IBM calculations far from stability and phase transitions.

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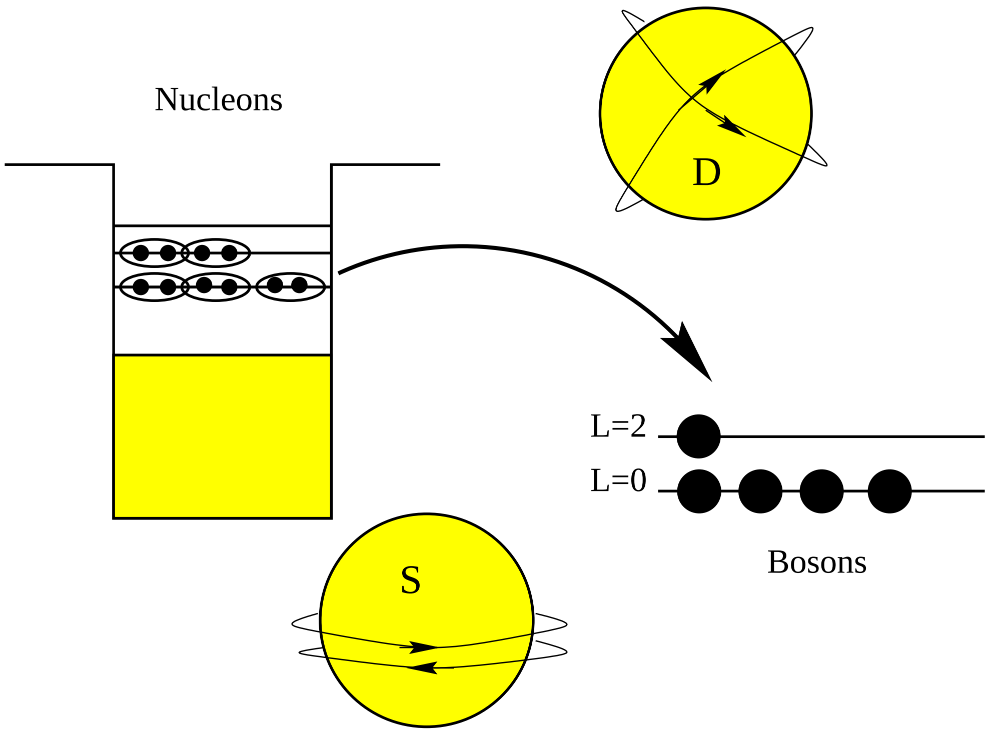
The *I*nteracting *B*oson *M*odel

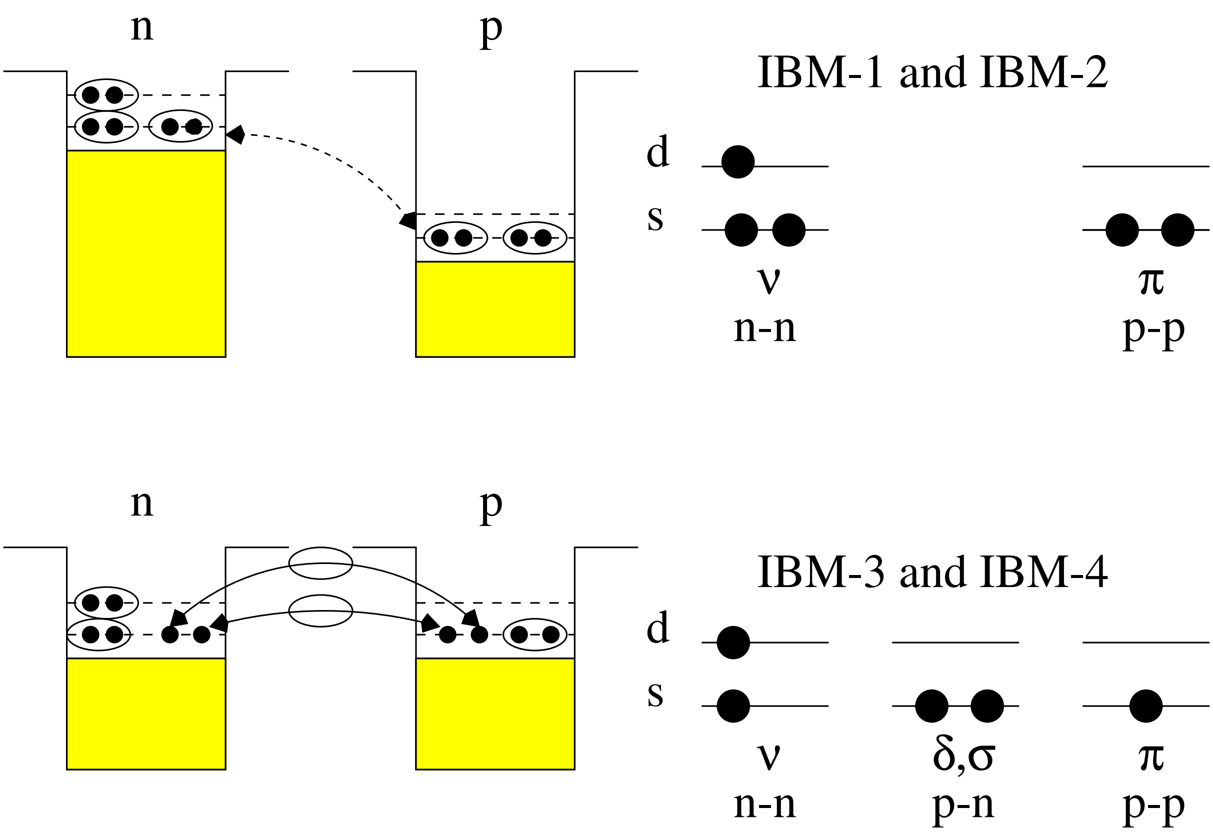
- The IBM is a model which describes the low lying collective states of medium mass and heavy nuclei.
- It can be considered as an approximation to the Shell Model. Two steps are necessary: truncation of the Shell Model space and bosonization of the nucleon pairs.



- The IBM can also be considered as the second quantization of the shape variables of the Geometric Collective Model.

Differences between IBM-1,2 and IBM-3,4

- **IBM-1 and IBM-2 are applied to even-even medium-mass and heavy nuclei with $N >> Z$.**



- **IBM-3 and IBM-4 are applied to even-even medium-mass and light nuclei with $N \approx Z$. Besides IBM-4 can be applied to odd-odd nuclei.**

Algebraic structure of IBM-1

$$\begin{array}{ccc} s^\dagger, d^\dagger_m(m=0,\pm1,\pm2) & & \gamma^\dagger_{lm}, \gamma_{lm} \\ & \longrightarrow & \\ s, d_m(m=0,\pm1,\pm2) & & (l=0,2; \quad -l \leq m \leq l) \end{array}$$

$$[\gamma_{lm}, \gamma^\dagger_{l'm'}] = \delta_{ll'}\delta_{mm}, [\gamma^\dagger_{lm}, \gamma^\dagger_{l'm'}] = 0, [\gamma_{lm}, \gamma_{l'm'}] = 0$$

- **The dynamical algebra of the IBM is U(6).**

  Generators $U(6)$: $\hat{G}_{ij} = \gamma^\dagger_i \gamma_j$, with $i, j = 1, \ldots, 6$.

$$[\hat{G}_{ij}, \hat{G}_{kl}] = \hat{G}_{il}\delta_{jk} - \hat{G}_{jk}\delta_{il}$$

- **Every dynamic operator can be written in terms of $U(6)$ generators.**

$$\hat{H} = \sum_{ij} \varepsilon_{ij} \gamma^\dagger_i \gamma_j + \sum_{ijkl} V_{ijkl} \gamma^\dagger_i \gamma^\dagger_j \gamma_k \gamma_l$$

$$\hat{T} = \sum_{ij} t_{ij} \gamma^\dagger_i \gamma_j$$

Algebraic structure of IBM-3

- **The IBM-3 is an isospin conserving model ⇒ Isospin triplet T=1.**

$$\tau = -1 \quad \nu \ \text{boson} \ \ \text{(neutron-neutron pair)}$$
$$\tau = 0 \quad \delta \ \text{boson} \ \ \text{(proton-neutron pair)}$$
$$\tau = 1 \quad \pi \ \text{boson} \ \ \text{(proton-proton pair)}$$

- **Creation and annihilation operators.**

$$\begin{matrix} s^\dagger_\tau, d^\dagger_{m\tau} \\ s_\tau, d_{m\tau} \end{matrix} \begin{pmatrix} m = 0, \pm 1, \pm 2 \\ \tau = -1, 0, 1 \end{pmatrix} \longrightarrow \ \gamma^\dagger_{lm,1\tau}, \gamma_{lm,1\tau}$$

$$[\gamma_{lm,1\tau}, \gamma^\dagger_{l'm',1\tau'}] = \delta_{ll'}\delta_{mm}\delta_{\tau\tau'}, \quad [\gamma^\dagger_{lm,1\tau}, \gamma^\dagger_{l'm',1\tau}] = 0$$
$$[\gamma_{lm,1\tau}, \gamma_{l'm',1\tau'}] = 0$$

- **Dynamical algebra: U(18).**

Generators of $U(18)$: $\quad \gamma^\dagger_{lm,1\tau}\gamma_{l'm',1\tau'}.$

More on the algebraic structure of IBM-3

- **Groups and subgroups.**

$$
\begin{array}{ccccccc}
U(18) & \supset & U_L(6) & \otimes & (SU_T(3) & \supset & O_T(3)) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
[N] & & [N_1, N_2, N_3] & & (\lambda_T, \mu_T) & & T
\end{array}
$$

- **States.**

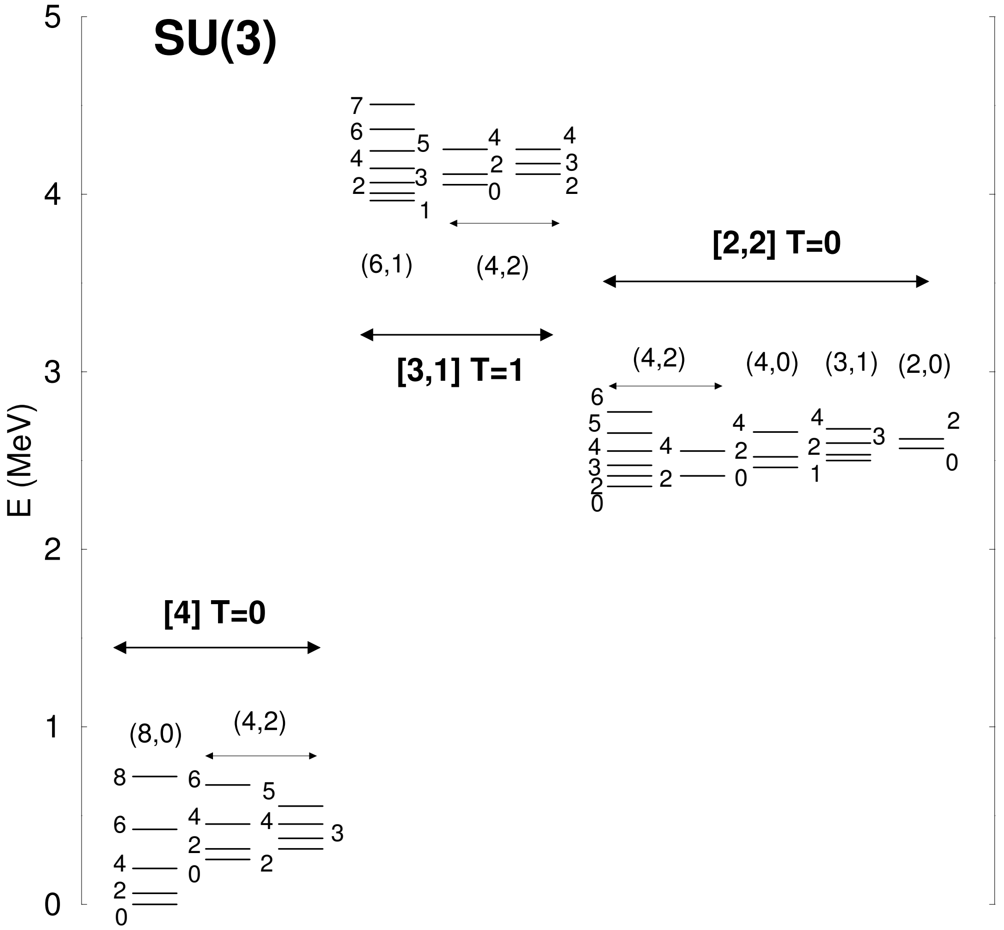
$$|[N_1, N_2, N_3]\phi L M_L; T M_T\rangle$$

- **The $SU(3)$ limit**

$$
\begin{array}{ccccccc}
U_L(6) & \supset & SU_L(3) & \supset & O_L(3) & \supset & O_L(2) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
[N_1, N_2, N_3] & \beta & (\lambda, \mu) & \kappa & L & & M_L
\end{array}
$$

$$|[N_1, N_2, N_3]\beta(\lambda, \mu)\kappa L M_L; T M_T\rangle$$

$SU(3)$ schematic spectrum



$$
\begin{aligned}
\hat{H}_{\mathrm{SU(3)}} &= -0.175\hat{C}_2[U_L(6)] - 0.006\hat{C}_2[SU_L(3)] + 0.010\hat{C}_2[O_L(3)] \\
&+ \alpha_2\hat{C}_2[SU_T(3)] + 1.2\hat{C}_2[O_T(3)].
\end{aligned}
$$

All the coefficients in MeV. The number of bosons is $N_p = 2$ and $N_n = 2$.

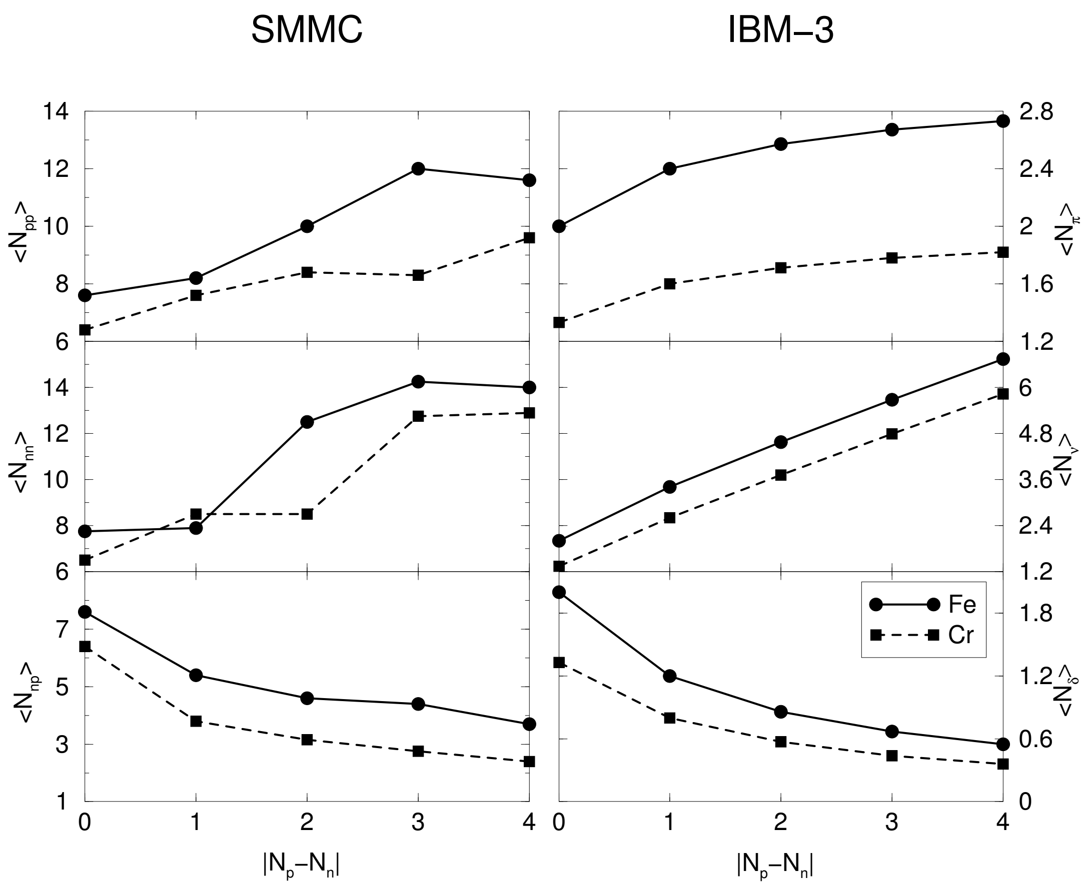
Why is important the limit $SU_T(3) \otimes U_L(6)$?

- **The $SU_T(3) \otimes U_L(6)$ symmetry is very appropriate for labeling states.**
- **It is simple to establish a connection with both IBM-1 and IBM-2.**
- **$SU_T(3) \otimes U_L(6)$ symmetry is approximately followed by sets of states.**

Nucleus	States	Composition (%)	
		[N]	[N-1,1]
^{44}Ti	2_M^+	0	81
	2_1^+	97	0
^{46}Ti	3_M^+	5	93
	2_M^+	18	80
	2_1^+	93	1
^{48}Ti	3_M^+	0	91
	2_M^+	16	78
	2_1^+	94	0

M. Abdelaziz, M.J. Thompson, J.P. Elliott and J.A. Evans, J. Phys. G **14**, 219 (1988).

Boson number in the ground state



$$\langle [N]\phi L; T,-T|\hat{N}_\pi|[N]\phi L; T,-T\rangle = \frac{(T+1)(N-T)}{2T+3}$$

$$\langle [N]\phi L; T,-T|\hat{N}_\delta|[N]\phi L; T,-T\rangle = \frac{N-T}{2T+3}$$

$$\langle [N]\phi L; T,-T|\hat{N}_\nu|[N]\phi L; T,-T\rangle = \frac{T(N+T)+(N+2T)}{2T+3}$$

Schematic calculations in the $f_{7/2}$ shell

- **Hamiltonian.**

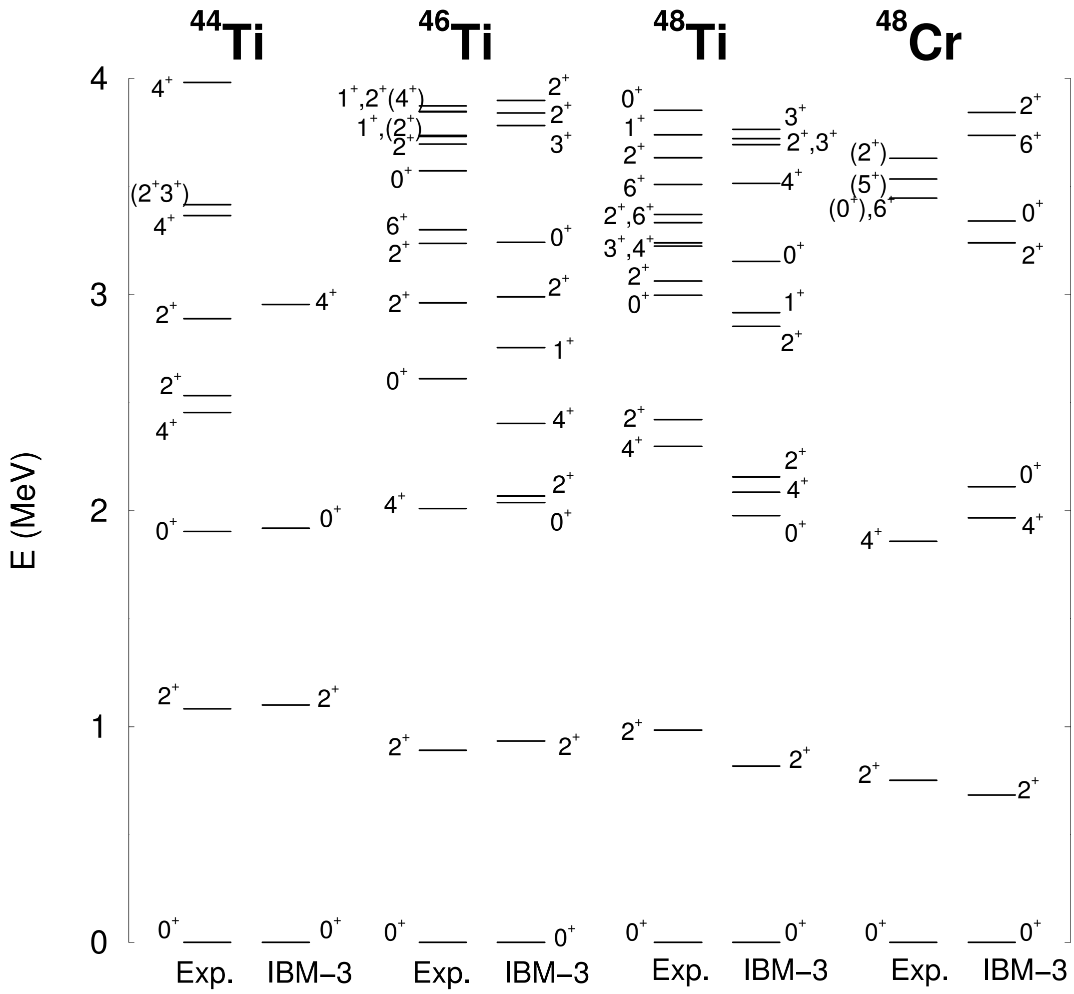
$$\hat{H} = \epsilon_d \hat{n}_d + \kappa_0 \mathcal{N}[\hat{Q}^0 : \hat{Q}^0 + \frac{2}{3}\hat{Q}^1 : \hat{Q}^1] + t\hat{T}^2,$$

where $\hat{Q}^T = [s^\dagger \tilde{d} + d^\dagger \tilde{s} + \chi(d^\dagger \times \tilde{d})]^{L=2,T}$.

- **Calculating the parameters of the Hamiltonian.**
  - $^{40-42}$Ca, $^{41-42}$Sc and ^{42}Ti $\longrightarrow$ ϵ_d.
  - ^{44,46,48}Ti and ^{48}Cr $\longrightarrow$ κ_0, χ.
  - SM Calculations $\longrightarrow$ t.

- **Parameters of the Hamiltonian.**
  - ⊙ $\epsilon_d = 1.5$ MeV
  - ⊙ $\kappa_0 = -0.2$ MeV
  - ⊙ $\chi = -2.4$
  - ⊙ $t = 1.2$ MeV

Spectra for ^{44,46,48}Ti and ^{48}Cr



Energy of non-symmetric states

Nuclei	States	Energy (MeV)	
		Observed	IBM-3
^{44}Ti	1_1^+	5.7[a]	5.2
	2_M^+	6.6	4.8
^{46}Ti	1_1^+	4.3	2.8
	2_M^+	2.5[a]	2.1
	3_M^+	3.6[a]	3.8
^{48}Ti	1_1^+	3.7	2.9
	2_M^+	2.4	2.2
	3_M^+	3.2	4.3
^{48}Cr	1_1^+	5.5[a]	5.4

[a] Calculated in M. Abdelaziz *et al.*, J. Phys. G **14**, 219 (1988); M. Abdelaziz *et al.*, Nucl. Phys. A **503**, 452 (1989).

B(M1) transitions

Nuclei	Transitions	B(M1) (μ_N^2)		
		Observed	Shell Model[a]	IBM-3
^{44}Ti	$2^+_M \rightarrow 2^+_1$		1.14	1.14
	$0^+_1 \rightarrow 1^+_1$		2.40	1.75
^{46}Ti	$2^+_M \rightarrow 2^+_1$		0.73	0.73
	$3^+_M \rightarrow 2^+_1$		0.07	0.20
	$3^+_M \rightarrow 4^+_1$		0.20	0.41
	$0^+_1 \rightarrow 1^+_1$	1.01		1.15
^{48}Ti	$2^+_M \rightarrow 2^+_1$	0.50(10)	0.58	0.90
	$3^+_M \rightarrow 2^+_1$	0.08(3)	0.003	0.30
	$3^+_M \rightarrow 4^+_1$	0.42(16)	0.32	0.49
	$4^+_M \rightarrow 4^+_1$	1.4(5)	1.50	
	$0^+_1 \rightarrow 1^+_1$	0.50(8)	0.54	1.82
^{48}Cr	$0^+_1 \rightarrow 1^+_1$		3.05	4.82

[a] Calculated in M. Abdelaziz *et al.*, J. Phys. G **14**, 219 (1988); M. Abdelaziz *et al.*, Nucl. Phys. A **503**, 452 (1989); E. Caurier *et al.*, Phys. Rev. C **50**, 225 (1994).

$$g_1 = 1.20\mu_N \text{ and } g_2 = 0.58\mu_N.$$

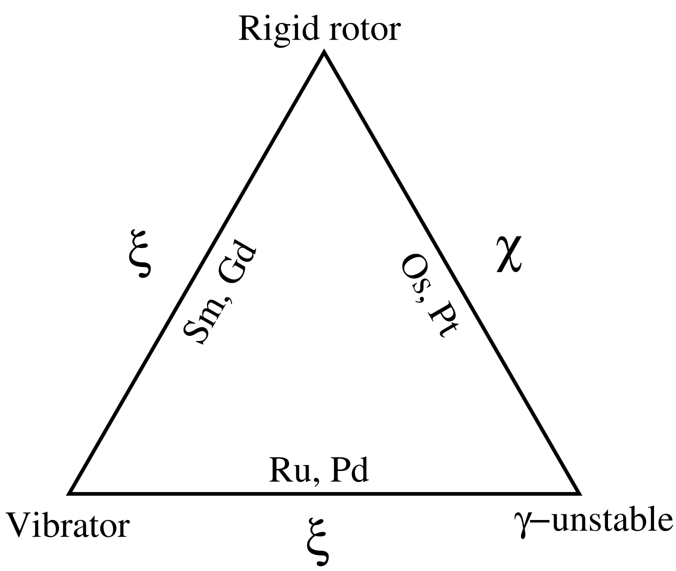
Transitional regions in the IBM

$$\hat{H} = \kappa(N\frac{1-\xi}{\xi}\hat{n}_d - \hat{Q}\cdot\hat{Q}) + \kappa'\hat{L}\cdot\hat{L}$$

$\hat{n}_d = d$ boson number operator

$$\hat{L} = \sqrt{10}(d^\dagger \times \tilde{d})^{(1)},$$

$$\hat{Q} = s^\dagger\tilde{d} + d^\dagger\tilde{s} + \chi(d^\dagger \times \tilde{d})^{(2)}$$



Binding energy and S_{2n}

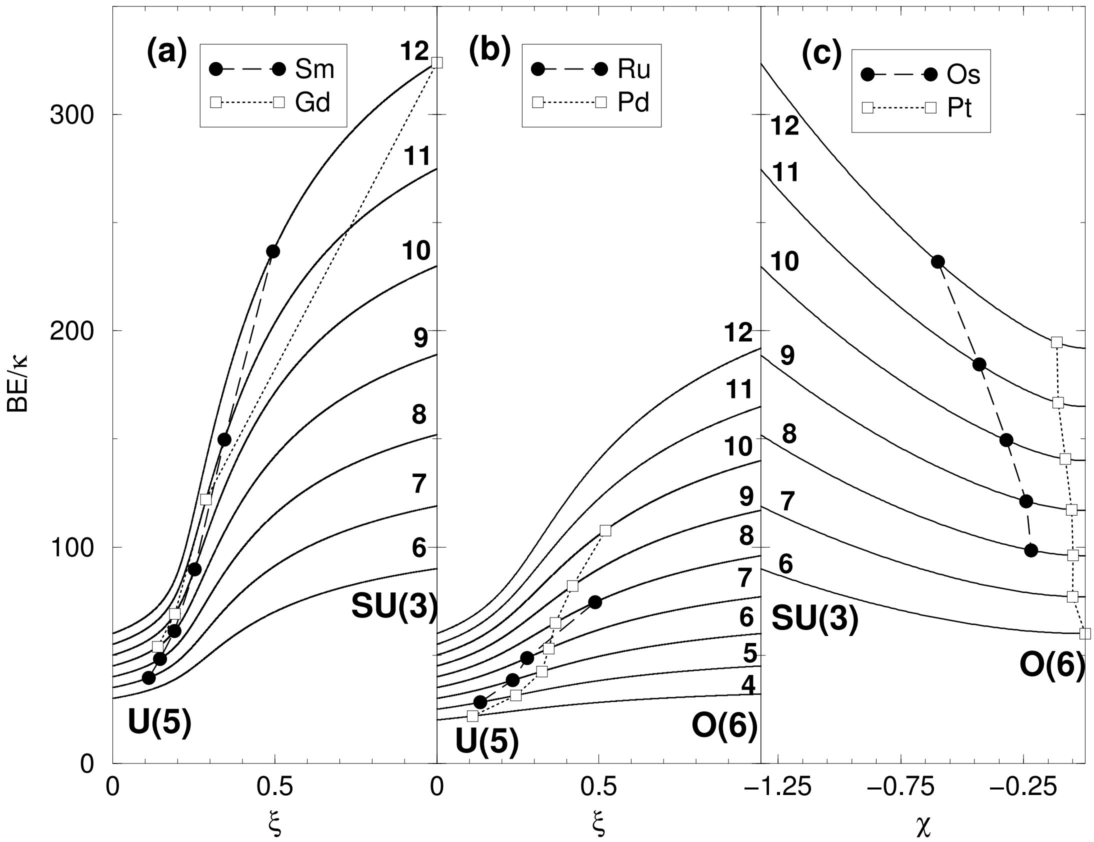
$$H = E_0 + AN + \frac{B}{2}N(N-1) + \hat{H}_{IBM}$$

$$S_{2n}(N) = BE(N) - BE(N-1)$$

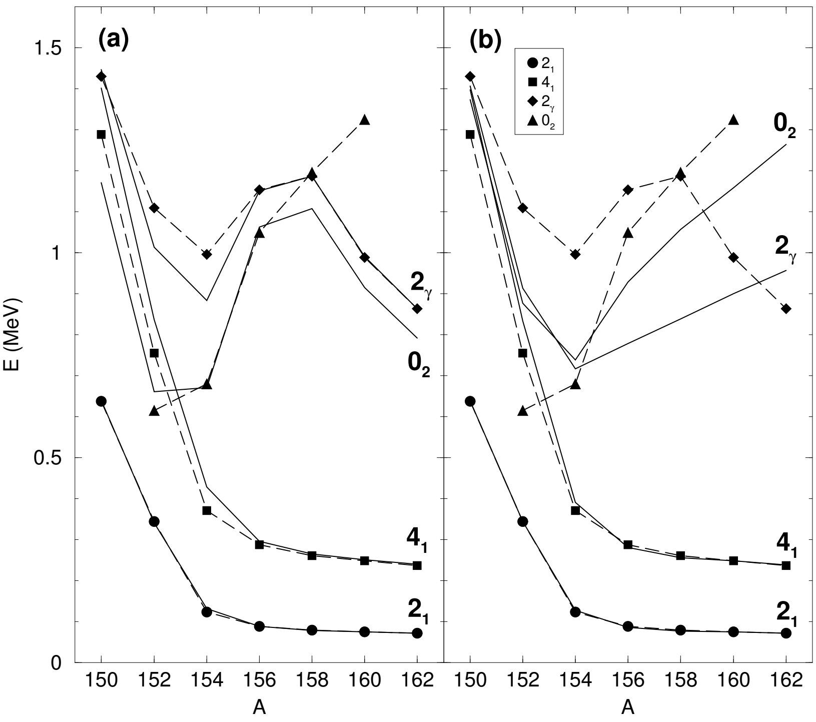
$$\begin{aligned} S_{2n}(N) &= (A - B/2) + BN \\ &+ BE_{IBM}(N) - BE_{IBM}(N-1) \end{aligned}$$

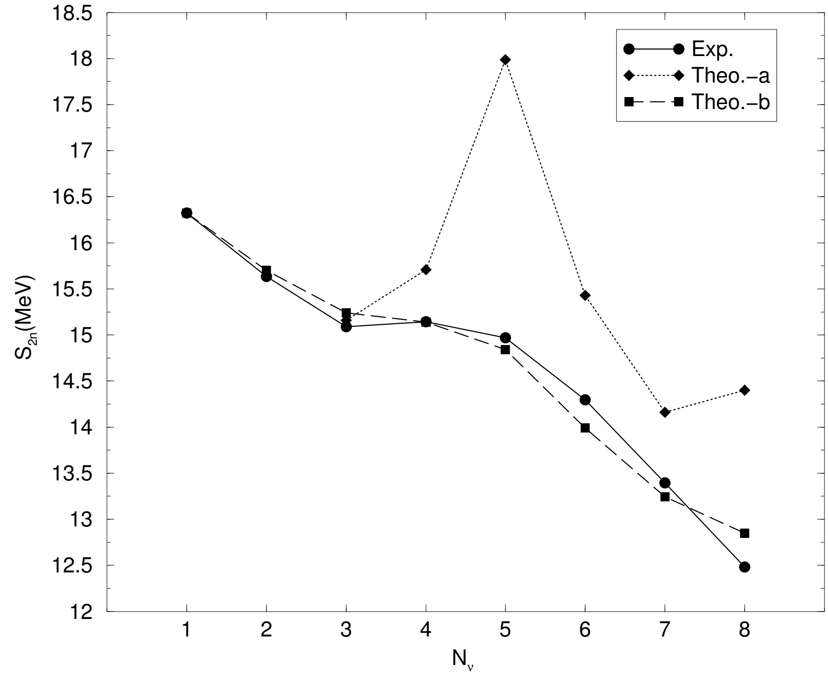
A and $B \approx$ constant

“Phase” diagrams

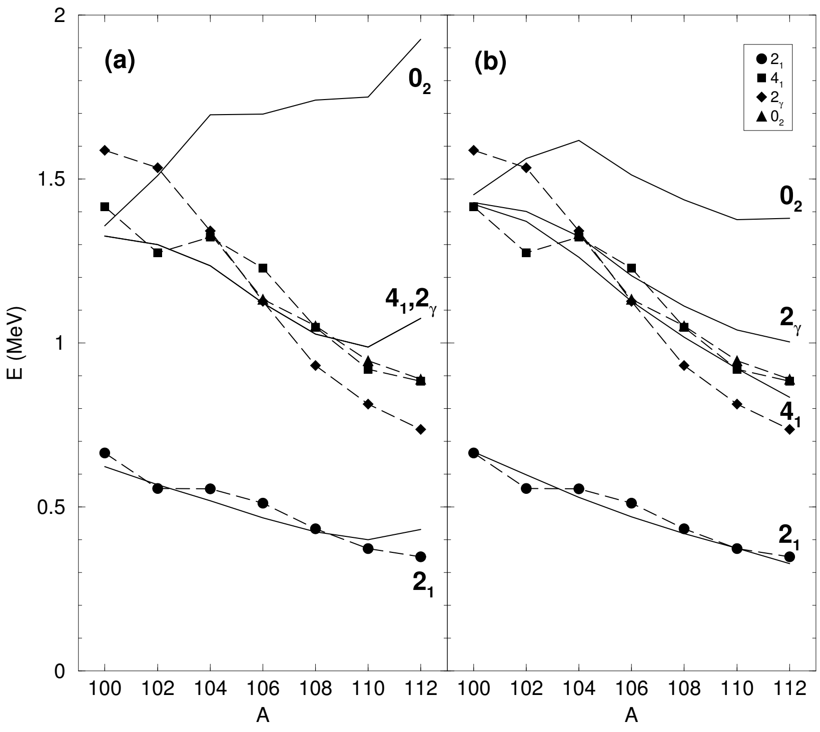


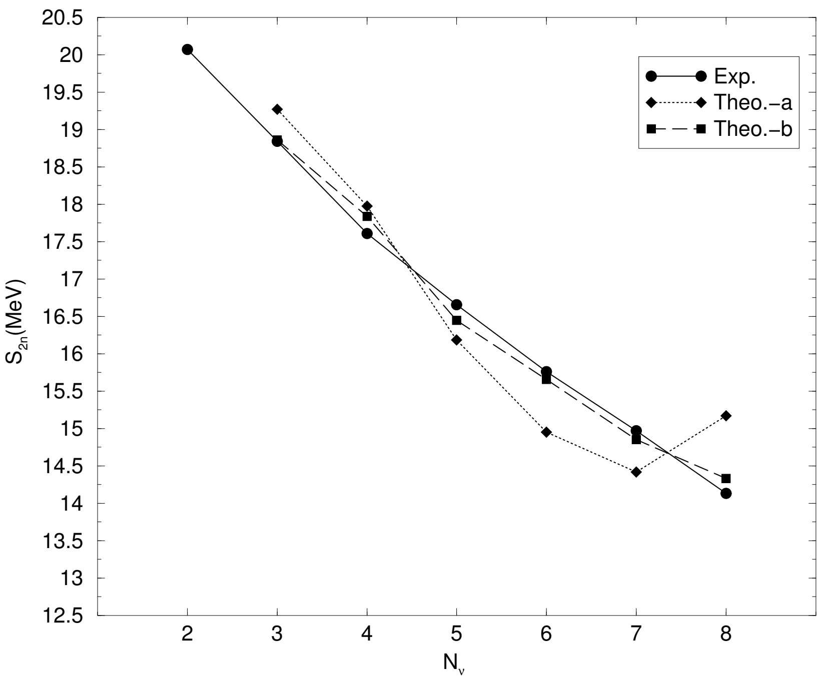
Gd isotopes





Pd isotopes


(a)
(b)
0_2
$4_1,2_\gamma$
2_1
2_γ
4_1
2_1
4_1
2_γ
0_2
E (MeV)
A


Exp.
Theo.–a
Theo.–b
S_{2n}(MeV)
N_ν

Pt isotopes