Unified description of seagull cancellations and infrared finiteness of gluon propagators

A.C.Aguilar, D.Binosi, C.T.Figueiredo, and J.P.

Phys. Rev. D94, no. 4, 045002 (2016), arXiv:1604.08456

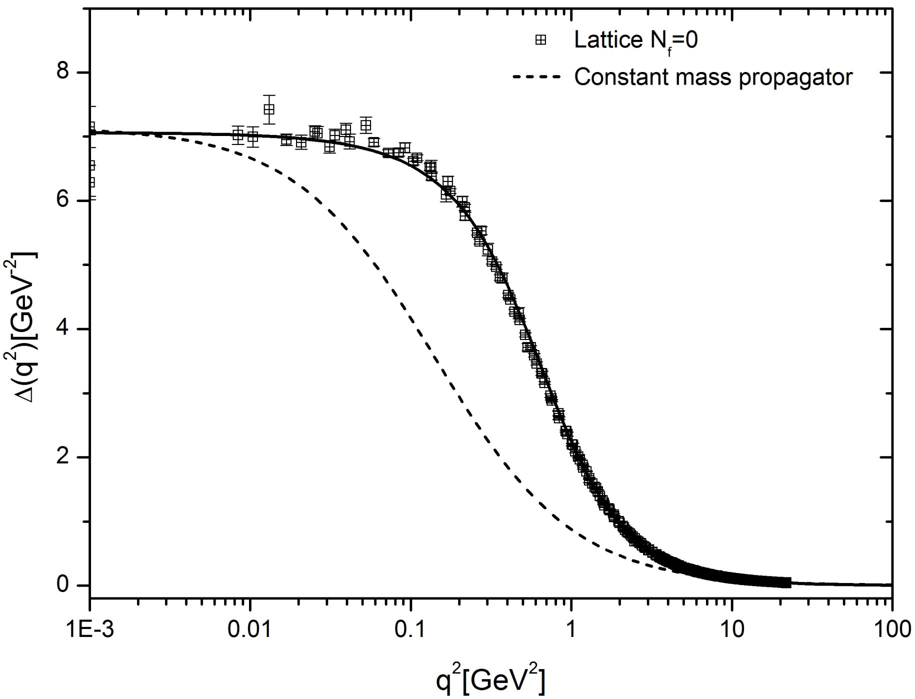
Joannis Papavassiliou

Department of Theoretical Physics and IFIC
University of Valencia-CSIC

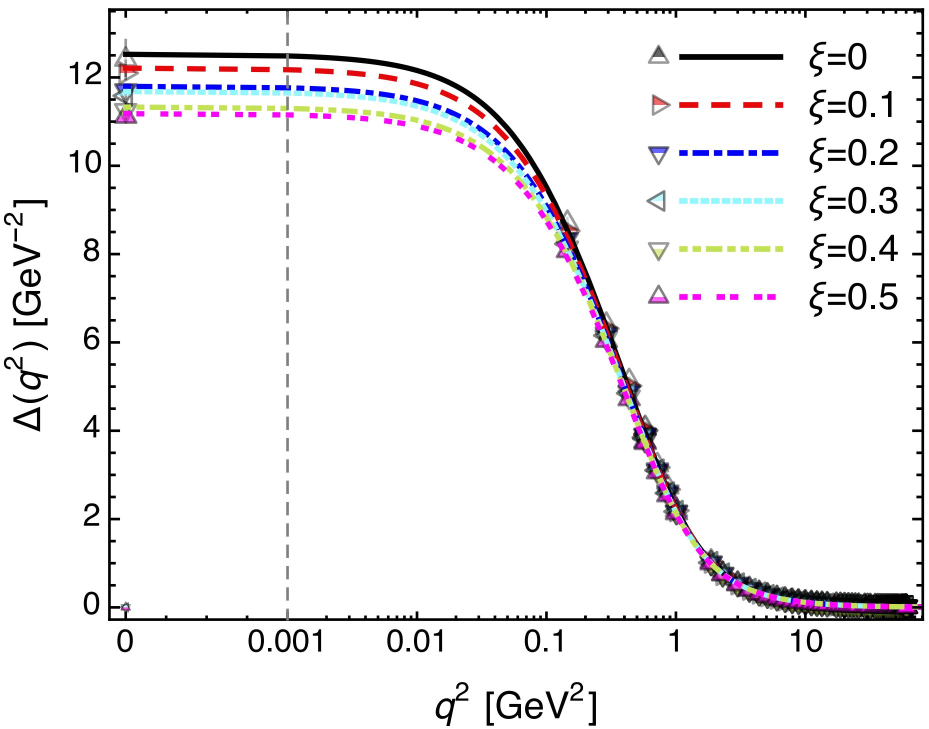
Large volume lattice simulations

The gluon propagator saturates in the deep infrared,
both in the Landau gauge and away from it !

I.L.Bogolubsky, et al , PoS LAT2007, 290 (2007)



P. Bicudo et al, Phys. Rev.D 92, 114514 (2015)



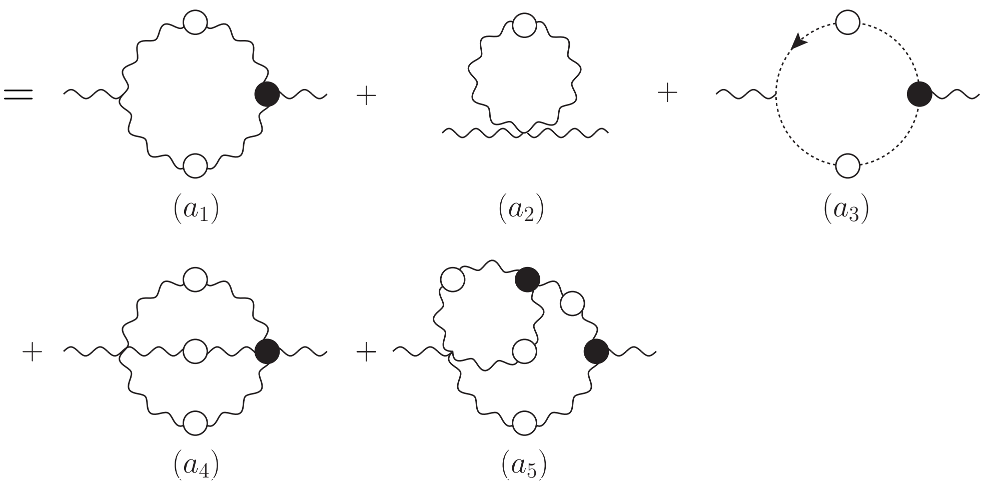
Saturation explained through dynamical gluon mass generation

$$\Delta^{-1}(0) = m^2(0)$$

Non-perturbative explanation in the continuum : Schwinger-Dyson equations

Gluon self-energy
in linear covariant gauges

$$\Pi_{\mu\nu}(q) =$$



The fully-dressed vertices satisfy their own complicated SDEs

Truncations and approximations seem unavoidable

But poorly controlled truncations tend to compromise certain important properties

For a better behaved truncation scheme

switch from the linear covariant gauges to Background Field Method

BFM reminder: Basic concepts

- The BFM is a special quantization scheme: Split the gauge field

$$A^a_\mu \to \widetilde{A}^a_\mu + A^a_\mu$$

$\widetilde{A}^a_\mu$ → background field; A^a_μ → quantum (fluctuating) field;

- In the generating functional integrate only over A^a_μ
- The gauge fixing condition $G(A)$ is chosen to be → $G(A) = D^\mu A^a_\mu$
  where $D_\mu = \partial_\mu - i\widetilde{A}^a_\mu t^a$ instead of the "standard"→ $G(A) = \partial^\mu A^a_\mu$

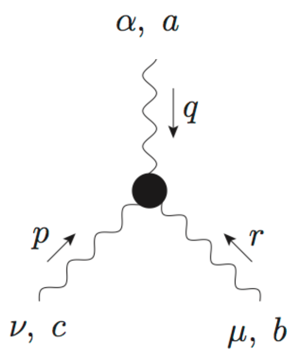
- The gauge-fixed $\mathcal{L}$ is invariant under the transformations:

$$\widetilde{A}^a_\mu \to \widetilde{A}^a_\mu + D_\mu \theta^a \qquad\qquad A^a_\mu \to A^a_\mu - f^{abc} A^a_\mu \theta^b$$

- $\widetilde{A}^a_\mu$ carries the local gauge transformation $\partial^\mu \theta^a$
- A^a_μ transforms as matter field in the adjoint.

B. S. DeWitt, Phys. Rev. 162 (1967) 195—1239
G. 't Hooft, In *Karpacz 1975, Proceedings, Acta Universitatis Wratislaviensis No.368, 1976, 345-369
L. F. Abbott, Nucl. Phys. B185 (1981) 189

Abelian-like Slavnov-Taylor identities



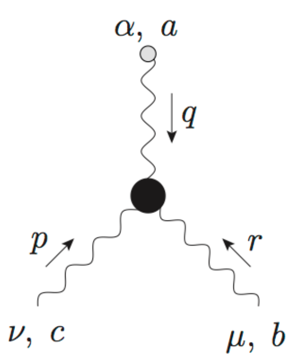
$F(q^2)$: Ghost dressing function

$D(q^2) = F(q^2)/q^2$: Ghost propagator

$H_{\mu\nu}(q,r,p)$: Ghost-gluon kernel

$$q^\alpha \Gamma_{\alpha\mu\nu}(q,r,p) = iF(q)\left[\Delta^{-1}_{\sigma\nu}(r) H^\sigma_\mu(q,r,p) - \Delta^{-1}_{\sigma\mu}(p) H^\sigma_\nu(q,r,p)\right]$$

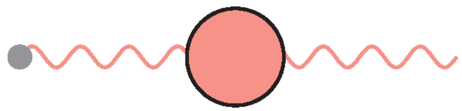
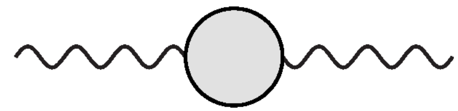
INSTEAD



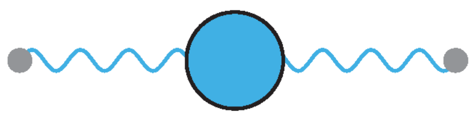
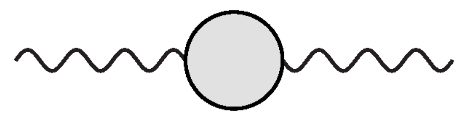
$$q^\alpha \widetilde{\Gamma}_{\alpha\mu\nu}(q,r,p) = i\Delta^{-1}_{\mu\nu}(r) - i\Delta^{-1}_{\mu\nu}(p)$$

Abelian-like ("ghost-free")

Background and quantum propagators are related !

 $= (\mathbf{1}+\mathbf{G})\otimes$ 

$$\widetilde{\Delta}(q) = [\mathbf{1}+\mathbf{G}(\mathbf{q})]\Delta(q)$$

 $= (\mathbf{1}+\mathbf{G})^{\mathbf{2}}\otimes$ 

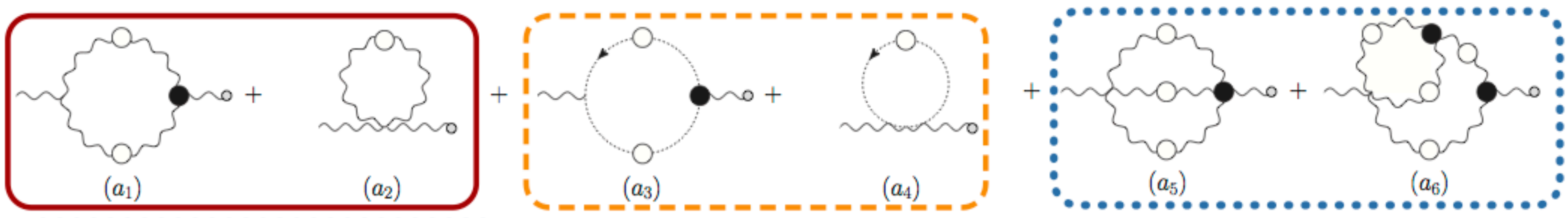
$$\widehat{\Delta}(q) = [\mathbf{1}+\mathbf{G}(\mathbf{q})]^{\mathbf{2}}\Delta(q)$$

Known function in the Landau gauge

P. A. Grassi, T. Hurth and M. Steinhauser, Annals Phys. 288 , 197 (2001)
D. Binosi and J. P., Phys. Rev. D66, 025024 (2002)

SDEs in the BFM framework

$\widetilde{\Pi}_{\mu\nu}(q) =$



$$\Delta^{-1}(q^2)P_{\mu\nu}(q) = \frac{q^2 P_{\mu\nu}(q) + i\sum_{i=1}^{6}(a_i)_{\mu\nu}}{1+G(q^2)},$$

Transversality is enforced separately for gluon and ghost loops, and order by order in the “dressed-loop” expansion!

$$q^\mu \widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p) \quad \Rightarrow \quad q^\mu[(a_1)+(a_2)]_{\mu\nu} = 0$$

$$q^\mu \widetilde{\Gamma}_\mu(q,r,-p) = D^{-1}(p) - D^{-1}(r) \quad \Rightarrow \quad q^\mu[(a_3)+(a_4)]_{\mu\nu} = 0$$

$$q^\mu \widetilde{\Gamma}^{mnrs}_{\mu\alpha\beta\gamma} = f^{mse}f^{ern}\Gamma_{\alpha\beta\gamma} + f^{mne}f^{esr}\Gamma_{\beta\gamma\alpha} + f^{mre}f^{ens}\Gamma_{\gamma\alpha\beta} \quad \Rightarrow \quad q^\mu[(a_5)+(a_6)]_{\mu\nu} = 0$$

A.C. Aguilar and J.P. ,JHEP 0612, 012 (2006)
D. Binosi and J. P. , Phys.Rev. D 77, 061702 (2008); JHEP 0811:063,2008.

From Takahashi to Ward identities

$$q^\mu \widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

Taylor expansion of both sides around $q = 0$

assuming no poles $1/q^2$ in the form factors of $\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p)$

$$\Downarrow$$

$$\widetilde{\Gamma}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial\Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu}$$

with

$$\Delta^{-1}_{\alpha\beta}(r) = i\left[g_{\alpha\beta} - \frac{r_\alpha r_\beta}{r^2}\right]\Delta^{-1}(r^2) + \frac{i}{\xi}\, r_\alpha r_\beta$$

Ward identities and vertex form factors

$$\widetilde{\Gamma}^{\mu\alpha\beta}(q,r,p) = \sum_{i=1}^{14} \widetilde{A}_i(q^2,r^2,q\cdot r)\, b_i^{\mu\alpha\beta}$$

$$b_1^{\mu\alpha\beta} = q^\mu g^{\alpha\beta};\quad b_2^{\mu\alpha\beta} = q^\mu q^\alpha q^\beta;\quad b_3^{\mu\alpha\beta} = q^\mu q^\alpha r^\beta;\quad b_4^{\mu\alpha\beta} = q^\mu r^\alpha q^\beta;\quad b_5^{\mu\alpha\beta} = q^\mu r^\alpha r^\beta,$$
$$b_6^{\mu\alpha\beta} = r^\mu g^{\alpha\beta};\quad b_7^{\mu\alpha\beta} = r^\mu q^\alpha q^\beta;\quad b_8^{\mu\alpha\beta} = r^\mu q^\alpha r^\beta;\quad b_9^{\mu\alpha\beta} = r^\mu r^\alpha q^\beta;\quad b_{10}^{\mu\alpha\beta} = r^\mu r^\alpha r^\beta,$$
$$b_{11}^{\mu\alpha\beta} = q^\alpha g^{\beta\mu};\quad b_{12}^{\mu\alpha\beta} = q^\beta g^{\alpha\mu};\quad b_{13}^{\mu\alpha\beta} = r^\alpha g^{\beta\mu};\quad b_{14}^{\mu\alpha\beta} = r^\beta g^{\alpha\mu}.$$

$$\widetilde{\Gamma}^{\mu\alpha\beta}(0,r,-r) = \widetilde{A}_6(r^2) r^\mu g^{\alpha\beta} + \widetilde{A}_{10}(r^2) r^\mu r^\alpha r^\beta + \widetilde{A}_{13} r^\alpha g^{\beta\mu} + \widetilde{A}_{14} r^\beta g^{\alpha\mu}$$

From Ward identity

$$\widetilde{A}_6(r^2) = 2\frac{\partial \Delta^{-1}(r^2)}{\partial r^2},\quad \widetilde{A}_{10}(r^2) = -2\frac{\partial}{\partial r^2}\left(\frac{\Delta^{-1}(r^2)}{r^2}\right),\quad \widetilde{A}_{13} = \widetilde{A}_{14} = \frac{1}{\xi} - \frac{\Delta^{-1}(r^2)}{r^2}$$

Seagull identity

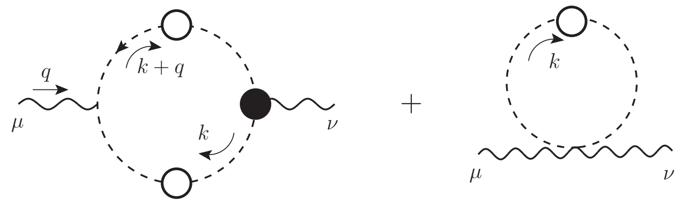
In dimensional regularization, any function that satisfies the criterion of Wilson

$$\int_k f(k^2) = \frac{1}{(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \int_0^\infty dy y^{\frac{d}{2}-1} f(y) = \text{finite}, \qquad \text{for } 0 < d < d^*$$

$$\longrightarrow \quad \int_k k^2 \frac{\partial f(k^2)}{\partial k^2} + \frac{d}{2} \int_k f(k^2) = 0$$

Integrate by parts, drop the surface term, do analytic continuation

One-loop example: scalar QED

$\Pi_{\mu\nu}(q) =$ 

$$\int_k \frac{k^2}{(k^2+m^2)^2} = -(4\pi)^{\frac{d}{2}} \left(\frac{d}{2}\right) \Gamma\left(1-\frac{d}{2}\right) (m^2)^{\frac{d}{2}-1}$$

$$\int_k \frac{1}{k^2+m^2} = (4\pi)^{\frac{d}{2}} \Gamma\left(1-\frac{d}{2}\right) (m^2)^{\frac{d}{2}-1}$$

$$\int_k \frac{k^2}{(k^2+m^2)^2} + \frac{d}{2}\int_k \frac{1}{k^2+m^2} = 0$$

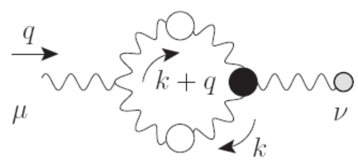
Seagull identity

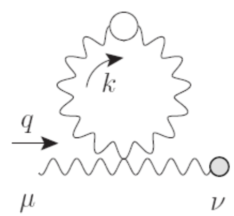
$$f(k^2) = \frac{1}{k^2+m^2}$$

Gluon propagator at the origin

$$\widetilde{\Gamma}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial\Delta^{-1}_{\alpha\beta}(r)}{\partial r^{\mu}} \quad \Rightarrow \quad \Delta^{\alpha\rho}(k)\Delta^{\beta\sigma}(k)\widetilde{\Gamma}^{\mu}_{\sigma\rho}(0,k,-k) = \frac{\partial\Delta^{\alpha\beta}(k)}{\partial k^{\mu}}$$

$$\Delta^{-1}(0) = \lim_{q\to 0}\mathrm{Tr}\Bigg\{$$





$$\Bigg\} \sim \int_k k^2 \frac{\partial\Delta(k^2)}{\partial k^2} + \frac{d}{2}\int_k \Delta(k) = 0$$

Ward Identities (No poles)

Seagull identity

BFM
Ward identites

+

Seagull identity

$$\Rightarrow \quad \Delta^{-1}(0) = 0$$

No "gluon mass"

Exact result from BFM Schwinger-Dyson equation !

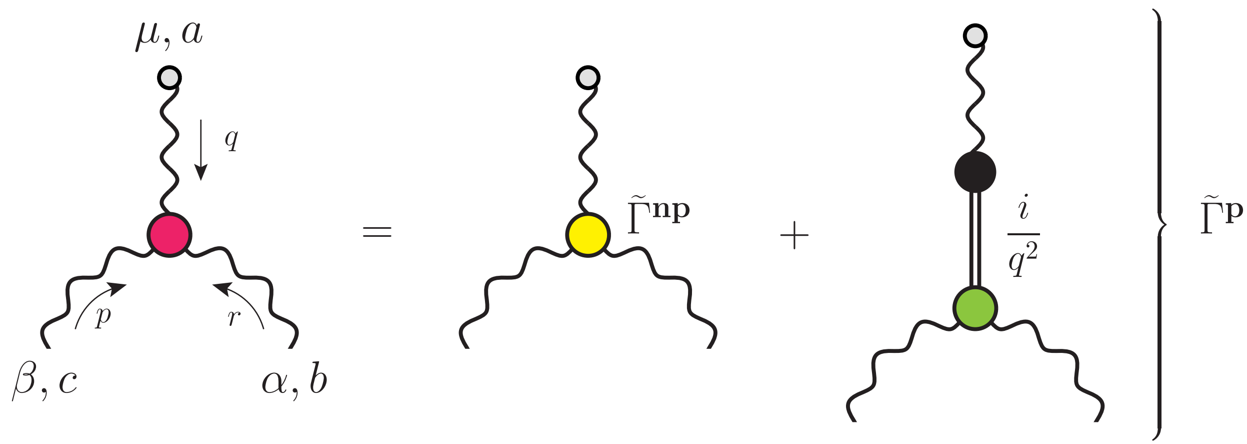
Vertices with massless poles

To evade the previous result, one must relax one of the underlying assumptions

In particular, the derivation of the WI hinges on the absence of poles $1/q^2$

Therefore, let us introduce poles in $\widetilde{\Gamma}_{\mu\alpha\beta}$

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p)$$



Explicit implementation of the Schwinger mechanism in Yang-Mills theories

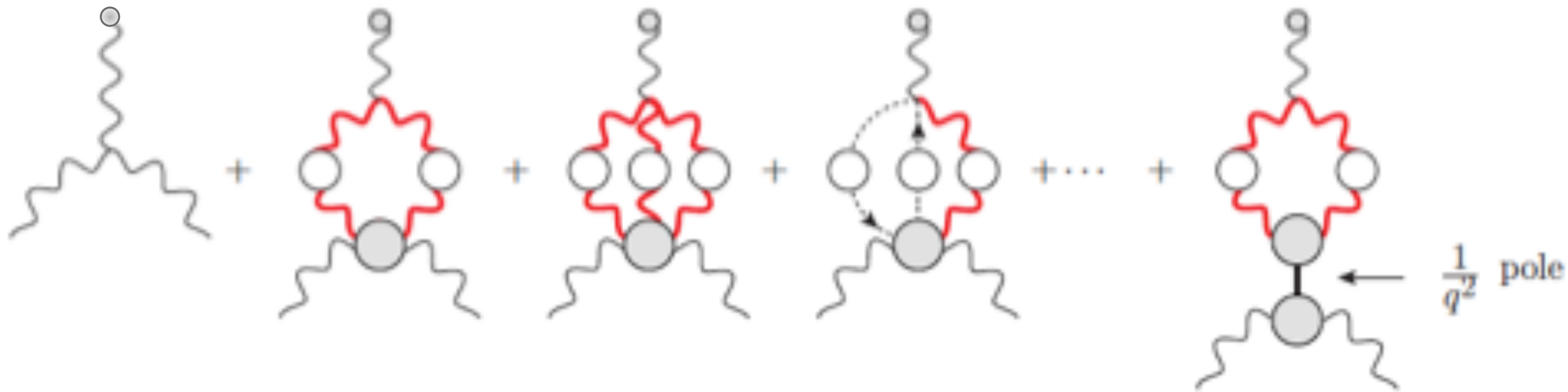
J.S. Schwinger
Phys. Rev.125, 397 (1962);
Phys.Rev.128, 2425 (1962).

R. Jackiw and K. Johnson, Phys. Rev. D8, 2386 (1973)

E. Eichten and F. Feinberg, Phys. Rev. D10, 3254 (1974)

The poles correspond to massless bound-state (colored) excitations

$$\widetilde{\Gamma}_{\mu\alpha\beta} =$$



$\widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}$ $\widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}$

The poles are longitudinally coupled

Act as composite Nambu-Goldstone bosons

$$\widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p) = \frac{q_\mu}{q^2}\widetilde{C}_{\alpha\beta}(q,r,p)$$

$$= \frac{q^\mu}{q^2}\left[\widetilde{C}_1\, g^{\alpha\beta} + \widetilde{C}_2\, q^\alpha q^\beta + \widetilde{C}_3\, q^\alpha r^\beta + \widetilde{C}_4\, r^\alpha q^\beta + \widetilde{C}_5\, r^\alpha r^\beta\right]$$

$\Rightarrow$ $\widetilde{A}_i = \widetilde{A}_i^{\mathbf{np}} + \dfrac{\widetilde{C}_i}{q^2}\,,\ \ i = 1, ..., 5$ while $\widetilde{A}_i = \widetilde{A}_i^{\mathbf{np}}\,,\ \ i = 6, ..., 14$

Ward identities in the presence of poles

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \frac{q_\mu}{q^2}\widetilde{C}_{\alpha\beta}(q,r,p)$$

$$q^\mu \widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

Same ST identity !

$$q^\mu \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{C}_{\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p),$$

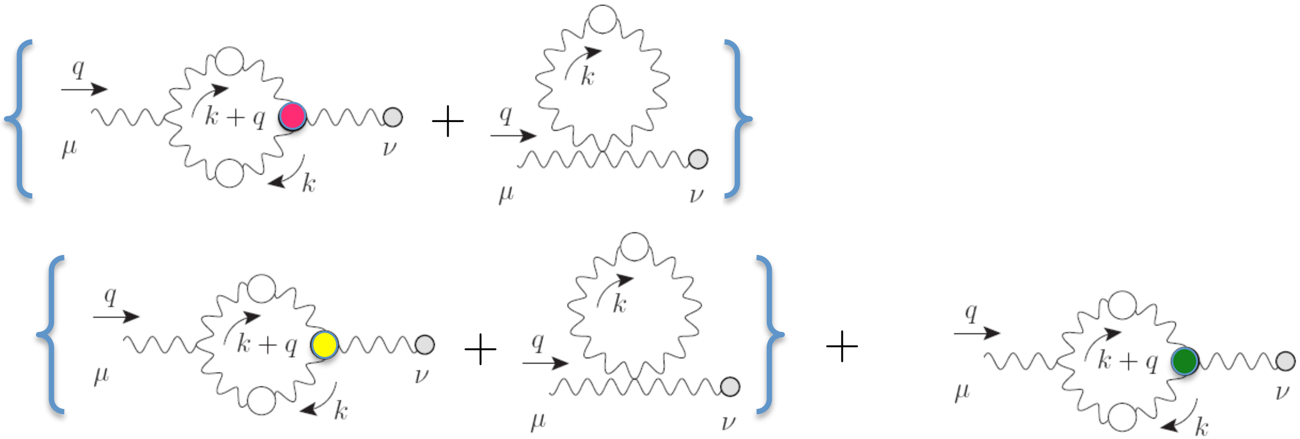
$$q^\mu \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) + \widetilde{C}_{\alpha\beta}(0,r,-r) + q^\mu \left\{\frac{\partial}{\partial q^\mu}\widetilde{C}_{\alpha\beta}(q,r,p)\right\}_{q=0} = -i\frac{\partial \Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu}$$

$$\widetilde{C}_{\alpha\beta}(0,r,-r) = 0$$

$$\widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial \Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu} - \left\{\frac{\partial}{\partial q^\mu}\widetilde{C}_{\alpha\beta}(q,r,p)\right\}_{q=0}$$

Evading the seagull identity

$$\Delta^{-1}(0) = \lim_{q\to 0} \mathrm{Tr}$$



Triggers seagull identity exactly as before

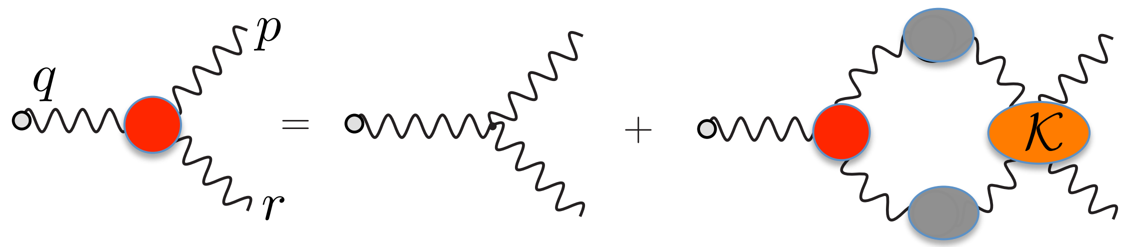
Vanishes identically

$$\Delta^{-1}(0) \sim \int_k k^2 \Delta^2(k^2) \widetilde{C}_1'(k^2)$$

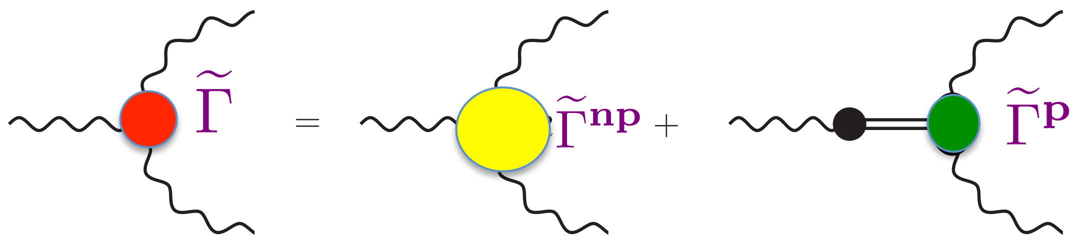
A.C.Aguilar, D.Binosi, C.T.Figueiredo and J.P., Phys. Rev. D 94, no. 4, 045002 (2016)

Dynamical formation of massless poles

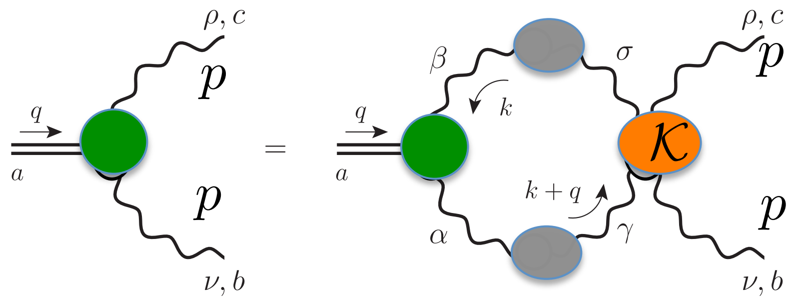
Bethe-Salpeter equation
for the full vertex



Substitute:



$q \to 0$

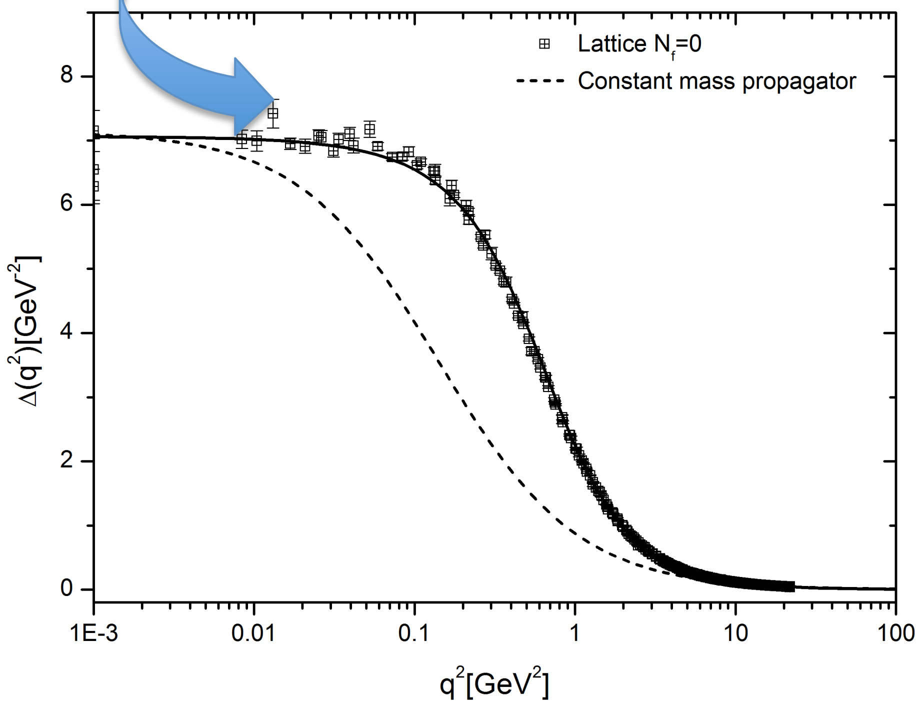


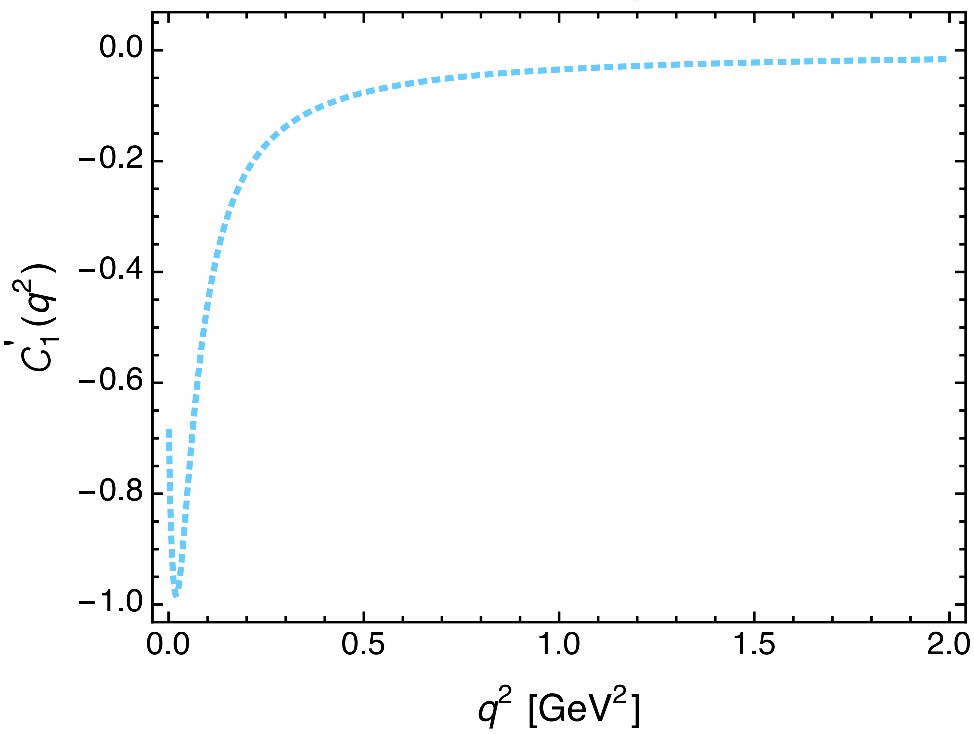
Dynamical equation for massless pole formation

$$\widetilde{C}_1'(p^2) = \lambda \int_k \widetilde{C}_1'(k^2)\Delta^2(k)\Delta(k+p)\,\mathcal{K}(k,p)$$

Homogeneous integral equation coupled with

$$\Delta^{-1}(0) = -\rho \int_k k^2 \Delta^2(k^2)\widetilde{C}_1'(k^2)$$





… but are the poles "really" there ?

$$\widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial\Delta^{-1}_{\alpha\beta}(r)}{\partial r^{\mu}} - \left\{\frac{\partial}{\partial q^{\mu}}\widetilde{C}_{\alpha\beta}(q,r,p)\right\}_{q=0}$$

$$\widetilde{C}_{\alpha\beta}(q,r,p) = \widetilde{C}_{1}(q,r,p)g_{\alpha\beta} + \ldots$$

(Landau gauge)

$$\left\{\frac{\partial}{\partial q^{\mu}}\widetilde{C}_{\alpha\beta}(q,r,p)\right\}_{q=0} = 2\widetilde{C}_{1}'(r^2)r_{\mu}g_{\alpha\beta}$$

$$\widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) = \widetilde{A}^{\mathbf{np}}_{6}(r^2)r_{\mu}g_{\alpha\beta} + \widetilde{A}^{\mathbf{np}}_{10}(r^2)r_{\mu}r_{\alpha}r_{\beta} + \widetilde{A}^{\mathbf{np}}_{13}r_{\alpha}g_{\beta\mu} + \widetilde{A}^{\mathbf{np}}_{14}r_{\beta}g_{\alpha\mu}$$

$\Rightarrow$ In the limit $q \to 0$, $\widetilde{A}^{\mathbf{p}}_{1}$ "talks" to $\widetilde{A}^{\mathbf{np}}_{6}$!

Ward identity (without poles)

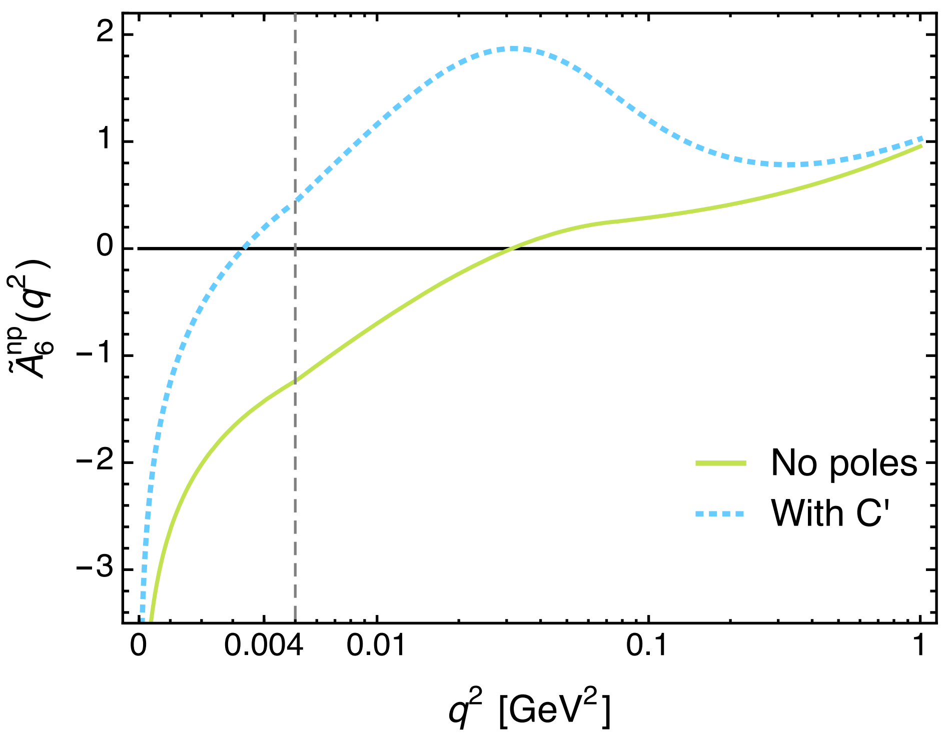
$$\widetilde{A}^{\mathbf{np}}_{6}(r^2) = 2\frac{\partial\Delta^{-1}(r^2)}{\partial r^2}$$

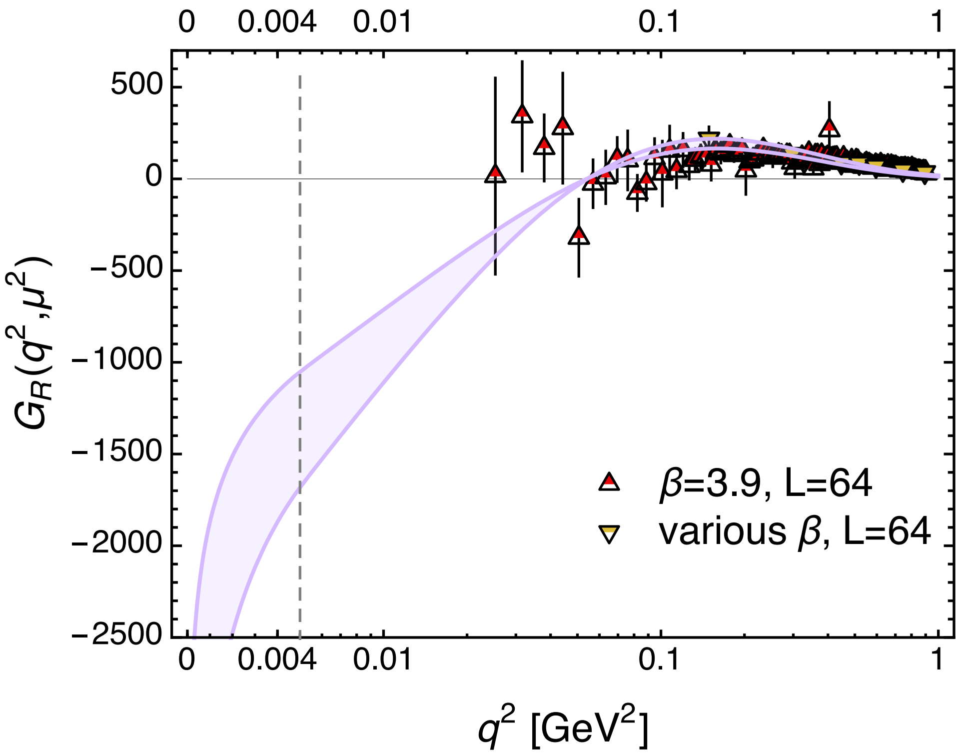
Ward identity (with poles!)

$$\widetilde{A}^{\mathbf{np}}_{6}(r^2) = 2\left[\frac{\partial\Delta^{-1}(r^2)}{\partial r^2} - \widetilde{C}_{1}'(r^2)\right].$$

The “existence” of the poles can be tested on the lattice (in principle)

Falsifiable mechanism !





Catch: The BFM three gluon vertex is not the same as the normal one !

$$\widetilde{A}_6^{\mathbf{np}} = (1+G)\,A_6^{\mathbf{np}} + \text{“stuff”}$$

The effect may be “masked”…