ECT* European Centre for Theoretical Studies in Nuclear Physics and Related Areas

FONDAZIONE BRUNO KESSLER

INFN Istituto Nazionale di Fisica Nucleare

TIFPA Trento Institute for Fundamental Physics and Applications

Somewhere over the rainbow

Daniele Binosi

ECT* - Fondazione Bruno Kessler, Italy

Non-perturbative QCD 2016

October 17-21, 2016

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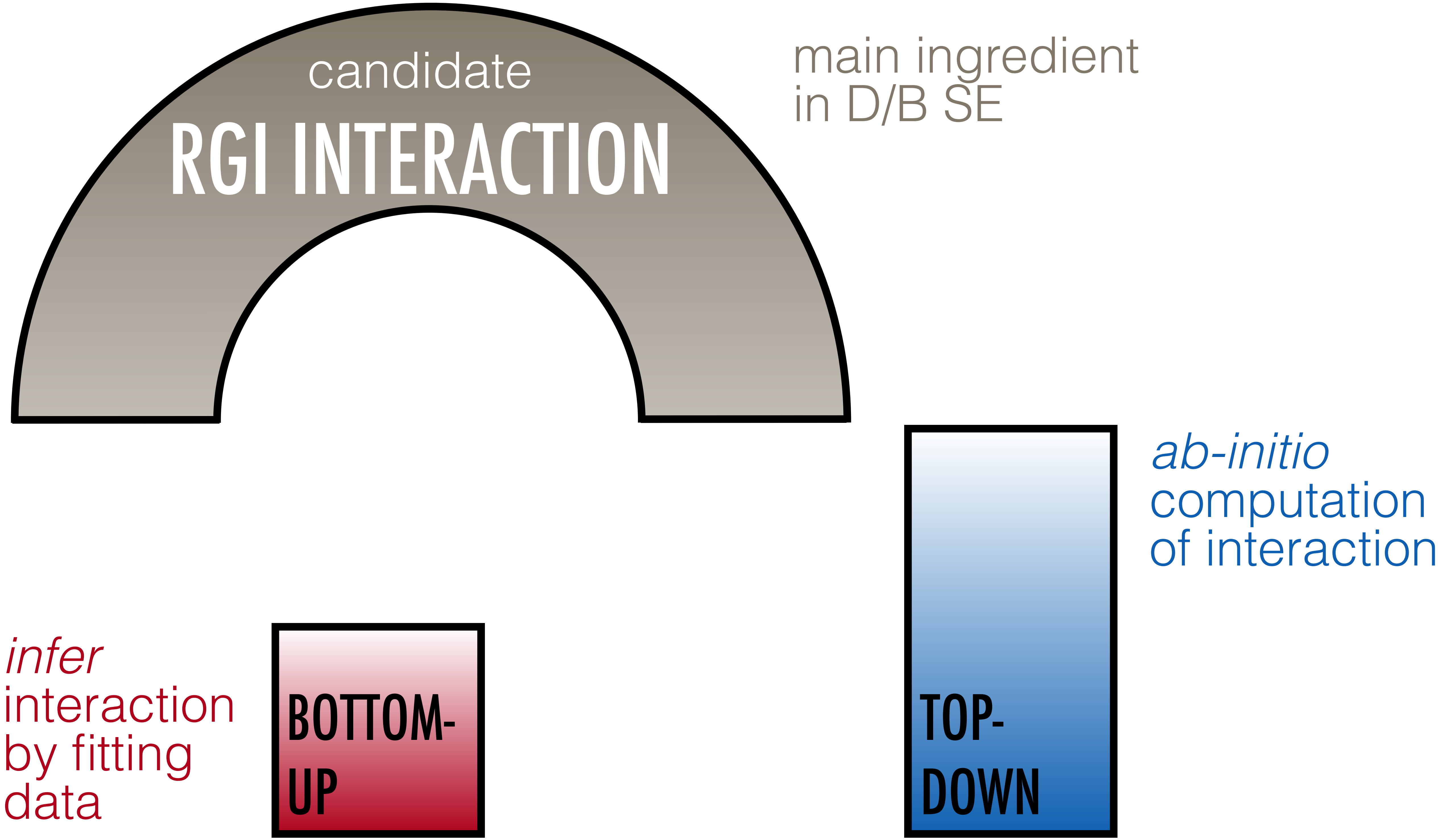
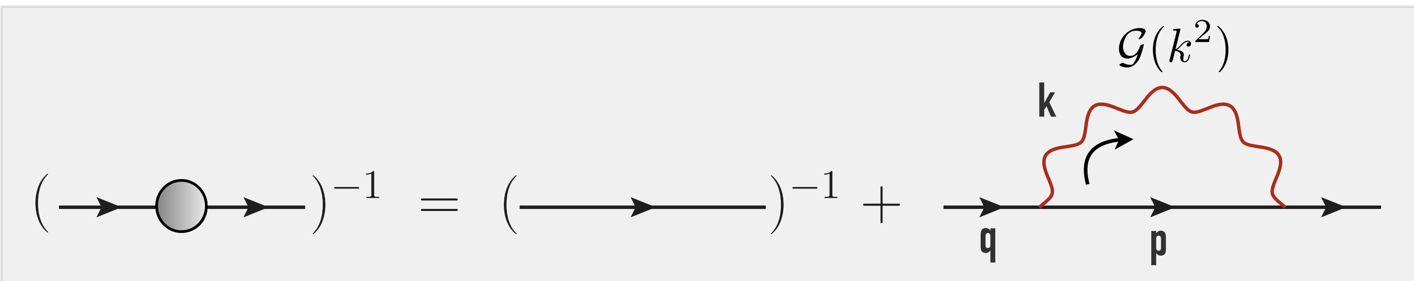
Gap equation's interaction kernel

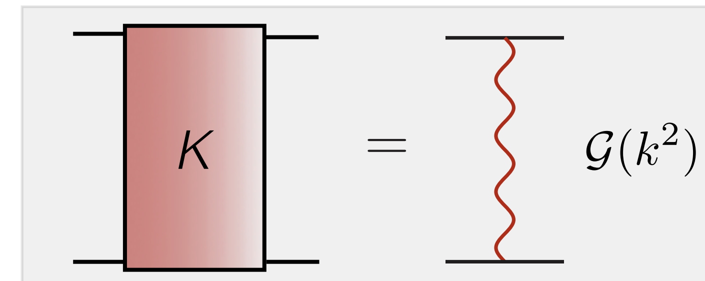
Gap equation’s interaction kernel



RGI interaction: bottom-up approach - I

- **Use rainbow-ladder truncation:**
  tree level vertex + one gluon exchange effective kernel



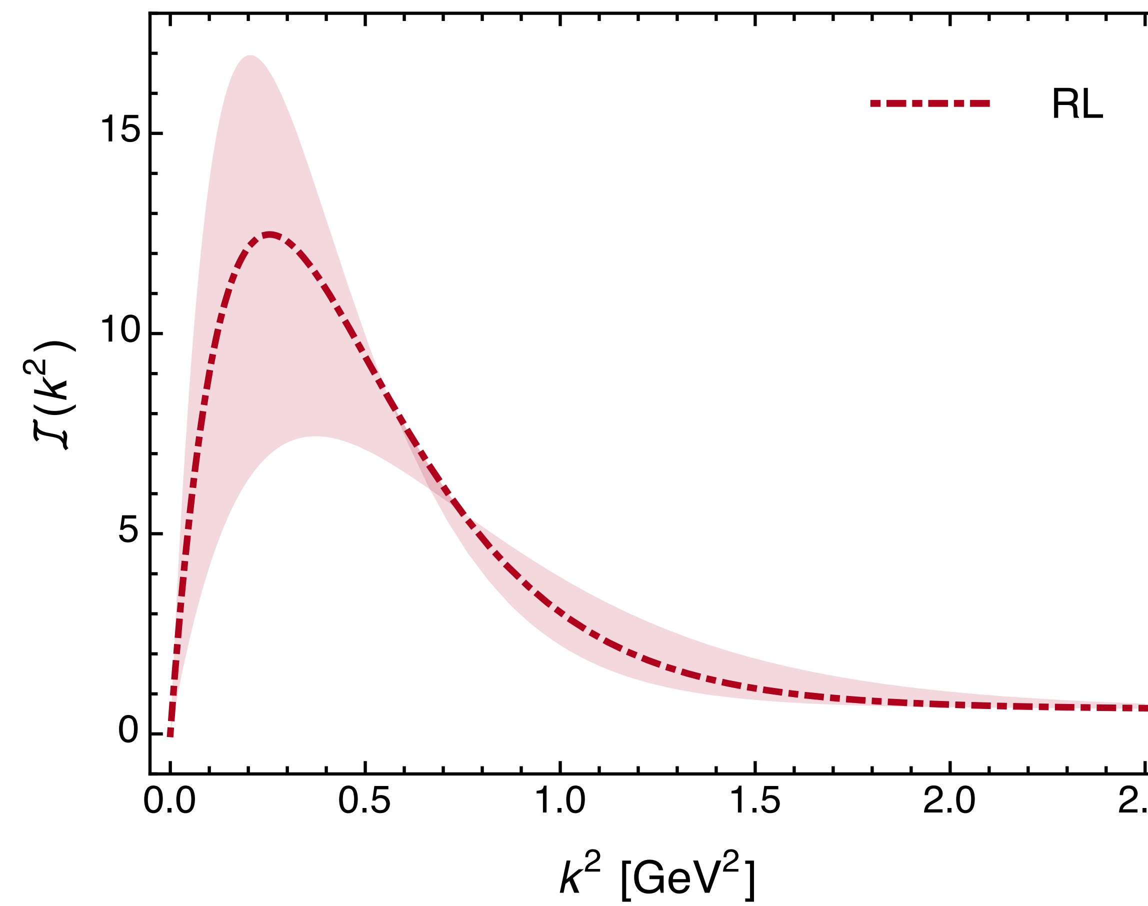


- **Interaction strength Ansatz**
  IR: Gaussian; UV: perturbative tail
  Maris, Roberts, Tandy, PRC 56 (1997); PRC 60 (1999)

$$\mathcal{I}(k^2) = k^2 \frac{\mathcal{G}_{\rm IR}(k^2) + \mathcal{G}_{\rm UV}(k^2)}{4\pi}$$

$$\mathcal{G}_{\rm IR}(k^2) = \frac{8\pi^2}{\omega^5}\varsigma^3 {\rm e}^{-k^2/\omega^2}$$

$$\mathcal{G}_{\rm UV}(k^2) = \frac{96\pi^2}{25}\frac{1-e^{-k^2/1[{\rm GeV}^2]}}{k^2\log[{\rm e}^2-1+(1+k^2/\Lambda^2)^2]}$$

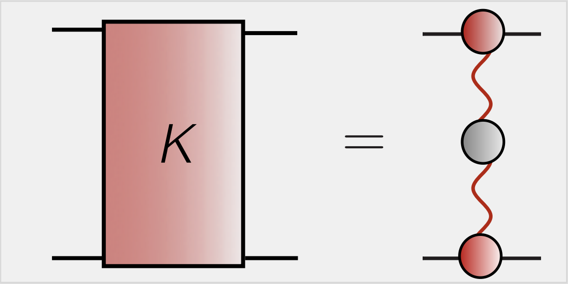


- **One parameter interaction** ($\Lambda_{\rm MS}$=234 [MeV])
  ς fitted to obtain pion decay constant

$$\varsigma_{\rm RL} = 0.87\ [{\rm GeV}]$$

- **Depends on truncation used:** $\Gamma_\nu = \gamma_\nu$
- **Many observables invariant while** $\omega \in [0.4, 0.6]$

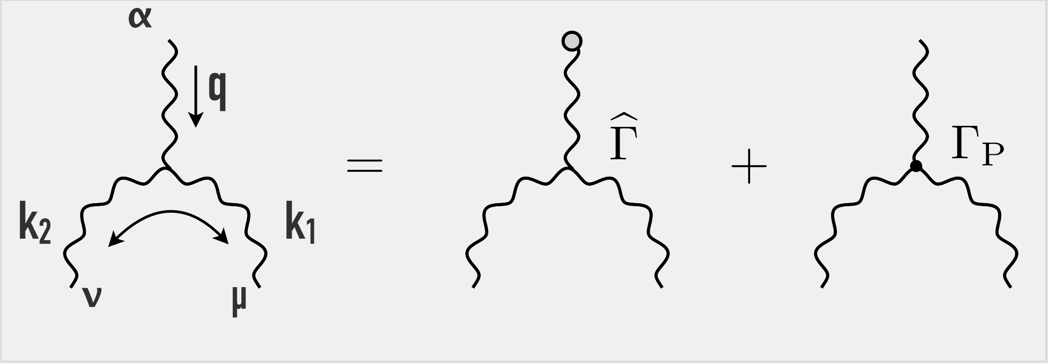
RGI interaction: top-down approach



- **Universal (process-independent) contribution:**
  originates entirely from the gauge sector

- **Fundamental quantities: PT-BFM propagators/vertices**
  satisfy Abelian-like Slavnov-Taylor (ST) identities

- **How to get them?**
  use the PT algorithm
  Cornwall, Papavassiliou, PRD 40 (1989)

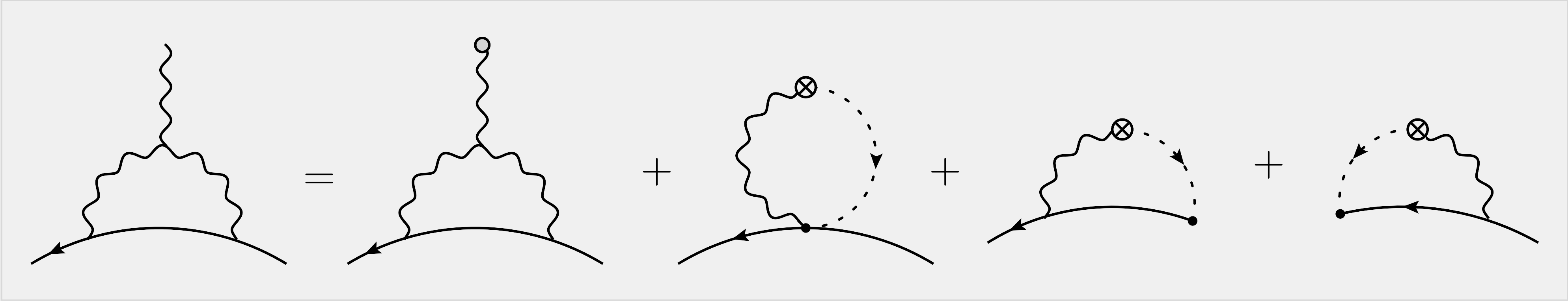


$$\widehat{\Gamma}^{\alpha\mu\nu} = (k_2 - k_1)^\alpha g^{\mu\nu} + 2q^\nu g^{\alpha\mu} - 2q^\mu g^{\alpha\nu}$$

$$\Gamma_{\mathrm{P}}^{\alpha\mu\nu} = k_1^\mu g^{\alpha\nu} - k_2^\nu g^{\alpha\mu}$$

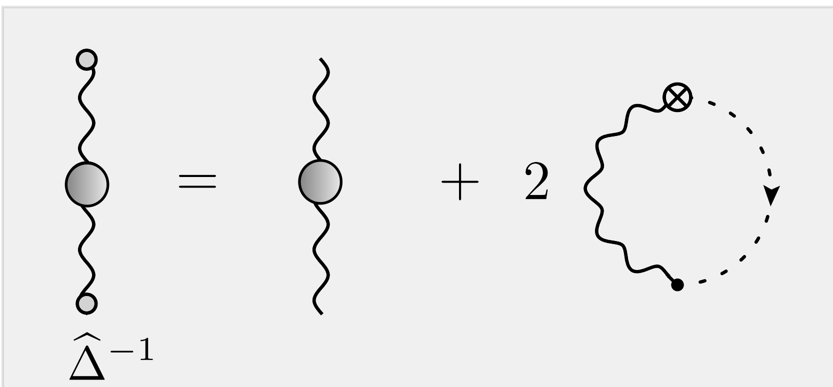
- **longitudinal momenta**
  trigger elementary Ward identities

- **Apply the PT to the quark-gluon vertex**
  one loop result:

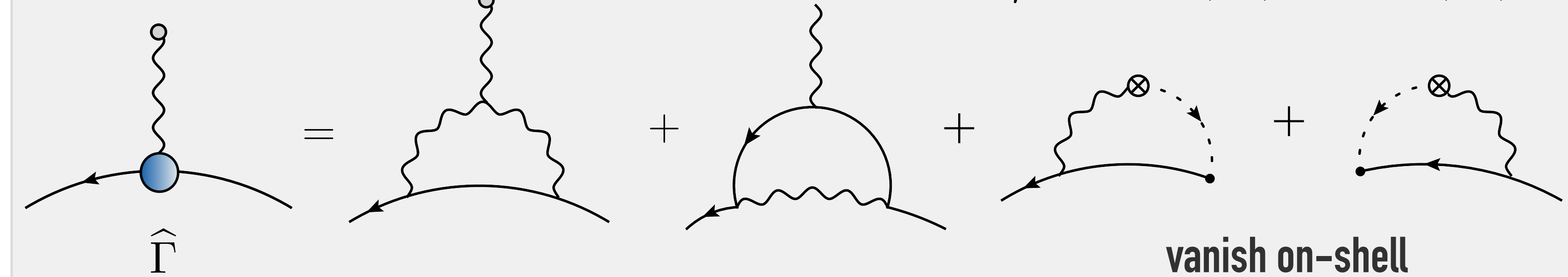


RGI interaction: top-down approach

- **Allot pieces to different Green's functions**
  construct $\widehat{\Delta}$ and $\widehat{\Gamma}_\mu$



$$q^\mu \widehat{\Gamma}_\mu = S^{-1}(p_1) - S^{-1}(p_2)$$



- **Crucial all-order equivalence: PT=BFM**
  yields Feynman rules for systematic calculation

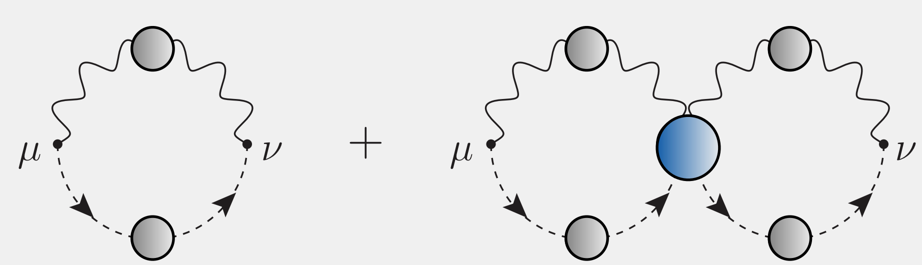
$$\widehat{\Delta} \sim \frac{1}{q^2[1 + bg^2 \log q^2/\mu^2]}; \qquad b = 11C_A/48\pi^2$$

- **Absorbs all the RG logs**
  as the photon in QED
- **Renormalizes as** Z_g^{-2}

- **An additional equivalence holds: antiBRST+BRST=BFM**
  plethora of symmetry identities, in particular BQ identities
  DB, Quadri, PRD 88 (2013)

$$\Delta(q^2) = [1 + G(q^2)]^2 \widehat{\Delta}(q^2)$$

$\Lambda_{\mu\nu}(q) =$



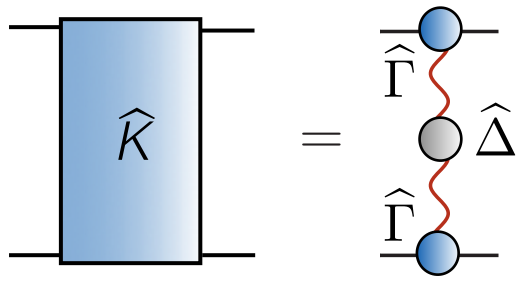
$$= G(q^2) g_{\mu\nu} + L(q^2) \frac{q_\mu q_\nu}{q^2}$$

- ***G* special PT-BFM function:**
  determined by ghost-gluon dynamics
- **Combination** 1+*G* **appears in *all* BQIs**
  fundamental non-Abelian quantity
- ***G* is related (Landau gauge) to the ghost dressing:**
  use ghost gap equation to constrain *1+G, L*

$$F^{-1}(q^2) = 1 + G(q^2) + L(q^2)$$

RGI interaction: top-down approach

- **Convert vertices/propagators into PT-BFM ones**
  new RG invariant combination appears

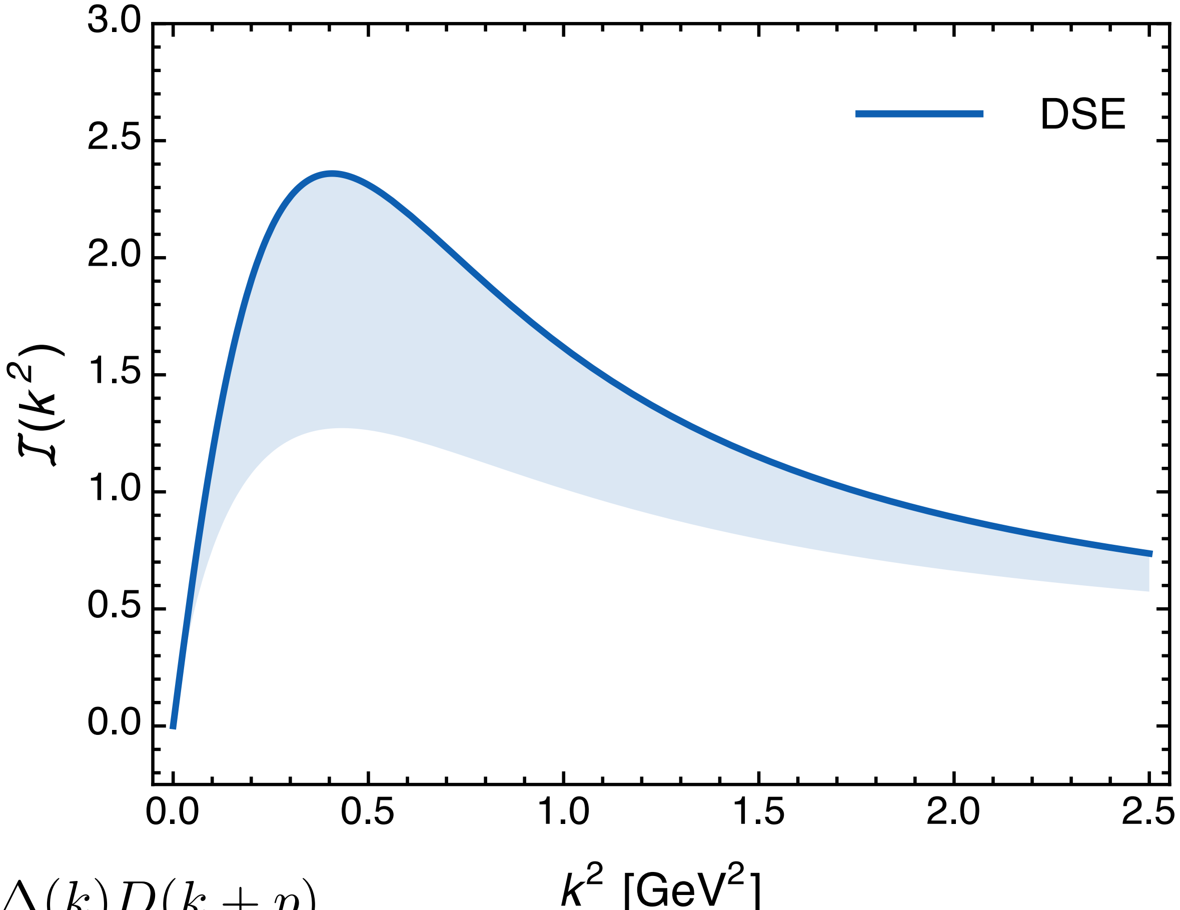


$$\widehat{d}(k^2) = \alpha(\mu^2)\widehat{\Delta}(k^2;\mu^2)$$

- **Use symmetry identity**
  to identify the interaction strength
  Aguilar, DB, Papavassiliou, Rodriguez-Quintero, PRD 90 (2009)
  DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)

$$\mathcal{I}(k^2) = k^2\widehat{d}(k^2)$$

$$\widehat{d}(k^2) = \frac{\alpha(\mu^2)\Delta(k^2;\mu^2)}{[1+G(k^2;\mu^2)]^2}$$



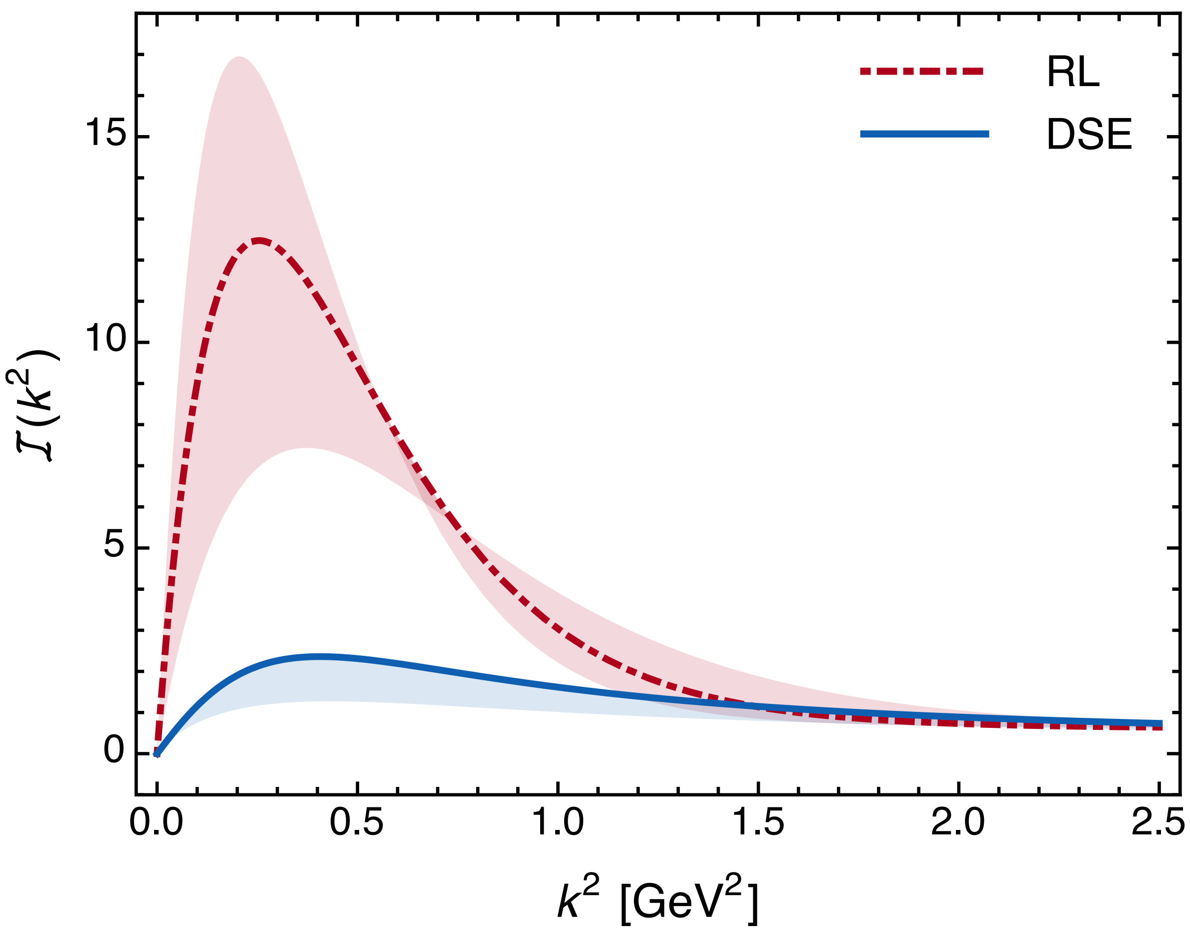
- **1+*G* and *L* determined by their own SDEs**
  under simplifying assumptions:

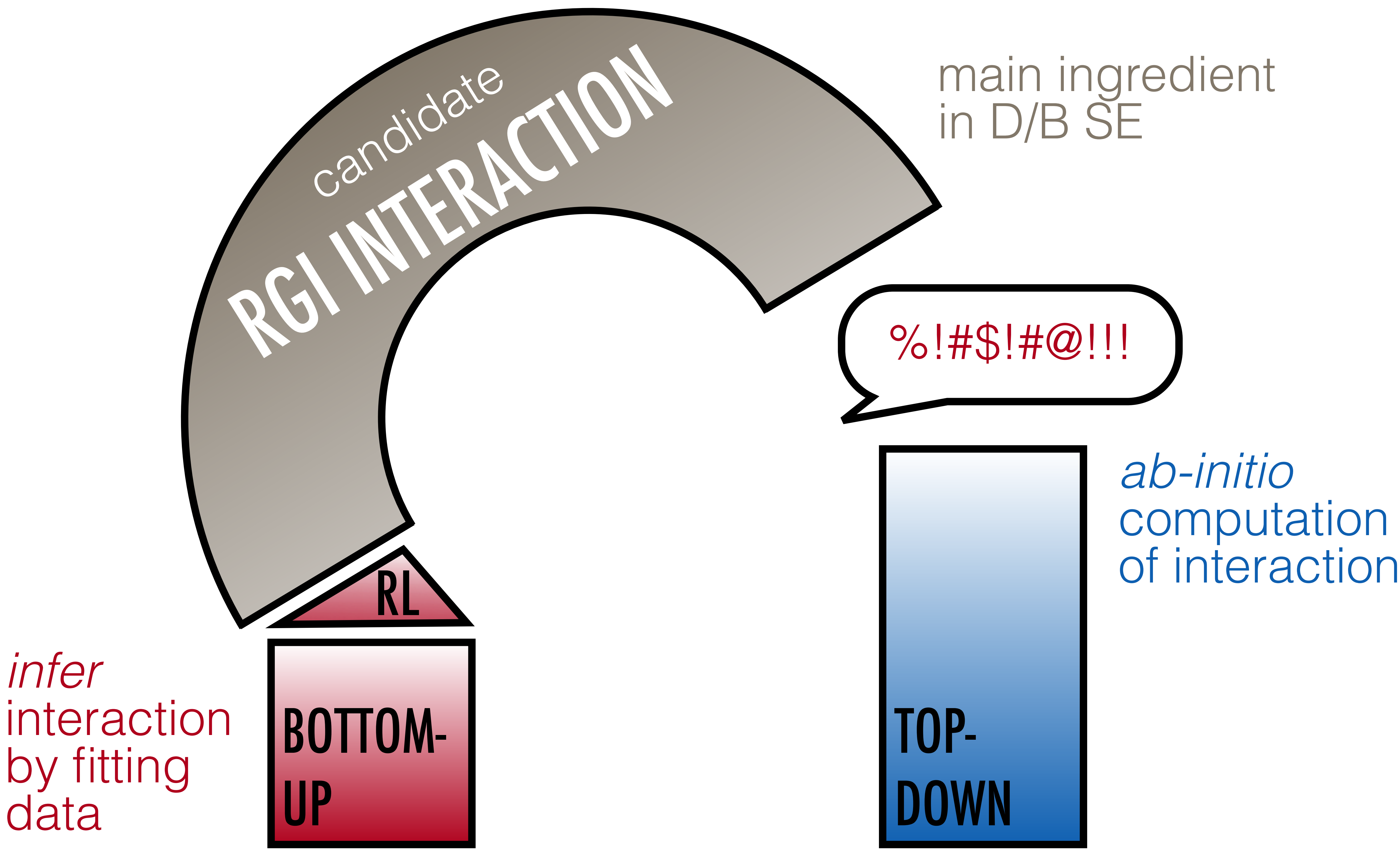
$$1+G(p^2) = Z_c + \frac{g^2 C_{\text{A}}}{d-1}\int_k \left[(d-2) + \frac{(k\cdot p)^2}{k^2p^2}\right] B_1(-k,0,k)\Delta(k)D(k+p)$$

$$L(p^2) = \frac{g^2 C_{\text{A}}}{d-1}\int_k \left[1 - \frac{(k\cdot p)^2}{k^2p^2}\right] B_1(-k,0,k)\Delta(k)D(k+p)$$

- **Main source of uncertainties:**
  needs assumptions on ghost vertex behavior
- **Parametrized by** δ∈[0,1]
  lower bound (δ=0): *1/F=1+G*

Top-down vs bottom-up comparison - I




candidate
RGI INTERACTION
main ingredient
in D/B SE
%!#$!#@!!!
RL
infer
interaction
by fitting
data
BOTTOM-
UP
TOP-
DOWN
ab-initio
computation
of interaction

RGI interaction: bottom-up approach - II

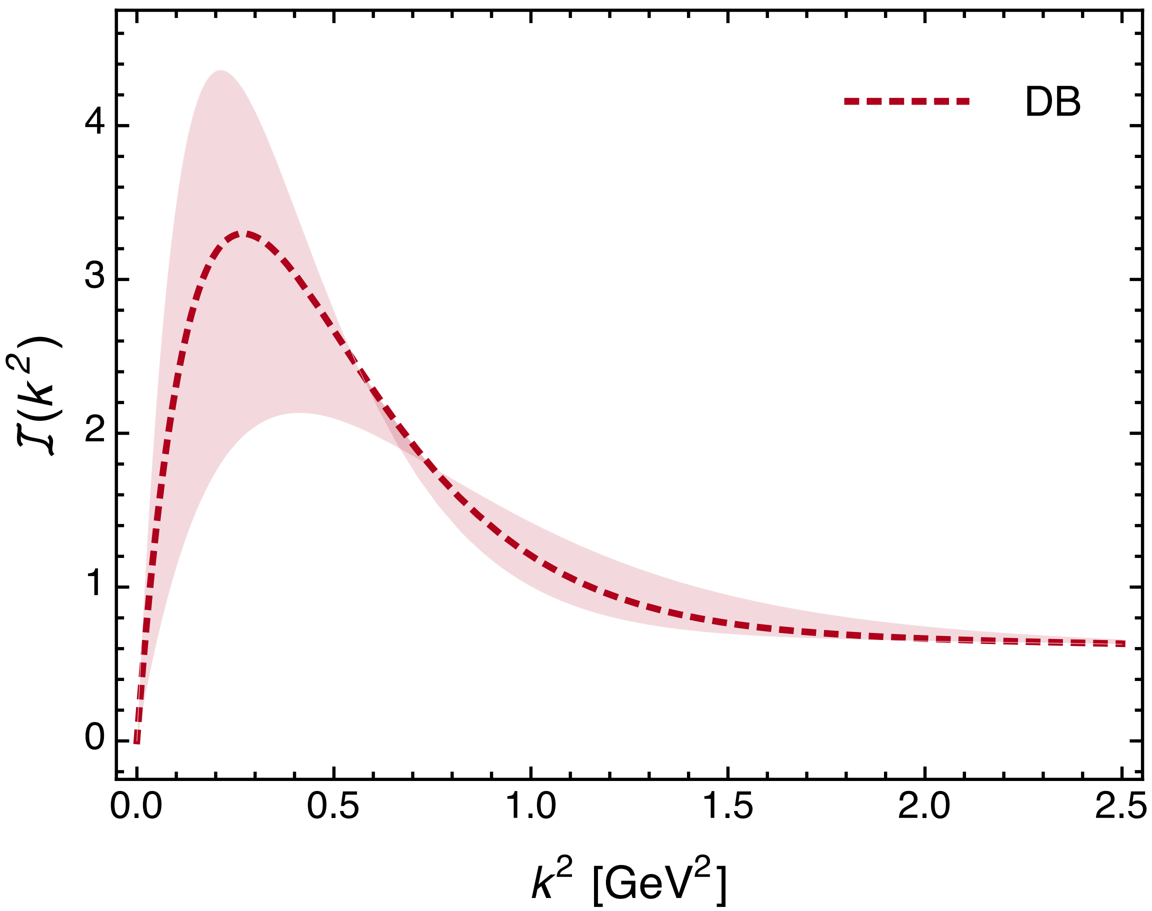
- **Go beyond RL truncation:**
  implement dynamical symmetry breaking into bound-state equations

- **Same RL interaction strength Ansatz**
  IR: Gaussian; UV: perturbative tail
  Qin, Chang, Liu, Roberts, Wilson, PRC 84 (2011)

$$\mathcal{I}(k^2) = k^2 \frac{\mathcal{G}_{\mathrm{IR}}(k^2) + \mathcal{G}_{\mathrm{UV}}(k^2)}{4\pi}$$

$$\mathcal{G}_{\mathrm{IR}}(k^2) = \frac{8\pi^2}{\omega^5} \varsigma^3 \mathrm{e}^{-k^2/\omega^2}$$

$$\mathcal{G}_{\mathrm{UV}}(k^2) = \frac{96\pi^2}{25} \frac{1 - e^{-k^2/1[\mathrm{GeV}^2]}}{k^2 \log[\mathrm{e}^2 - 1 + (1 + k^2/\Lambda^2)^2]}$$



- **Quark-gluon vertex:**

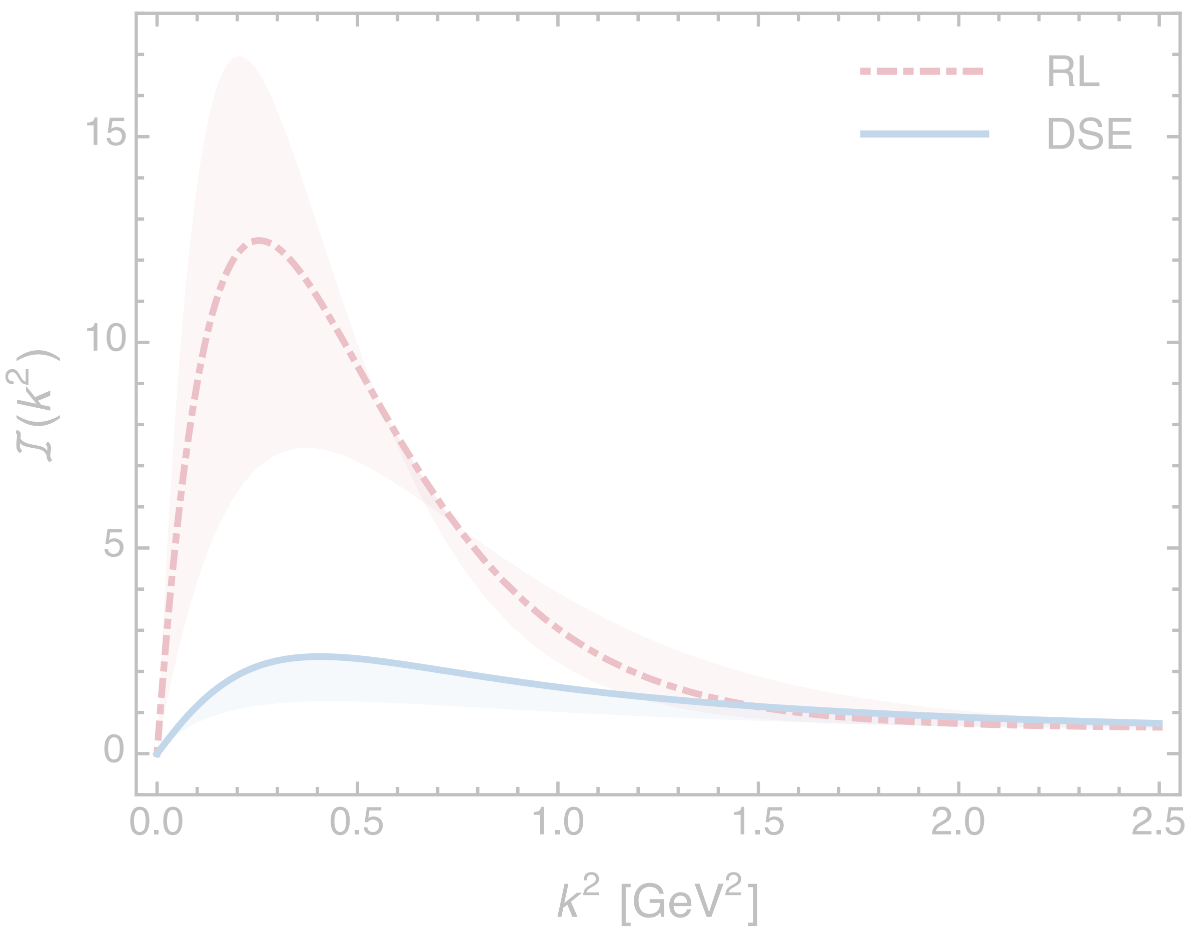
$$\Gamma_\nu = \Gamma_\nu^{\mathrm{BC}} + \Gamma_\nu^{\mathrm{ACM}}$$

  - **Ball-Chiu vertex**
    completely determined by fermion propagator
    Ball, Chiu PRD 22 (1980)
  - **Anomalous chromo-magnetic vertex**
    transverse part (undetermined by STI)

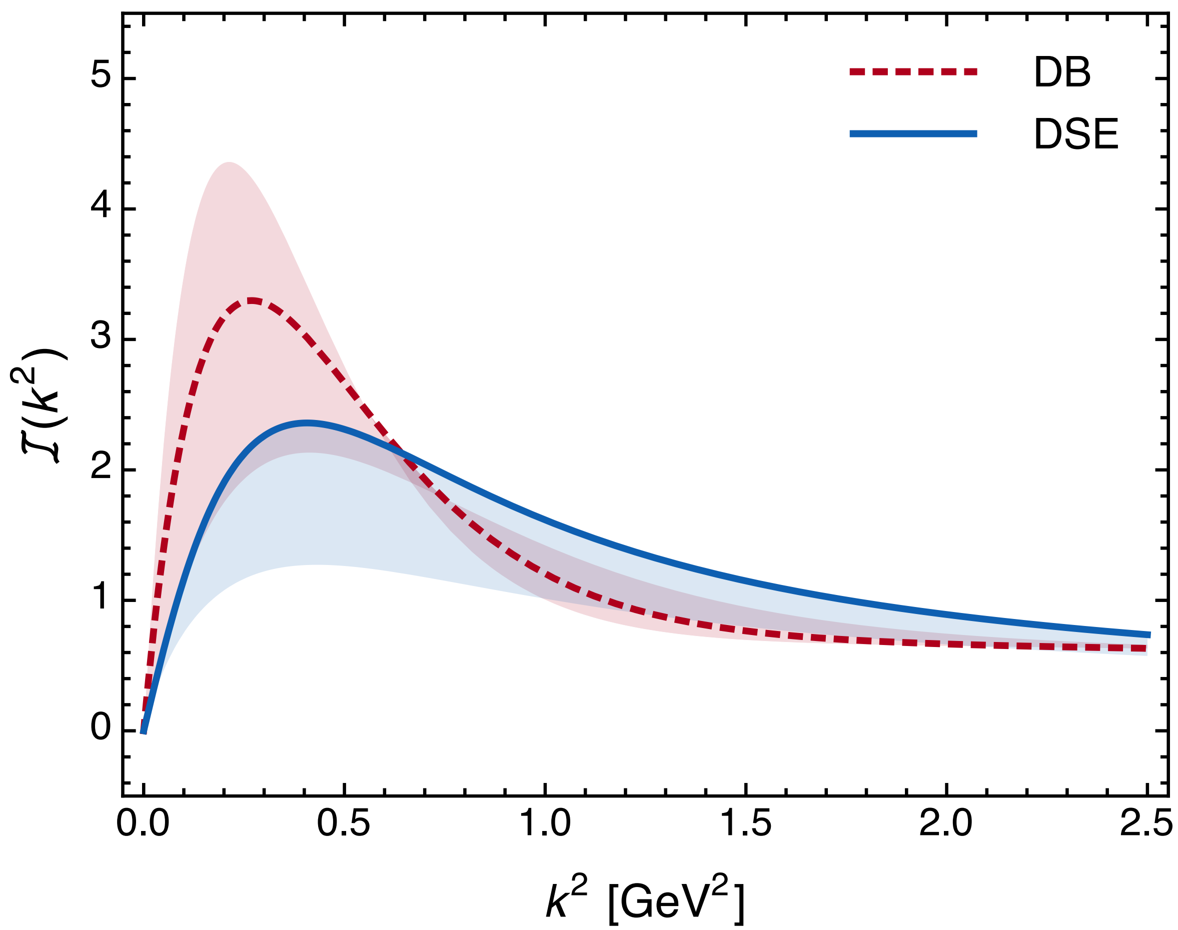
- **ς fitted to**

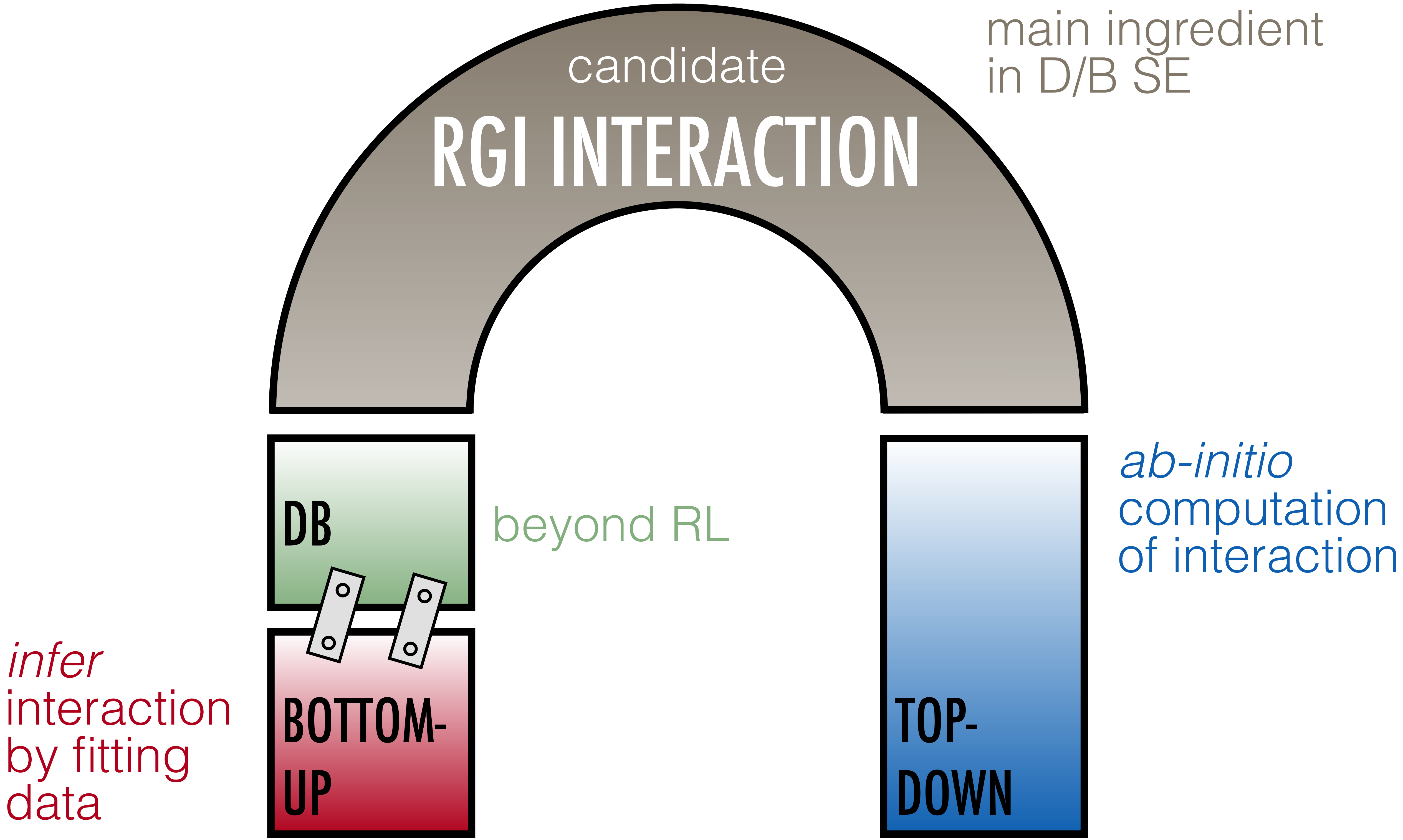
$$\varsigma_{\mathrm{DB}} = 0.55 \ [\mathrm{GeV}]$$

Top-down vs bottom-up comparison - II



DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)




candidate
RGI INTERACTION
main ingredient
in D/B SE
DB
beyond RL
BOTTOM-
UP
infer
interaction
by fitting
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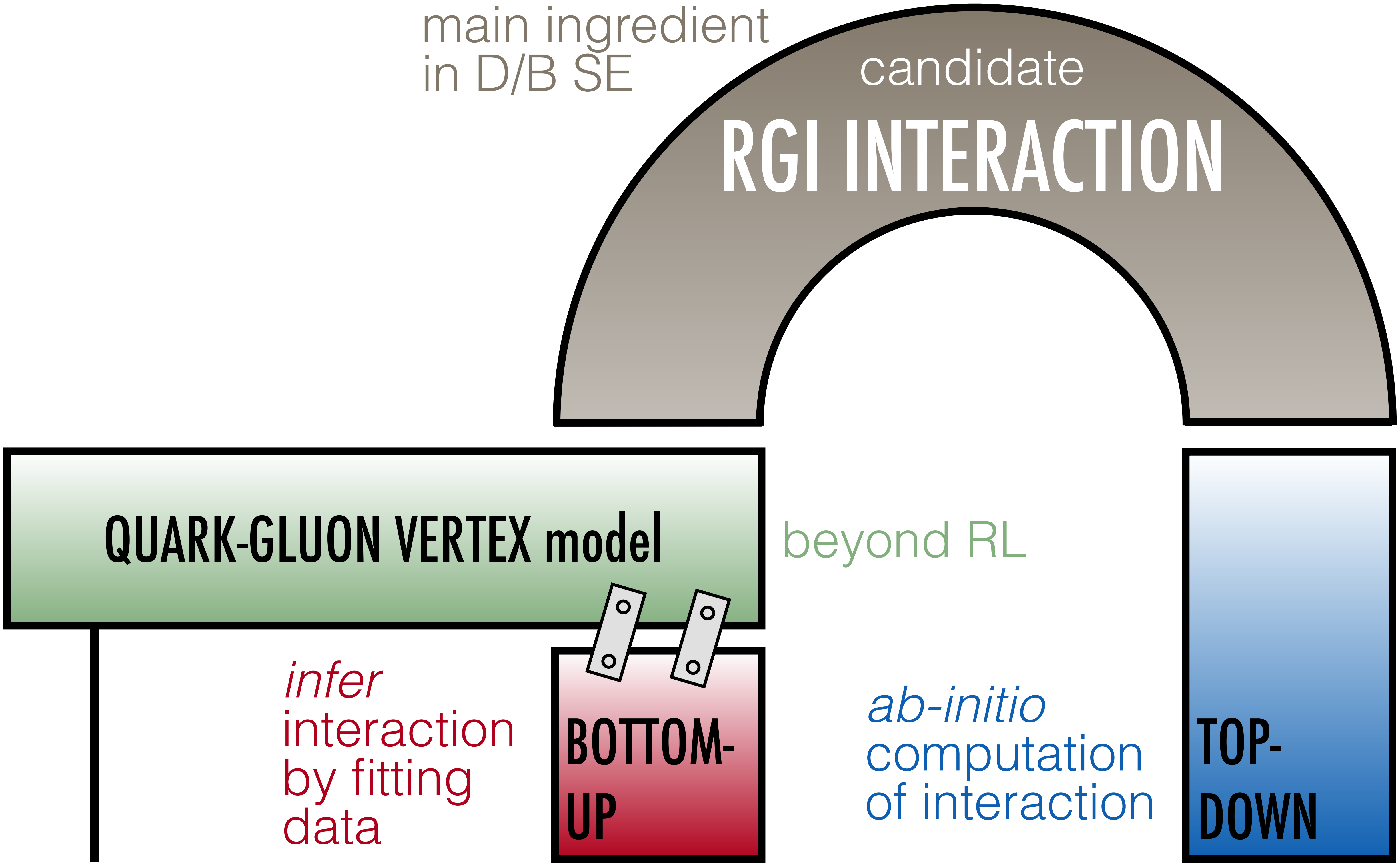
main ingredient
in D/B SE
candidate
RGI INTERACTION
QUARK-GLUON VERTEX model
beyond RL
infer
interaction
by fitting
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BOTTOM-
UP
ab-initio
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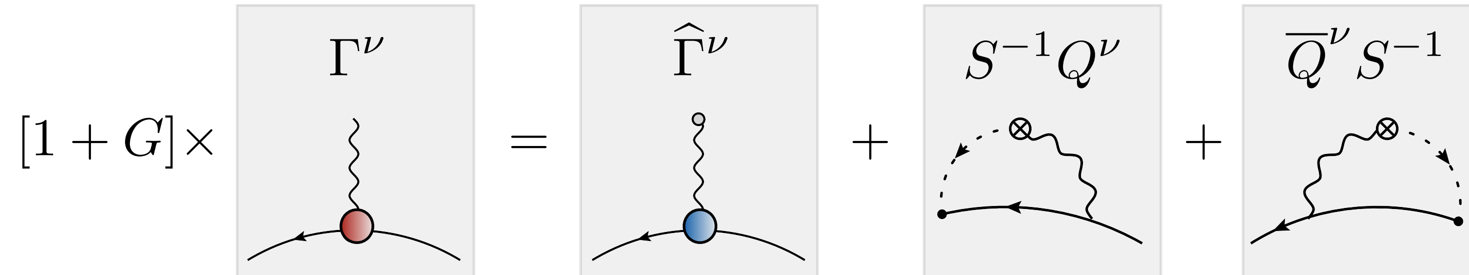
Quark-gluon vertex

- **Complete quark-gluon vertex** solves all symmetry identities

Aguilar, DB, Ibañez, Papavassiliou, PRD 90 (2014)



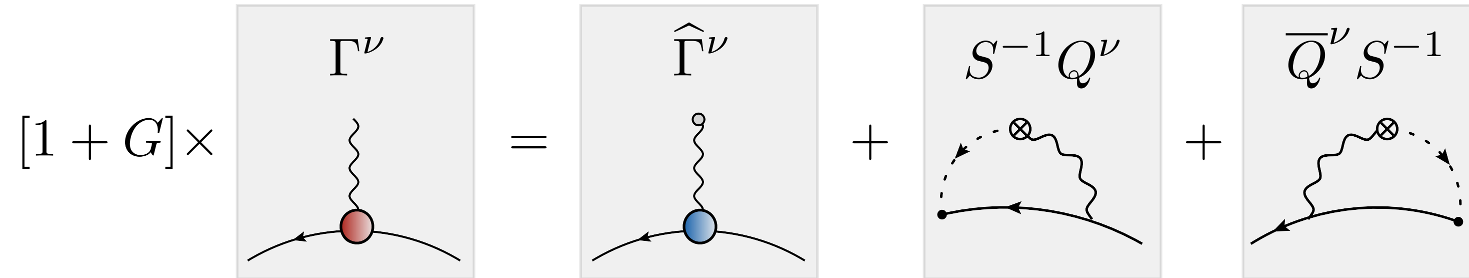
$$
\begin{aligned}
\mathcal{G}_q\Gamma_1^L &= [1-L(q^2)F(q^2)]\left\{\widehat{\Gamma}_1^L + A_1\left[\frac{1}{2}(q\cdot t)K_3^L-(p_1\cdot t)K_4^L\right]-B_1K_1^L\right.\\
&\left.\quad +A_2\left[-\frac{1}{2}(q\cdot t)\overline{K}_3^L+(p_2\cdot t)\overline{K}_4^L\right]-B_2\overline{K}_1^L\right\},\\
\mathcal{G}_q\Gamma_2^L &= [1-L(q^2)F(q^2)]\left\{\widehat{\Gamma}_2^L + A_1\left[\frac{1}{2}K_3^L+\frac{p_1\cdot q}{q\cdot t}K_4^L\right]-B_1K_2^L\right.\\
&\left.\quad +A_2\left[\frac{1}{2}\overline{K}_3^L-\frac{p_2\cdot q}{q\cdot t}\overline{K}_4^L\right]-B_2\overline{K}_2^L\right\},\\
\mathcal{G}_q\Gamma_3^L &= [1-L(q^2)F(q^2)]\left\{\widehat{\Gamma}_3^L + A_1\left[\frac{p_1\cdot q}{q\cdot t}K_1^L+(p_1\cdot t)K_2^L\right]-B_1K_3^L\right.\\
&\left.\quad +A_2\left[\frac{p_2\cdot q}{q\cdot t}\overline{K}_1^L+(p_2\cdot t)\overline{K}_2^L\right]-B_2\overline{K}_3^L\right\},\\
\mathcal{G}_q\Gamma_4^L &= [1-L(q^2)F(q^2)]\left\{\frac{A_1}{2}\left[-K_1^L+(q\cdot t)K_2^L\right]-B_1K_4^L\right.\\
&\left.\quad +\frac{A_2}{2}\left[\overline{K}_1^L+(q\cdot t)\overline{K}_2^L\right]-B_2\overline{K}_4^L\right\},\\
\mathcal{G}_q\Gamma_1^T &= \widehat{\Gamma}_1^T + A_1\left[-\frac{1}{q\cdot t}K_1^L+(p_1\cdot t)K_2^T+K_3^T-K_6^T\right]-B_1K_1^T\\
&\quad + A_2\left[\frac{1}{q\cdot t}\overline{K}_1^L+(p_2\cdot t)\overline{K}_2^T+\overline{K}_3^T+\overline{K}_6^T\right]-B_2\overline{K}_1^T\\
&\quad + \frac{2}{q^2}L(q^2)F(q^2)\left\{A_1\left[\frac{p_1\cdot q}{q\cdot t}K_1^L+(p_1\cdot t)K_2^L\right]-B_1K_3^L+\right.\\
&\left.\quad + A_2\left[\frac{p_2\cdot q}{q\cdot t}\overline{K}_1^L+(p_2\cdot t)\overline{K}_2^L\right]-B_2\overline{K}_3^L-\frac{B_1-B_2}{q\cdot t}\right\}\\
\mathcal{G}_q\Gamma_2^T &= \widehat{\Gamma}_2^T + A_1\left[-\frac{1}{q\cdot t}K_4^L+\frac{1}{2}K_1^T+\frac{1}{2}(p_1\cdot q)K_4^T-\frac{1}{2}K_7^T\right]-B_1K_2^T\\
&\quad + A_2\left[-\frac{1}{q\cdot t}\overline{K}_4^L+\frac{1}{2}\overline{K}_1^T-\frac{1}{2}(p_2\cdot q)\overline{K}_4^T-\frac{1}{2}\overline{K}_7^T\right]-B_2\overline{K}_2^T\\
&\quad + \frac{2}{q^2}L(q^2)F(q^2)\left\{A_1\left[\frac{1}{2}K_3^L+\frac{p_1\cdot q}{q\cdot t}K_4^L\right]-B_1K_2^L+\right.\\
&\left.\quad + A_2\left[\frac{1}{2}\overline{K}_3^L-\frac{p_2\cdot q}{q\cdot t}\overline{K}_4^L\right]-B_2\overline{K}_2^L+\frac{A_1-A_2}{2(q\cdot t)}\right\},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{G}_q\Gamma_3^T &= \widehat{\Gamma}_3^T + \frac{A_1}{2}\left[\frac{1}{2}(q\cdot t)K_1^T-\frac{1}{2}(p_1\cdot t)(q\cdot t)K_4^T-K_5^T\right]-B_1K_3^T\\
&\quad + \frac{A_2}{2}\left[-\frac{1}{2}(q\cdot t)\overline{K}_1^T+\frac{1}{2}(p_2\cdot t)(q\cdot t)\overline{K}_4^T-\overline{K}_5^T\right]-B_2\overline{K}_3^T\\
&\quad + \frac{1}{q^2}L(q^2)F(q^2)\left\{A_1\left[\frac{1}{2}(q\cdot t)K_3^L-(p_1\cdot t)K_4^L\right]-B_1K_1^L\right.\\
&\left.\quad + A_2\left[-\frac{1}{2}(q\cdot t)\overline{K}_3^L+(p_2\cdot t)\overline{K}_4^L\right]-B_2\overline{K}_1^L+\frac{1}{2}(A_1+A_2)\right\},\\
\mathcal{G}_q\Gamma_4^T &= \widehat{\Gamma}_4^T + A_1\left[K_2^T-\frac{2}{q\cdot t}K_3^T+\frac{1}{q\cdot t}K_8^T\right]-B_1K_4^T+A_2\left[\overline{K}_2^T+\frac{2}{q\cdot t}\overline{K}_3^T-\frac{1}{q\cdot t}\overline{K}_8^T\right]-B_2\overline{K}_4^T\\
&\quad + \frac{4}{q^2(q\cdot t)}L(q^2)F(q^2)\left\{\frac{A_1}{2}\left[-K_1^L+(q\cdot t)K_2^L\right]-B_1K_4^L+\frac{A_2}{2}\left[\overline{K}_1^L+(q\cdot t)\overline{K}_2^L\right]-B_2\overline{K}_4^L\right\},\\
\mathcal{G}_q\Gamma_5^T &= \widehat{\Gamma}_5^T + \frac{A_1}{2}\left[-K_1^L-q^2K_3^T-(q\cdot t)K_6^T-(p_1\cdot t)K_8^T\right]-B_1K_5^T\\
&\quad + \frac{A_2}{2}\left[-\overline{K}_1^L-q^2\overline{K}_3^T-(q\cdot t)\overline{K}_6^T-(p_2\cdot t)\overline{K}_8^T\right]-B_2\overline{K}_5^T,\\
\mathcal{G}_q\Gamma_6^T &= \widehat{\Gamma}_6^T + \frac{A_1}{2}\left[-K_3^L-\frac{q^2}{2}K_1^T+\frac{q^2}{2}(p_1\cdot t)K_4^T-K_5^T-(p_1\cdot t)K_7^T\right]-B_1K_6^T\\
&\quad + \frac{A_2}{2}\left[\overline{K}_3^L+\frac{q^2}{2}\overline{K}_1^T-\frac{q^2}{2}(p_2\cdot t)\overline{K}_4^T+\overline{K}_5^T+(p_2\cdot t)\overline{K}_7^T\right]-B_2\overline{K}_6^T,\\
\mathcal{G}_q\Gamma_7^T &= \widehat{\Gamma}_7^T + A_1\left[-K_2^L-\frac{q^2}{q\cdot t}K_3^T-K_6^T+\frac{p_1\cdot q}{q\cdot t}K_8^T\right]-B_1K_7^T\\
&\quad + A_2\left[-\overline{K}_2^L+\frac{q^2}{q\cdot t}\overline{K}_3^T+\overline{K}_6^T+\frac{p_2\cdot q}{q\cdot t}\overline{K}_8^T\right]-B_2\overline{K}_7^T\\
&\quad + \frac{2}{q\cdot t}L(q^2)F(q^2)\left\{\frac{A_1}{2}\left[-K_1^L+(q\cdot t)K_2^L\right]-B_1K_4^L+\frac{A_2}{2}\left[\overline{K}_1^L+(q\cdot t)\overline{K}_2^L\right]-B_2\overline{K}_4^L\right\},\\
\mathcal{G}_q\Gamma_8^T &= \widehat{\Gamma}_8^T + A_1\left[K_4^L-K_5^T+\frac{1}{2}(q\cdot t)K_7^T\right]-B_1K_8^T+A_2\left[-\overline{K}_4^L-\overline{K}_5^T-\frac{1}{2}(q\cdot t)\overline{K}_7^T\right]-B_2\overline{K}_8^T.
\end{aligned}
$$

Quark-gluon vertex

- **Complete quark-gluon vertex** solves all symmetry identities

  Aguilar, DB, Ibañez, Papavassiliou, PRD 90 (2014)



Cristina's talk

$$\mathcal{G}_q\Gamma_1^L = [1-L(q^2)F(q^2)]\left\{\widehat{\Gamma}_1^L + A_1\left[\frac{1}{2}(q\cdot t)K_3^L - (p_1\cdot t)K_4^L\right] - B_1K_1^L \right.$$
$$\left. + A_2\left[-\frac{1}{2}(q\cdot t)\overline{K}_3^L + (p_2\cdot t)\overline{K}_4^L\right] - B_2\overline{K}_1^L\right\},$$
$$\mathcal{G}_q\Gamma_2^L = [1-L(q^2)F(q^2)]\left\{\widehat{\Gamma}_2^L + A_1\left[\frac{1}{2}K_3^L + \frac{p_1\cdot q}{q\cdot t}K_4^L\right] - B_1K_2^L \right.$$
$$\left. + A_2\left[\frac{1}{2}\overline{K}_3^L - \frac{p_2\cdot q}{q\cdot t}\overline{K}_4^L\right] - B_2\overline{K}_2^L\right\},$$
$$\mathcal{G}_q\Gamma_3^L = [1-L(q^2)F(q^2)]\left\{\widehat{\Gamma}_3^L + A_1\left[\frac{p_1\cdot q}{q\cdot t}K_1^L + (p_1\cdot t)K_2^L\right] - B_1K_3^L \right.$$
$$\left. + A_2\left[\frac{p_2\cdot q}{q\cdot t}\overline{K}_1^L + (p_2\cdot t)\overline{K}_2^L\right] - B_2\overline{K}_3^L\right\},$$
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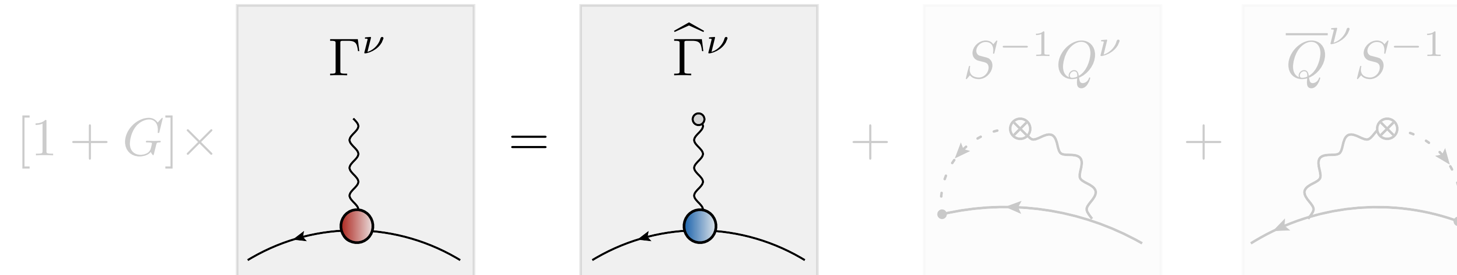
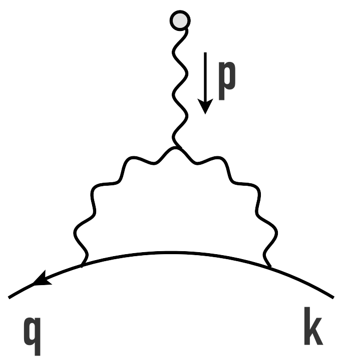
$$\left. + \frac{A_2}{2}\left[\overline{K}_1^L + (q\cdot t)\overline{K}_2^L\right] - B_2\overline{K}_4^L\right\},$$
$$\mathcal{G}_q\Gamma_1^T = \widehat{\Gamma}_1^T + A_1\left[-\frac{1}{q\cdot t}K_1^L + (p_1\cdot t)K_2^T + K_3^T - K_6^T\right] - B_1K_1^T$$
$$+ A_2\left[\frac{1}{q\cdot t}\overline{K}_1^L + (p_2\cdot t)\overline{K}_2^T + \overline{K}_3^T + \overline{K}_6^T\right] - B_2\overline{K}_1^T$$
$$+ \frac{2}{q^2}L(q^2)F(q^2)\left\{A_1\left[\frac{p_1\cdot q}{q\cdot t}K_1^L + (p_1\cdot t)K_2^L\right] - B_1K_3^L + \right.$$
$$\left. + A_2\left[\frac{p_2\cdot q}{q\cdot t}\overline{K}_1^L + (p_2\cdot t)\overline{K}_2^L\right] - B_2\overline{K}_3^L - \frac{B_1-B_2}{q\cdot t}\right\}$$
$$\mathcal{G}_q\Gamma_2^T = \widehat{\Gamma}_2^T + A_1\left[-\frac{1}{q\cdot t}K_4^L + \frac{1}{2}K_1^T + \frac{1}{2}(p_1\cdot q)K_4^T - \frac{1}{2}K_7^T\right] - B_1K_2^T$$
$$+ A_2\left[-\frac{1}{q\cdot t}\overline{K}_4^L + \frac{1}{2}\overline{K}_1^T - \frac{1}{2}(p_2\cdot q)\overline{K}_4^T - \frac{1}{2}\overline{K}_7^T\right] - B_2\overline{K}_2^T$$
$$+ \frac{2}{q^2}L(q^2)F(q^2)\left\{A_1\left[\frac{1}{2}K_3^L + \frac{p_1\cdot q}{q\cdot t}K_4^L\right] - B_1K_2^L + \right.$$
$$\left. + A_2\left[\frac{1}{2}\overline{K}_3^L - \frac{p_2\cdot q}{q\cdot t}\overline{K}_4^L\right] - B_2\overline{K}_2^L + \frac{A_1-A_2}{2(q\cdot t)}\right\},$$

$$\mathcal{G}_q\Gamma_3^T = \widehat{\Gamma}_3^T + \frac{A_1}{2}\left[\frac{1}{2}(q\cdot t)K_1^T - \frac{1}{2}(p_1\cdot t)(q\cdot t)K_4^T - K_5^T\right] - B_1K_3^T$$
$$+ \frac{A_2}{2}\left[-\frac{1}{2}(q\cdot t)\overline{K}_1^T + \frac{1}{2}(p_2\cdot t)(q\cdot t)\overline{K}_4^T - \overline{K}_5^T\right] - B_2\overline{K}_3^T$$
$$+ \frac{1}{q^2}L(q^2)F(q^2)\left\{A_1\left[\frac{1}{2}(q\cdot t)K_3^L - (p_1\cdot t)K_4^L\right] - B_1K_1^L \right.$$
$$\left. + A_2\left[-\frac{1}{2}(q\cdot t)\overline{K}_3^L + (p_2\cdot t)\overline{K}_4^L\right] - B_2\overline{K}_1^L + \frac{1}{2}(A_1+A_2)\right\},$$
$$\mathcal{G}_q\Gamma_4^T = \widehat{\Gamma}_4^T + A_1\left[K_2^T - \frac{2}{q\cdot t}K_3^T + \frac{1}{q\cdot t}K_8^T\right] - B_1K_4^T + A_2\left[\overline{K}_2^T + \frac{2}{q\cdot t}\overline{K}_3^T - \frac{1}{q\cdot t}\overline{K}_8^T\right] - B_2\overline{K}_4^T$$
$$+ \frac{4}{q^2(q\cdot t)}L(q^2)F(q^2)\left\{\frac{A_1}{2}\left[-K_1^L + (q\cdot t)K_2^L\right] - B_1K_4^L + \frac{A_2}{2}\left[\overline{K}_1^L + (q\cdot t)\overline{K}_2^L\right] - B_2\overline{K}_4^L\right\},$$
$$\mathcal{G}_q\Gamma_5^T = \widehat{\Gamma}_5^T + \frac{A_1}{2}\left[-K_1^L - q^2K_3^T - (q\cdot t)K_6^T - (p_1\cdot t)K_8^T\right] - B_1K_5^T$$
$$+ \frac{A_2}{2}\left[-\overline{K}_1^L - q^2\overline{K}_3^T - (q\cdot t)\overline{K}_6^T - (p_2\cdot t)\overline{K}_8^T\right] - B_2\overline{K}_5^T,$$
$$\mathcal{G}_q\Gamma_6^T = \widehat{\Gamma}_6^T + \frac{A_1}{2}\left[-K_3^L - \frac{q^2}{2}K_1^T + \frac{q^2}{2}(p_1\cdot t)K_4^T - K_5^T - (p_1\cdot t)K_7^T\right] - B_1K_6^T$$
$$+ \frac{A_2}{2}\left[\overline{K}_3^L + \frac{q^2}{2}\overline{K}_1^T - \frac{q^2}{2}(p_2\cdot t)\overline{K}_4^T + \overline{K}_5^T + (p_2\cdot t)\overline{K}_7^T\right] - B_2\overline{K}_6^T,$$
$$\mathcal{G}_q\Gamma_7^T = \widehat{\Gamma}_7^T + A_1\left[-K_2^L - \frac{q^2}{q\cdot t}K_3^T - K_6^T + \frac{p_1\cdot q}{q\cdot t}K_8^T\right] - B_1K_7^T$$
$$+ A_2\left[-\overline{K}_2^L + \frac{q^2}{q\cdot t}\overline{K}_3^T + \overline{K}_6^T + \frac{p_2\cdot q}{q\cdot t}\overline{K}_8^T\right] - B_2\overline{K}_7^T$$
$$+ \frac{2}{q\cdot t}L(q^2)F(q^2)\left\{\frac{A_1}{2}\left[-K_1^L + (q\cdot t)K_2^L\right] - B_1K_4^L + \frac{A_2}{2}\left[\overline{K}_1^L + (q\cdot t)\overline{K}_2^L\right] - B_2\overline{K}_4^L\right\},$$
$$\mathcal{G}_q\Gamma_8^T = \widehat{\Gamma}_8^T + A_1\left[K_4^L - K_5^T + \frac{1}{2}(q\cdot t)K_7^T\right] - B_1K_8^T + A_2\left[-\overline{K}_4^L - \overline{K}_5^T - \frac{1}{2}(q\cdot t)\overline{K}_7^T\right] - B_2\overline{K}_8^T.$$

Quark-gluon vertex model

- **Complete quark-gluon vertex** solves all symmetry identities
  Aguilar, DB, Ibañez, Papavassiliou, PRD 90 (2014)





t=q+k

p=q-k

- **Abelian-like ST identity** determines vertex up to transverse parts

$$\widehat{\Gamma}^\nu = \Gamma^\nu_{\mathrm{BC}} + \widehat{\Gamma}^\nu_{\mathrm{T}}$$

- **Ball-Chiu vertex:** completely specified by the fully dressed quark propagator
  Ball, Chiu PRD 22 (1980)

$$\Gamma^\nu_{\mathrm{BC}} = \gamma^\nu \Sigma_A + \frac{1}{2} t_\nu \left[(\gamma\cdot t)\Delta_A - i\Delta_B\right]$$

$$\Sigma_\Phi(q^2,k^2) = \frac{1}{2}[\Phi(q^2)+\Phi(k^2)]$$

$$\Delta_\Phi(q^2,k^2) = \frac{\Phi(q^2)-\Phi(k^2)}{q^2-k^2}$$

- **Transverse part:** minimum required for meson spectrum description, including scalar meson and radial excitations
  Sixue's talk

$$a^\nu_T = a^\nu + (a\cdot p)p^\nu/p^2$$

$$\tau_1(q,k) = a_1\Delta_A \qquad T^\nu_1(q,k) = \frac{i}{2} t^\nu_T;$$

$$T^\nu_2(q,k) = \frac{1}{2}(\gamma\cdot t)t^\nu_T;$$

$$\tau_3(q,k) = -2(k\cdot q)a_3\Delta_A \qquad T^\nu_3(q,k) = \gamma^\nu_T;$$

$$\tau_4(q,k) = a_4\frac{4\Delta_B}{t_T\cdot t_T} \qquad T^\nu_4(q,k) = -\frac{1}{2}t^\nu_T k^\mu q^\rho \sigma_{\mu\rho};$$

$$T^\nu_5(q,k) = \sigma^{\nu\mu}p_\mu \qquad \tau_5(q,k) = a_5\Delta_B$$

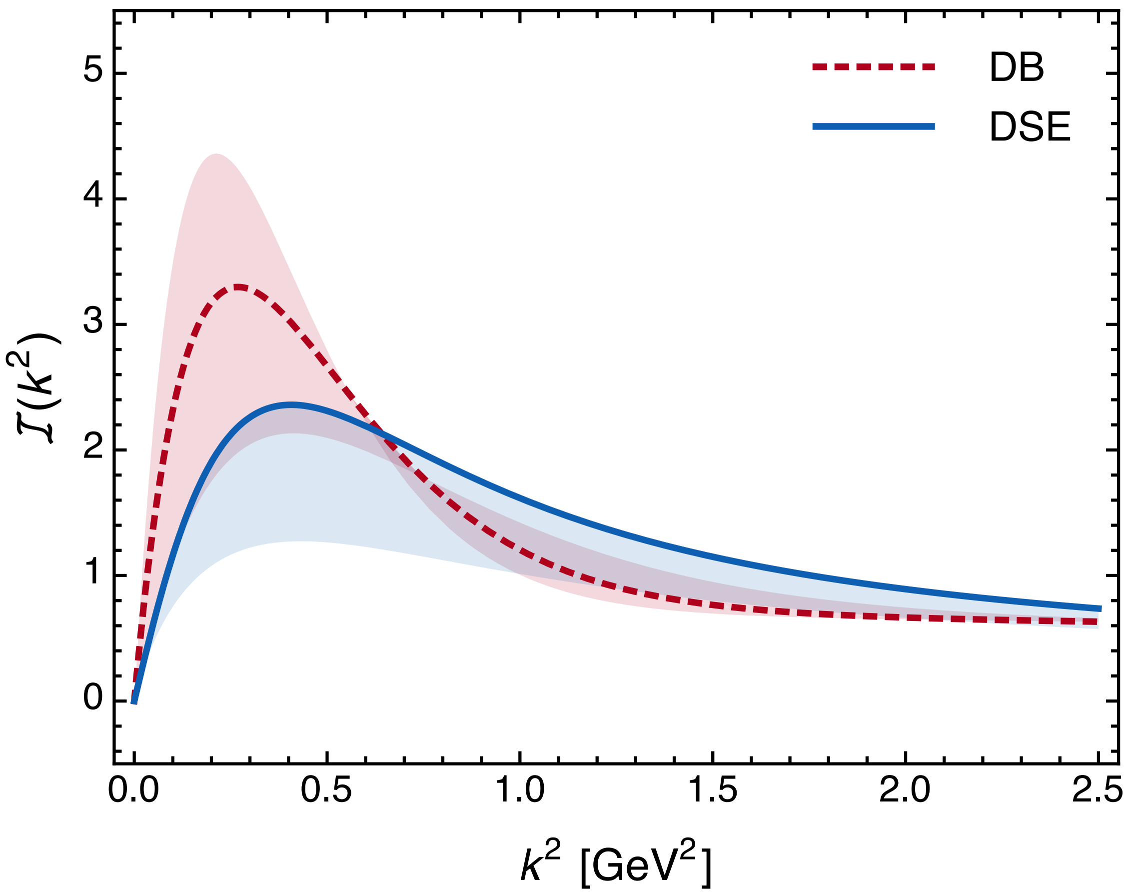
$$T^\nu_6(q,k) = -\gamma^\nu(q^2-k^2) + t^\nu \gamma\cdot p$$

$$T^\nu_7(q,k) = \frac{i}{2}(q^2-k^2)[\gamma^\nu(\gamma\cdot t) - t^\nu] + t^\nu k^\mu q^\rho \sigma_{\mu\rho}$$

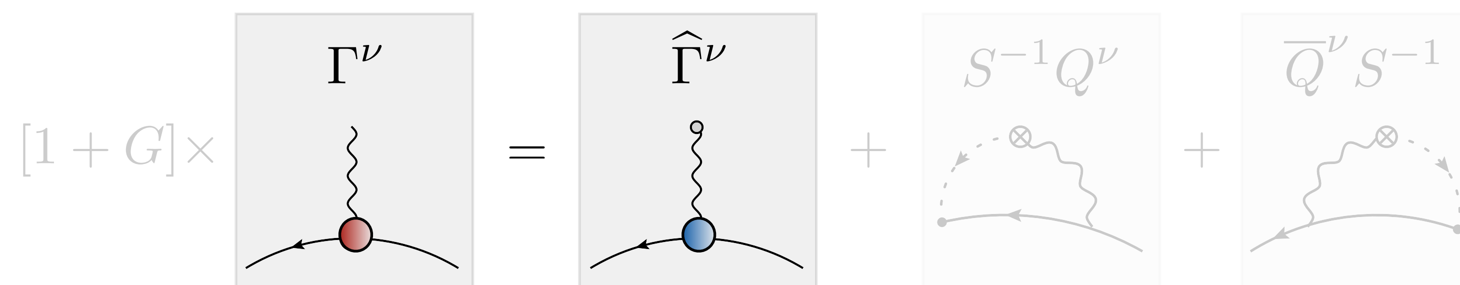
$$T^\nu_8(q,k) = q^\nu \gamma\cdot k - k^\nu(\gamma\cdot q) - i\gamma^\nu k^\mu q^\rho \sigma_{\mu\rho} \qquad \tau_8(q,k) = a_8\Delta_A$$

Natural constraints on the vertex

- **Interaction strength**



- **Interaction vertex**



$$\widehat{\Gamma}^\nu = \Gamma^\nu_{\mathrm{BC}} + \widehat{\Gamma}^\nu_{\mathrm{T}}$$

$$\Gamma^\nu_{\mathrm{BC}} = \gamma^\nu \Sigma_A + \frac{1}{2} t_\nu \left[(\gamma\cdot t)\Delta_A - i\Delta_B\right]$$

$$\widehat{\Gamma}^\nu_T = a_1 \Delta_A T^\nu_1 - 2(k\cdot q) a_3 \Delta_B T^\nu_3 + 4a_4 \frac{\Delta_B}{t^T\cdot t^T} + a_5 T^\nu_5 + a_8 \Delta_A T^\nu_8$$

- **Solve gap equation**
  in this new scheme
  DB, Chang, Papavassiliou, Qin, Roberts arXiv:1609.02568

- **Many degeneracies**
  form factors enter in the following combinations
  $\tau_1 - \Delta_B \qquad \Sigma_A + \tau_3 \qquad \tau_4 - 3\tau_5$
- **Scan parameter space**
  look for solutions varying $a_{1,3,45,8}$ ($a_{45}= a_4-3a_5$)

$$\mathbb{V}_4 = \{(a_1, a_3, a_{45}, a_8) \,|\, a_1, a_3 \in [-1,1],\; a_{45} \in [-7,5],\; a_8 \in [-5,1]\}$$

Natural constraints on the vertex

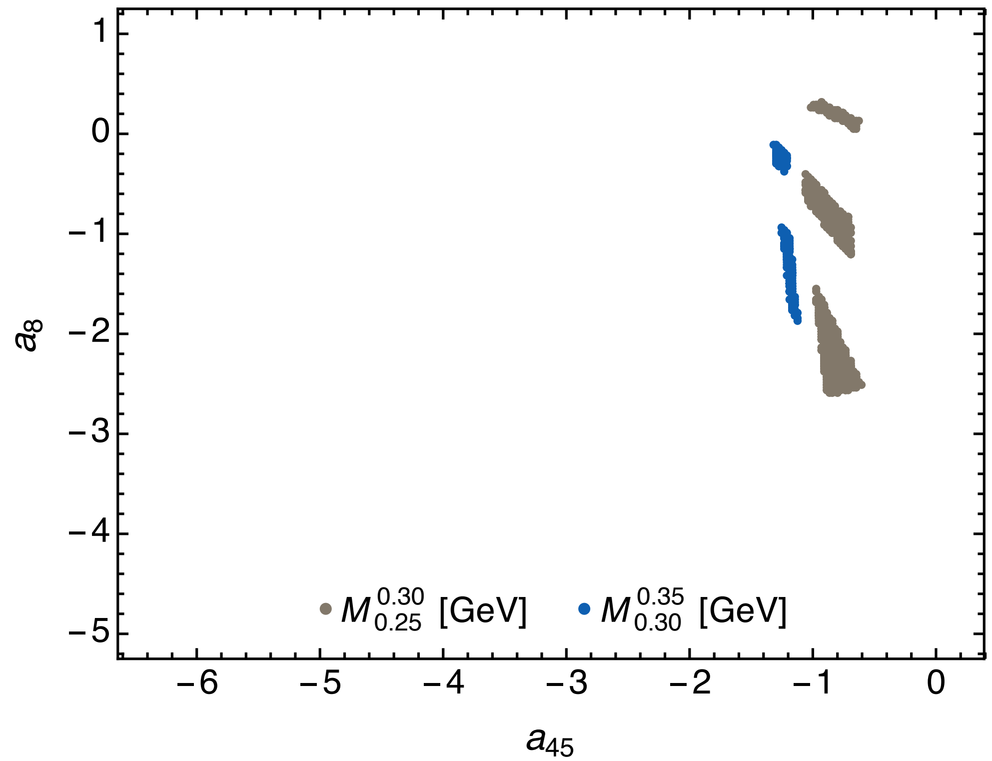
- **Scanning method**
  1. Generate a quadruplet $\mathfrak{q} = (a_1, a_3, a_{45}, a_8)$
  2. Construct the quark-gluon vertex ${}^{\mathfrak{q}}\Gamma_\nu$
  3. Solve the gap equation with the t-d/b-u RGI interaction
  4. Repeat 1.66 million times

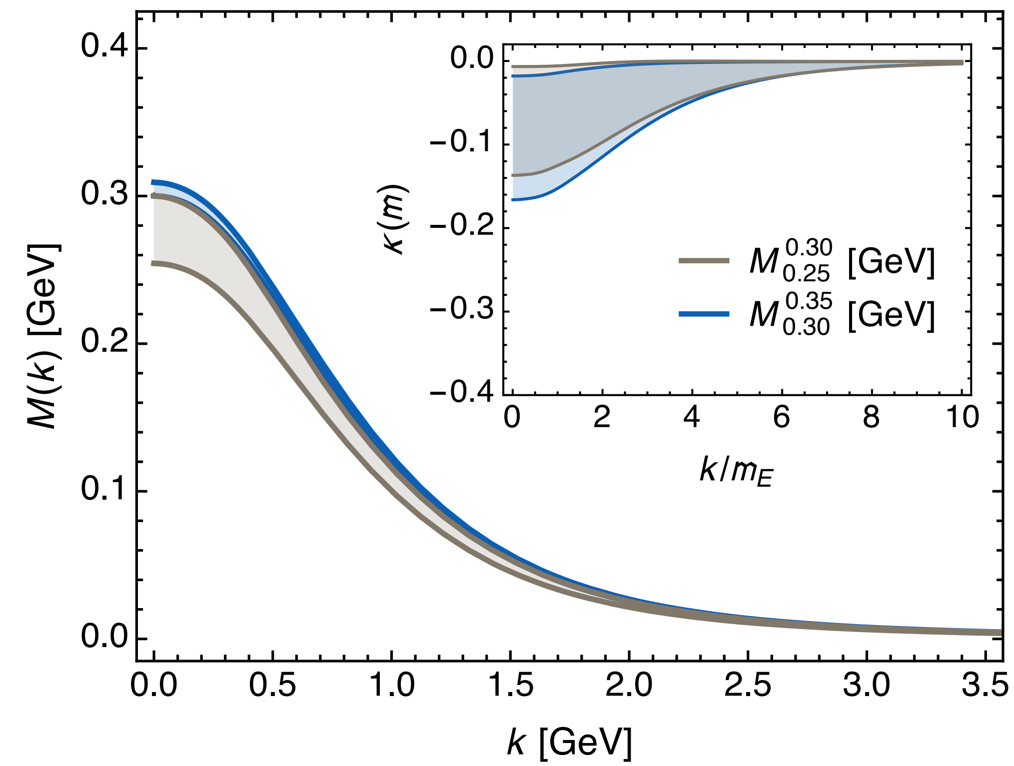
- **Filtering method**
  1. Check if solution has converged
  2. Categorise the solutions as acceptable iff:
     (i) express DCSB of sufficient strength: $M^{0.45}_{0.25}$ [GeV]
     (ii) have non-negative ACM distribution
     $$\kappa(\mathfrak{m}) = 2\mathfrak{m}\frac{(a_5 - 1 + a_1/2)\delta_B + \mathfrak{m}(1-a_8)\delta_A}{\sigma_A + 2\mathfrak{m}^2(a_3-1)\delta_A + 2\mathfrak{m}(1-a_1/2)\delta_B}$$
     (iii) f_π within 5% of chiral value (0.088 [GeV])

Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex Ansätze

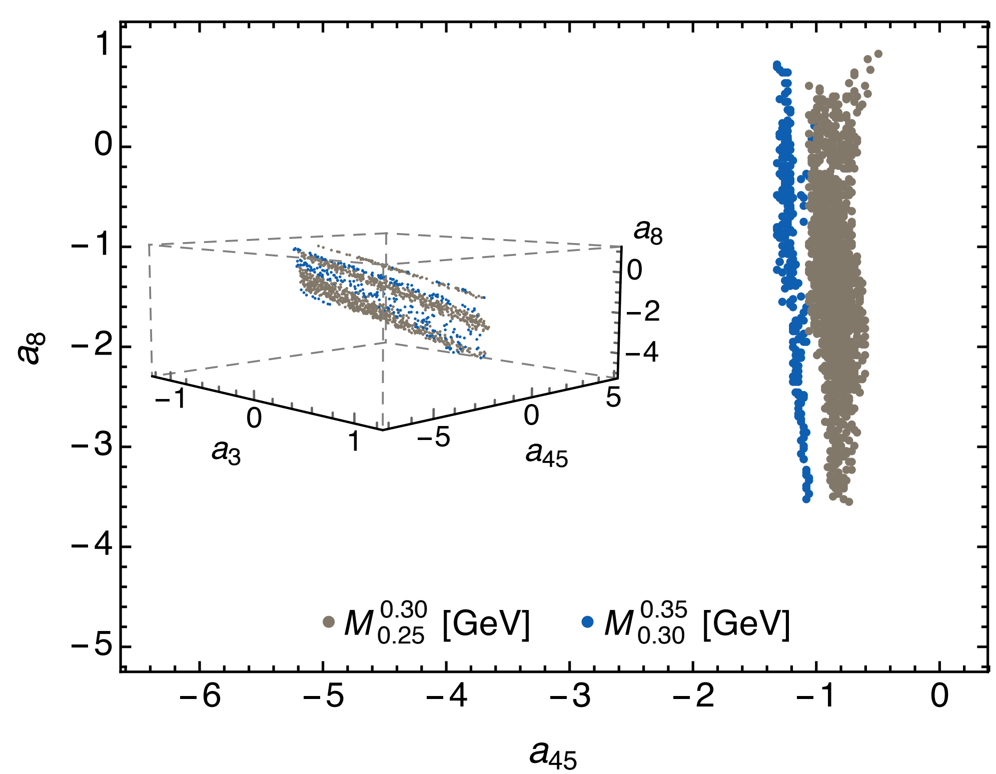
Natural constraints on the vertex

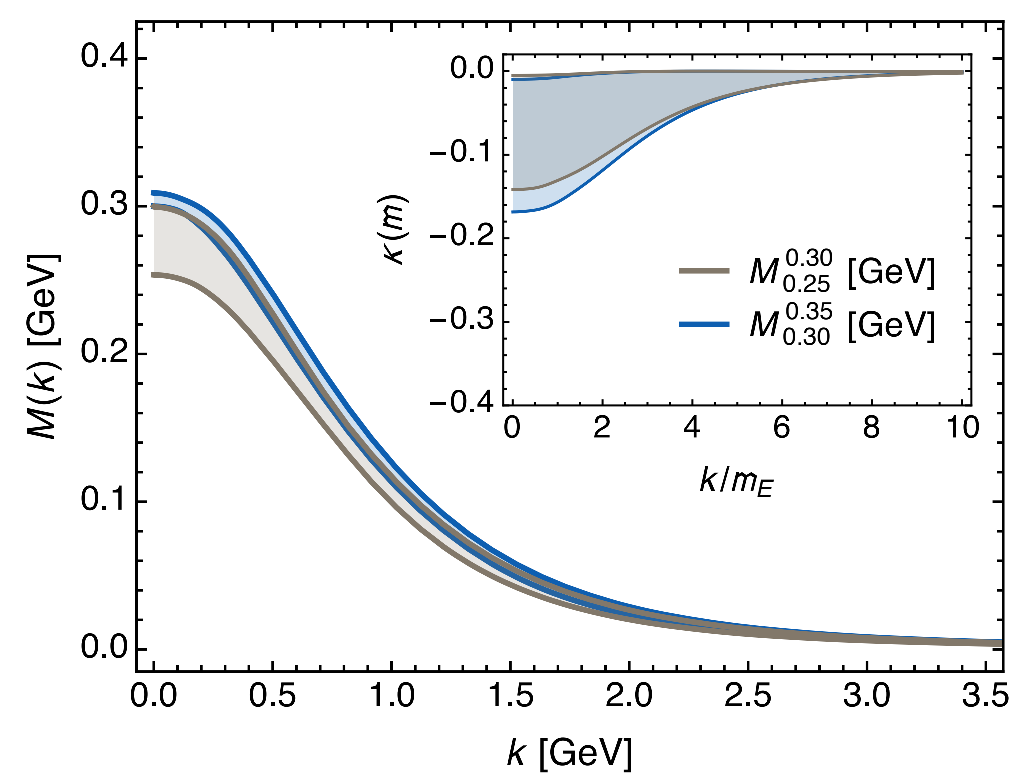
- **Case A** ($a_1=a_3=0$, 360k $\mathfrak{q}$)





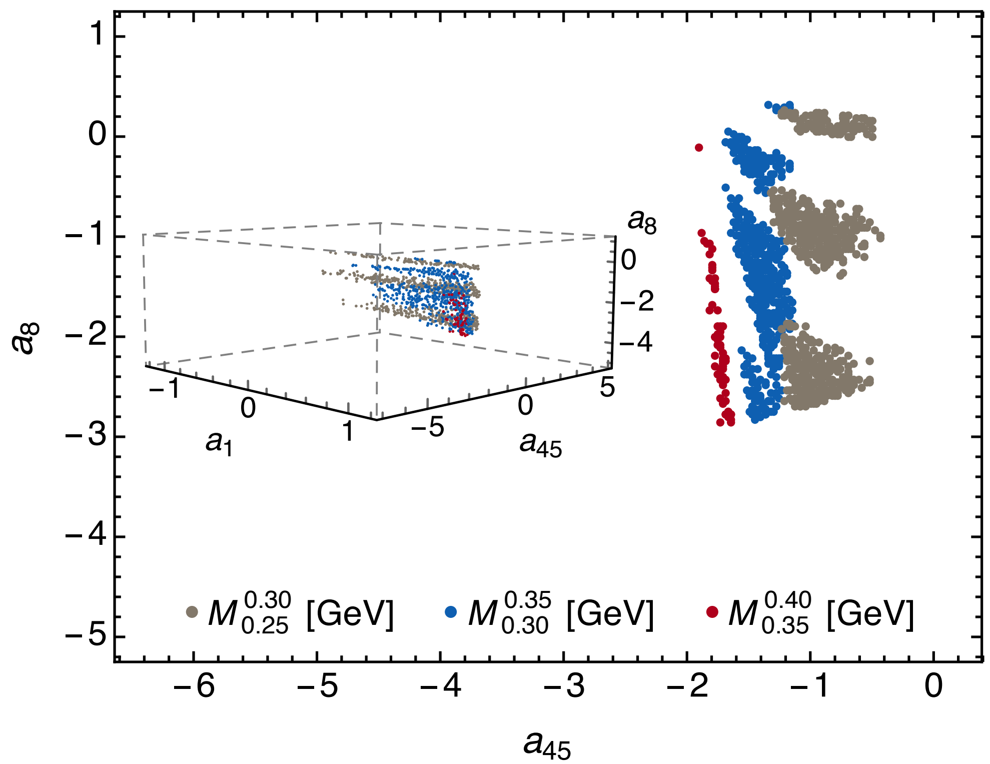
- **Case B** ($a_1=0$, 400k $\mathfrak{q}$)

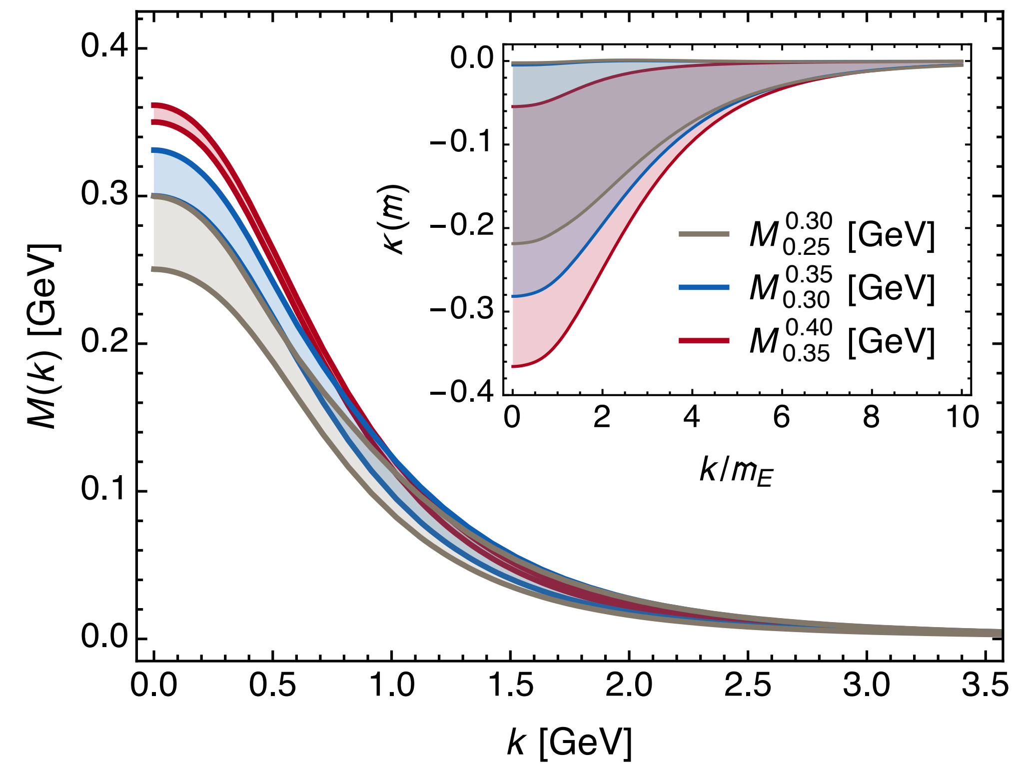




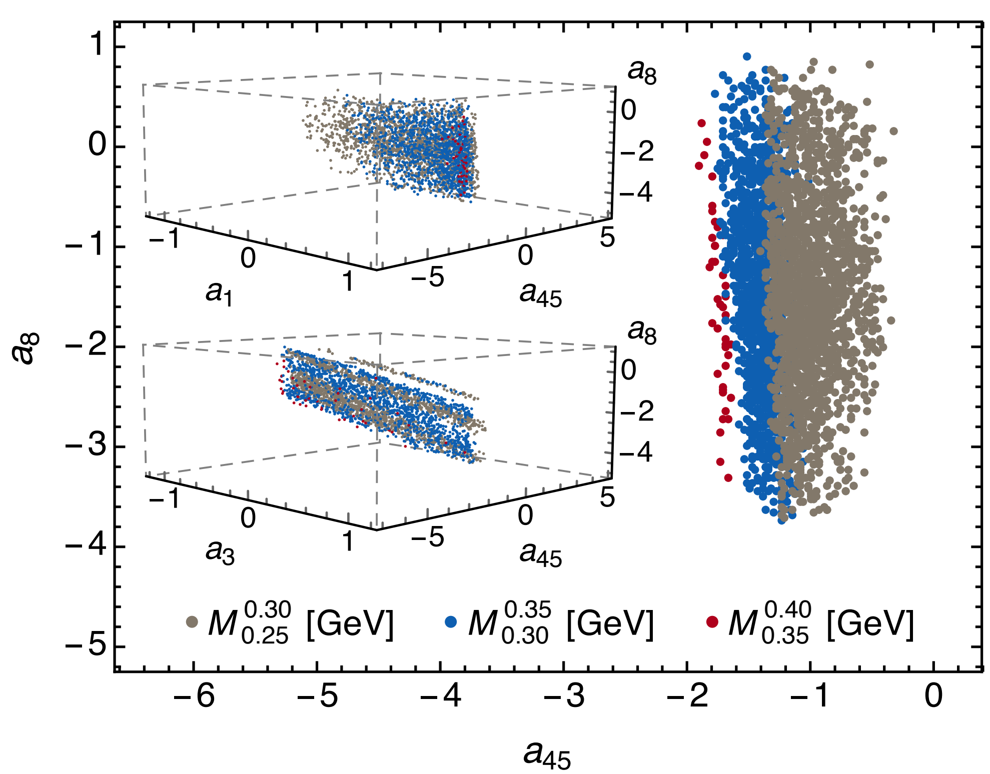
Natural constraints on the vertex

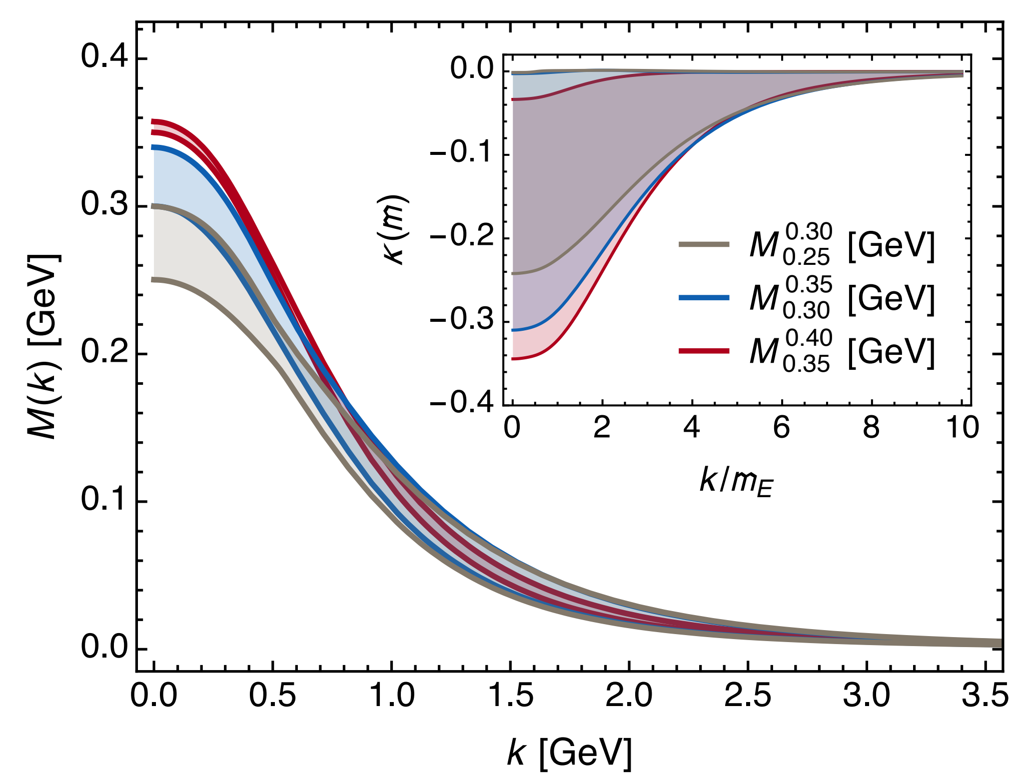
- **Case C** (a_3=0, 360k q)





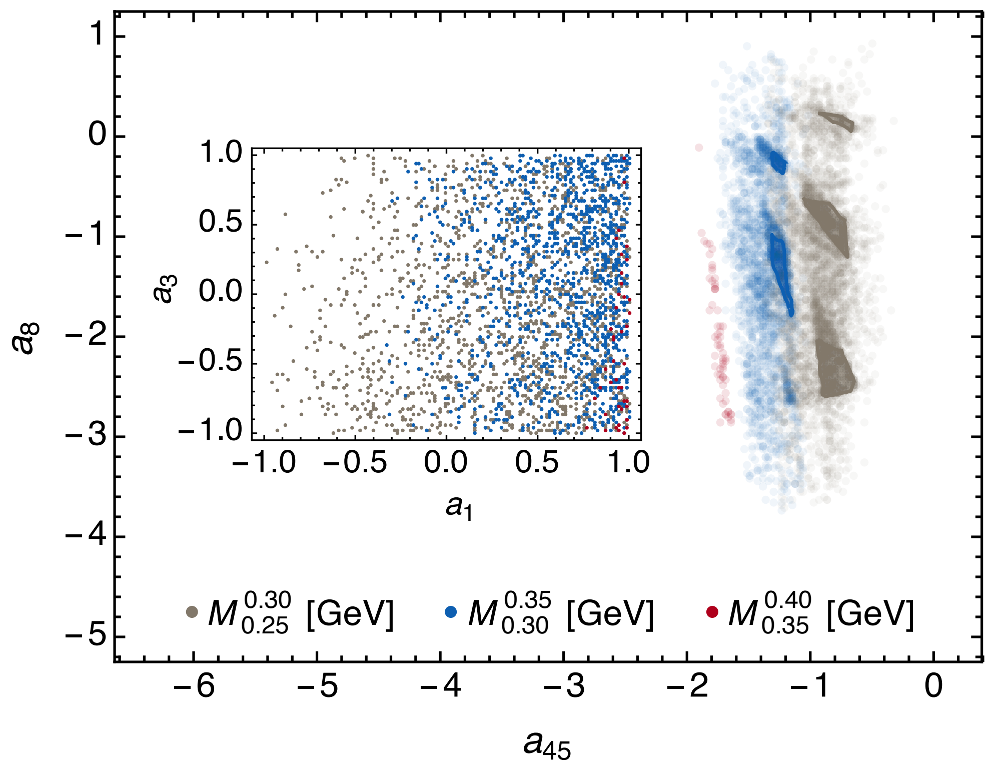
- **Case D** ($a_{1,3}\neq 0$, 560kq)



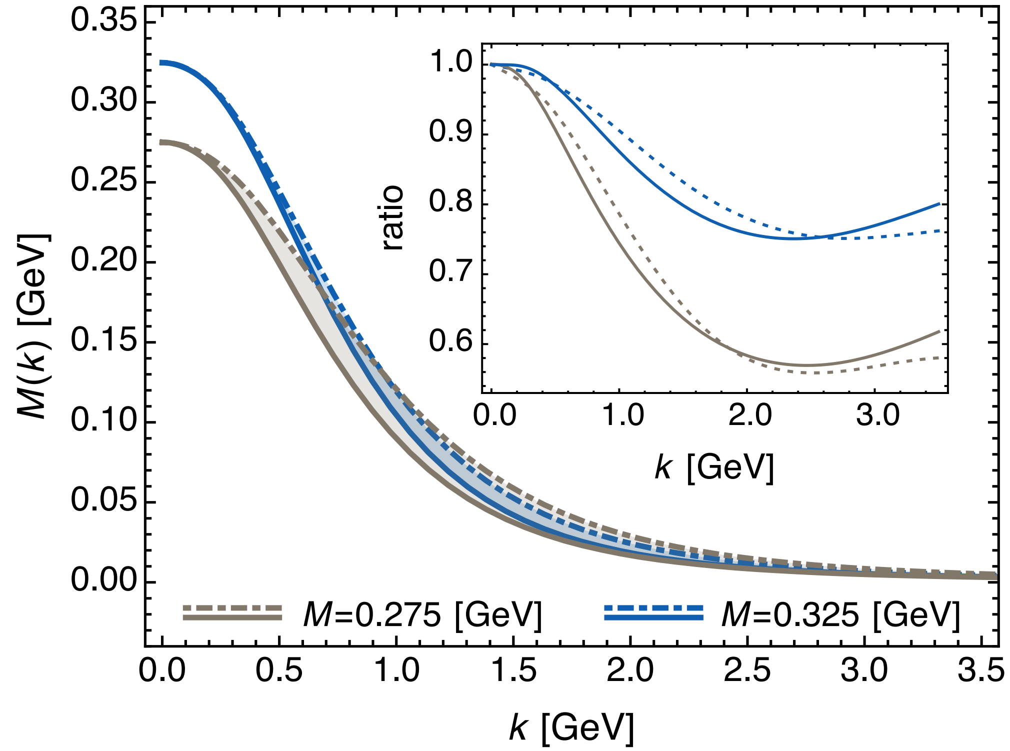


Natural constraints on the vertex

- **All domains**

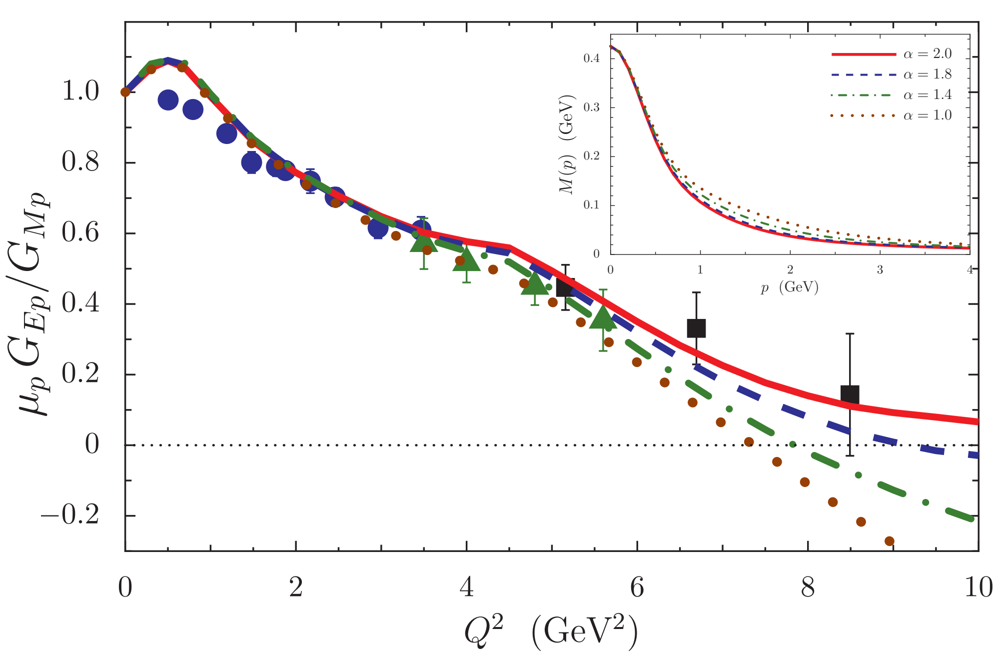


- **Running**

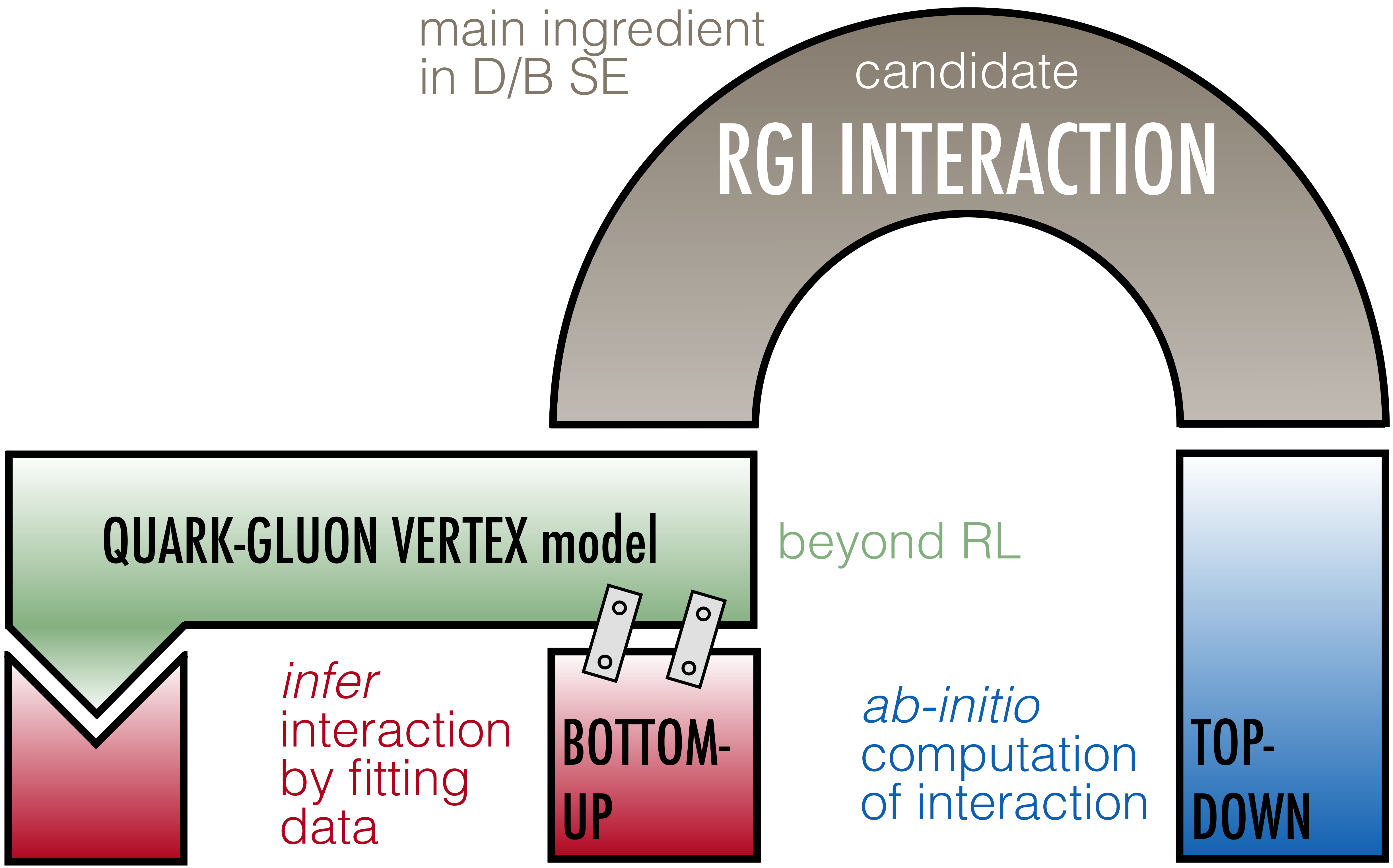


- **Impacts on observable:**
possible existence of a zero in the ratio of p elastic form factors

- **Probe the mass running rate**
and ultimately the structure of the quark-gluon vertex



Cloët, Roberts, Thomas PRL 111 (2013)

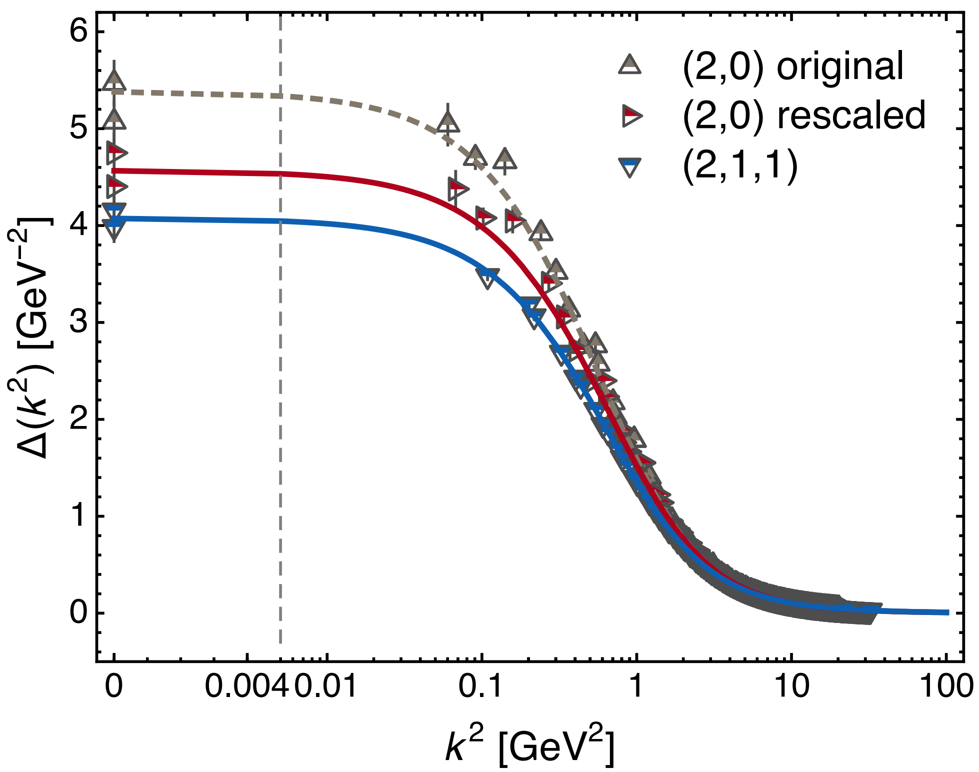

main ingredient
in D/B SE
candidate
RGI INTERACTION
QUARK-GLUON VERTEX model
beyond RL
infer
interaction
by fitting
data
BOTTOM-
UP
ab-initio
computation
of interaction
TOP-
DOWN

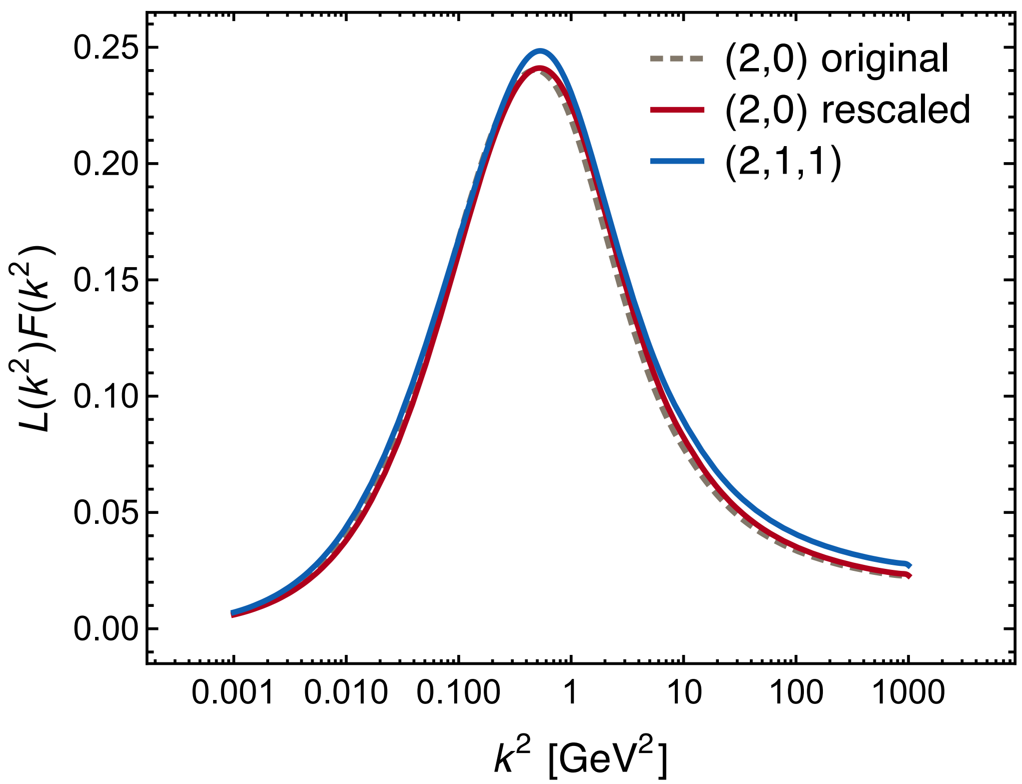
Flavour dependence

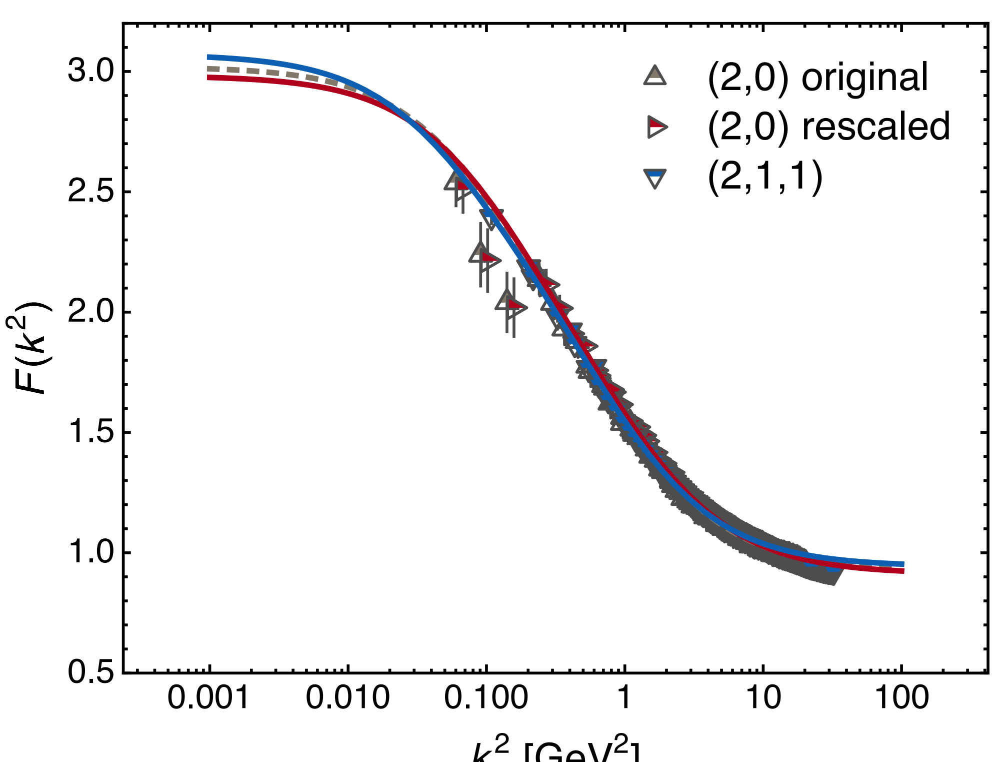
- **Top-down kernel:**
  how does flavour dependence get communicated between gauge/quark sectors?
  - **Gauge sector**
    only *quantitative* differences - lattice (2,0) and (2,1,1)
    Ayala, Bashir, DB, Cristoforetti, Rodriguez-Quintero, Phys. Rev. D86 (2012)
  - **Quark sector**
    qualitative differences: chiral symmetry restoration
- **However:**
  calculation of $\mathcal{I}$ using directly lattice results yields $\mathcal{I}_{(2,0)} > \mathcal{I}_0 \sim \mathcal{I}_{(2,1,1)}$
- **Scale setting problem**
  lattice spacing has not been correctly computed for quenched and (2,0) theories
- **Scale resetting**
  use PT charge requiring decoupling of heavy flavors at low enough p

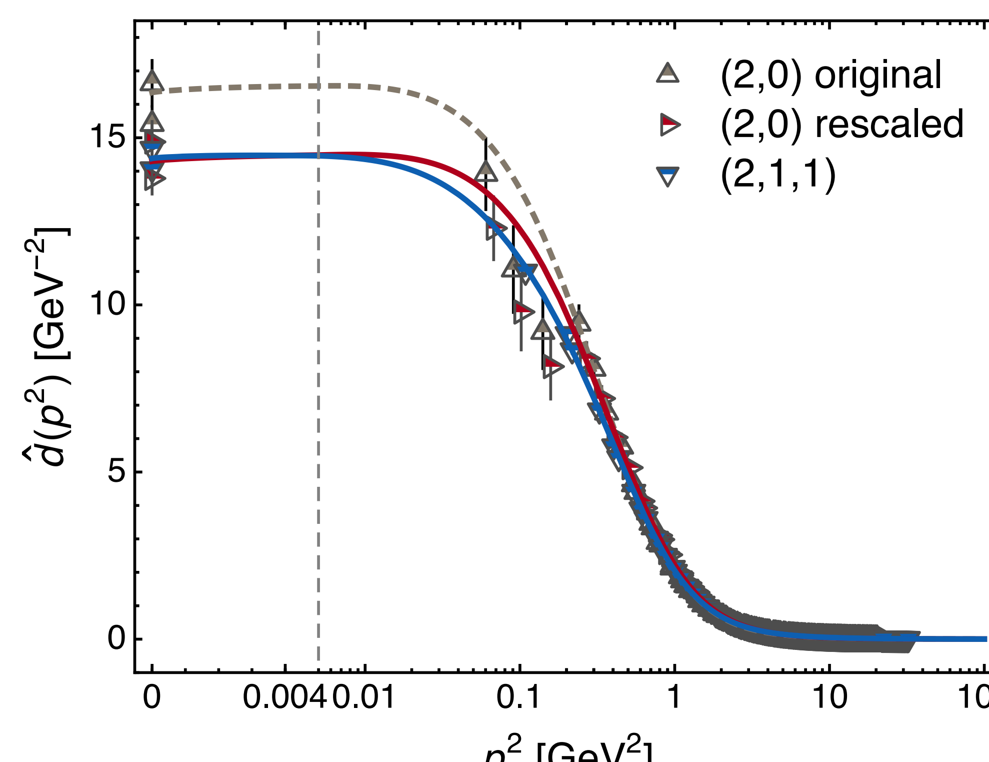
$$\lim_{p^2\to 0}\frac{\mathcal{I}_{n_\mathrm{f}}(p^2)}{p^2} = \lim_{p^2\to 0}\frac{\mathcal{I}_{N_\mathrm{f}}(p^2)}{p^2} \qquad \Leftrightarrow \qquad \widehat{d}_{n_\mathrm{f}}(0) = \widehat{d}_{N_\mathrm{f}}(0)$$

Flavour dependence

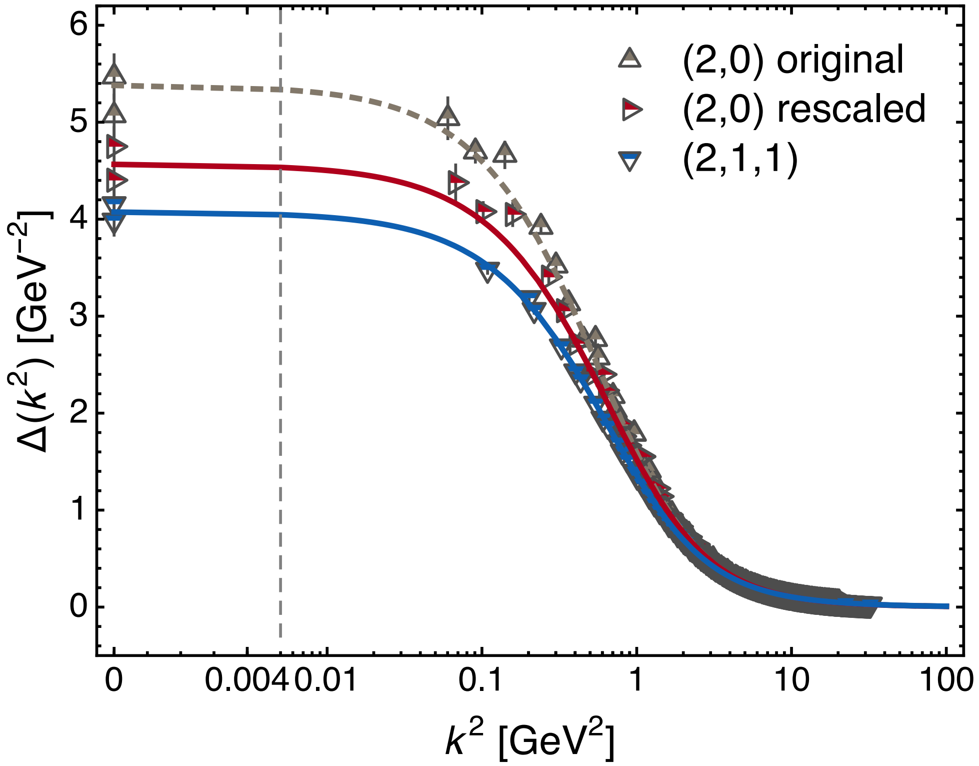




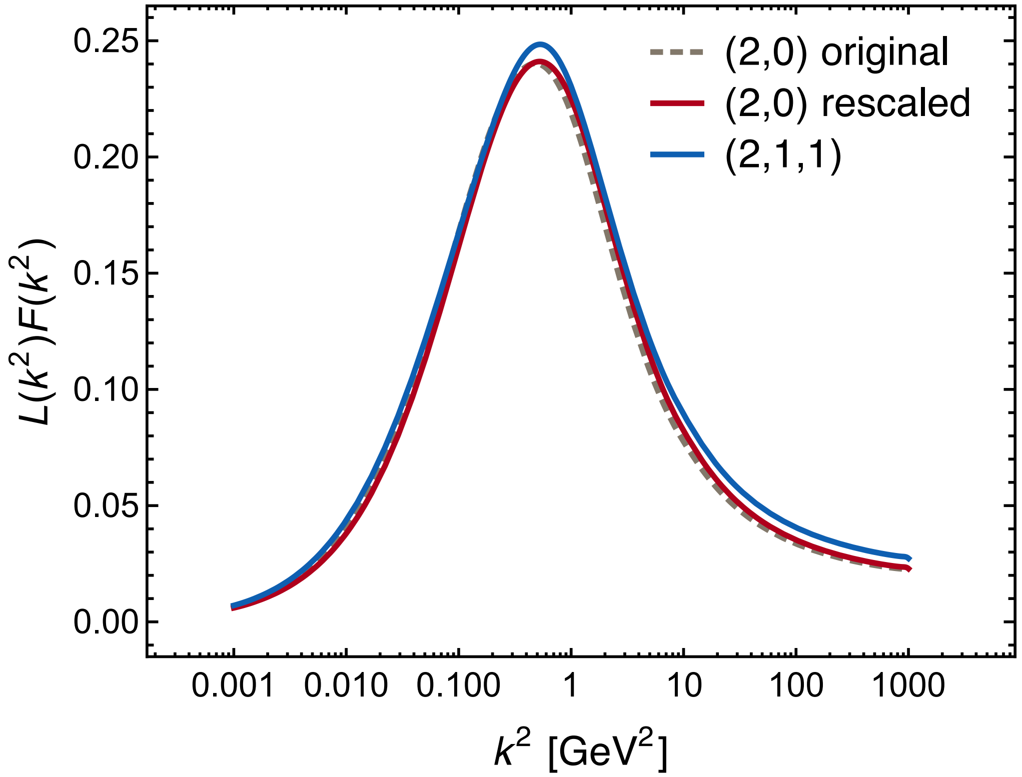


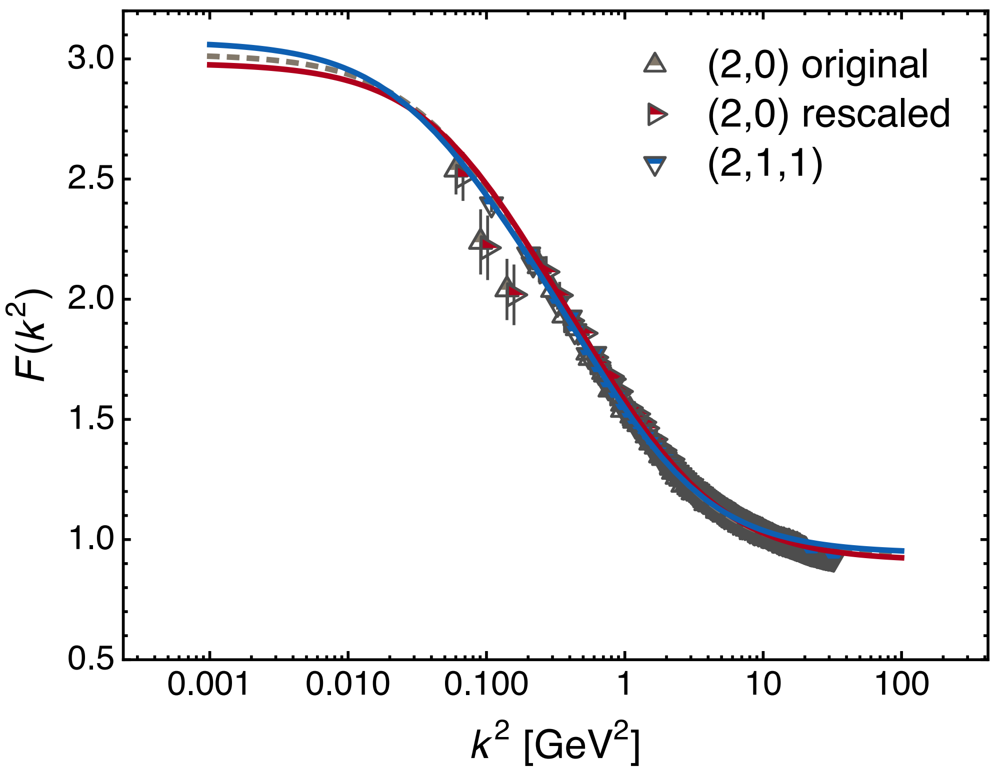


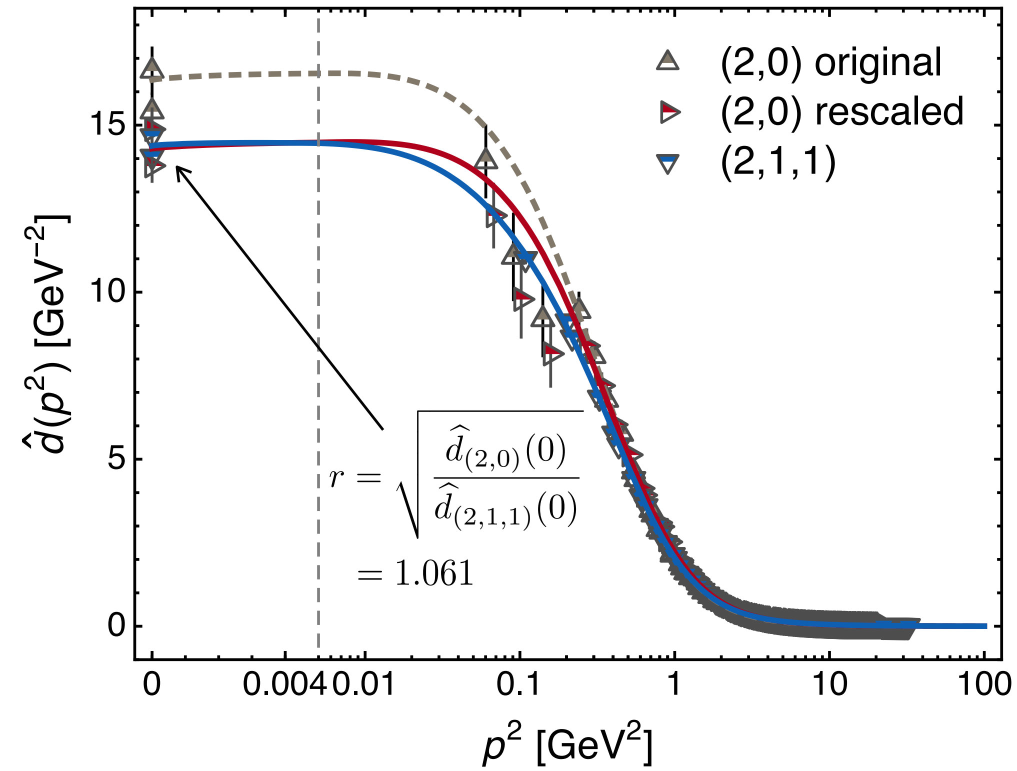
Flavour dependence



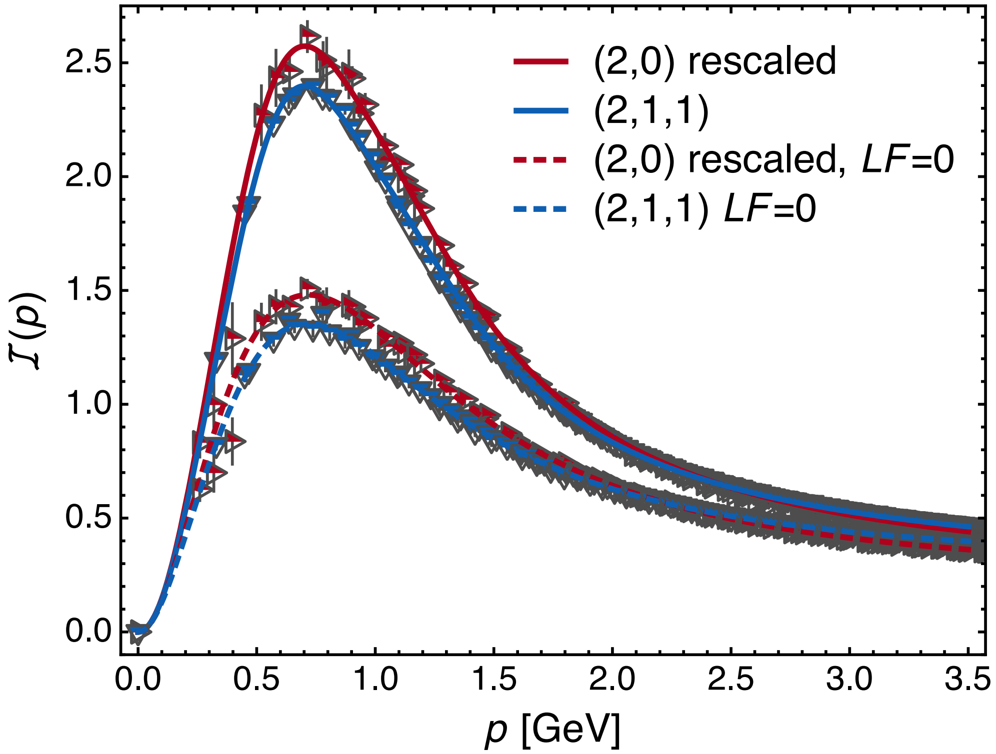
$(1 - LF)/F = 1 + G$

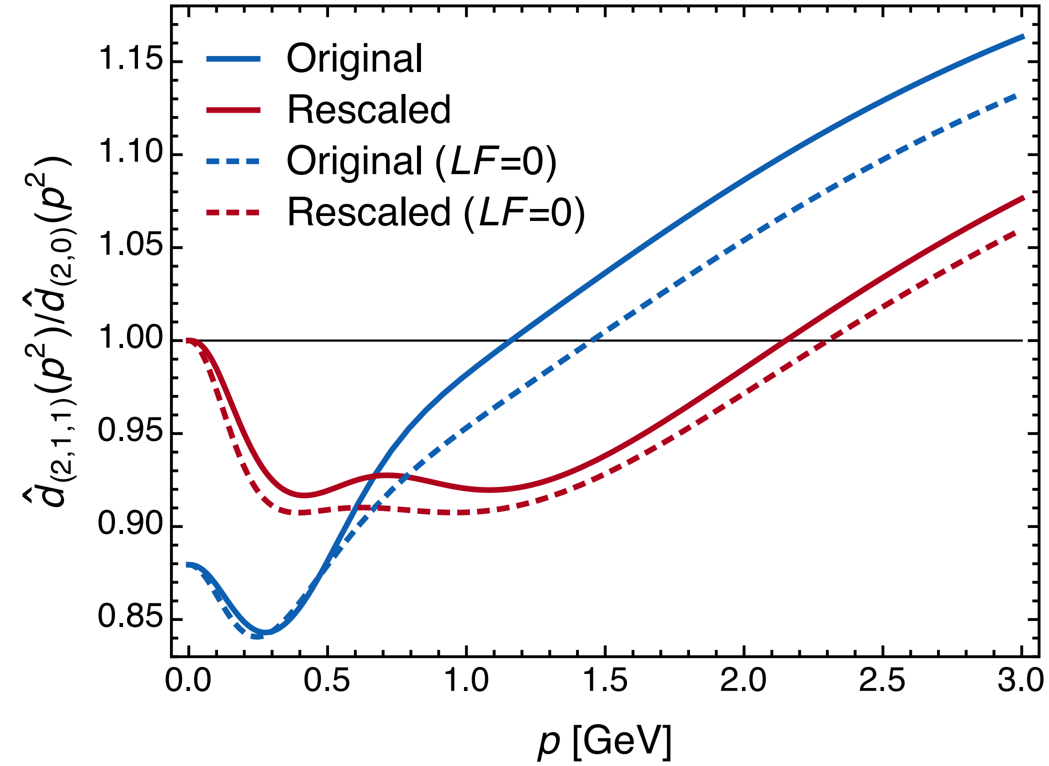






Flavour dependence

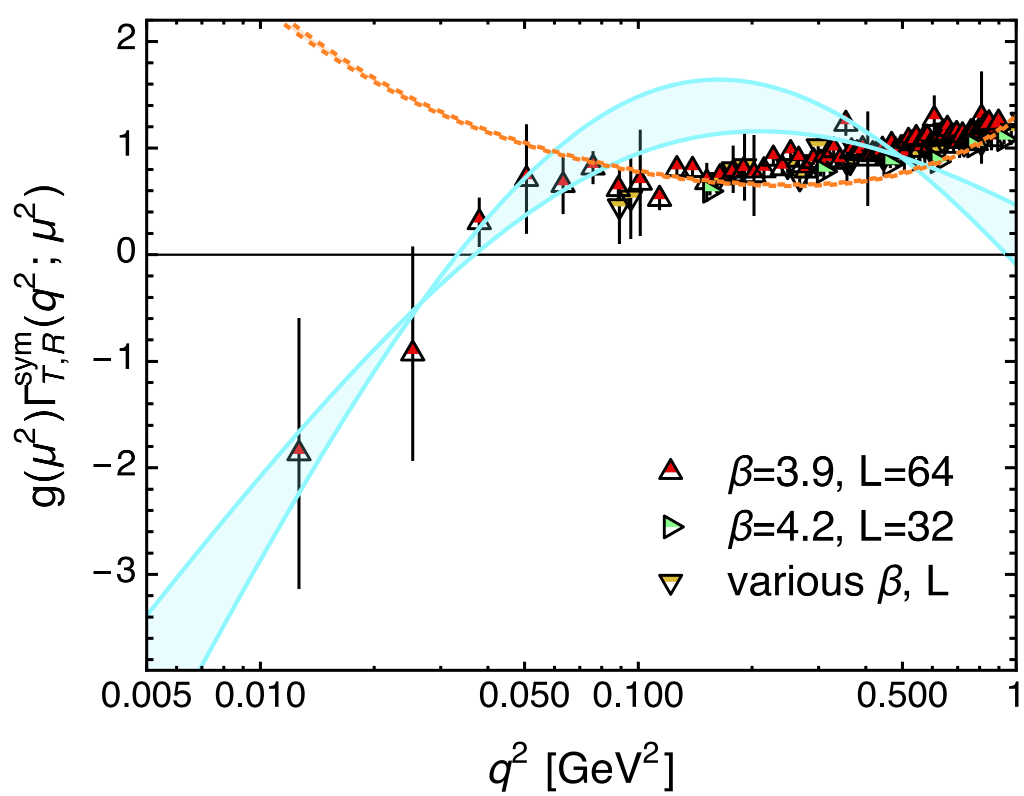




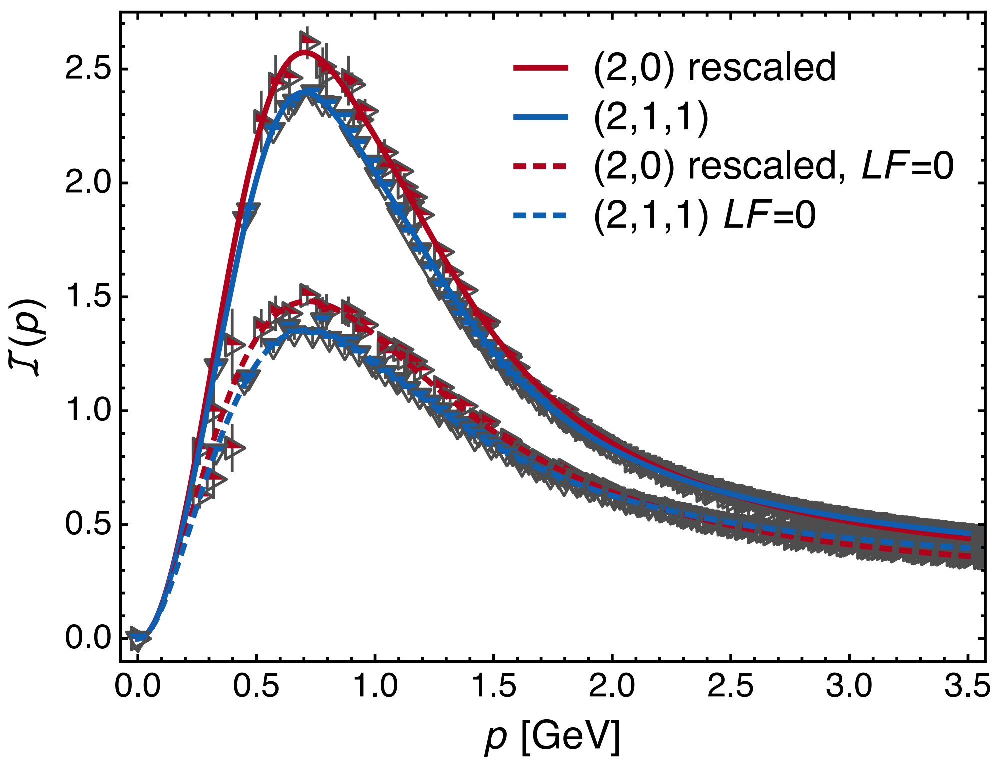
- **IR/UV asymptotics** described by

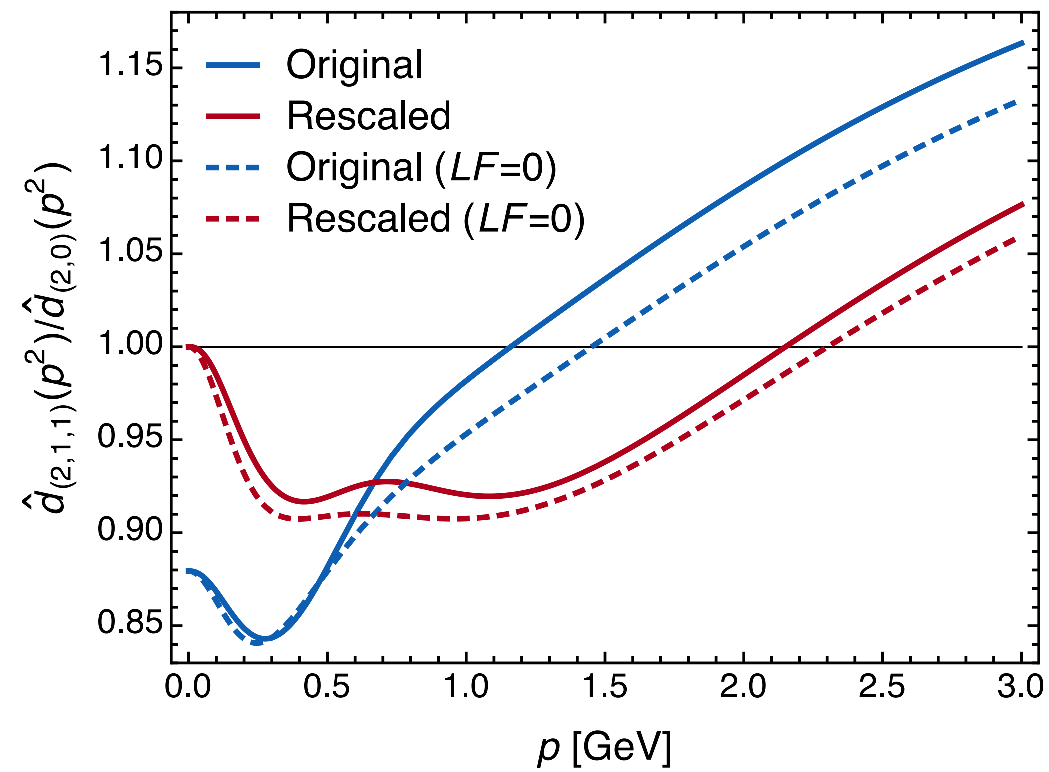
$$\mathcal{I}(p^2) \underset{p^2/\Lambda_\mathrm{T}^2 \ll 1}{\simeq} p^2 \widehat{d}(0) \left[1 - \left(\frac{\widehat{d}(0)}{8\pi} + \frac{c}{M^2}\right) p^2 \ln \frac{p^2}{\Lambda_\mathrm{T}^2}\right]$$

$$\mathcal{I}(p^2) \underset{p^2/\Lambda_\mathrm{T}^2 \gg 1}{\simeq} \alpha_\mathrm{T}(p^2) \underset{p^2/\Lambda_\mathrm{T}^2 \gg 1}{\simeq} \frac{4\pi}{\beta_0 \ln\left(p^2/\Lambda_\mathrm{T}^2\right)}$$



Flavour dependence





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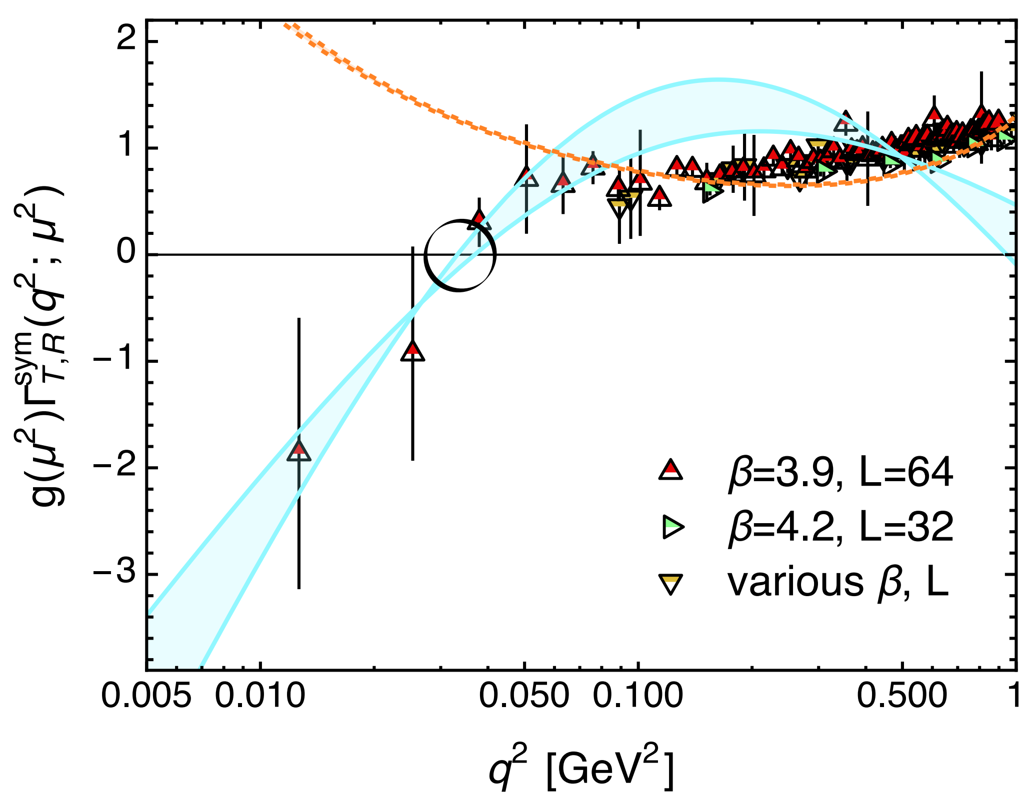
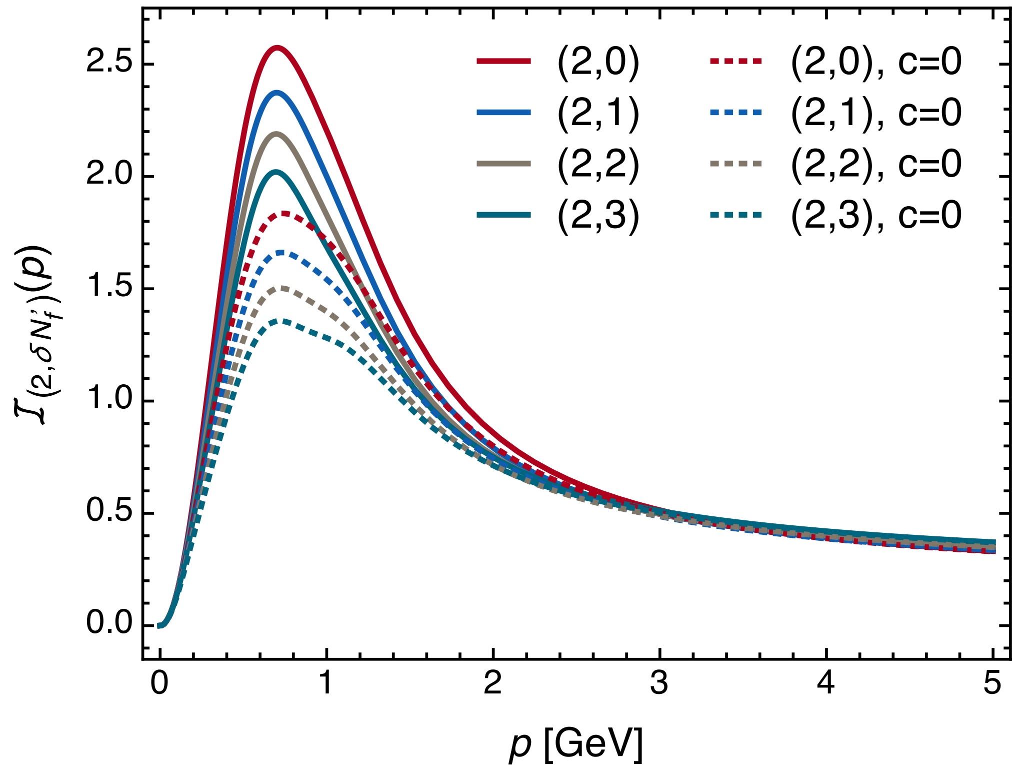
$$\mathcal{I}(p^2) \underset{p^2/\Lambda_{\mathrm{T}}^2 \ll 1}{\simeq} p^2 \widehat{d}(0)\left[1-\left(\frac{\widehat{d}(0)}{8\pi}+\frac{ⓒ}{M^2}\right)p^2 \ln\frac{p^2}{\Lambda_{\mathrm{T}}^2}\right]$$

$$\mathcal{I}(p^2) \underset{p^2/\Lambda_{\mathrm{T}}^2 \gg 1}{\simeq} \alpha_{\mathrm{T}}(p^2) \underset{p^2/\Lambda_{\mathrm{T}}^2 \gg 1}{\simeq} \frac{4\pi}{\beta_0 \ln\left(p^2/\Lambda_{\mathrm{T}}^2\right)}$$



Chiral symmetry restoration

- **Interaction bulk:**
  located where strange quark is active (no charm)
  - **Suppression of (2,1,1) wrt (2,0)**
    mainly due to the strange quark
  - **Expand interaction**
    around $N_\mathrm{f} + \delta N_\mathrm{f}'$ where $m_{\delta N_\mathrm{f}'} = 0.095\mathrm{GeV}$
  - **Match with (2,1,1) theory**
    for $\delta N_\mathrm{f}' = 1$



- **Study chiral symmetry restoration**
  solving the quark gap equation
  - **Use previous vertex Ansätz**
    set a_1=a_3=0
  - **Use parameters in the common region**
    $\mathbb{V}_2 = \{(a_{\widehat{45}}, a_8)\,|\, a_{\widehat{45}} \in [-0.95, -0.7], a_8 \in [-1.3, -0.73]\}$
  - **Build ratio** ${}^{\mathrm{q}}M_{(2,\delta N_\mathrm{f}')}(0)/{}^{\mathrm{q}}M_{(2,0)}(0)$
    and extrapolate linearly to x intercept
  - **Critical number of flavours 9 ✔**
    unacceptably low number (5) without ghost enhancement