New description of anharmonic double-γ excitations in deformed nuclei using the IBM.

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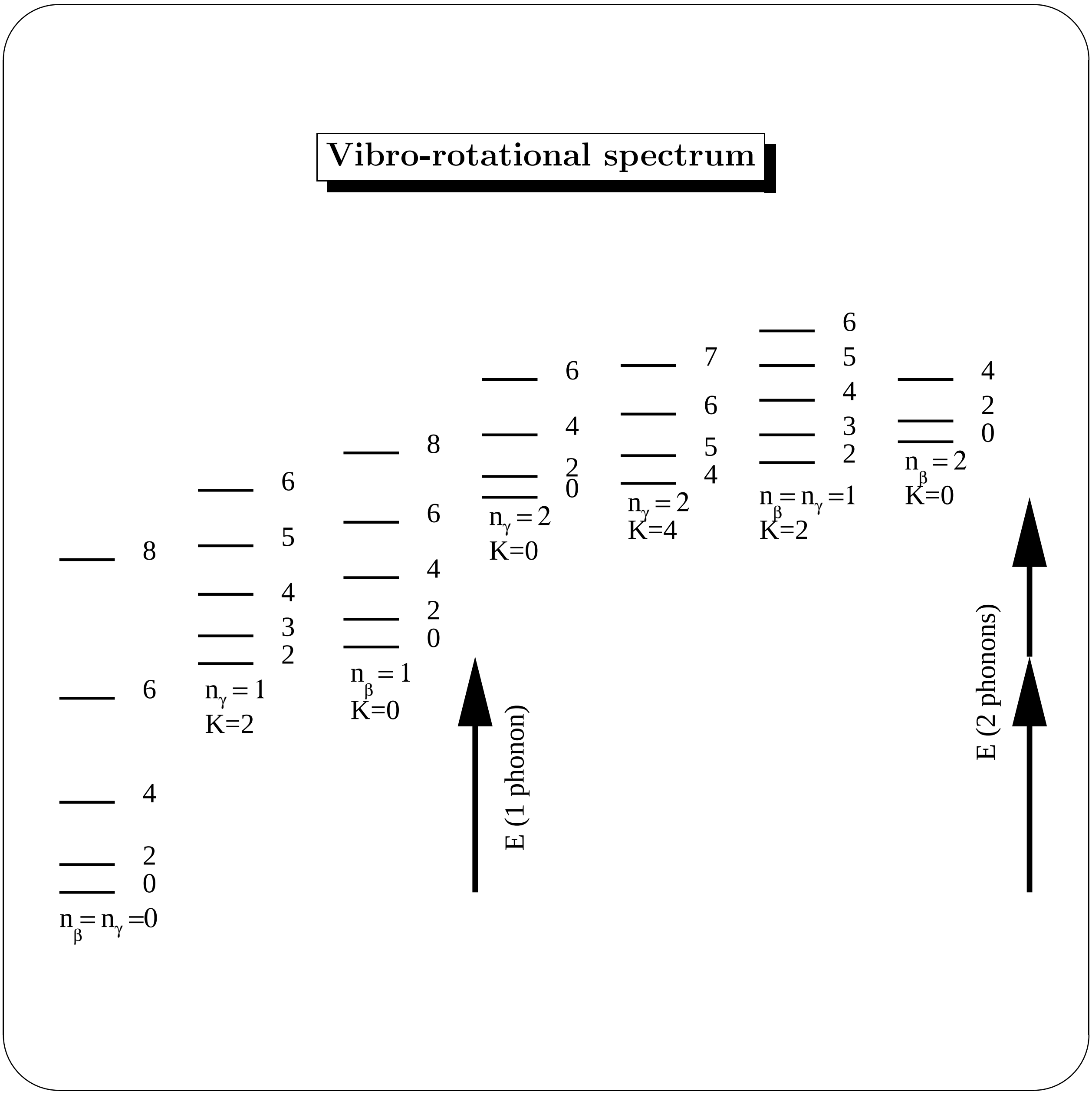
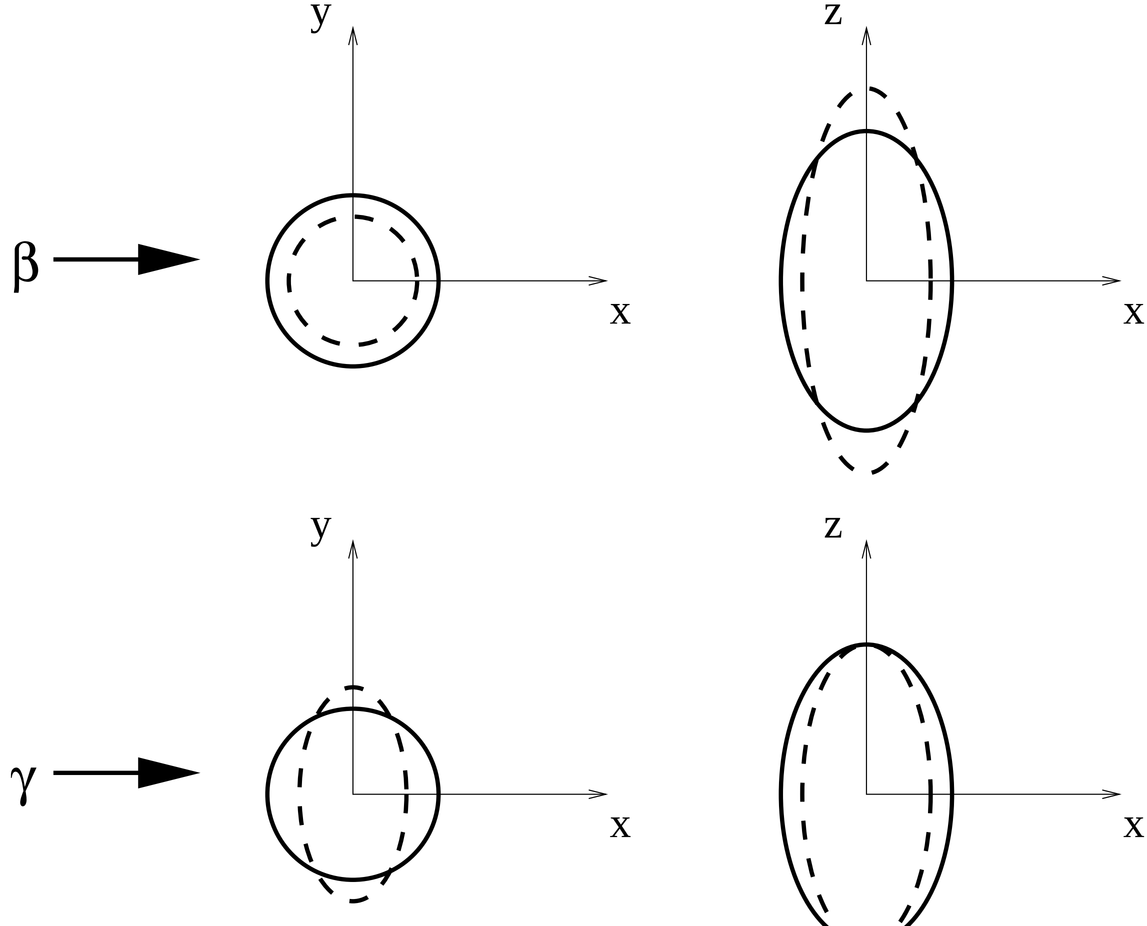


Vibro-rotational spectrum
8
6
4
2
0
$n_\beta = n_\gamma = 0$
6
5
4
3
2
$n_\gamma = 1$
K=2
8
6
4
2
0
$n_\beta = 1$
K=0
E (1 phonon)
6
4
2
0
$n_\gamma = 2$
K=0
7
6
5
4
$n_\gamma = 2$
K=4
6
5
4
3
2
$n_\beta = n_\gamma = 1$
K=2
4
2
0
$n_\beta = 2$
K=0
E (2 phonons)

Geometric representation of β and γ vibrations



Some experimental data

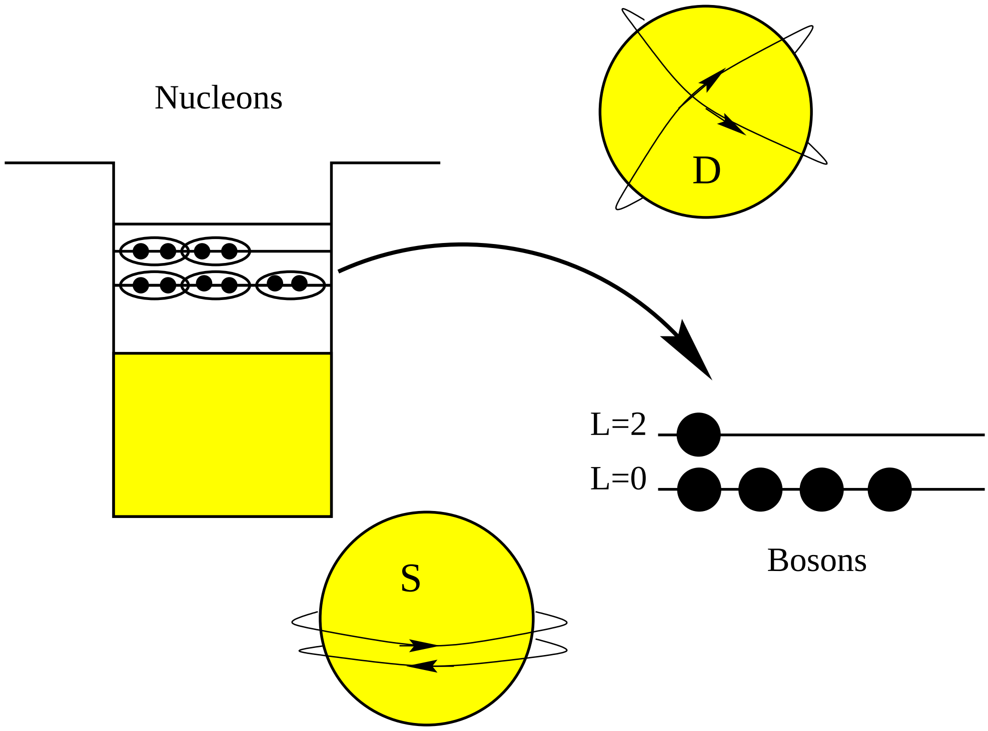
- ^{106}Mo: 1435 keV $(K^\pi = 4^+)$
- $^{154-156}$Gd: $(K^\pi = 4^+)$
- ^{164}Dy: 2173 $(K^\pi = 4^+)$
- ^{166}Er: 1943 keV $(K^\pi = 0^+)$, 2028 keV $(K^\pi = 4^+)$
- ^{168}Er: 2055 keV $(K^\pi = 4^+)$
- $^{186-192}$Os, ^{194}Pt

Degree of anharmonicity

- ^{164}Dy: $\dfrac{E^*(4^+_{\gamma^2})}{E^*(2^+_\gamma)} = \mathbf{2.95}$
- ^{166}Er: $\dfrac{E^*(0^+_{\gamma^2})}{E^*(2^+_\gamma)} = \mathbf{2.47}$
- ^{166}Er: $\dfrac{E^*(4^+_{\gamma^2})}{E^*(2^+_\gamma)} = \mathbf{2.58}$
- ^{168}Er: $\dfrac{E^*(4^+_{\gamma^2})}{E^*(2^+_\gamma)} = \mathbf{2.50}$

The *I*nteracting *B*oson *M*odel

- **The IBM is a model which describes the low lying collective states of medium mass and heavy nuclei.**
- **It can be considered as an approximation to the Shell Model. Two steps are necessary: truncation of the Shell Model space and bosonization of the nucleon pairs.**



- **The IBM can also be considered as the second quantization of the shape variables of the Geometric Collective Model.**

Algebraic structure of the IBM

$$s^\dagger, d_m^\dagger (m = 0, \pm 1, \pm 2) \qquad \gamma_{lm}^\dagger, \gamma_{lm}$$
$$\longrightarrow$$
$$s, d_m (m = 0, \pm 1, \pm 2) \qquad (l = 0, 2; \quad -l \leq m \leq l)$$

$$[\gamma_{lm}, \gamma_{l'm'}^\dagger] = \delta_{ll'}\delta_{mm}, [\gamma_{lm}^\dagger, \gamma_{l'm'}^\dagger] = 0, [\gamma_{lm}, \gamma_{l'm'}] = 0$$

- **The dynamical algebra of the IBM is U(6).**

  Generators $U(6)$: $\hat{G}_{ij} = \gamma_i^\dagger \gamma_j$, with $i, j = 1, \ldots, 6$.

$$[\hat{G}_{ij}, \hat{G}_{kl}] = \hat{G}_{il}\delta_{jk} - \hat{G}_{jk}\delta_{il}$$

- **Every dynamic operator can be written in terms of $U(6)$ generators.**

$$\hat{H} = \sum_{ij} \varepsilon_{ij}\gamma_i^\dagger \gamma_j + \sum_{ijkl} V_{ijkl}\gamma_i^\dagger \gamma_j^\dagger \gamma_k \gamma_l$$

$$\hat{T} = \sum_{ij} t_{ij}\gamma_i^\dagger \gamma_j$$

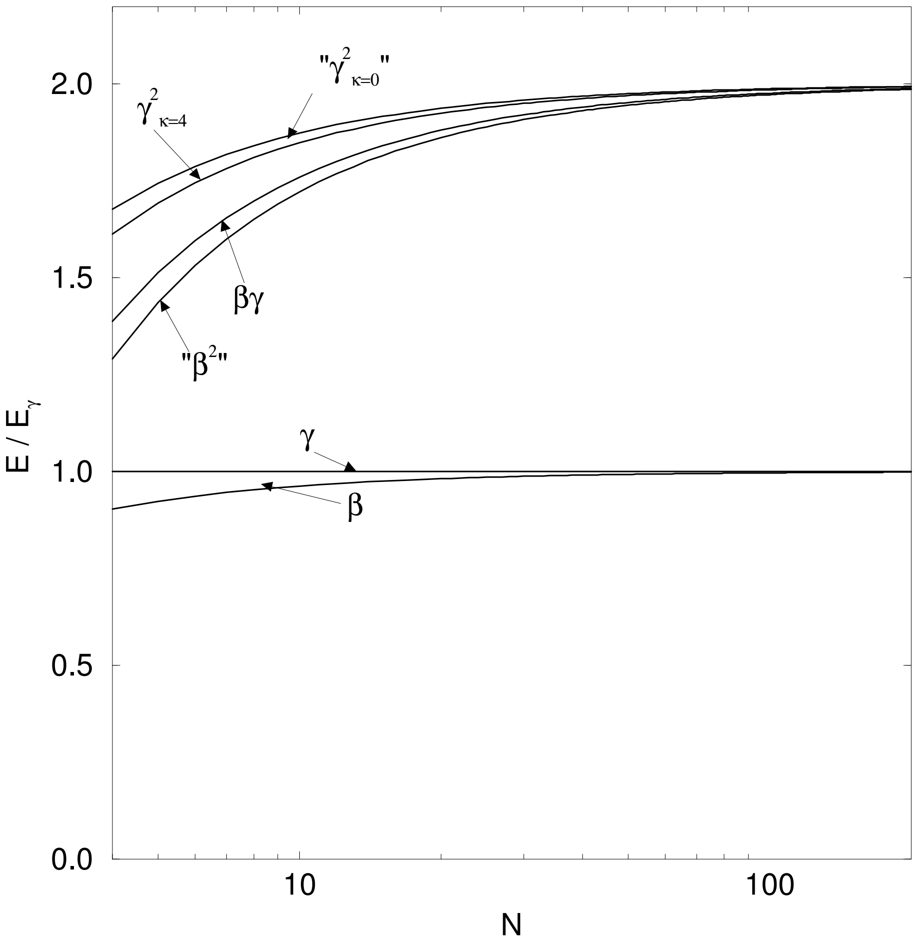
Generic Hamiltonian

$$
\begin{aligned}
\hat{H} &= \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d + \kappa_0 \hat{P}^\dagger \hat{P} + \kappa_1 \hat{L} \cdot \hat{L} \\
&+ \kappa_2 \hat{Q} \cdot \hat{Q} + \kappa_3 \hat{T}_3 \cdot \hat{T}_3 + \kappa_4 \hat{T}_4 \cdot \hat{T}_4,
\end{aligned}
$$

where $\hat{n}_s$ and $\hat{n}_d$ are the s and d boson number operators, respectively, and

$$
\begin{aligned}
\hat{P}^\dagger &= \frac{1}{2} d^\dagger \cdot d^\dagger - \frac{1}{2} s^\dagger \cdot s^\dagger, \\
\hat{L} &= \sqrt{10} (d^\dagger \times \tilde{d})^{(1)}, \\
\hat{Q} &= s^\dagger \tilde{d} + d^\dagger \tilde{s} + \chi (d^\dagger \times \tilde{d})^{(2)}, \\
\hat{T}_3 &= (d^\dagger \times \tilde{d})^{(3)} \\
\hat{T}_4 &= (d^\dagger \times \tilde{d})^{(4)}.
\end{aligned}
$$

$SU(3)$ limit



$\hat{H} = -\hat{Q} \cdot \hat{Q}$, with $\chi = \pm\sqrt{7}/2$

Some definitions

Energy ratios.

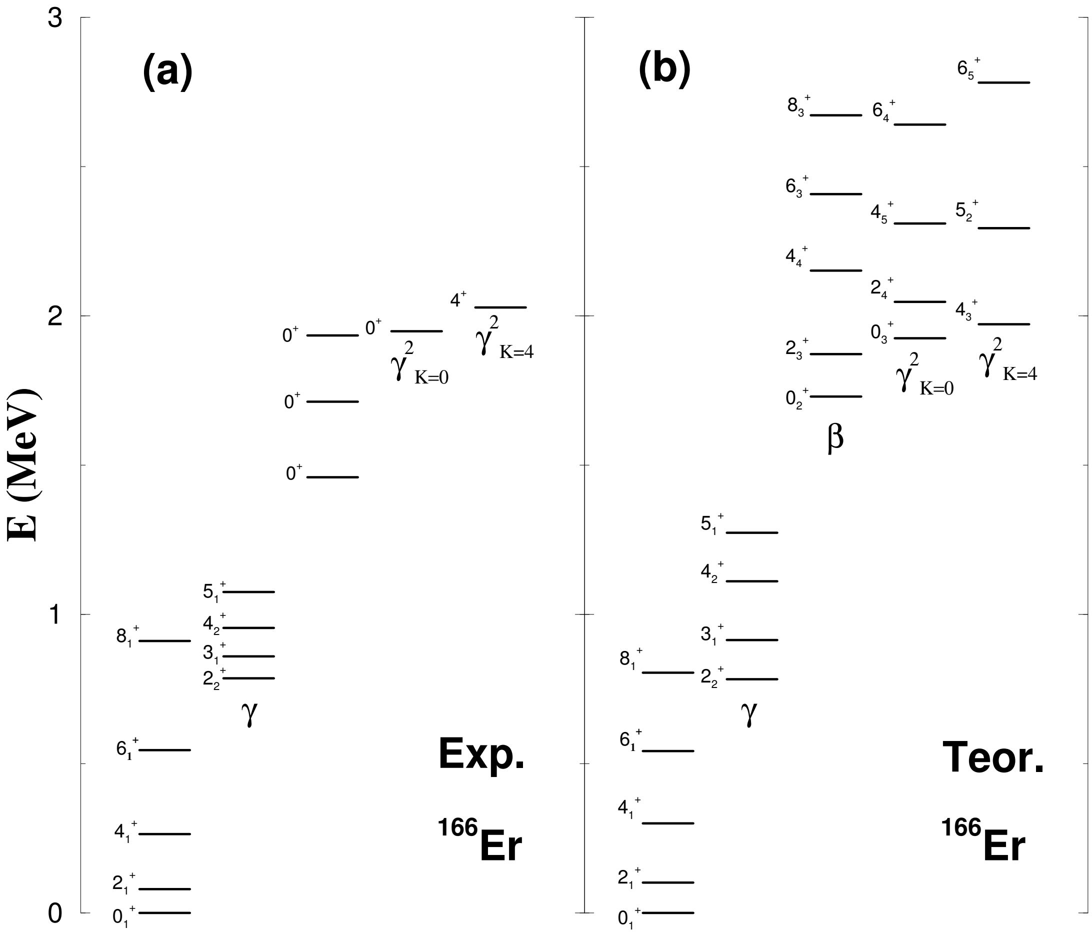
$$R_0^\gamma \equiv \frac{E_\mathrm{x}(0^+_{\gamma\gamma})}{E_\mathrm{x}(2^+_\gamma) - E_\mathrm{x}(2^+_1)}, \qquad R_4^\gamma \equiv \frac{E_\mathrm{x}(4^+_{\gamma\gamma}) - E_\mathrm{x}(4^+_1)}{E_\mathrm{x}(2^+_\gamma) - E_\mathrm{x}(2^+_1)}$$

First extension: 3-body terms

Hamiltonian.

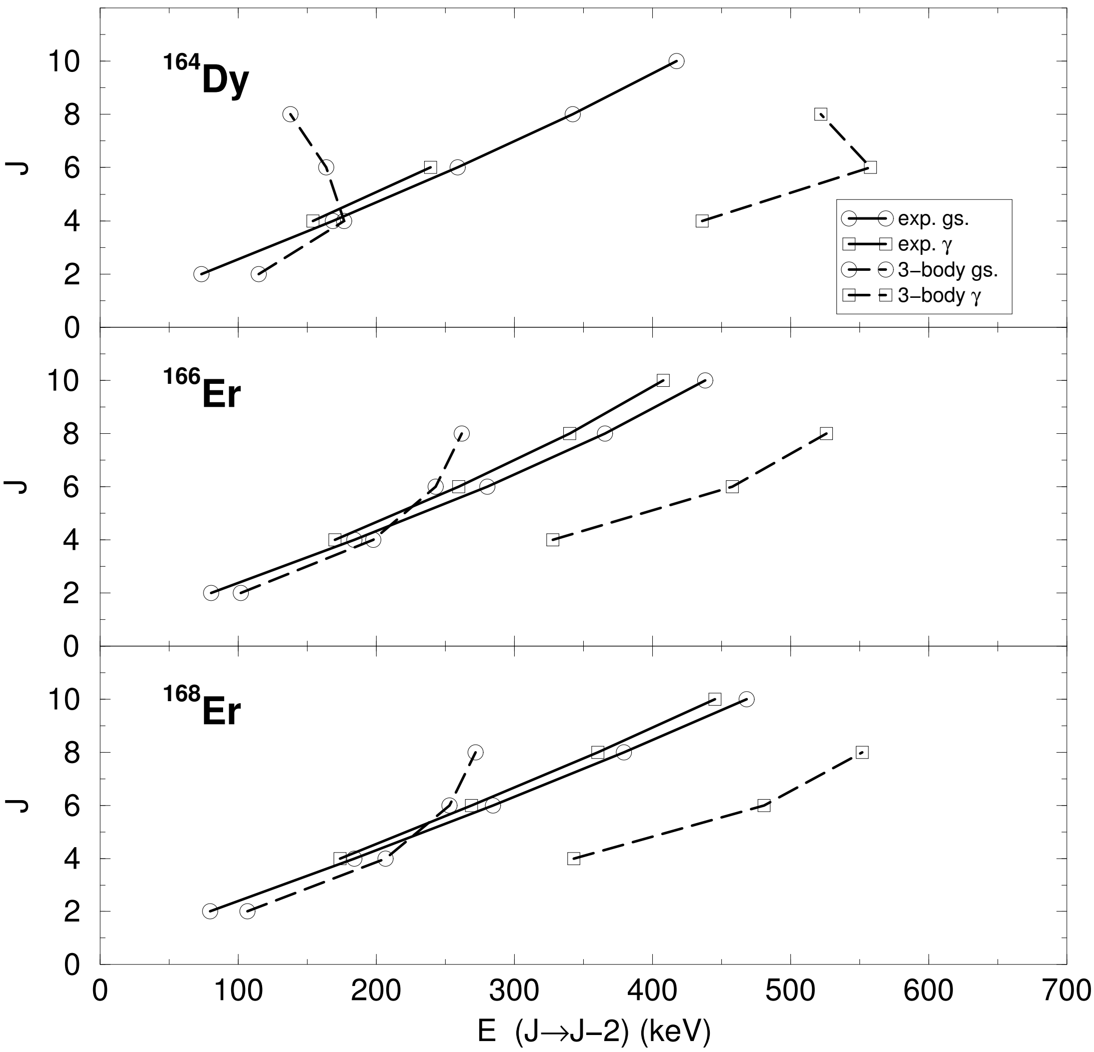
$$\begin{aligned}\hat{H} &= -\kappa\hat{Q}\cdot\hat{Q}+\kappa'\hat{L}\cdot\hat{L} \\ &+ \sum_{l=0,2,3,4,6}\theta_l\left((d^\dagger\times d^\dagger)^{(k)}\times d^\dagger\right)^{(l)}\cdot\left((\tilde{d}\times\tilde{d})^{(k)}\times\tilde{d}\right)^{(l)}\end{aligned}$$

Realistic calculation for ^{166}Er. Three-body Hamiltonian



Experimental (a) and theoretical (b) spectra for ^{166}Er. Hamiltonian parameters: $\kappa = 23.8$ keV, $\chi = -0.55$, $\kappa' = -1.9$ keV, and $\theta_4 = 31.3$ keV. $N = 15$.

Moment of inertia with a 3-body Hamiltonian



Moment of inertia: $\mathcal{I} \approx 2\hbar \frac{dJ}{dE_\gamma}$.

Beyond 3-body terms: 4-body terms?

- **An analytic approximation. The $SU(3)$ limit.**

$$\hat{H} = a\ \hat{C}_2[SU(3)] + b\ \hat{C}_3[SU(3)] + c\ \hat{C}_2[SU(3)]^2$$

Eigenvalues.

$$\begin{aligned} \langle(\lambda,\mu)|\hat{H}|(\lambda,\mu)\rangle &= a(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu) \\ &+ b(\lambda-\mu)(2\lambda+\mu+3)(\lambda+2\mu+3) \\ &+ c(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu)^2 \end{aligned}$$

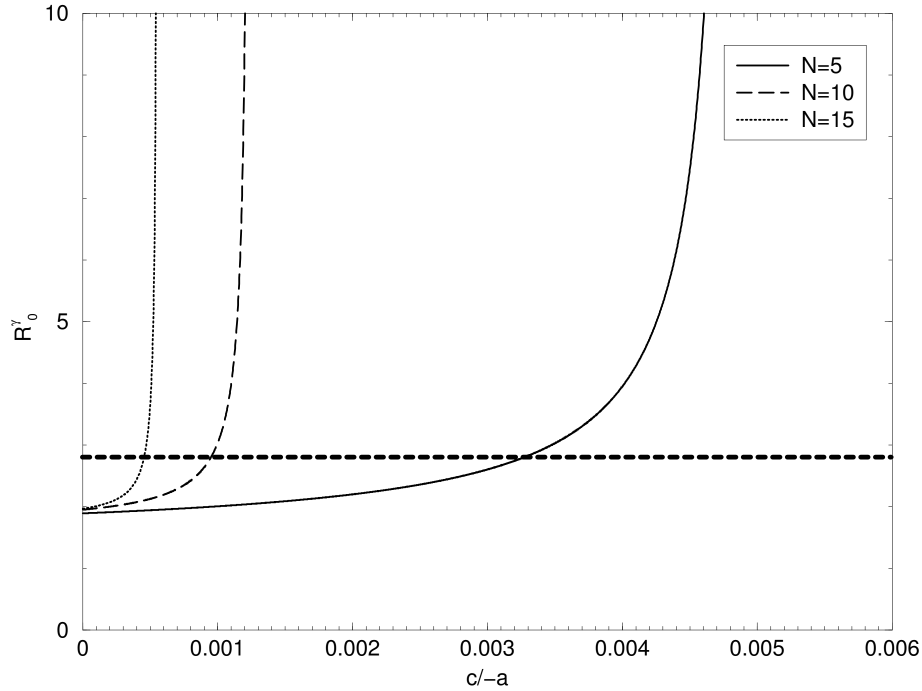
- **Q expansion.**

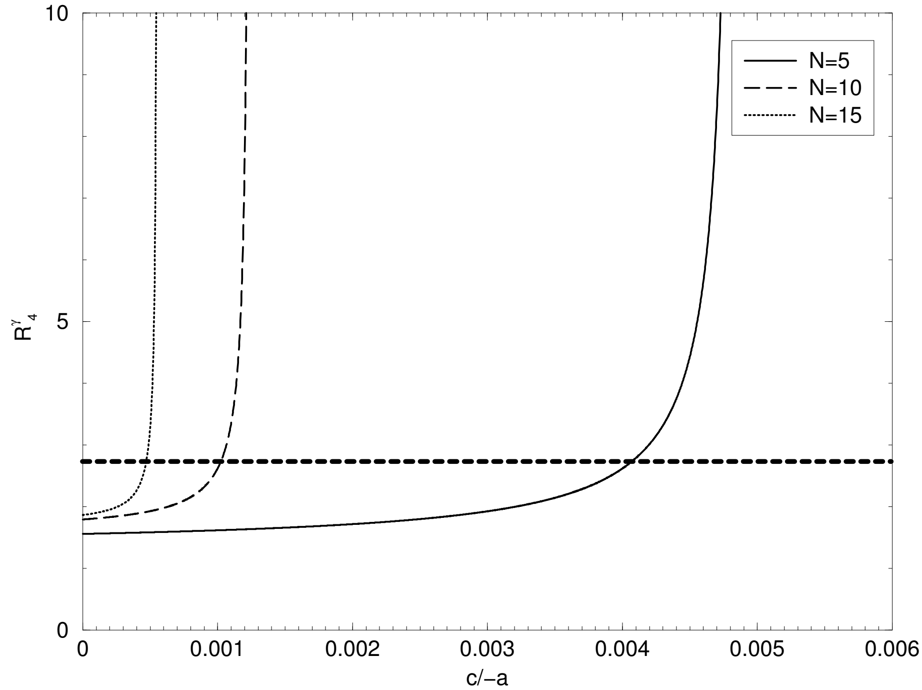
$$\begin{aligned} \hat{H}_Q &= \kappa'\ \hat{L}\cdot\hat{L} + a\ \hat{Q}\cdot\hat{Q} + b\ (\hat{Q}\times\hat{Q}\times\hat{Q})^{(0)} \\ &+ c\ (\hat{Q}\cdot\hat{Q})(\hat{Q}\cdot\hat{Q}) \end{aligned}$$

- **Pseudo-Casimir expansion.**

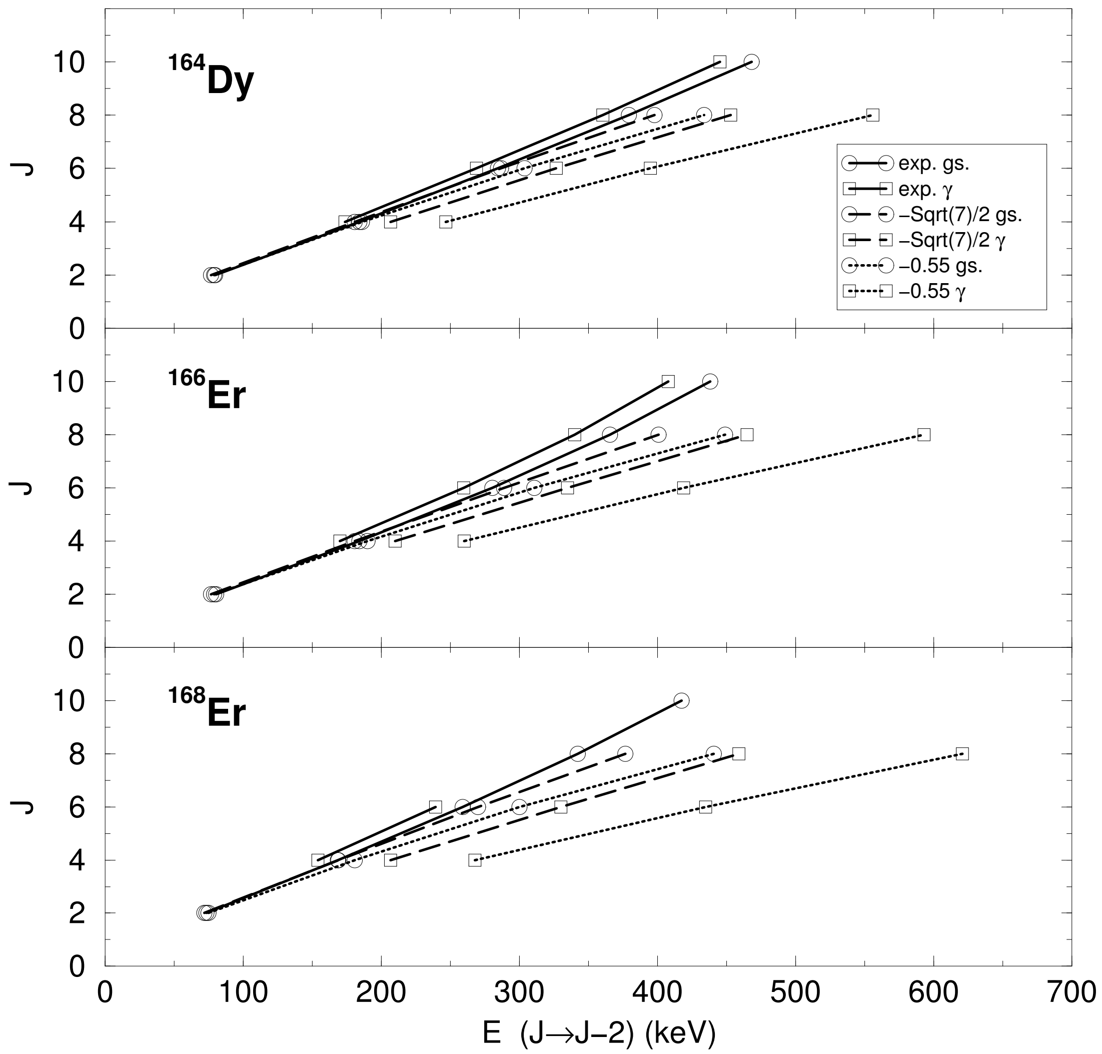
$$\begin{aligned} \hat{H}_{pC} &= \kappa'\ \hat{L}\cdot\hat{L} + a\ \hat{C}_2[SU(3)]_\chi + b\ \hat{C}_3[SU(3)]_\chi \\ &+ c\ \hat{C}_2[SU(3)]_\chi^2 \end{aligned}$$

Energy ratios in the $SU(3)$ limit



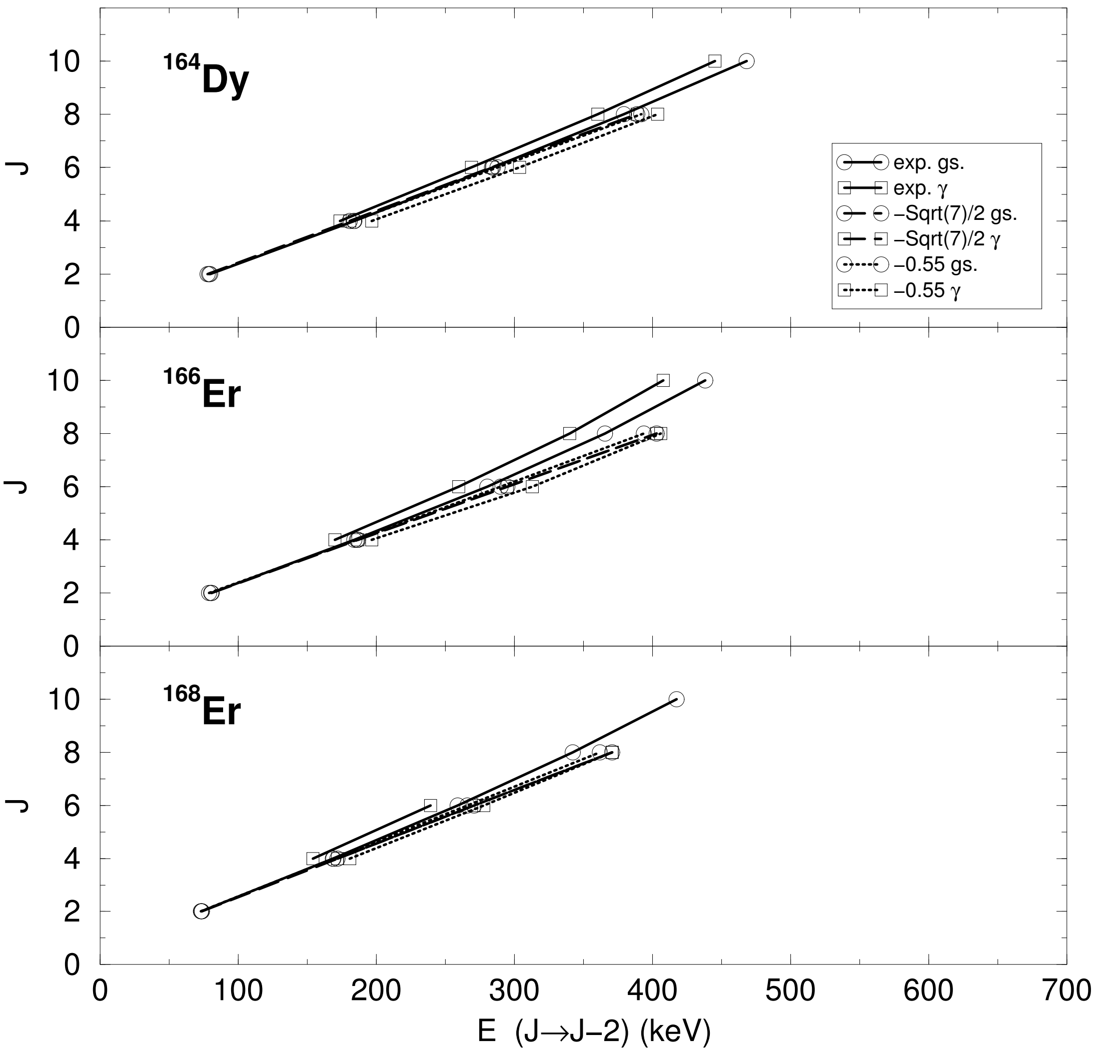


Moment of inertia with a $\hat{Q}$ expansion



Moment of inertia: $\mathcal{I} \approx 2\hbar \frac{dJ}{dE_\gamma}$.

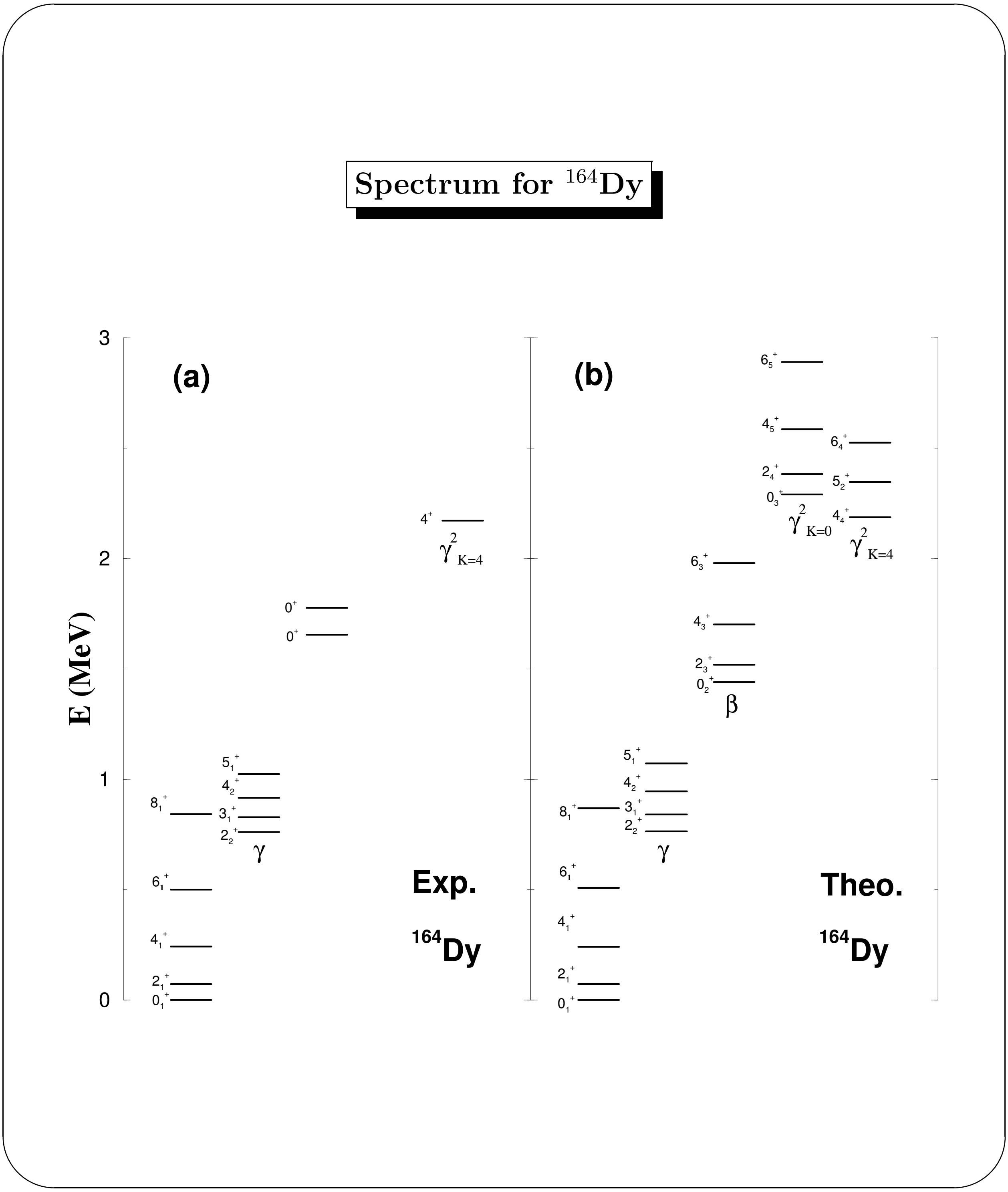
Moment of inertia with a pseudo–Casimir expansion



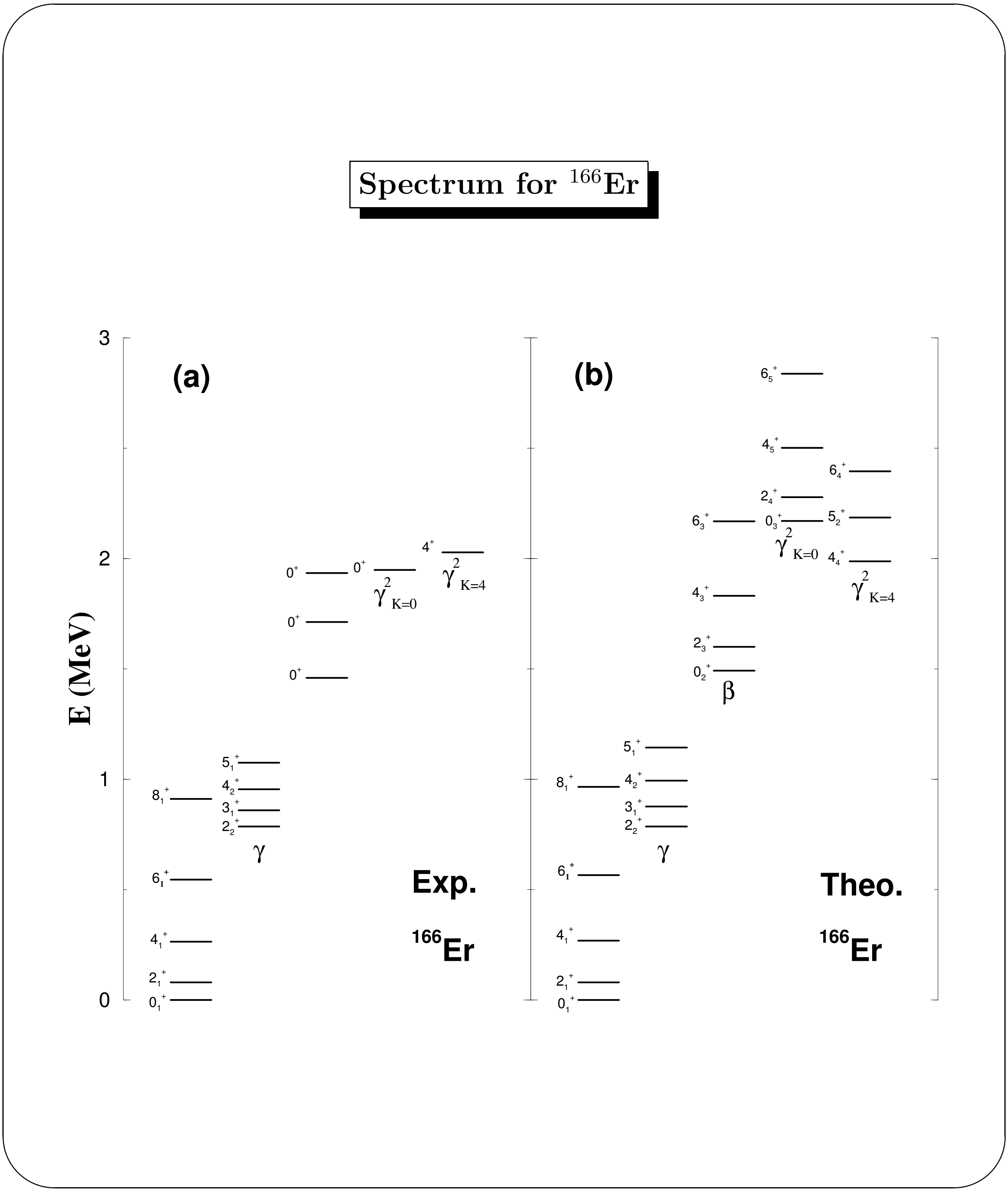
Moment of inertia: $\mathcal{I} \approx 2\hbar \frac{dJ}{dE_\gamma}$.

Final calculations for ^{164}Dy, ^{166}Er, and ^{168}Er

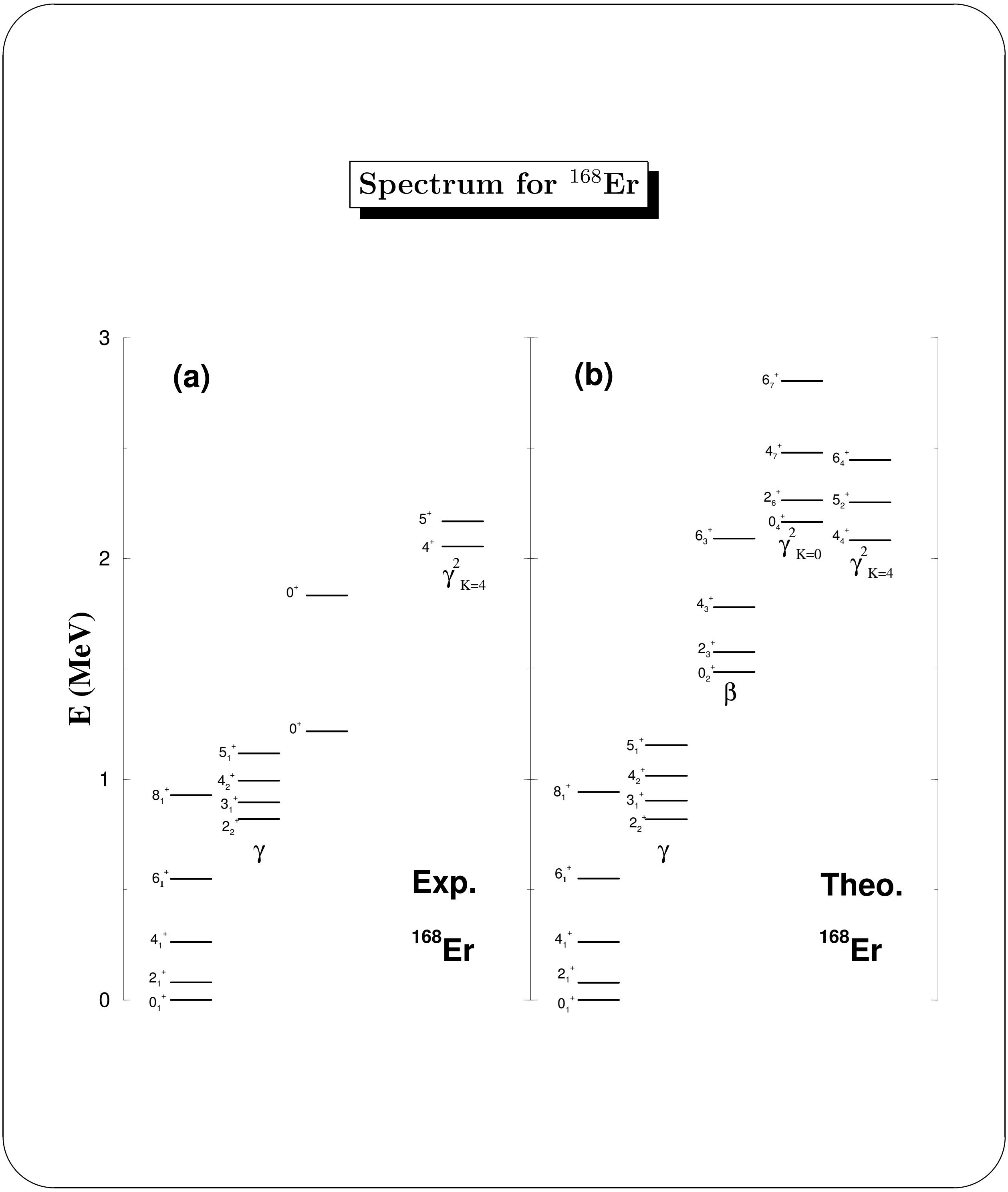
Nucleus	κ' (keV)	a (keV)	c (keV)	χ	N
^{164}Dy	12.18	-82.90	0.05150	-0.55	16
^{166}Er	13.55	-75.40	0.05286	-0.45	15
^{168}Er	13.23	-67.25	0.04080	-0.50	16



Spectrum for ^{164}Dy
E (MeV)
0
1
2
3
(a)
(b)
Exp.
^{164}Dy
Theo.
^{164}Dy
0_1^+
2_1^+
4_1^+
6_1^+
8_1^+
2_2^+
3_1^+
4_2^+
5_1^+
γ
0^+
0^+
4^+
$\gamma^2_{K=4}$
0_2^+
2_3^+
4_3^+
6_3^+
β
0_3^+
2_4^+
4_5^+
6_5^+
$\gamma^2_{K=0}$
4_4^+
5_2^+
6_4^+
$\gamma^2_{K=4}$



Spectrum for 166Er
E (MeV)
(a)
(b)
3
2
1
0
0_1^+
2_1^+
4_1^+
6_1^+
8_1^+
2_2^+
3_1^+
4_2^+
5_1^+
γ
0^+
0^+
0^+
0^+
γ^2 K=0
4^+
γ^2 K=4
Exp.
166Er
0_2^+
2_3^+
4_3^+
6_3^+
β
0_3^+
2_4^+
4_5^+
6_5^+
4_4^+
5_2^+
6_4^+
Theo.
166Er


Spectrum for 168Er
E (MeV)
0
1
2
3
(a)
(b)
γ
β
Exp.
168Er
Theo.
168Er
0+
5+
4+
γ2 K=4
γ2 K=0
0₁+
2₁+
4₁+
6₁+
8₁+
2₂+
3₁+
4₂+
5₁+
0₂+
2₃+
4₃+
6₃+
0₄+
2₆+
4₇+
6₇+
4₄+
5₂+
6₄+