

Theoretical contributions to mass measurements

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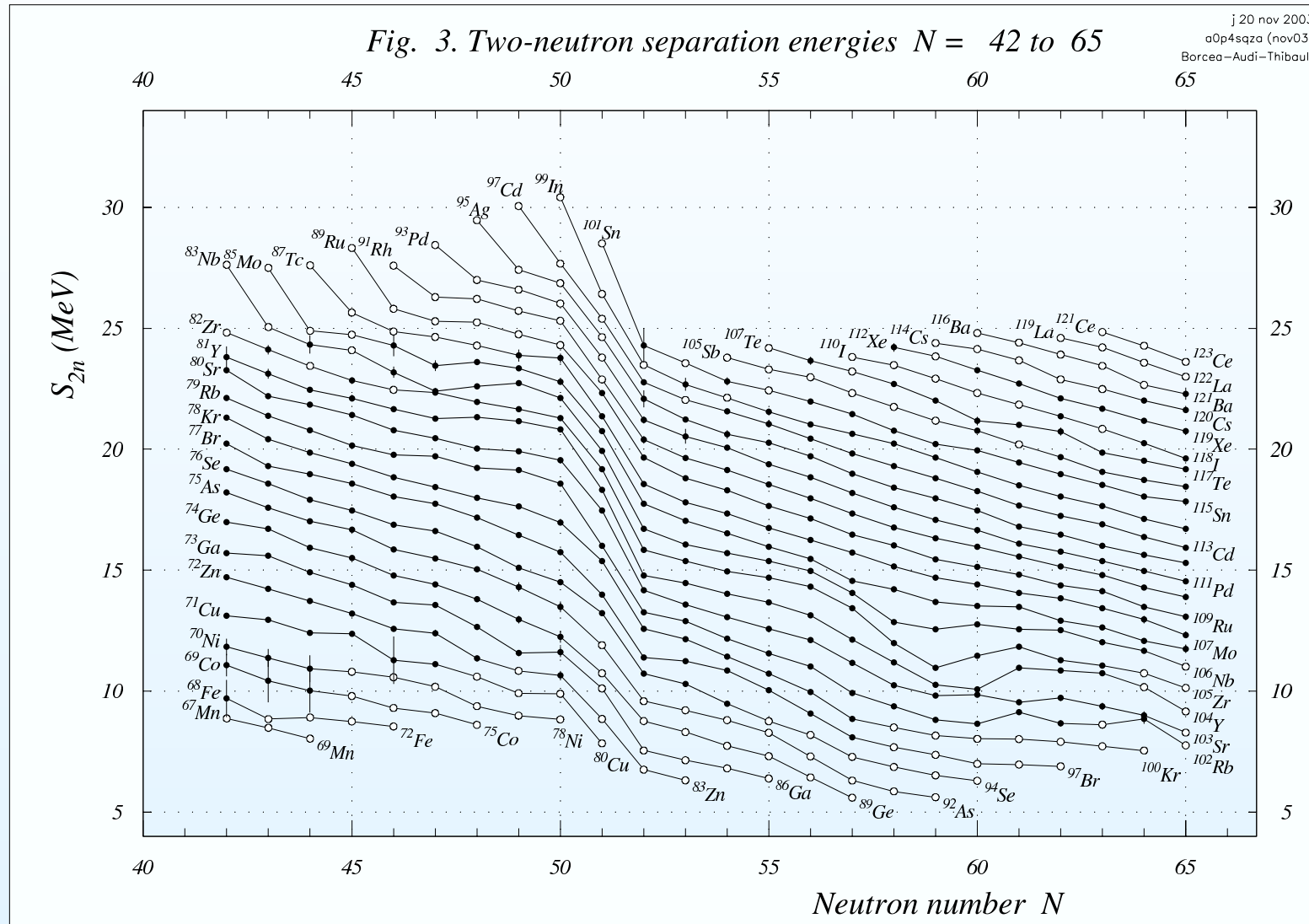
Index

- What can we learn on nuclear structure from nuclear masses?
- Models for describing nuclear masses: global versus local models.
- Simple conclusions from the Liquid Drop Model and from the Nuclear Shell Model.
- The Interacting Boson Model (IBM).
 - Introduction to the IBM.
 - How to calculate binding energies: global and local part.
 - Some examples.
 - Limitations of the model.
 - Extentions: inclusion of shape coexistence.
- Some relevant references.
- Conclusions.

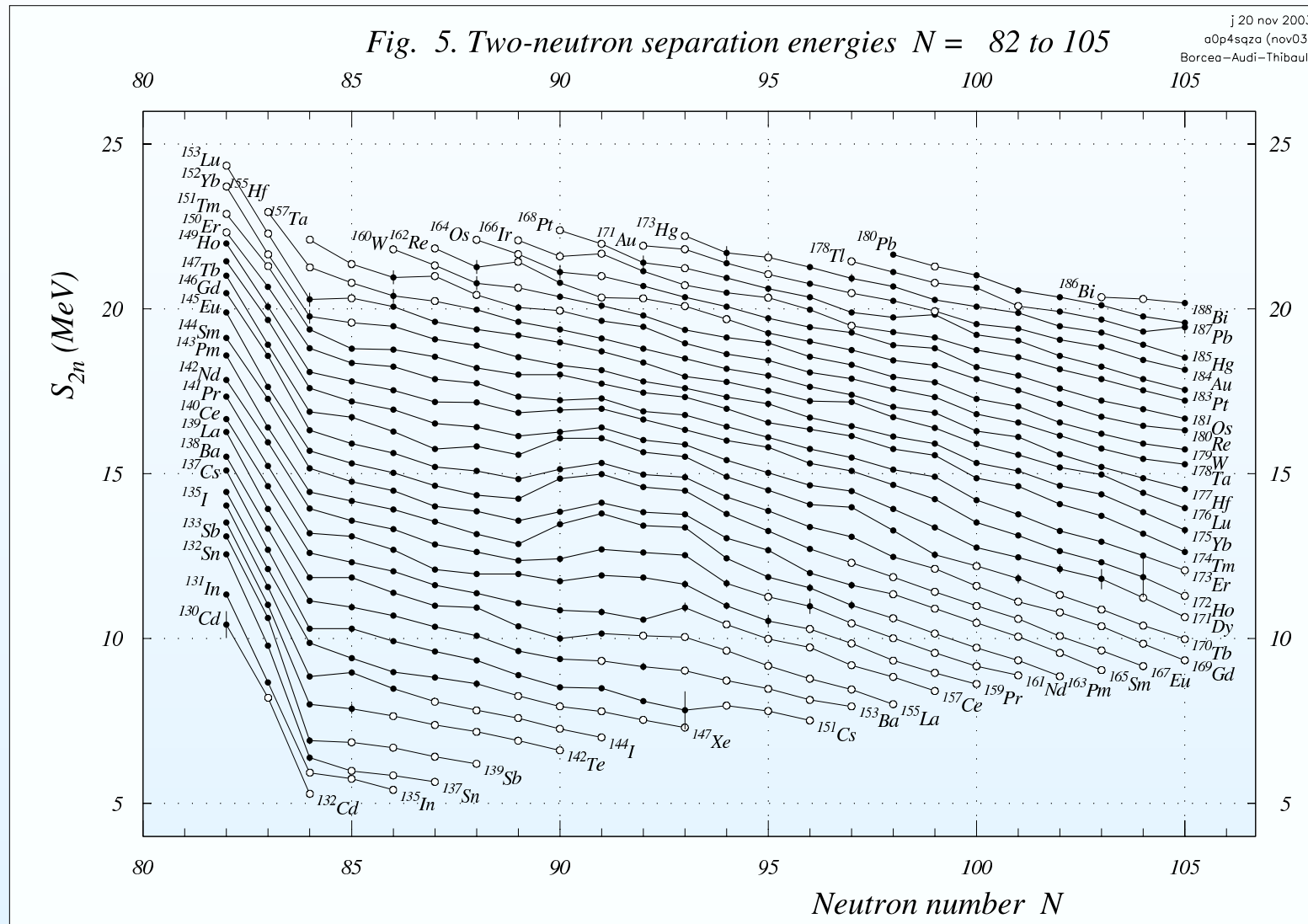
What can we learn from nuclear masses?

1. Existence of nuclear shells.
2. Regions of deformation.
3. Mixing of the ground state with intruder states.
4. New magic numbers.
5. Pairing with $T=0$.

Experimental two neutron separation energies (I)



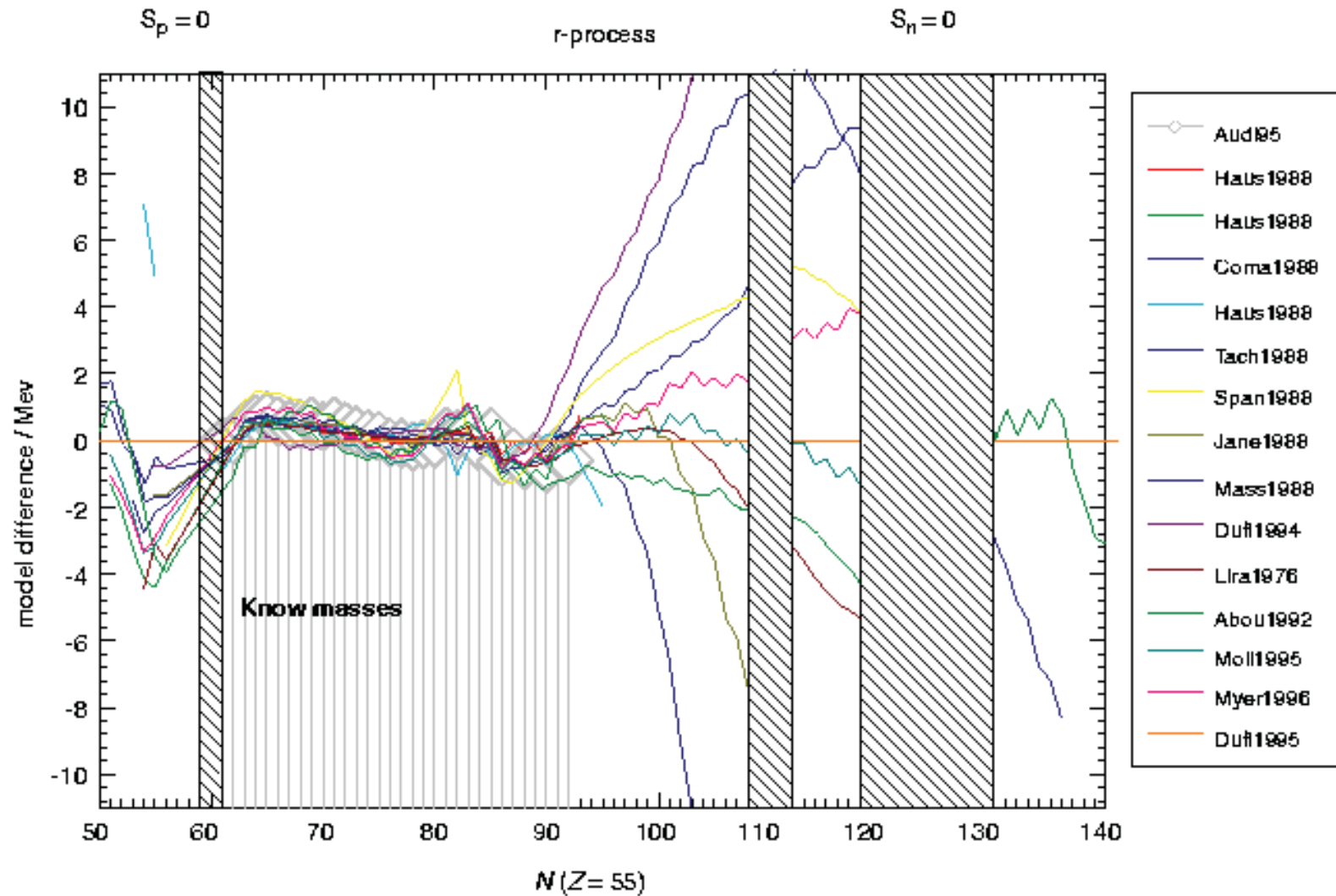
Experimental two neutron separation energies (II)



Global models for describing nuclear masses.

- Semiempirical models
 - The Liquid Drop Model and the Weizsäcker semiempirical mass formula.
 - Shell Model using monopolar term (Duflo-Zuker mass formula).
- Microscopic models
 - Self-consistent models: Hartree-Fock-Bogoliubov plus symmetry restoration, generator coordinate method.
 - Shell model (only for light nuclei or near to close shell nuclei).
- Microscopic-macroscopic models
 - Finite range droplet model (FRDM).
 - Extended Thomas-Fermi plus Strutinsky integral (ETFSI).
 - TF-FRDM.

Extrapolation of the models



The linear behavior of the S_{2n} . LDM (I)

- Definition:

$$S_{2n}(A, Z) = BE(A, Z) - BE(A - 2, Z).$$

- Binding energy:

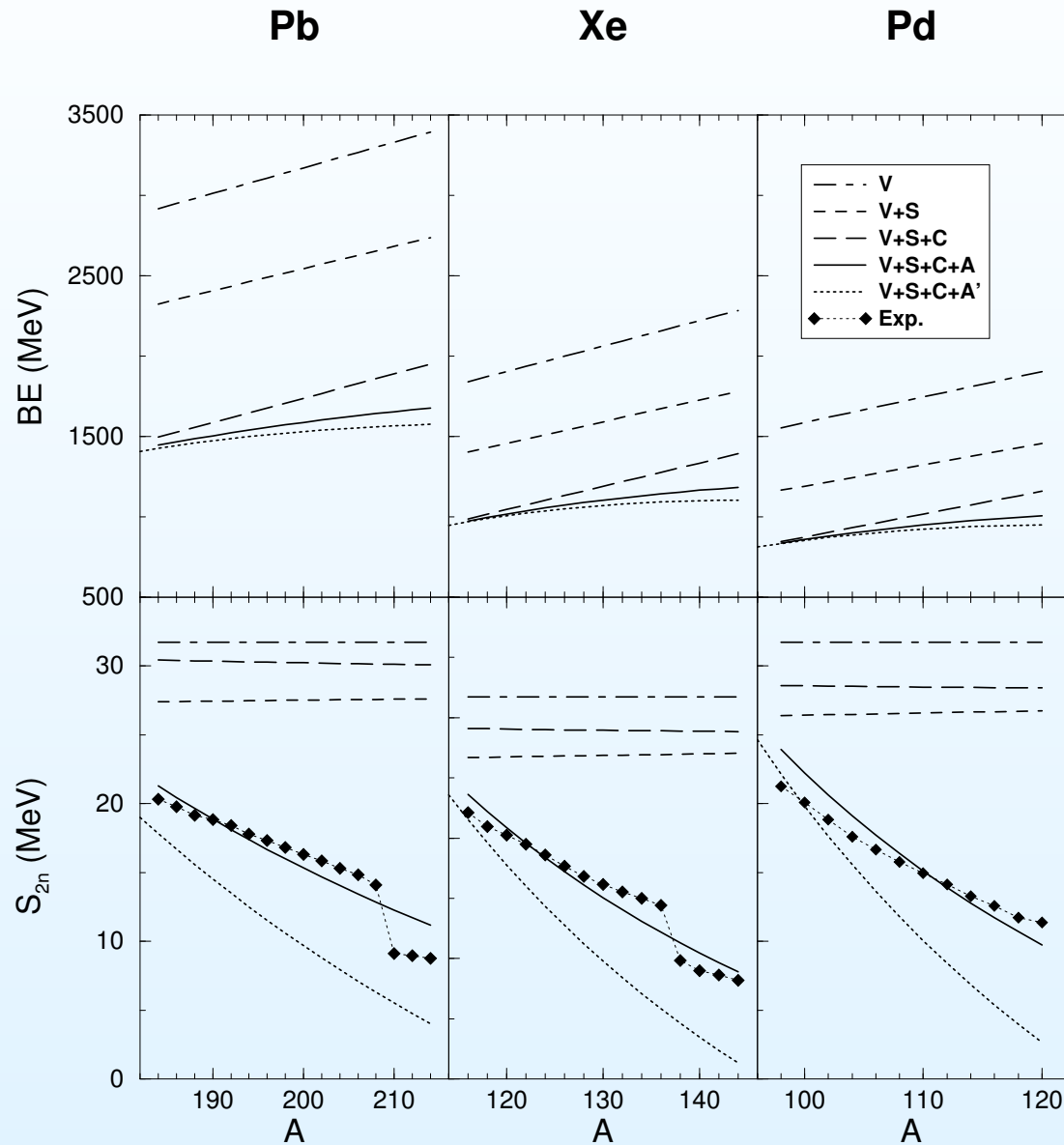
$$BE(A, Z) = a_V A - a_S A^{\frac{2}{3}} - a_C Z(Z - 1)A^{-\frac{1}{3}} - a_A (A - 2Z)^2 A^{-1}.$$

with: $a_V = 15.85$ MeV, $a_C = 0.71$ MeV, $a_S = 18.34$ MeV and $a_A = 23.22$ MeV (A.H. Wapstra 1958).

- S_{2n} within the LDM:

$$S_{2n} \approx 2(a_V - a_A) - \frac{4}{3}a_S A^{-\frac{1}{3}} + \frac{2}{3}a_C Z(Z - 1)A^{-\frac{4}{3}} + 8a_A \frac{Z^2}{A(A - 2)}$$

The linear behavior of the S_{2n} . LDM (II)



The linear behavior of the S_{2n} . Shell Model (I)

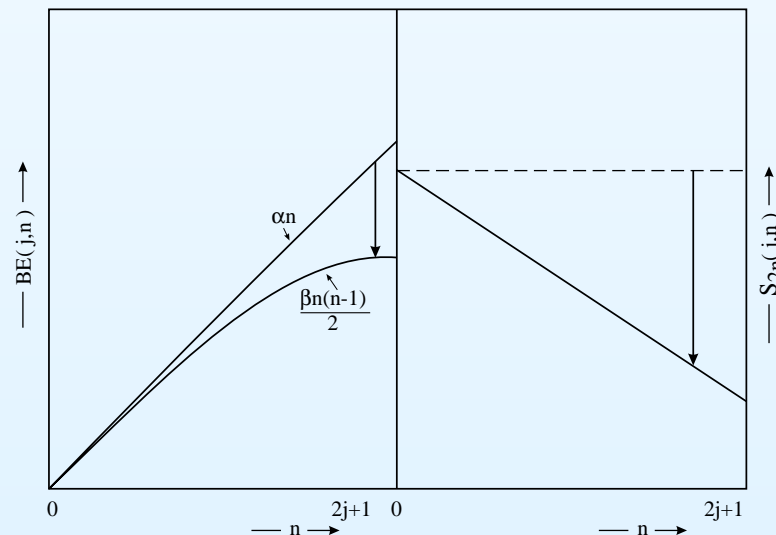
- For a zero-range interaction, in a single-j shell, using the seniority as a good quantum number:

$$BE(j, n) = n\varepsilon_j + \frac{n}{2}V_0,$$

where $V_0 = \langle j^2, v = 0, J = 0 | V | j^2, v = 0, J = 0 \rangle$.

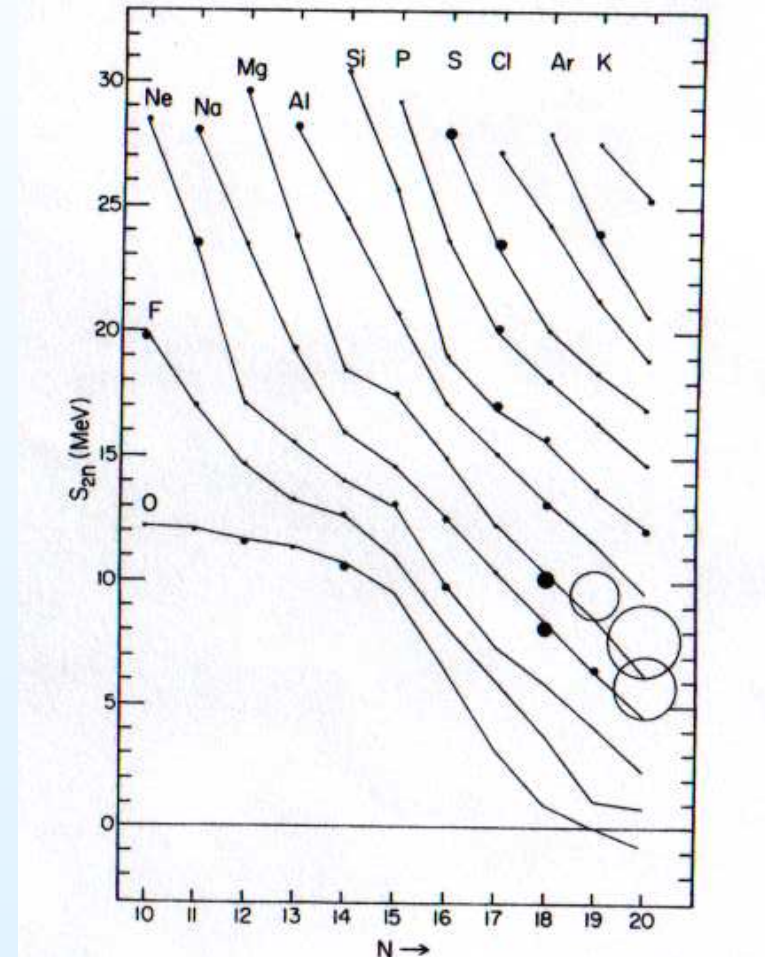
- For a more general interaction, including pairing:

$$BE(j, n) = C + \alpha n + \beta \frac{n(n-1)}{2} + \left[\frac{n}{2}\right]P, \quad \beta < 0, |\beta| \ll \alpha$$



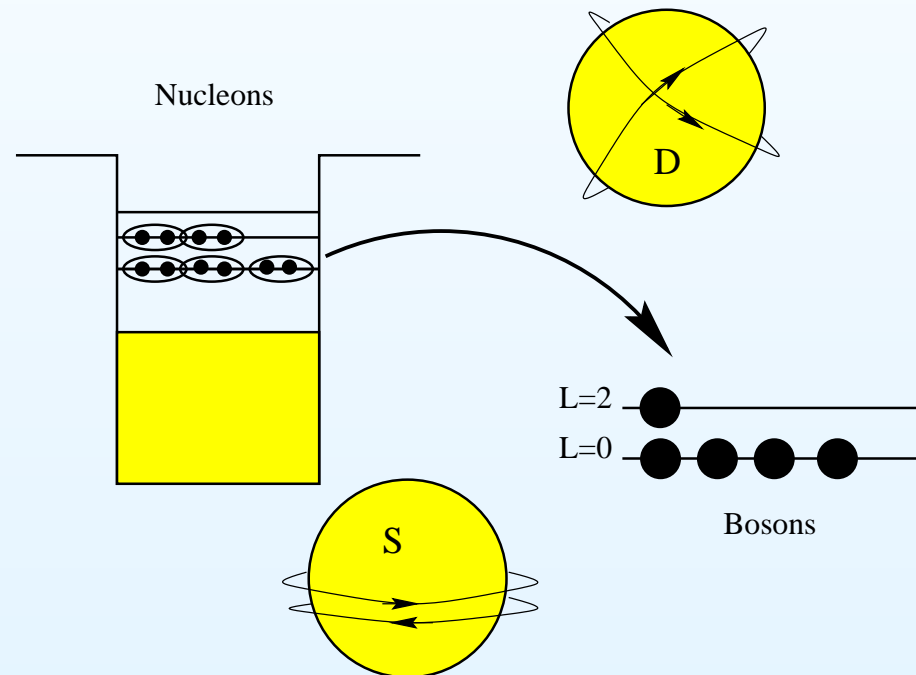
The linear behavior of the S_{2n} . Shell Model (II)

- For obtaining a good shell-model description of the binding energies it is needed a good description of the evolution of the single-particle energies.
- Also it is needed to use the monopole term in order to correctly reproduce saturation properties.
- Example for the *sd* shell (Wildenthal 1984).
The size of the circles is proportional to the deviation between theory and experimental data.
Global rms=185 keV.



The Interacting Boson Model (IBM)

- The IBM is a model which describes the low lying collective states of medium mass and heavy nuclei.
- It can be considered as an approximation to the Shell Model. Two steps are necessary: truncation of the Shell Model space and bosonization of the nucleon pairs.



- The IBM can also be considered as the second quantization of the shape variables of the Geometric Collective Model.

Algebraic structure

- Creation and annihilation operators

$$\begin{array}{l} s^\dagger, d_m^\dagger (m = 0, \pm 1, \pm 2) \\ s, d_m (m = 0, \pm 1, \pm 2) \end{array} \longrightarrow \begin{array}{l} \gamma_{lm}^\dagger, \gamma_{lm} \\ (l = 0, 2; -l \leq m \leq l) \end{array}$$

$$[\gamma_{lm}, \gamma_{l'm'}^\dagger] = \delta_{ll'} \delta_{mm'}, [\gamma_{lm}^\dagger, \gamma_{l'm'}^\dagger] = 0, [\gamma_{lm}, \gamma_{l'm'}] = 0$$

- $U(6)$ as the dynamical algebra of the model

Generators of $U(6)$: $\hat{G}_{ij} = \gamma_i^\dagger \gamma_j$, con $i, j = 1, \dots, 6$.

$$[\hat{G}_{ij}, \hat{G}_{kl}] = \hat{G}_{il} \delta_{jk} - \hat{G}_{jk} \delta_{il}$$

- Every dynamic operator can be written in terms of $U(6)$ generators.

$$\hat{H} = \sum_{ij} \epsilon_{ij} \gamma_i^\dagger \gamma_j + \sum_{ijkl} V_{ijkl} \gamma_i^\dagger \gamma_j^\dagger \gamma_k \gamma_l$$

Algebraic structure

- Chain of subgroups.

$$\begin{array}{ccccccc}
 & U(6) & \supset & U(5) & \supset & O(5) & \supset & O(3) \\
 U(5) & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 & | [N] & & n_d & & \tau & \nu_\Delta & L \rangle
 \end{array}$$

$$\begin{array}{ccccccc}
 & U(6) & \supset & SU(3) & \supset & O(3) & & \\
 SU(3) & \downarrow & & \downarrow & & \downarrow & & \\
 & | [N] & & (\lambda, \mu) & & \kappa & & L \rangle
 \end{array}$$

$$\begin{array}{ccccccc}
 & U(6) & \supset & O(6) & \supset & O(5) & \supset & O(3) \\
 O(6) & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 & | [N] & & \sigma & & \tau & \nu_\Delta & L \rangle
 \end{array}$$

$U(5)$ spectrum

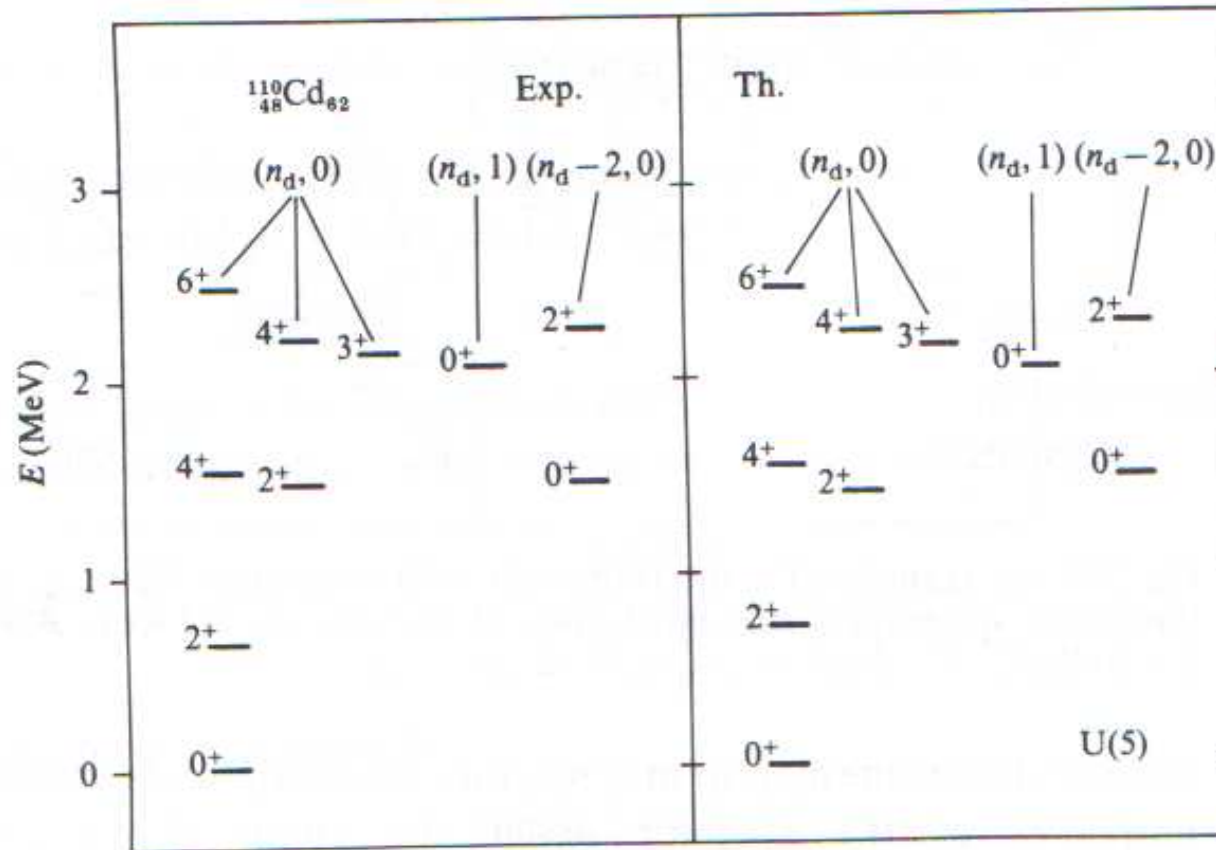


Fig. 2.5. An example of a spectrum with $U(5)$ symmetry: $^{110}_{48}\text{Cd}_{62}$, $N=7$. The theoretical spectrum is calculated using (2.79) and (2.82) with $\varepsilon' = 722$ KeV, $c_0 = 29$ KeV, $c_2 = -42$ KeV, $c_4 = 98$ KeV.

$SU(3)$ spectrum

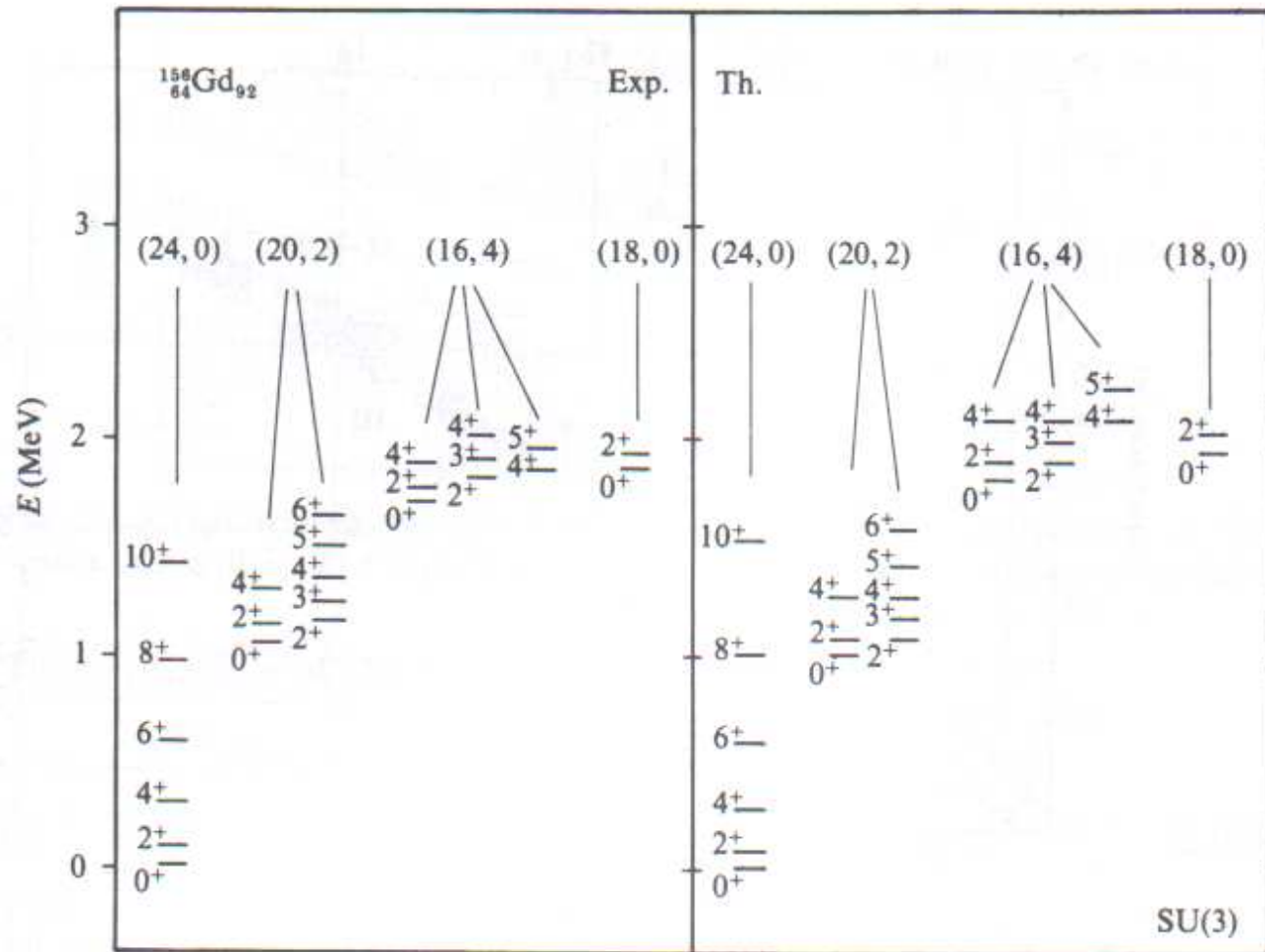


Fig. 2.6. An example of a spectrum with $SU(3)$ symmetry: $^{156}_{64}\text{Gd}_{92}$, $N = 12$. The theoretical spectrum is calculated using (2.84) with $\gamma = 3.8$ KeV, $\delta = -20.1$ KeV.

O(6) spectrum

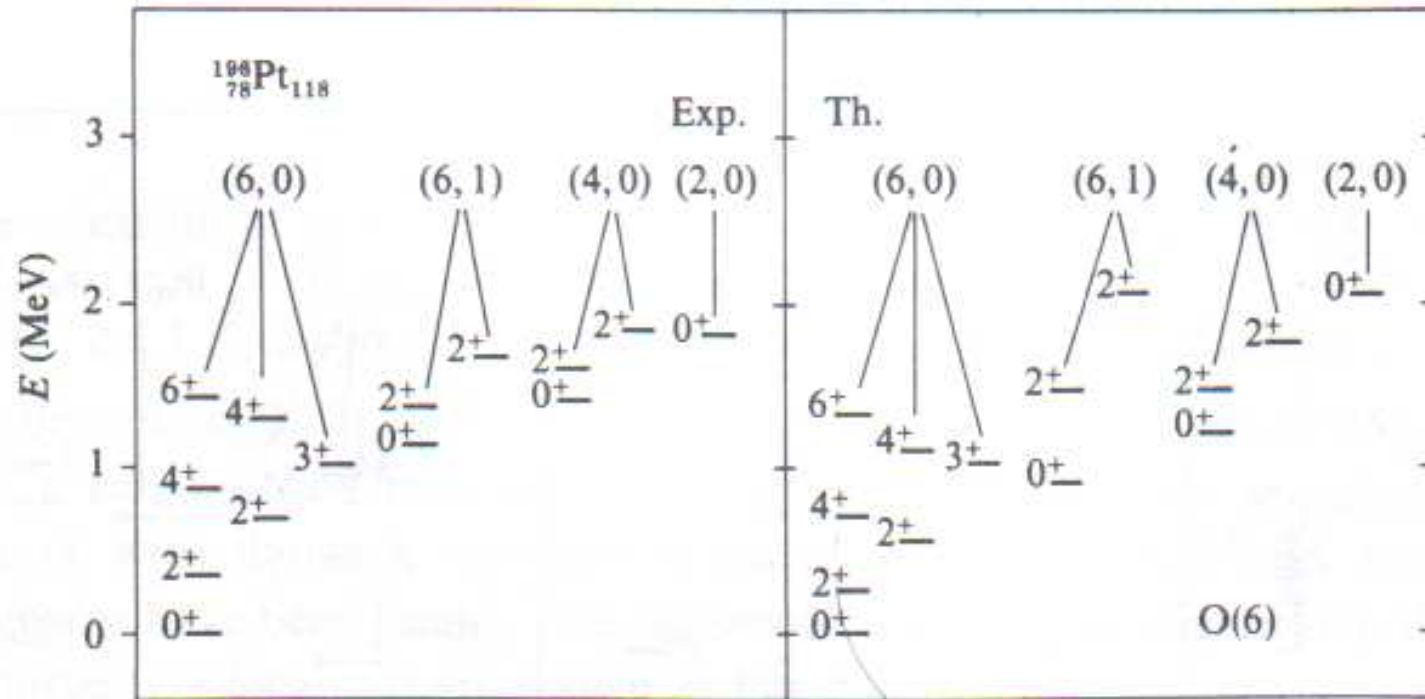


Fig. 2.7. An example of a spectrum with O(6) symmetry: $^{196}\text{Pt}_{118}$, $N=6$. The theoretical spectrum is calculated using (2.92) with $A = 171$ KeV, $B/6 = 50$ KeV, $C = 10$ KeV.

General (“Local”) Hamiltonian

- General

$$\begin{aligned}\hat{H} &= \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d + \kappa_0 \hat{P}^\dagger \hat{P} + \kappa_1 \hat{L} \cdot \hat{L} \\ &+ \kappa_2 \hat{Q} \cdot \hat{Q} + \kappa_3 \hat{T}_3 \cdot \hat{T}_3 + \kappa_4 \hat{T}_4 \cdot \hat{T}_4,\end{aligned}$$

where \hat{n}_s and \hat{n}_d are the s and d boson number operators, respectively, and

$$\begin{aligned}\hat{P}^\dagger &= \frac{1}{2} d^\dagger \cdot d^\dagger - \frac{1}{2} s^\dagger \cdot s^\dagger, \\ \hat{L} &= \sqrt{10} (d^\dagger \times \tilde{d})^{(1)}, \\ \hat{Q} &= s^\dagger \tilde{d} + d^\dagger \tilde{s} + \chi (d^\dagger \times \tilde{d})^{(2)}, \\ \hat{T}_3 &= (d^\dagger \times \tilde{d})^{(3)}, \\ \hat{T}_4 &= (d^\dagger \times \tilde{d})^{(4)}.\end{aligned}$$

Schematic (“Local”) Hamiltonian

- Hamiltonian

$$\hat{H} = \varepsilon \hat{n}_d - \kappa \hat{Q} \cdot \hat{Q} + \kappa' \hat{L} \cdot \hat{L}$$

- The same \hat{Q} operator appearing in the Hamiltonian is used in the E2 transition operator.

The “global” part of the IBM Hamiltonian

- It is related with the Casimir operators of the $U(6)$ group.

$$\hat{H}^{gl} = -E_0 - \mathcal{A} \hat{N} - \frac{\mathcal{B}}{2} \hat{N}(\hat{N} - 1).$$

$$BE^{gl}(N) = E_0 + \mathcal{A} N + \frac{\mathcal{B}}{2} N(N - 1).$$

$$BE(N) = BE^{lo}(N) + BE^{gl}(N).$$

$$S_{2n}^{gl}(N) = (\mathcal{A} - \mathcal{B} / 2) + \mathcal{B} N. \quad S_{2n}(N) = BE(N) - BE(N - 1).$$

- **Why are the coefficients \mathcal{A} and \mathcal{B} constants?**
 - Analogy with the Liquid drop model.
 - Shell model argument (SDI and MSDI).
 - Good description of long chains of isotopes with IBM using this *ansatz*.

Deriving the “local” part of the IBM

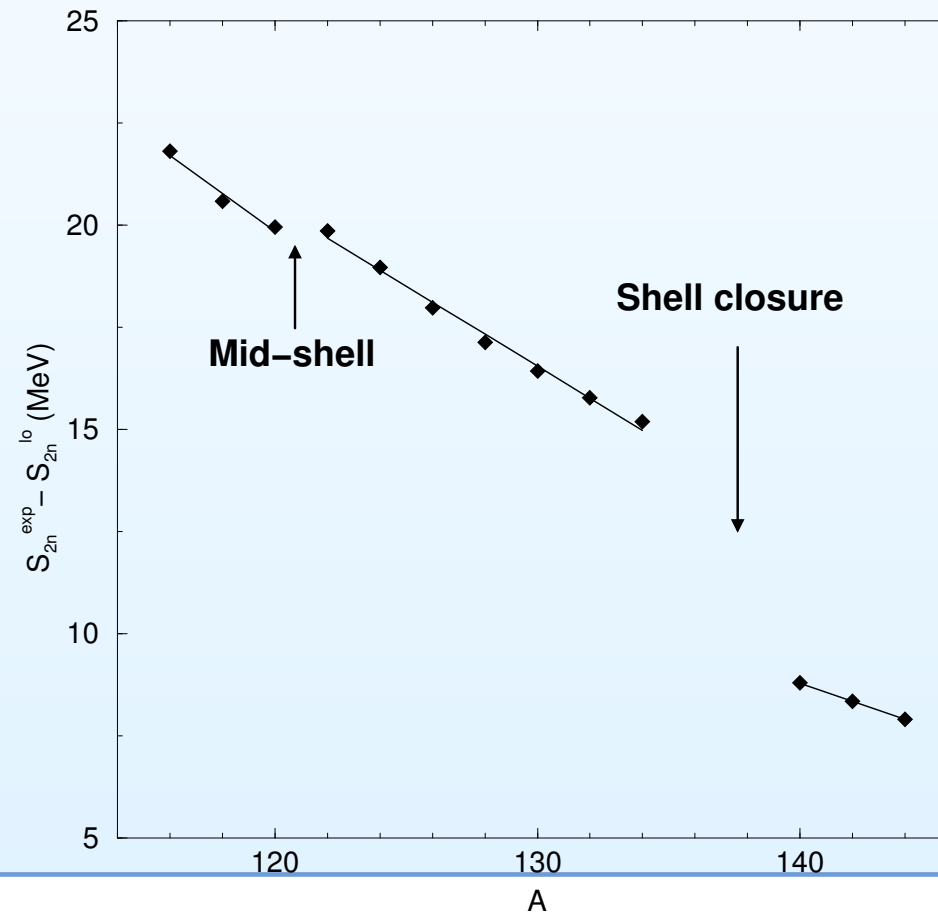
- Construction of a schematic IBM Hamiltonian (W.T. Chou *et al*, Phys. Rev. C **56**, 829 (1997)) for reproducing several excitation energies and transition rates: $E(2_1^+)$, $E(4_1^+)/E(2_1^+)$, $E(2_\gamma^+)$, $E(0_2^+)/(E(2_\gamma^+) - E(2_1^+))$, $B(E2; 2_\gamma^+ \rightarrow 0_1^+)/B(E2; 2_\gamma^+ \rightarrow 2_1^+)$, and $B(E2; 2_\gamma^+ \rightarrow 0_1^+)/B(E2; 2_1^+ \rightarrow 0_1^+)$.
- The parameters of the Hamiltonian are fixed for a wide set of isotopes $^{114-144}_{54}\text{Xe}$, $^{120-148}_{56}\text{Ba}$, $^{124-152}_{58}\text{Ce}$, $^{128-154}_{60}\text{Nd}$, $^{132-160}_{62}\text{Sm}$, $^{138-162}_{64}\text{Gd}$, $^{148-166}_{66}\text{Dy}$, $^{150-168}_{68}\text{Er}$, $^{152-178}_{70}\text{Yb}$, $^{158-184}_{72}\text{Hf}$, $^{166-188}_{74}\text{W}$, $^{170-196}_{76}\text{Os}$, and $^{176-200}_{78}\text{Pt}$.
- The obtained parameter have a very smooth behavior. The parameter $\kappa \approx 30$ keV for all nuclei and κ' , ε and χ present a clear trend for each series of isotopes.
- **The parametrization has been found without considering S_{2n} values.**

Getting \mathcal{A} and \mathcal{B}

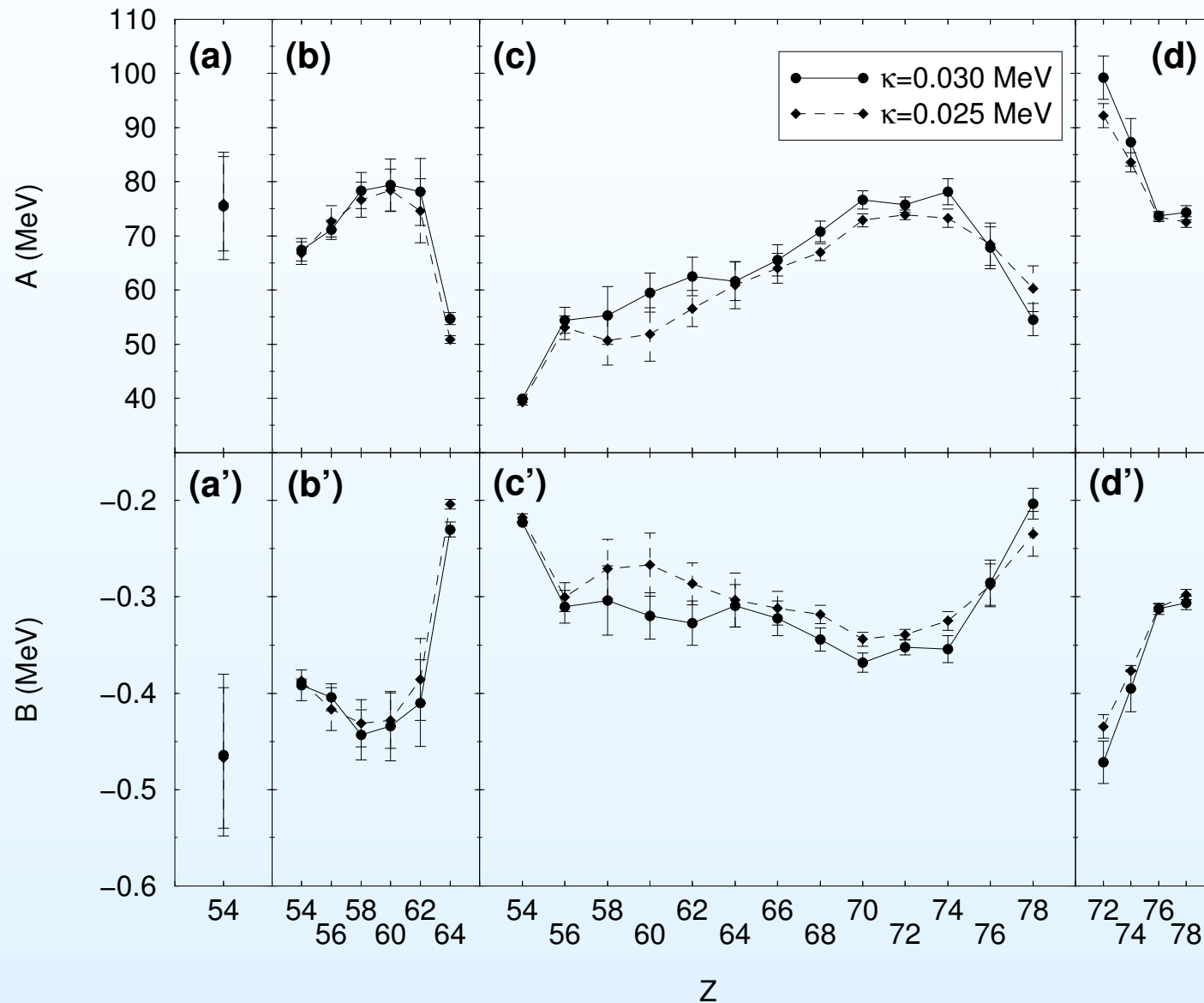
- Global S_{2n} :

$$S_{2n}^{gl} \equiv \mathcal{A} + \mathcal{B} \tilde{N} = S_{2n}^{exp} - S_{2n}^{lo}$$

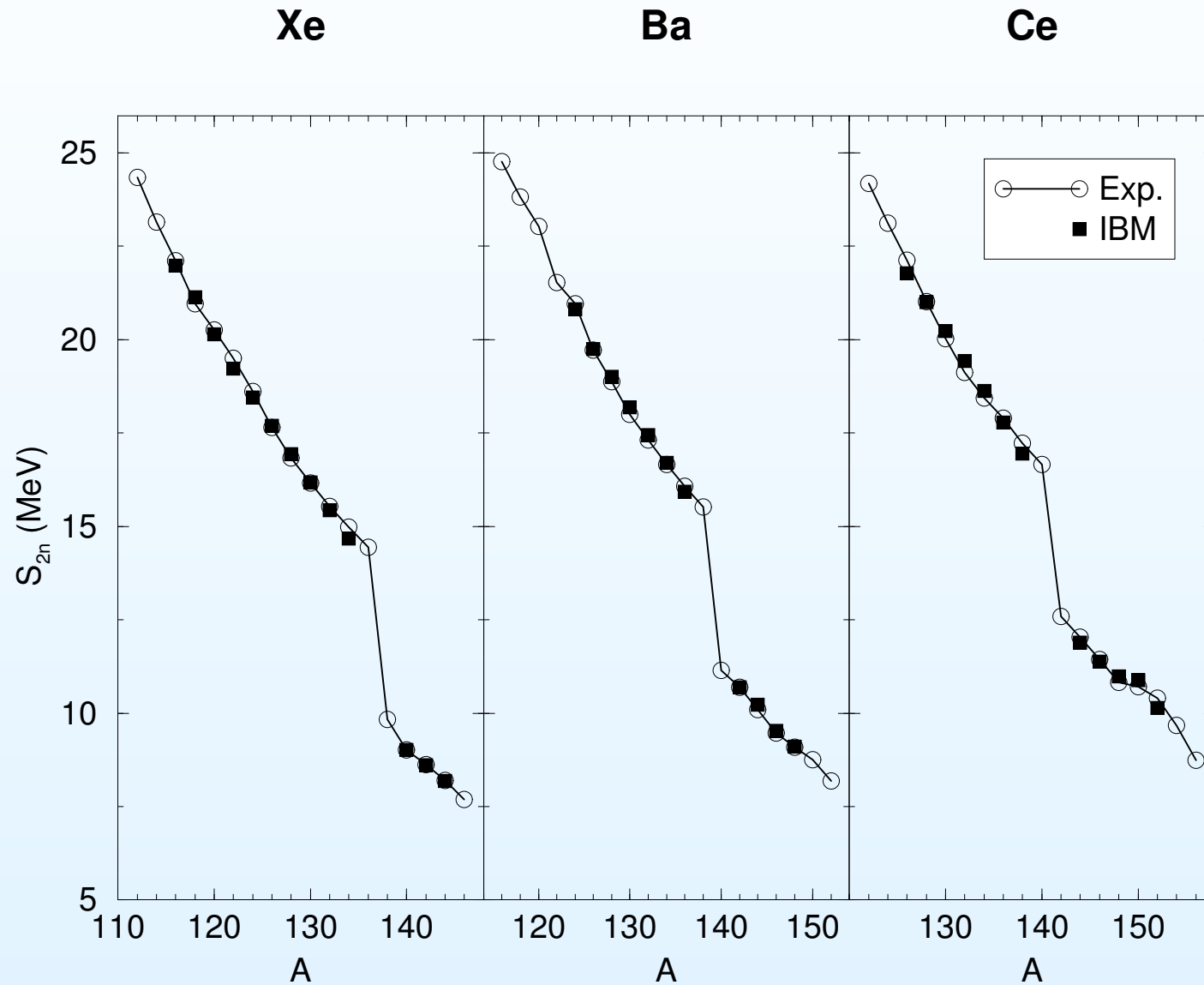
- Case of Xe isotopes



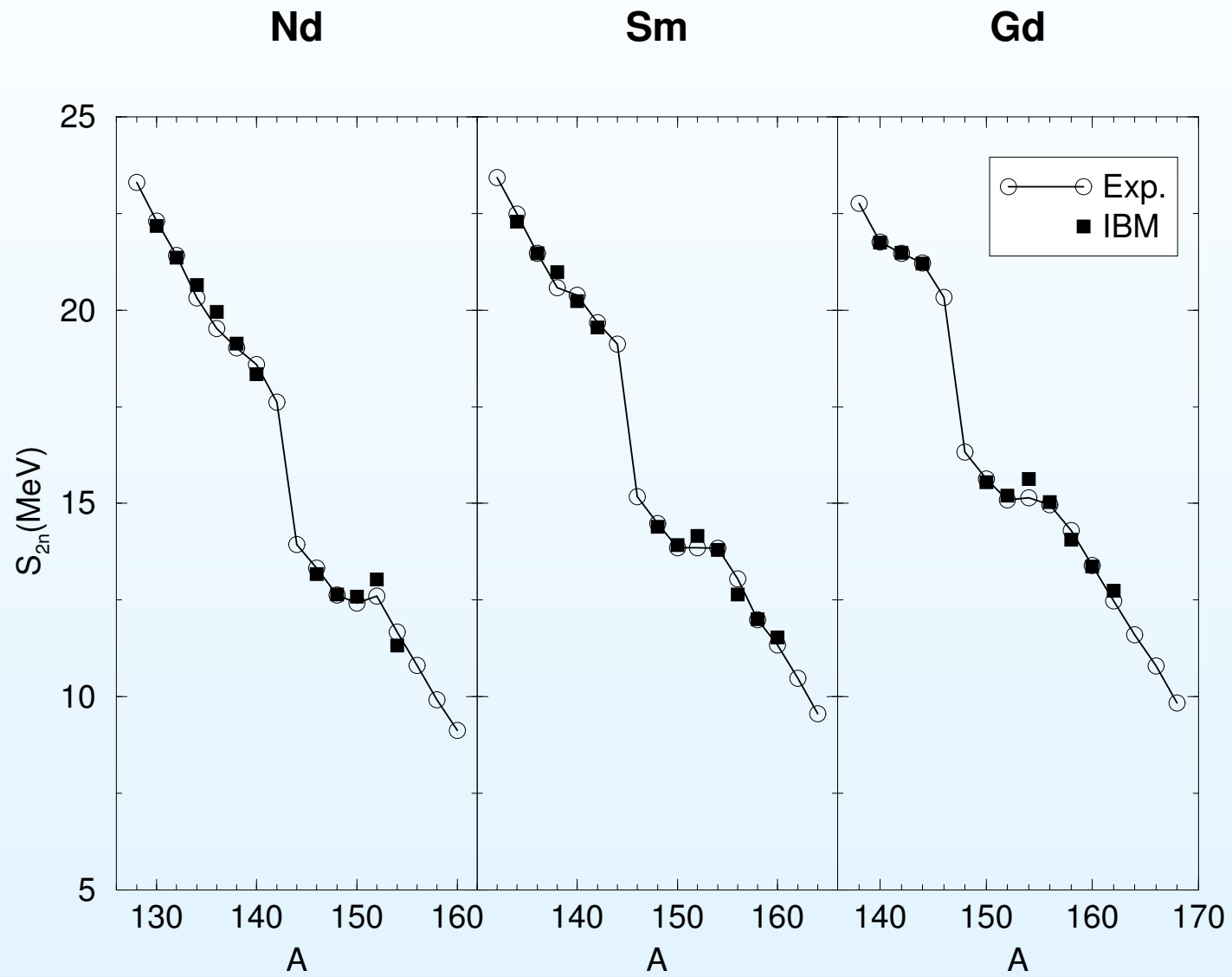
Values of \mathcal{A} and \mathcal{B}



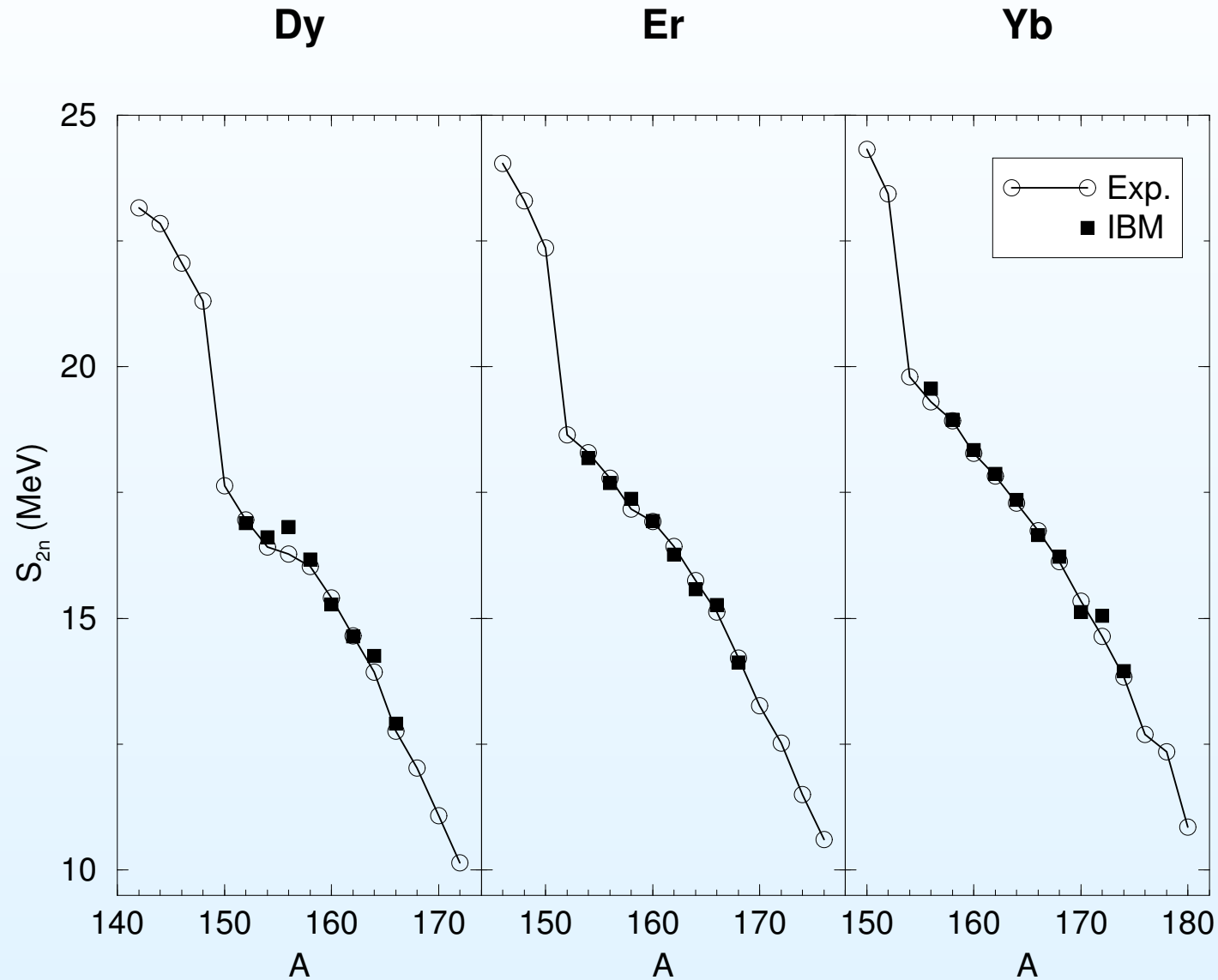
Results I



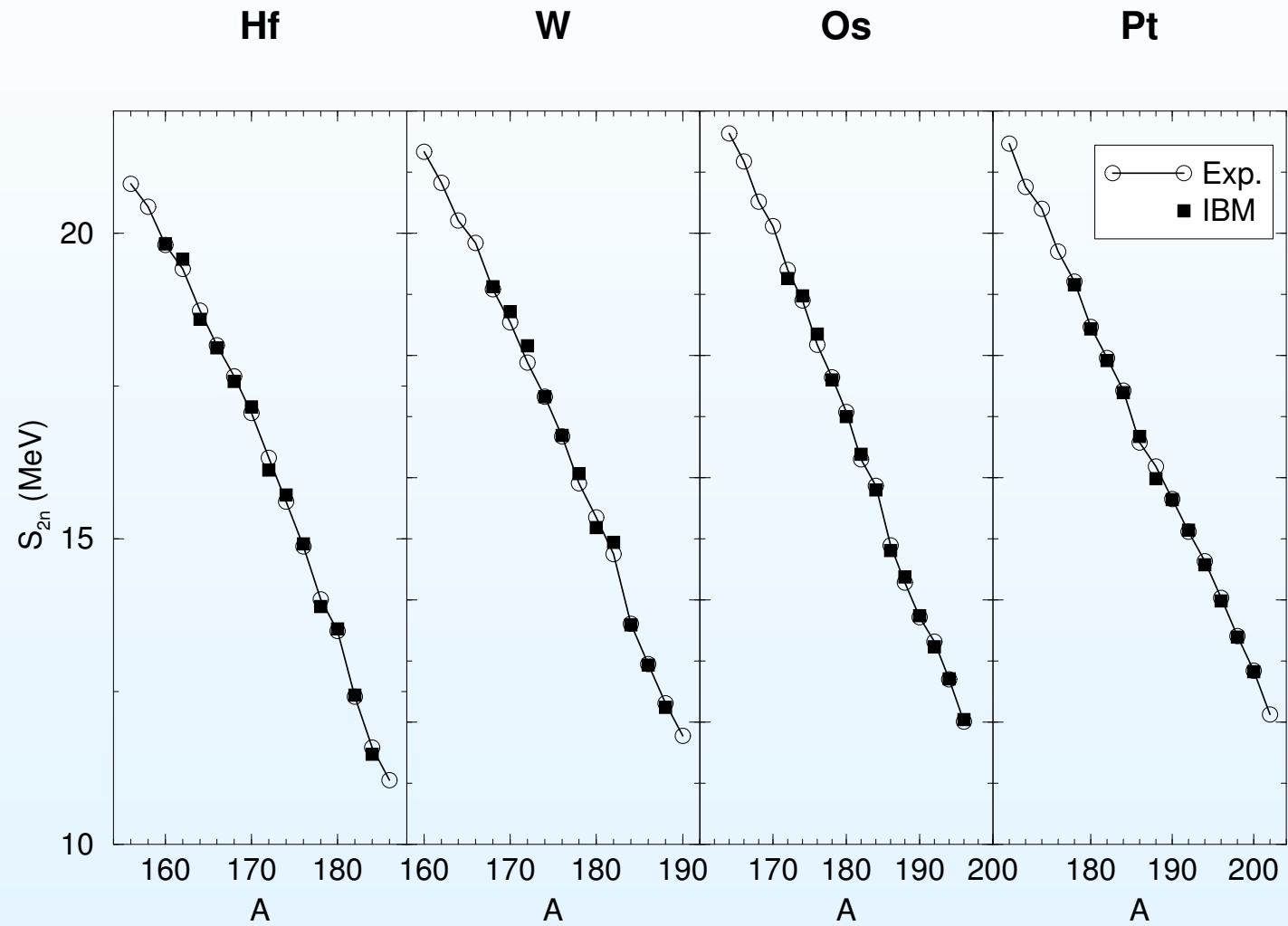
Results II



Results III

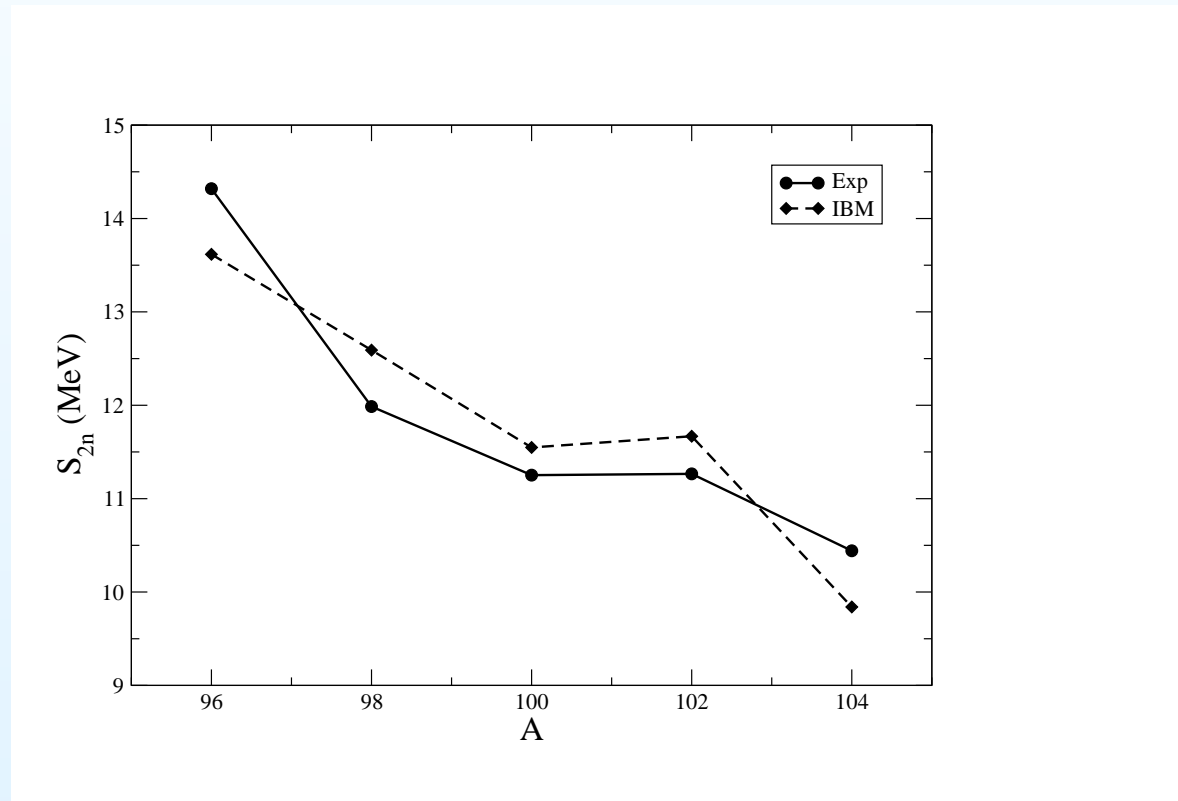


Results IV

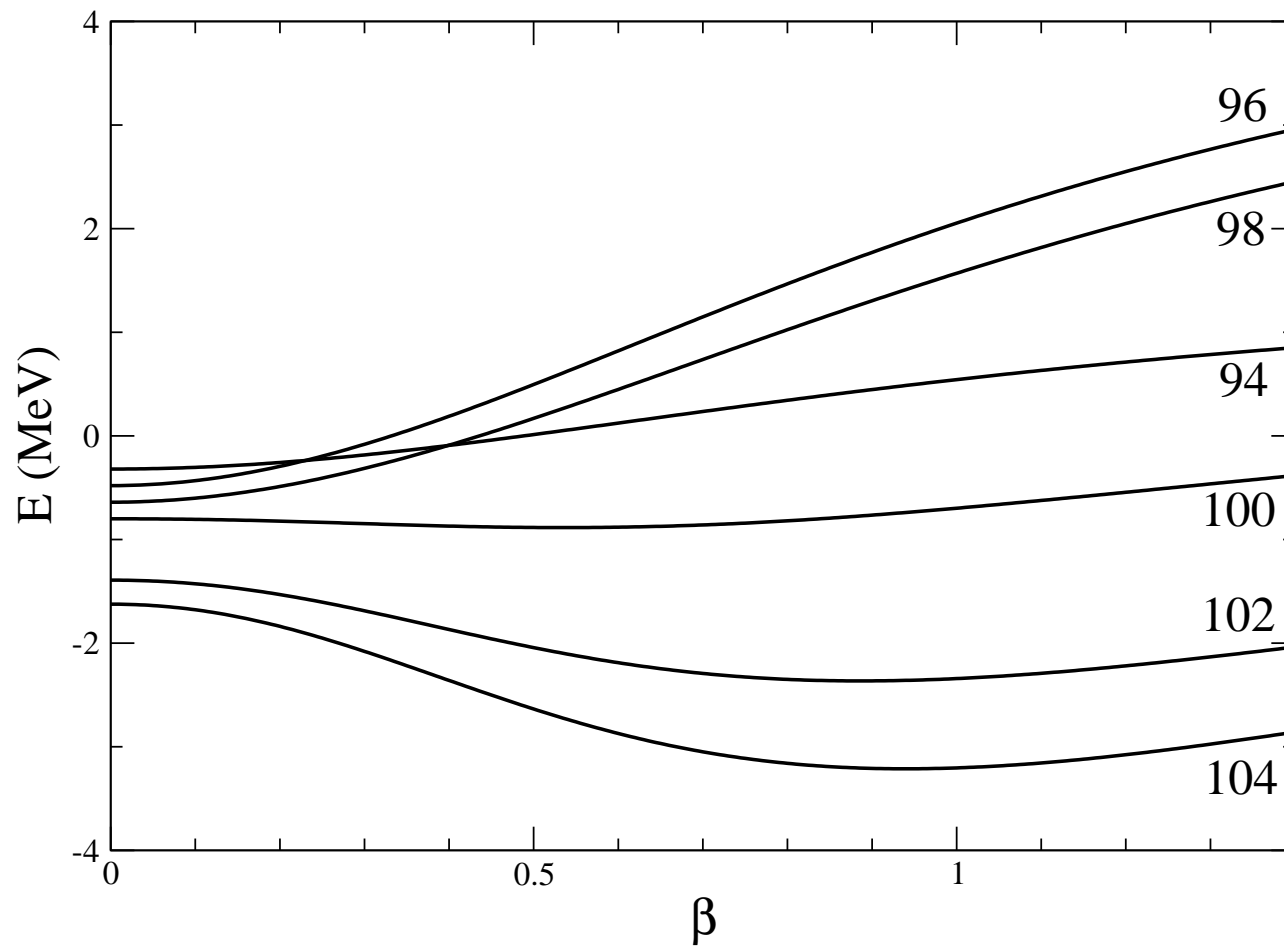


A case for neutron rich nuclei

- Zr isotopes motivated by the work S. Rinta-Antila, *et al*, Phys. Rev. C **70**, 011301(R) (2004).
- Published in J.E. García-Ramos, *et al*, Eur. Phys. J. **26**, 221 (2005).



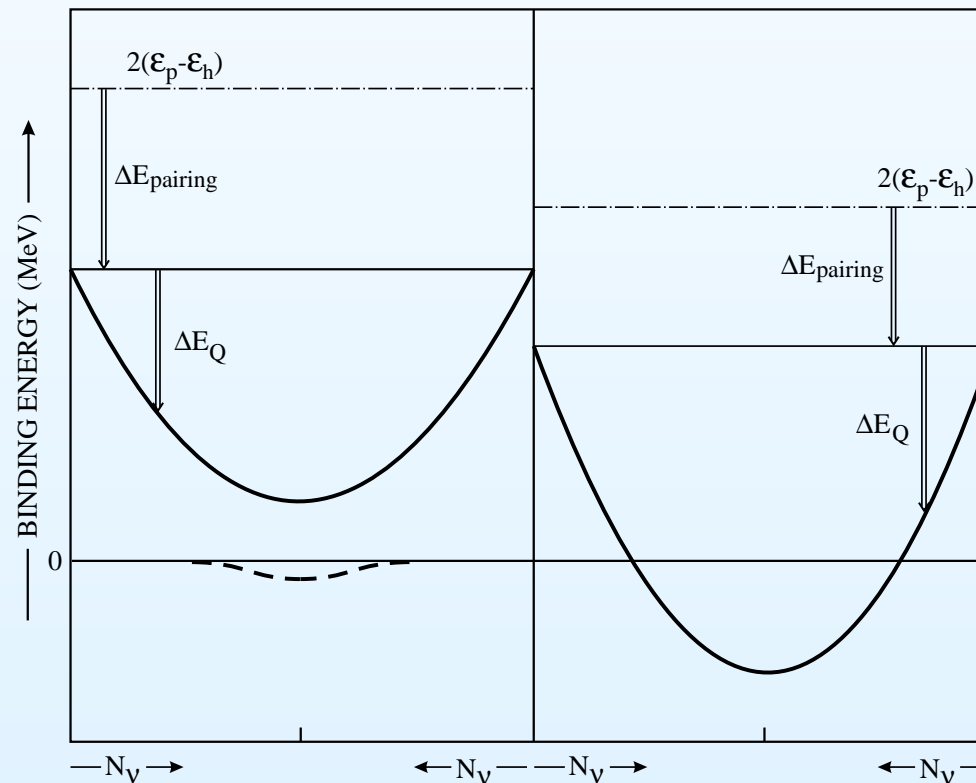
Energy surfaces for Zr



Extensions of the model (I)

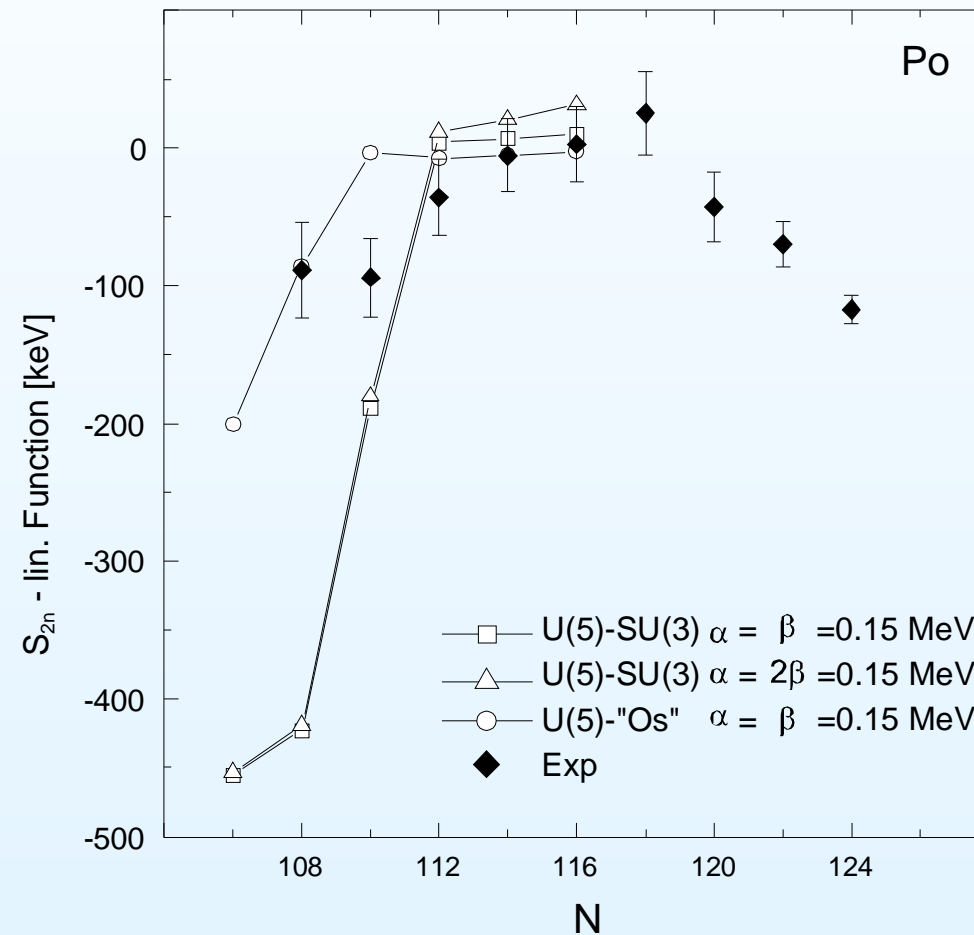
- Intruder excitations: in the IBM this states can be taken into account mixing the $[N]$ space with the $[N+2]$ space. Their main influence appears in the mid shell

$$\Delta E_Q \simeq 2\kappa\Delta N_\pi N_\nu$$

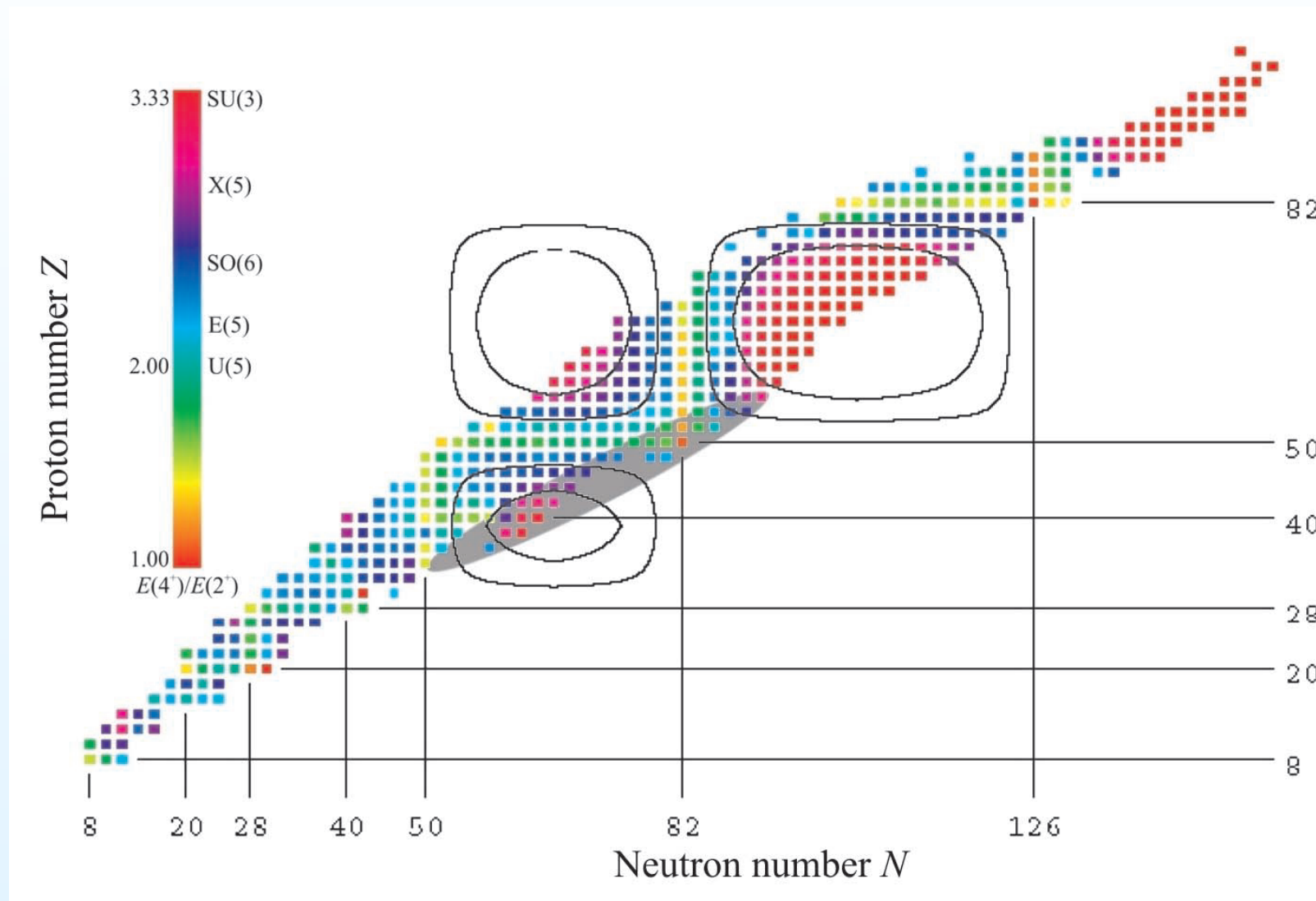


Extensions of the model (II)

- Calculations for Po isotopes assuming three different structures for the mixing states.



Regions of possible nuclear phase transitions



(taken from Franco Iachello)

Where the IBM works better

- Even-even medium mass and heavy nuclei.
- Around mid-shell, where the number of bosons is maximum.
- It is needed to connect a series of isotopes.
- For doing a consistent calculation it is needed to know excited states data.

Some relevant references

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- “Status of the nuclear shell model”, B.A. Brown and B.H. Wildenthal, Ann. Rev. Nucl. Part. Sci. **38**, 29 (1988).
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Summary and conclusions

- All the known nuclear models can be hardly extrapolated to unknown areas.
- The experimental accuracy of masses is much higher than the theoretical one.
- The IBM works as a local model for calculating masses and its main advantage is the simplicity. It provides an accuracy similar to other more complex models.
- The IBM allows to describe consistently masses and excited state properties.
- There is room in IBM for improving the description of masses, *e.g.*, including intruder states.