Non-perturbative Interactions of Quarks and Gluons

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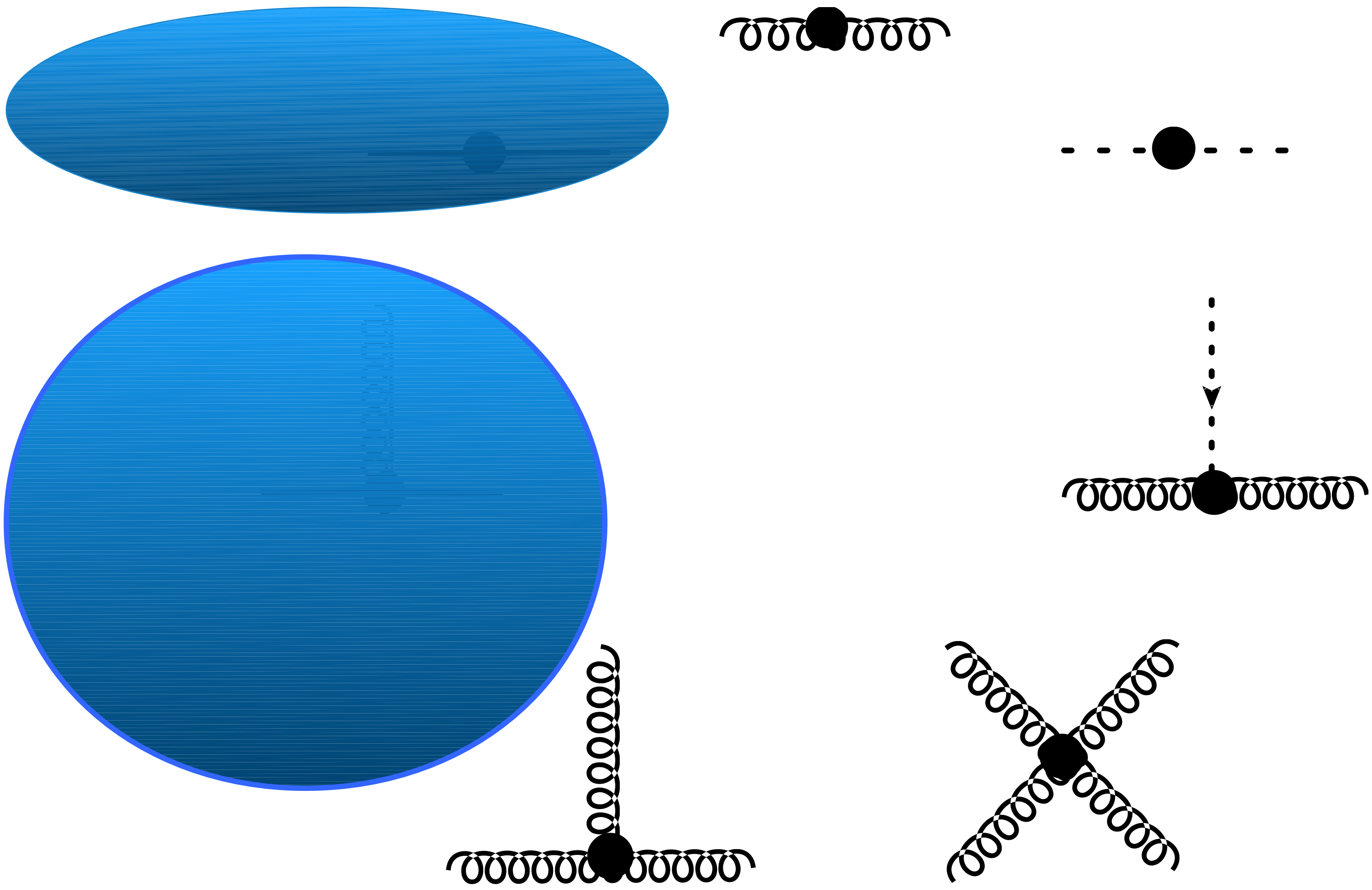
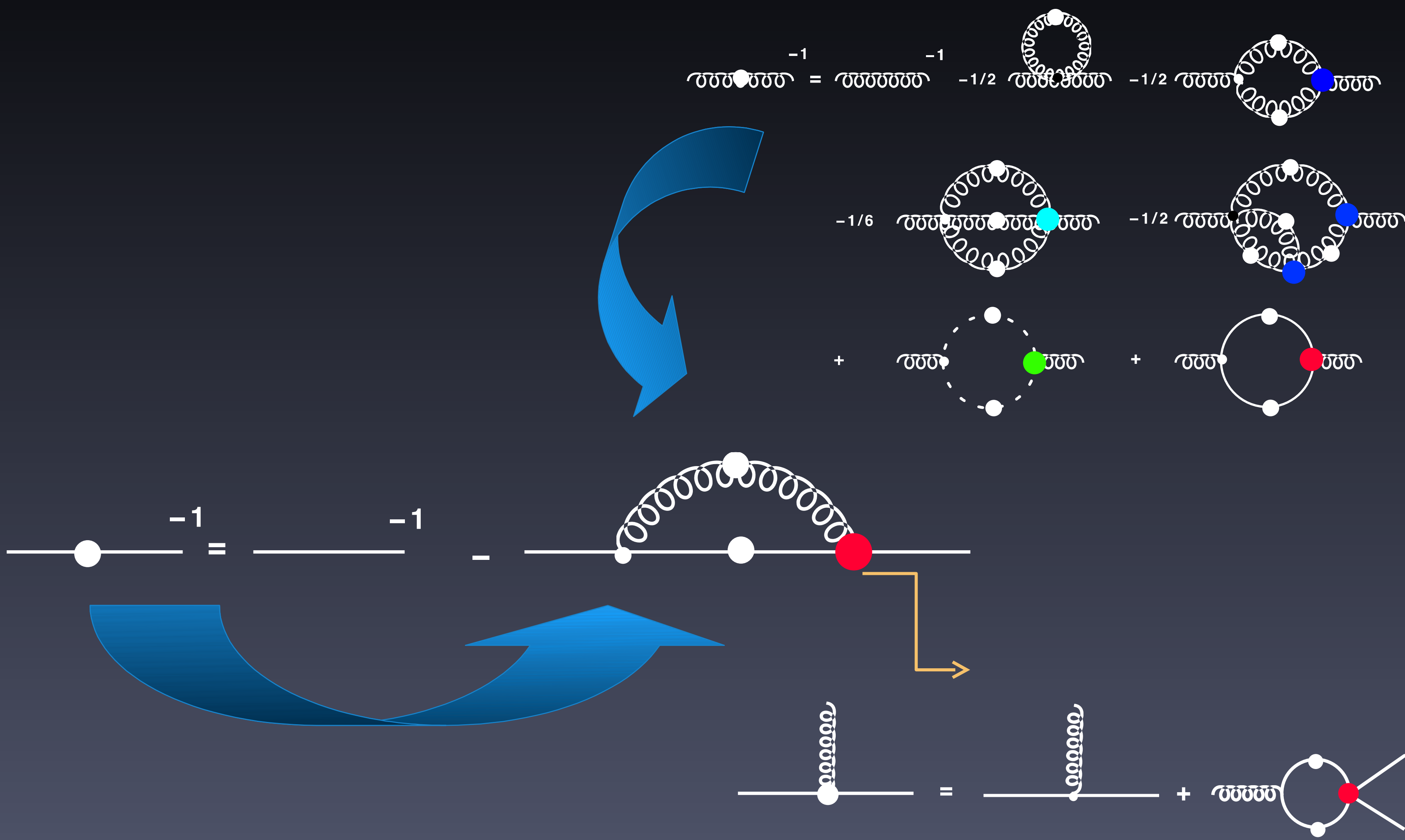
SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER



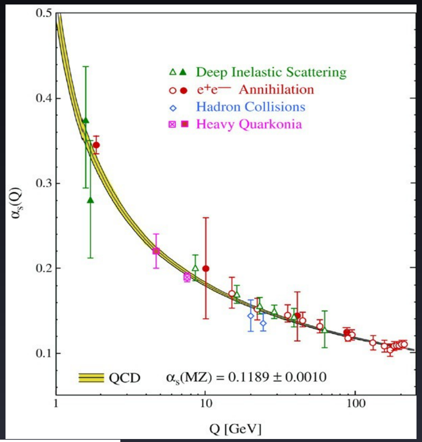
Schwinger-Dyson Equations



Schwinger-Dyson Equations

QED Action:

$$S = \int dx \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \overline{\Psi}(i \not\partial - m)\Psi - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - e\overline{\Psi}\gamma_\mu A^\mu \Psi \right]$$



$$\boxed{\int \mathcal{D}\Phi \frac{\delta}{\delta\Phi} \equiv 0}$$

$$S_F(p) = \frac{Z(p^2)}{\not p - M(p^2)}$$

provides information about dynamical chiral symmetry breaking, quark confinement, hadron spectroscopy

$$\Gamma^\mu = \sum_{i=1}^{12} f^i V_i^\mu$$

$$S_F(p) = \frac{Z(p^2)}{\not{p} - M(p^2)} = \frac{1}{A(p^2)\ \not{p} - B(p^2)}$$

$$S^L(pa) = \frac{Z^L(pa)}{ia\ \not{K}(p) + aM^L(pa)}$$

$$K_\mu(p) \equiv \frac{1}{a} sin(p_\mu a)$$

$$Z \equiv 1/A$$

$$M \equiv B/A$$

$$Z^L$$

$$M^L$$

$$S_{SW} = S_W - i\frac{a}{4}g_0 c_{SW} \sum_x \sum_{\mu\nu} \overline{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

$$\overline{\psi}(x)\left(i \not{D} + m\right)\psi(x) + \overline{\psi}(x)D^2\psi(x) + \overline{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

$$S(x,y) \equiv < \psi(x)\overline{\psi}(y) >$$

$$\begin{aligned}\psi &\rightarrow (1 + b_q am)(1 - c_q a \not{D})\psi \\ \overline{\psi} &\rightarrow \overline{\psi}(1 + b_q am)(1 + c_q a \overleftarrow{\not{D}})\end{aligned}$$

$$\boxed{S(x,y) =< (1 + b_q am)^2(1 - c_q a \not{D}(x))S_0(x,y;U)(1 + c_q a \not{D}(y)) >}$$

$$c_q = b_q = 1/4$$

$$S^L(pa) = \frac{Z^L(pa)}{ia \not{K}(p) + aM^L(pa)}$$

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$$Z(ap) = \frac{Z^L(ap)}{Z^{(0)}(ap)} \qquad aM(p) = \frac{M^L(pa) - a\Delta M^{(-)}(pa)}{Z_m^{(+)}}$$

β	κ	a [fm]	V	m_π [MeV]	m_q [MeV]	N_{cfg}
5.20	0.13596	0.081	$32^3 \times 64$	280	6.2	900
5.29	0.13620	0.071	$32^3 \times 64$	422	17.0	900
5.29	0.13632	0.071	$32^3 \times 64$	295	8.0	908
			$64^3 \times 64$	290		750
	0.13640		$64^3 \times 64$	150	2.1	400
5.40	0.13647	0.060	$32^3 \times 64$	426	18.4	900

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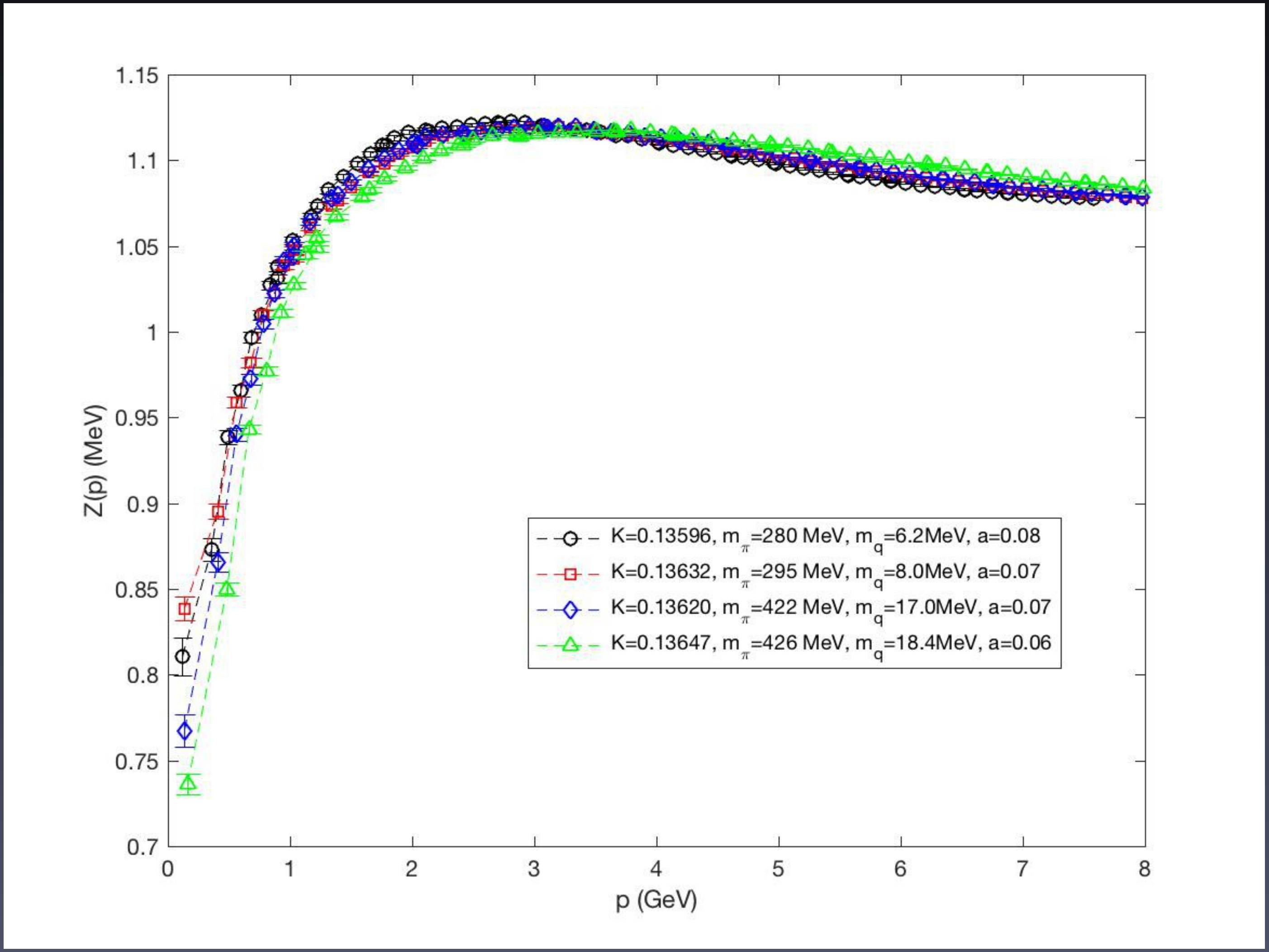
Quark wave function renormalisation in Landau Gauge

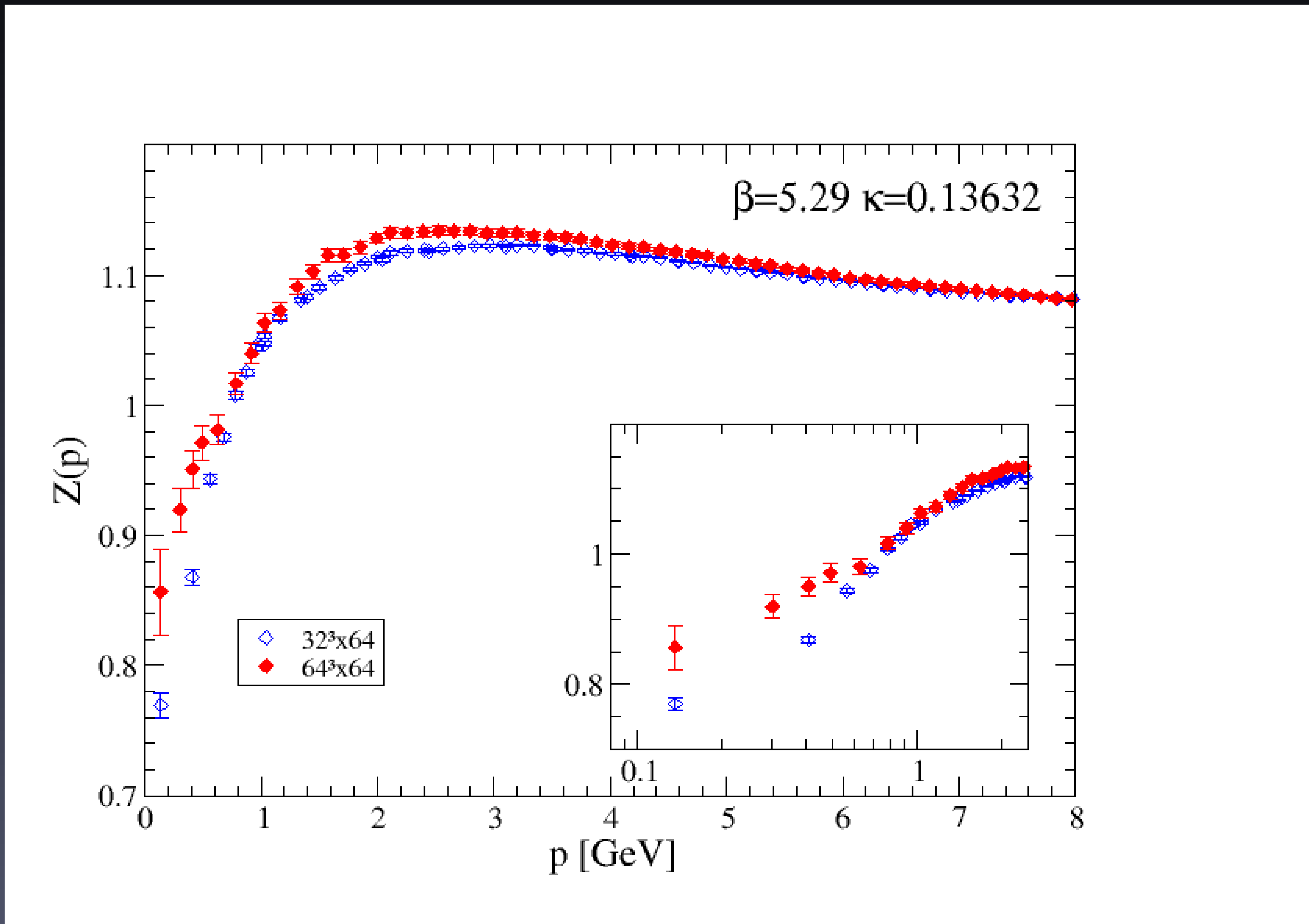
All 4 ensembles on the 323 x 64 volume, using the tree-level corrected Z(p)

$$S_F(p) = \frac{Z(p^2)}{\not{p} - M(p^2)}$$



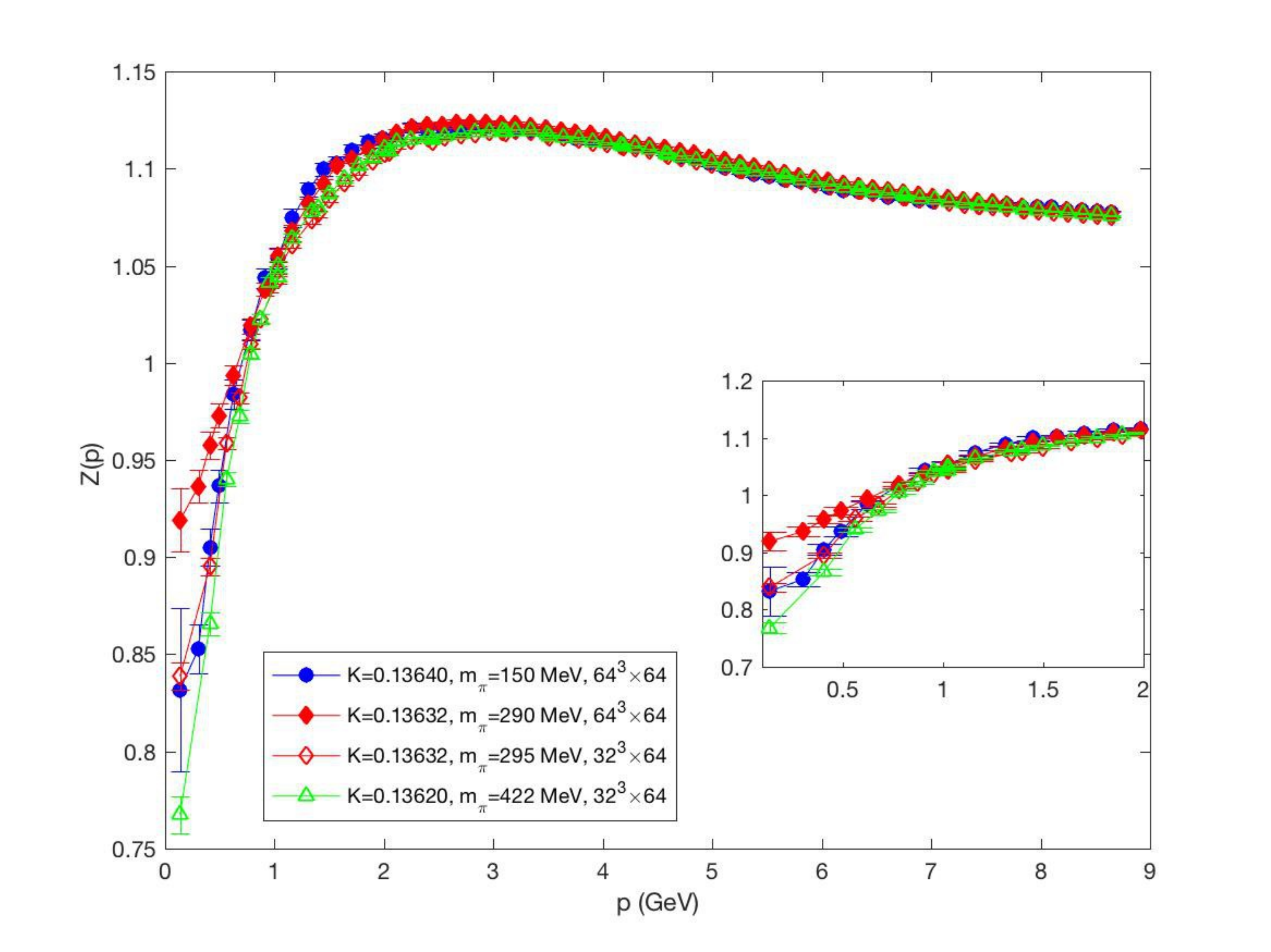
Finite Volume Effects

Quark Wave-function for two lattice volumes at b = 5.29, k =0.13632



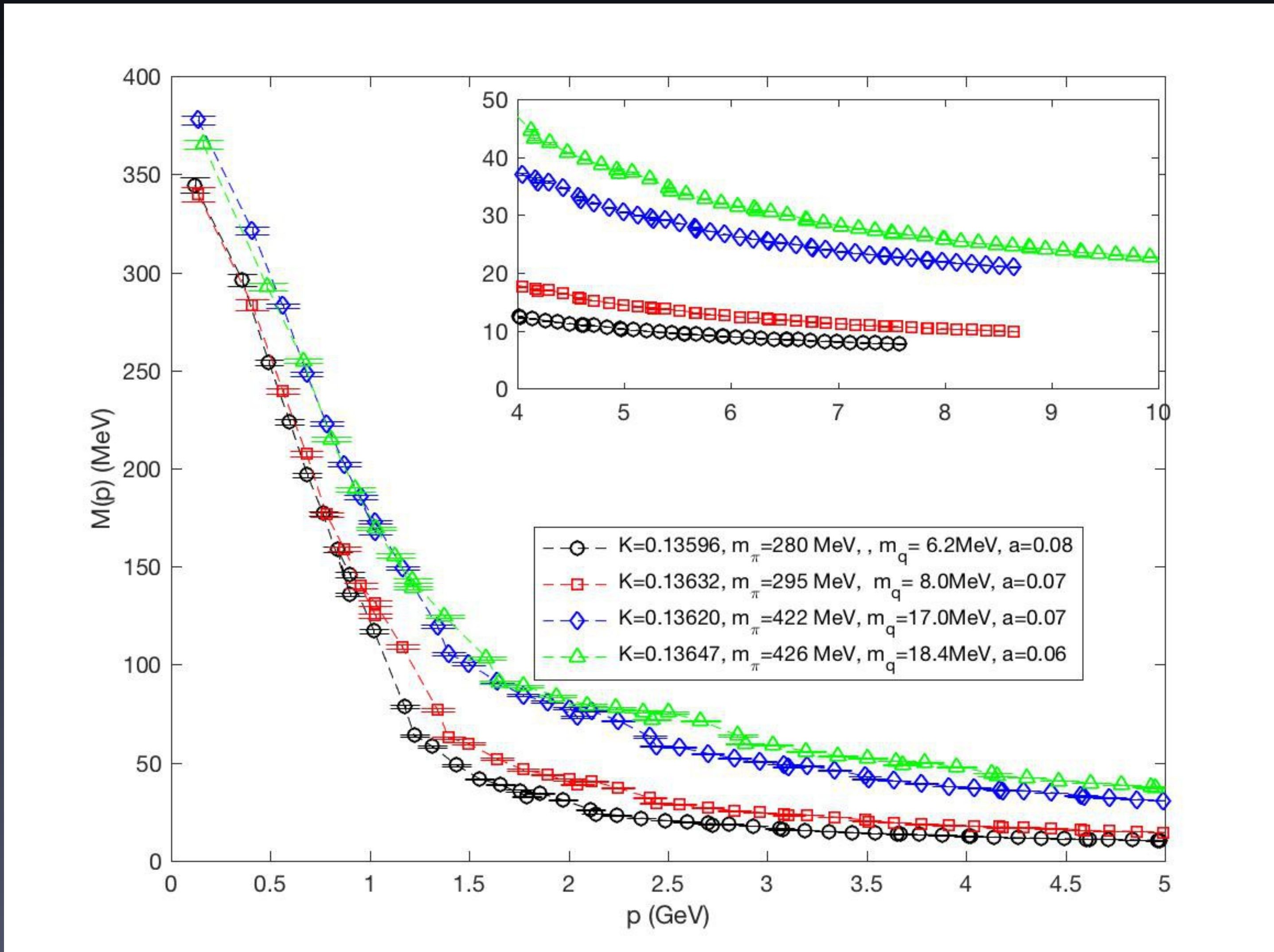
Quark Wave Function for the lighter quark masses

b = 5.29, a=0.07



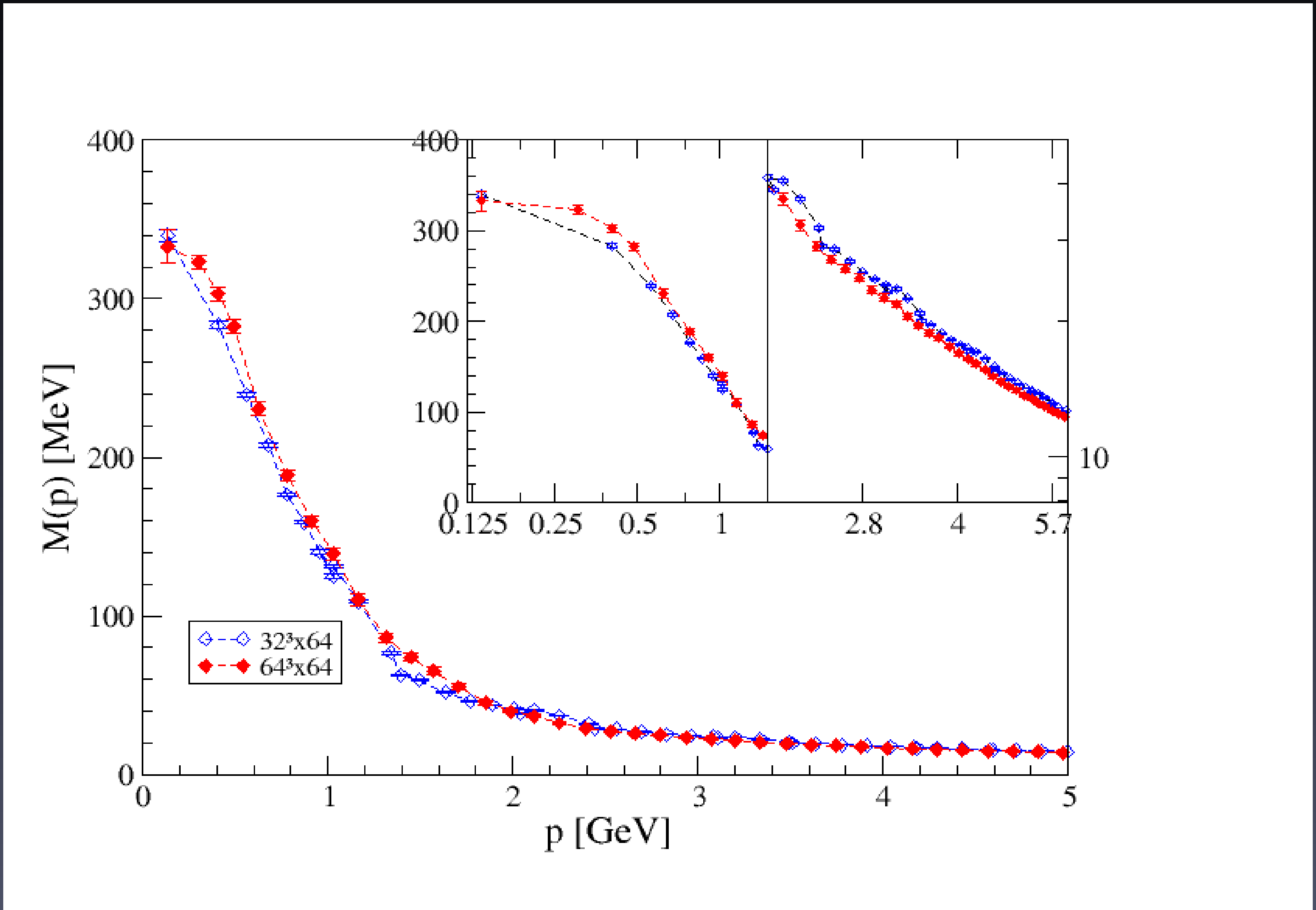
Mass function

All 4 ensembles on the 323 x 64 volume, using the hybrid correction scheme for M(p).



Finite Volume Effects

Mass function for two lattice volumes at b = 5.29, k =0.13632

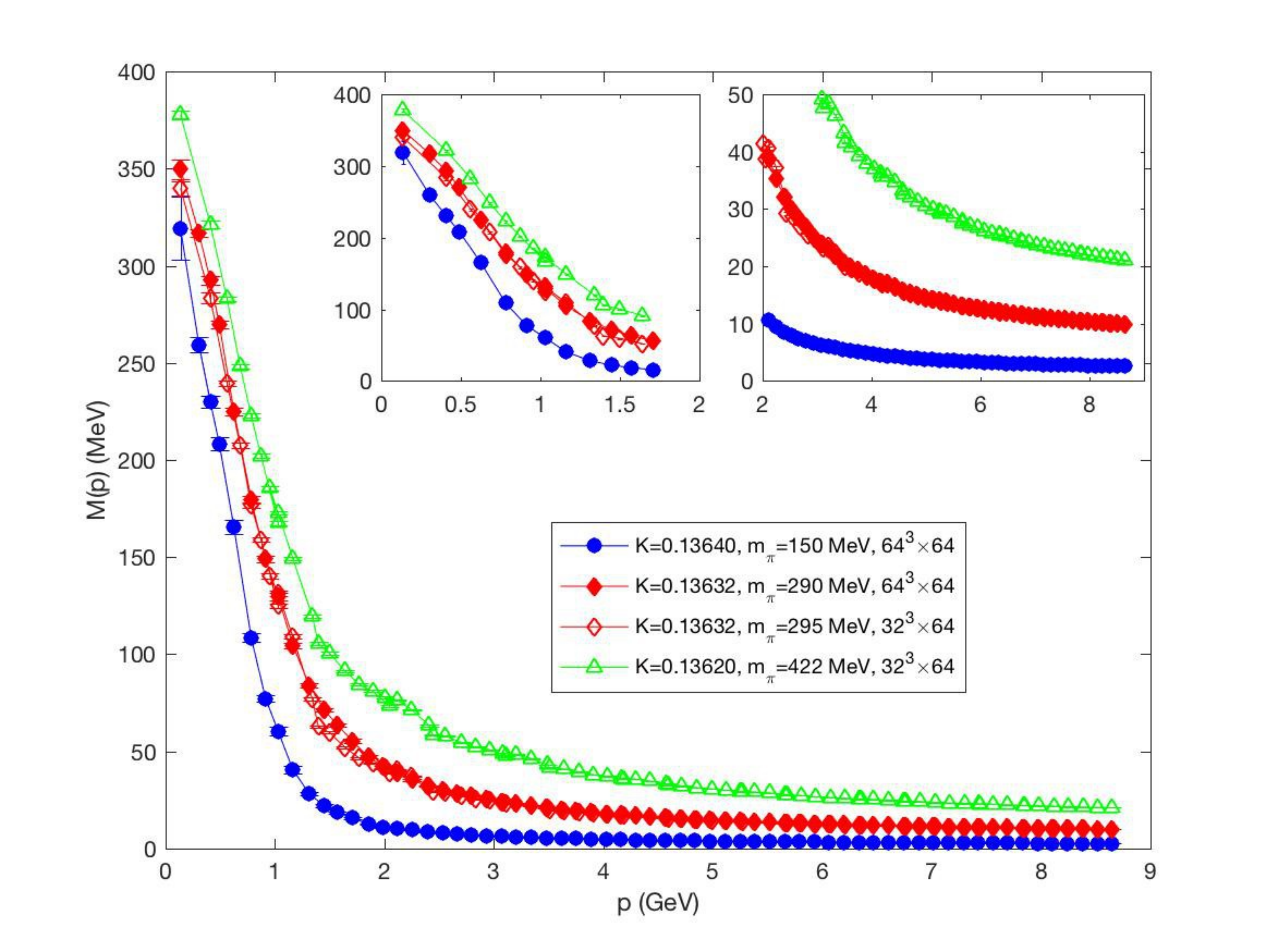


Quark Mass Function for the lighter quark masses

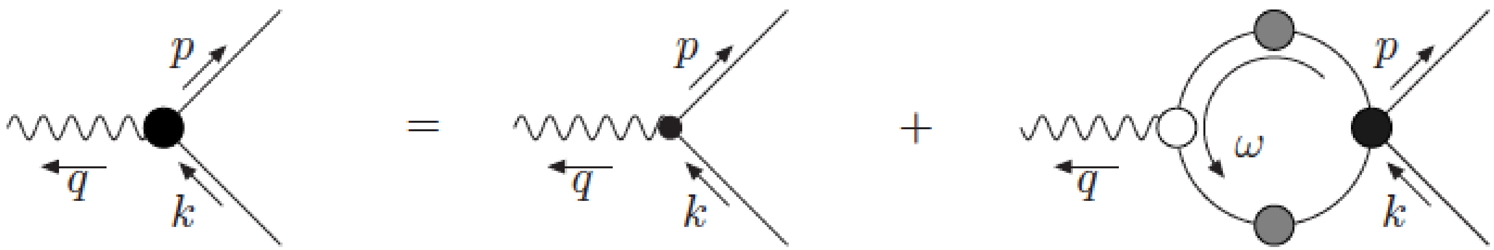
b = 5.29, a=0.07

M = 349.7 ± 5.2 MeV @ p= 136 MeV for mp=290 MeV

M = 319 ± 16 MeV @ p= 136 MeV for mp=150 MeV



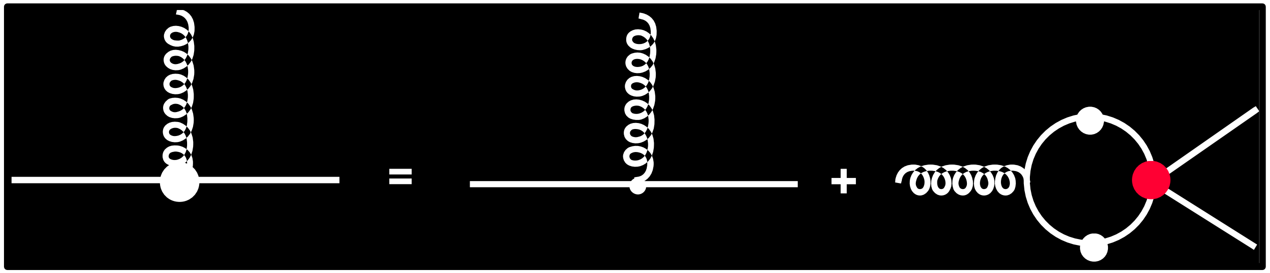
Quark Gluon Vertex


p
q
k
=
+
ω

A.

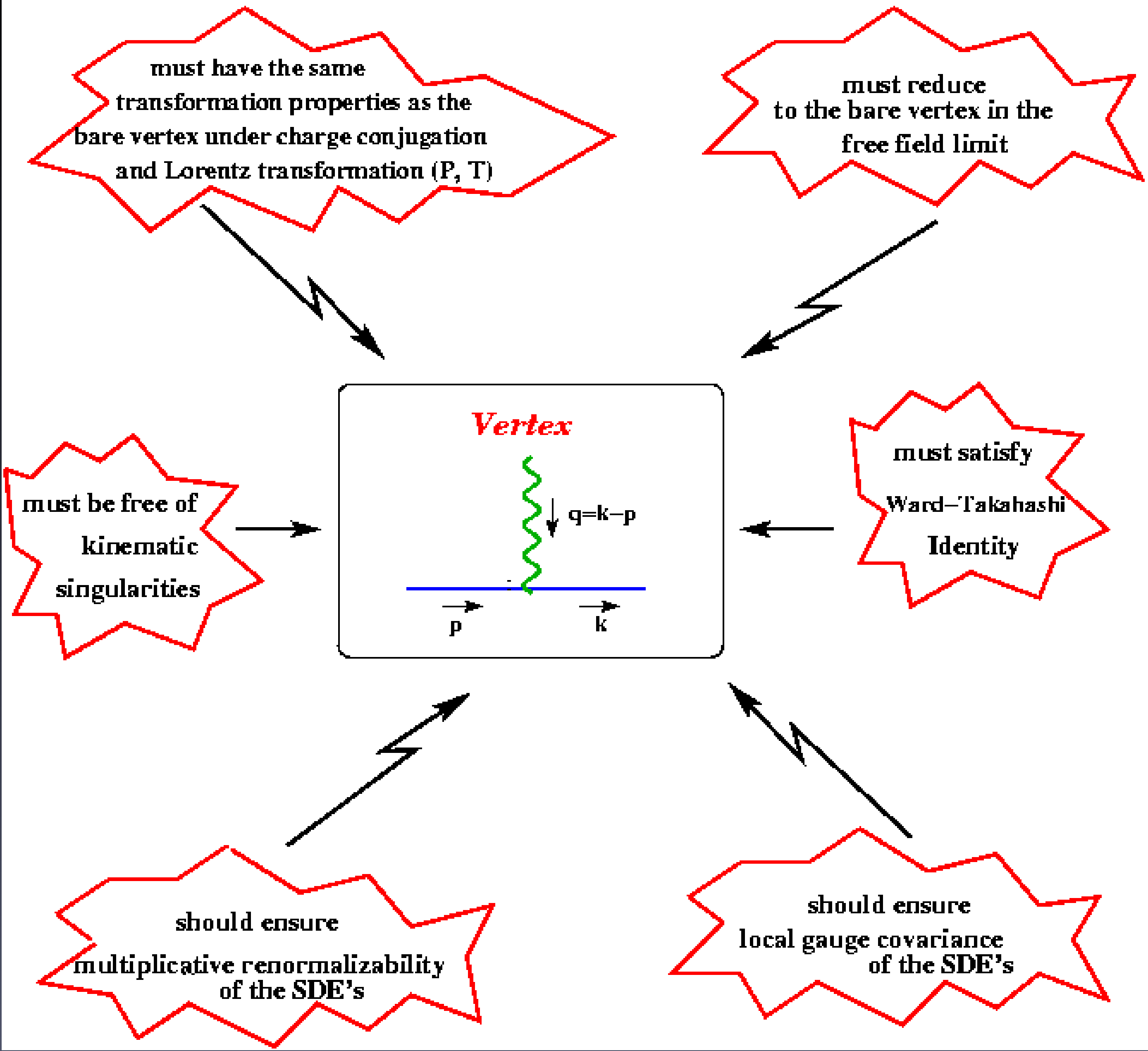
B.

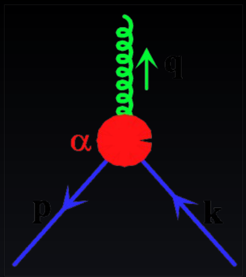
C.



A.

What are the requirements of the "VERTEX FUNCTION"?

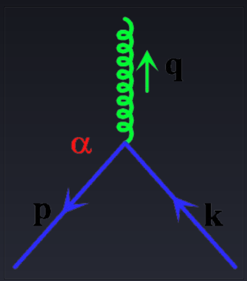




Slavnov_Taylor Identity:

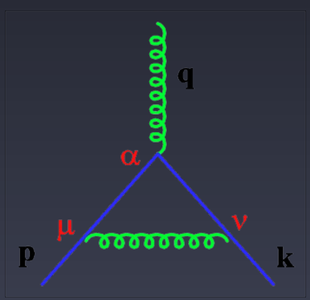
$$q^\mu \Gamma_\mu(p,q) = G_P(q^2)\ \left[\bar{H}(k,-p,-q)S^{-1}(k) - S^{-1}(p)H(-p,k,-q)\right]$$

Ghost-Quark Scattering Kernel



Zeroth order:

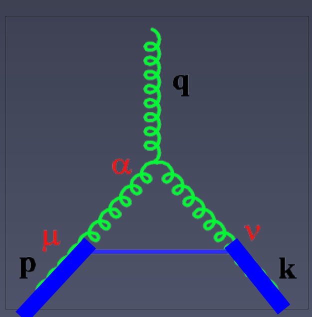
$$\left[S^{-1}(k) - S^{-1}(p)\right]^{(0)}$$



One-loop ABELIAN (QED) contribution:

$$+\left(C_F - \frac{C_A}{2}\right)\ C_F^{-1}\left[[S^{-1}(k)]^{(1)} - [S^{-1}(p)]^{(1)}\right]$$

WTI



One-loop NON- ABELIAN contribution:

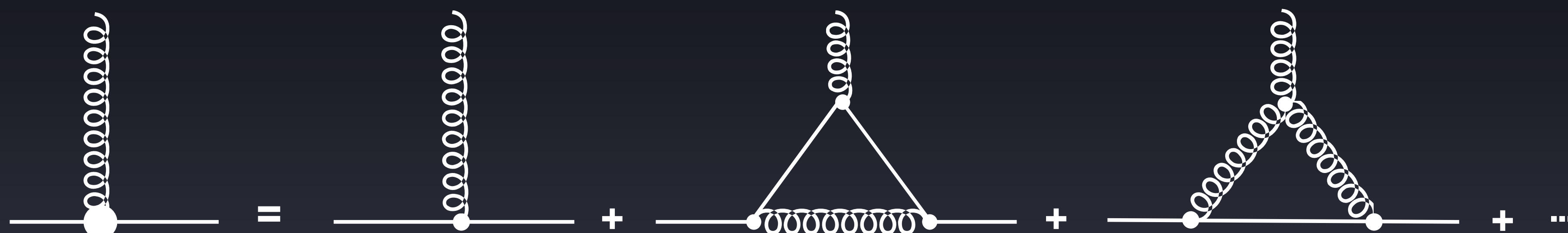
$$+\quad \frac{C_A}{2}C_F^{-1}\ \left[[S^{-1}(k)]^{(1)}H^{(0)} - H^{(0)}[S^{-1}(p)]^{(1)}\right]$$

$$+\quad \left[[S^{-1}(k)]^{(0)}H^{(1)} - H^{(1)}[S^{-1}(p)]^{(0)}\right]$$

$$+\quad G_P^{(1)}\ \left[S^{-1}(k)]H - HS^{-1}(p)\right]^{(0)}$$

In Perturbation theory:

$$M(p^2) \sim m_0 \, [\ldots.]$$



$$\left(C_F - \frac{C_A}{2}\right) \qquad\qquad (C_A)$$

$$\boxed{\lambda_i^{QCD} = \left(C_F - \frac{C_A}{2}\right)\lambda_i^a + C_A \lambda_i^b}$$

(Color factors in Fundamental and Adjoint Rep.) $C_A = N, \; C_F = \frac{N^2-1}{2N}$

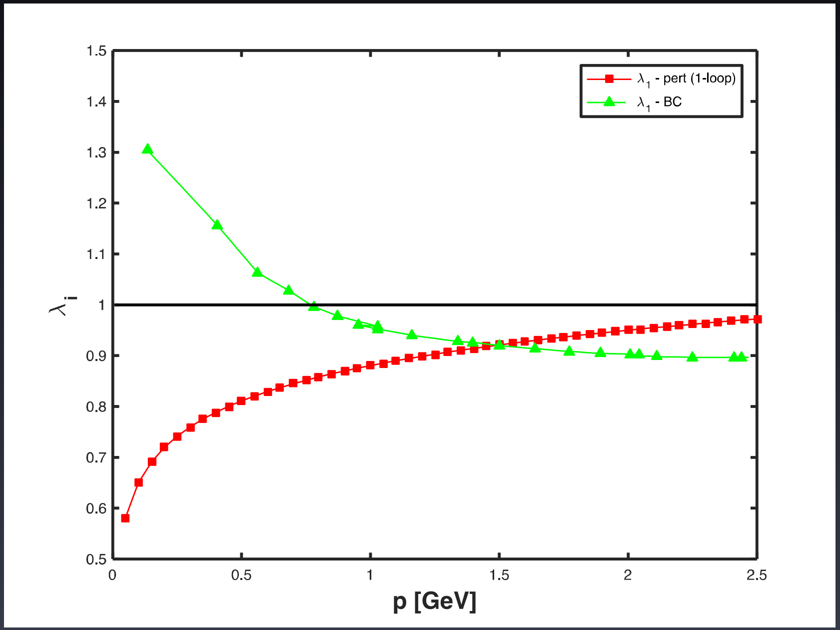
$C_A = 0, C_F = 1$ (for QED)

What do we know about the Form Factors?

In Continuum:

In QED:

$$\lambda_1(\mathbf{k^2}, \mathbf{q^2}, \mathbf{p^2}) = \frac{\mathbf{1}}{\mathbf{2}}\left[\mathbf{A}(\mathbf{k^2}) + \mathbf{A}(\mathbf{p^2})\right]$$

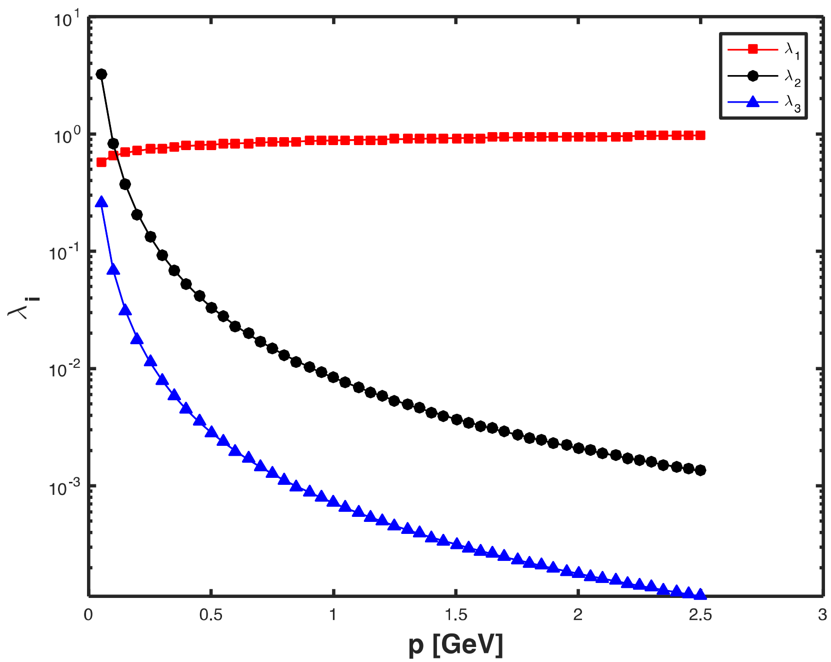


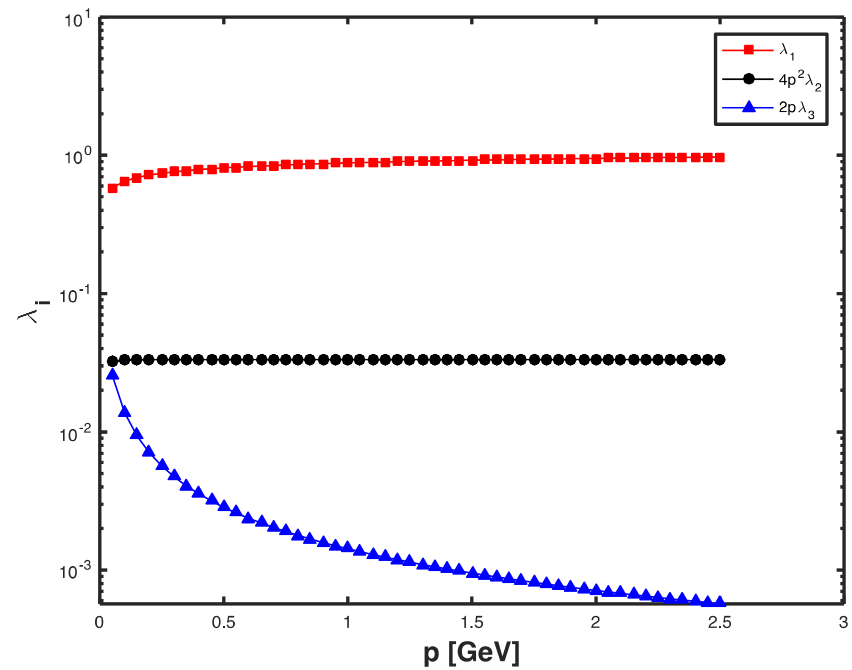
In QCD:

$$\lambda_1(p^2,0,p^2) = G(0)\Big[A(p^2)\chi_0(p^2,0,p^2) + B(p^2)\left(\chi_1(p^2,0,p^2)+\chi_2(p^2,0,p^2)\right) - 2p^2 A(p^2)\chi_3(p^2,0,p^2)\Big]$$

Form Factors of ghost-quark scattering kernel

$$q_\mu = 0 \qquad k_\mu = p_\mu = \frac{P_\mu}{2}$$



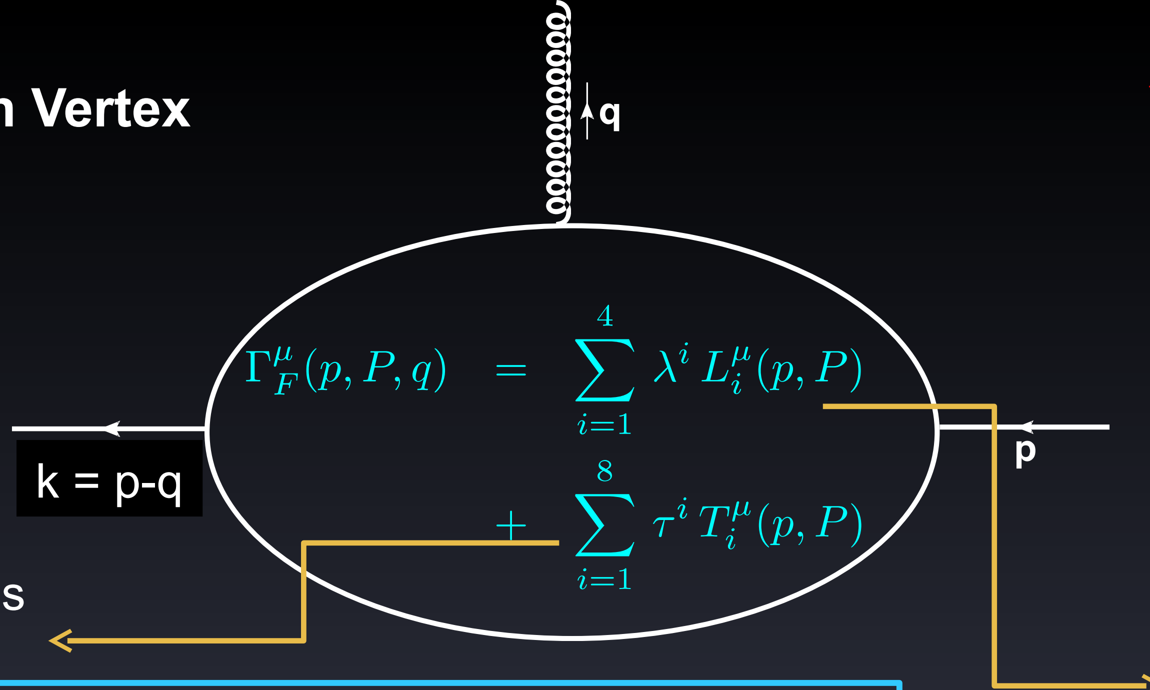


A.

Quark Gluon Vertex

$$P = k + p$$

$$q = k - p$$



$$\Gamma_F^\mu(p,P,q) = \sum_{i=1}^{4} \lambda^i L_i^\mu(p,P) + \sum_{i=1}^{8} \tau^i T_i^\mu(p,P)$$

Transverse Basis

$$
\begin{aligned}
T_{1_\mu} &= -i\,[(pq)k_\mu - (kq)p_\mu] \\
T_{2_\mu} &= -\not{P}\,[(pq)k_\mu - (kq)p_\mu] \\
T_{3_\mu} &= \not{q}\,q_\mu - q^2\gamma_\mu \\
T_{4_\mu} &= -i\left[q^2\sigma_{\mu\nu}P_\nu + 2q_\mu\sigma_{\nu\lambda}p_\nu k_\lambda\right] \\
T_{5_\mu} &= -i\,\sigma_{\mu\nu}q_\nu \\
T_{6_\mu} &= (qP)\gamma_\mu - \not{q}\,P_\mu \\
T_{7_\mu} &= -\frac{i}{2}(qP)\sigma_{\mu\nu}P_\nu - i\,P_\mu\,\sigma_{\nu\lambda}\,p_\nu\,k_\lambda \\
T_{8_\mu} &= -\gamma_\mu\,\sigma_{\nu\lambda}\,p_\nu\,k_\lambda - \not{p}\,k_\mu + \not{k}\,p_\mu
\end{aligned}
$$

Longitudinal Basis

$$
\begin{aligned}
L_{1_\mu} &= \gamma_\mu \\
L_{2_\mu} &= -\not{P}P_\mu \\
L_{3_\mu} &= -i\,P_\mu \\
L_{4_\mu} &= -i\,\sigma_{\mu\nu}P_\nu
\end{aligned}
$$

$$\Gamma^T_\mu(p,k,q) = P^T_{\mu\nu}(q)\Gamma_\mu = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Gamma_\mu(p,k,q)$$

$$\Lambda^{a,P}_\mu(p,q) \equiv P_{\mu\nu}\Lambda^{a,lat.}_\nu(p,q) = S(p)^{-1}V^a_\nu(p,q)S(p+q)^{-1}D(q^2)^{-1}$$

$$\boxed{\begin{aligned}\Gamma^{T^\mu} = -\frac{1}{q^2}\left[\Lambda_1\, T^\mu_3 + \Lambda_2\, T^\mu_2 + \Lambda_3\, T^\mu_1 + \Lambda_4\, T^\mu_4\right] \\ + \tau_5\, T^\mu_5 + \tau_6\, T^\mu_6 + \tau_7\, T^\mu_7 + \tau_8\, T^\mu_8\end{aligned}}$$

$$\Lambda_1 = \lambda_1 - q^2\tau_3 \qquad \Lambda_2 = \Lambda_2 - \frac{q^2}{2}\tau_2$$

$$\Lambda_3 = \lambda_3 - \frac{q^2}{2}\tau_1 \qquad \Lambda_4 = \lambda_4 + q^2\tau_4$$

$$\mathbf{V^a_\mu(x,y,z)^{ij}_{\alpha\beta}} = < \mathbf{\Psi^i_a(x)\bar{\Psi}(z)A^a_\mu(y)} > = << \mathbf{S^{ij}_{\alpha\beta}(x,z)A^a}\mu\mathbf{(y)} >>$$

$$\mathbf{\Lambda^{a,lat.}_\mu(p,q) = S(p)^{-1}V^a_\nu(p,q)S(p+q)^{-1}D(q)^{-1}_{\nu\mu}}$$

$$(\mathbf{\Lambda^a_\mu})^{\mathbf{ij}}_{\alpha\beta} \; = \; -\mathbf{g_0 t^a_{ij}}(\mathbf{\Gamma}_\mu)_{\alpha\beta}$$

$$D_{\mu\nu}(q) = \frac{-1}{q^2}\left[G(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \xi\frac{q_\mu q_\nu}{q^2}\right]$$

$$\boxed{\begin{array}{ll} \Lambda_1 = \lambda_1 - q^2\tau_3 & \Lambda_2 = \Lambda_2 - \frac{q^2}{2}\tau_2 \\ \Lambda_3 = \lambda_3 - \frac{q^2}{2}\tau_1 & \Lambda_4 = \lambda_4 + q^2\tau_4 \end{array}}$$

$$Tr_4[\mathbf{I}\,\Gamma_\mu^T] = i\,\Lambda_3\,\frac{1}{q^2}\,[(q\cdot P)q_\mu - q^2 P_\mu]$$

$$Tr_4[\gamma_\nu\,\Gamma_\mu^T]$$

$$= \Lambda_1\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \Lambda_2\,\frac{2}{q^2}\,P_\mu\,\ell_\nu + \tau_6\,(q\cdot P)\,\left(\delta_{\mu\nu} - \frac{q_\mu P_\nu}{qP}\right)$$

$$Tr_4[\gamma_{\mathbf{5}}\gamma_\alpha\Gamma_\mu^T] \;\; = \tau_8\,\epsilon_{\alpha\mu\kappa\lambda}k_\kappa p_\lambda$$

$$Tr_4[\sigma_{\alpha\beta}\Gamma_\mu^T]$$

$$\begin{aligned} = \quad & i\left[\Lambda_4 + \frac{1}{2}(q\cdot P)\,\tau_7\right](\delta_{\alpha\mu}P_\beta - \delta_{\beta\mu}P_\alpha) \\ + \quad & i\left[\Lambda_4\frac{q_\mu}{q^2} + \frac{1}{2}\,\tau_7 P_\mu\right](P_\alpha q_\beta - P_\beta q_\alpha) + i\,\tau_5\,(\delta_{\alpha\mu}q_\beta - \delta_{\beta\mu}q_\alpha) \end{aligned}$$

$$\boxed{P_\mu = p_\mu + k_\mu = 2p_\mu + q_\mu}$$

$$Tr_4[\mathbf{I}\,\Gamma^T_\mu] = i\,\Lambda_3\,\frac{1}{q^2}\left[(q\cdot P)q_\mu - q^2 P_\mu\right]$$

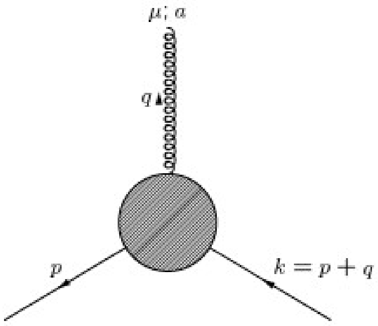
$$\boldsymbol{\Lambda_3 \equiv -\left(\lambda_3 - \frac{q^2}{2}\,\tau_1\right) = -i\,\frac{q^2}{\left[(q\cdot P)^2 - q^2P^2\right]}\,\mathbf{Tr_4}\,[\mathbf{I}\,\Gamma_\mu]\,\mathbf{P}_\mu}$$

$$\boxed{\begin{aligned} K_\mu(p) &= \frac{1}{a}\sin p_\mu a \\ Q_\mu(p) &= \frac{2}{a}\sin\left(\frac{p_\mu a}{2}\right) \\ C_\mu(p) &= \cos(p_\mu a) \end{aligned}}$$

$Q_\mu(P)$ or $K_\mu(P)$

$$\begin{aligned}
\frac{\mathrm{Tr}_4\left(I\,\Gamma^T_\mu\right)}{R} &= -2c_q\,\frac{i}{Q^2(q)}\left[K(q)\cdot K(P)Q_\mu(q) - Q^2(q)\bar{C}_\mu(q)K_\mu(P)\right] \\
&+ \frac{1}{2}\,\frac{i}{Q^2(q)}\left[\left(1-c_q^2\,K^2(p)\right)\left(1-c_q^2\,K^2(k)\right) - 4c_q^2K(p)\cdot K(k)\right]\Big(Q(q)\cdot Q(P)Q_\mu(q) - Q^2(q)Q_\mu(P \\
&+ 4ic_q^2c_{sw}\,\bar{C}_\mu(q)\left[K(k)\cdot K(q)K_\mu(p) - K(p)\cdot K(q)K_\mu(k)\right] \\
&+ \frac{c_q^3}{2}\,i\left[K(P)\cdot K(q)\left\{2\bar{c}_\mu^2 - 1 + \frac{Q^2(P)}{Q^2(q)}\right\}Q_\mu(q) - 2\left[K^2(k)+K^2(p)\right]\bar{C}_\mu(q)K_\mu(P)\right]
\end{aligned}$$

$$q_\mu = 0 \qquad k_\mu = p_\mu = \frac{P_\mu}{2}$$

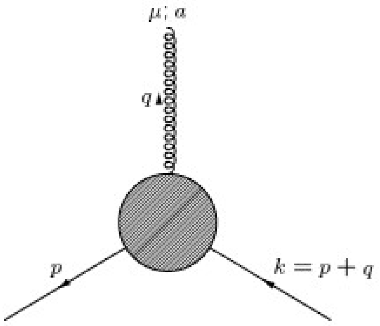


$$(\mathbf{\Lambda}^{\mathbf{a}}_\mu)^{\mathbf{ij}}_{\alpha\beta} \;=\; -\mathbf{i g_0 t^a_{ij}} (\mathbf{\Gamma}^{\mathbf{T}}_\mu)_{\alpha\beta}$$

$$(\mathbf{\Lambda}^{\mathbf{a}}_\mu)^{\mathbf{ij}}_{\alpha\beta} \;=\; -\mathbf{i g_0 t^a_{ij}} (\lambda^{\mathbf{E}}_{\mathbf{1}}\,\gamma_\mu - \mathbf{4}\,\lambda^{\mathbf{E}}_{\mathbf{2}}\;\not{\mathbf{p}}\mathbf{p}_\mu - \mathbf{2i}\lambda^{\mathbf{E}}_{\mathbf{3}}\,\mathbf{p}_\mu - \mathbf{2i}\lambda^{\mathbf{E}}_{\mathbf{4}}\,\sigma_{\mu\nu}\mathbf{p}_\nu)_{\alpha\beta}$$

$$\left.\begin{aligned} Tr_4[\mathbf{I}\,\Gamma^T_\mu] \quad &= -\,2\,i\,\lambda^E_3\,p_\mu \\[2em] Tr_4[\gamma_\nu\,\Gamma^T_\mu] \quad &= \lambda^E_1 \delta_{\alpha\mu} - 4\,\lambda^E_2 p_\alpha p_\mu \\[2em] Tr_4[\sigma_{\alpha\beta}\Gamma^T_\mu] \quad &= -2i\lambda^E_4\,(p_\alpha\delta_{\beta\mu} - p_\beta\delta_{\alpha\mu}) \end{aligned}\right\} \quad \begin{aligned} &\lambda^E_1, \lambda^E_2, \lambda^E_3 \\ &\lambda^E_4 = 0 \end{aligned}$$

- $$q_\mu = 0 \qquad k_\mu = p_\mu = \frac{P_\mu}{2}$$

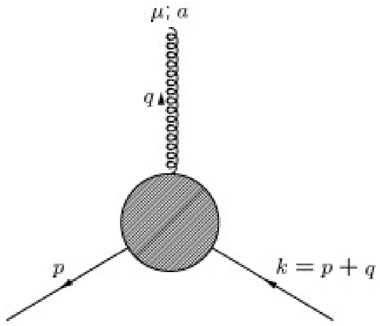


$$Tr_4[\mathbf{I}\,\Gamma^T_\mu] \qquad = -\,2\,i\,\lambda_3^E\,p_\mu$$

$$\lambda_3(P, q=0) \quad = \quad \frac{1}{Q^2(P)}\; iQ_\mu(P)\left(\frac{\mathrm{Tr}_4\left(\Gamma^T_\mu\right)}{R}\right)\Bigg|_{q=0}$$

$$\boxed{\lambda_3(P, q=0) \quad = \quad \frac{1}{2}\left\{\left(1 - \frac{c_q^2}{4}\,Q^2(P)\right)^2 - c_q^2 Q^2(P)\;-4c_q\,\left(1 - \frac{1}{4}c_q^2 Q^2(P)\right)\,\frac{Q(P)\cdot K(P)}{Q^2(P)}\right\}}$$

$$q_\mu = 0 \qquad k_\mu = p_\mu = \frac{P_\mu}{2}$$



$$Tr_4[\gamma_\alpha \, \Gamma^T_\mu] = \lambda^E_1 \delta_{\alpha\mu} - 4\, \lambda^E_2 p_\alpha p_\mu$$

$$\lambda_1(P, q=0) = \frac{1}{3}\left\{ \left.\left(\frac{\mathrm{Tr}_4\left(\gamma_\mu \Gamma^T_\mu\right)}{R} \right)\right|_{q=0} - \frac{Q_\mu(P) Q_\alpha(P)}{Q^2(P)} \left.\left(\frac{\mathrm{Tr}_4\left(\gamma_\alpha \Gamma^T_\mu\right)}{R} \right)\right|_{q=0} \right\}$$

$$\lambda_1(P, q=0) = \frac{1}{3}\left[4 - \frac{\overline{Q}^2(P)}{8} - \frac{K(P)\cdot Q(P)}{Q^2(P)} \right] \left[\left(1 - \frac{c_q^2}{4} Q^2(P) \right)^2 + c_q^2 Q^2(P) \right]$$

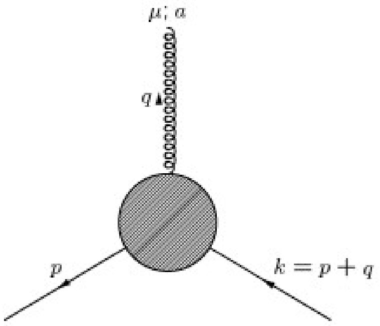
$$\lambda_1\Big|_{\substack{q=0, P_\mu=0, \alpha=\mu \\ \text{no sum over } \mu}} = \left.\left(\frac{\mathrm{Tr}_4\left(\gamma_\alpha \Gamma^T_\mu\right)}{R} \right)\right|_{\substack{q=0,, \alpha=\mu \\ \text{no sum over } \mu}}$$

Tree Level Expression:

$$\lambda_1\Big|_{\substack{q=0, P_\mu=0, \alpha=\mu \\ \text{no sum over } \mu}} = \left(1 - \frac{1}{4} c_q^2 Q^2(P) \right)^2 + c_q^2 Q^2(P)$$

$$q_\mu = 0 \qquad k_\mu = p_\mu = \frac{P_\mu}{2}$$

$$Tr_4[\gamma_\alpha\, \Gamma^T_\mu] = \lambda^E_1 \delta_{\alpha\mu} - 4\, \lambda^E_2 p_\alpha p_\mu$$



$$\lambda_2(P, q=0) = \frac{1}{3\, Q^2(P)} \left\{ \left(\frac{\mathrm{Tr}_4\left(\gamma_\mu \Gamma^T_\mu\right)}{R} \right)\Bigg|_{q=0} - 4\, \frac{Q_\mu(P) Q_\alpha(P)}{Q^2(P)} \left(\frac{\mathrm{Tr}_4\left(\gamma_\alpha \Gamma^T_\mu\right)}{R} \right)\Bigg|_{q=0} \right\}$$

$$\lambda_2(P, q=0) = \frac{1}{3} \left\{ \frac{4}{Q^2(P)} \left(1 - \frac{c_q^2}{4} Q^2(P)\right)^2 \left(1 - \frac{K(P)\cdot Q(P)}{Q^2(P)} - \frac{\overline{Q}^2(P)}{32}\right) \right.$$
$$\left. + 4 c_q^2 \left(1 + \frac{1}{2} \frac{K(P)\cdot Q(P)}{Q^2(P)} - \frac{\overline{Q}^2(P)}{32}\right) - 3 c_q \left(1 - \frac{c_q^2}{4} Q^2(P)\right) \right\}$$

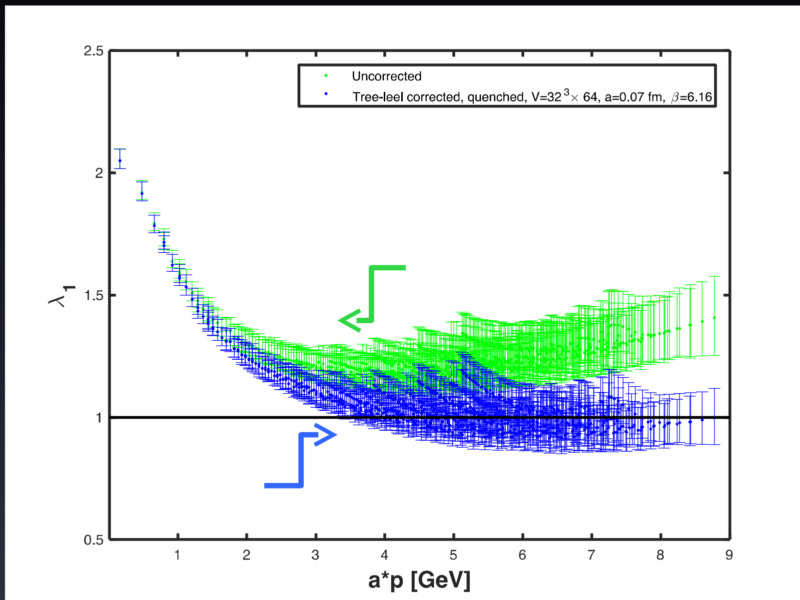
$$\lambda_2\Big|_{q=0,\alpha\neq\beta} = \left(\frac{\mathrm{Tr}_4\left(\gamma_\alpha \Gamma^T_\mu\right)}{R} \right)\Bigg|_{q=0,\alpha\neq\beta}$$

$$\lambda_2\Big|_{q=0,\alpha\neq\beta} = -c_q \left(1 - \frac{1}{4} c_q^2 Q^2(P)\right) - 2 c_q^2 \left(1 - \frac{\overline{Q}^2_\mu(P)}{8}\right)$$

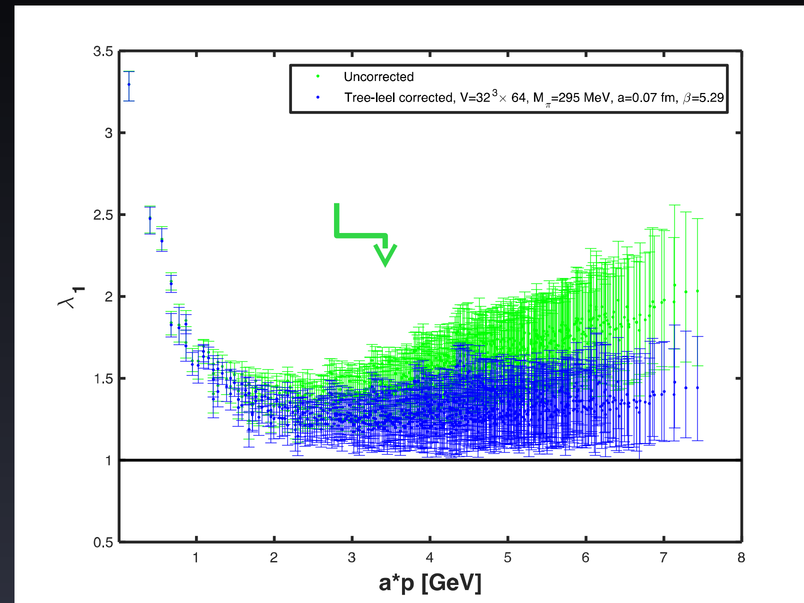
Soft gluon Kinematics (Asymmetric)

l1 with and without tree-level correction

Quenched, Nf = 0 323x64

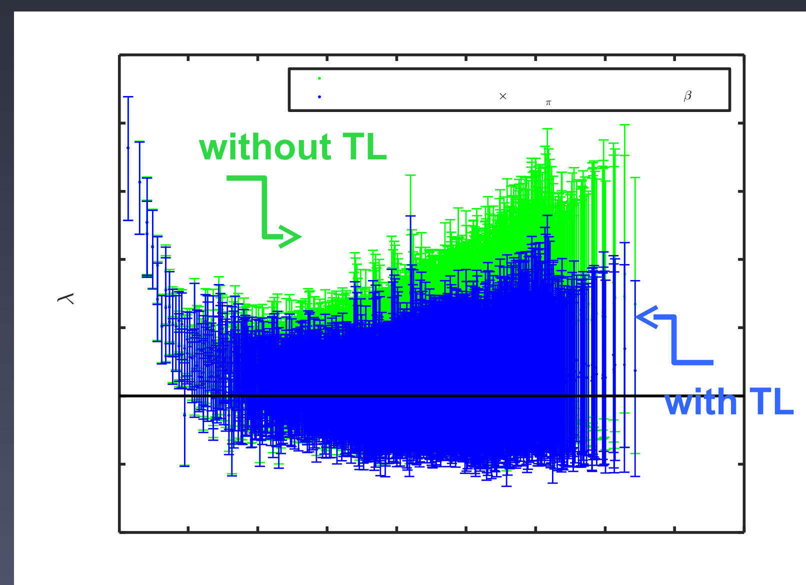


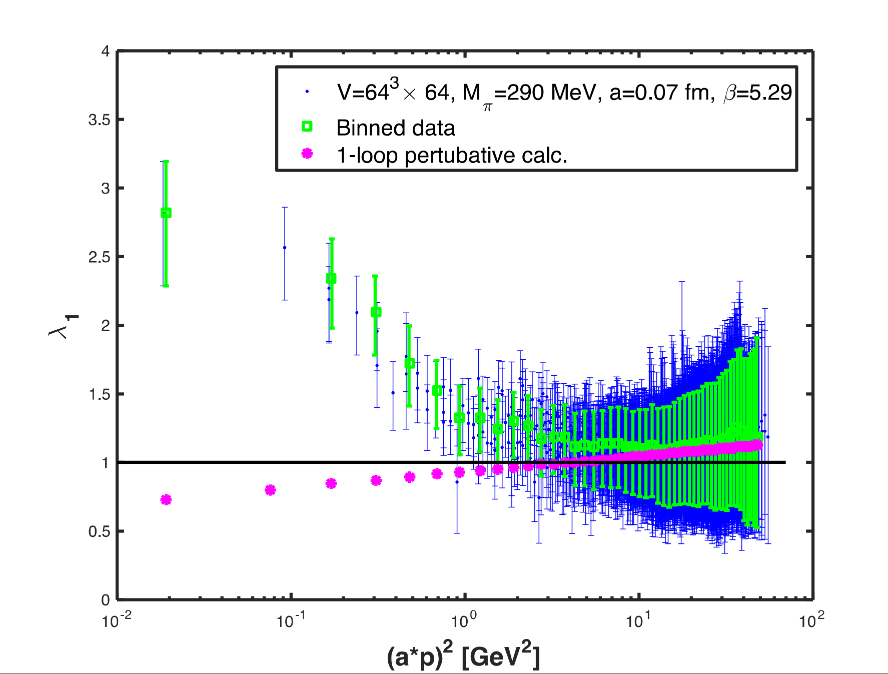
Unquenched, Nf = 2 323x64

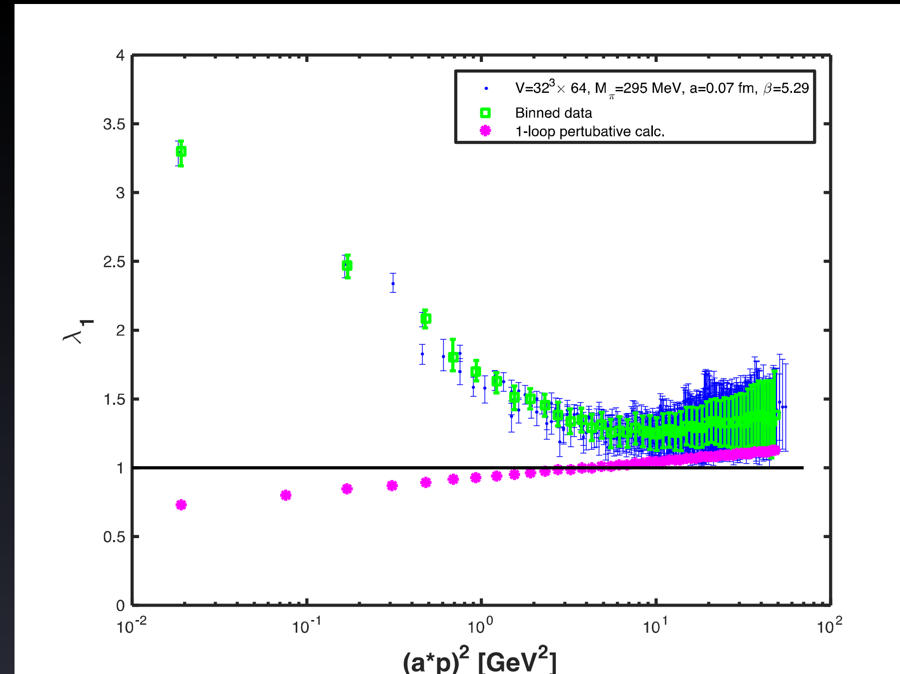


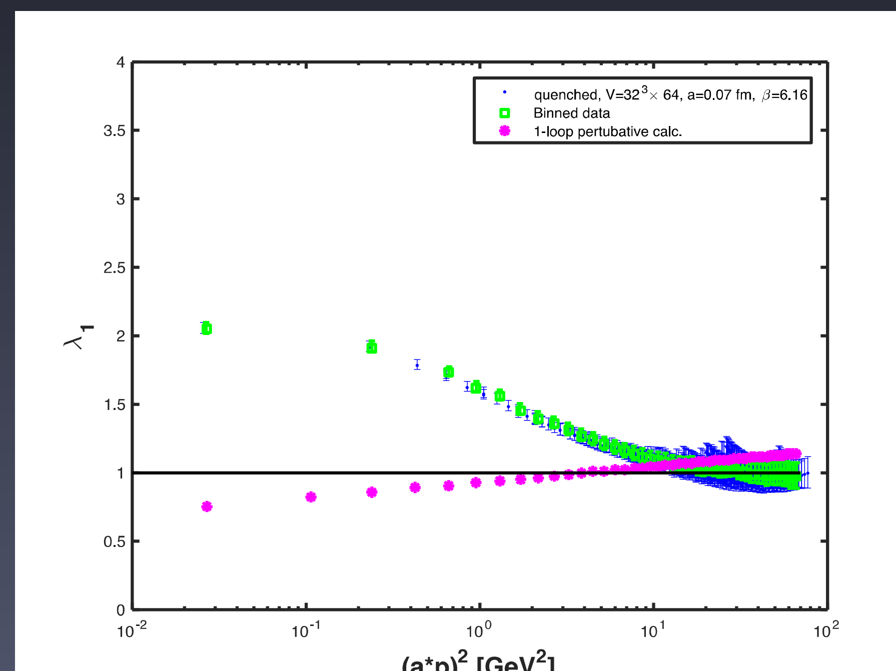
Unquenched, Nf = 2

643x64

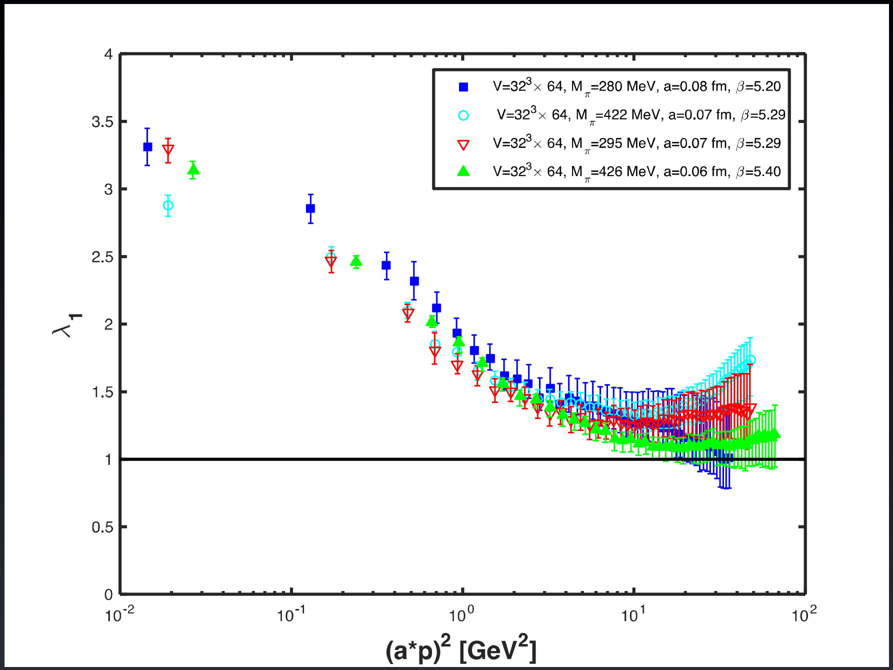


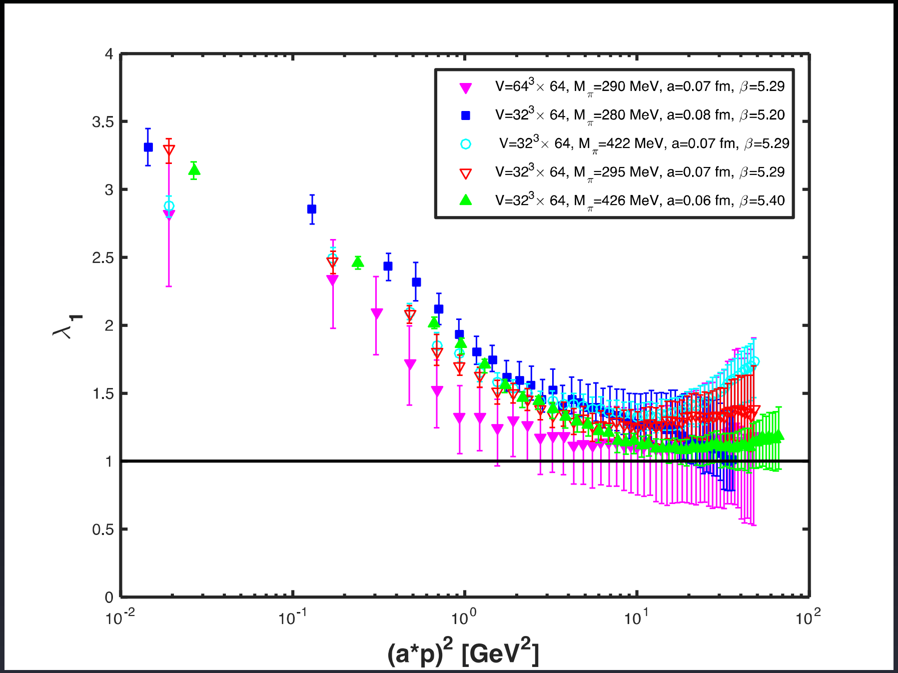

V=64^3 × 64, M_π=290 MeV, a=0.07 fm, β=5.29
Binned data
1-loop pertubative calc.
λ_1
(a*p)^2 [GeV^2]

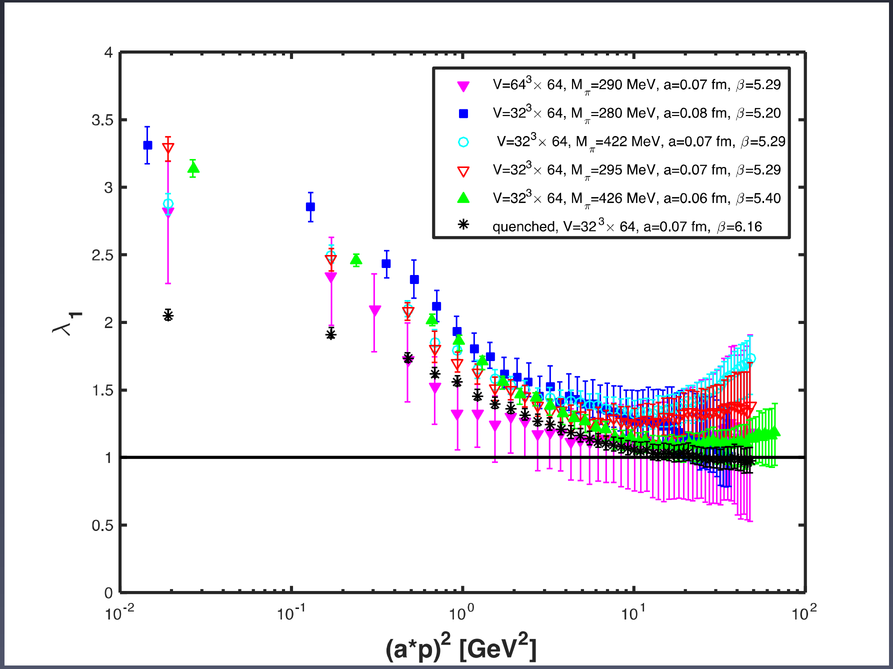

V=32^3 × 64, M_π=295 MeV, a=0.07 fm, β=5.29
Binned data
1-loop pertubative calc.
λ_1
(a*p)^2 [GeV^2]

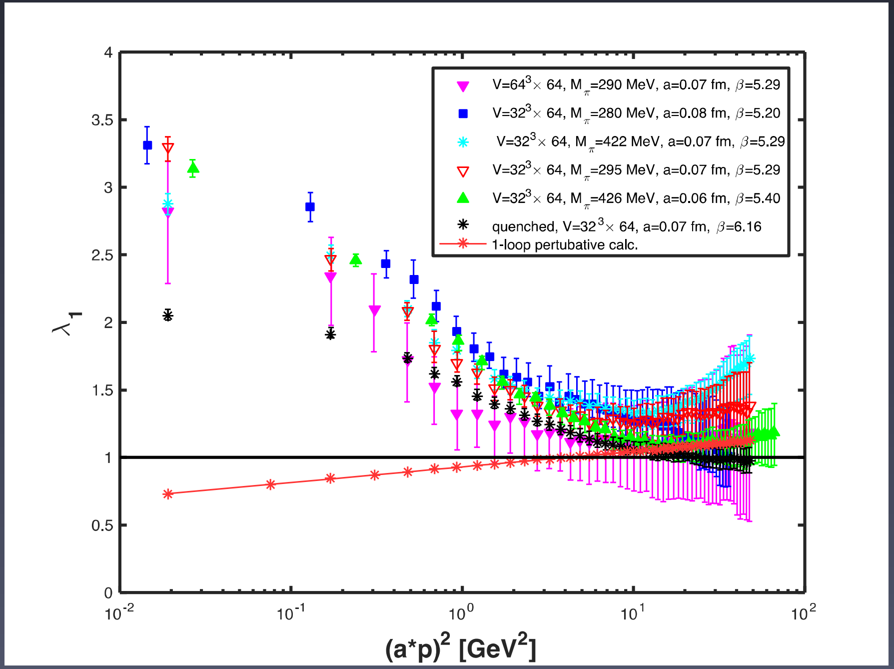

quenched, V=32^3 × 64, a=0.07 fm, β=6.16
Binned data
1-loop pertubative calc.
λ_1
(a*p)^2 [GeV^2]

TL corrected, binned data



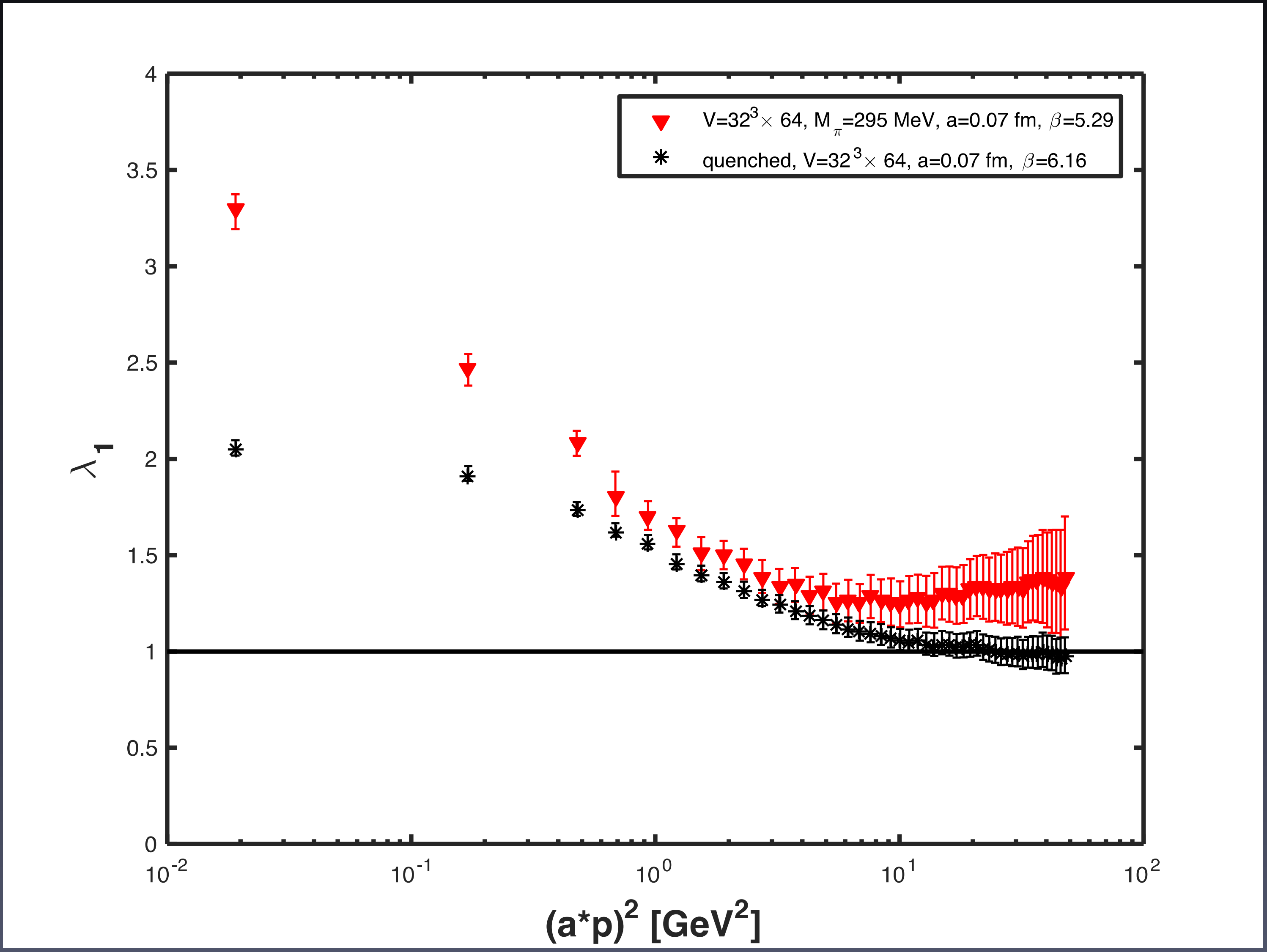






Soft gluon Kinematics (Asymmetric)

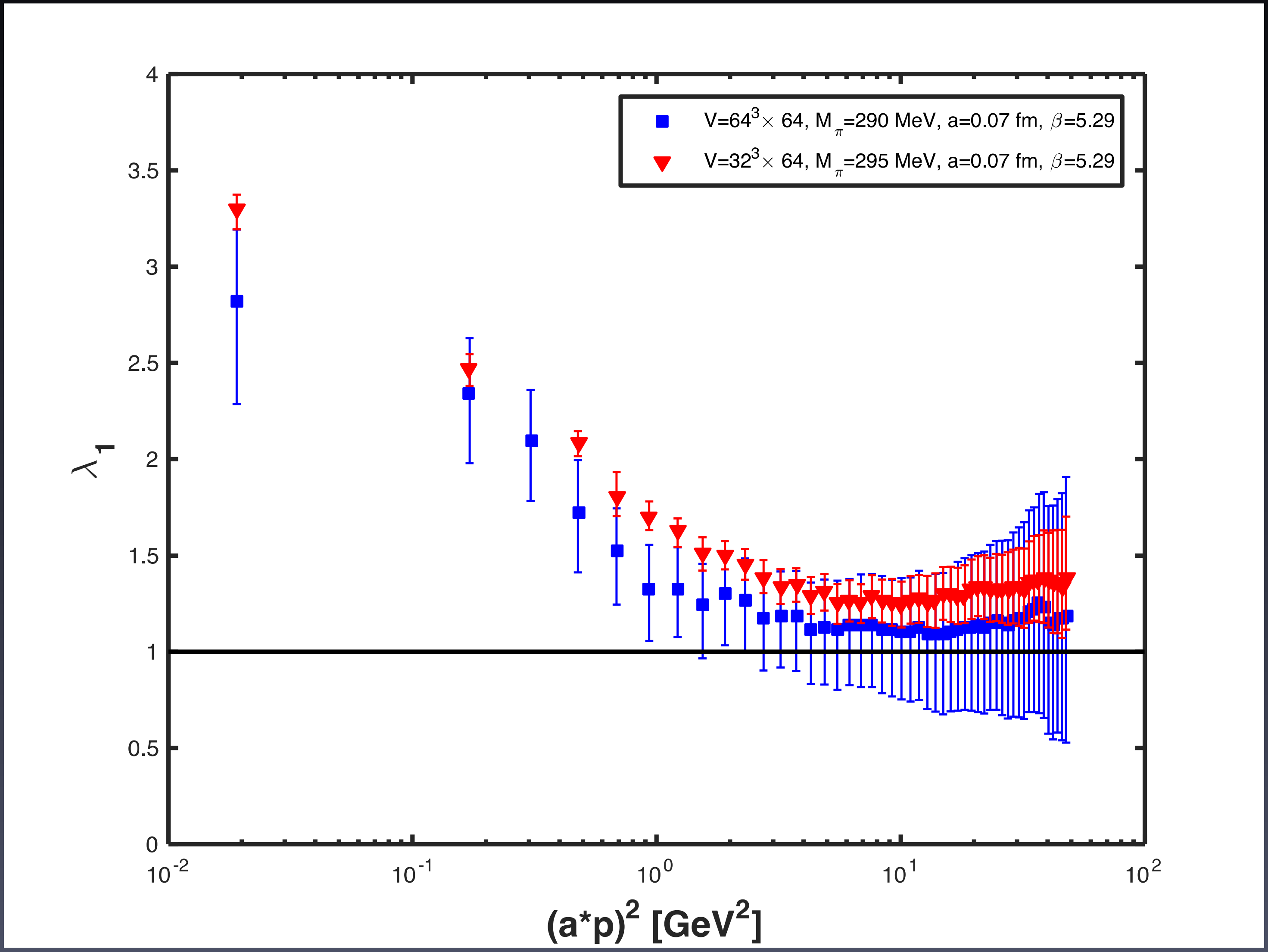
l_1 Without tree-level correction



Soft gluon Kinematics (Asymmetric)

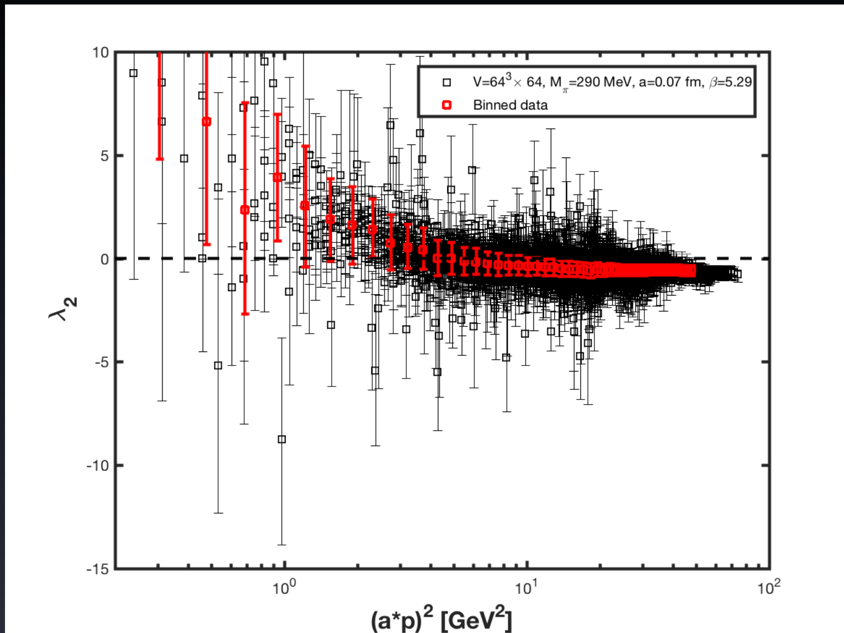
Volume dependence

l1 With tree-level correction

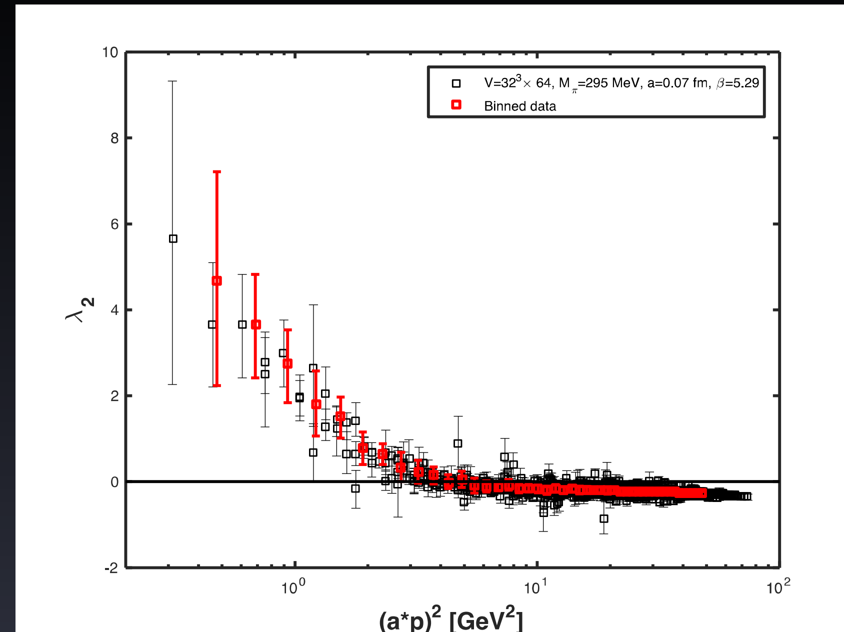


l2 without tree-level correction

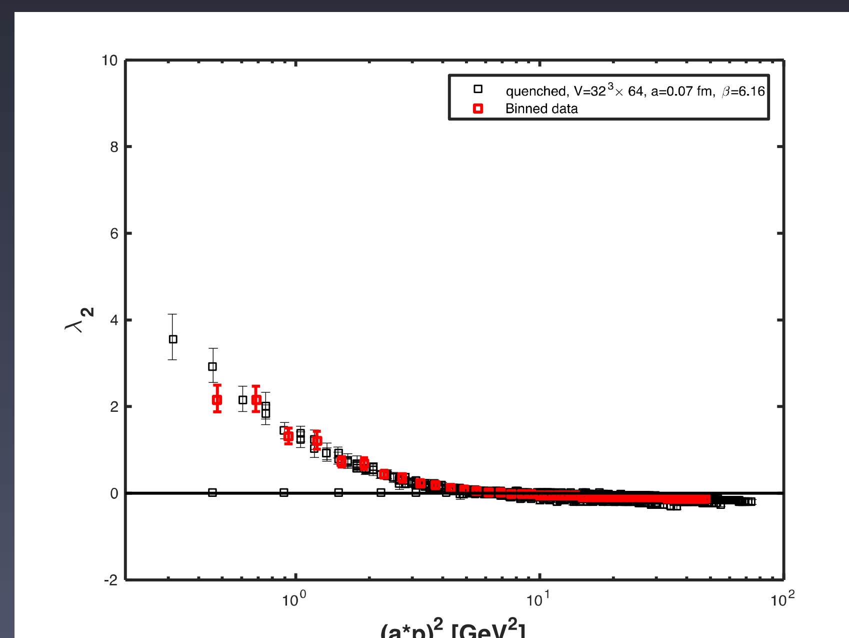
Unquenched, Nf = 2



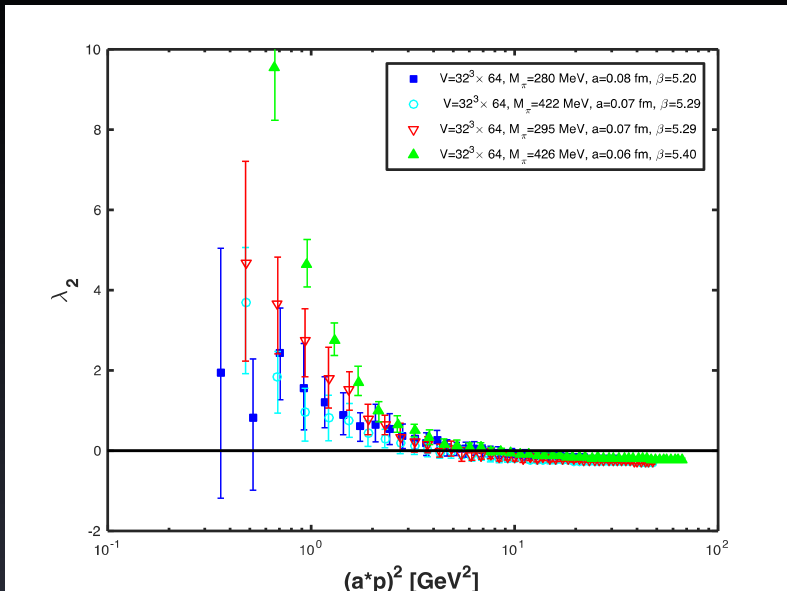
Unquenched, Nf = 2

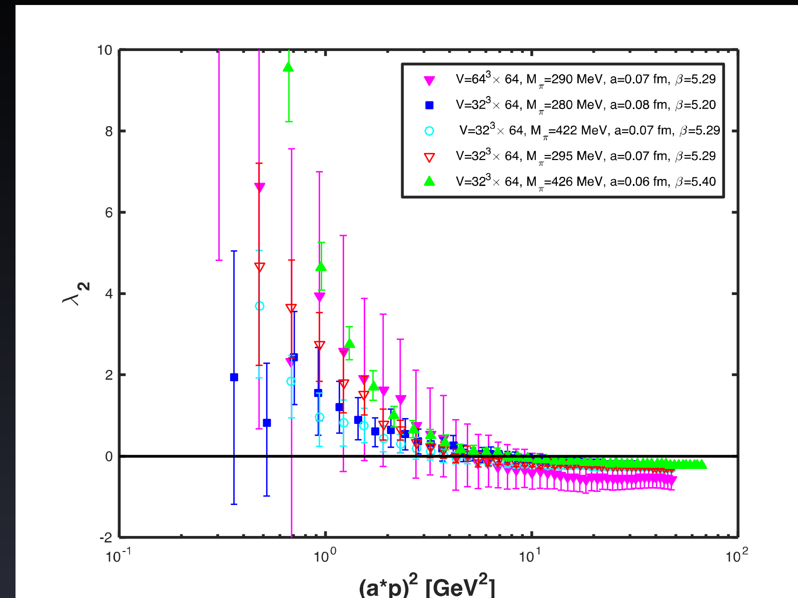


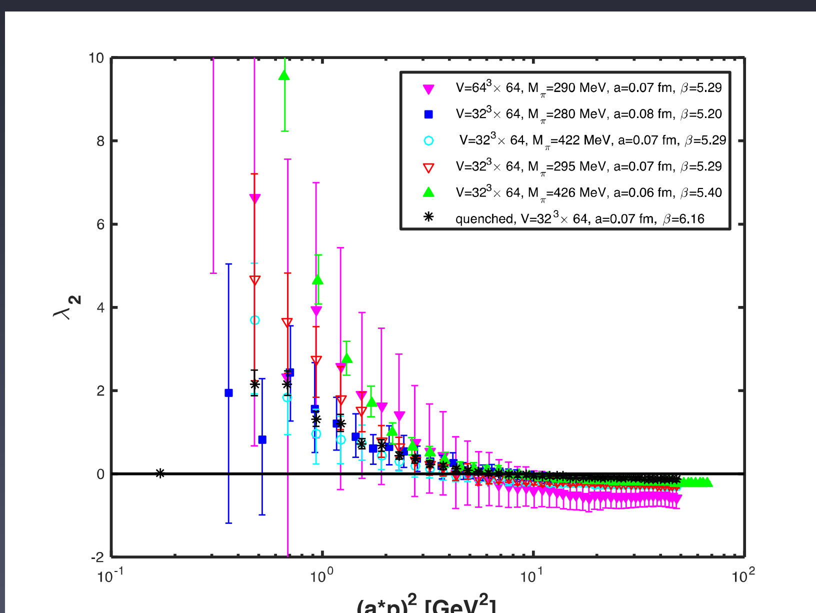
Quenched, $Nf = 0$

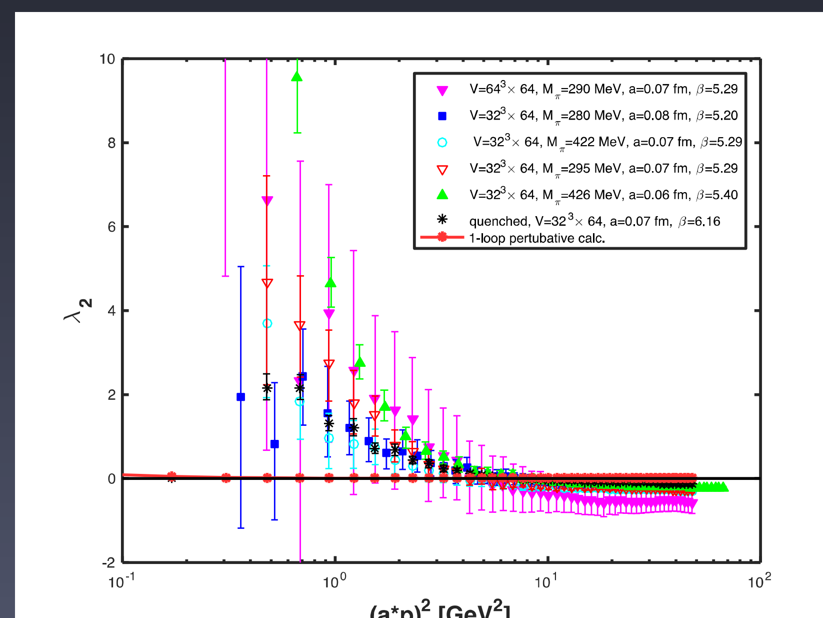


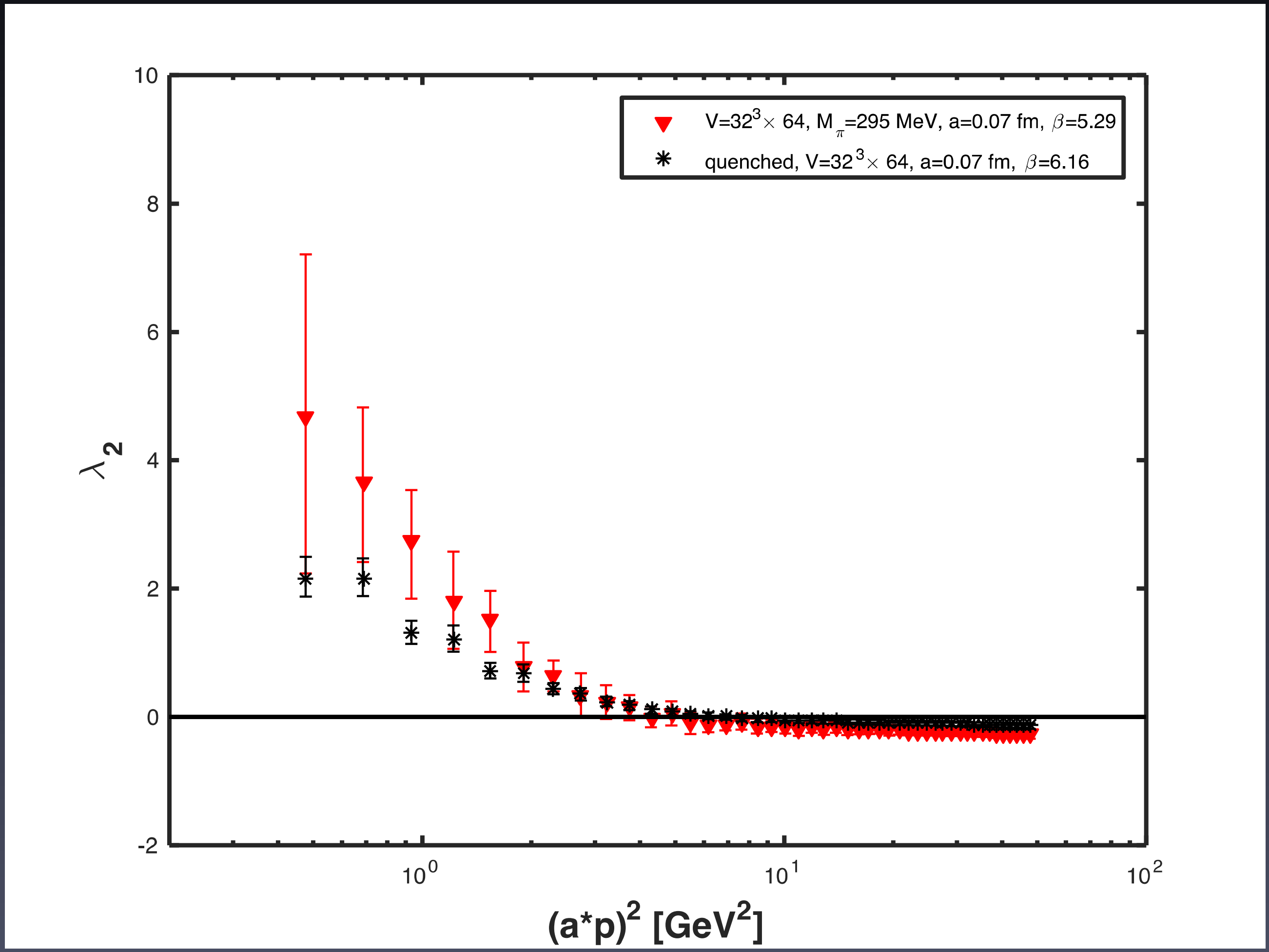
|2 without tree-level correction









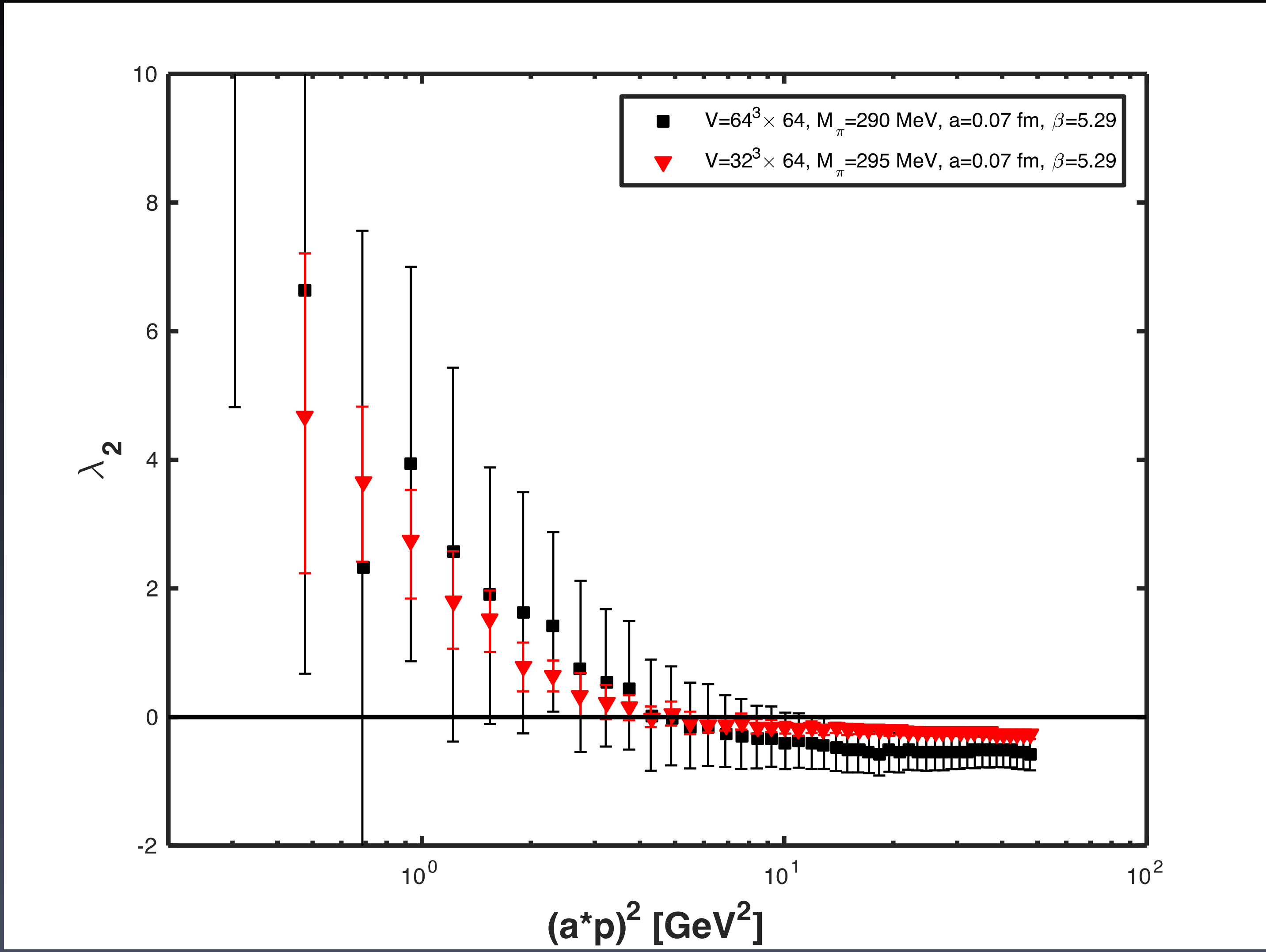


V=32^3× 64, M_π=295 MeV, a=0.07 fm, β=5.29
quenched, V=32^3× 64, a=0.07 fm, β=6.16
λ_2
(a*p)^2 [GeV^2]
10
8
6
4
2
0
-2
10^0
10^1
10^2

Volume dependence

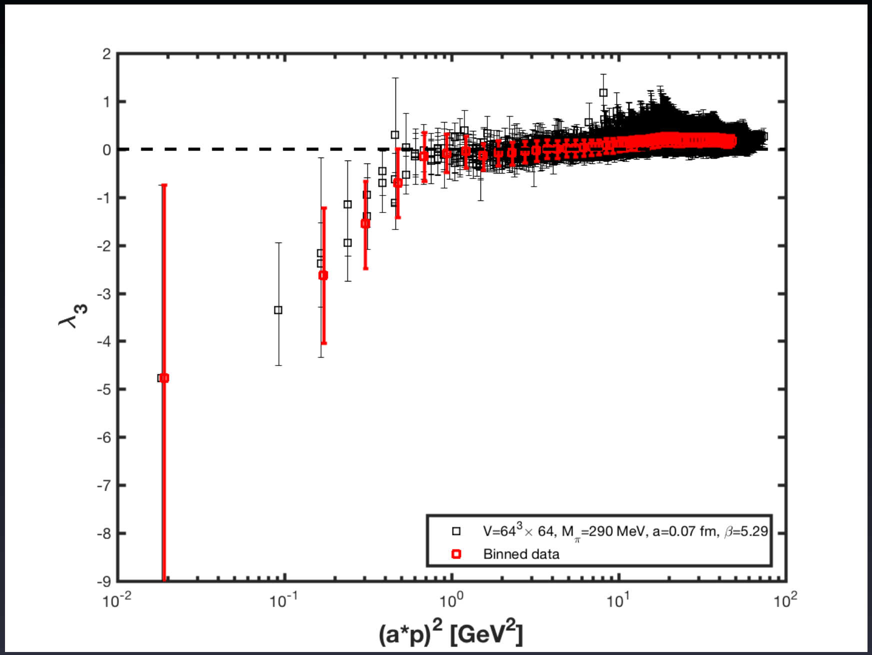
l2 **Without tree-level correction**

Soft gluon Kinematics (Asymmetric)

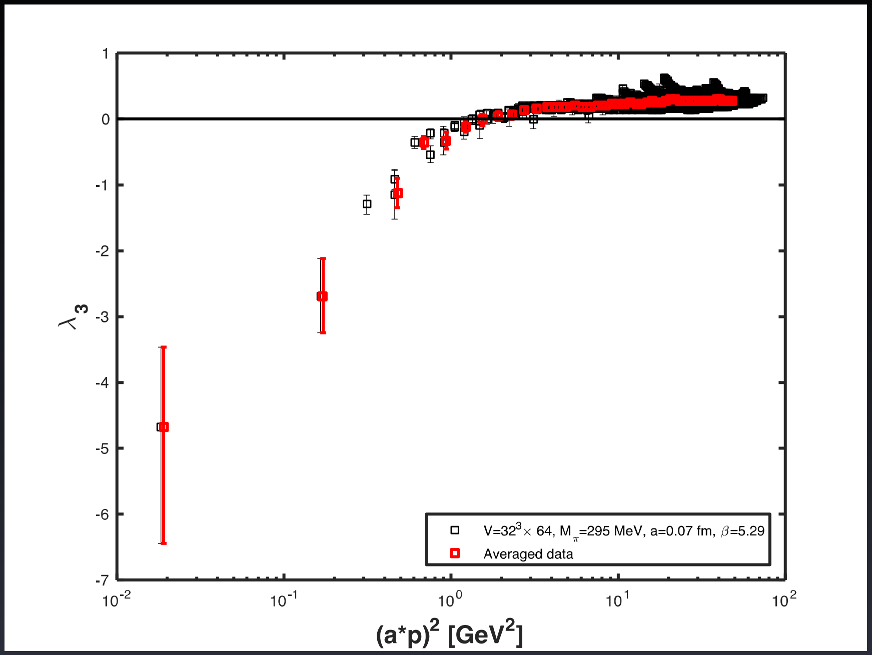


l3 without tree-level correction

Unquenched, Nf = 2

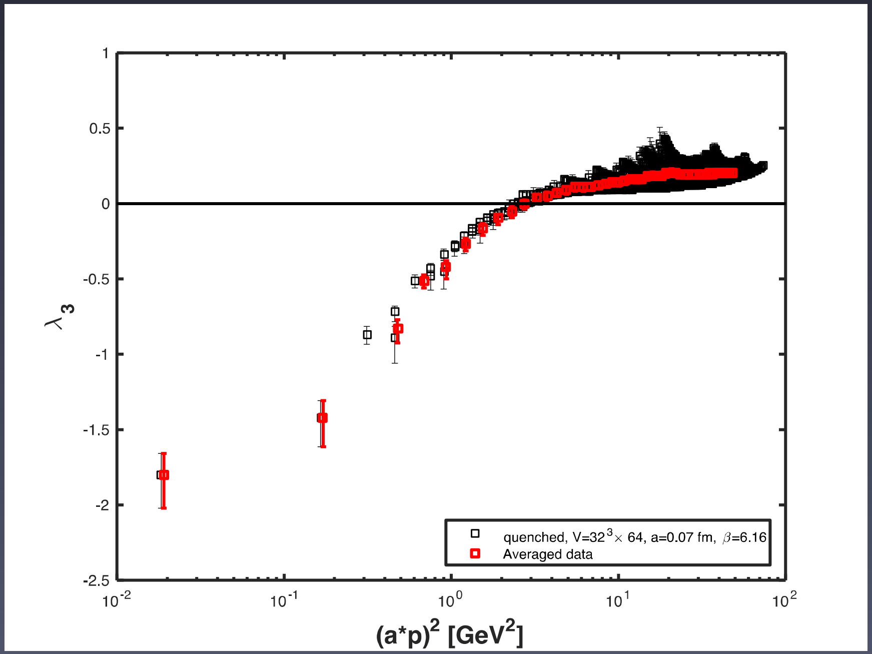


Unquenched, Nf = 2

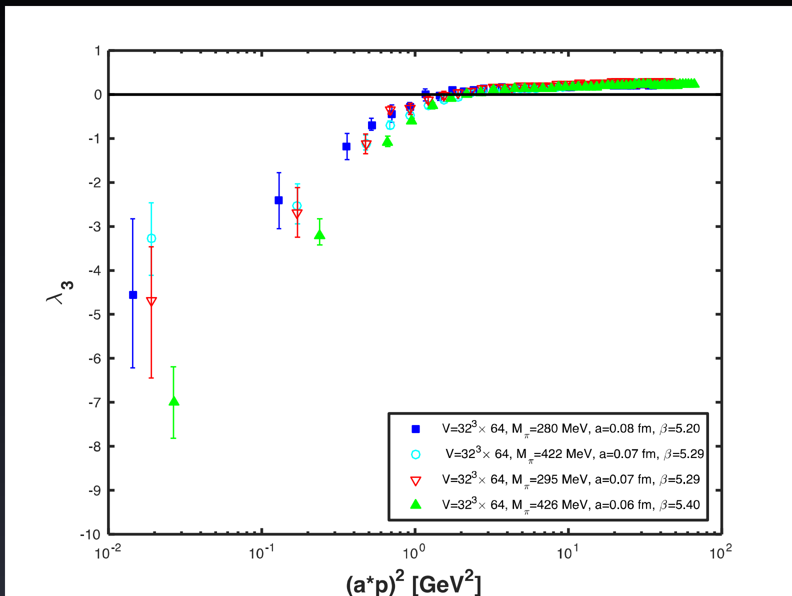


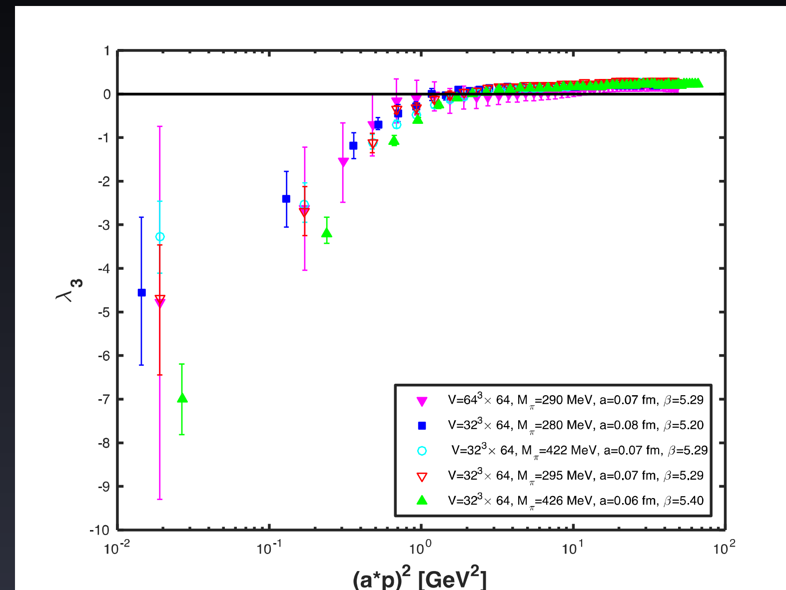
Quenched, Nf = 2

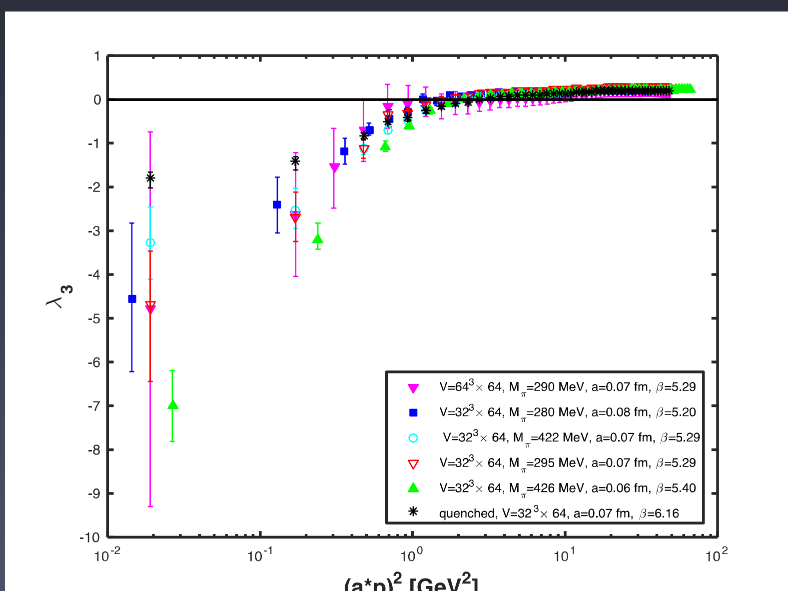
323x64

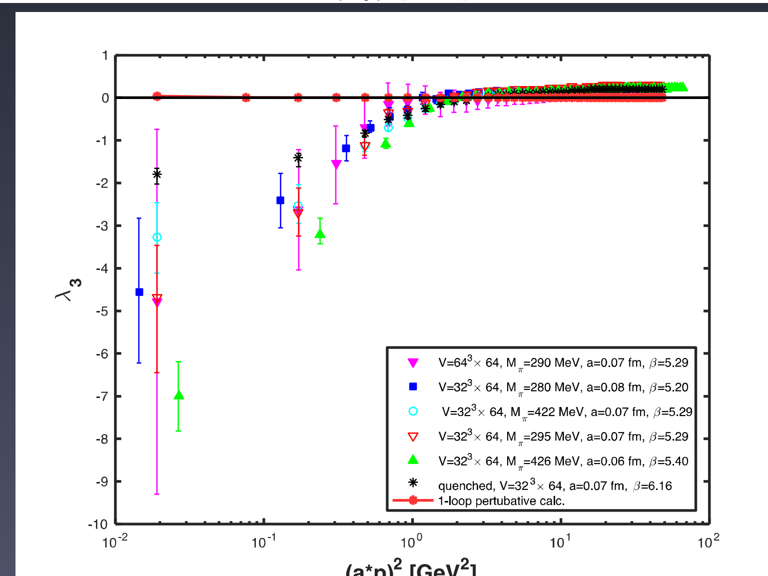


l3 Without tree-level correction





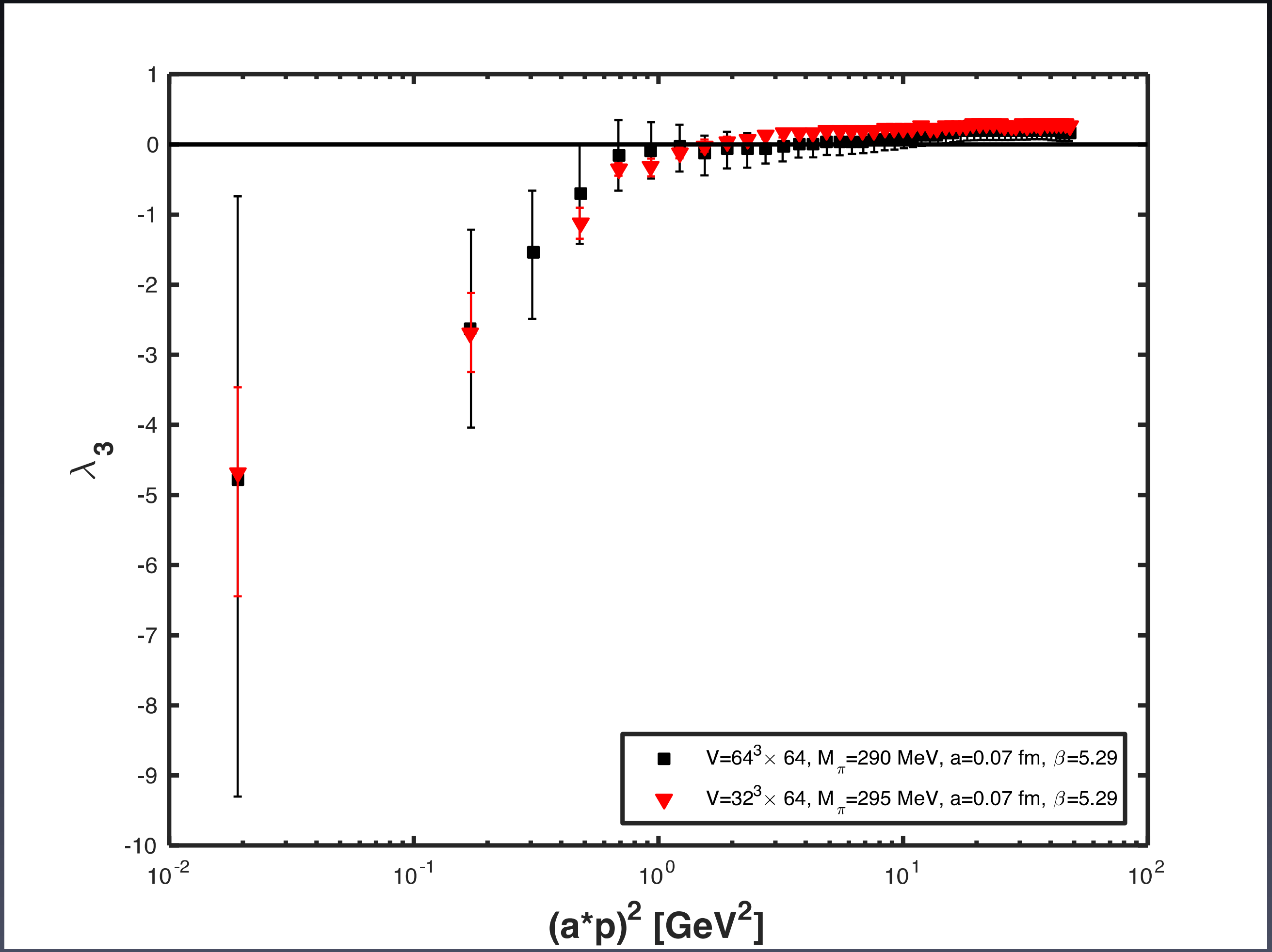




Soft gluon Kinematics (Asymmetric)

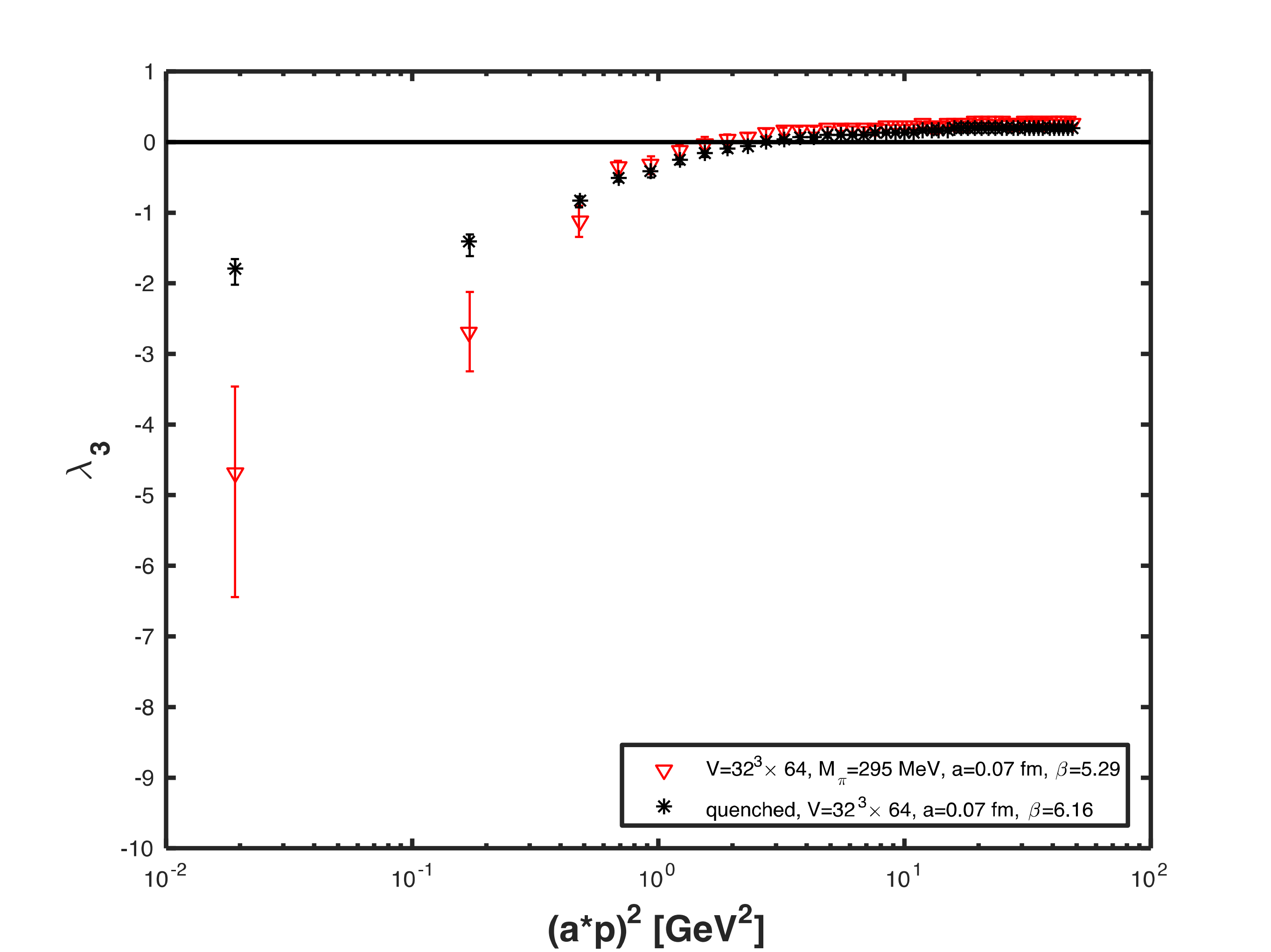
Volume dependence

l3 Without tree-level correction



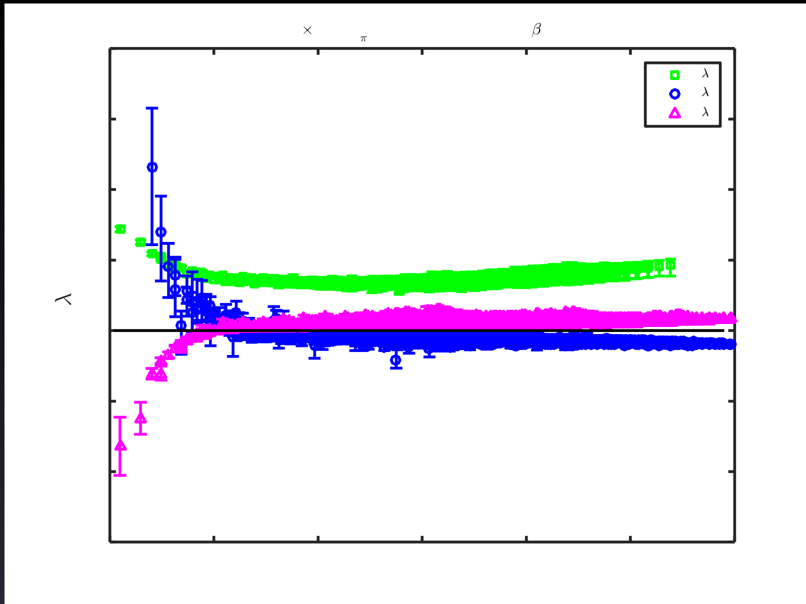
Unquenched versus Quenched

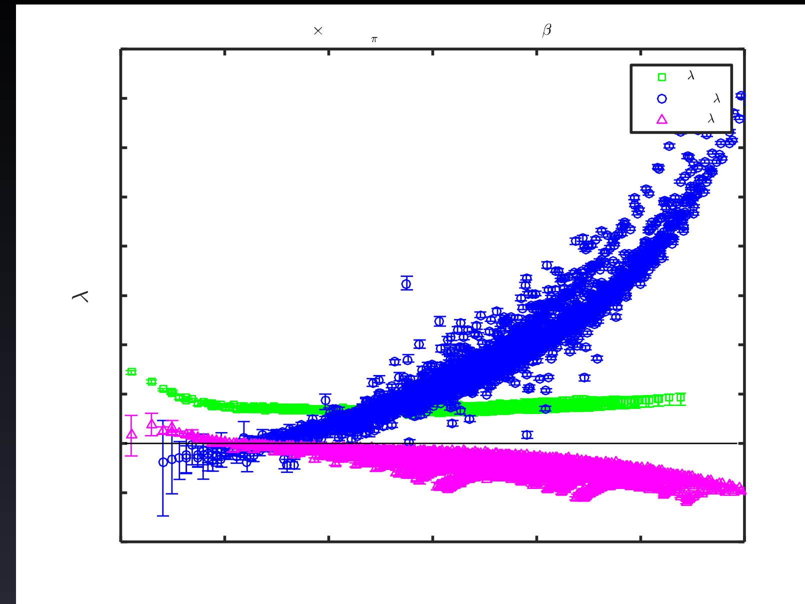
l3 With tree-level correction



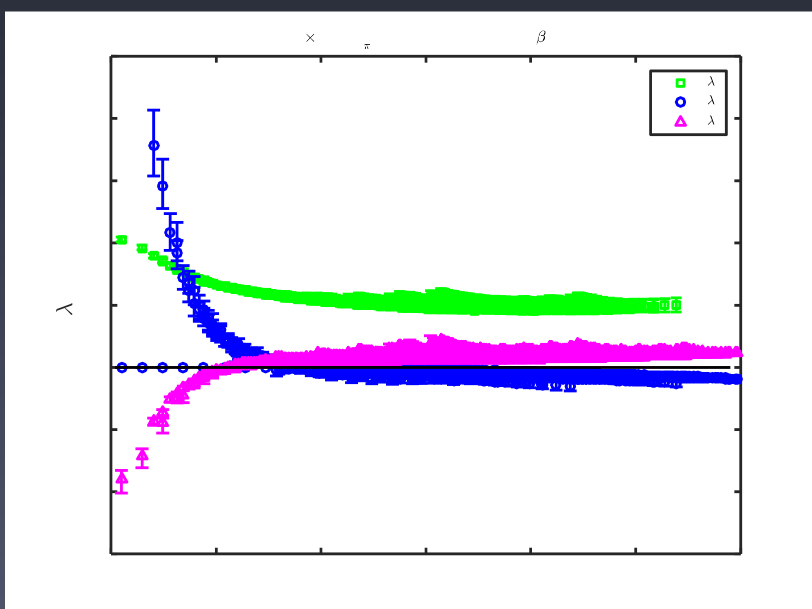
Soft gluon Kinematics (Asymmetric)

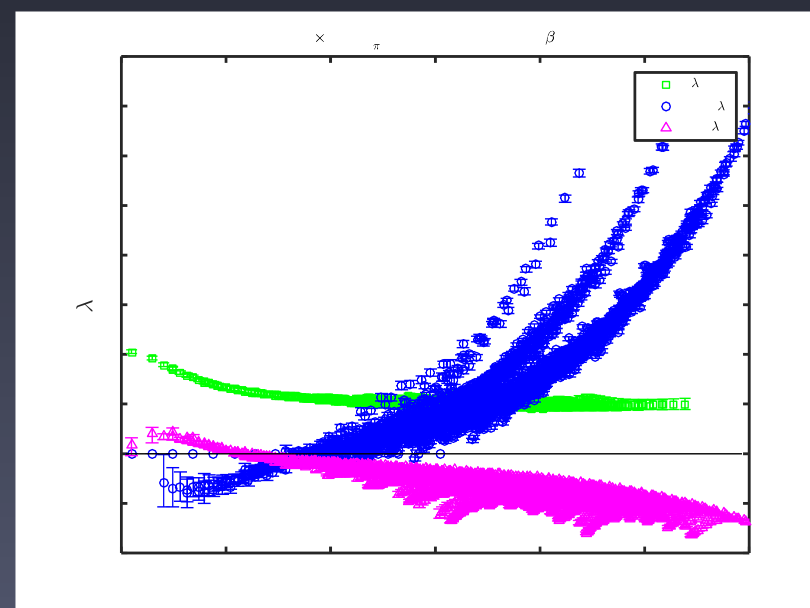
Hirarchy of Form Factors



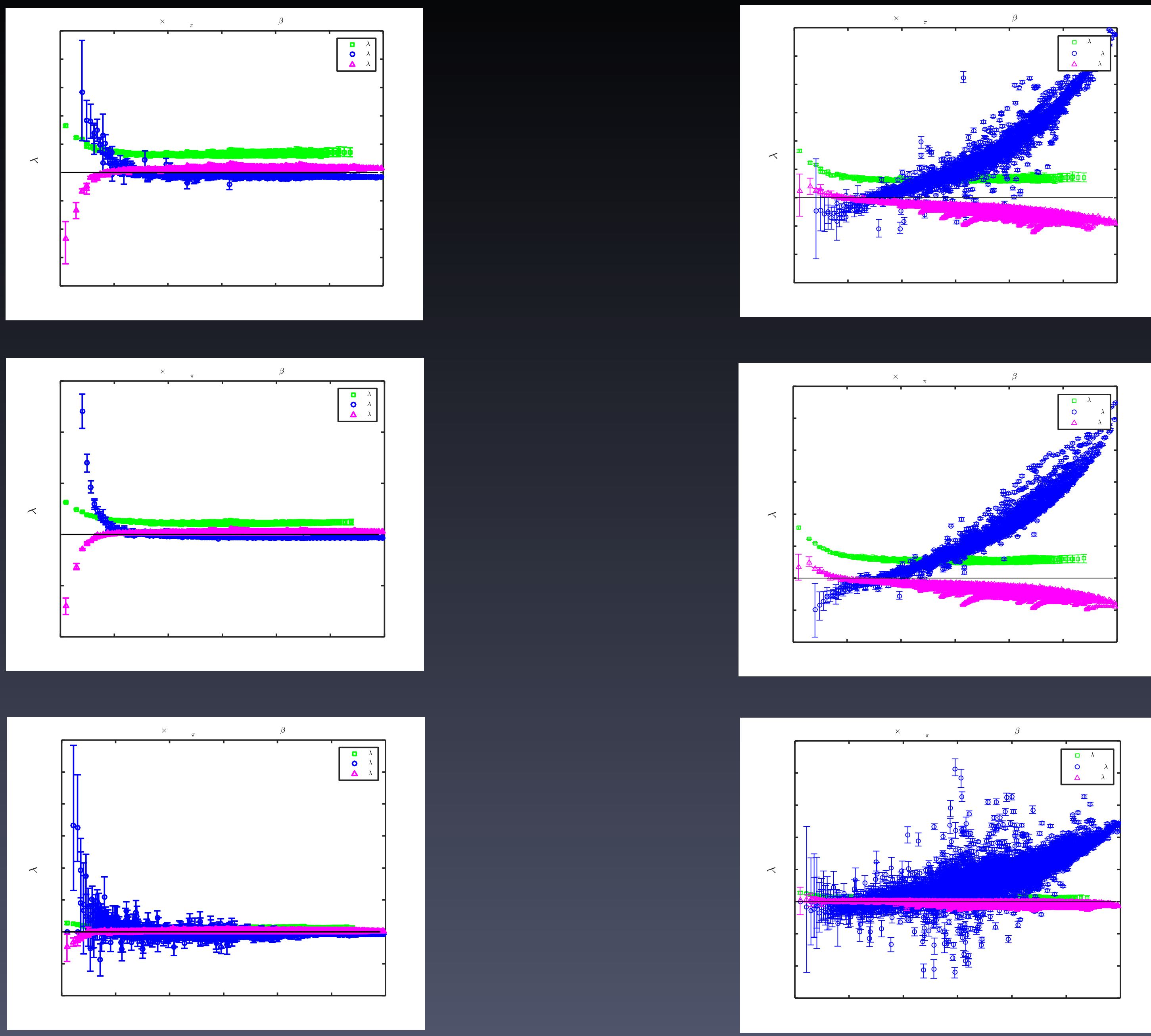


$$\mathbf{(\Lambda^{a}_{\mu})^{ij}_{\alpha\beta} = -i g_0 t^{a}_{ij}}(\lambda^{E}_{1}\,\gamma_\mu - 4\,\lambda^{E}_{2}\;\not{p}\,p_\mu - 2i\lambda^{E}_{3}\,p_\mu - 2i\lambda^{E}_{4}\,\sigma_{\mu\nu}p_\nu)_{\alpha\beta}$$





Hirarchy of Form Factors

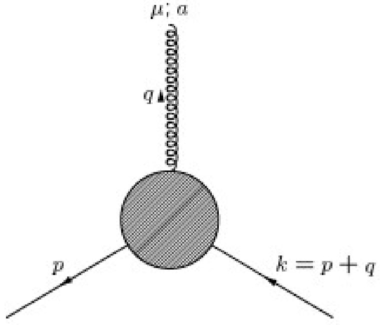


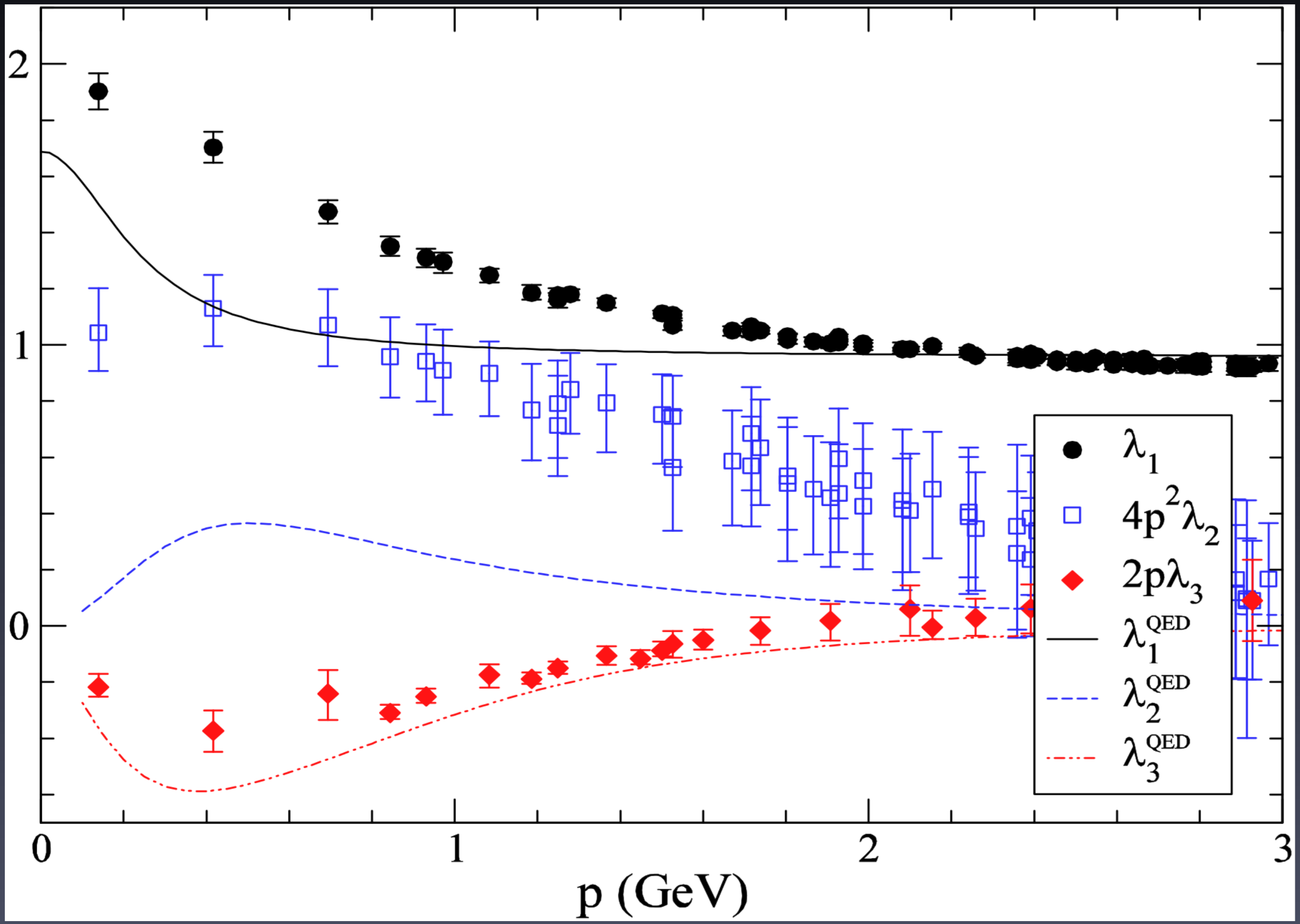
$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 = 0$$

$$\Lambda_1, \tau_5$$

$$\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \tau_5, \tau_7$$

$$\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \tau_5, \tau_6, \tau_7, \tau_8$$





2
1
0
0
1
2
3
p (GeV)
λ_1
$4p^2\lambda_2$
$2p\lambda_3$
λ_1^{QED}
λ_2^{QED}
λ_3^{QED}

A.Kizilersu and M.R. Pennington,Phys.Rev.D79,(2009),125020

Kizilersu-Pennington Unquenched Vertex

$$\begin{aligned}
\tau_2^E(p^2,k^2,q^2) =& -\frac{4}{3}\,\frac{1}{(k^4-p^4)}\,\left(A(k^2)\,-\,A(p^2)\right) \\
& -\frac{1}{3}\,\frac{1}{(k^2+p^2)^2}\,\left(A(k^2)+A(p^2)\right)\,\ln\left[\left(\frac{A(k^2)\,A(p^2)}{\boxed{A(q^2)^2}}\right)\right] \\
\tau_3^E(p^2,k^2,q^2) =& -\frac{5}{12}\,\frac{1}{(k^2-p^2)}\,\left(A(k^2)\,-\,A(p^2)\right) \\
& -\frac{1}{6}\,\frac{1}{(k^2+p^2)}\,\left(A(k^2)+A(p^2)\right)\,\ln\left[\left(\frac{A(k^2)\,A(p^2)}{\boxed{A(q^2)^2}}\right)\right] \\
\tau_6^E(p^2,k^2,q^2) =& \frac{1}{4}\,\frac{1}{(k^2+p^2)}\,\left(A(k^2)\,-\,A(p^2)\right) \\
\tau_8^E(p^2,k^2,q^2) =& 0
\end{aligned}$$

- Unquenched transverse vertex with q2 dependence
- It respects MR and gauge invariance
- It satisfies the fermion and photon SDE's in all orders in leading logarithms for massless QED.