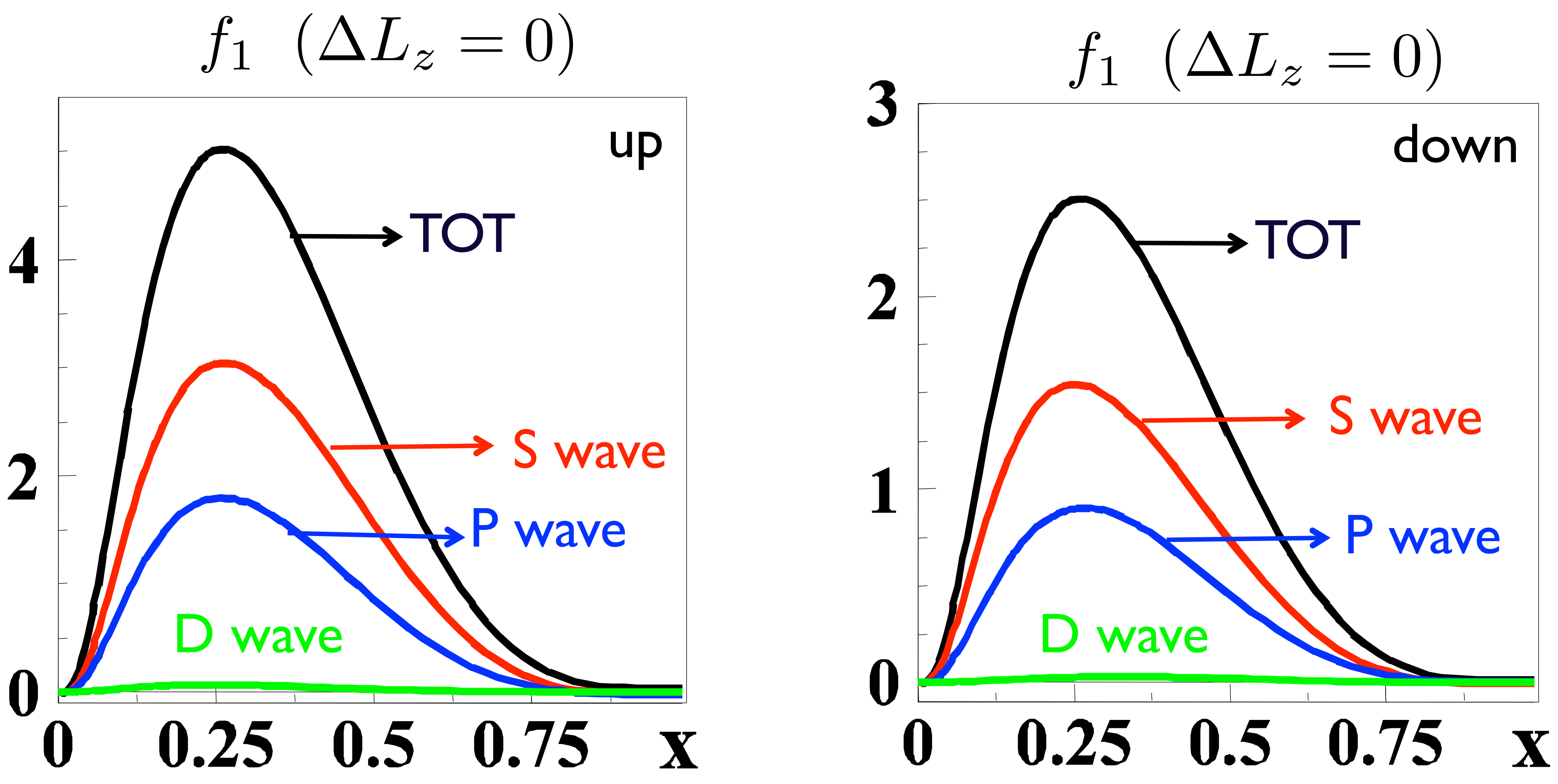
OAM content of TMDs

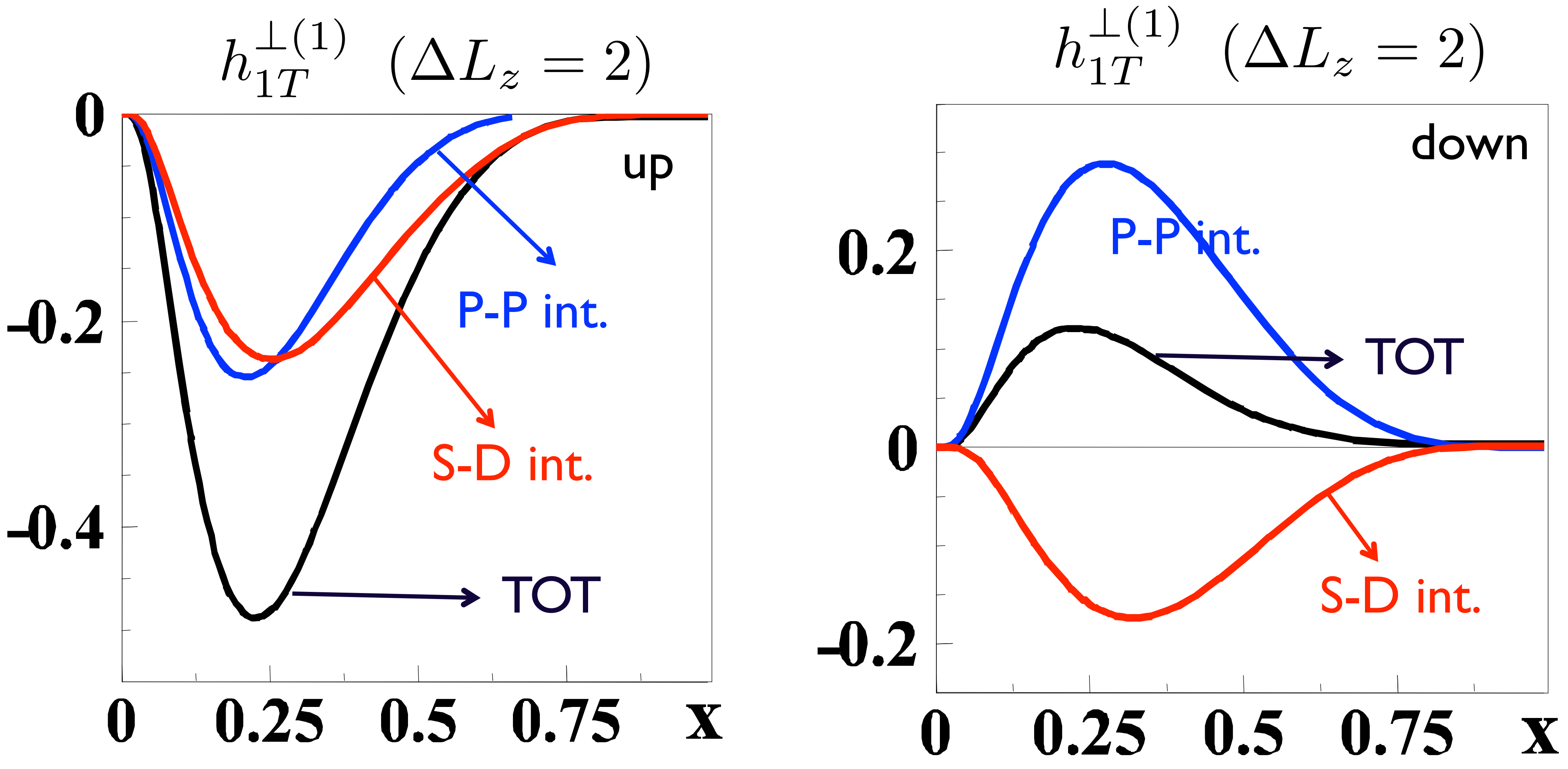


$$j^{(1)}(x) = \int \mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, j(x, k_\perp^2)$$

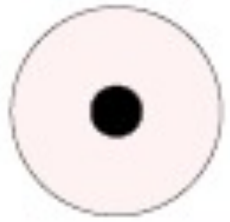


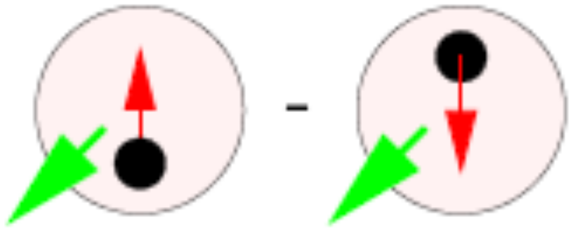
OAM content of TMDs



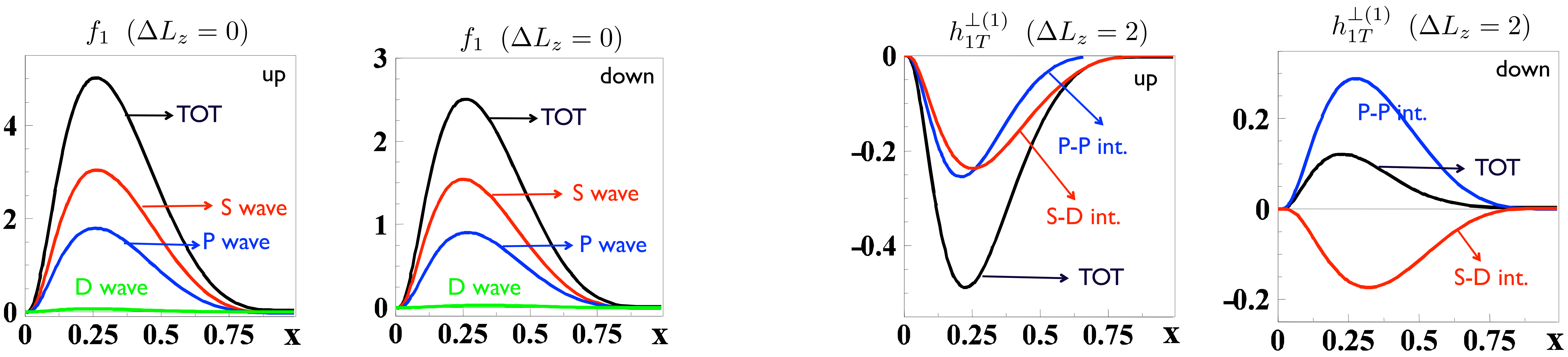


"pretzelosity"

$f_1 =$ 

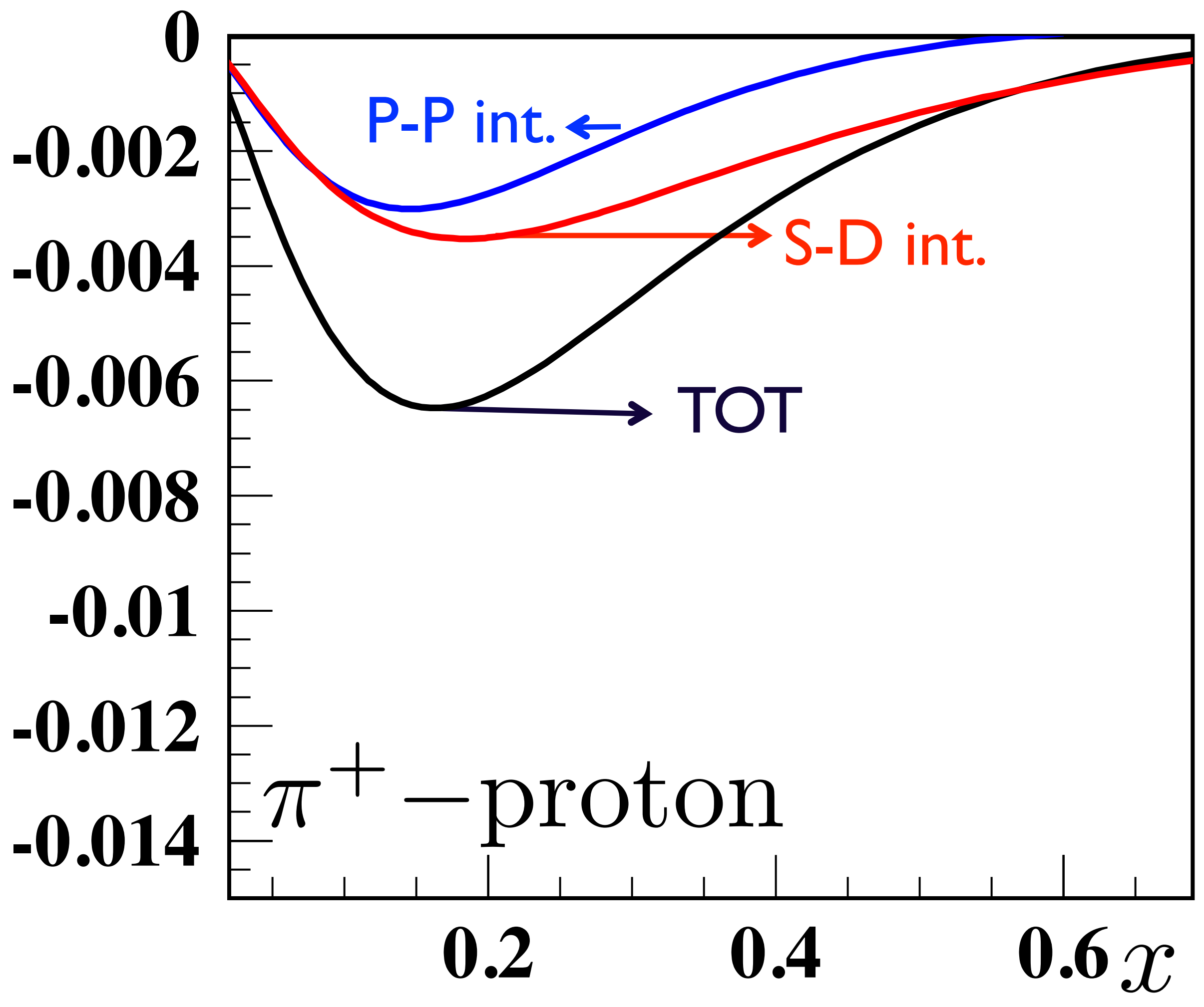
$h_{1T}^{\perp} =$ 

OAM content of TMDs

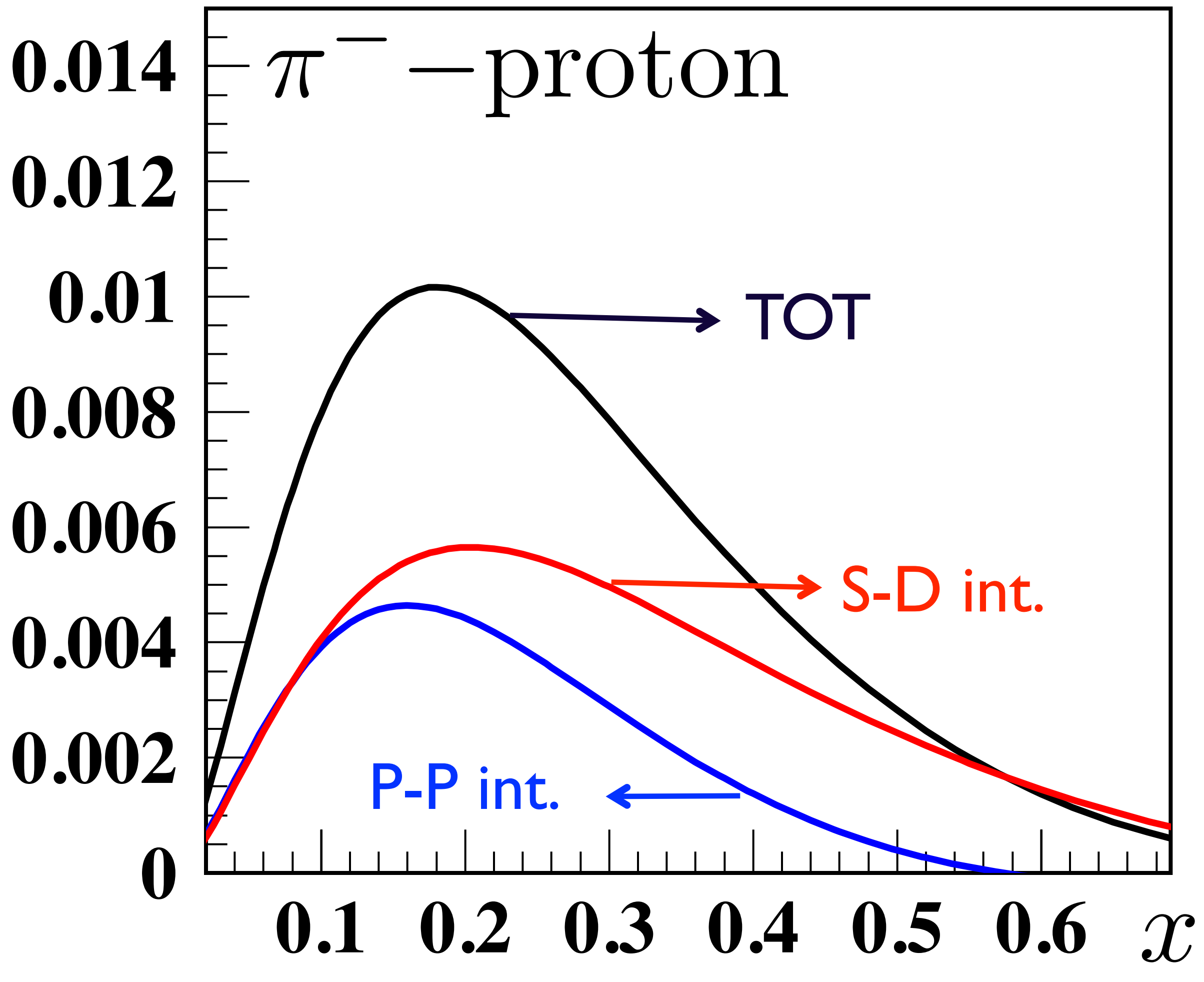


- ✦ Effects on SIDIS observables

$$A_{UT}^{\sin(3\phi-\phi_S)} \sim \frac{h_{1T}^{\perp} \otimes H_1}{f_1 \otimes D_1}$$



$\langle Q^2 \rangle = 2.5 \ \mathrm{GeV}^2$



Boffi, Efremov, BP, Schweitzer, PRD79(2009)

Quark OAM from Pretzelosity

“pretzelosity”

model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2} h_{1T}^{\perp}(x, k_\perp^2)$$

first derived in LF-diquark model and bag model

[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

Quark OAM from Pretzelosity

"pretzelosity"

model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

first derived in LF-diquark model and bag model

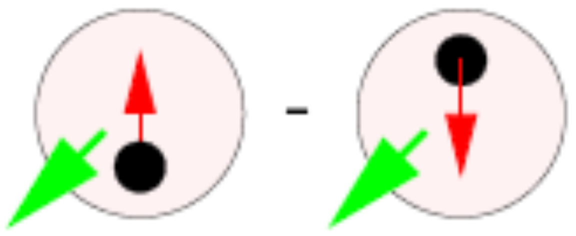
[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

$\mathcal{L}_z$	$h_{1T}^{\perp}$
chiral even and charge even	chiral odd and charge odd
$\Delta L_z = 0$	$\lvert\Delta L_z\rvert = 2$

no operator identity
relation at level of matrix elements of operators

Quark OAM from Pretzelosity

$h_{1T}^{\perp} =$



“pretzelosity”

model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

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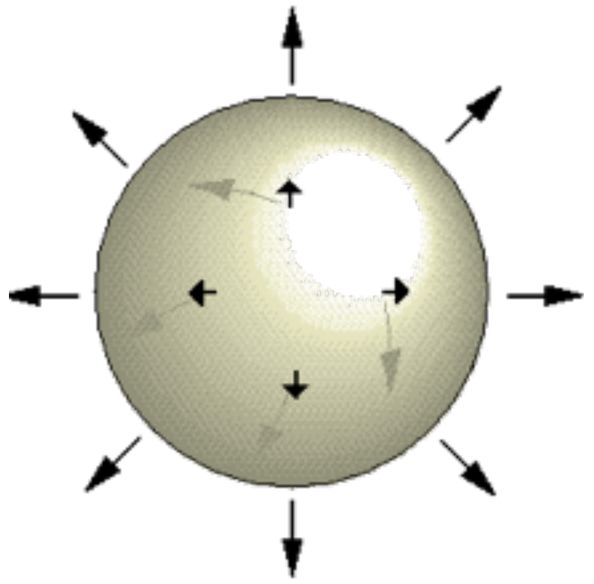
chiral even and charge even	chiral odd and charge odd
$\Delta L_z = 0$	$\|\Delta L_z\| = 2$

no operator identity
relation at level of matrix elements of operators

⇩



valid in all quark models with spherical symmetry in the rest frame

[Lorcé, BP, PLB (2012)]

Relations among T-even TMDs

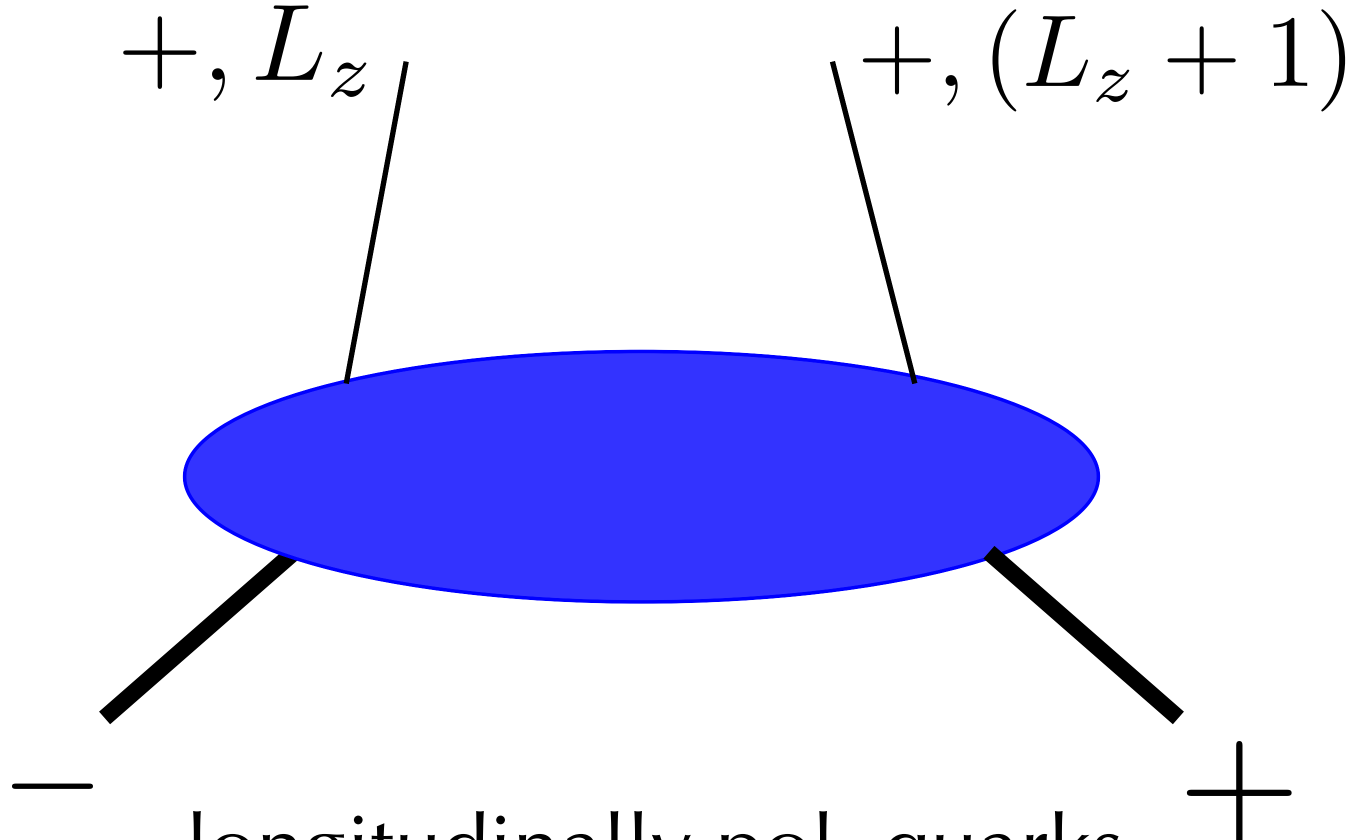
[Avakian, Efremov, Schweitzer, Yuan, 2008]
[Lorcé, Pasquini, 2011]

*=SU(6)	Linear Relations		Quadratic Relations	
Flavor dependent $D^u = \frac{2}{3},\ D^d = -\frac{1}{3}$	$D^1\, f_1^q + g_{1L}^q = 2\,h_1^q$	* * *		
Flavor independent	$g_{1T}^q = -h_{1L}^{\perp q}$	* * * ● ● ●	$2\,h_1^q\,h_{1T}^{\perp q} = -(g_{1T}^q)^2$	* * ● ● ●
	$g_{1L}^q - h_1^q = \frac{k_\perp^2}{2M^2} h_{1T}^{\perp q}$	* * * ● ● ●		

Bag	[Jaffe, Ji 1991); Signal (1997); Barone & al. (2002); Avakian & al., (2008-2010)]
χ QSM	[Lorcé, Pasquini, Vanderhaeghen (2011)]
LCQM	[Pasquini & al. (2008)]
S Diquark	[Ma & al. (1996-2009); Jakob & al. (1997); Bacchetta & al. (2008)]
AV Diquark	[Ma & al. (1996-2009); Jakob & al. (1997); Bacchetta & al. (2008)]
Cov. Parton	[Efremov & al. (2009)]
Quark Target	[Meissner & al. (2007)]

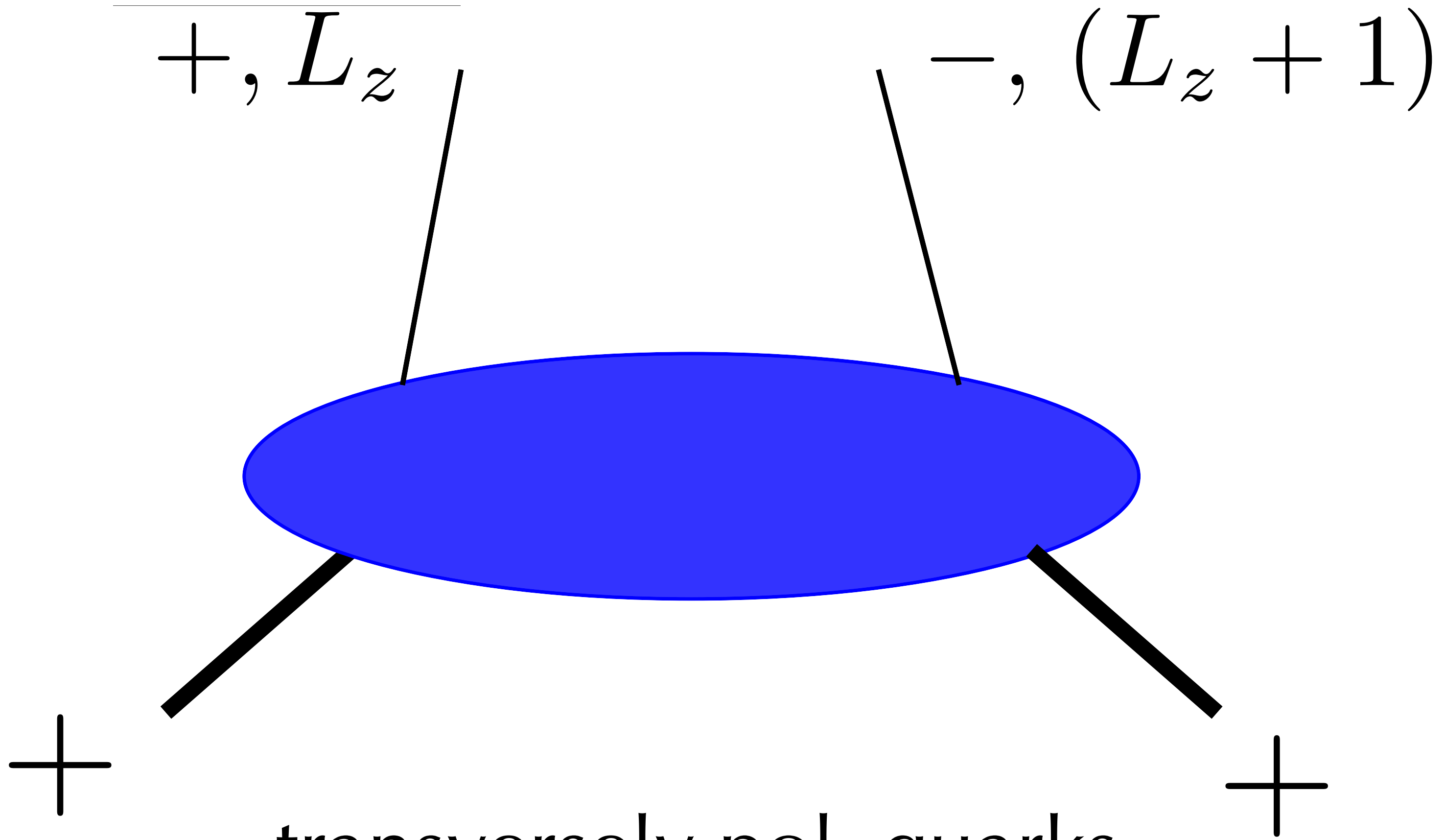
The Worm-Gear functions

g_{1T}



longitudinally pol. quarks
in transversely pol. nucleon

$h_{1L}^{\perp}$

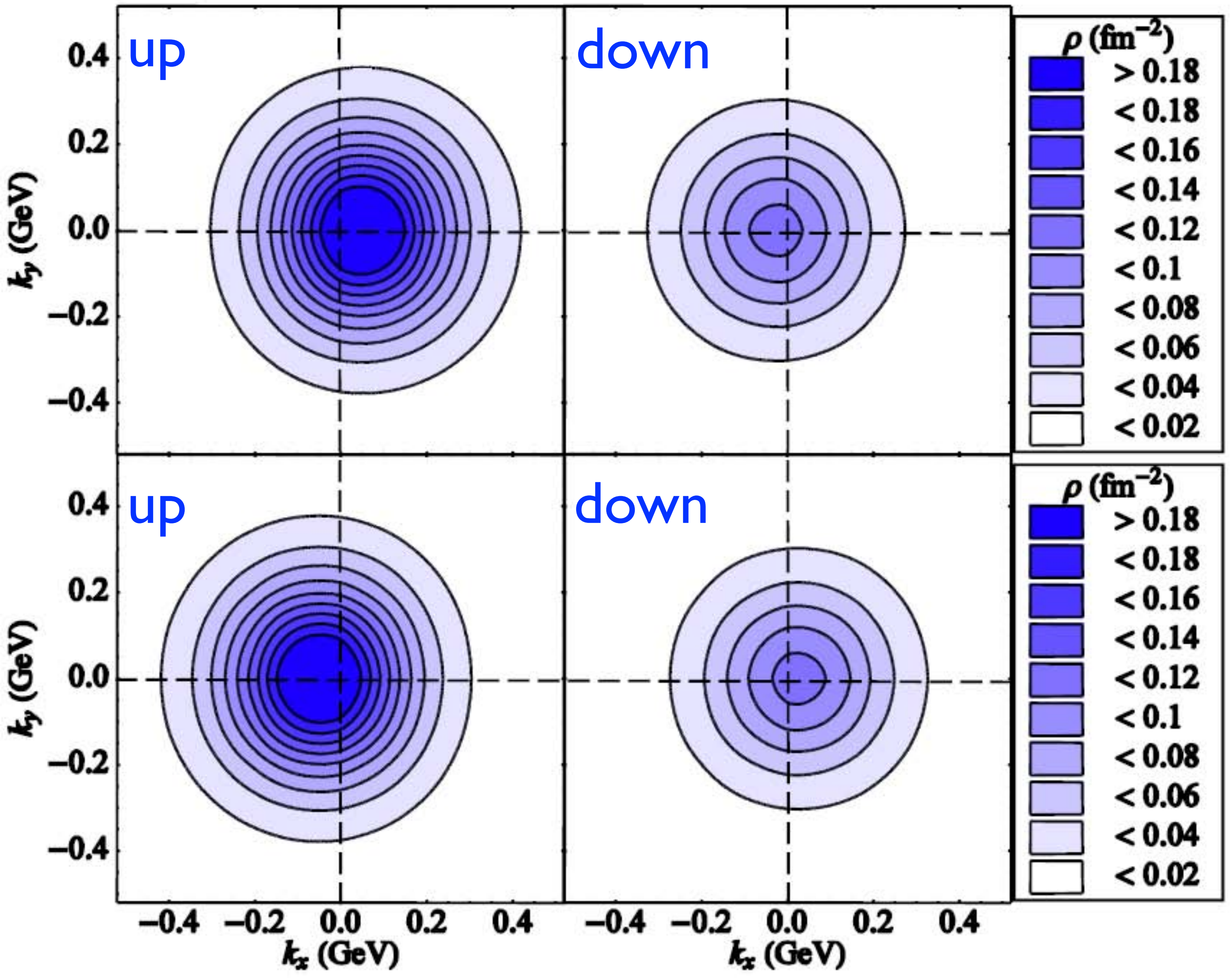


transversely pol. quarks
in longitudinal pol. nucleon

~~$g_{1T}, h_{1L}^{\perp} \longleftrightarrow$ GPDs~~

genuine effect of intrinsic transverse momentum of quarks!

Light-Front Quark Model

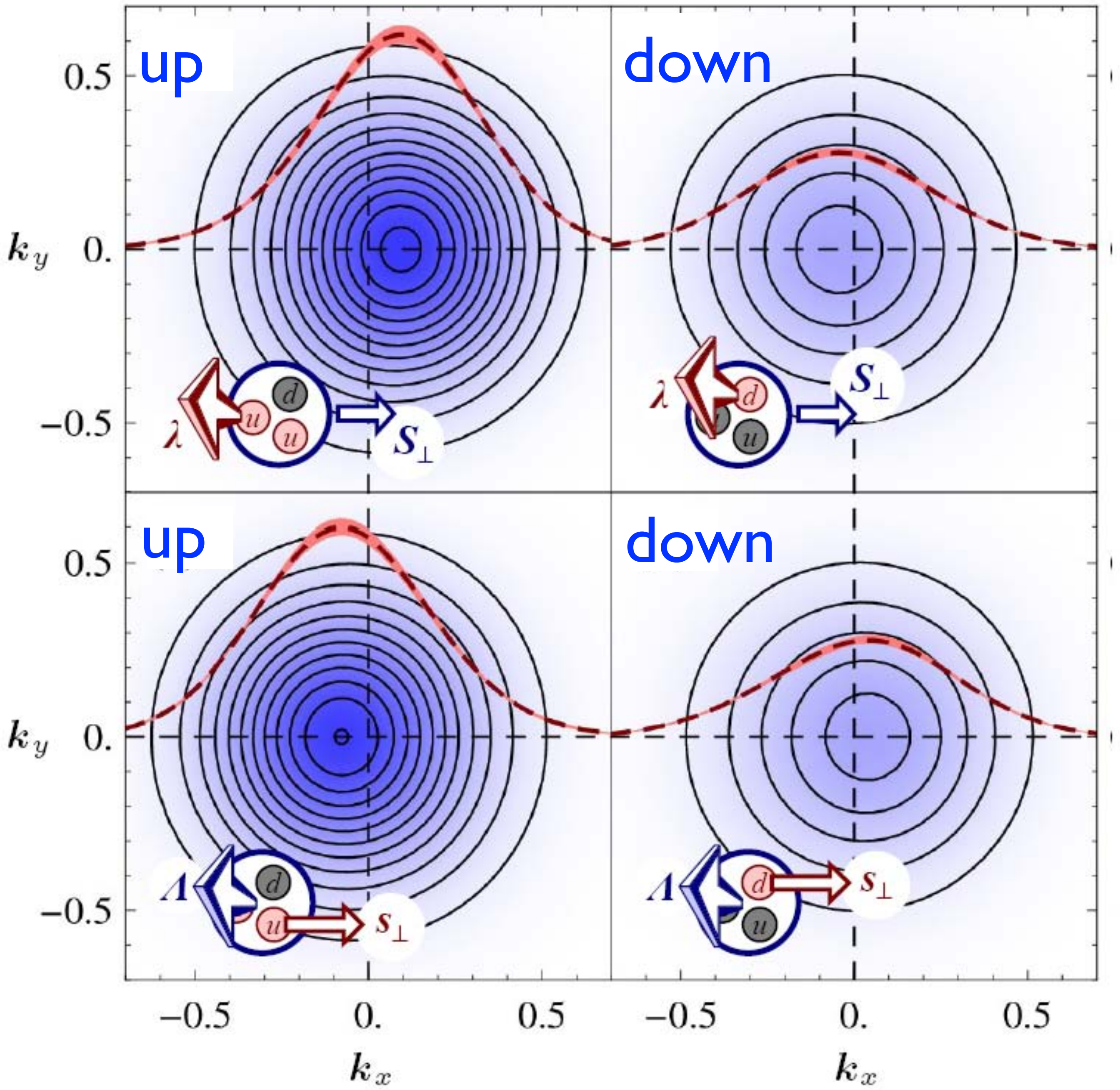


BP, Cazzaniga, Boffi, PRD78 (2008)

Lattice QCD

$g_{1T}^{(1)}(x)$

$h_{1L}^{\perp(1)}(x)$



Musch, Haegler, Negele, Schaefer, PRD83 (2011)

Model-dependent relation:

$$g_{1T}(x, k_\perp^2) = -h_{1L}^\perp(x, k_\perp^2)$$

supported by lattice calculation

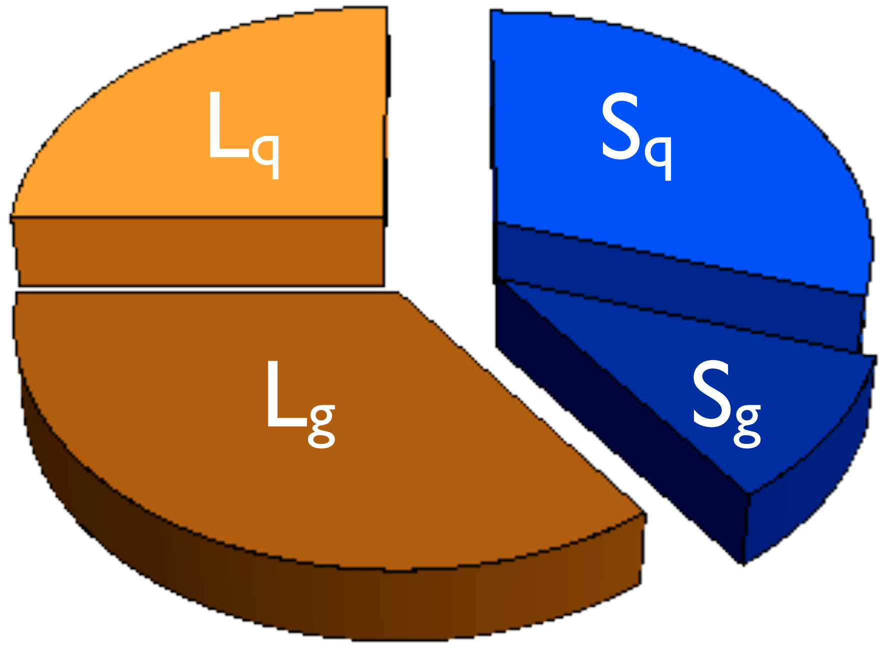
$h_{1L}^\perp$: $\langle k_x^u \rangle = -55.8$ MeV $\quad \langle k_x^d \rangle = 27.9$ MeV

g_{1T} : $\langle k_x^u \rangle = 67(5)$ MeV $\quad \langle k_x^d \rangle = -30(5)$ MeV

$h_{1L}^\perp$: $\langle k_x^u \rangle = -60(5)$ MeV $\quad \langle k_x^d \rangle = 16(5)$ MeV

Different definitions of OAM

Jaffe-Manohar



Pros:

- Satisfies canonical relations
- Complete decomposition

Cons:

- Gauge-variant decomposition
- Missing observables for the OAM

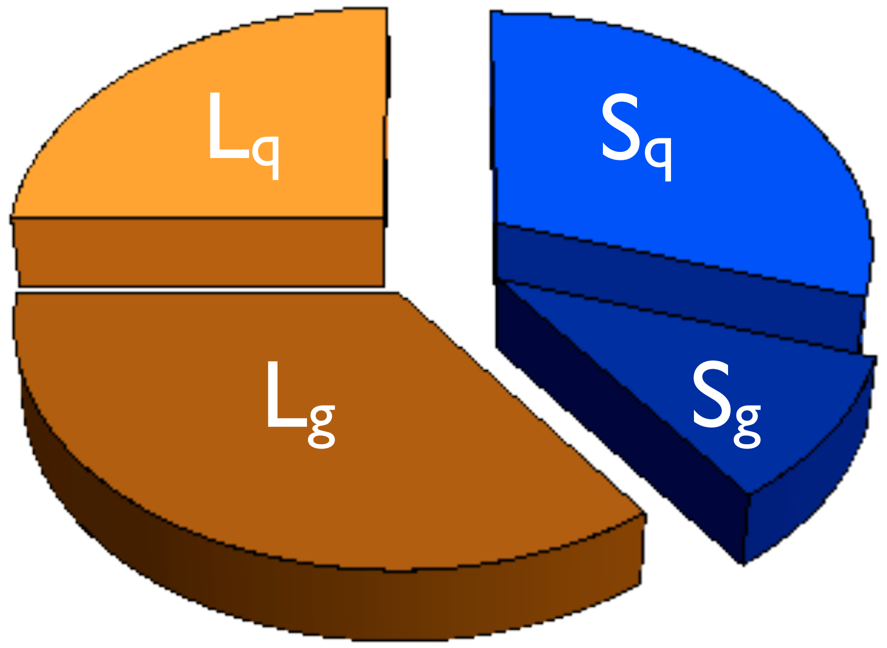
(Δg and $\Delta\Sigma$ measured by COMPASS, HERMES , RHIC)

Improvements:

- OAM accessible via Wigner distributions and it can be calculated on the lattice

Different definitions of OAM

Jaffe-Manohar



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(Δg and $\Delta\Sigma$ measured by COMPASS, HERMES , RHIC)

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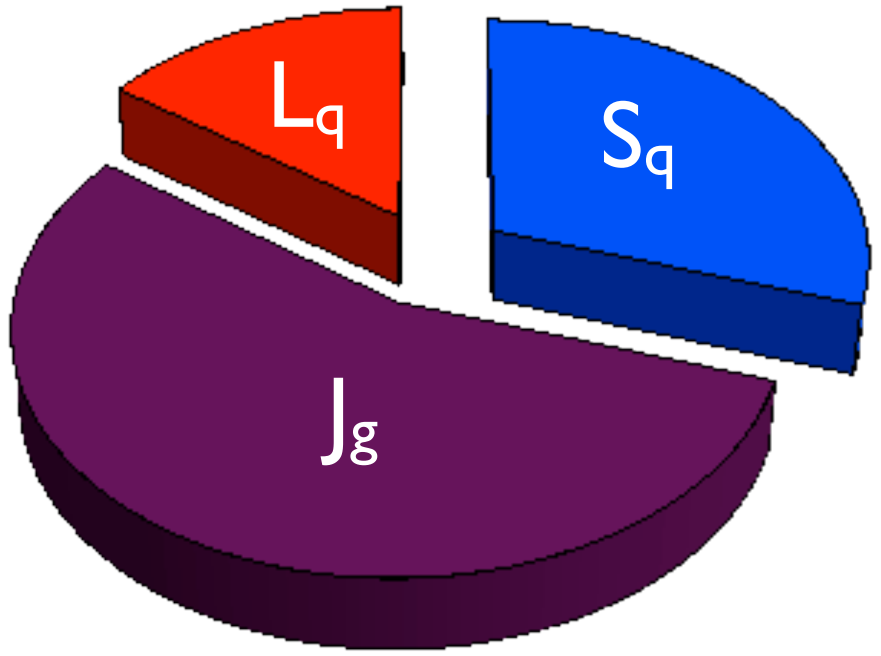
- OAM accessible via Wigner distributions and it can be calculated on the lattice

Ji's relation: $J^{q,g} = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x \left(H^{q,g}(x,0,0) + E^{q,g}(x,0,0)\right)$



Pros:

- Each term is gauge invariant
- Accessible in DIS and DVCS
- Can be calculated in Lattice QCD

Cons:

- No decomposition of J_g in spin and orbital part

Improvements:

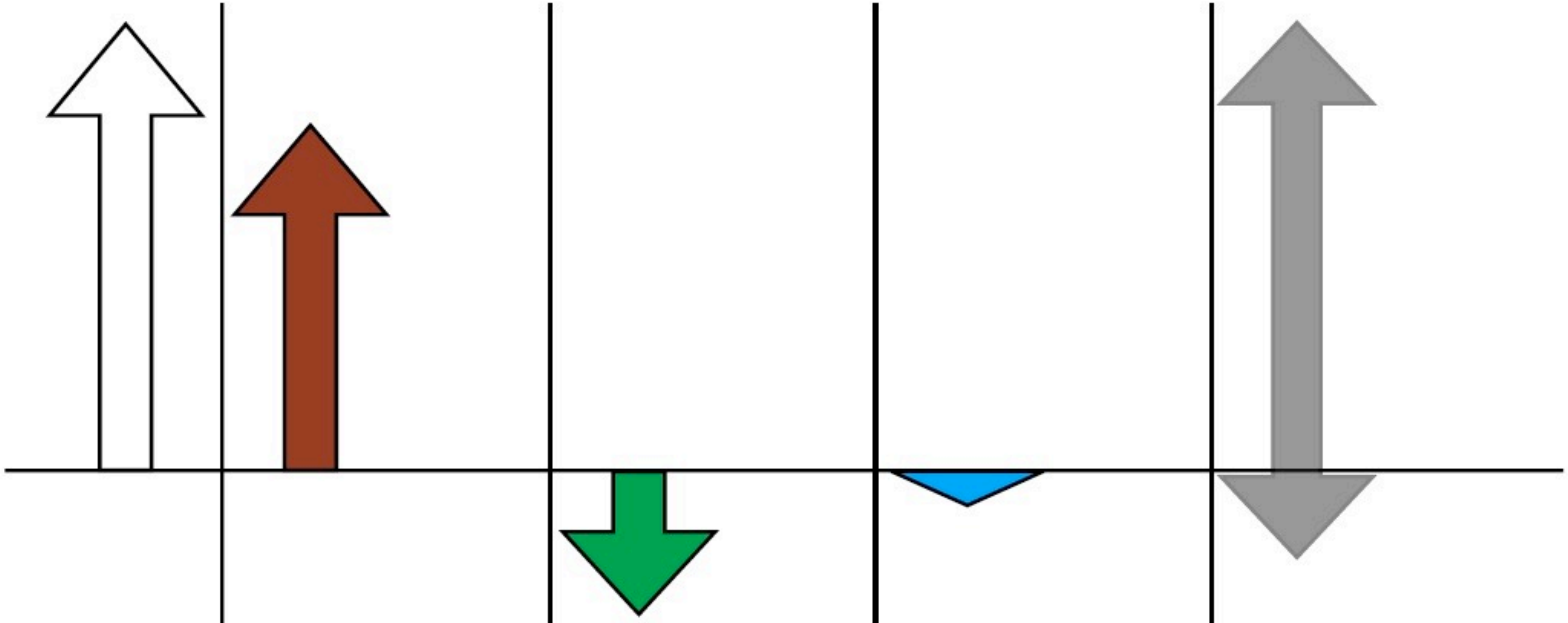
- Complete decomposition: $J^g = L^g + \Delta g$
- quark OAM from twist-3 GPD:

$$L_z^q = -\int \mathrm{d}x\, x\, G_2^q(x,0,0)$$

$\longrightarrow$ see talk of S. Liuti

Lorcé, Leader, Phys. Rep. ***541*** *(2014) 163*

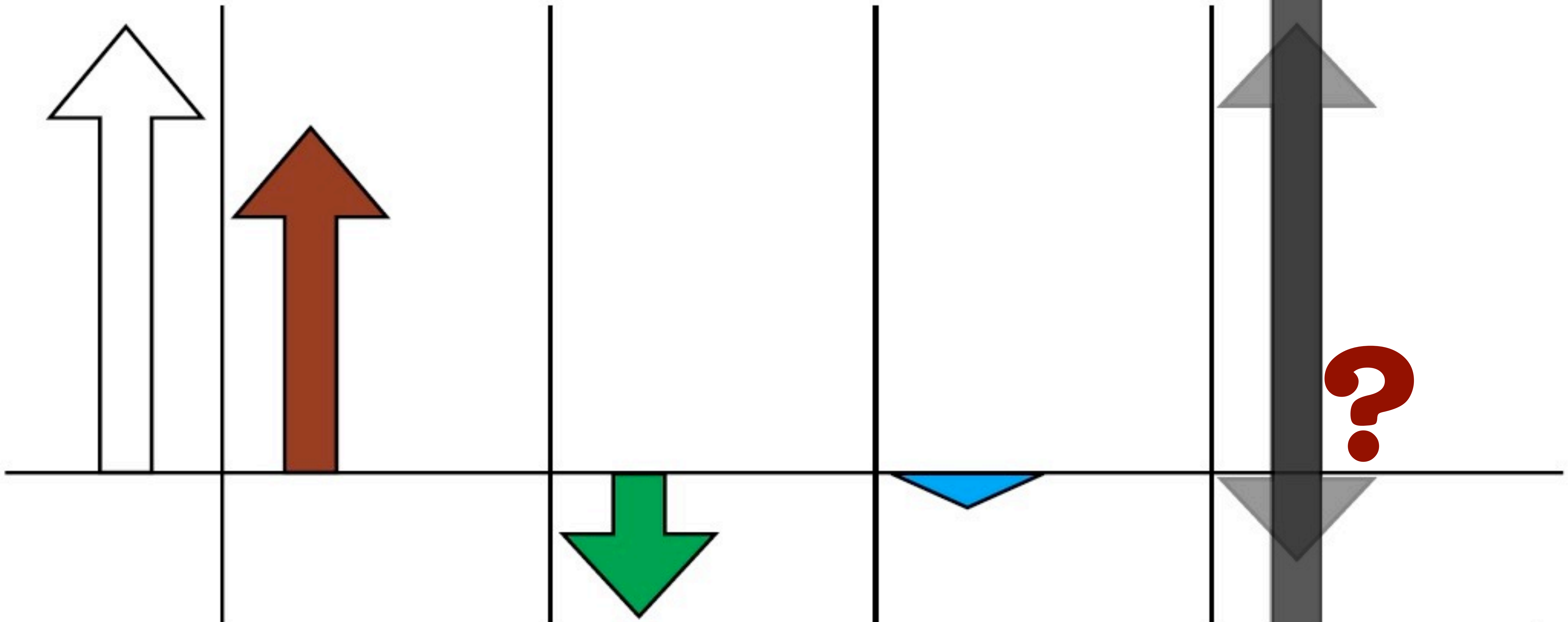
Status of spin sum rule



Proton spin	$u+\bar{u}$		$d+\bar{d}$		$s+\bar{s}$		g		
	spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	
	82 ± 2		-45 ± 4		-11 ± 10		40 ± 60		DSSV

de Florian, Sassot, Stratmann, Vogelsang, PRL 113 (14)
NNPDF, Ball... Nocera... NPB 887 (14), Tab. 12, 13

Status of spin sum rule

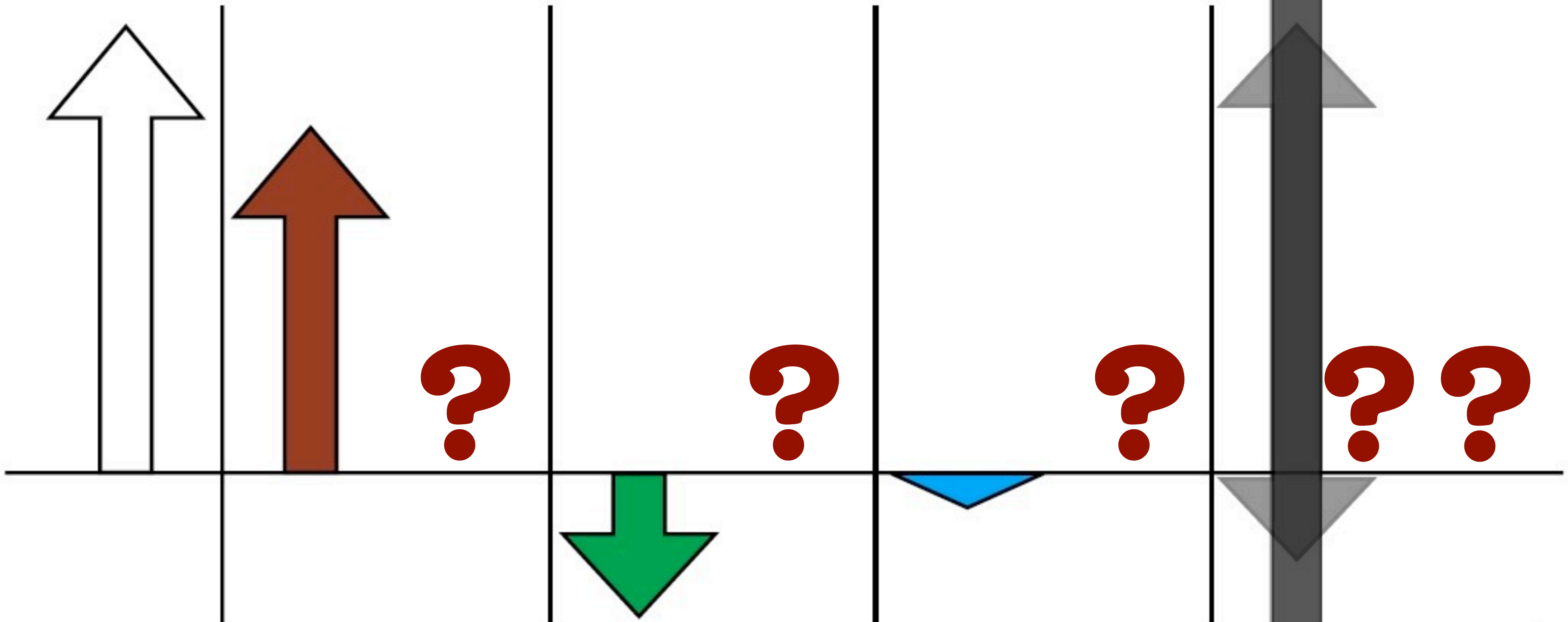


Proton spin	$u+\bar{u}$		$d+\bar{d}$		$s+\bar{s}$		g		
	spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	
	82 ± 2		-45 ± 4		-11 ± 10		40 ± 60		DSSV
	79 ± 7		-47 ± 7		-12 ± 10		3 ± 324		NNPDF

de Florian, Sassot, Stratmann, Vogelsang, PRL 113 (14)

NNPDF, Ball... Nocera... NPB 887 (14), Tab. 12, 13

Status of spin sum rule



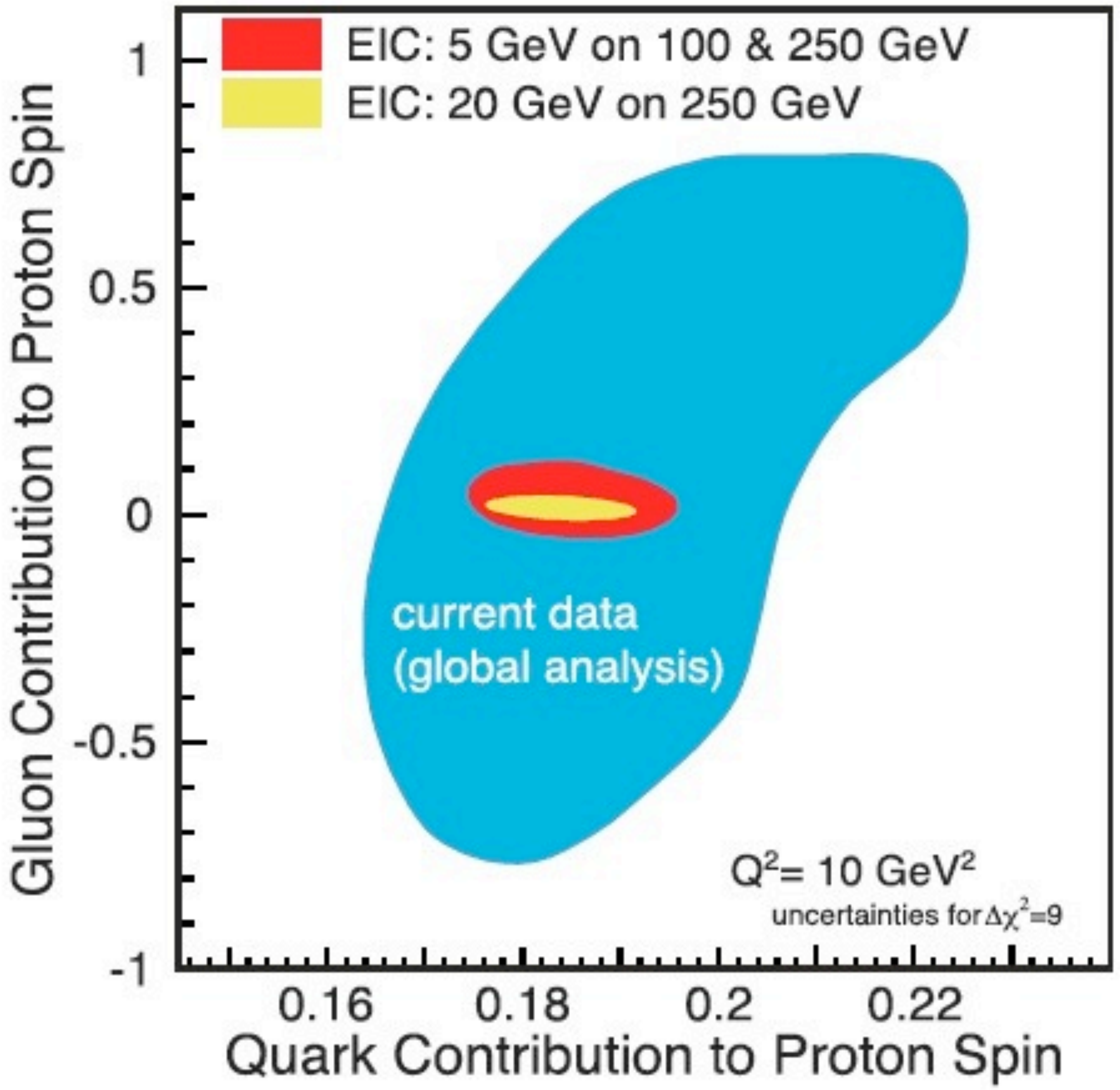
Proton spin

$u + \bar{u}$		$d + \bar{d}$		$s + \bar{s}$		g		
spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	
82 ± 2		-45 ± 4		-11 ± 10		40 ± 60		DSSV
79 ± 7		-47 ± 7		-12 ± 10		3 ± 324		NNPDF

de Florian, Sassot, Stratmann, Vogelsang, PRL 113 (14)

NNPDF, Ball... Nocera... NPB 887 (14), Tab. 12, 13

Impact of EIC on proton spin

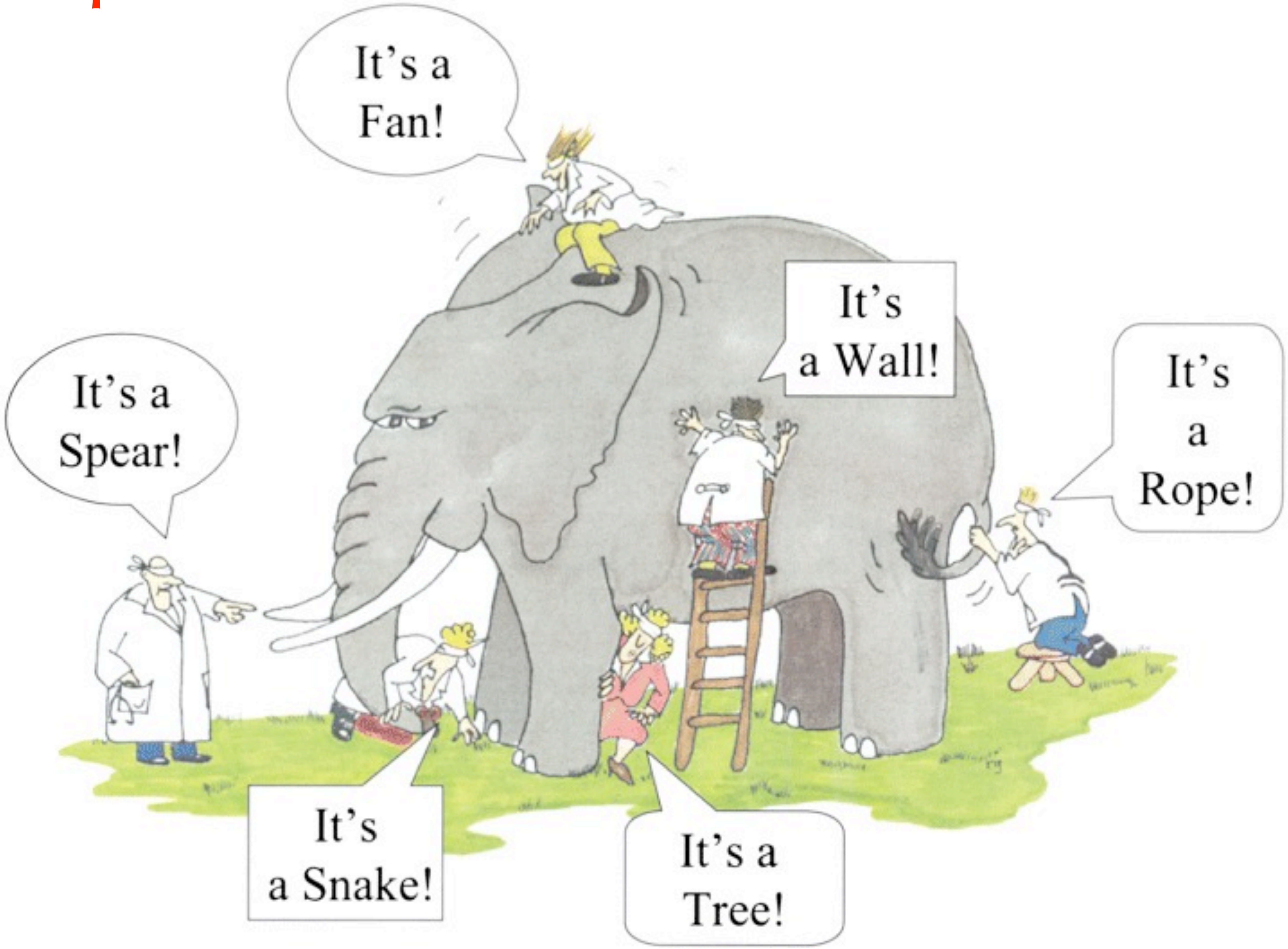


Aschenauer, Stratmann, Sassot,PRD86 (2012)

Geesaman, et al., Reaching for the horizon: The 2015 long range plan for nuclear science (2015)

The blind men and the elephant

from H. Avakian

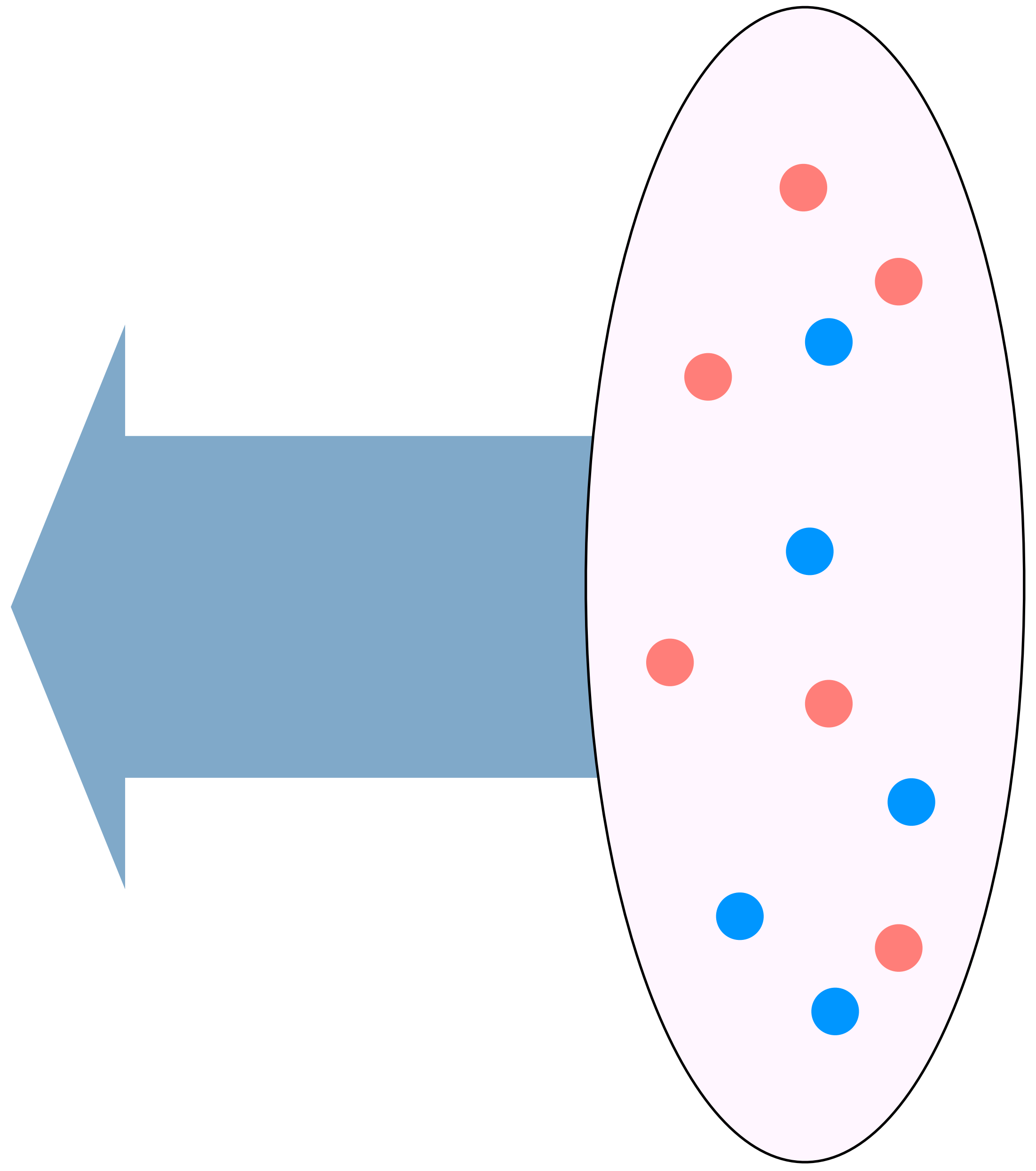


TMDs, GPDs and GTMDs provide different and complementary information and need to talk to each other to reconstruct the full multidimensional picture of the nucleon

Backup

Model relation TMD $\longleftrightarrow$ GPD

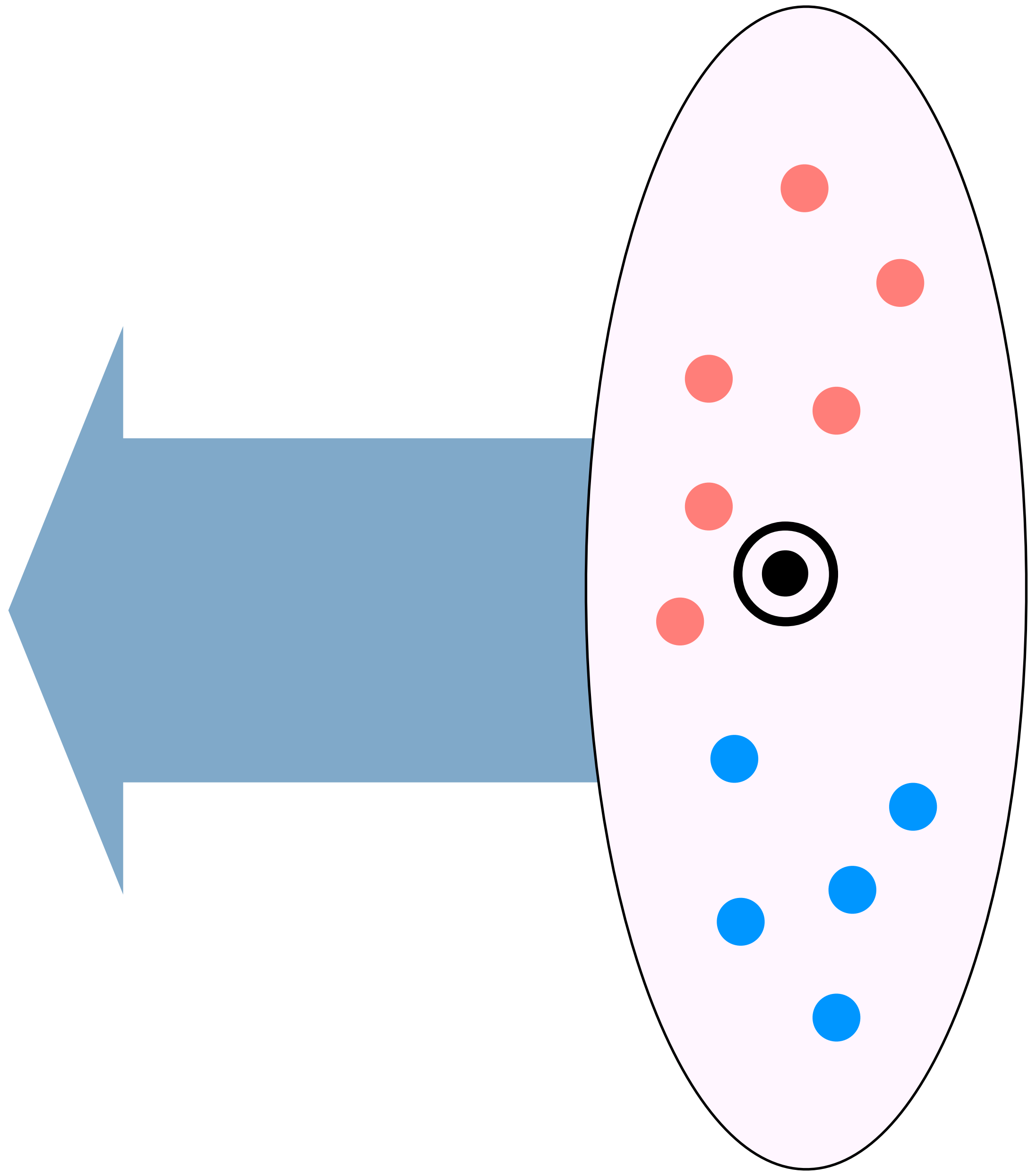
unpolarized quark in unpolarized nucleon



*Burkardt, PRD **66** (2002) 114005*

Model relation TMD ⟷ GPD

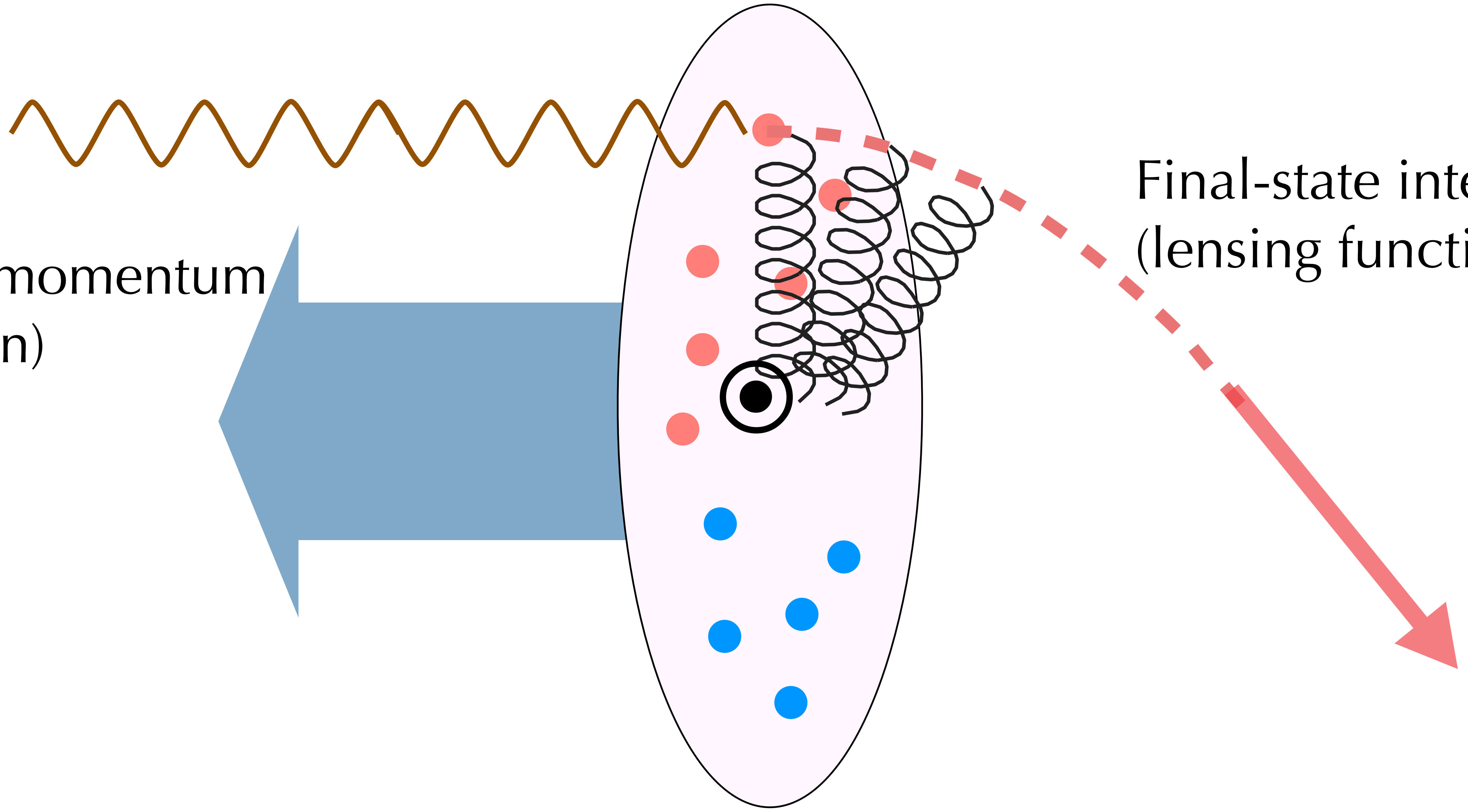
unpolarized quark in transversely pol. nucleon



Distortion in impact parameter
(related to GPD E)

Burkardt, PRD ***66*** *(2002) 114005*

Model relation TMD ⟷ GPD

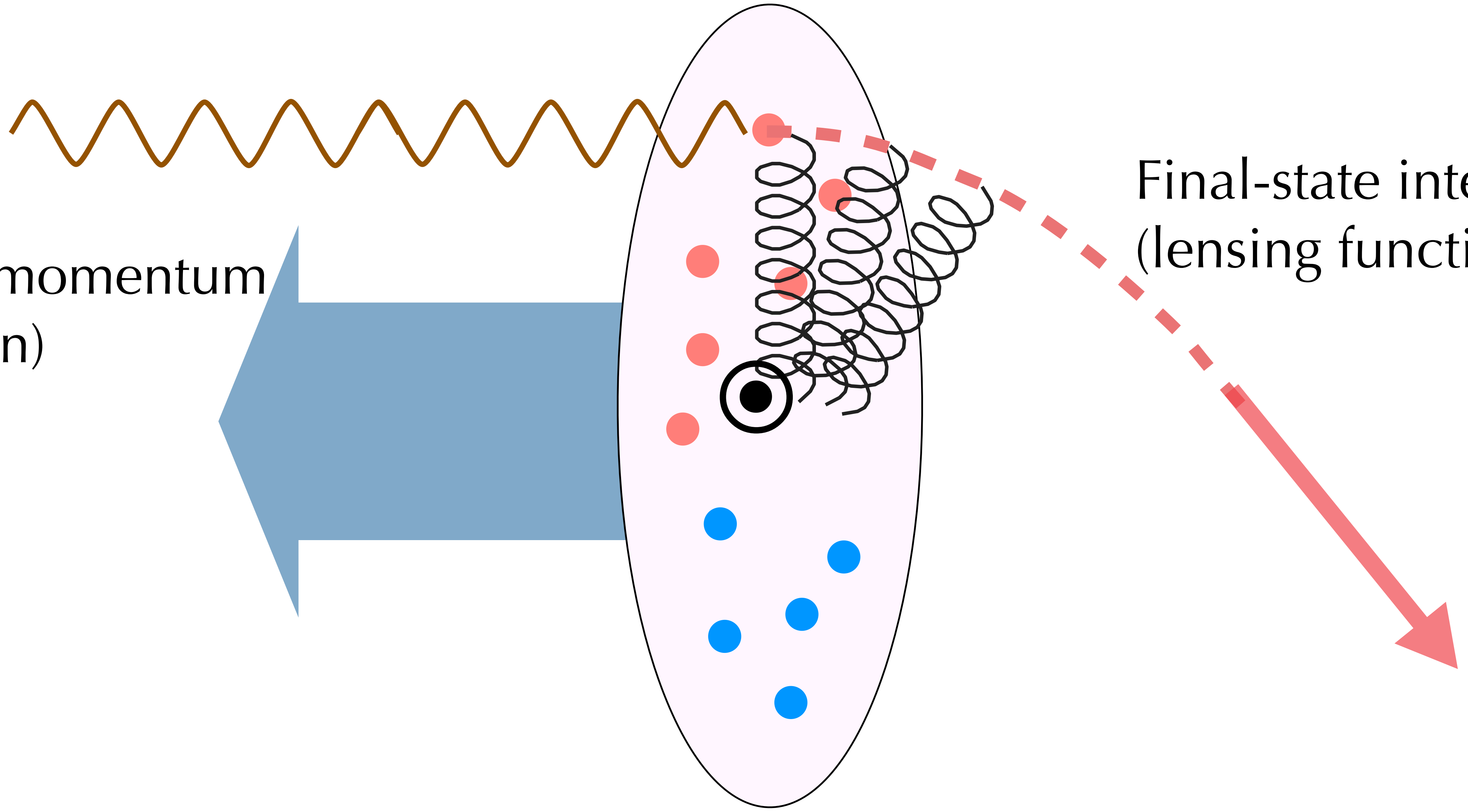


Final-state interaction
(lensing function)

Distortion in transverse momentum
(related to Sivers function)

*Burkardt, PRD **66** (2002) 114005*

Model relation TMD ⟷ GPD



Distortion in transverse momentum
(related to Sivers function)

Final-state interaction
(lensing function)

*Burkardt, PRD **66** (2002) 114005*

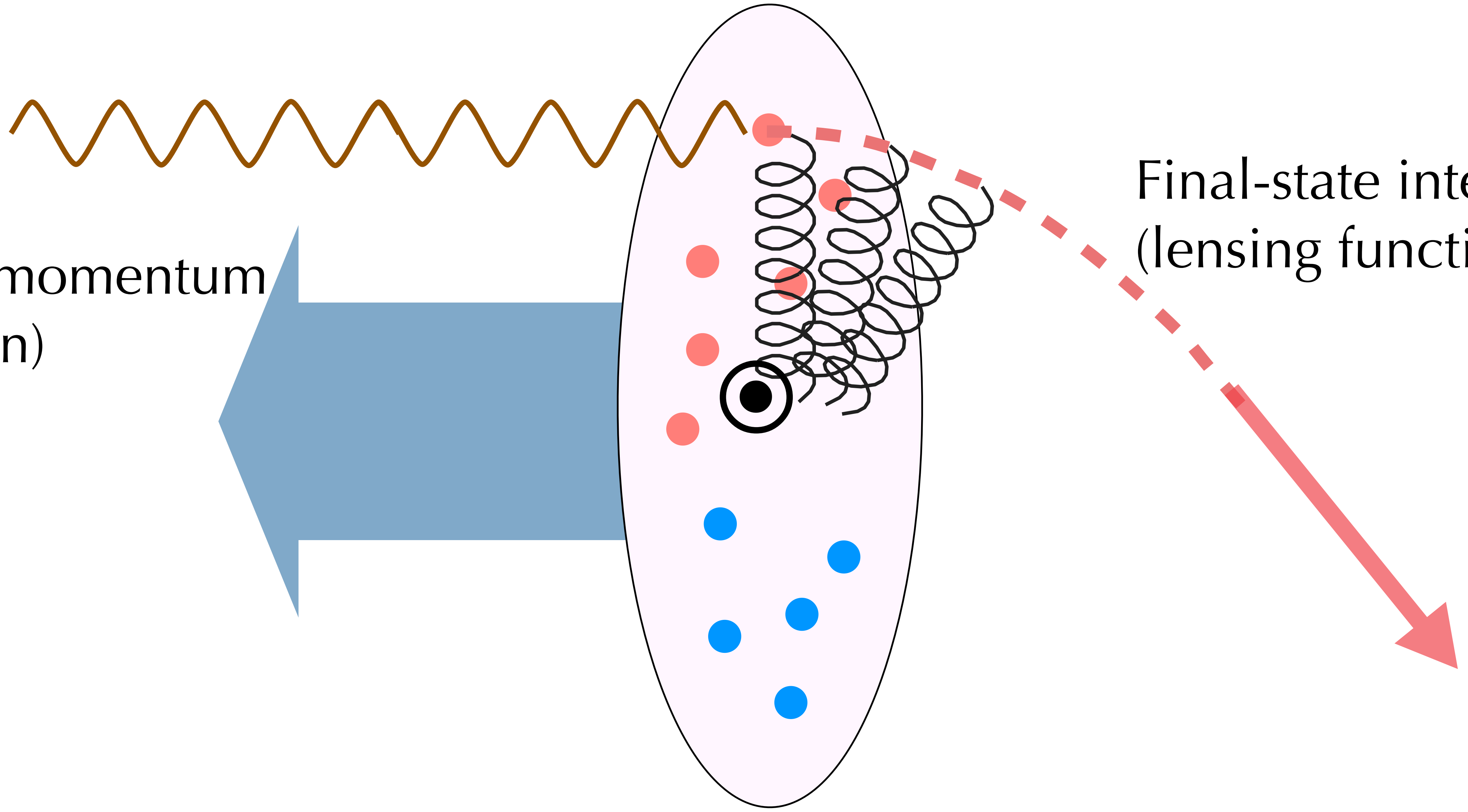
$$-\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^2) \simeq \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x,\vec{b}_T^2)\right)'$$

Sivers function

Lensing function

F.T. of E(x,0,t)

Model relation TMD ⟷ GPD



Distortion in transverse momentum
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*Burkardt, PRD **66** (2002) 114005*

$$-\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^2) \simeq \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x,\vec{b}_T^2)\right)'$$

Sivers function

Lensing function

F.T. of E(x,0,t)

inspired from model results

*Bacchetta, Radici, PRL **107** (2011)*

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\, E^a(x,0,0;Q_L^2)$$

fitted to SIDIS data
(COMPASS,HERMES,JLab)

flavor independent

first moment constrained
from anomalous magnetic moment

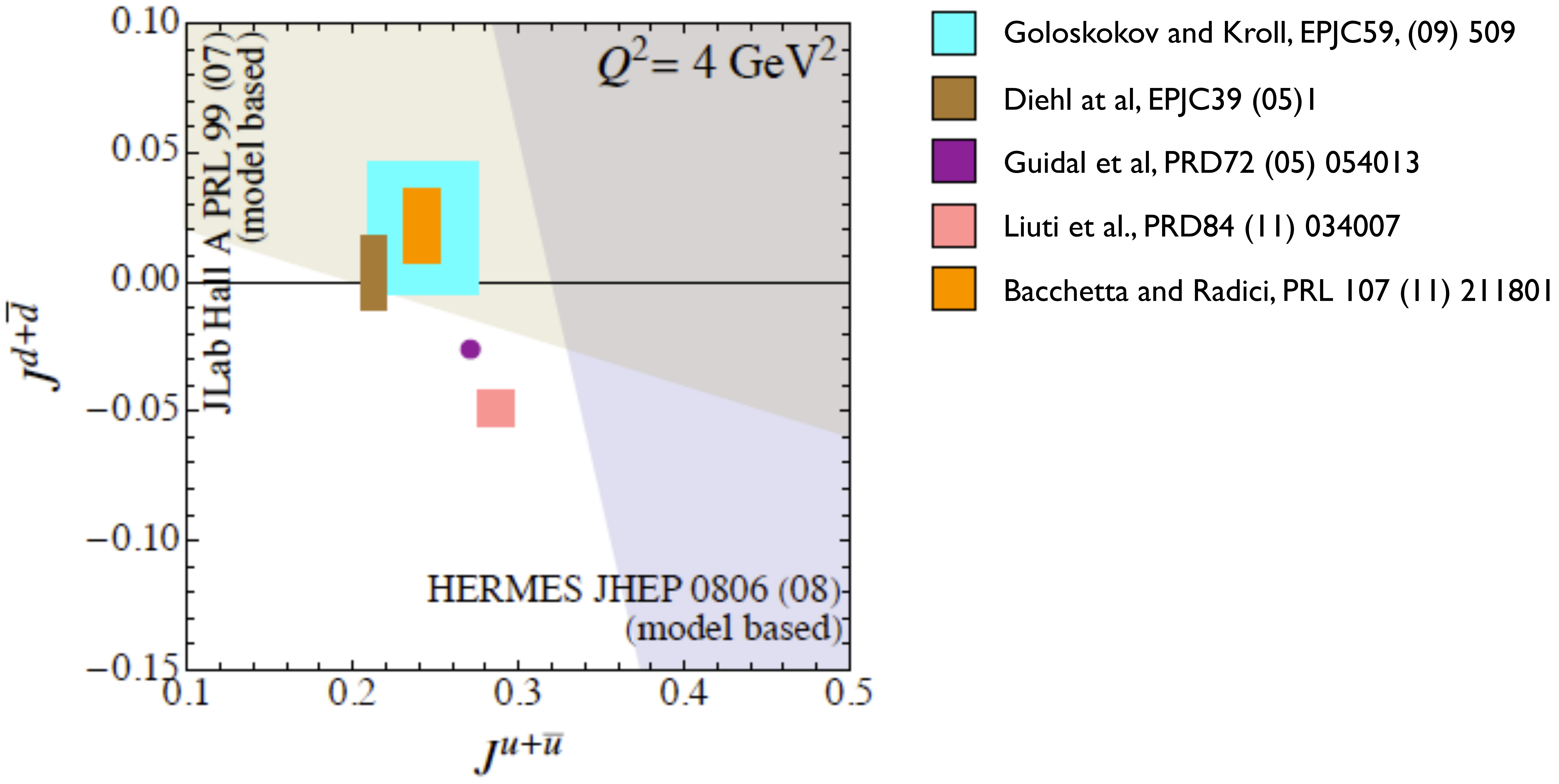
◉ Results from Sivers ⇦ lensing ⇨ GPD $J^q = \frac{1}{2}\int \mathrm{d}x\, x\,[H^q(x,0,0) + E^q(x,0,0)]$

Bacchetta, Radici, PRL107(2011)

$$
\begin{aligned}
J^u &= 0.229 \pm 0.002^{+0.008}_{-0.012}, & J^{\bar{u}} &= 0.015 \pm 0.003^{+0.001}_{-0.000},\\
J^d &= -0.007 \pm 0.003^{+0.020}_{-0.005}, & J^{\bar{d}} &= 0.022 \pm 0.005^{+0.001}_{-0.000},\\
J^s &= 0.006^{+0.002}_{-0.006}, & J^{\bar{s}} &= 0.006^{+0.000}_{-0.005}.
\end{aligned}
$$

$(Q^2 = 4 \text{ GeV}^2)$

◉ Comparing with GPD models and parametrizations

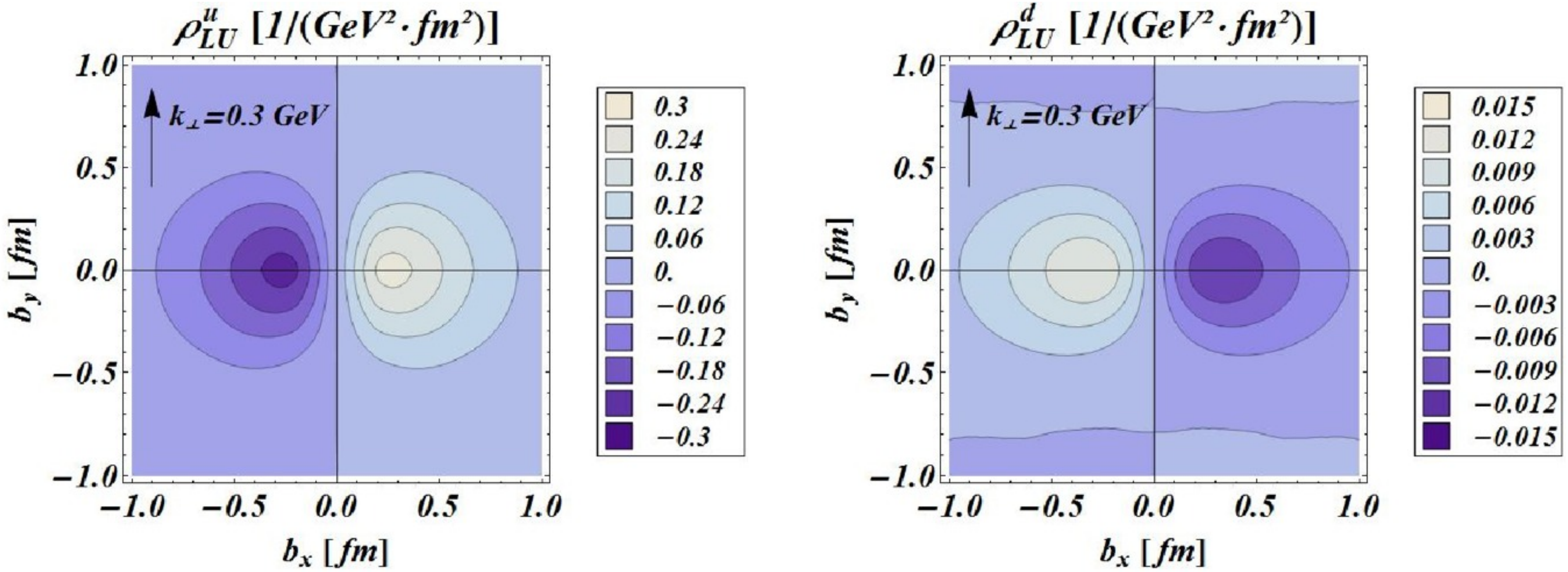


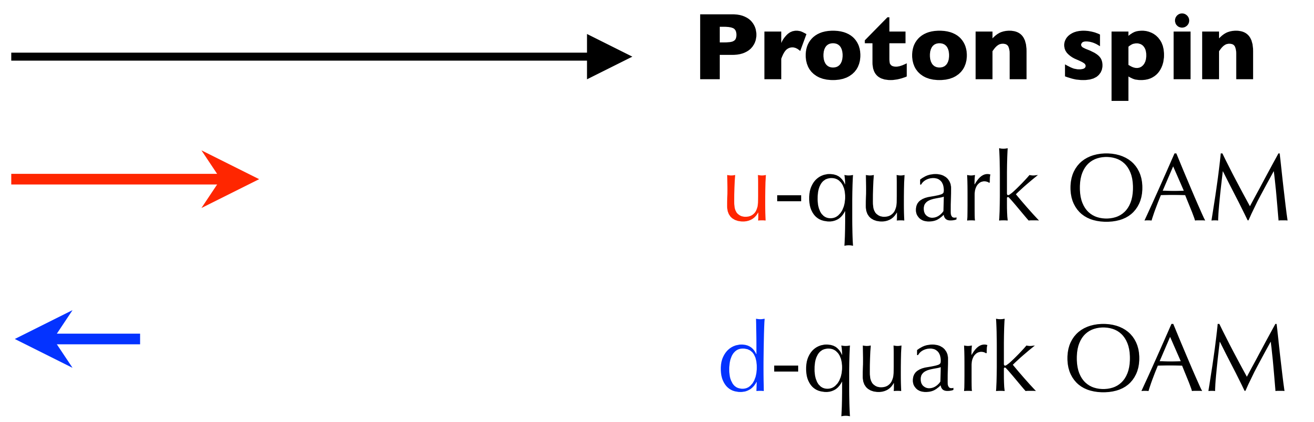
Unpol. quark in Long. pol. Proton

fixed $\vec{k}_\perp \uparrow$
integrated over x

up quark

down quark





Proton spin

u-quark OAM

d-quark OAM

★ projection to GPD and TMD is vanishing

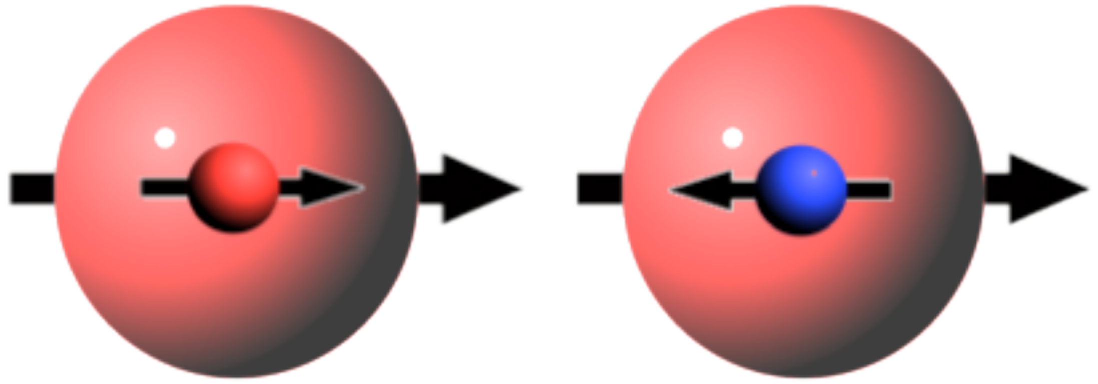
unique information on OAM from Wigner distributions

Lorcé, Pasquini (2011)

Longitudinal

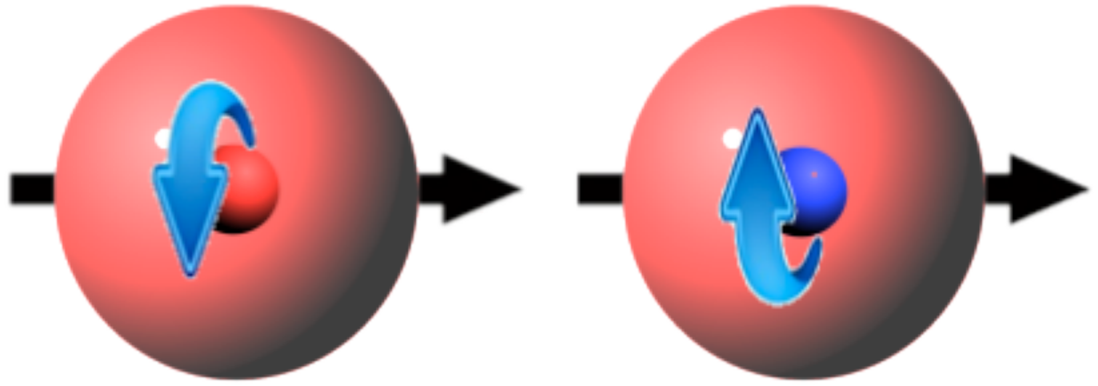
$\boxed{\vec{k}_\perp}$ $\boxed{\vec{b}_\perp}$

$g_{1L}^q \leftrightarrow \tilde{\mathcal{H}}^q$



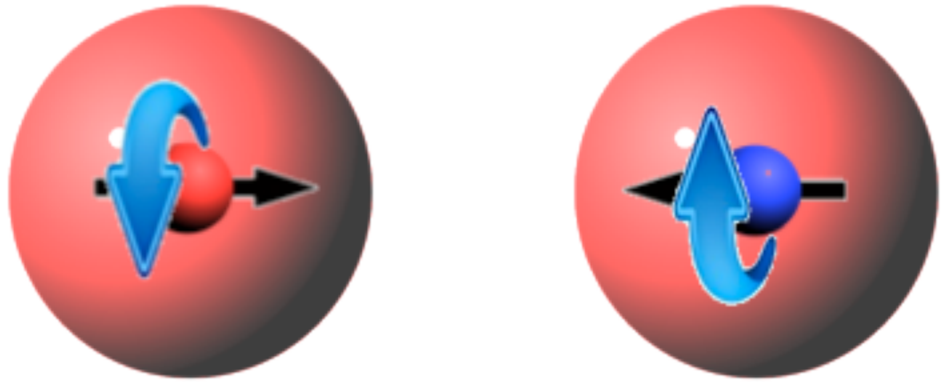
$\boxed{\vec{b}_\perp, \vec{k}_\perp}$

$\ell_z^q \leftrightarrow \tilde{\mathcal{F}}_{14}^q$



$\boxed{\vec{b}_\perp, \vec{k}_\perp}$

$C_z^q \leftrightarrow \tilde{\mathcal{G}}_{11}^q$

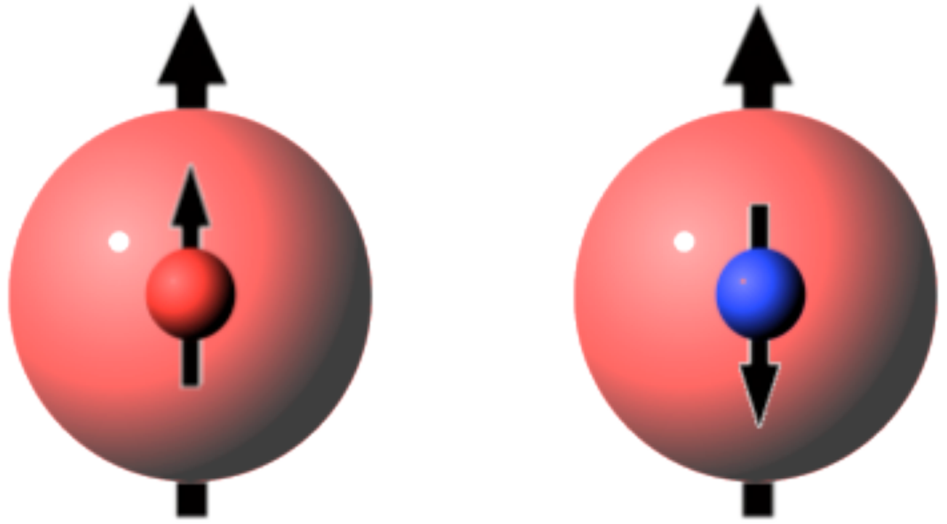


[Lorce', Pasquini (2011)
Meissner, Metz, Schlegel (2009)]

Transverse

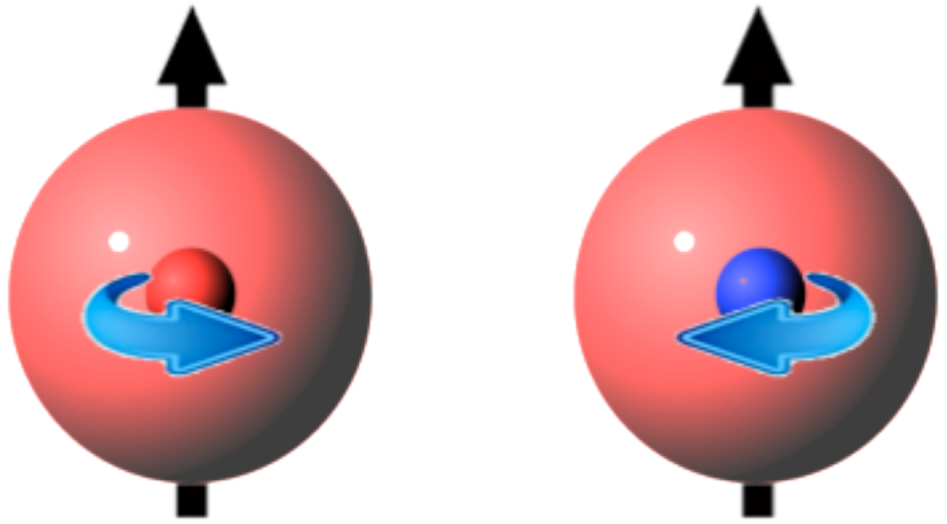
$\boxed{\vec{k}_\perp}$ $\boxed{\vec{b}_\perp}$

$h_1^q \leftrightarrow \mathcal{H}_T^q$



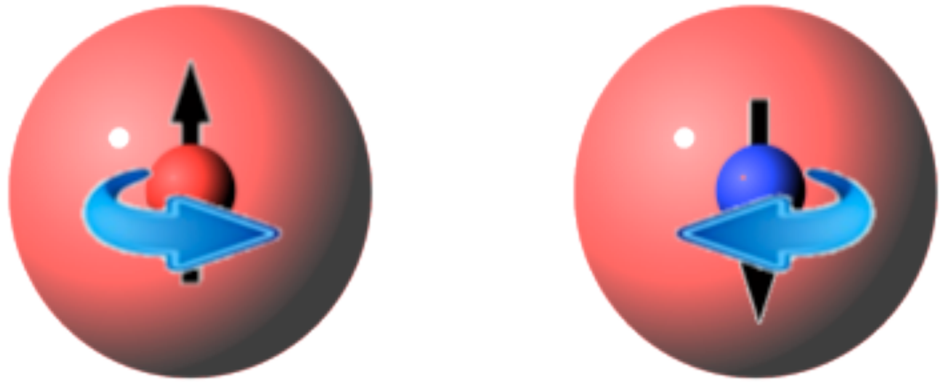
$\boxed{\vec{k}_\perp}$ $\boxed{\vec{b}_\perp}$

$f_{1T}^{\perp q} \leftrightarrow \mathcal{E}^q$



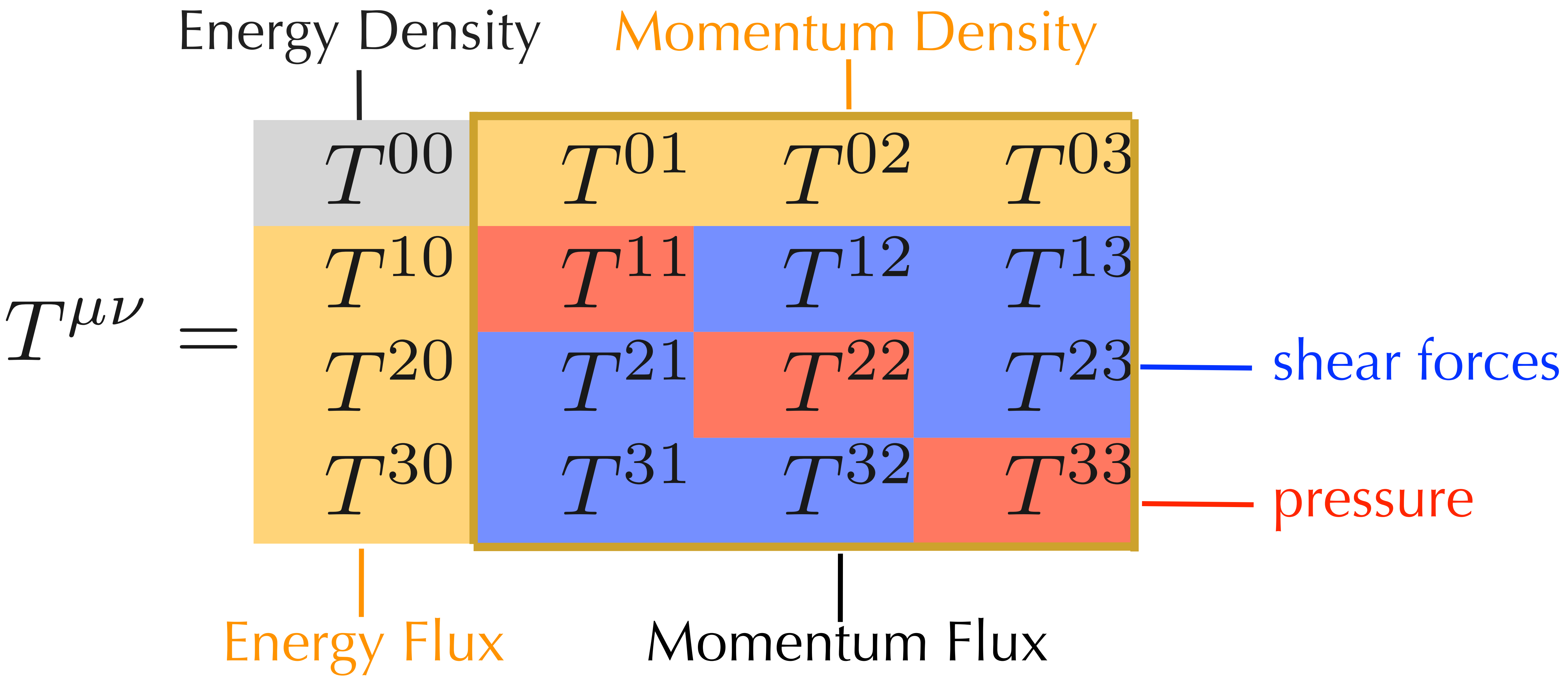
$\boxed{\vec{k}_\perp}$ $\boxed{\vec{b}_\perp}$

$h_1^{\perp q} \leftrightarrow \mathcal{E}_T^q$



[Burkardt (2005)]
[Barone et al. (2008)]

Form factors of Energy Momentum tensor



Energy Density

Momentum Density

$$T^{\mu\nu} = \begin{array}{cccc} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{array}$$

shear forces

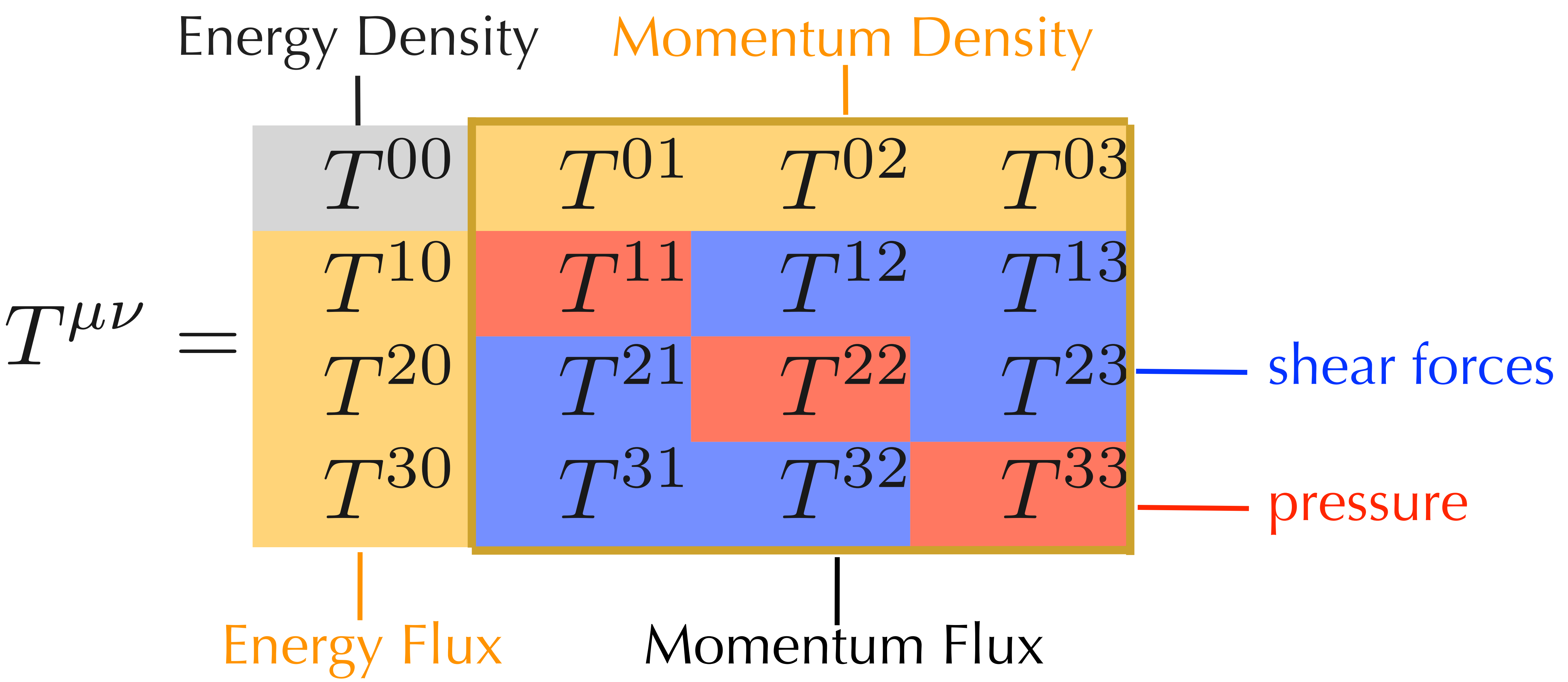
pressure

Energy Flux

Momentum Flux

$$\langle P'|T^{Q,G}_{\mu\nu}|P\rangle = \bar{u}(P')[M_2^{Q,G}(t)\,\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}]u(P)$$

Form factors of Energy Momentum tensor



$$T^{\mu\nu} = \begin{array}{cccc} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{array}$$

$$\langle P'|T^{Q,G}_{\mu\nu}|P\rangle = \bar{u}(P')[M_2^{Q,G}(t)\,\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}]u(P)$$

Relation with second-moments of GPDs:

$$\sum_q \int \mathrm{d}x\, x\, H^q(x,\xi,t) = M_2^Q(t) + \frac{4}{5}\, d_1^Q(t)\xi^2$$

$$\sum_q \int \mathrm{d}x\, x\, E^q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5}\, d_1^Q(t)\xi^2$$

"Charges" of the EM Tensor Form Factors at t=0

$M_2(0)$ nucleon momentum carried by parton

$J(0)$ angular momentum of partons

$d_1(0)$ D-term related to "stability" of the nucleon

Fourier transform in coordinate space

$$T^Q_{ij}(\vec{r}) = s(\vec{r})\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(\vec{r})\,\delta_{ij}$$

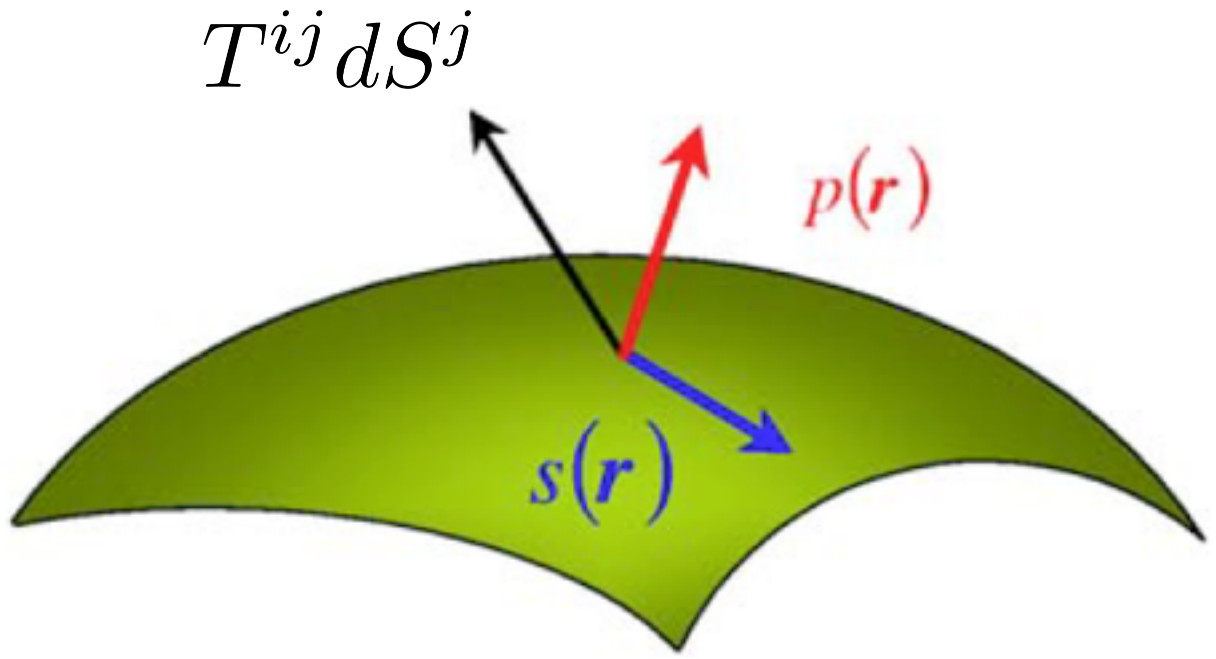
↓ shear forces

↓ pressure

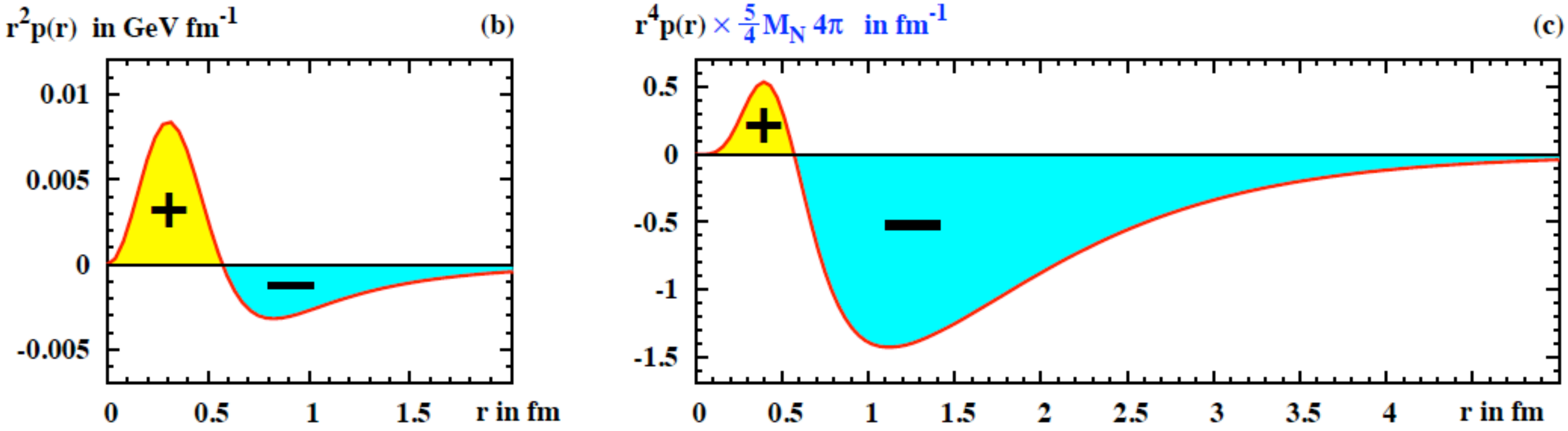
↓

$$\boxed{d_1^Q(0) = 5\pi M_N \int_0^\infty \mathrm{d}r\, r^4\, p(r)}$$

“mechanical properties” of nucleon



*M. Polyakov, PLB **555** (2003) 57*



$$\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0$$

$$\int_0^\infty \mathrm{d}r\, r^4\, p(r) < 0$$

*Schweitzer et al., PRD **75** (2007) 094021*

Quark spin and OAM

GTMDs

Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, G_{14}^q(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

polarized PDF
inclusive DIS

$$\ell_z^q = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{\vec{k}_\perp^2}{M^2}\,F_{14}^q(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

[Lorce, BP(2011)]
[Hatta (2011)]
[Lorce',BP, et al. (2012)]

$$\ell_{iz}^{\text{int}} = \vec{b}_{i\perp} \times \vec{k}_{i\perp}$$

TMDs

Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, g_{1L}^q(x,\vec{k}_\perp)$$

polarized PDF
inclusive DIS

$$\mathcal{L}_z^q(x,\vec{k}_\perp) = -\frac{\vec{k}_\perp^2}{2M^2}\,h_{1T}^{\perp q}(x,\vec{k}_\perp^2)$$

[Burkardt (2007)]
[Efremov et al. (2008,2010)]
[She, Zhu, Ma (2009)]
[Avakian et al. (2010)]
[Lorce', BP (2011)]

- **Model-dependent**
- **Not intrinsic!**

⚠ $$\mathcal{L}_{iz} = \vec{r}_{i\perp} \times \vec{k}_{i\perp}$$

GPDs

Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\tilde{H}^q(x,0,0)$$

polarized PDF
inclusive DIS

Ji sum rule

$$J^q = \tfrac{1}{2}\int \mathrm{d}x\,x\,[H^q(x,0,0) + E^q(x,0,0)]$$

$$L^q = J^q - S_z^q$$

[Ji (1997)]

Twist-3

$$L_z^q = -\int \mathrm{d}x\,x\,G_2^q(x,0,0)$$

Pure twist-3!

[Penttinen et al. (2000)]

OAM and origin dependence

OAM from Pretzelosity

$$\mathcal{L}_{iz} = \vec{r}_{i\perp} \times \vec{k}_{i\perp}$$

"naive" OAM

⚠ **Depends on proton position**

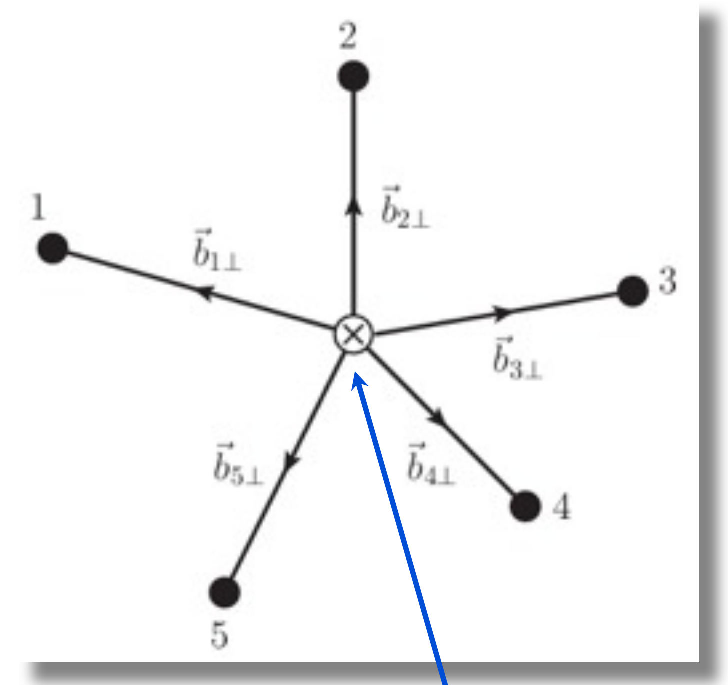
Momentum conservation

$$\boxed{\sum_{i=1}^{N} \vec{k}_{i\perp} = \vec{0}_\perp}$$

OAM from GTMDs

$$\ell_{iz}^{\text{int}} = \vec{b}_{i\perp} \times \vec{k}_{i\perp}$$

intrinsic OAM



Transverse center of momentum

$$\vec{R}_\perp = \sum_{i=1}^{N} x_i \, \vec{r}_{i\perp}$$

equivalence for TOTAL OAM

Model	LCCQM			χQSM		
q	u	d	Total	u	d	Total
ℓ_z^q	0.131	−0.005	0.126	0.073	−0.004	0.069
L_z^q	0.071	0.055	0.126	−0.008	0.077	0.069
$\mathcal{L}_z^q$	0.169	−0.042	0.126	0.093	−0.023	0.069

$$\mathcal{L}_{iz} \neq \ell_{iz}^{\text{int}} \xrightarrow{\text{momentum conservation}} \mathcal{L}_z = l_z^{\text{int}}$$

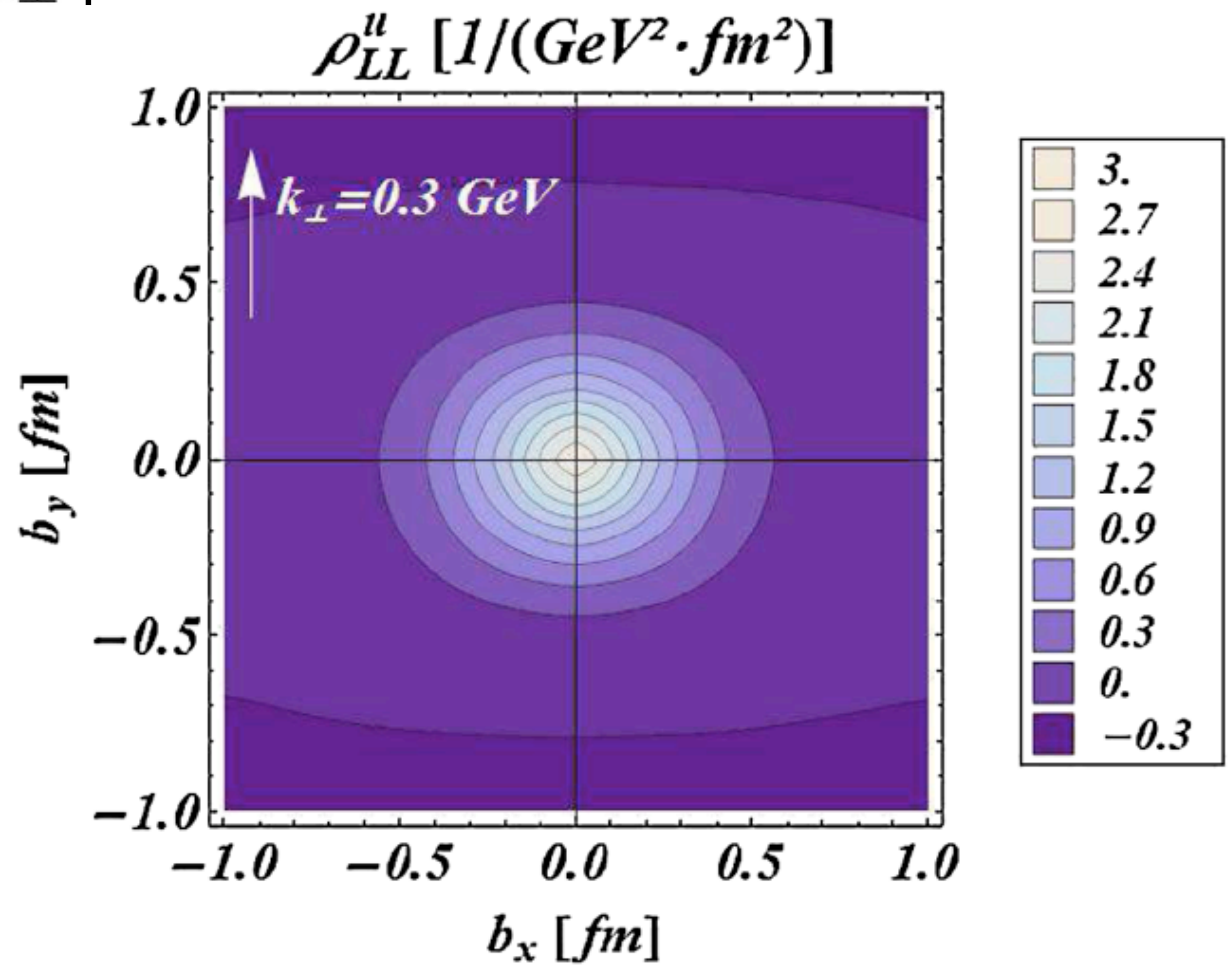
$$\sum_{i=1}^{N} \vec{b}_{i\perp} \times \vec{k}_{i\perp} = \sum_{i=1}^{N} \left(\vec{r}_{i\perp} - \vec{R}_\perp \right) \times \vec{k}_{i\perp} = \sum_{i=1}^{N} \vec{r}_{i\perp} \times \vec{k}_{i\perp}$$

Intrinsic **Naive**

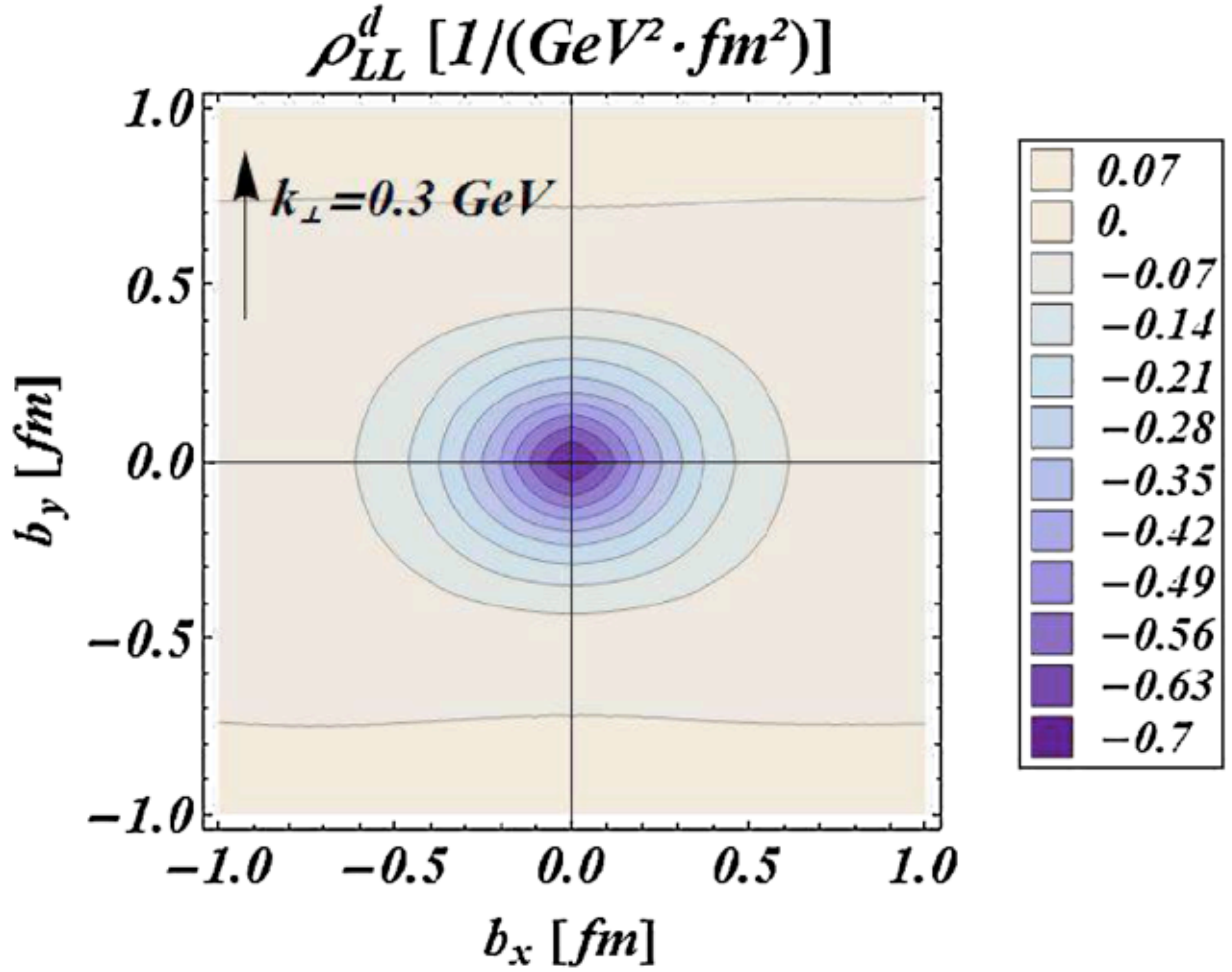
Long. pol. quark in Long. pol. Proton

fixed $\vec{k}_\perp$ ↑

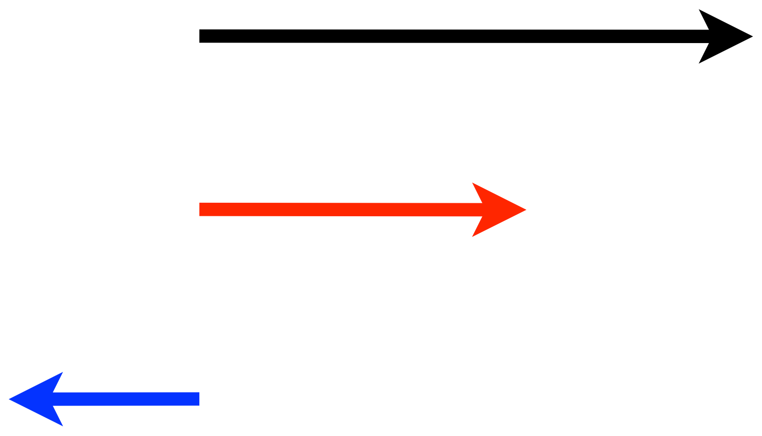
up quark



down quark



Quark spin - Nucleon spin correlation



Proton spin

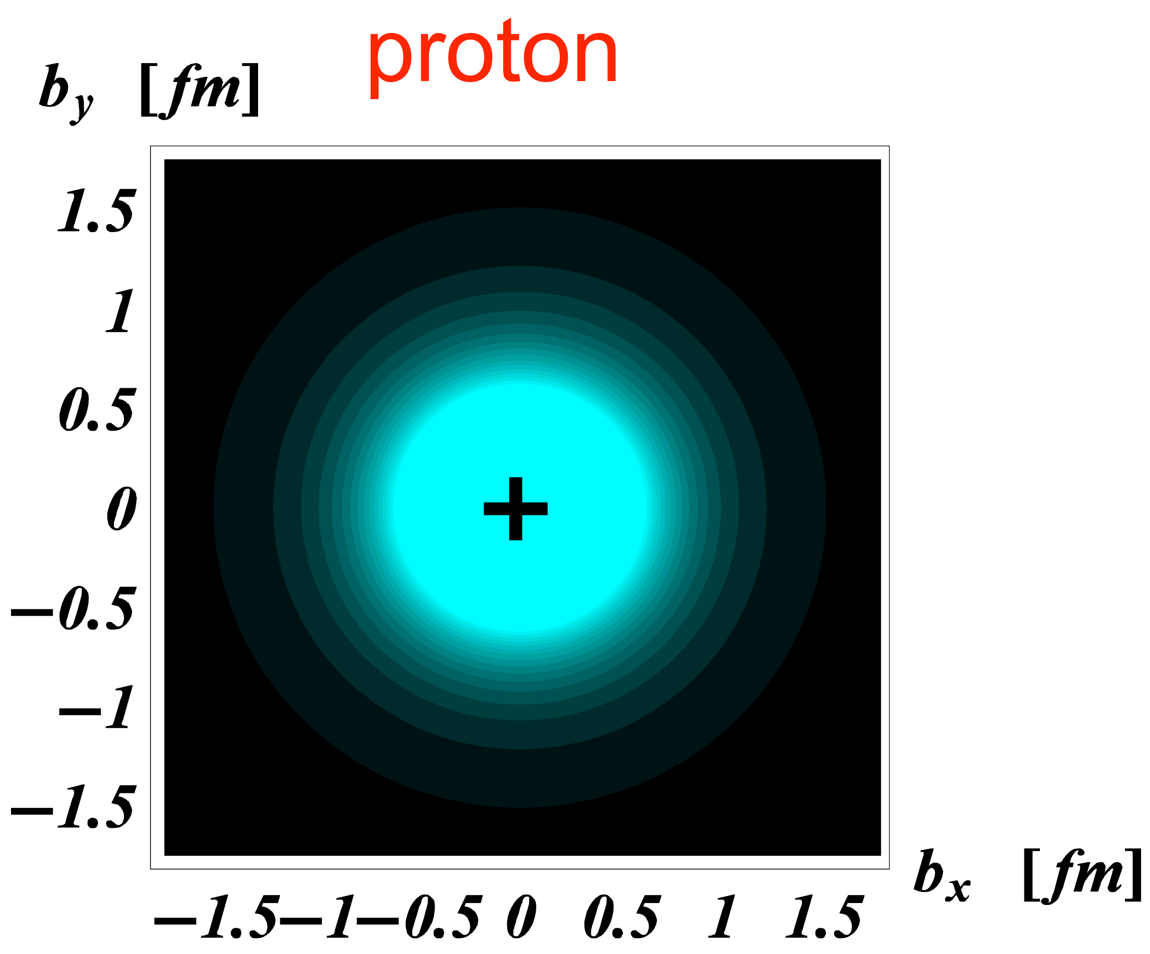
u-quark spin

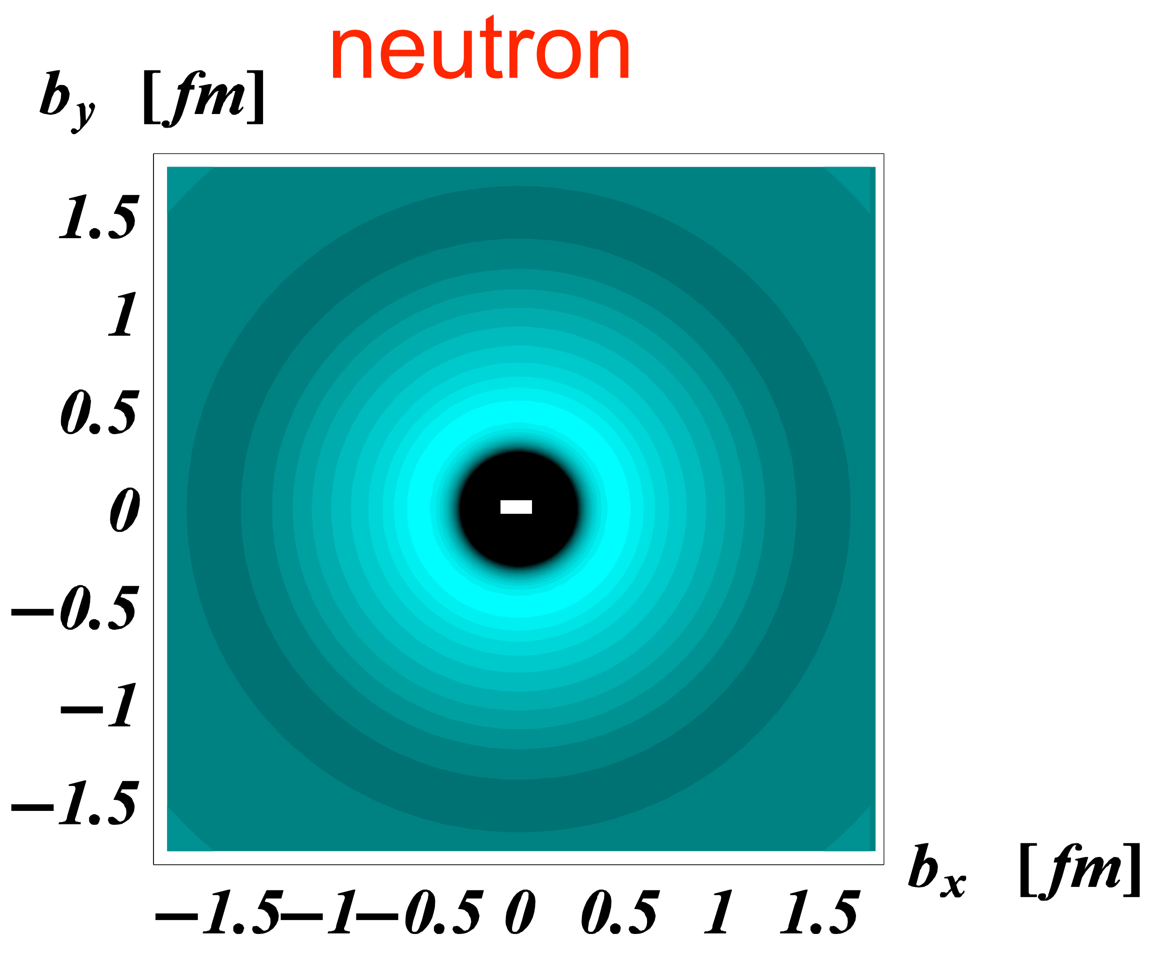
d-quark spin

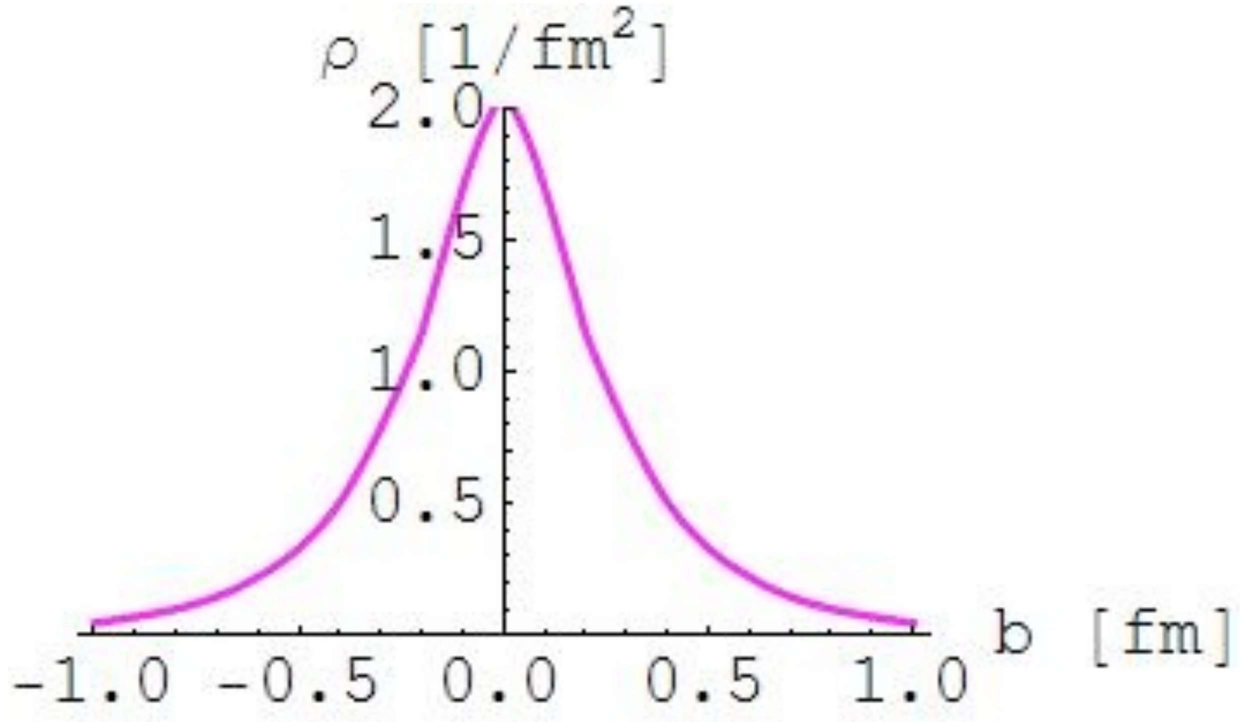
Charge density of partons in the transverse plane

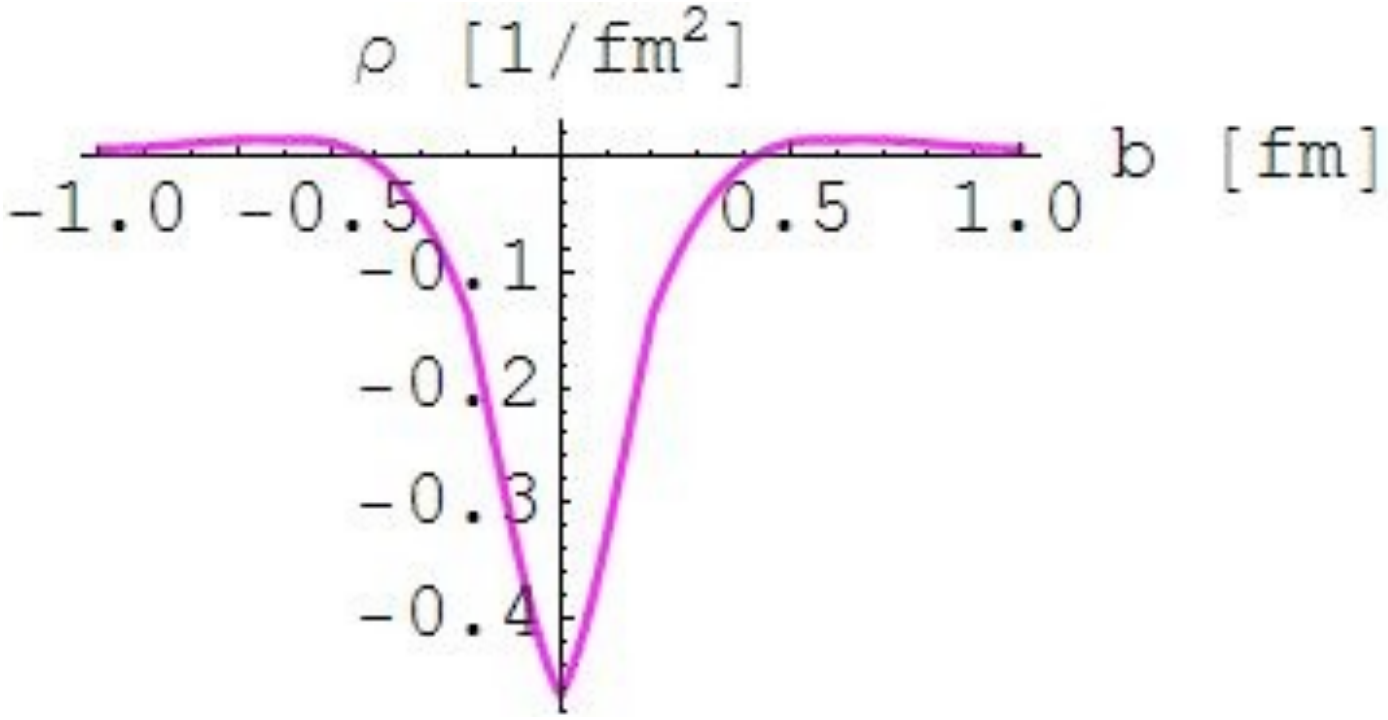
$$\rho^q(b_\perp^2) = e^q \int \mathrm{d}^2\Delta_\perp e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} F_1^q(\Delta_\perp^2)$$

charge distribution in the transverse plane









[Miller (2007); Burkardt (2007)]

SIDIS   l N → l' h X

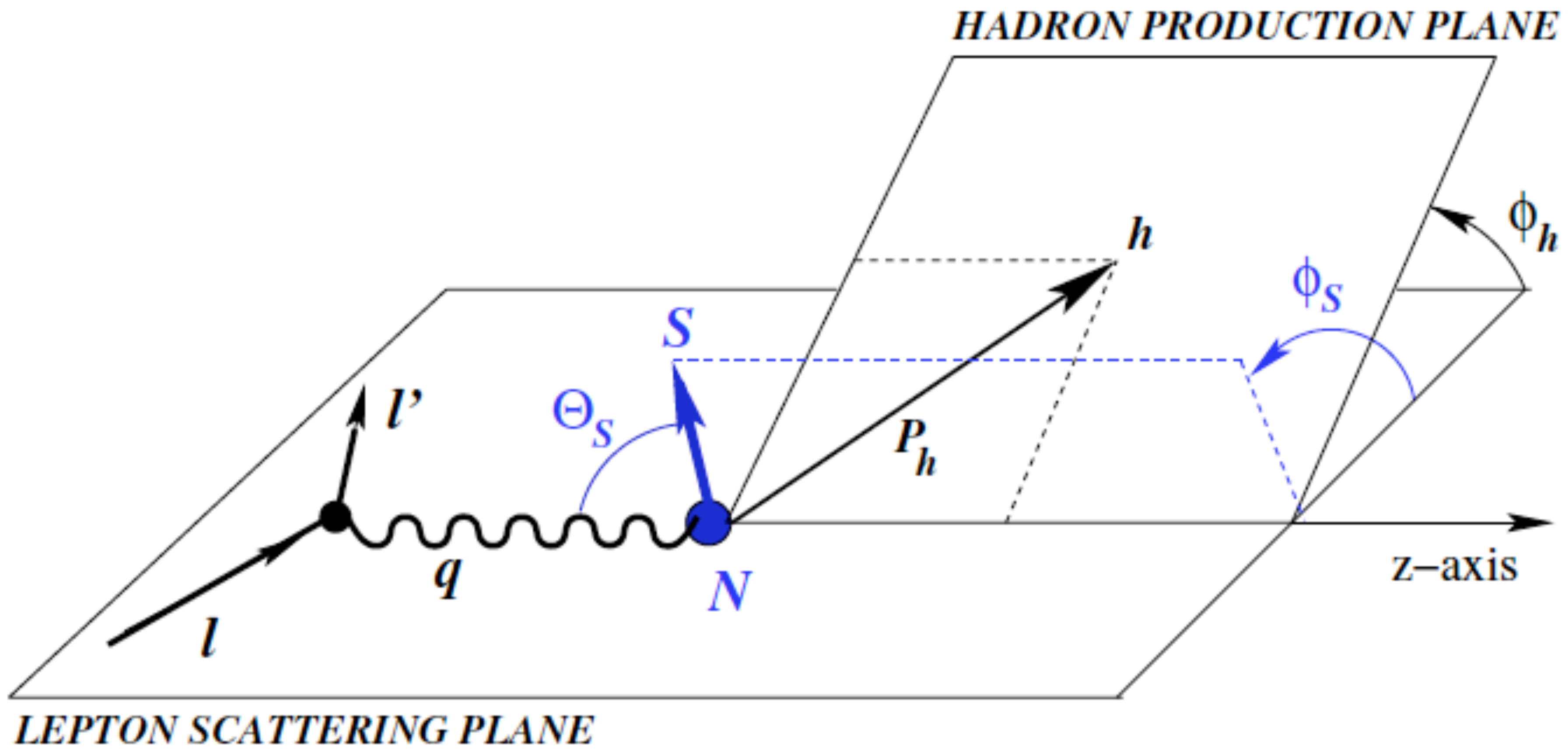
$$\frac{d^4\sigma}{dx\,dy\,dz\,d\phi_h} = \frac{d^4\sigma_0}{dx\,dy\,dz\,d\phi_h}\Big\{1 + \cos(2\phi_h)\,p_1(y)\,A_{UU}^{\cos(2\phi_h)} + S_L\sin(2\phi_h)\,p_1(y)\,A_{UL}^{\sin(2\phi_h)}$$

$$+\lambda\,S_L\,p_2(y)\,A_{LL} + \lambda\,S_T\cos(\phi_h-\phi_S)\,p_2(y)\,A_{LT}^{\cos(\phi_h-\phi_S)} + S_T\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)}$$

$$+S_T\sin(\phi_h+\phi_S)\,p_1(y)\,A_{UT}^{\sin(\phi_h+\phi_S)} + S_T\sin(3\phi_h-\phi_S)\,p_1(y)\,A_{UT}^{\sin(3\phi_h-\phi_S)}\Big\} + \ldots$$

$$A_{XY}^{\text{weight}} = \frac{F_{XY}^{\text{weight}}}{F_{UU}}$$

X=beam polarization
Y=target polarization
weight=ang. distr. hadron



$$F_{UU} \propto \sum_a e_a^2\, f_1^a \otimes D_1^a$$

$$F_{LL} \propto \sum_a e_a^2\, g_1^a \otimes D_1^a$$

$$F_{LT}^{\cos(\phi_h-\phi_S)} \propto \sum_a e_a^2\, g_{1T}^{\perp a} \otimes D_1^a$$

$$F_{UT}^{\sin(\phi_h-\phi_S)} \propto \sum_a e_a^2\, f_{1T}^{\perp a} \otimes D_1^a$$

$$F_{UU}^{\cos(2\phi_h)} \propto \sum_a e_a^2\, h_1^{\perp a} \otimes H_1^{\perp a}$$

$$F_{UL}^{\sin(2\phi_h)} \propto \sum_a e_a^2\, h_{1L}^{\perp a} \otimes H_1^{\perp a}$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} \propto \sum_a e_a^2\, h_1^a \otimes H_1^{\perp a}$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} \propto \sum_a e_a^2\, h_{1T}^{\perp a} \otimes H_1^{\perp a}$$

SIDIS   l N → l’ h X

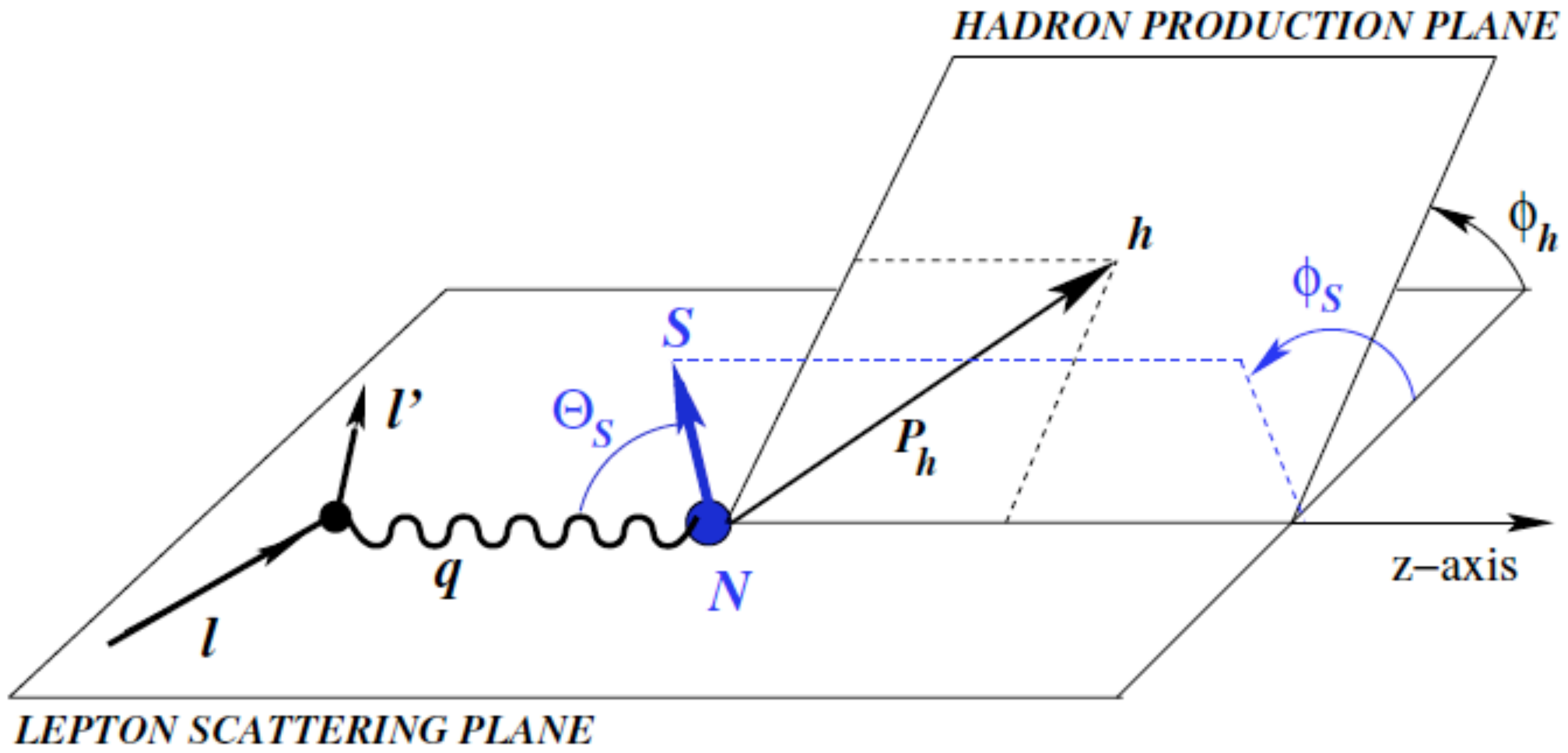
$$\frac{d^4\sigma}{dx\,dy\,dz\,d\phi_h} = \frac{d^4\sigma_0}{dx\,dy\,dz\,d\phi_h}\Big\{1 + \cos(2\phi_h)\,p_1(y)\,A_{UU}^{\cos(2\phi_h)} + S_L\sin(2\phi_h)\,p_1(y)\,A_{UL}^{\sin(2\phi_h)}$$

$$+\lambda\,S_L\,p_2(y)\,A_{LL} + \lambda\,S_T\cos(\phi_h-\phi_S)\,p_2(y)\,A_{LT}^{\cos(\phi_h-\phi_S)} + S_T\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)}$$

$$+S_T\sin(\phi_h+\phi_S)\,p_1(y)\,A_{UT}^{\sin(\phi_h+\phi_S)} + S_T\sin(3\phi_h-\phi_S)\,p_1(y)\,A_{UT}^{\sin(3\phi_h-\phi_S)}\Big\} + \ldots$$

$$A_{XY}^{\text{weight}} = \frac{F_{XY}^{\text{weight}}}{F_{UU}}$$

X=beam polarization
Y=target polarization
weight=ang. distr. hadron



$$F_{UU} \propto \sum_a e_a^2\, f_1^a \otimes D_1^a$$

$$\boxed{F_{LL}} \propto \sum_a e_a^2\, g_1^a \otimes D_1^a$$

$$\boxed{F_{LT}^{\cos(\phi_h-\phi_S)}} \propto \sum_a e_a^2\, g_{1T}^{\perp a} \otimes D_1^a$$

$$F_{UT}^{\sin(\phi_h-\phi_S)} \propto \sum_a e_a^2\, f_{1T}^{\perp a} \otimes D_1^a$$

$$F_{UU}^{\cos(2\phi_h)} \propto \sum_a e_a^2\, h_1^{\perp a} \otimes H_1^{\perp a}$$

$$\boxed{F_{UL}^{\sin(2\phi_h)}} \propto \sum_a e_a^2\, h_{1L}^{\perp a} \otimes H_1^{\perp a}$$

$$\boxed{F_{UT}^{\sin(\phi_h+\phi_S)}} \propto \sum_a e_a^2\, h_1^a \otimes H_1^{\perp a}$$

$$\boxed{F_{UT}^{\sin(3\phi_h-\phi_S)}} \propto \sum_a e_a^2\, h_{1T}^{\perp a} \otimes H_1^{\perp a}$$

Collins SSA

gaussian ansatz $\Longrightarrow$ $$A_{UT}^{\sin(\phi_h+\phi_S)}(x) = \frac{\sum_a e_a^2\, x\, h_1^a(x)\, \langle B_1\, H_1^{\perp(1/2)a} \rangle}{\sum_a e_a^2\, x\, f_1^a(x)\, \langle D_1^a \rangle}$$

- $h_1(x)$ from Light-Cone CQM evolved at Q^2=2.5 GeV^2, $f_1(x)$ from GRV at Q^2=2.5 GeV^2
- $H_1^{\perp\,(1/2)}$ from HERMES & BELLE data

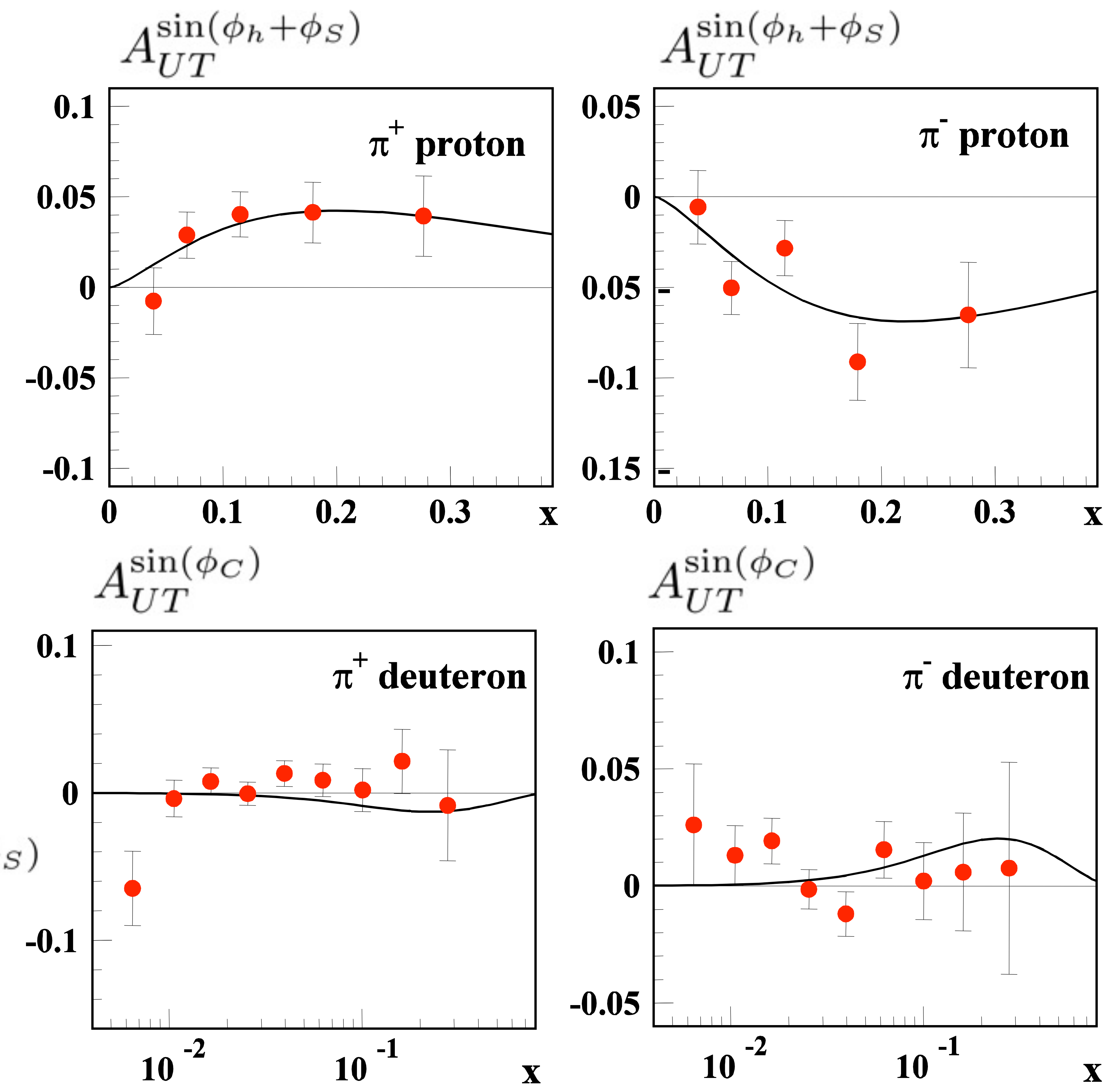
Efremov, Goeke, Schweitzer, PRD73 (2006);
Anselmino et al., PRD75 (2007); Vogelsang, Yuan, PRD72 (2005)

● HERMES data:
Diefenthaler, hep-ex/0507013

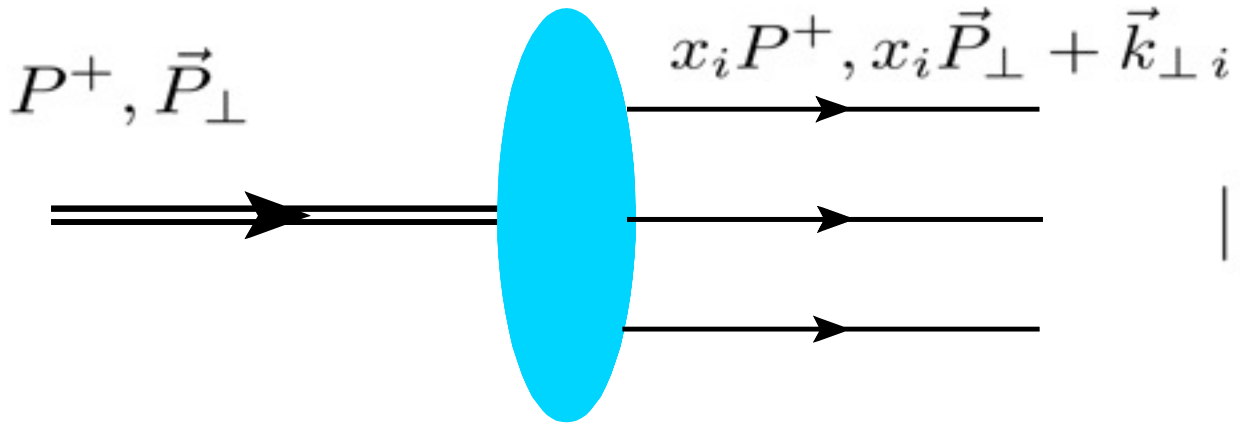
More recent HERMES and BELLE data not included in the fit of Collins function

● COMPASS data:
Alekseev et al,, PLB673, (2009)

$$\phi_C = \phi_h + \phi_S + \pi$$
$$A_{UT}^{\sin(\phi_C)} = -A_{UT}^{\sin(\phi_h+\phi_S)}$$



Three Quark Light Cone Amplitudes



$$|\,P,\lambda\rangle \;=\; \sum_\beta \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\; \Psi^f_{\lambda,\beta}(x_i,\vec{k}_{\perp,i})\, \frac{\varepsilon^{ijk}}{\sqrt{6}}\, u^\dagger_{i\lambda_1}(1)\, u^\dagger_{j\lambda_2}(2)\, d^\dagger_{k\lambda_3}(3)|\,0\rangle$$

- classification of LCWFs in angular momentum components

[Ji, J.P. Ma, Yuan, 03;
Burkardt, Ji, Yuan, 02]

$$|\,P,\uparrow\rangle \;=\; |\,P,\uparrow\rangle^{L_z=2}_{-\frac{3}{2}} + |\,P,\uparrow\rangle^{L_z=1}_{-\frac{1}{2}} + |\,P,\uparrow\rangle^{L_z=0}_{\frac{1}{2}} + |\,P,\uparrow\rangle^{L_z=-1}_{\frac{3}{2}}$$

total quark helicity J^q

$J_z = J_z^q + L_z^q$

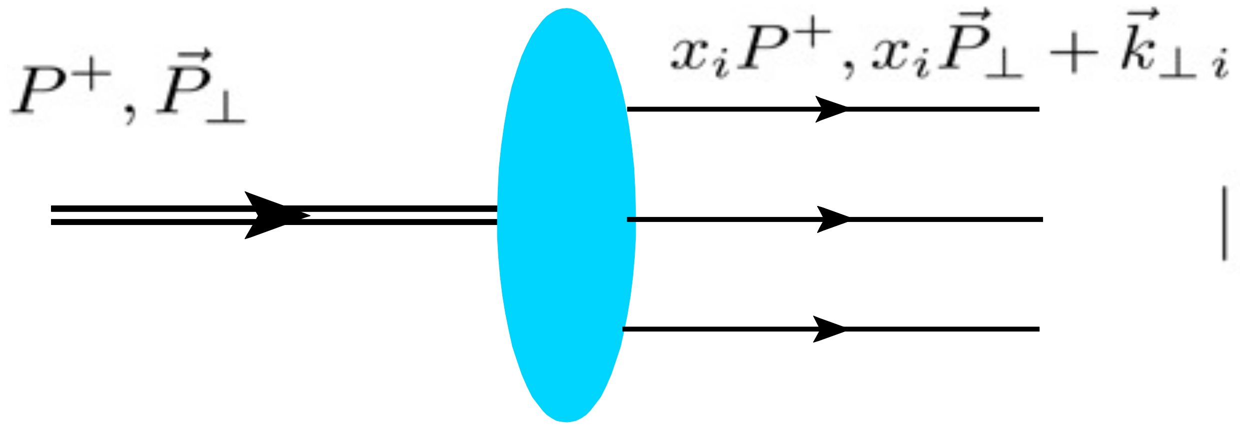
$L_z^q = -1$	$L_z^q = 0$	$L_z^q = 1$	$L_z^q = 2$
$(\uparrow\uparrow\uparrow)_{LC}$	$(\uparrow\uparrow\downarrow)_{LC}$	$(\uparrow\downarrow\downarrow)_{LC}$	$(\downarrow\downarrow\downarrow)_{LC}$

$$\langle 0\,|\epsilon^{ijk}\, u^\dagger_{i\lambda_i}(1)\,\Gamma\, u^\dagger_{j\lambda_j}(2) d^\dagger_{k\lambda_k}(3)|\,P\rangle$$

parity
time reversal
isospin symmetry

6 independent wave function amplitudes: $\psi^{(i)}\; i = 1, .., 6$

Three Quark Light Cone Amplitudes



$$|\, P, \lambda\rangle \;=\; \sum_\beta \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\; \Psi^f_{\lambda,\,\beta}(x_i, \vec{k}_{\perp,i})\; \frac{\varepsilon^{ijk}}{\sqrt{6}}\; u^\dagger_{i\lambda_1}(1) u^\dagger_{j\lambda_2}(2) d^\dagger_{k\lambda_3}(3)|\, 0\rangle$$

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$$\langle 0\, |\epsilon^{ijk}\, u^\dagger_{i\lambda_i}(1)\, \Gamma\, u^\dagger_{j\lambda_j}(2) d^\dagger_{k\lambda_k}(3)|\; P\rangle$$

⇓

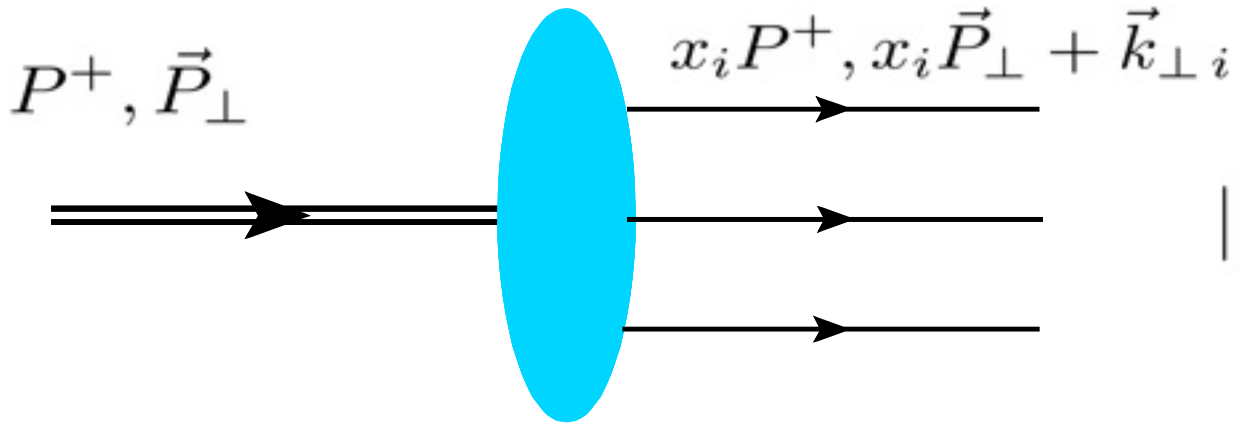
parity
time reversal
isospin symmetry

6 independent wave function amplitudes: $\psi^{(i)}\; i = 1, .., 6$

$L_z^q = 0$

$$|P\uparrow\rangle^{L_z=0}_{\frac{1}{2}} \;=\; \int d[1]d[2]d[3]\left(\psi^{(1)}(1,2,3) + i(k_1^x k_2^y - k_1^y k_2^x)\psi^{(2)}(1,2,3)\right)$$
$$\times \frac{\epsilon^{ijk}}{\sqrt{6}} u^\dagger_{i\uparrow}(1)\left(u^\dagger_{j\downarrow}(2) d^\dagger_{k\uparrow}(3) - d^\dagger_{j\downarrow}(2) u^\dagger_{k\uparrow}(3)\right)|0\rangle$$

Three Quark Light Cone Amplitudes



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parity
time reversal
isospin symmetry

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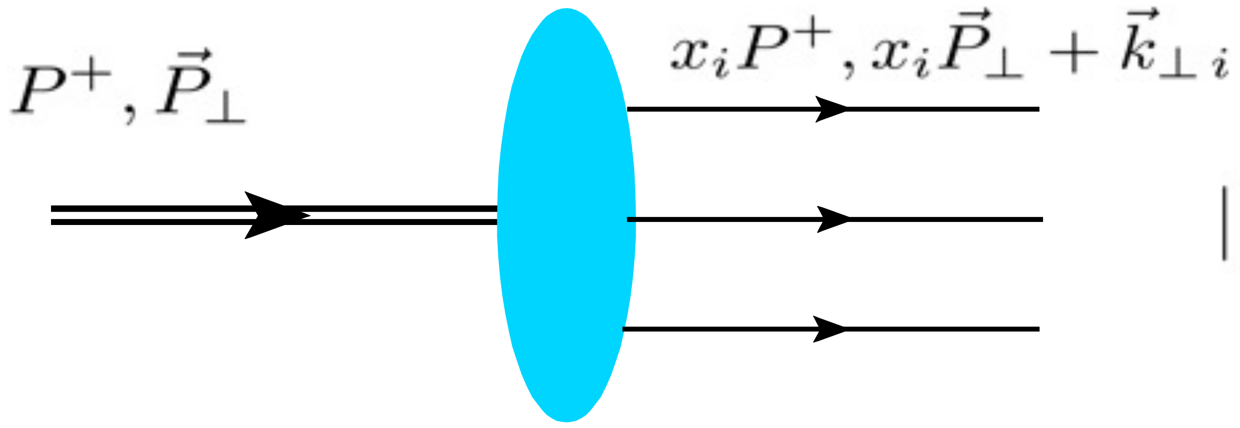
$L_z^q = 1$

$$|P\uparrow\rangle^{L_z=1}_{-\frac{1}{2}} \;=\; \int d[1]d[2]d[3]\left((k_1^x + ik_1^y)\psi^{(3)}(1,2,3) + (k_2^x + ik_2^y)\psi^{(4)}(1,2,3)\right)$$
$$\times \frac{\epsilon^{ijk}}{\sqrt{6}}\left(u^{\dagger}_{i\uparrow}(1)u^{\dagger}_{j\downarrow}(2)d^{\dagger}_{k\downarrow}(3) - d^{\dagger}_{i\uparrow}(1)u^{\dagger}_{j\downarrow}(2)u^{\dagger}_{k\downarrow}(3)\right)|0\rangle$$

Three Quark Light Cone Amplitudes



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$$\langle 0\, |\epsilon^{ijk}\, u^\dagger_{i\lambda_i}(1)\, \Gamma\, u^\dagger_{j\lambda_j}(2) d^\dagger_{k\lambda_k}(3)|\; P\rangle$$

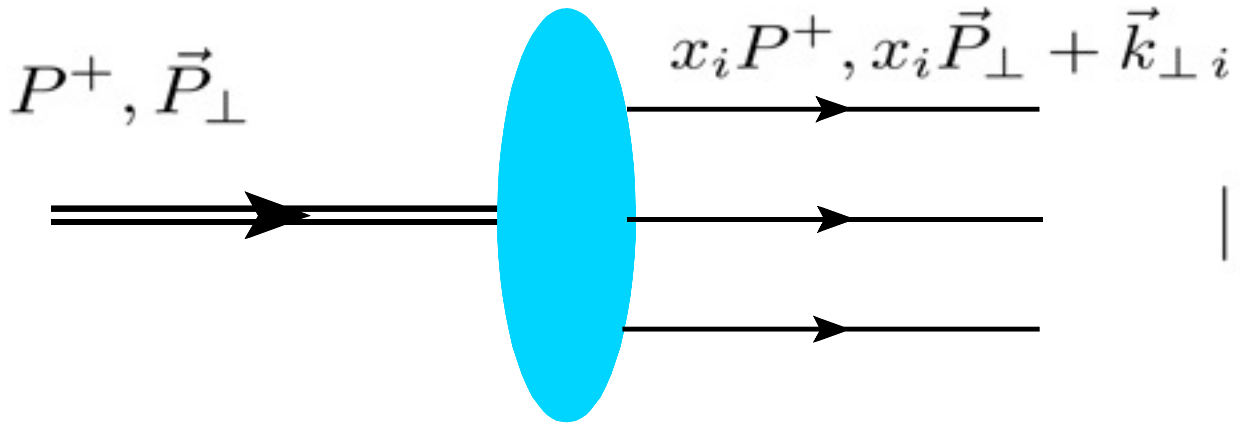
parity
time reversal
isospin symmetry

6 independent wave function amplitudes: $\psi^{(i)}\; i = 1, .., 6$

$L_z^q = -1$

$$|P\uparrow\rangle^{L_z=-1}_{\frac{3}{2}} \;=\; \int d[1]d[2]d[3]\;(k_2^x - ik_2^y)\psi^{(5)}(1,2,3) \times \frac{\epsilon^{ijk}}{\sqrt{6}} u^\dagger_{i\uparrow}(1)\left(u^\dagger_{j\uparrow}(2)d^\dagger_{k\uparrow}(3) - d^\dagger_{j\uparrow}(2)u^\dagger_{k\uparrow}(3)\right)|0\rangle$$

Three Quark Light Cone Amplitudes



$$|\, P, \lambda\rangle = \sum_\beta \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\; \Psi^f_{\lambda,\beta}(x_i, \vec{k}_{\perp,i})\, \frac{\varepsilon^{ijk}}{\sqrt{6}}\, u^\dagger_{i\lambda_1}(1) u^\dagger_{j\lambda_2}(2) d^\dagger_{k\lambda_3}(3)|\, 0\rangle$$

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[Ji, J.P. Ma, Yuan, 03;
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$$|\, P, \uparrow\rangle = |\, P, \uparrow\rangle^{L_z=2}_{-\frac{3}{2}} + |\, P, \uparrow\rangle^{L_z=1}_{-\frac{1}{2}} + |\, P, \uparrow\rangle^{L_z=0}_{\frac{1}{2}} + |\, P, \uparrow\rangle^{L_z=-1}_{\frac{3}{2}}$$

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$$\langle 0\, |\epsilon^{ijk}\, u^\dagger_{i\lambda_i}(1)\, \Gamma\, u^\dagger_{j\lambda_j}(2) d^\dagger_{k\lambda_k}(3)|\, P\rangle$$

parity
time reversal
isospin symmetry

6 independent wave function amplitudes: $\psi^{(i)}\; i = 1, .., 6$

$L_z^q = 2$

$$|P\uparrow\rangle^{L_z=2}_{-\frac{3}{2}} = \int d[1]d[2]d[3]\; (k_1^x + ik_1^y)(k_3^x + ik_3^y)\psi^{(6)}(1,2,3) \times \frac{\epsilon^{ijk}}{\sqrt{6}} u^\dagger_{i\downarrow}(1) \left(d^\dagger_{j\downarrow}(2) u^\dagger_{k\downarrow}(3) - u^\dagger_{j\downarrow}(2) d^\dagger_{k\downarrow}(3)\right) |0\rangle$$

Relations among TMDs in Quark Models

	Linear relations		Quadratic relation	
Flavor-dependent $D^u = \frac{2}{3},\ D^d = -\frac{1}{3}$	$D^q f_1^q + g_{1L}^q = 2h_1^q$	* * *		
Flavor-independent	$g_{1T}^q = -h_{1L}^{\perp q}$	* * ● * ● ●	$2h_1^q\, h_{1T}^{\perp q} = -(g_{1T}^q)^2$	● ● ● * * *
	$g_{1L}^q - h_1^q = \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp q}$	* * ● * ● ●		

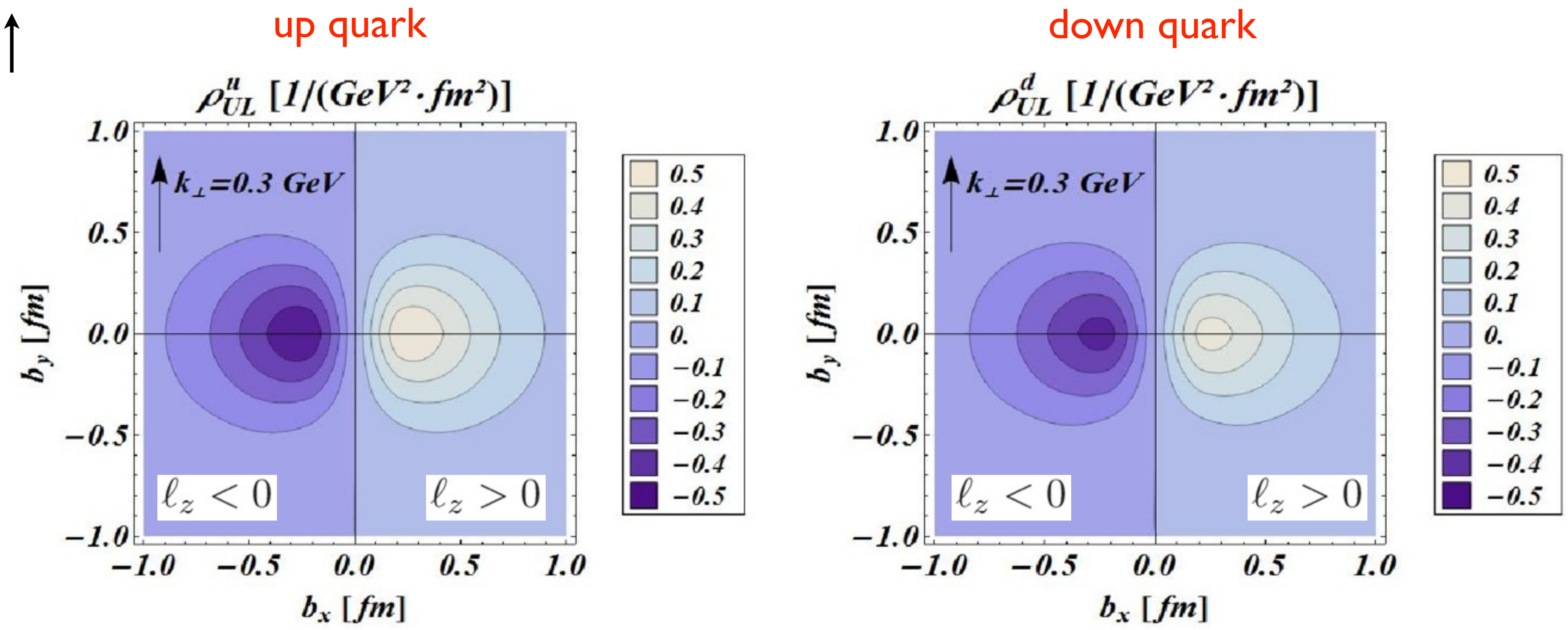
Bag	[Jaffe & Ji (1991), Signal (1997), Barone & *al.* (2002), Avakian & *al.* (2008-2010)]
ÂQSM	[Lorcé & Pasquini (in preparation)]
LCQM	[Pasquini & *al.* (2005-2008)]
S Diquark	[Ma & *al.* (1996-2009), Jakob & *al.* (1997), Bacchetta & al. (2008)]
AV Diquark	[Ma & *al.* (1996-2009), Jakob & *al.* (1997)] [Bacchetta & *al.* (2008)]
Cov. Parton	[Efremov & *al.* (2009)]
Quark Target	[Meißner & *al.* (2007)]

Common assumptions :

- ➢ No gluons
- ➢ Independent quarks

Long. pol. quark in Unpol. Proton

fixed $\vec{k}_\perp \uparrow$



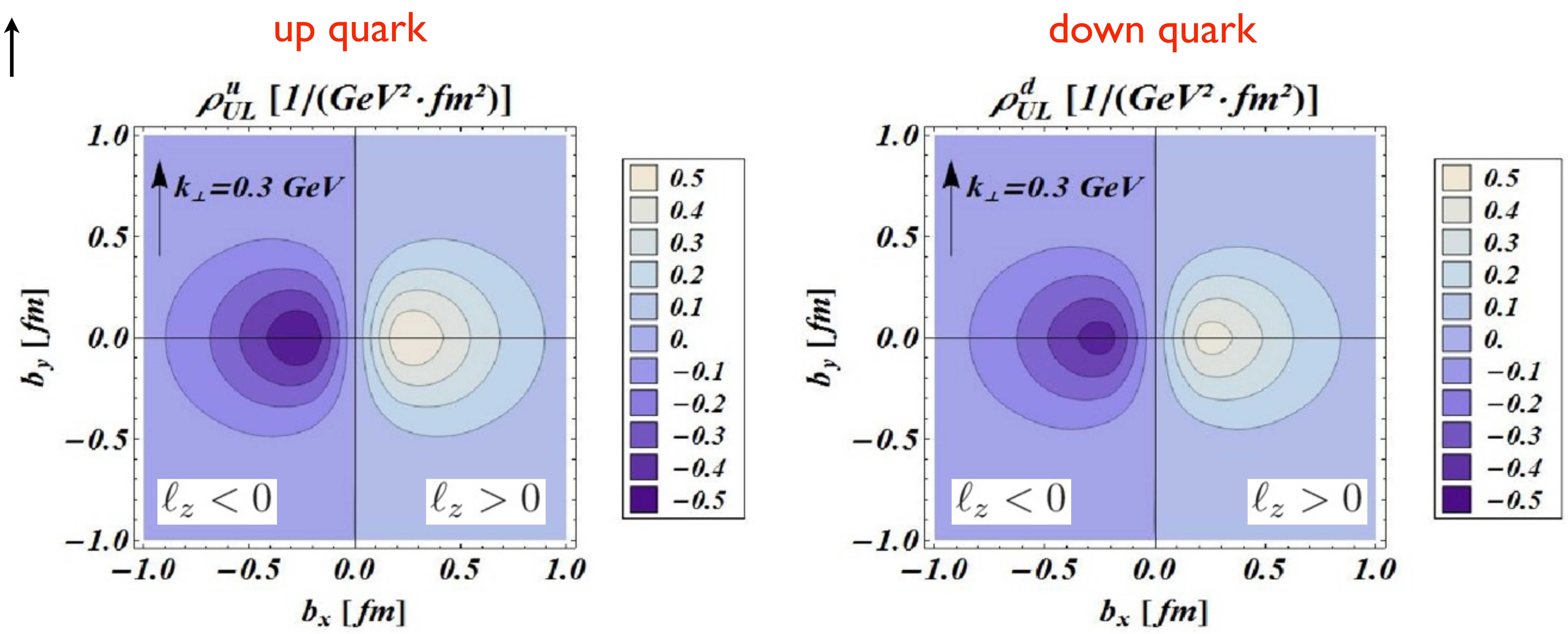
✦ projection to GPD and TMD is vanishing

➡ unique information on OAM from Wigner distributions

[Lorce', Pasquini (2011)]

Long. pol. quark in Unpol. Proton

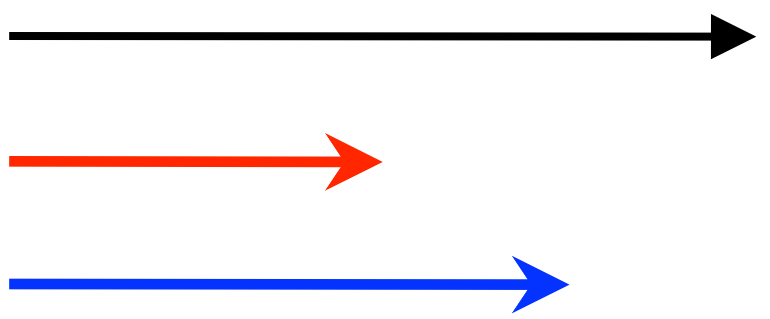
fixed $\vec{k}_\perp \uparrow$



correlation between quark spin and quark OAM

$$\boxed{C_z^q = \int \mathrm{d}x\, \mathrm{d}\vec{k}_\perp\, \mathrm{d}\vec{b}_\perp \left(\vec{b}_\perp \times \vec{k}_\perp\right) \rho^q_{UL}(x, \vec{k}_\perp, \vec{b}_\perp)}$$

	u-quark	d-quark
C_z^q	0.23	0.19



Quark spin

u-quark OAM

d-quark OAM

[Lorce', Pasquini (2011)]

Light-Cone Helicity and Canonical Spin

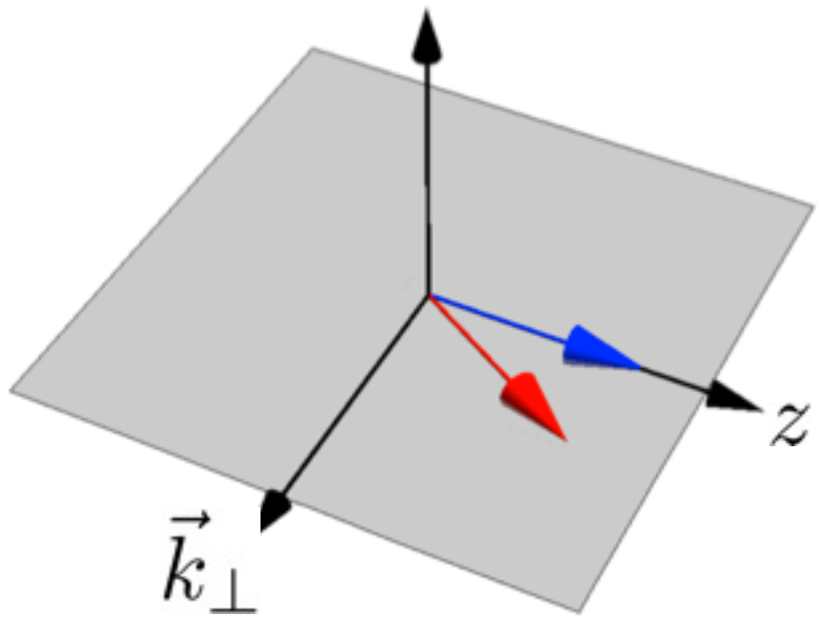
$$q_\lambda^{LC}(k) = D_{\lambda s}^{(1/2)*}\, q_s^{C}(k)$$

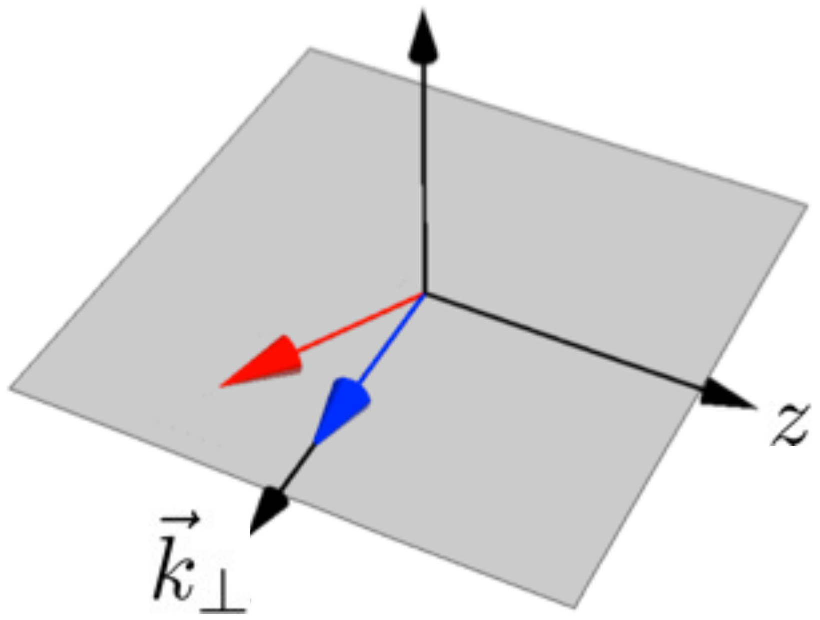
LF helicity

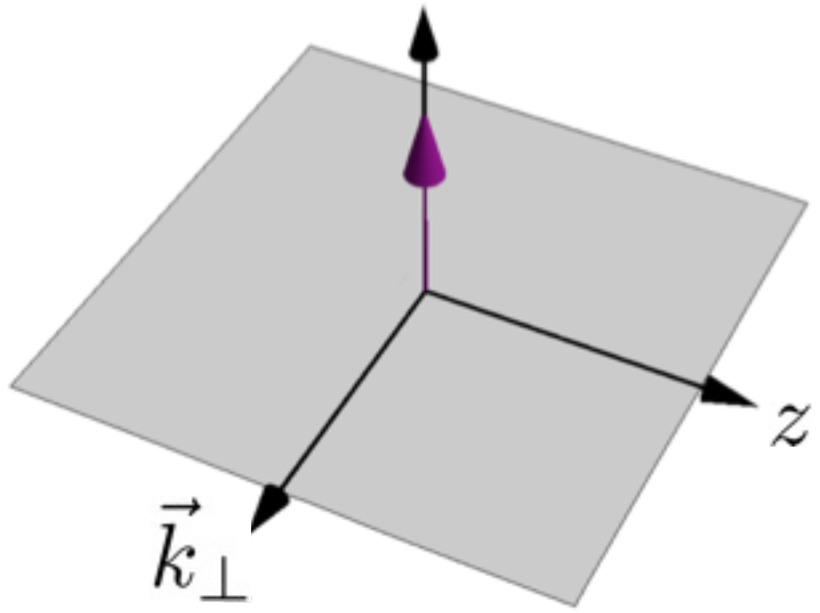
canonical spin

$$D_{\lambda s}^{(1/2)*}(k) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$

rotation around an axis
orthogonal to z and k⊥







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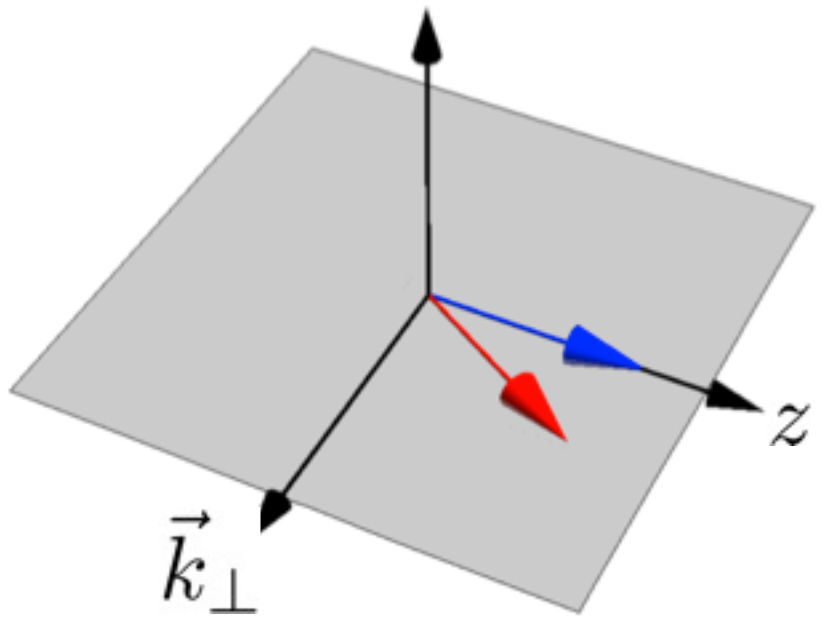
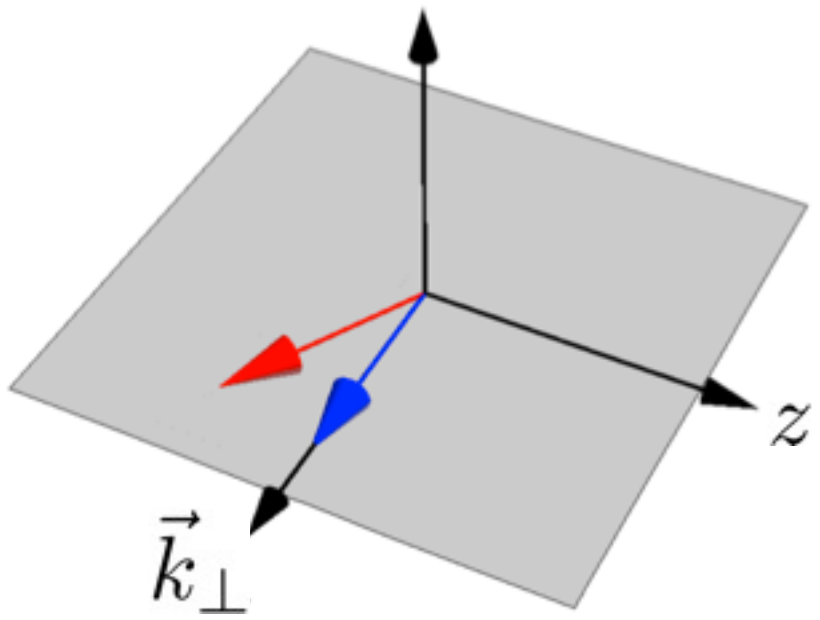
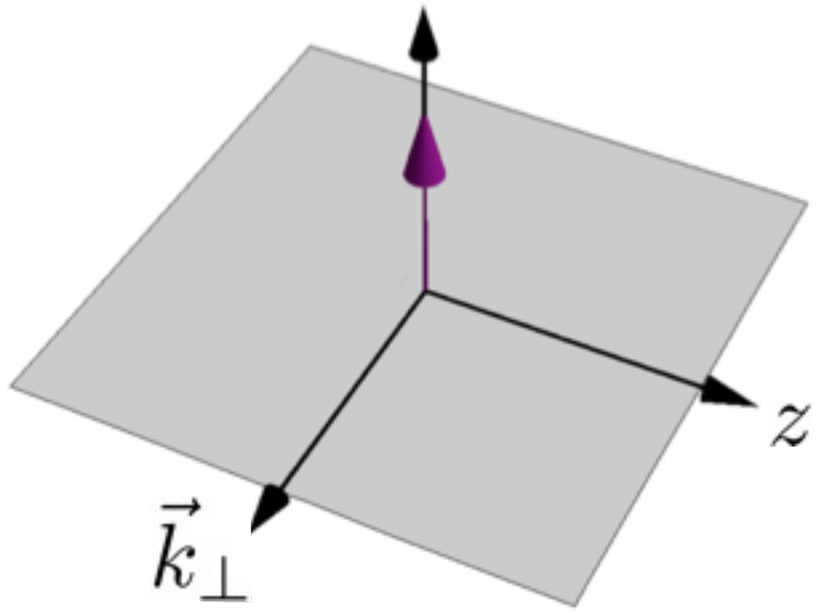






Light-Front CQM

$$K_z = m + x\mathcal{M}_0$$

$$\vec{K}_\perp = \vec{k}_\perp$$

$$k_z = x\mathcal{M}_0 - \sqrt{\vec{k}^2 + m^2}$$

(Melosh rotation)

Chiral Quark-Soliton Model

$$K_z = h(|\vec{k}|) + \frac{k_z}{|\vec{k}|}\, j(|\vec{k}|)$$

$$\vec{K}_\perp = \frac{\vec{k}_\perp}{|\vec{k}|}\, j(|\vec{k}|)$$

$$k_z = x\mathcal{M}_N - E_{\rm lev}$$

Bag Model

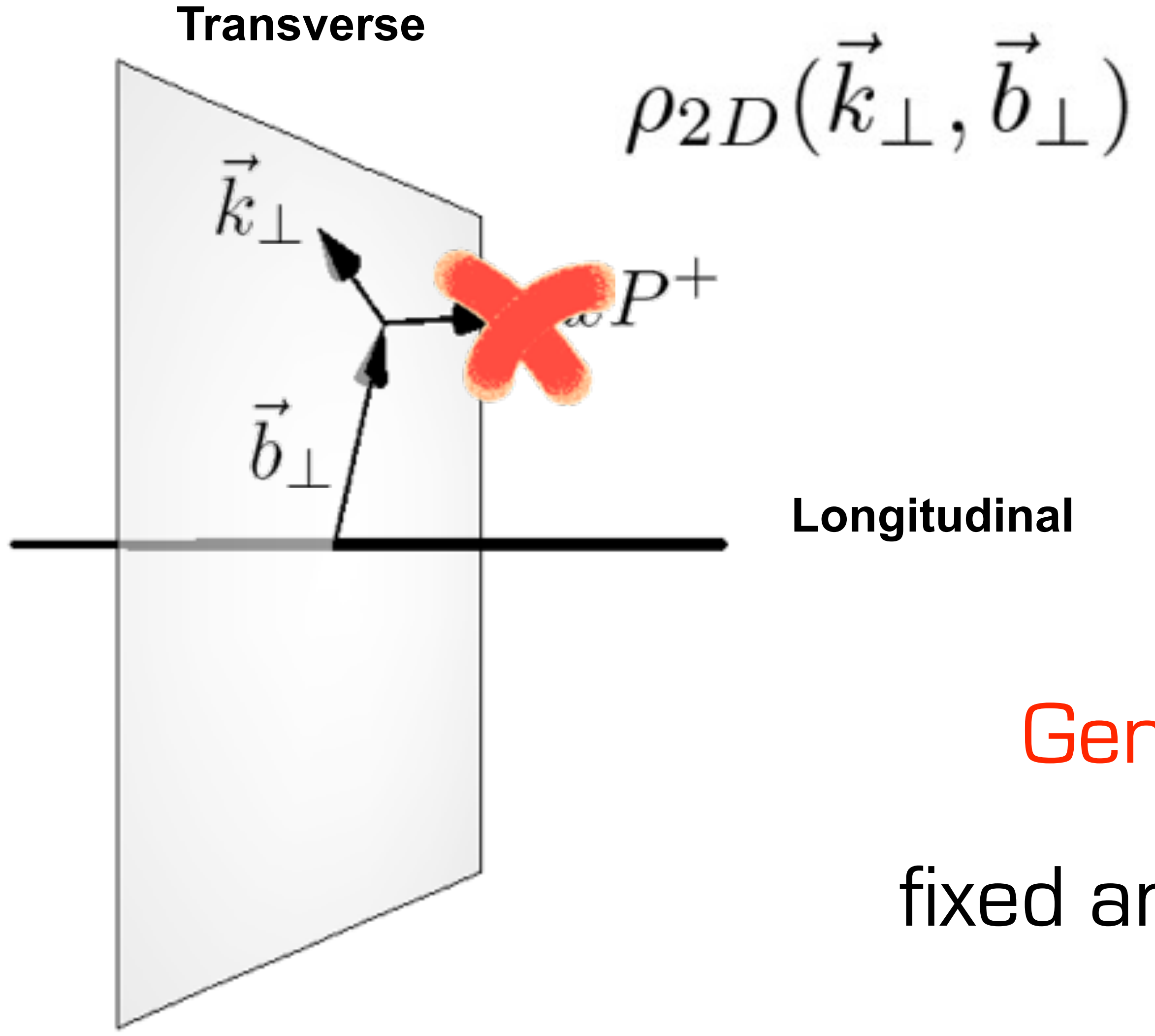
$$K_z = t_0(|\vec{k}|) + \frac{k_z}{|\vec{k}|}\, t_1(|\vec{k}|)$$

$$\vec{K}_\perp = \frac{\vec{k}_\perp}{|\vec{k}|}\, t_1(|\vec{k}|)$$

$$k_z = x\mathcal{M}_N - \omega/R_0$$

Unpol. up quark in Unpol. Proton

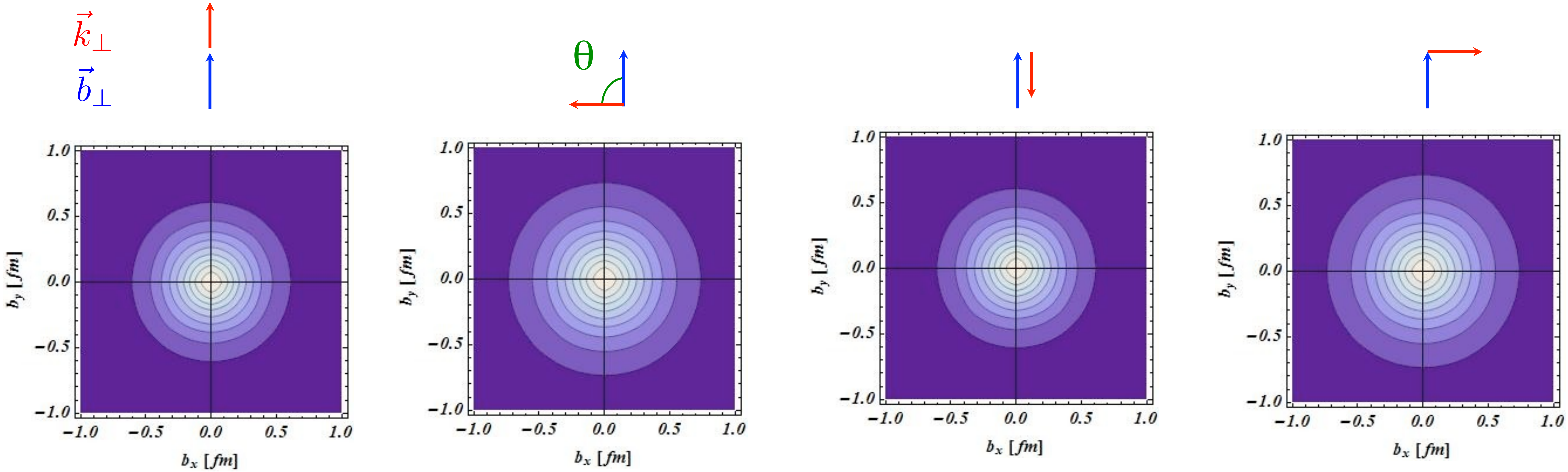
[Lorce', BP, PRD84 (2011)]



$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$

Generalized Transverse Charge Density

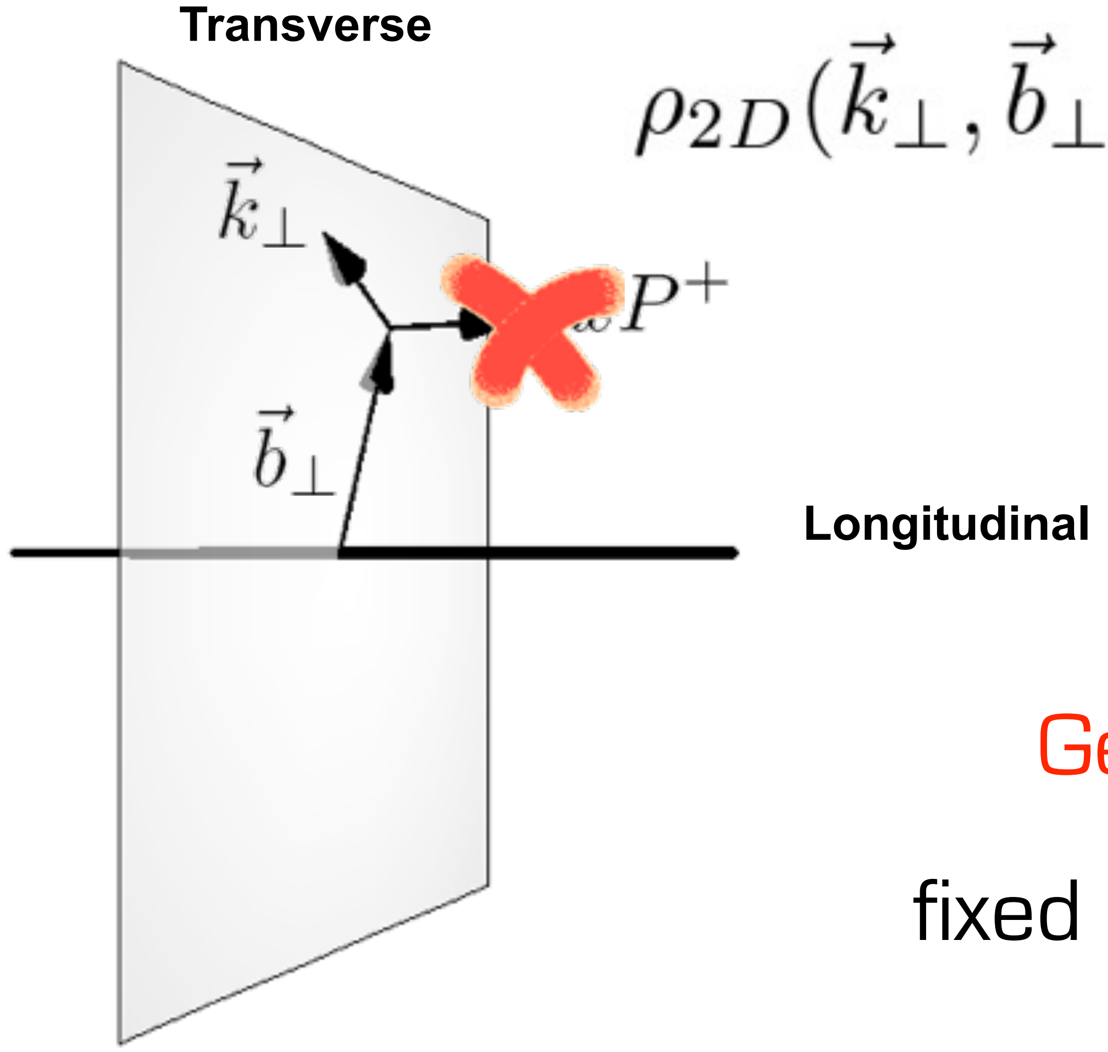
fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$



Unpol. up quark in Unpol. Proton

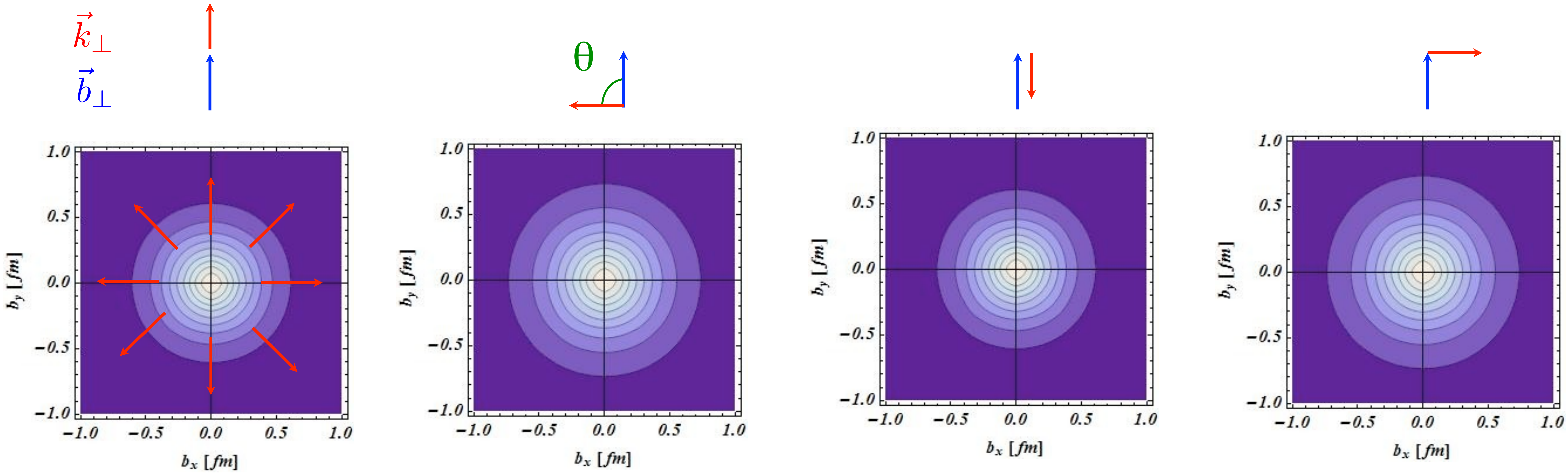
[Lorce', BP, PRD84 (2011)]

$$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$$



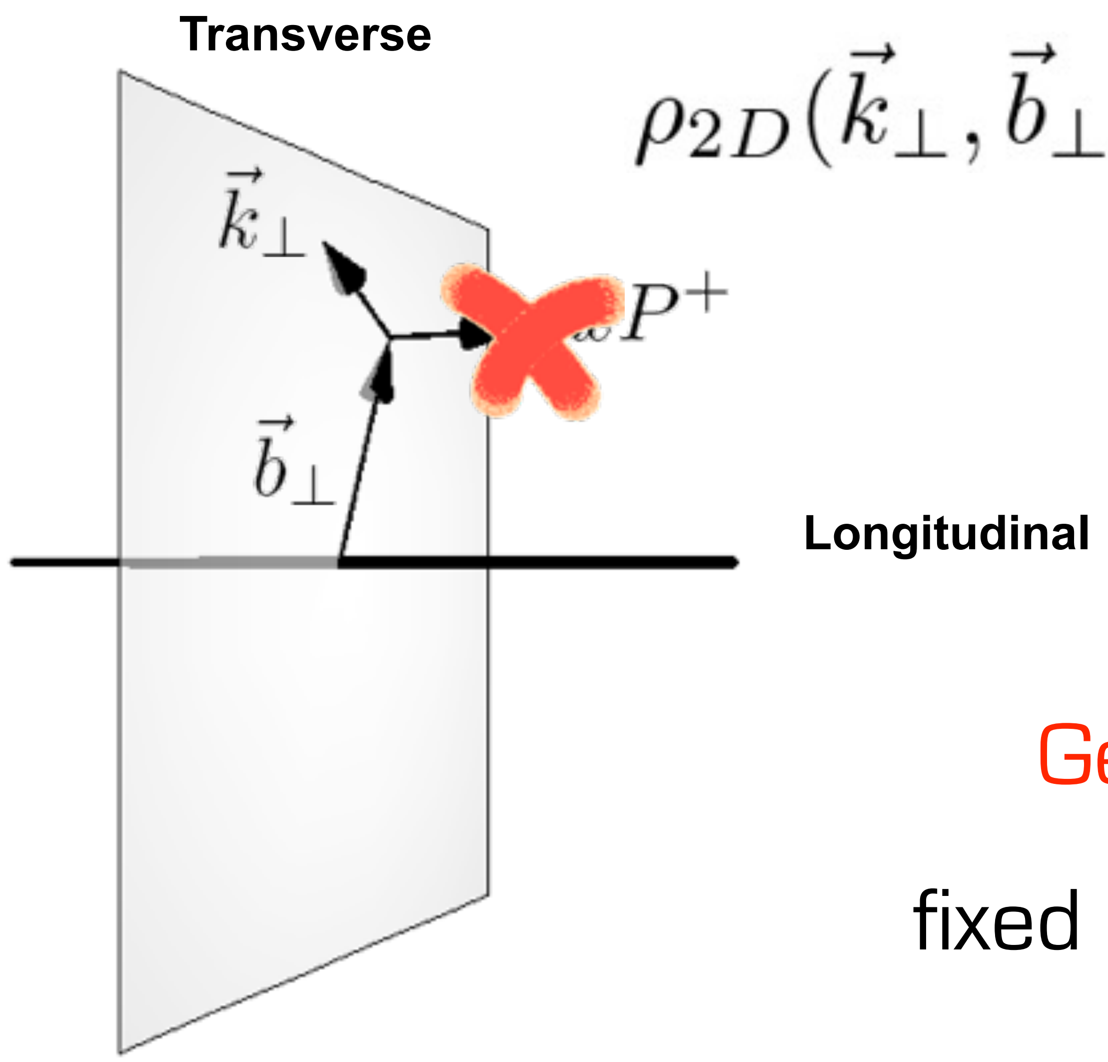
Generalized Transverse Charge Density

fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$



Unpol. up quark in Unpol. Proton

[Lorce', BP, PRD84 (2011)]



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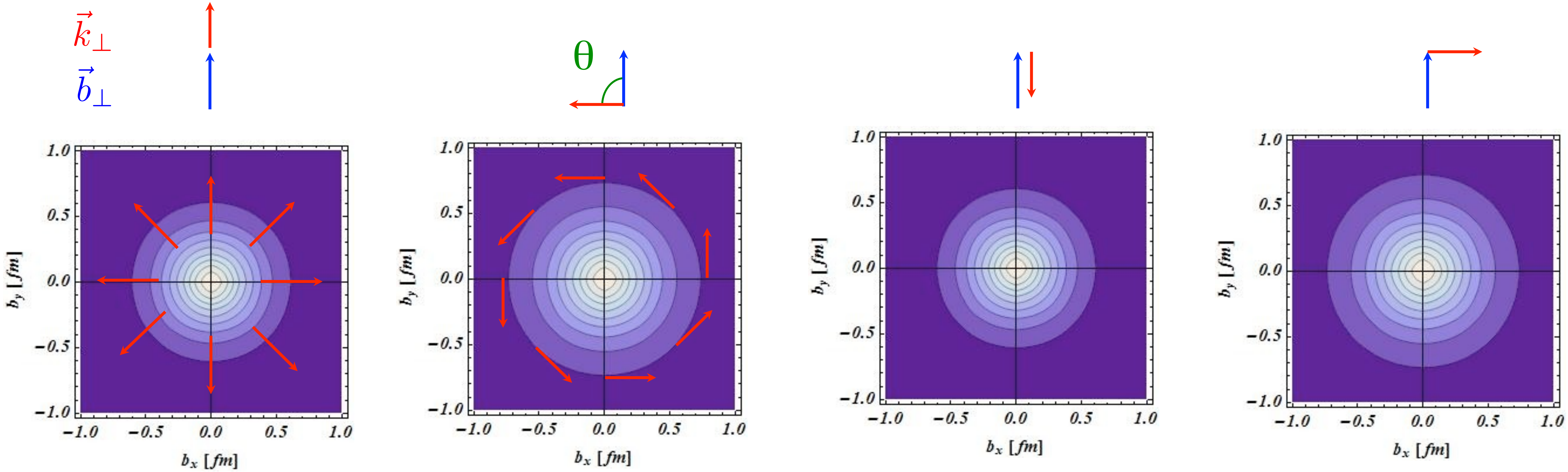
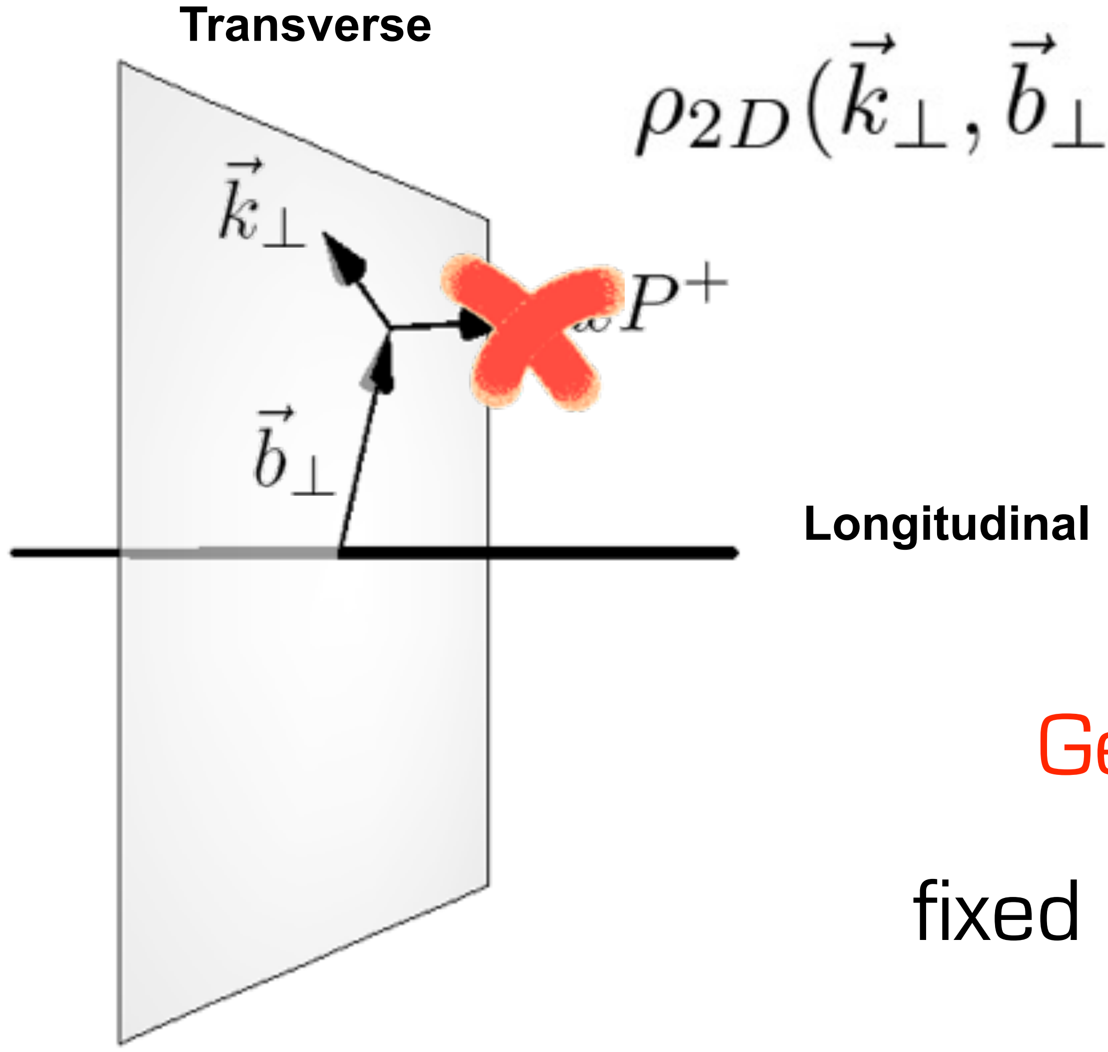


Unpol. up quark in Unpol. Proton

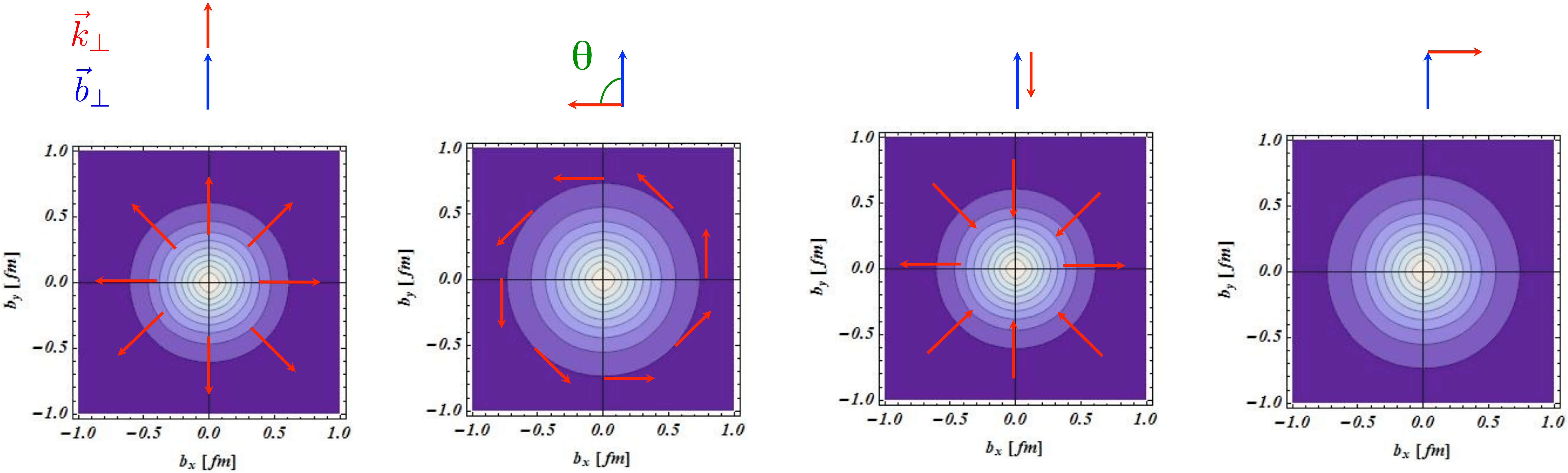
[Lorce', BP, PRD84
(2011)]



$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$

Generalized Transverse Charge Density

fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$



Unpol. up quark in Unpol. Proton

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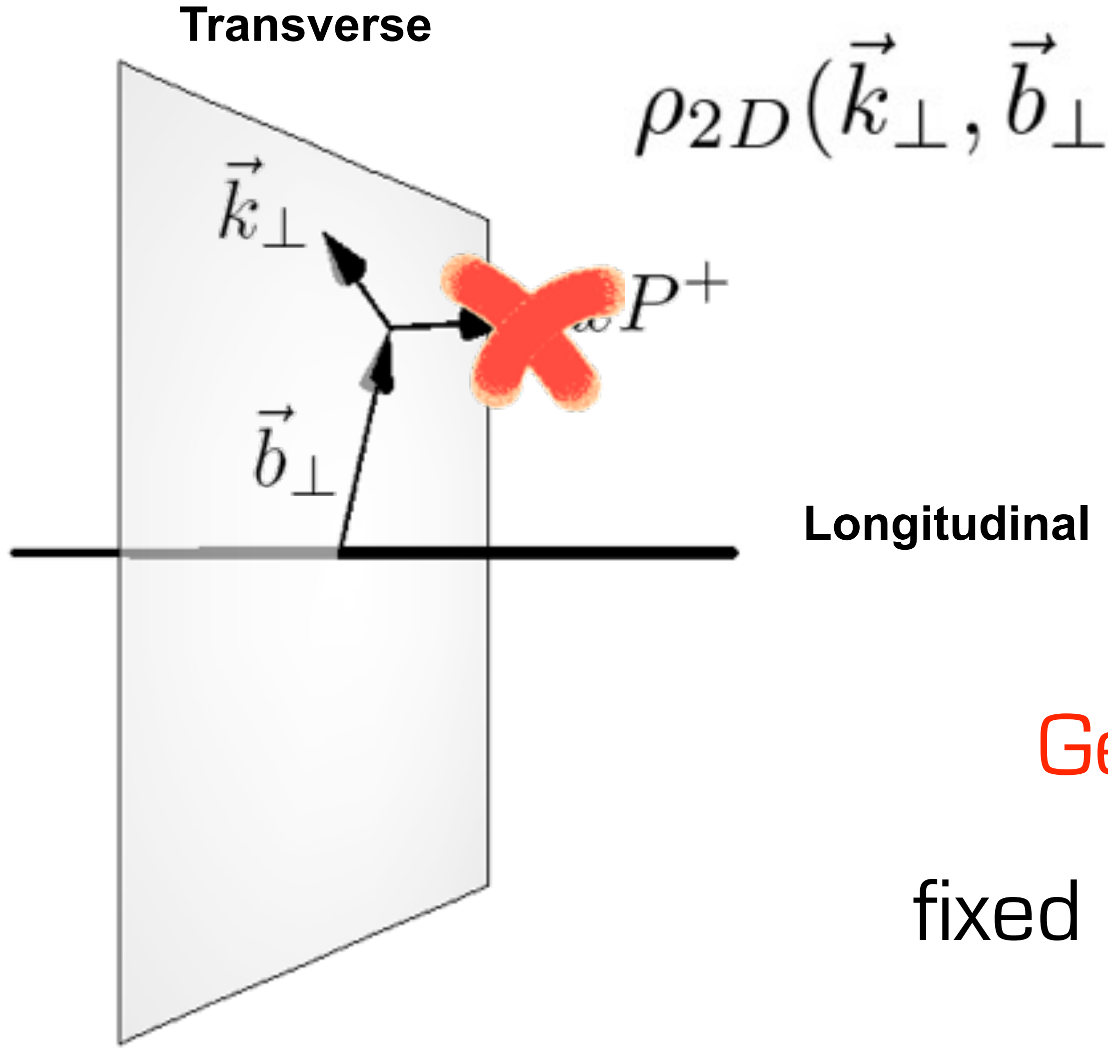
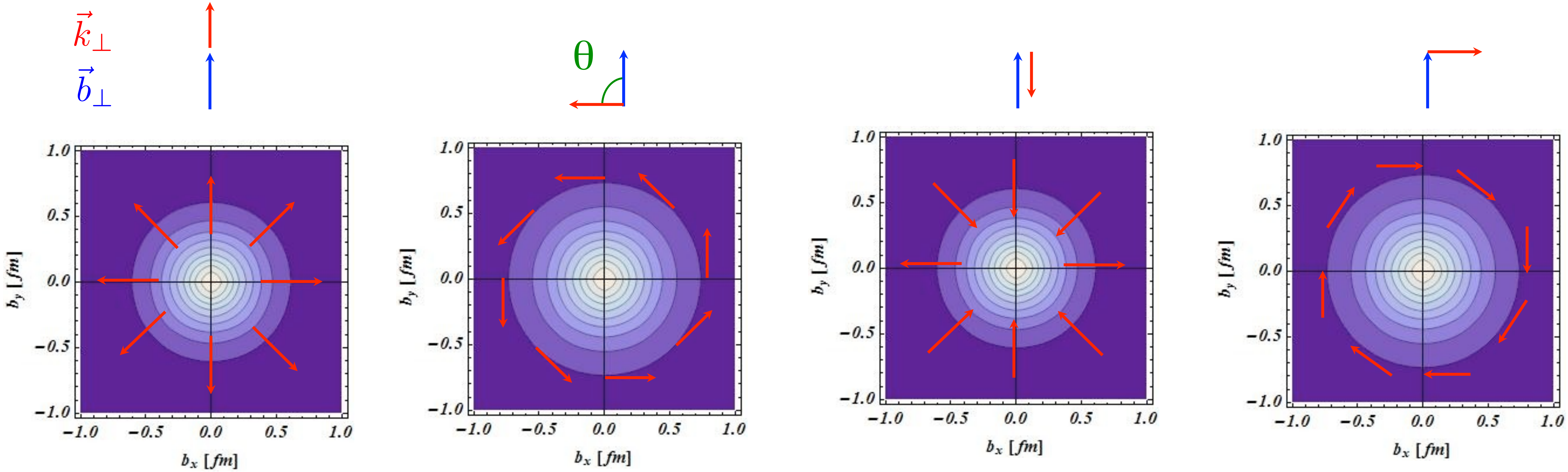


$$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$$

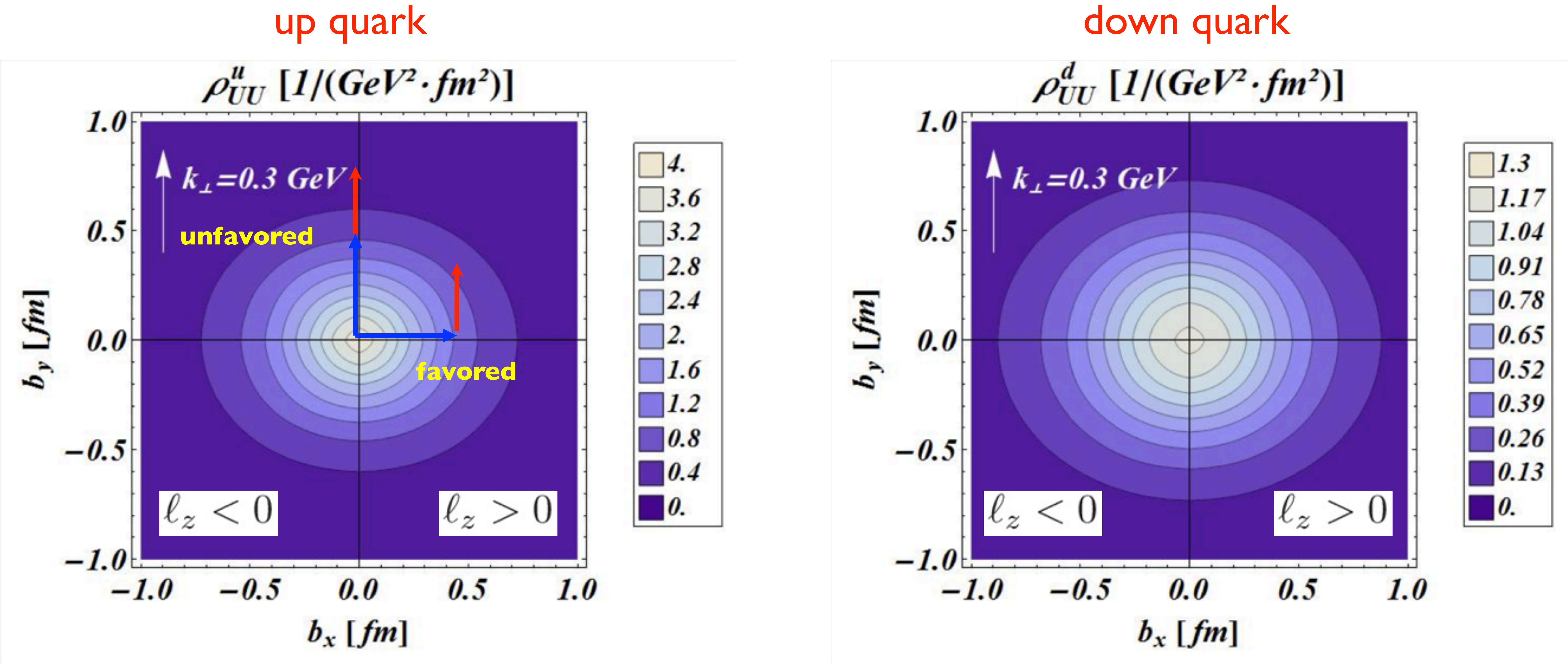
Generalized Transverse Charge Density

fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$



Unpol. quarks in Unpol. Proton

fixed $\vec{k}_\perp$: ↑



Distortion due to correlations between $\vec{k}_\perp$ and $\vec{b}_\perp$

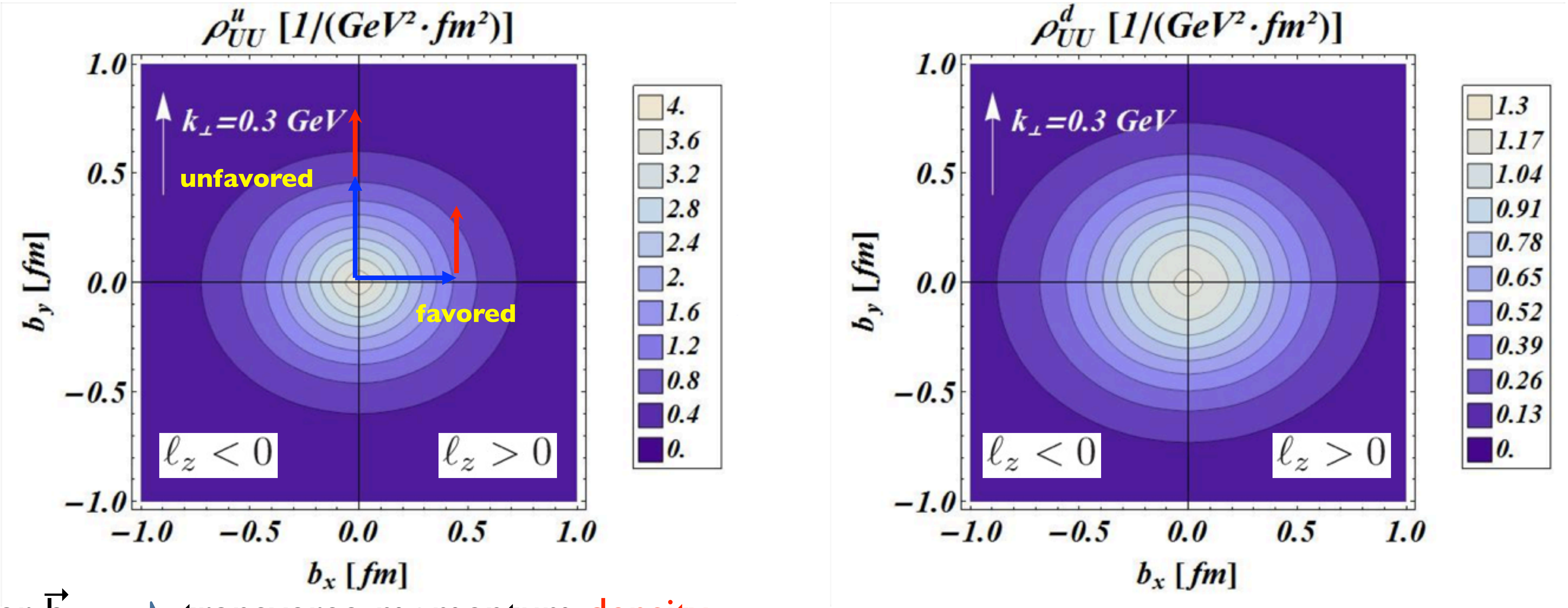
↳ absent in GPD and TMD !

Left-right symmetry → no net quark OAM

[Lorce', Pasquini (2011)]

Unpol. quarks in Unpol. Proton

fixed $\vec{k}_\perp$: ↑



up quark

down quark

✦ integrating over $\vec{b}_\perp$ ➡ transverse-momentum density

$$f_1^q(k_\perp^2) = \int \mathrm{d}x \, f_1^q(x, k_\perp^2)$$

✦ integrating over $\vec{k}_\perp$ ➡ charge density in the transverse plane $\vec{b}_\perp$

$$\rho^q(b_\perp^2) = e^q \int \mathrm{d}^2\Delta_\perp e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} F_1^q(\Delta_\perp^2)$$

Monopole

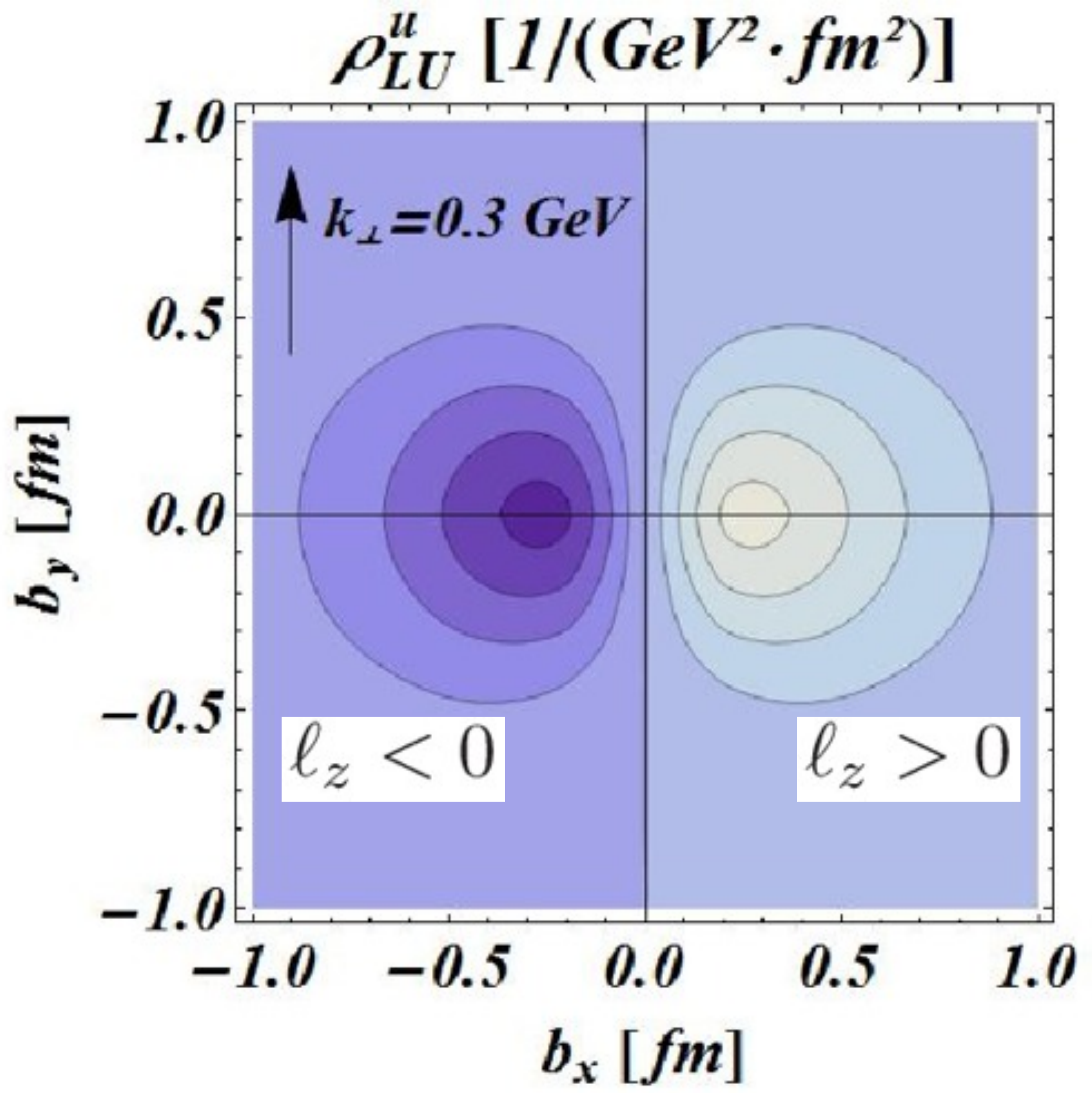
Distributions

[Miller (2007); Burkardt (2007)]

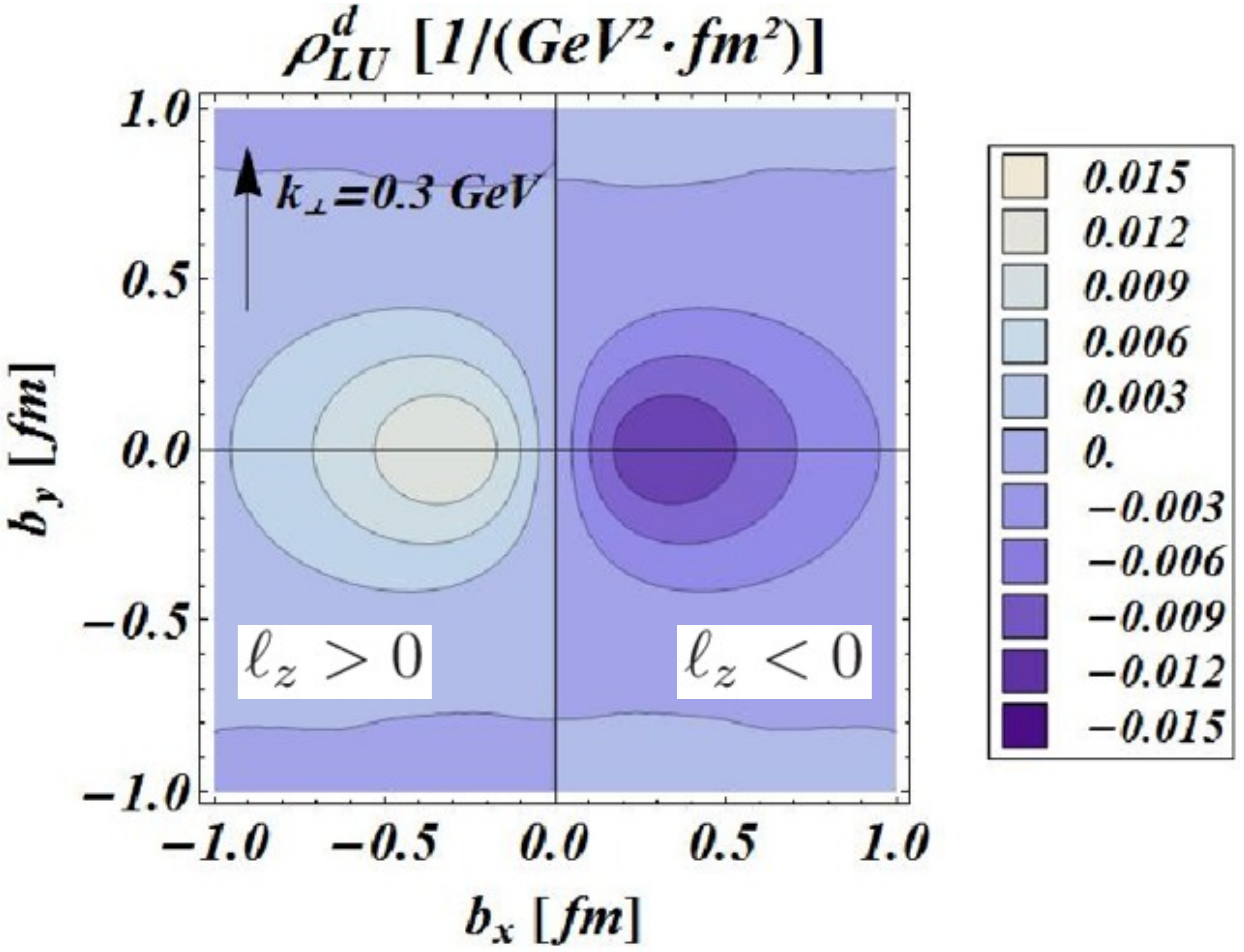
Unpol. quark in Long. pol. Proton

fixed $\vec{k}_\perp$ ↑

up quark



down quark



→ **Proton spin**

→ u-quark OAM

← d-quark OAM

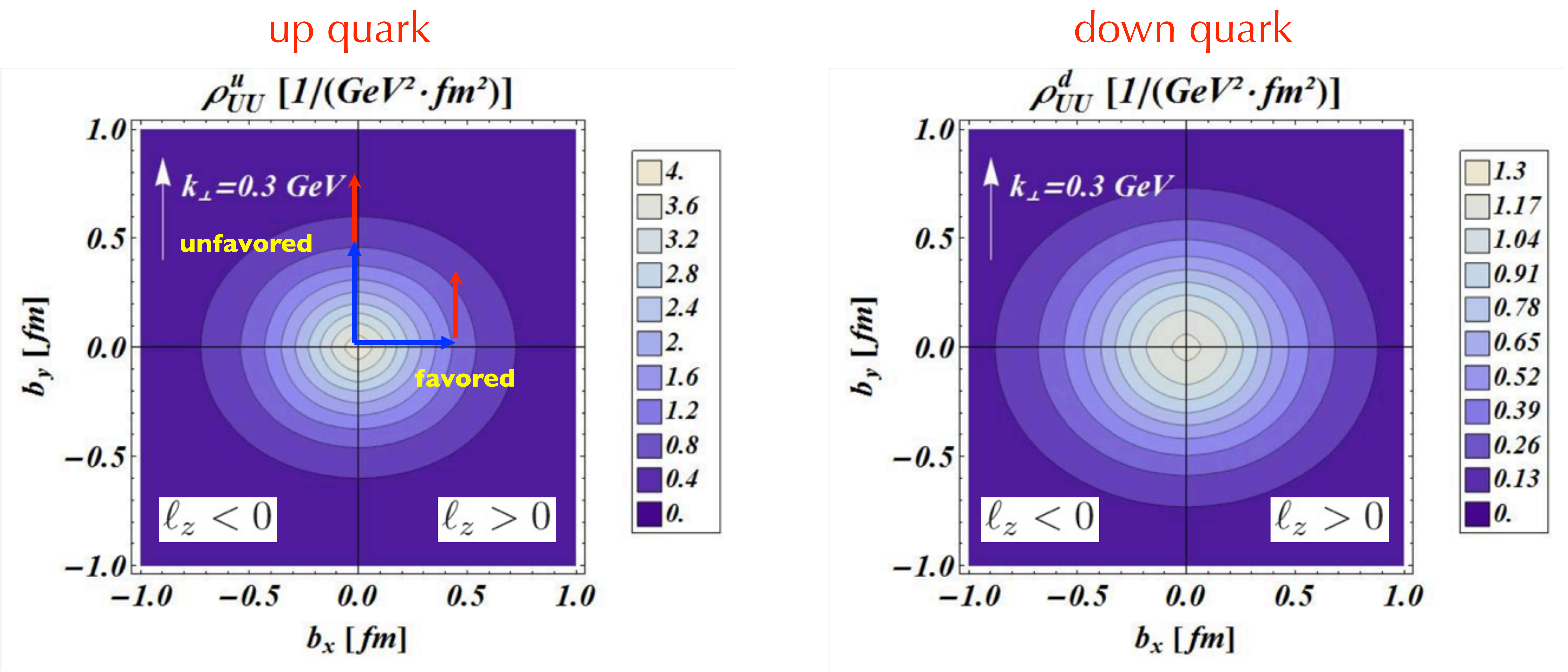
★ projection to GPD and TMD is vanishing

→ unique information on OAM from Wigner distributions

[Lorce', Pasquini (2011)]

Unpol. quarks in Unpol. Proton

fixed $\vec{k}_\perp$: ↑



Distortion due to correlations between $\vec{k}_\perp$ and $\vec{b}_\perp$

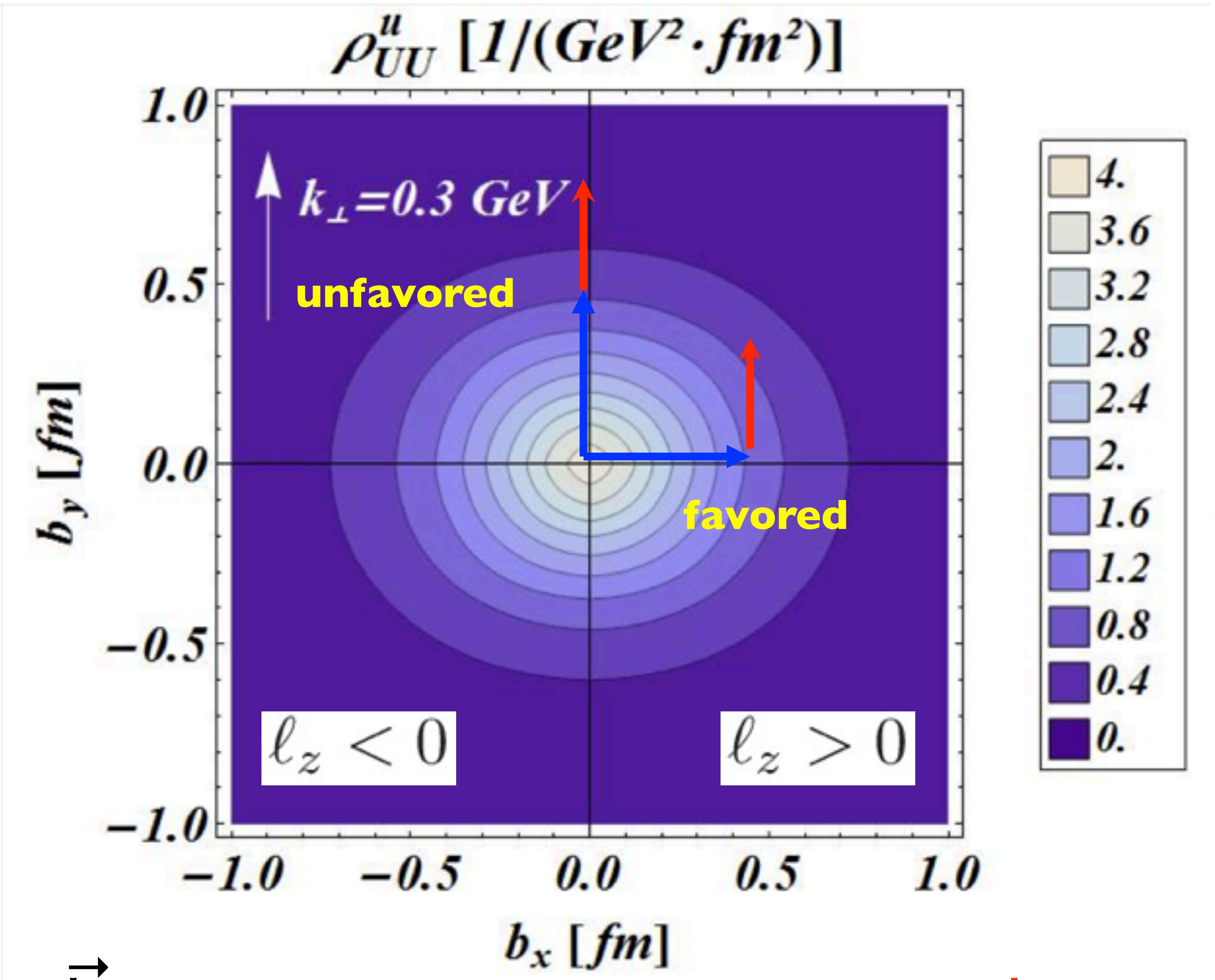
↳ absent in GPD and TMD !

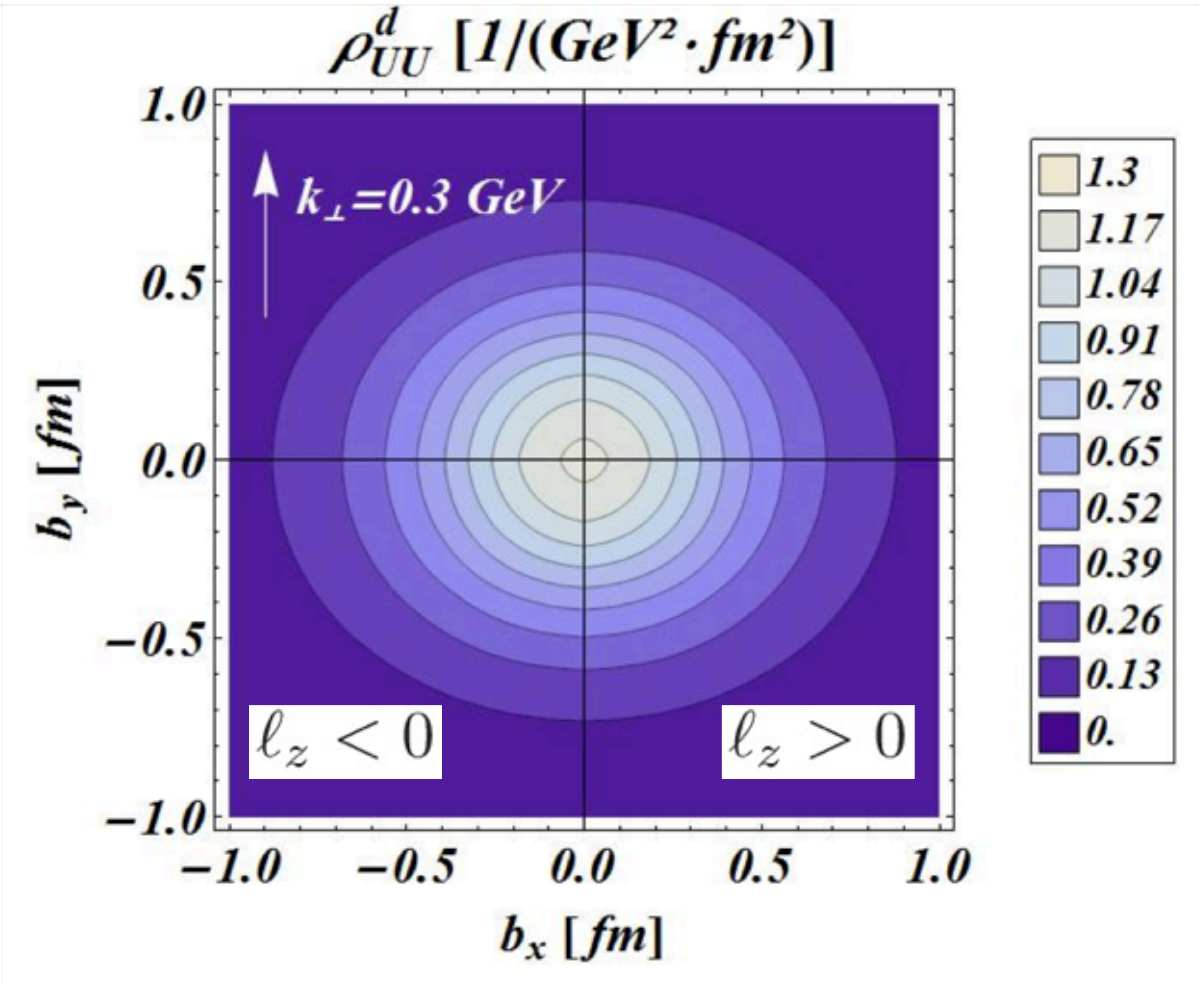
Left-right symmetry → no net quark OAM

Lorcé, Pasquini (2011)

Unpol. quarks in Unpol. Proton

fixed $\vec{k}_\perp$: ↑


up quark


down quark

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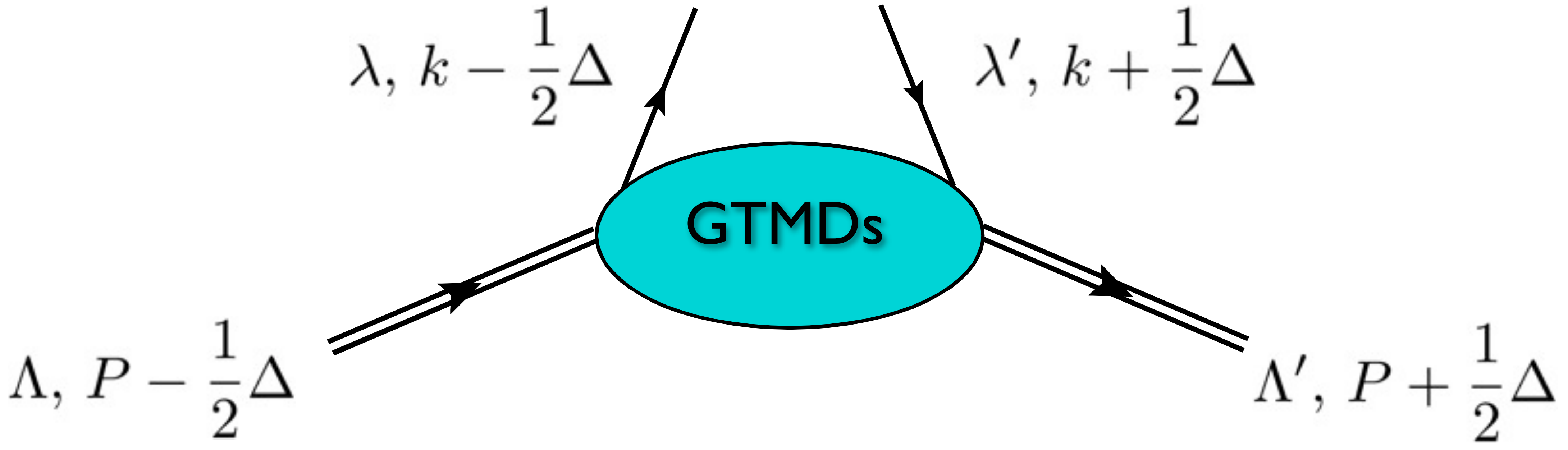
Monopole Distributions

Miller (2007); Burkardt (2007)

Generalized TMDs and Wigner Distributions



Quark polarization

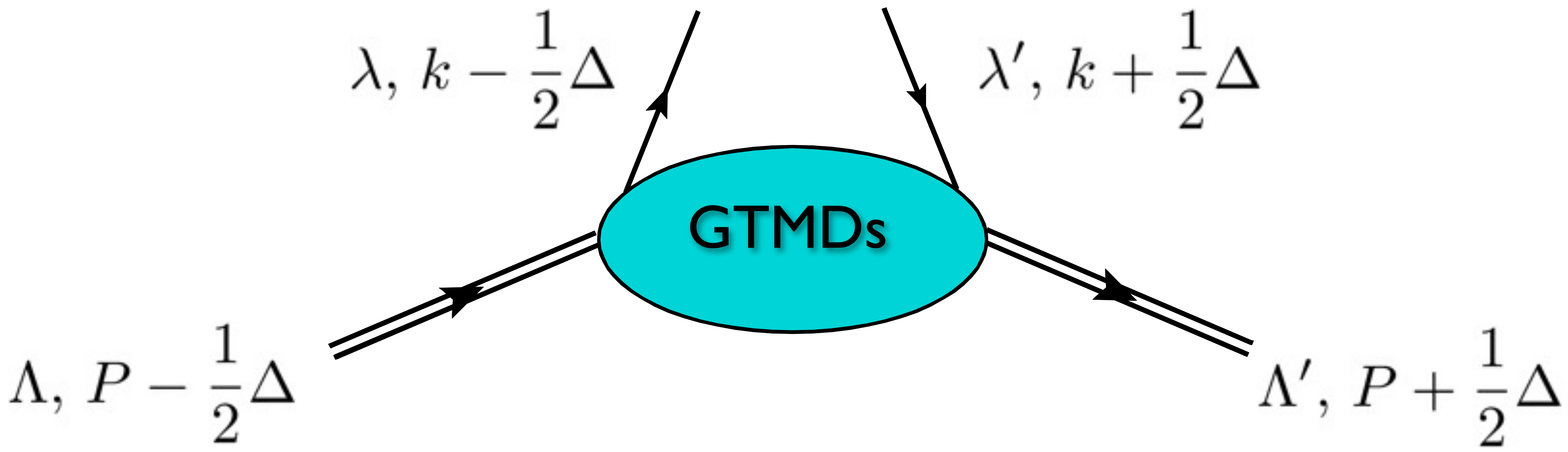
$$\boxed{W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2 z_\perp}{(2\pi)^3}\langle P',\Lambda'|\bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\psi^q_\lambda(\frac{z}{2})|P,\Lambda\rangle e^{i(xP^+z^- -\vec{k}_\perp\cdot\vec{z}_\perp)}}$$

Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

Generalized TMDs and Wigner Distributions



Quark polarization

$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{(2\pi)^3} \langle P', \Lambda' | \bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\, \mathcal{W}(-\frac{z}{2}, \frac{z}{2}|n)\psi^q_{\lambda}(\frac{z}{2})|P, \Lambda\rangle e^{i(xP^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)}$$

Nucleon polarization

4 X 4 =16 polarizations ↔ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp)$$

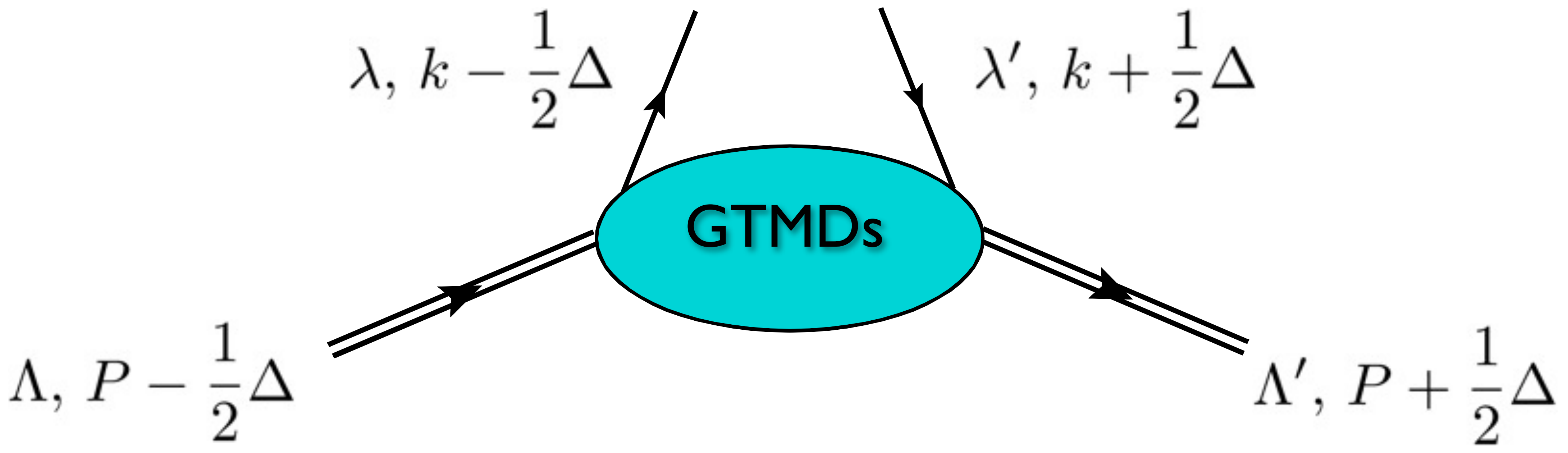
x: average fraction of quark longitudinal momentum

ξ: fraction of longitudinal momentum transfer

$\vec{k}_\perp$: average quark transverse momentum

$\vec{\Delta}_\perp$: nucleon transverse-momentum

Generalized TMDs and Wigner Distributions



Quark polarization

$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2 z_\perp}{(2\pi)^3}\langle P',\Lambda'|\bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\psi^q_{\lambda}(\frac{z}{2})|P,\Lambda\rangle e^{i(xP^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}$$

Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

↓ Fourier transform $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

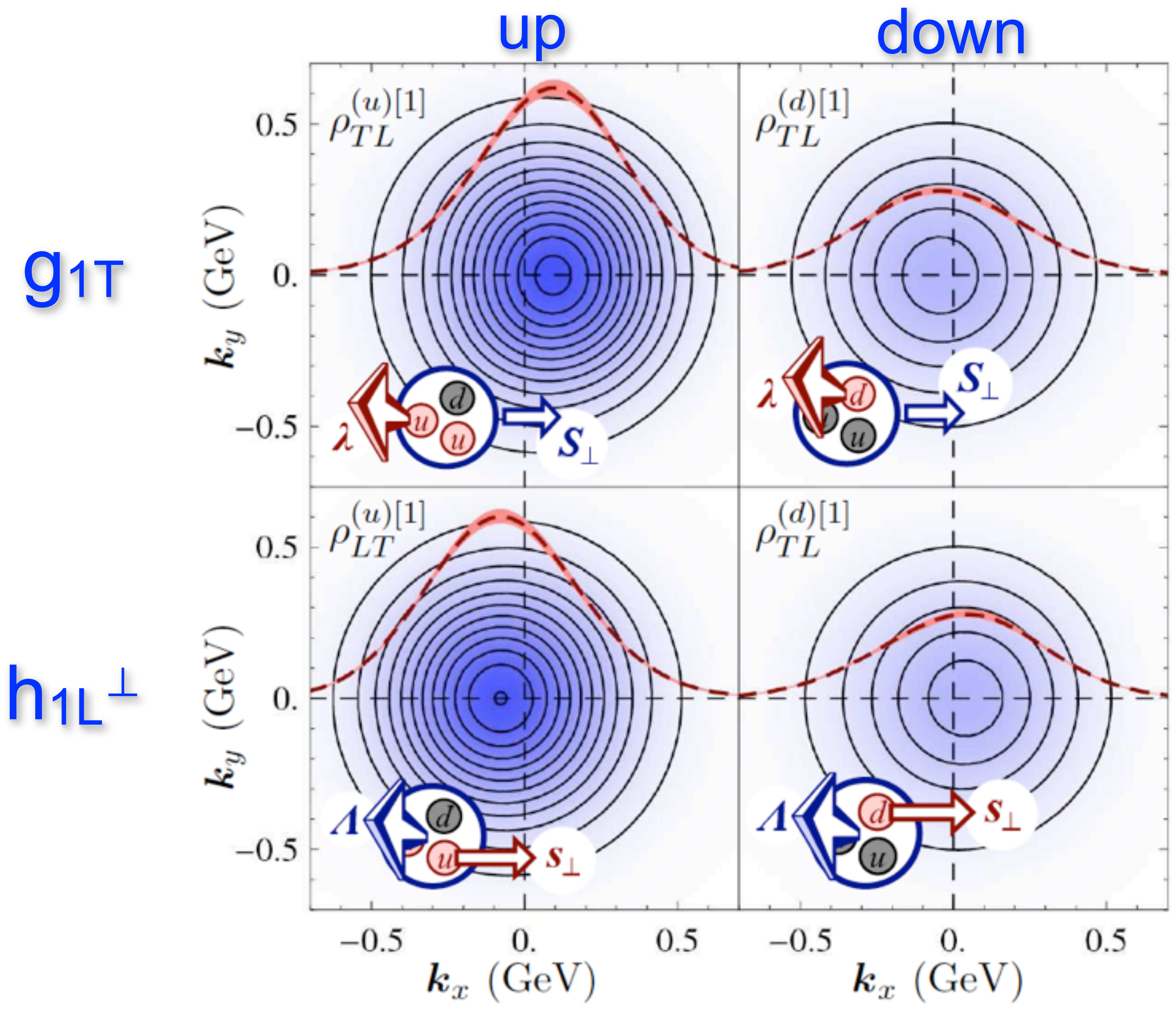
$\tilde{W}^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{b}_\perp)$ 16 real Wigner distributions

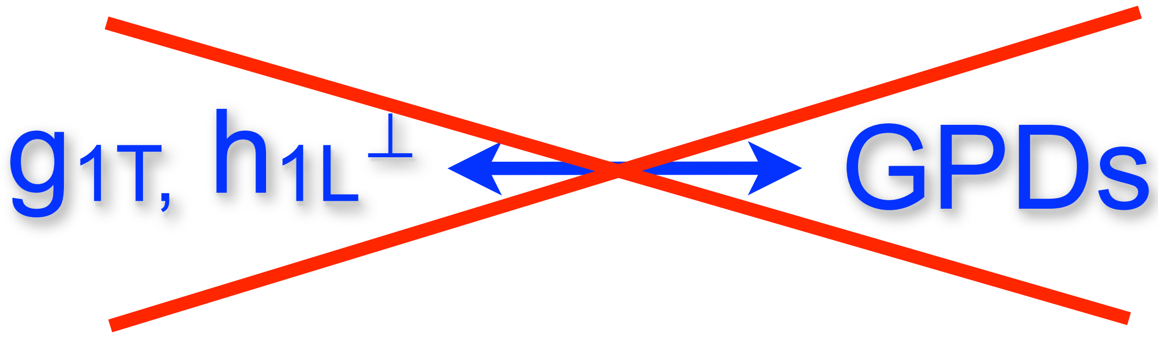
Pioneering lattice QCD studies

$$\langle k_x^q \rangle_{g_{1T}} \approx -\langle k_x^q \rangle_{h_{1L}^{\perp}}$$

consistent with model calculations

*BP, et al., PRD **78** (2008) 034025*



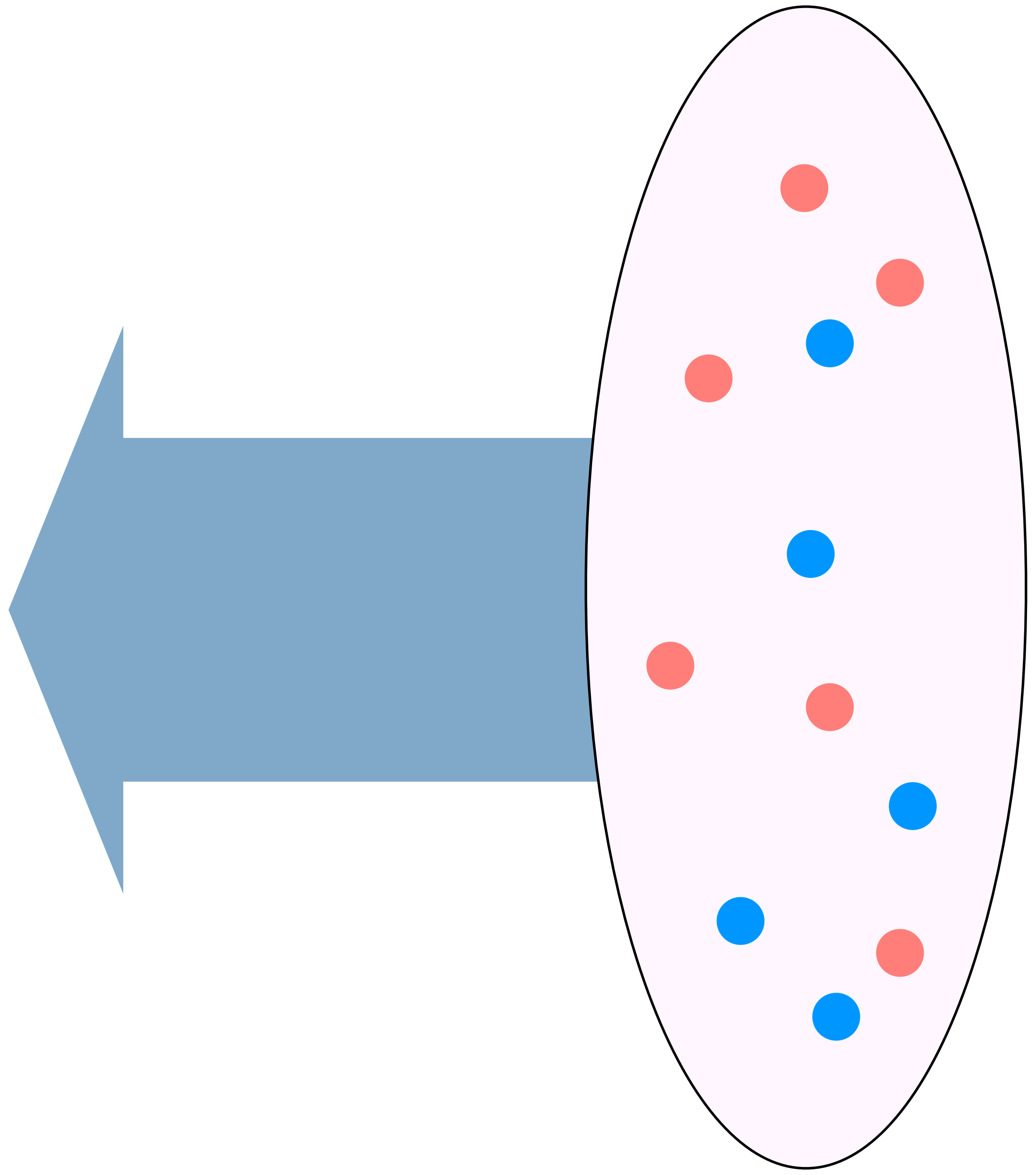

~~g_{1T}, $h_{1L}^{\perp}$ ⟷ GPDs~~

genuine effect of intrinsic transverse momentum of quarks!

*Musch, Hagler, Negele, Schaefer, Europhysics Lett. **88** (2009) 61001*

Model relation TMD ⟷ GPD

unpolarized quark in unpolarized nucleon



$$-\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^2) \simeq \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x,\vec{b}_T^2)\right)'$$

Sivers function

Lensing function

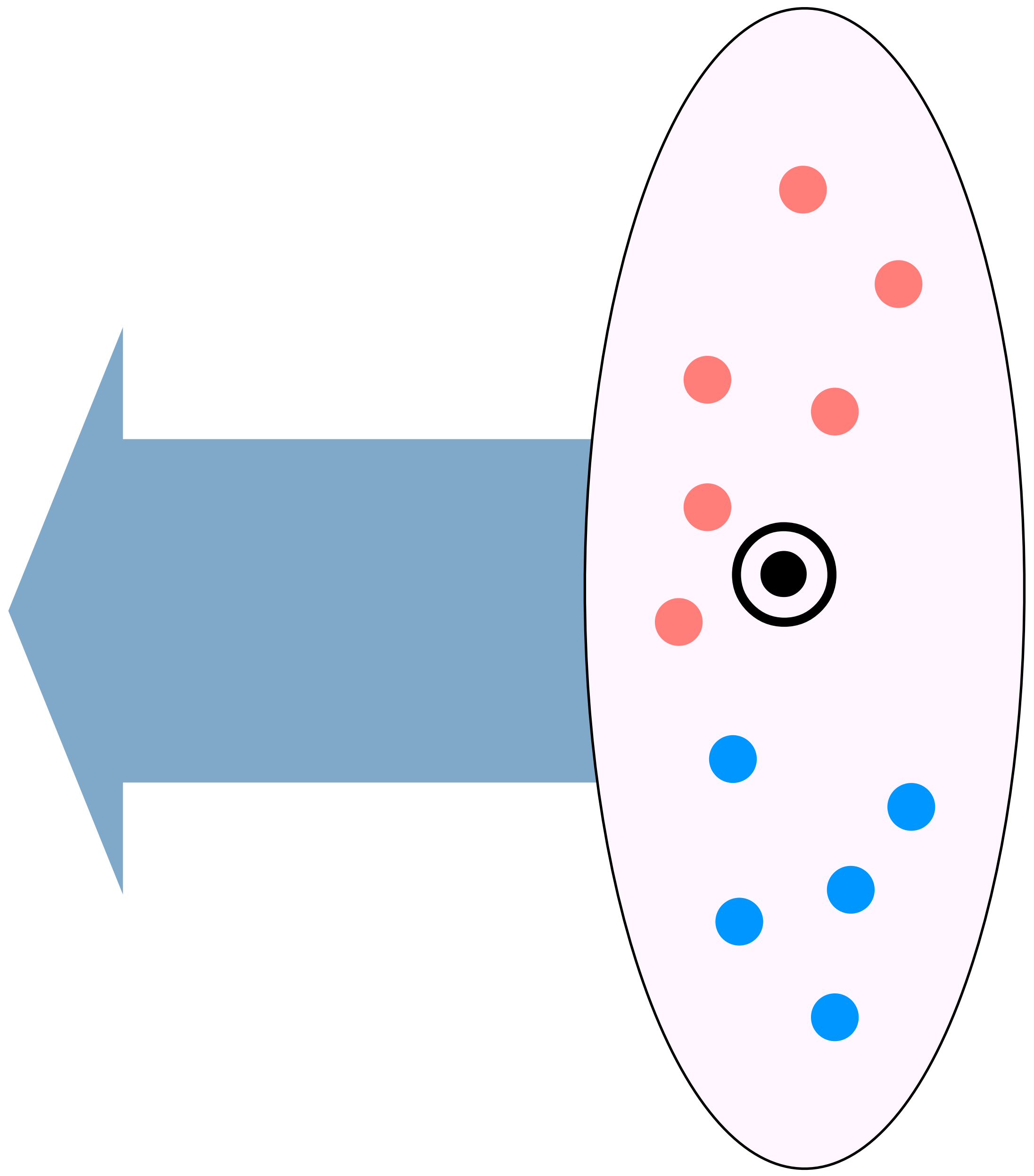
F.T. of E(x,0,t)

*Burkardt, PRD **66** (2002) 114005*

*Burkardt, Pasquini, EPJ A**52** (2016) 161*

Model relation TMD ⟷ GPD

unpolarized quark in transversely pol. nucleon



Distortion in impact parameter
(related to GPD E)

$$-\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^2) \;\simeq\; \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x,\vec{b}_T^2)\right)'$$

Sivers function

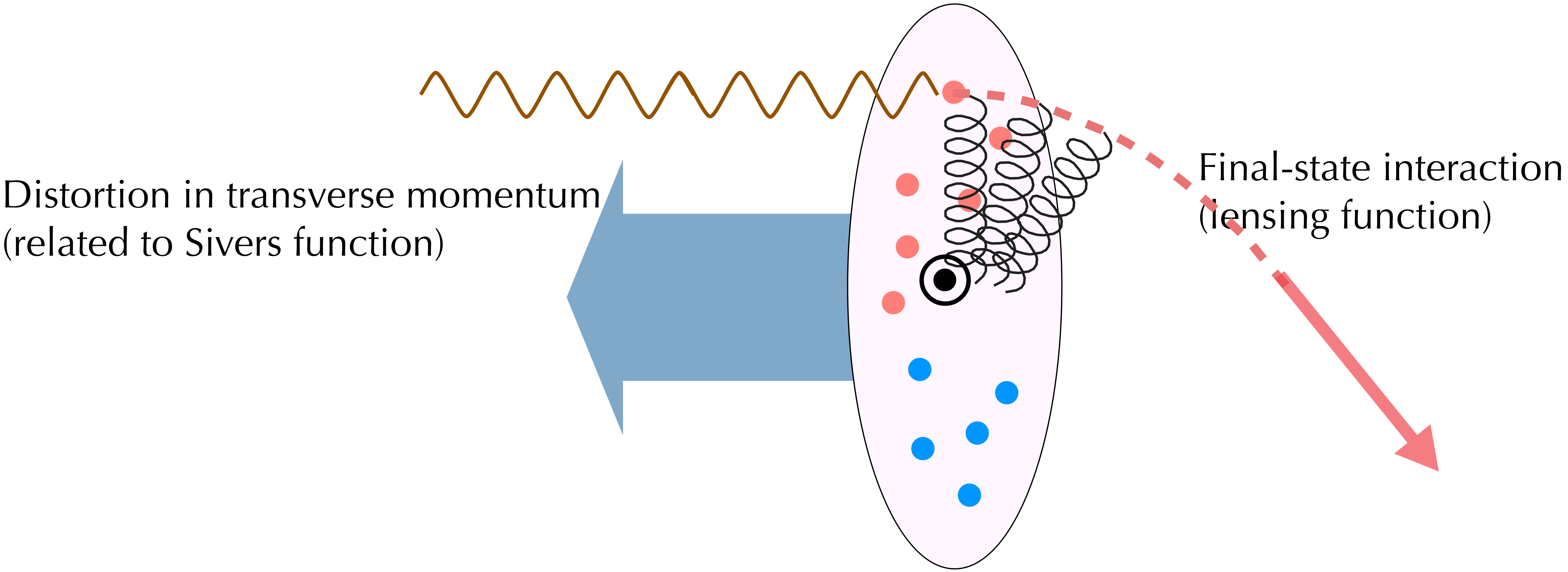
Lensing function

F.T. of E(x,0,t)

*Burkardt, PRD **66** (2002) 114005*

*Burkardt, Pasquini, EPJ A**52** (2016) 161*

Model relation TMD $\longleftrightarrow$ GPD



Distortion in transverse momentum
(related to Sivers function)

Final-state interaction
(lensing function)

$$-\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^2) \;\simeq\; \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x,\vec{b}_T^2)\right)'$$

Sivers function Lensing function F.T. of E(x,0,t)

Burkardt, PRD ***66*** *(2002) 114005*

*Burkardt, Pasquini, EPJ A****52*** *(2016) 161*

Key information from TMDs

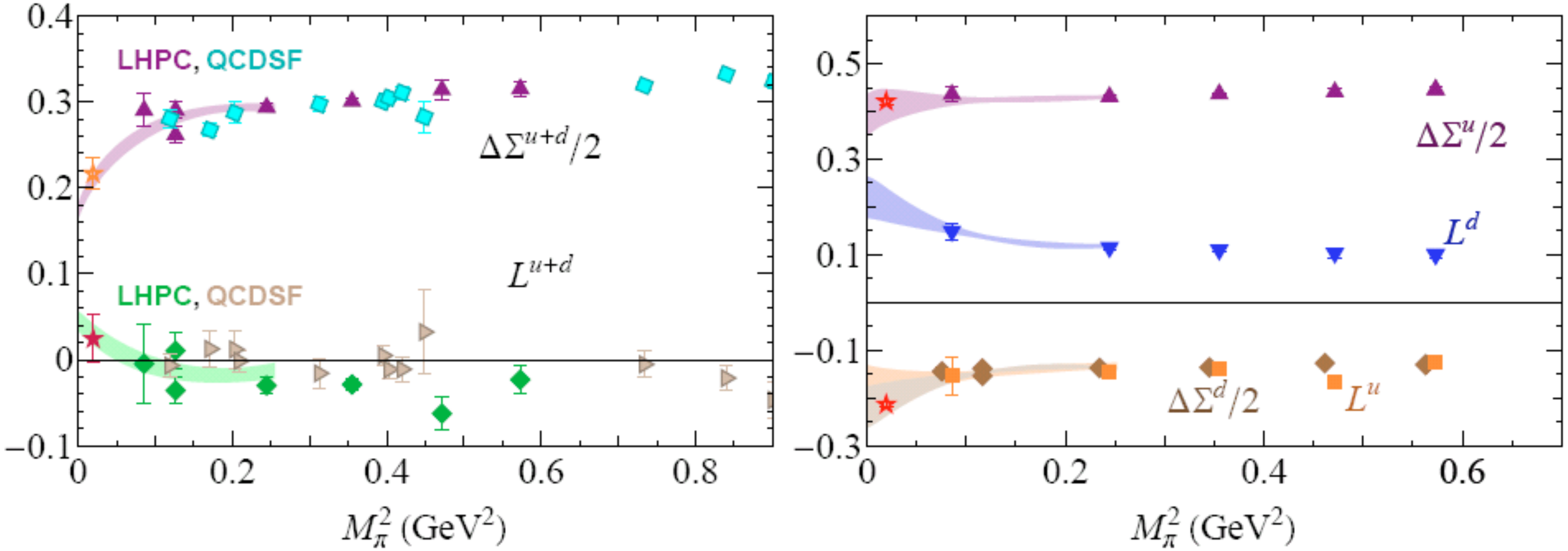
- Spin-Spin and Spin-Orbit Correlations of partons
- Transverse momentum size
- Test what we can calculate with QCD (perturbative and lattice)
- Non-perturbative structure we cannot calculate with QCD

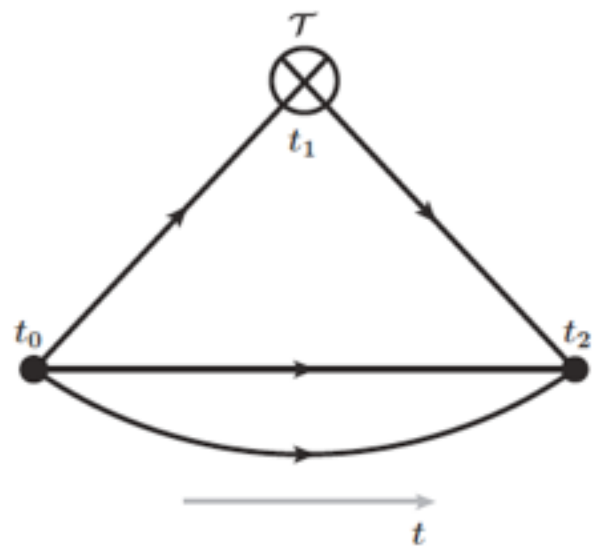
Lattice Calculations of Angular Momentum

$$L^u + L^d \approx 0$$

$$J^u > 0, \quad J^d \approx 0$$

without disconnected insertion





connected insertions (CI)

Lattice Calculations of Angular Momentum

without disconnected insertion

$$L^u + L^d \approx 0$$

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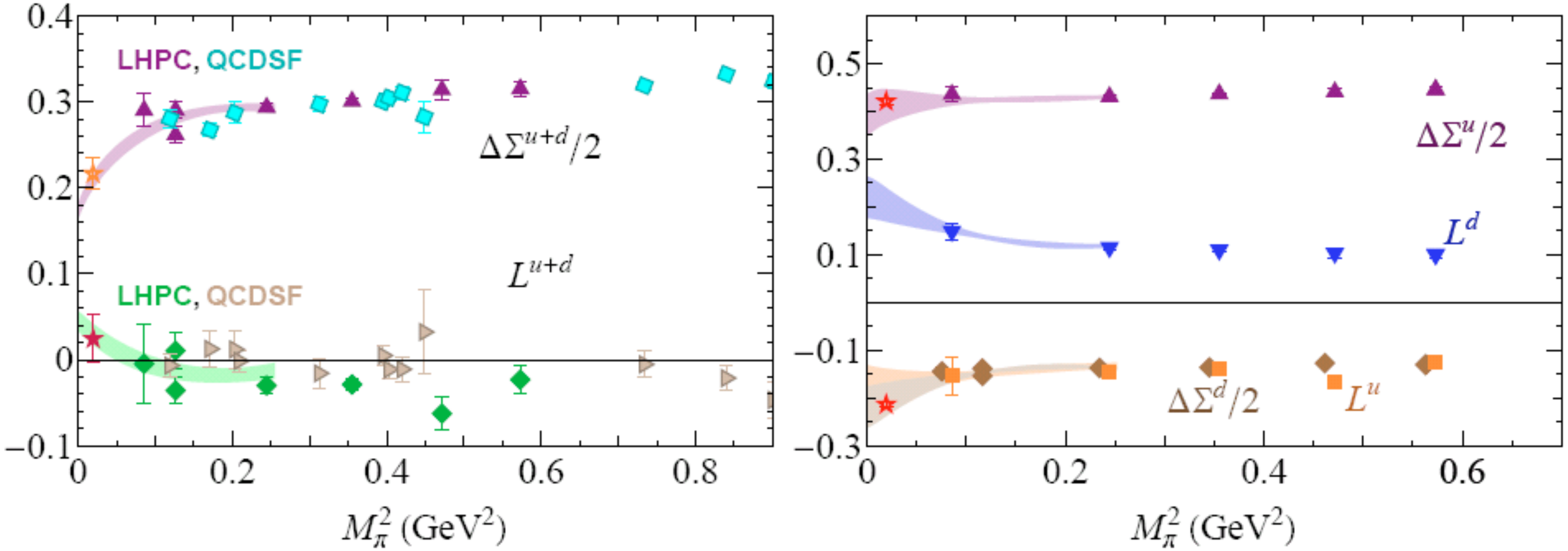
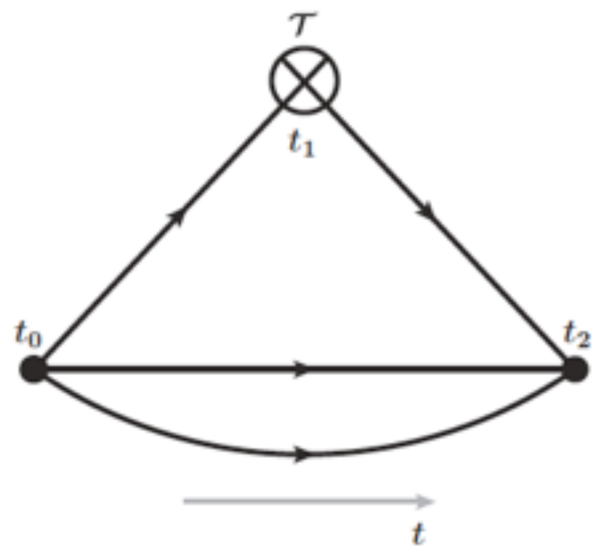
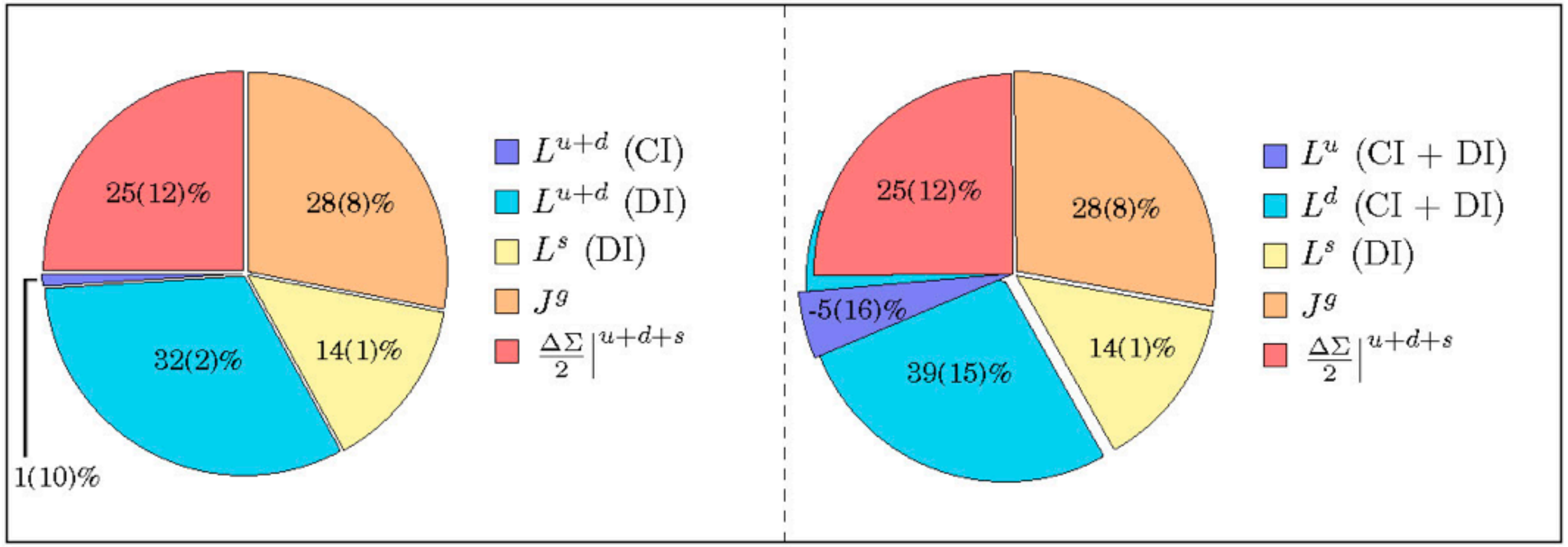


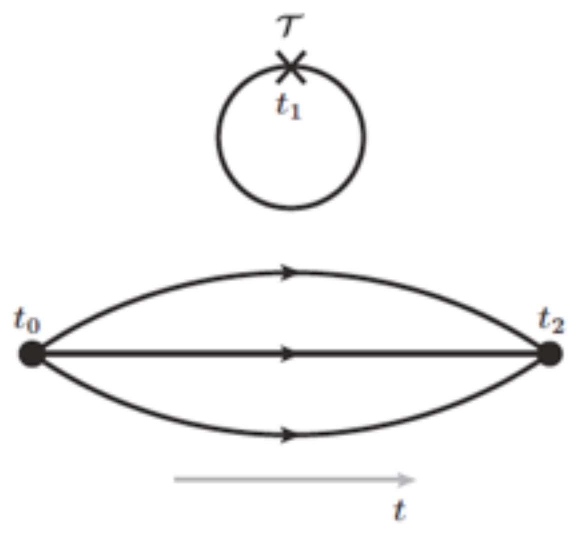


connected insertions (CI)

with disconnected insertion

$$L^u + L^d \approx 33\%$$





disconnected insertions (DI)

*Deka et al., PRD **91** (2015) 014505*

Angular Momentum Relation ("Ji's Sum Rule")

*X. Ji, PRL **78** (1997) 610*

quark and gluon contribution to the nucleon spin

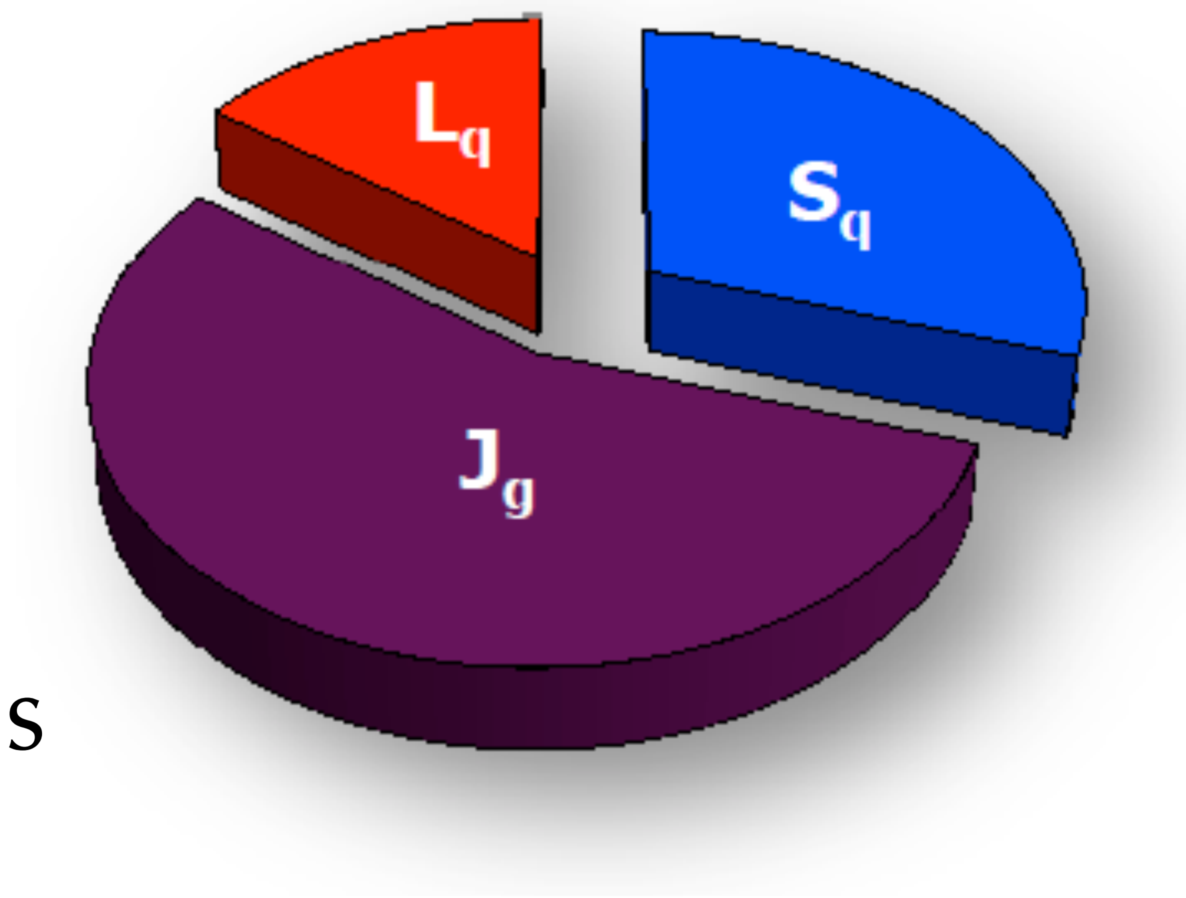
$$J^{q,g} = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x \left(\underset{\substack{\downarrow \\ \text{unpolarized PDF}}}{H^{q,g}(x,0,0)} + \underset{\substack{\downarrow \\ \text{not directly accessible}}}{E^{q,g}(x,0,0)} \right)$$

Proton spin decomposition

$$J^q = L^q + \overset{\substack{\frac{1}{2}\Delta\Sigma \text{ from DIS} \\ \uparrow}}{S^q}$$

gauge invariant decomposition

sum rule for L^q from twist-3 GPDs

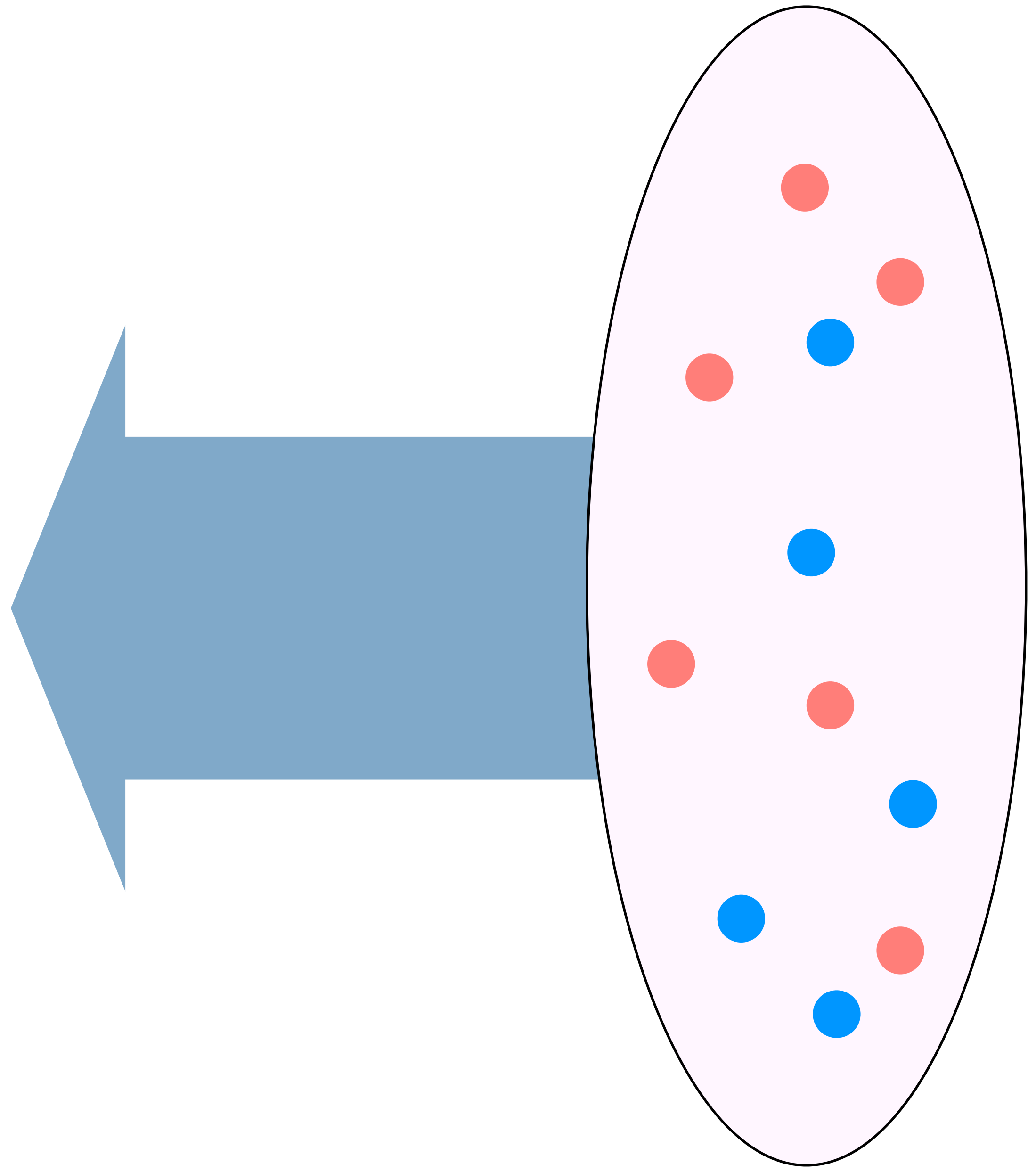


J^g

no further gauge-invariant decomposition

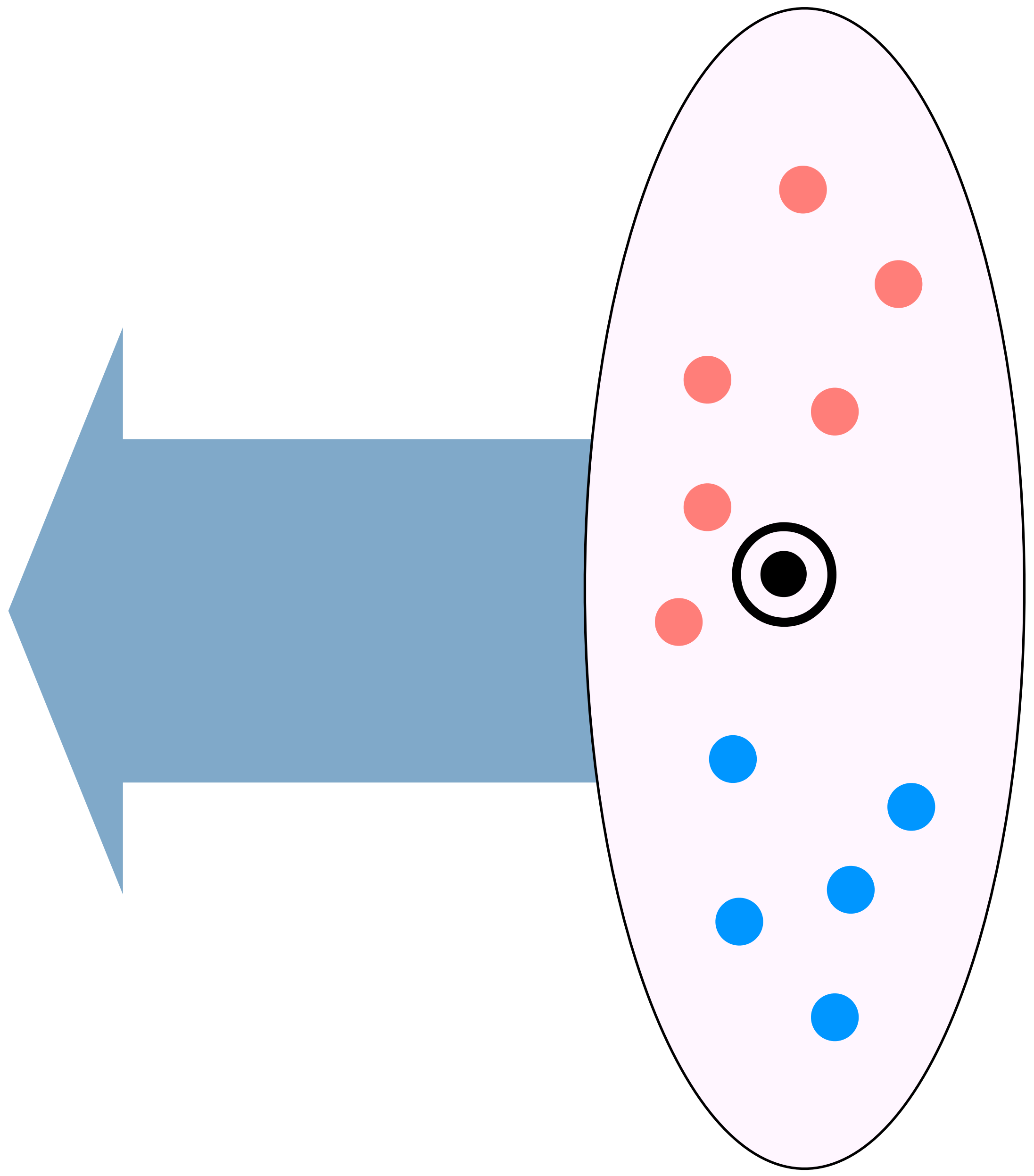
Model relation TMD ⟷ GPD

unpolarized quark in unpolarized nucleon



Burkardt, PRD ***66*** *(2002) 114005*

Model relation TMD ⟷ GPD

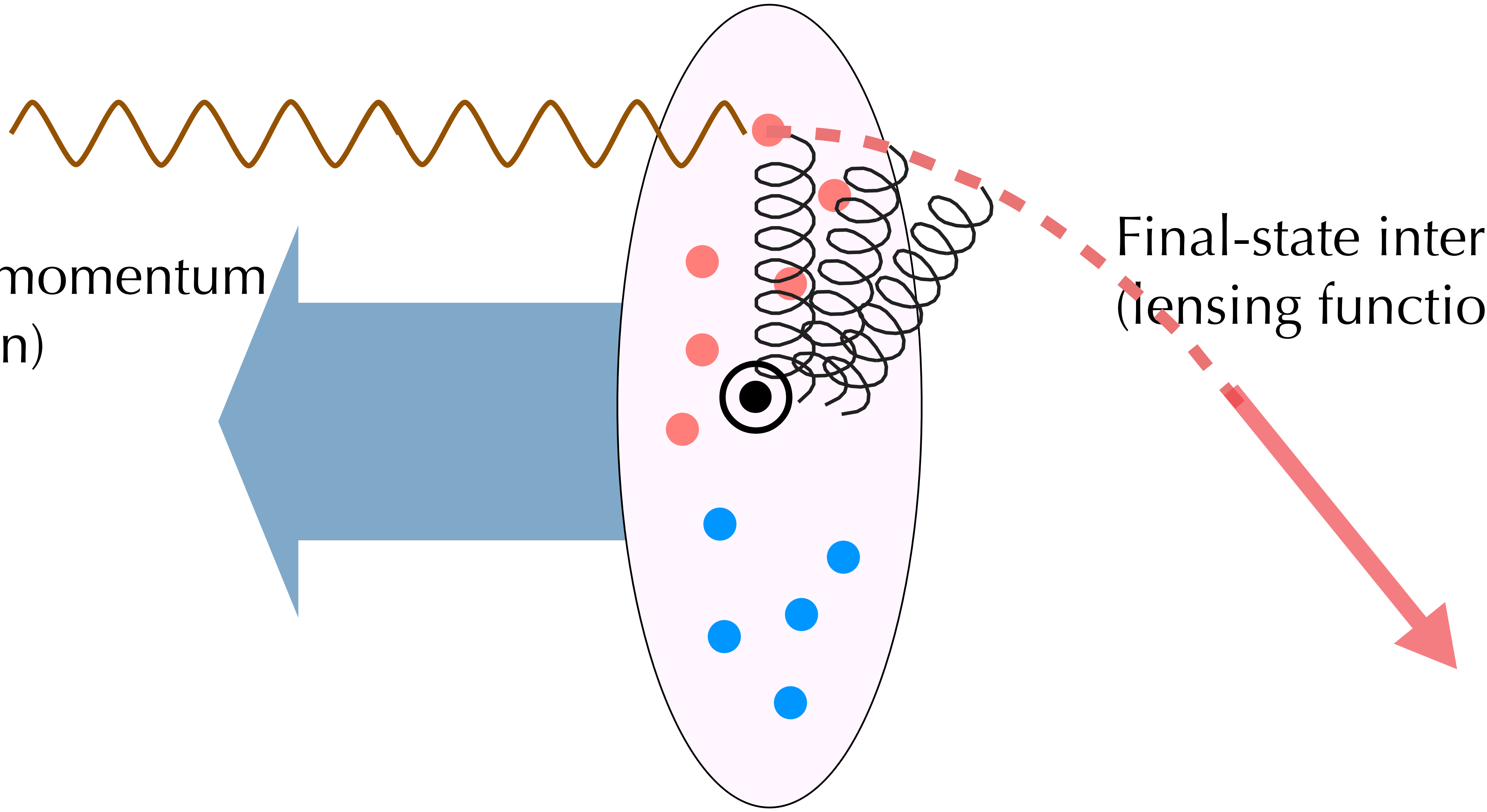


unpolarized quark in transversely pol. nucleon

Distortion in impact parameter
(related to GPD E)

*Burkardt, PRD **66** (2002) 114005*

Model relation TMD ⟷ GPD

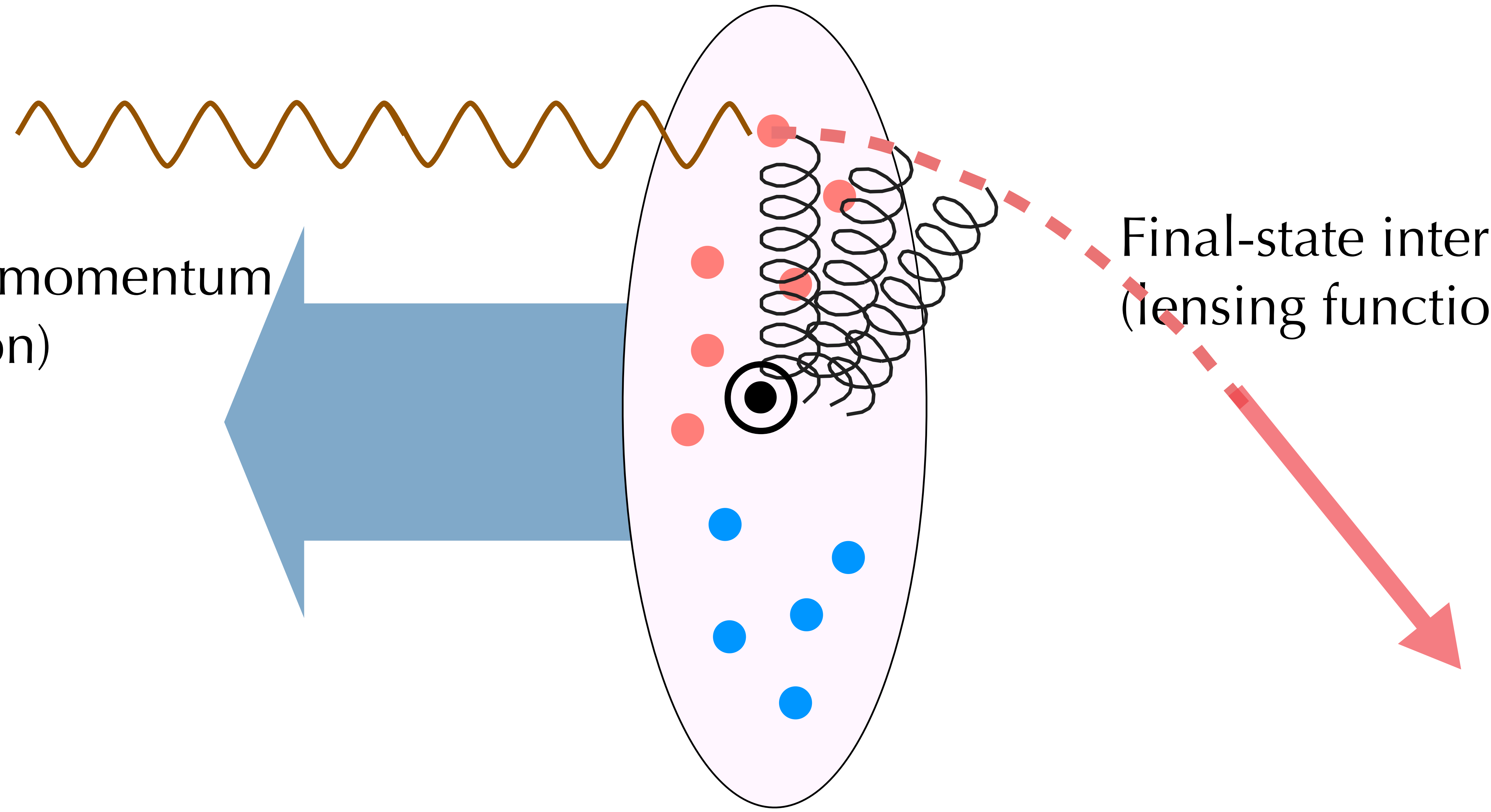


Distortion in transverse momentum
(related to Sivers function)

Final-state interaction
(lensing function)

Burkardt, PRD ***66*** *(2002) 114005*

Model relation TMD ⟷ GPD



Distortion in transverse momentum
(related to Sivers function)

Final-state interaction
(lensing function)

*Burkardt, PRD **66** (2002) 114005*

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Sivers function

Lensing function

F.T. of E(x,0,t)

Successful phenomenological applications:

*Bacchetta, Radici, PRL **107** (2011) 212001*

*Gamberg, Schlegel, PLB **685** (2010) 95*

Conclusions

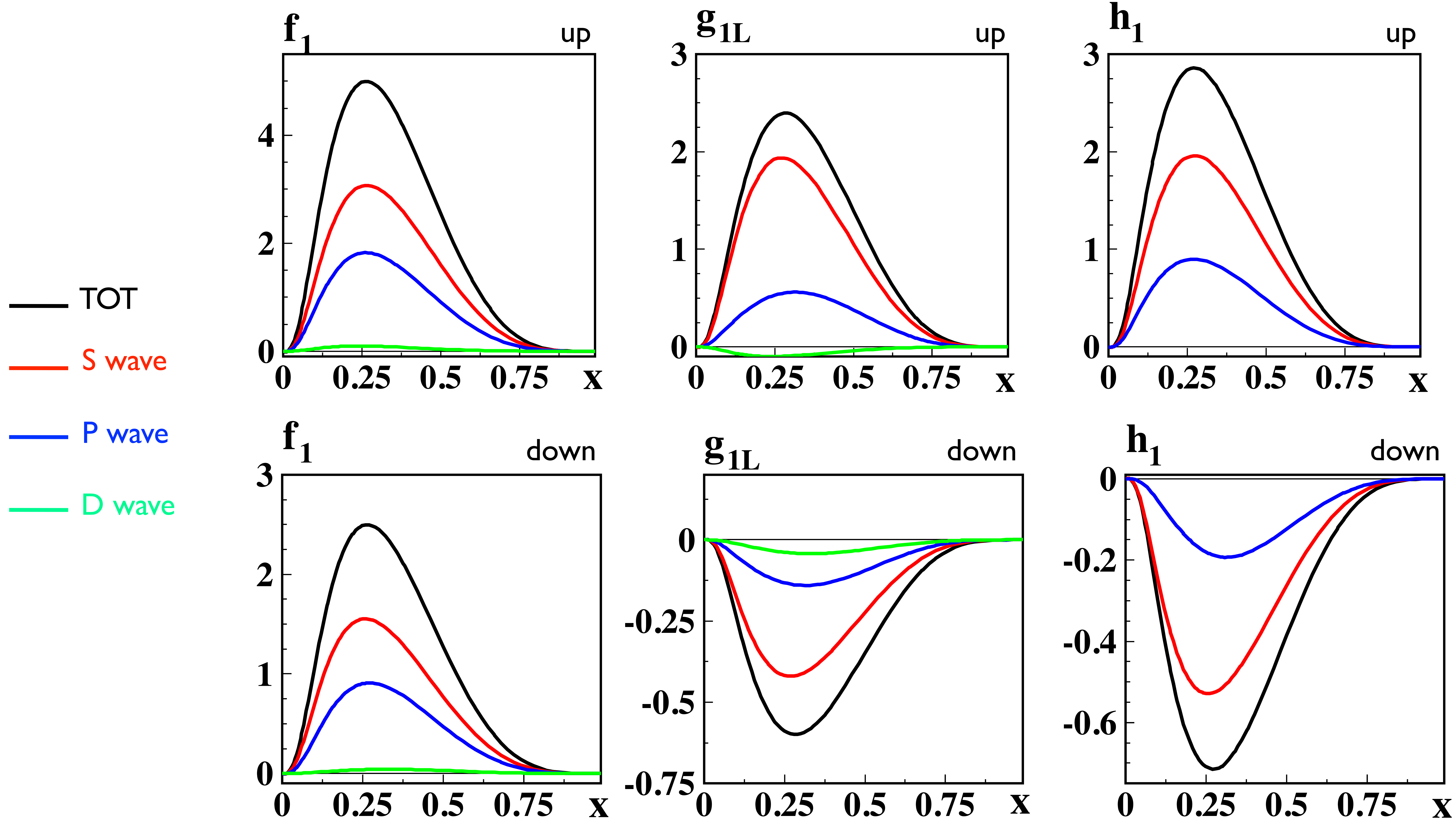
- TMDs and GPDs extend the concept of standard PDFs and provide a 3D description of the partonic structure of the nucleon

- TMDs and GPDs provide complementary information and allow us to investigate aspects of nucleon structure that are not accessible to standard collinear PDFs

- A lot of data is already available, but we expect more from e+e–, SIDIS at higher energies, Drell-Yan, DVCS, ...

- Some parametrizations of TMDs and GPDs are available, but we are a long way from anything similar to PDF global fits

OAM content of TMDs



OAM content of TMDs