Two neutron separation energies and phase transition in the Interacting Boson Model.

José Enrique García Ramos,

Department of Applied Physics, University of Huelva, Spain.

Ruben Fossion and Kris Heyde.

Department of Subatomic and Radiation Physics, University of Gent, Belgium.

“Mapping the triangle”, Jackson Lake Lodge, Wyoming, USA, May 22 – 25, 2002.

Motivation

- **Renewed interest in the study of transitional regions and phase transitions.**
- **The traditional IBM analysis of phase transitions can be improved.**
- **Better understanding of the evidences of phase transitions.**
- **Possible connection between excited states and ground state properties.**
- **Connection with shape and configuration mixing.**

Generic Hamiltonian

$$\hat{H} = \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d + \kappa_0 \hat{P}^\dagger \hat{P} + \kappa_1 \hat{L} \cdot \hat{L} + \kappa_2 \hat{Q} \cdot \hat{Q} + \kappa_3 \hat{T}_3 \cdot \hat{T}_3 + \kappa_4 \hat{T}_4 \cdot \hat{T}_4,$$

where $\hat{n}_s$ and $\hat{n}_d$ are the s and d boson number operators, respectively, and

$$\begin{aligned}
\hat{P}^\dagger &= \frac{1}{2} d^\dagger \cdot d^\dagger - \frac{1}{2} s^\dagger \cdot s^\dagger, \\
\hat{L} &= \sqrt{10} (d^\dagger \times \tilde{d})^{(1)}, \\
\hat{Q} &= s^\dagger \tilde{d} + d^\dagger \tilde{s} + \chi (d^\dagger \times \tilde{d})^{(2)}, \\
\hat{T}_3 &= (d^\dagger \times \tilde{d})^{(3)} \\
\hat{T}_4 &= (d^\dagger \times \tilde{d})^{(4)}.
\end{aligned}$$

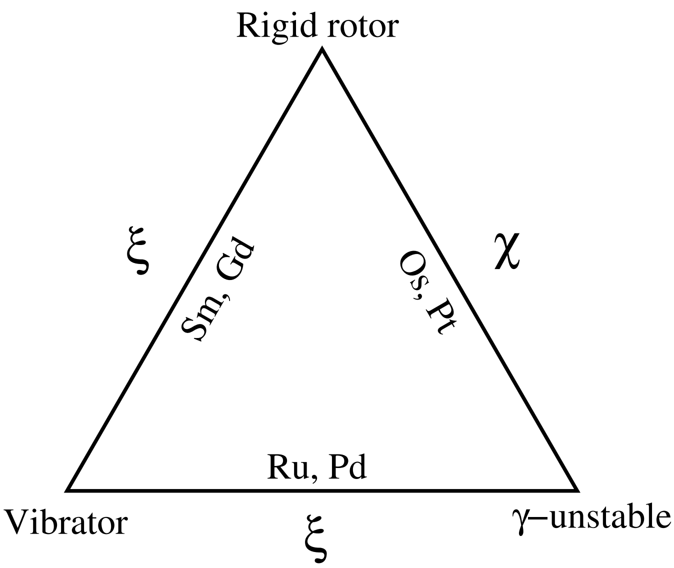
Transitional regions in the IBM

$$\hat{H} = \kappa(N\frac{1-\xi}{\xi}\hat{n}_d - \hat{Q}\cdot\hat{Q}) + \kappa'\hat{L}\cdot\hat{L}$$

$$\hat{n}_d = d \text{ boson number operator}$$

$$\hat{L} = \sqrt{10}(d^\dagger \times \tilde{d})^{(1)},$$

$$\hat{Q} = s^\dagger\tilde{d} + d^\dagger\tilde{s} + \chi(d^\dagger \times \tilde{d})^{(2)}$$



Intrinsic state in the IBM

- The trial wave function

$$|c\rangle = \frac{1}{\sqrt{N!}}(\Gamma_c^\dagger)^N|0\rangle,$$

where

$$\Gamma_c^\dagger = \frac{1}{\sqrt{1+\beta^2}}\left(s^\dagger + \beta\cos\gamma\, d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma\,(d_2^\dagger + d_{-2}^\dagger)\right).$$

- The energy surface

$$\begin{aligned}\langle c|H|c\rangle &= \frac{N}{5(1+\beta^2)}\Big(5\,\varepsilon_s + 25\,\kappa_2 + \beta^2\,(5\,\varepsilon_d - 3\,\kappa_1 + 5\,\kappa_2 + 5\,\chi^2\,\kappa_2 \\ &\qquad - 7\,\kappa_3 + 9\,\kappa_4)\Big) \\ &+ \frac{N(N-1)}{140(1+\beta^2)^2}\Big(35\,\kappa_0 + \beta^2(-70\,\kappa_0 + 560\,\kappa_2) \\ &\qquad - 80\sqrt{14}\,\beta^3\,\chi\,\cos(3\,\gamma)\,\kappa_2 + \beta^4(35\,\kappa_0 + 40\,\chi^2\,\kappa_2 + 72\,\kappa_4)\Big).\end{aligned}$$

- The symmetry limits

$U(5)$ limit $\to \beta = 0$.

$SU(3)$ limit $\to \beta = \sqrt{2}$, $\gamma = 0, \pi/3$.

$O(6)$ limit $\to \beta = 1$, γ unstable nucleus.

$\mathbf{S}_{2n}$ calculations using IBM

- Definition:

$$S_{2n}(N) = BE(N) - BE(N-1).$$

- "Global" part of the IBM Hamiltonian: It is related with the Casimir operators of the $U(6)$ group.

$$\hat{H}^{gl} = -E_0 - \mathcal{A} - \frac{\mathcal{B}}{2}\hat{N}(\hat{N}-1).$$

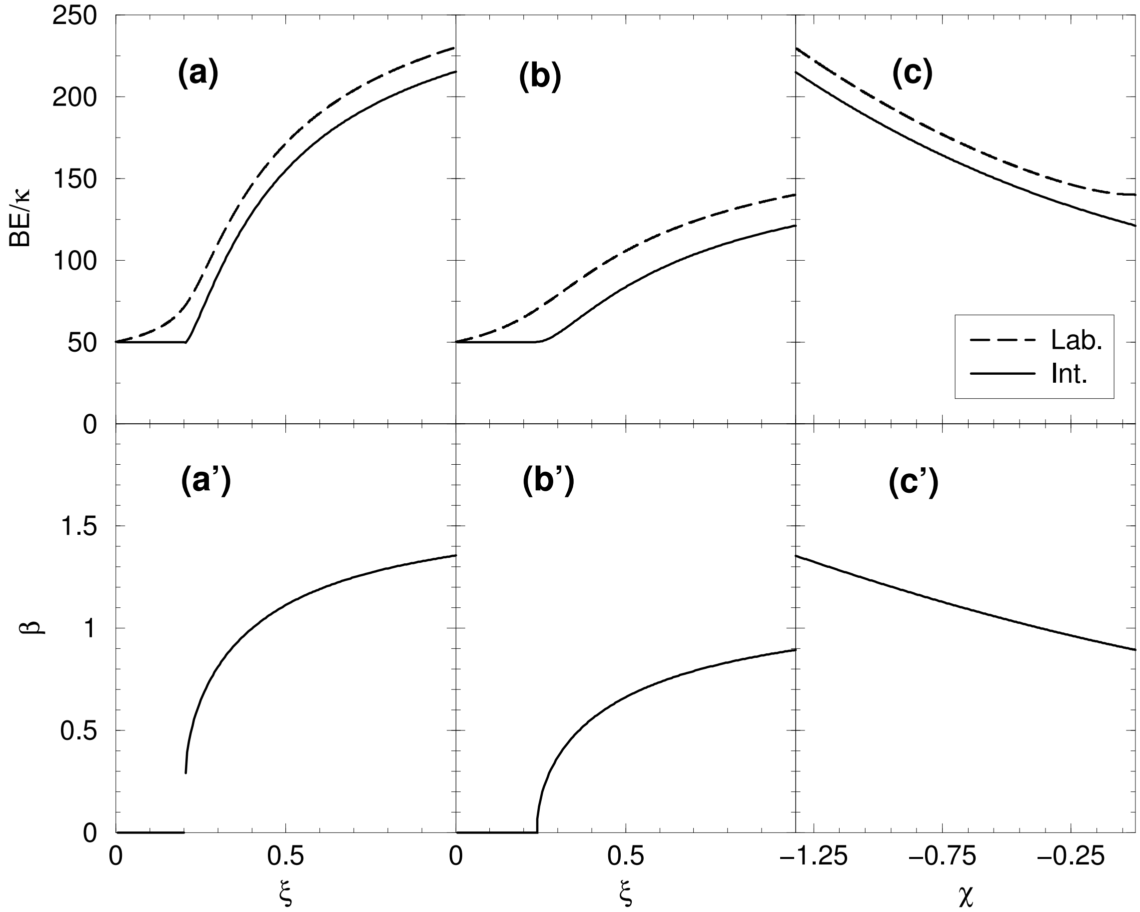
$$BE^{gl}(N) = E_0 + \mathcal{A}N + \frac{\mathcal{B}}{2}N(N-1).$$

$$S_{2n}^{gl}(N) = (\mathcal{A} - \mathcal{B}/2) + \mathcal{B}N.$$

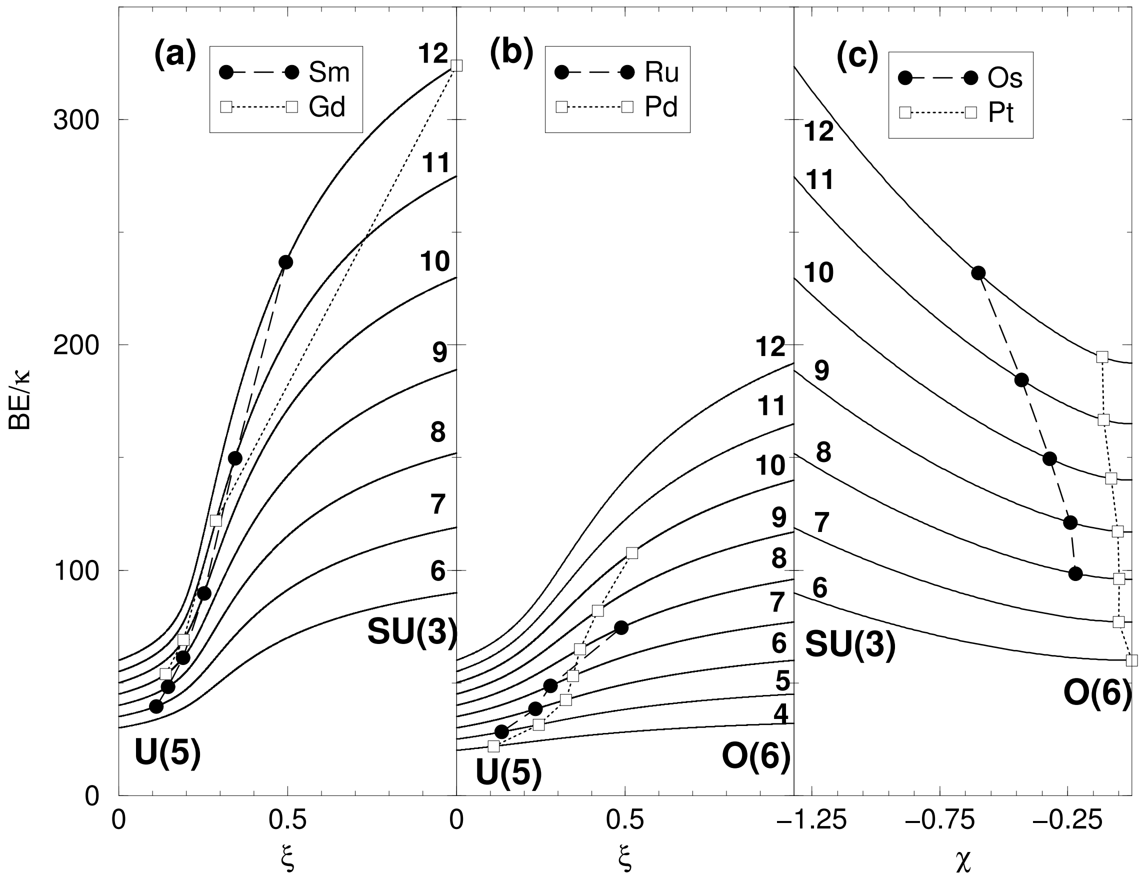
$$S_{2n} = S_{2n}^{gl} + S_{2n}^{lo}.$$

- $\mathcal{A}$ and $\mathcal{B}$ are kept as constants for a chain of isotopes at a given major shell.

Standard diagram “Binding energy vs. control parameter”



New diagram "Binding energy vs. N-control parameter"

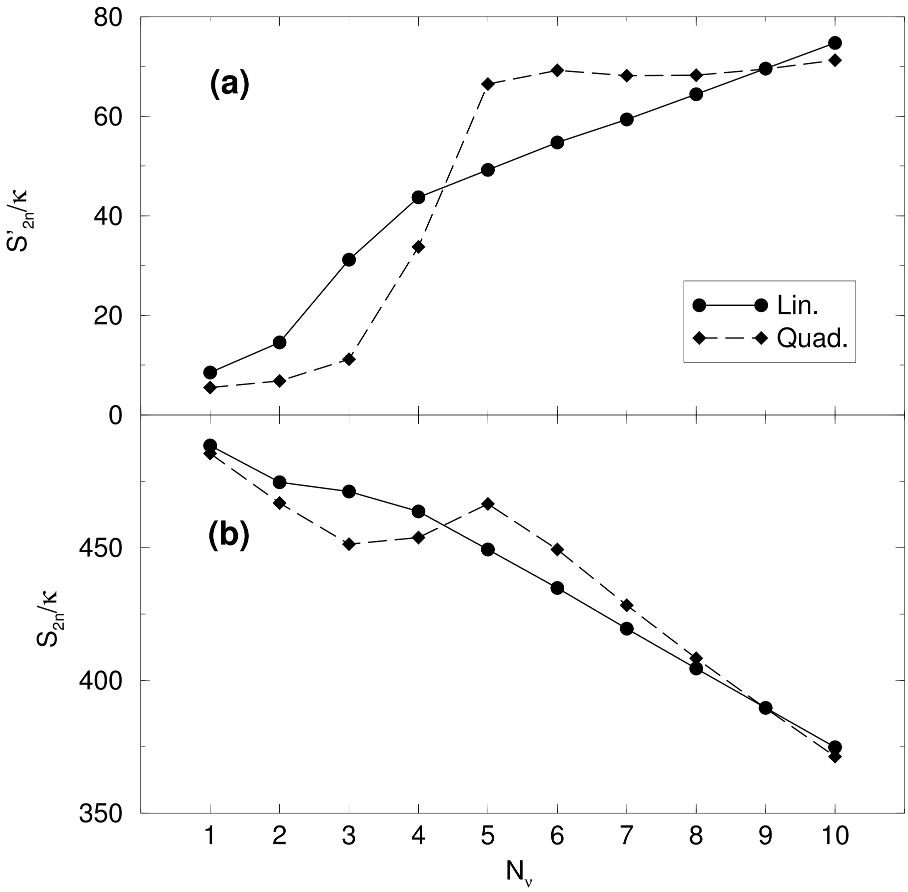


Simulation of a phase transition I

- Relation between the control parameter and the number of bosons.

$$\xi_{lin} = 0.099N_\nu + 0.01,$$
$$\xi_{qua} = 0.0099N_\nu^2 + 0.01.$$

- Linear part for S_{2n}: $S_{2n}^{lin}/\kappa = 200 - 20N_\nu$.
- $U(5) - SU(3)$.

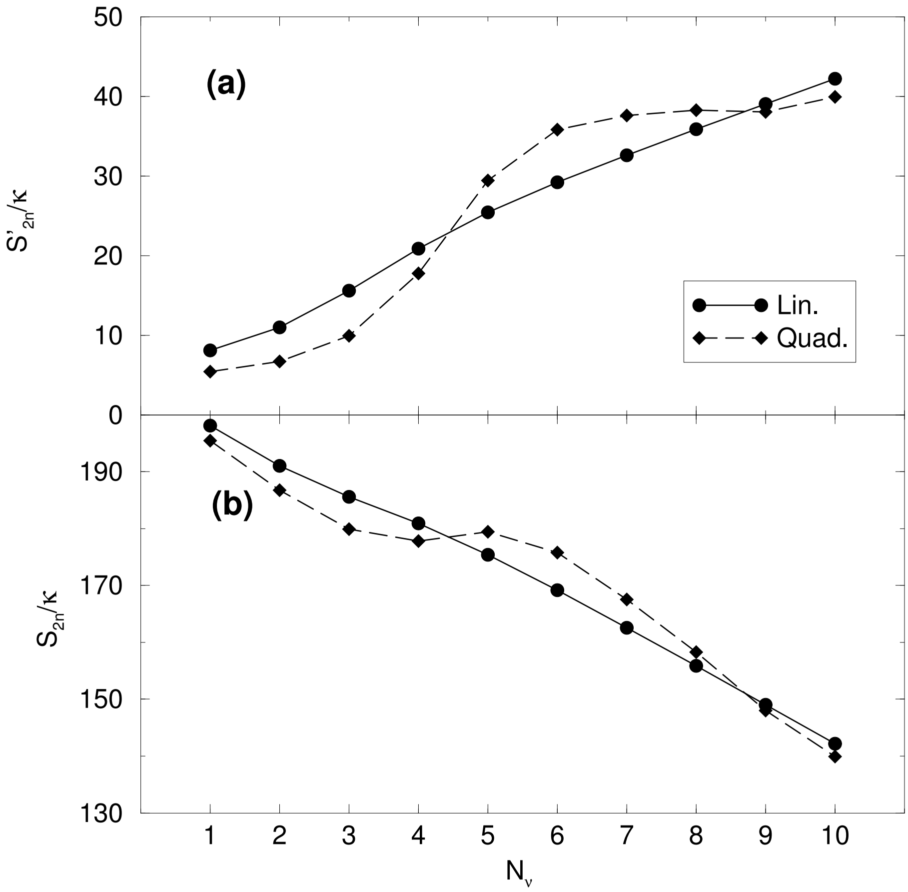


Simulation of a phase transition II

- Relation between the control parameter and the number of bosons.

$$\xi_{lin} = 0.099N_\nu + 0.01,$$
$$\xi_{qua} = 0.0099N_\nu^2 + 0.01.$$

- Linear part for S_{2n}: $S_{2n}^{lin}/\kappa = 200 - 10N_\nu$.
- $U(5) - O(6)$

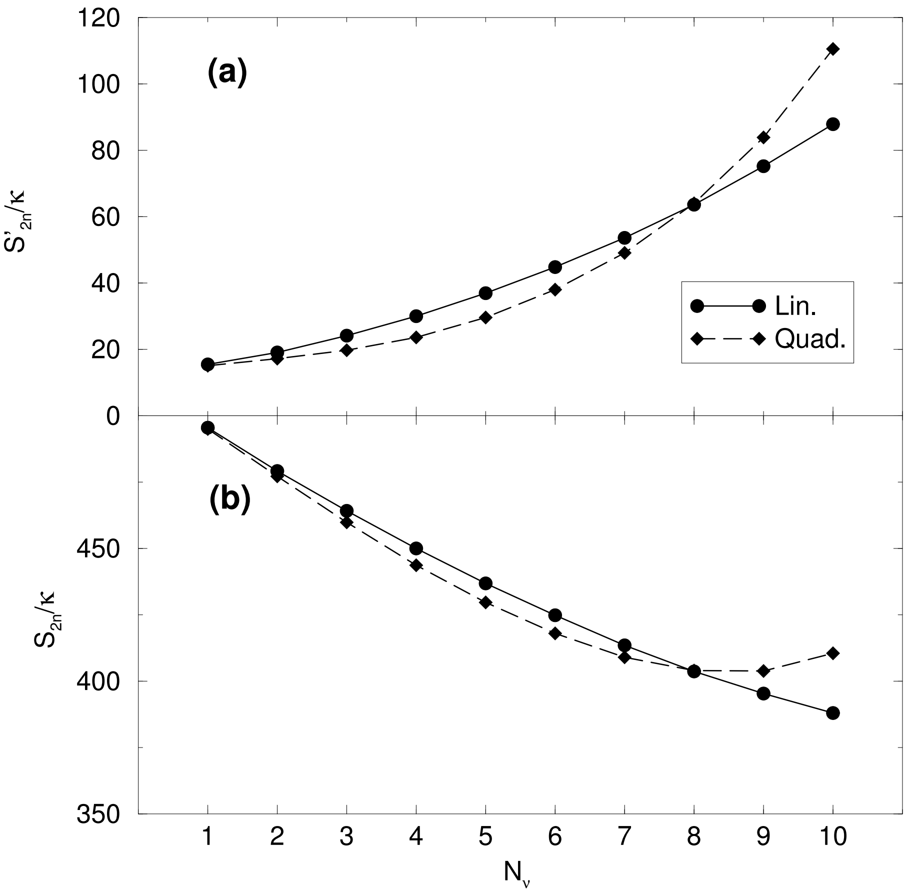


Simulation of a phase transition III

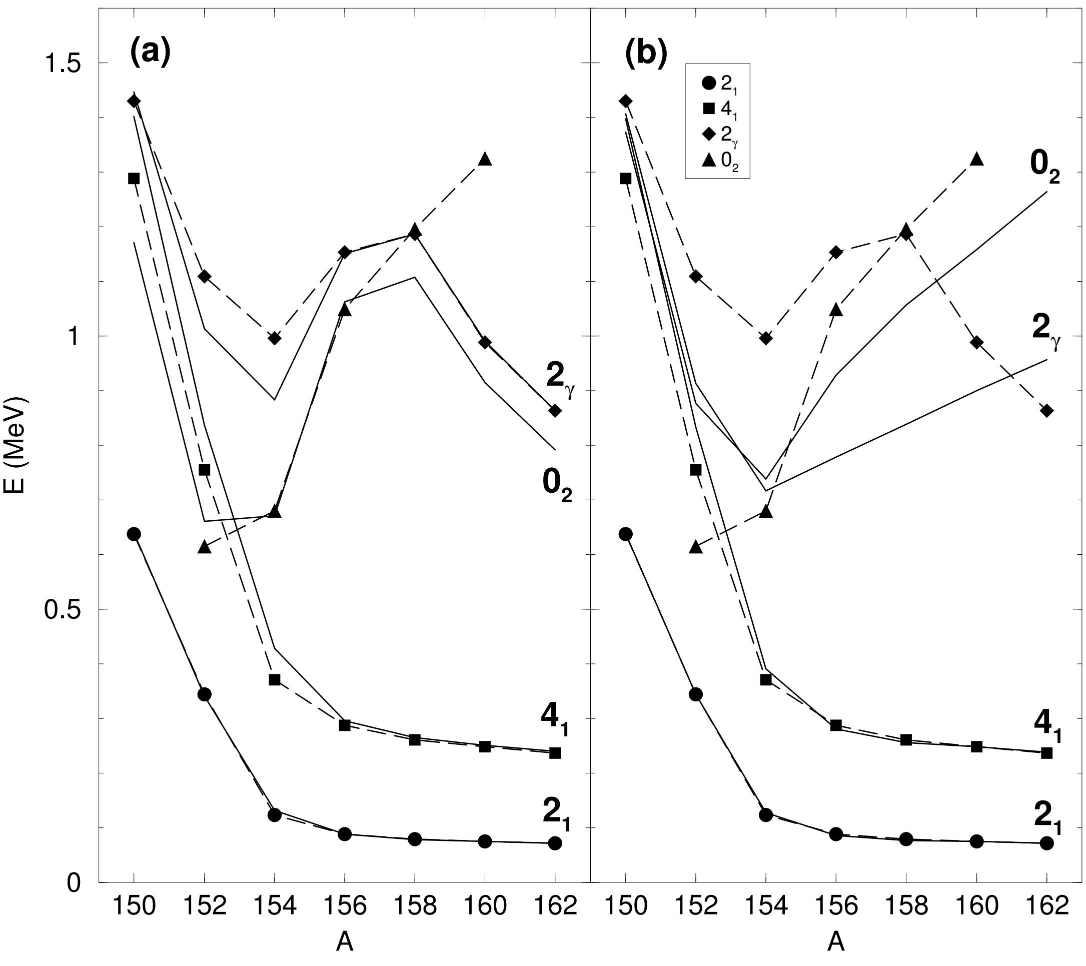
- Relation between the control parameter and the number of bosons.

$$\chi_{lin} = -\frac{\sqrt{7}}{20} N_\nu, \qquad \chi_{qua} = -\frac{\sqrt{7}}{200} N_\nu^2.$$

- Linear part for S_{2n}: $S_{2n}^{lin}/\kappa = 200 - 20N_\nu$.
- $SU(3) - O(6)$



Gd isotopes: spectra



Gd isotopes: parameters of the Hamiltonian

Theo.-a

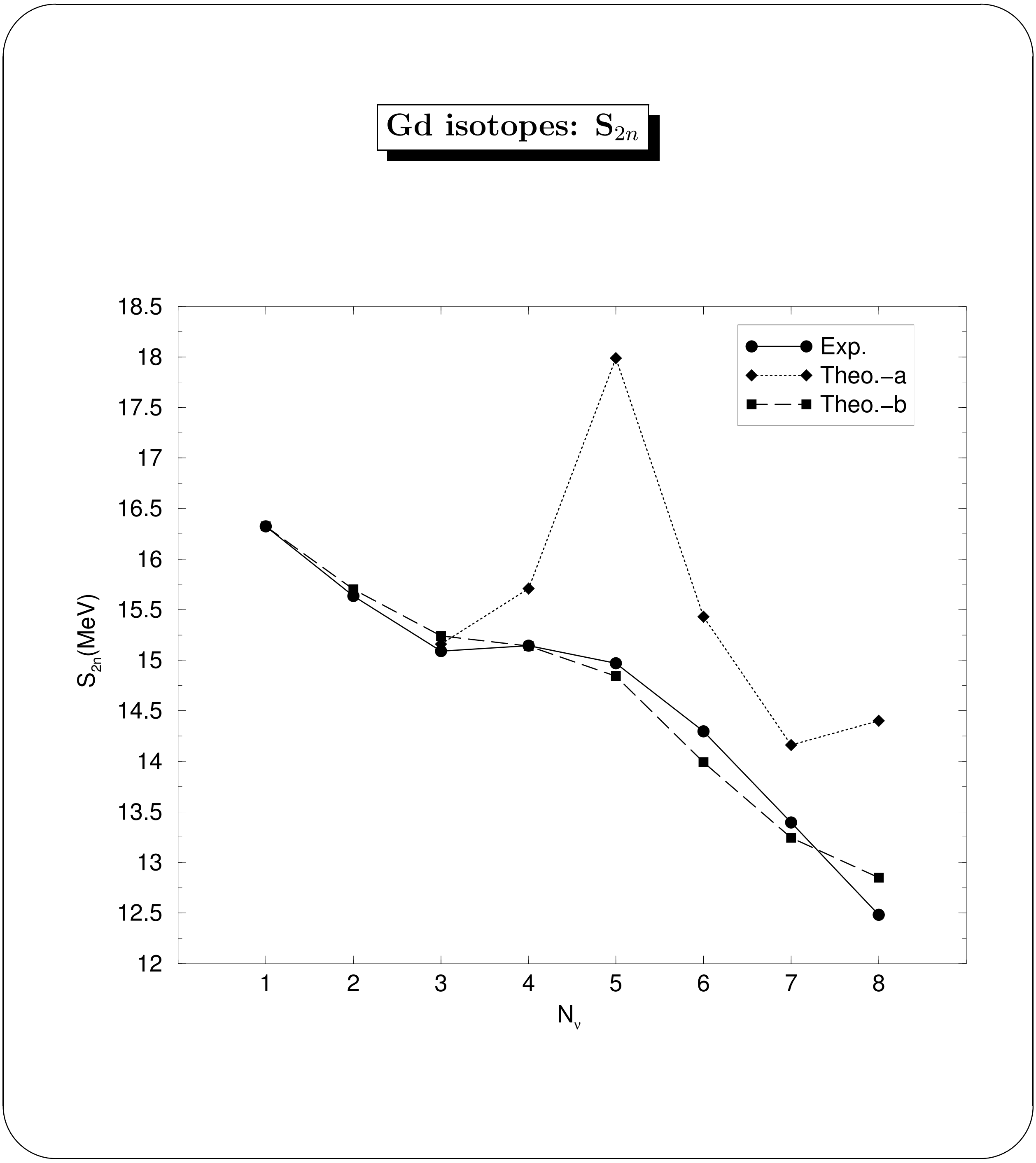
A	150	152	154	156	158	160	162
N_ν	2	3	4	5	6	7	8
κ	15.4	15.4	15.4	15.4	14.8	11.3	9.1
ξ	0.139	0.192	0.287	1	1	1	1
κ'	9.0	9.0	9.0	9.0	7.7	8.3	8.6

κ, κ' in keV and ξ dimensionless, $\chi = -\sqrt{7}/2$.

Theo-b

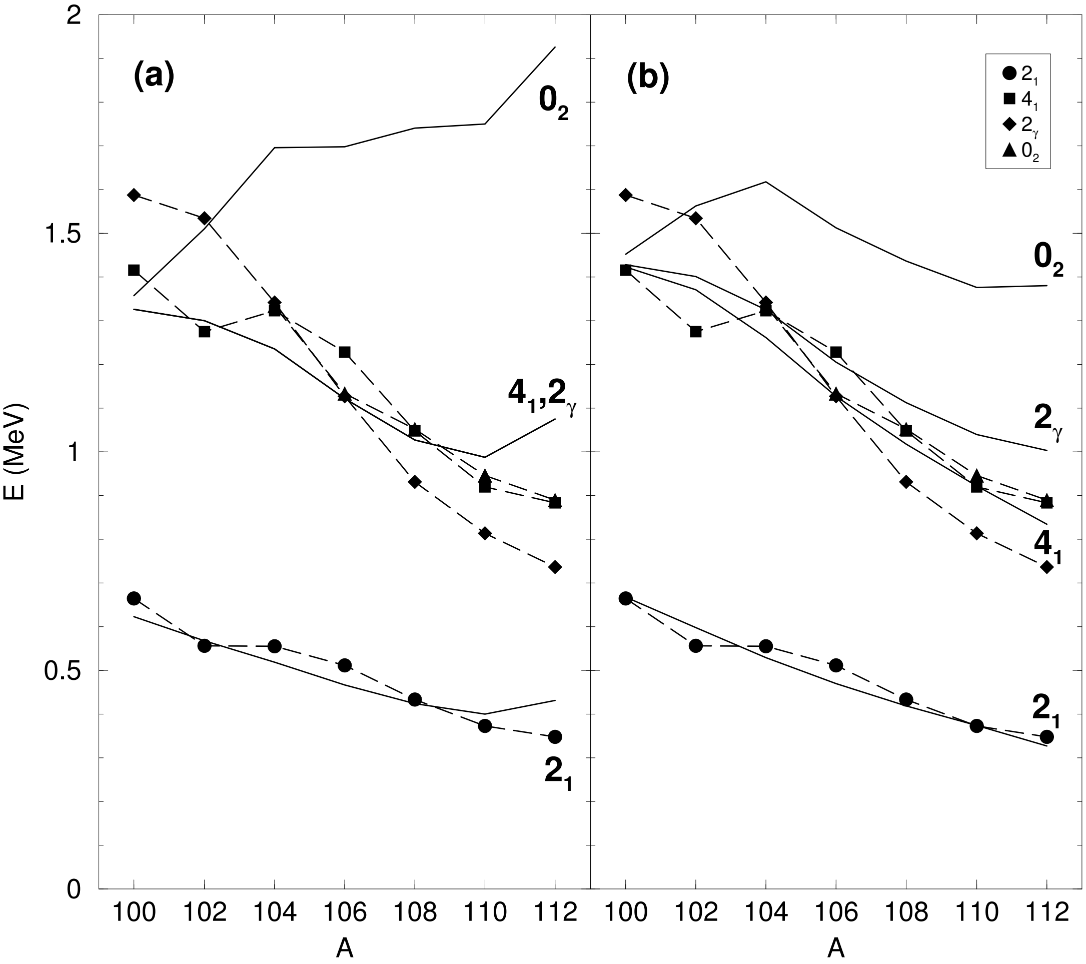
A	146	148	150	152	154	156	158	160	162
N_ν	0	1	2	3	4	5	6	7	8
ξ	0.60	0.137	0.166	0.236	0.373	0.535	0.625	0.658	0.724

$\kappa = 19.2$ keV, $\kappa' = 0$ and ξ dimensionless, $\chi = -0.6$.



Gd isotopes: S_{2n}
18.5
18
17.5
17
16.5
16
15.5
15
14.5
14
13.5
13
12.5
12
S_{2n}(MeV)
1
2
3
4
5
6
7
8
N_ν
Exp.
Theo.–a
Theo.–b

Pd isotopes: spectra



Pd isotopes: parameters of the Hamiltonian

Theo.-a

A	100	102	104	106	108	110	112
N_ν	2	3	4	5	6	7	8
κ	20.0	42.0	52.0	49.0	47.0	52.0	70.0
ξ	0.110	0.244	0.324	0.345	0.366	0.419	0.52

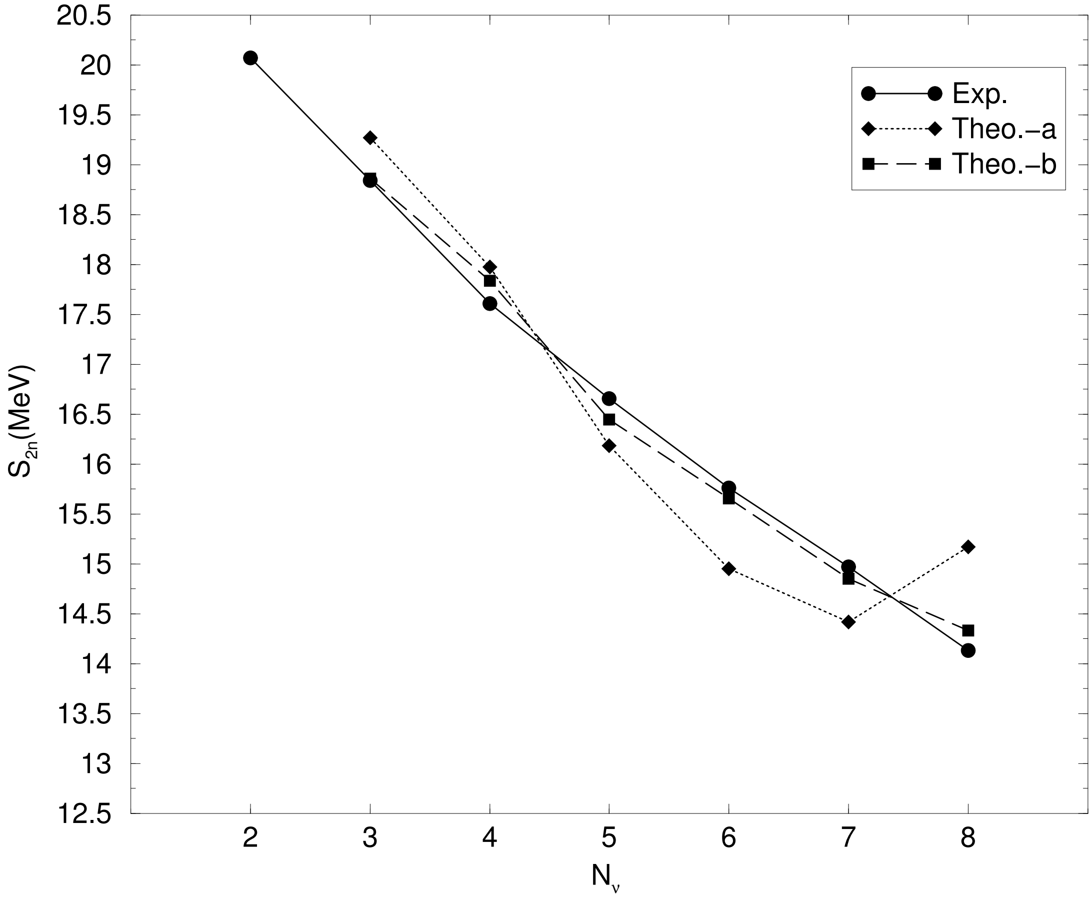
κ in keV, $\kappa' = 0$ and ξ dimensionless, $\chi = 0$.

Theo-b

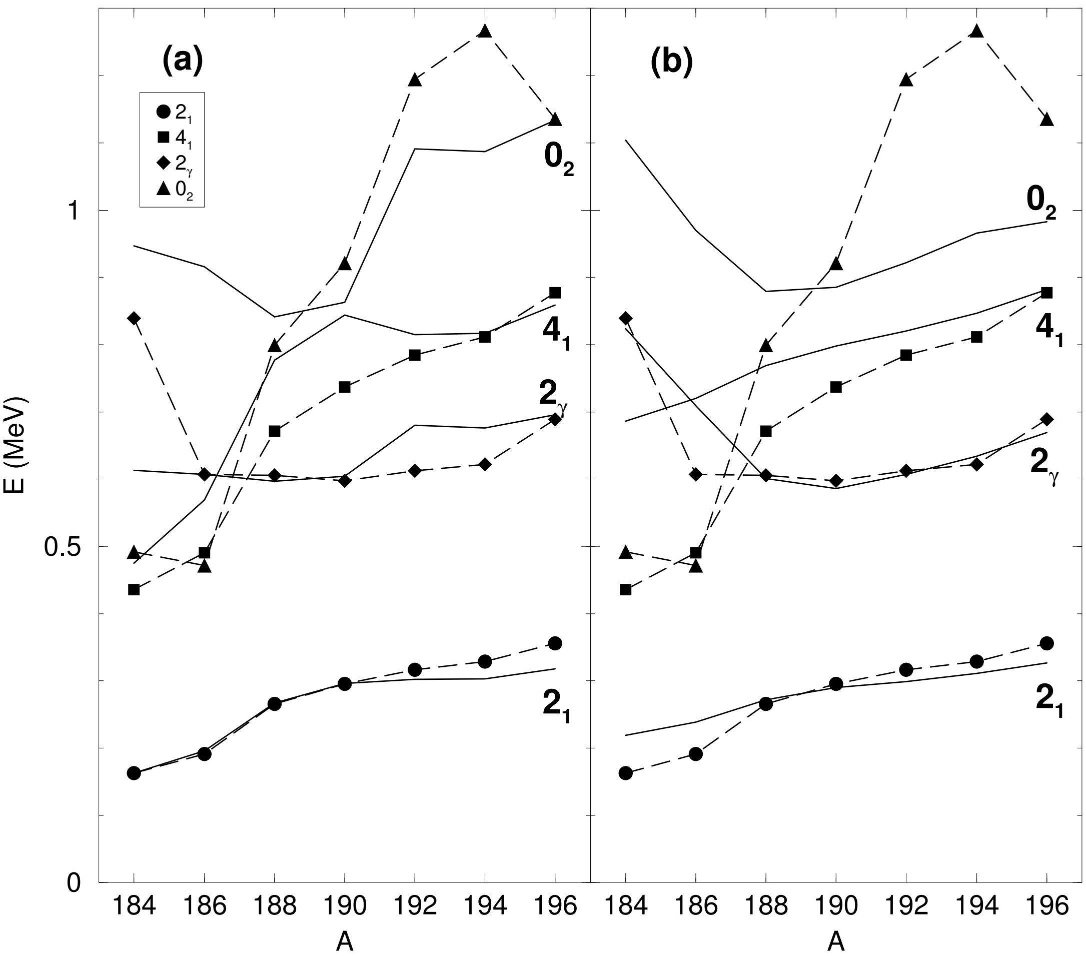
A	100	102	104	106	108	110	112	114
N_ν	2	3	4	5	6	7	8	9
κ	22.0	44.0	50.0	44.0	40.0	37.0	37.0	33.0
ξ	0.112	0.239	0.300	0.306	0.314	0.322	0.346	0.342

κ in keV, $\kappa' = 0$ and ξ dimensionless, $\chi = -0.3$.

Pd isotopes: S_{2n}



Pt isotopes: spectra



Pt isotopes: parameters of the Hamiltonian

Theo.-a

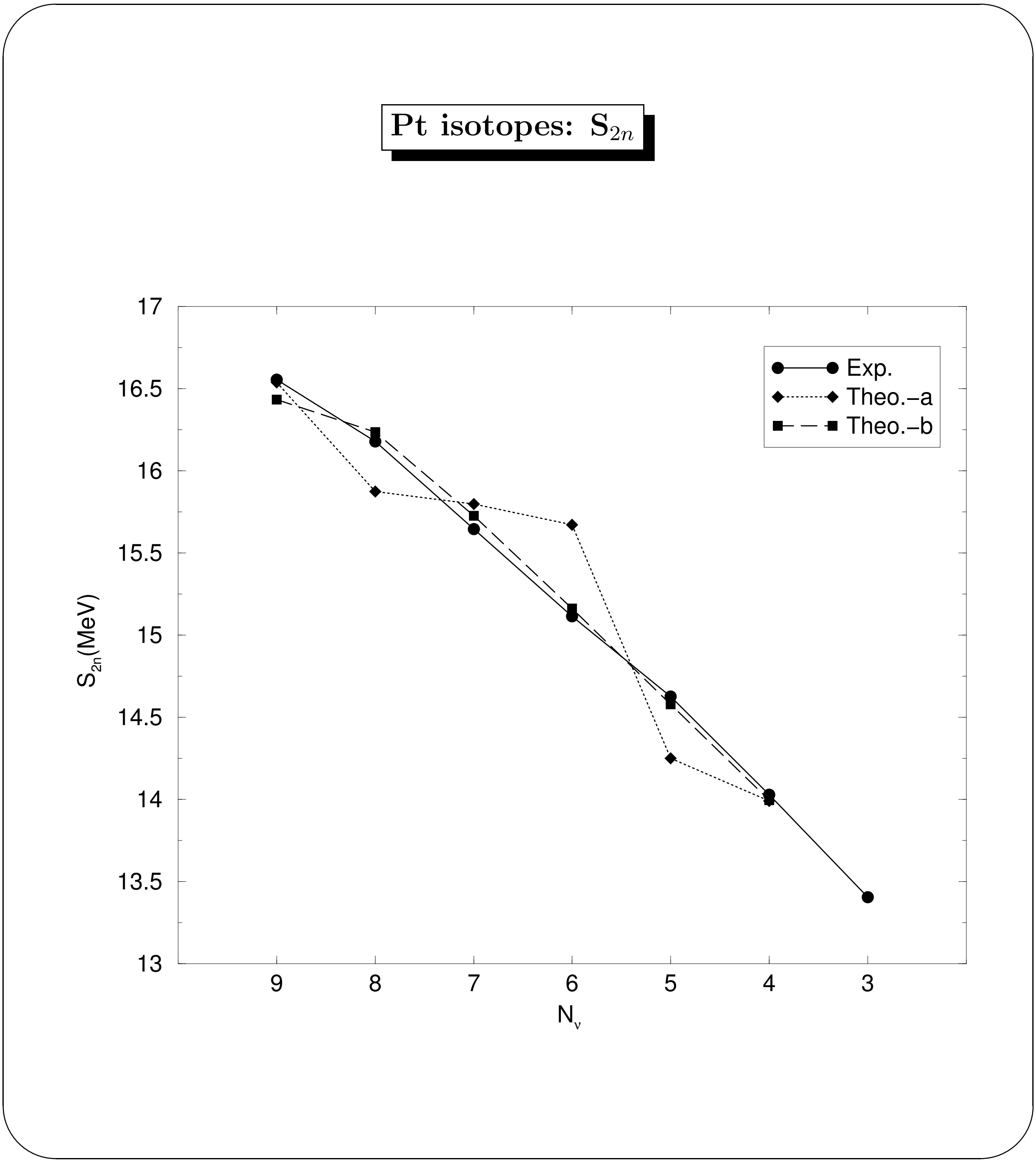
A	184	186	188	190	192	194	196
N_ν	10	9	8	7	6	5	4
κ	43.0	44.0	44.0	47.0	60.0	60.0	63.0
χ	-0.115	-0.110	-0.080	-0.055	-0.049	-0.050	0
κ'	4.2	0.8	17.6	19.0	11.0	11.0	11.0

κ and κ' in keV and ξ dimensionless, $\varepsilon_d = 0$.

Theo-b

A	184	186	188	190	192	194	196
N_ν	10	9	8	7	6	5	4
κ	33.5	33.5	33.5	33.5	33.5	33.5	33.5
χ	-0.25	-0.20	-0.10	0	0	0	0
κ'	15.2	15.2	15.2	15.2	15.2	15.2	15.2

κ and κ' in keV and ξ dimensionless, $\varepsilon_d = 25.7$ keV.


Pt isotopes: S$_{2n}$
17
16.5
16
15.5
15
14.5
14
13.5
13
9
8
7
6
5
4
3
S$_{2n}$(MeV)
N$_\nu$
Exp.
Theo.–a
Theo.–b

Conclusions

- **A new approach, that includes the connection between the control parameter and N, has been presented.**

- **The crossing of a phase transition point produces a deviation in the S_{2n} values from the overall linear background.**

- **The study of long chains of isotopes should include the analysis of ground state properties, *i.e.* binding energies.**

- **This method can be used for calculating S_{2n} values in extended areas.**