Evidential Reasoning Framework for Trust Services

Chan Li
Graduate Student
Rajendra P. Srivastava
Ernst & Young Distinguish Professor of Accounting
The University of Kansas
Presented at
Digital Accounting Research Conference
Oct. 13-14, 2005, University of Huelva

Outline

- Introduction and Objectives
- Evidential Reasoning Framework for Trust Services
- Dempster-Shafer Theory of Belief Functions
- Evidential Diagram
- Decision Theoretic Approach for Cost-Benefit Analysis.
- Conclusion and Discussion
Definition of Assurance Services: Collection, evaluation, and aggregation of evidence to decide whether the assertion on which an opinion is being provided is true or not true.

For a Financial Statements audit …to determine whether the Financial Statements are fairly stated.

Introduction

The main purposes of this paper:

- To provide a structured framework for aggregating evidence for making overall judgment about the system’s reliability in the context of Trust Services.
  - The framework is based on the evidential reasoning approach of Srivastava (1995) and uses the criteria established by AICPA/CICA for Trust Services to develop the evidential diagram.
- To develop a decision theoretical approach for cost-benefit analysis.
Introduction (Continued)

- Companies are now relying heavily on information systems more than ever before.
- It is imperative that IT systems be protected and reliable.
- In response to the growing concerns about reliabilities of systems, AICPA/CICA developed Trust Services, which incorporate “SysTrust” and “WebTrust” assurance services (AICPA 2003).
  - Five main assertion categories underlying reliable business systems: Security; Availability; Processing Integrity; Online Privacy; Confidentiality.

Introduction (Continued)

- A company’s information system is a complex system.
- The existing methodologies (see. Post and Diltz 1986; Rainer et al. 1991) are inadequate in properly incorporating complexities of risk.
- The representing uncertainties associated with items of evidence is another problem.
  - Probability theory does not provide a natural and logical way to model uncertainties encountered in the real world.
  - In contrast, Dempster-Shafer theory of belief functions provides a better framework for modeling such uncertainties (see, e.g., Gordon and Shortliffe 1990, Shafer and Srivastava 1990, and Srivastava and Mock 2000).
  - Empirical evidence shows the better performance of belief functions in mapping the uncertainty judgments (Harrison et al 2002; Curley and Golden 1994).
- The evidential reasoning approach under DS theory of belief functions provides a structured approach for evaluating and aggregating all the evidence gathered in the process.
An Evidential Diagram

Evidence for Z

Z: (z, ~z)

Evidence for X & Y

X: (x, ~x)

Evidence for X

Y: (y, ~y)

Evidence for Y

Relationship

Evidential Reasoning in Probability

Propagation of uncertainty through a Network

Z: {z, ~z}

X: {x, ~x}

Y: {y, ~y}

Relationship

\[ P_{XZ}(X|Z), P_{XYZ}(Z|XY) \]
Evidential Reasoning in Belief Functions

Propagation of uncertainty through a Network

\[
\begin{align*}
\text{Bel}_X(X) & \quad \text{Bel}_Y(Y) \quad \text{Bel}_Z(Z) \\
\{x_i \sim x\} & \quad \{y_i \sim y\} \\
\text{Bel}_X(Z) & \\
\{z_i \sim z\} & \quad \text{Bel}_Y(Y) \\
\end{align*}
\]

Relationship \( \text{Bel}_{XYZ}(XYZ) \)

Evidential Reasoning Framework for Trust Services

- Five steps in developing and using the evidential reasoning approach for Trust Services:
  - Step 1: Identify assertions and sub-assertions for the engagement.
  - Step 2: Connect these assertions and sub-assertions through logical relationships such as “and”, “or” as appropriate.
  - Step 3: Identify items of evidence pertaining to specific assertions and sub-assertions identified in Step 1 and link them to the corresponding assertions and sub-assertions.
  - Step 4: Collect and evaluate various items of evidence identified in Step 3 and express these evaluations in terms of belief functions.
  - Step 5: Aggregate all the items of evidence using Dempster’s rule of combination to obtain the overall belief that the assertion is met or not met.
Problems with Probability Framework: An Example

1. Problems with interpreting risks as probability numbers:
   SAS 47: AR = IR.CR.APR.DR

   Consider the following situations:
   (i) If the auditor does not want to depend on the inherent factors, the profession suggests to set IR = 1.
   What does this mean in probability theory? P(E) = 1
   (ii) If the auditor wants to depend on the inherent risk factors, which are favorable but not very strong, the profession suggests to set IR to a lower value, say, IR = 0.8.
   What does this mean in probability theory? P(E) = 0.8, P(~E) = 0.2
   (iii) The auditor feels that inherent factors are really strong and positive then the profession suggests to reduce IR further, say, IR = 0.5.
   What does this mean in probability theory? P(E) = 0.5, P(~E) = 0.5

Probability versus Belief Functions

- **Probability Framework:**
  Frame = \( \Theta = \{a_1, a_2, a_3\} \),
  \[ P(a_i) \geq 0 \]
  \[ P(a_1) + P(a_2) + P(a_3) = 1, \]
  Also, \( P(A) + P(\sim A) = 1 \).

- **Belief Function Framework:**
  Belief Mass: \( m(a_i), m(\{a_i, a_j\}), m(\{a_1, a_2, a_3\}) \geq 0, \)
  \[ \Sigma_{A \subseteq \Theta} m(A) = 1. \]

\( m(\ldots) \) represents the belief mass distribution function over the superset of the frame \( \Theta \). We refer to this function as the **basic belief assignment function**.
Belief Functions-Axioms

Axioms: If $\Theta$ is a frame of discernment, then a function $\text{Bel}: 2^\Theta \rightarrow [0, 1]$ is a belief function if and only if it satisfies the following conditions:

1. $\text{Bel}(\emptyset) = 0$.
2. $\text{Bel}(\Theta) = 1$.
3. For every positive integer $n$ and every collection $A_1, \ldots, A_n$ of subsets of $\Theta$,
   
   $$\text{Bel}(A_1 \cup \ldots \cup A_n) \geq \sum_{I \subseteq \{1, \ldots, n\}} (-1)^{|I|+1} \text{Bel}(\cap_{i \in I} A_i).$$

Belief Functions and Plausibility Functions

Let $m(B)$ be a mass function.

$$\text{Bel}(A) = \sum_{B \subseteq A} m(B)$$

$$\text{Pl}(A) = \sum_{A \cap B \neq \emptyset} m(B) = 1 - \text{Bel}(\neg A)$$
Simple Example

Analytical Procedure:
\[ m(a) = 0.3, \ m(\sim a) = 0, \ m(\{a,\sim a\}) = 0.7 \]

Based on the above evidence:
\[ \text{Bel}(a) = 0.3, \]
\[ \text{Bel}(\sim a) = 0, \]
\[ \text{Pl}(a) = 1.0, \]
\[ \text{Pl}(\sim a) = 0.7, \]

Plausibility that material errors are present defines the risk (Analytical Procedure Risk)

Complex Example: Belief Functions and Plausibility Functions

\[ \text{Bel}(A) = \sum_{B \subseteq A} m(B) \]

\[ m(a) = 0.1, \ m(\{a,b\}) = 0.2, \]
\[ m(c) = 0.3, \ m(\{a,b,c\}) = 0.2 \]
\[ m(e) = 0.1, \ m(\{a,b,c,d,e,f\}) = 0.1 \]

\[ \text{Bel}(\{a,b,c\}) = m(a) + m(\{a,b\}) + m(c) + m(\{a,b,c\}) = 0.1 + 0.2 + 0.3 + 0.2 = 0.8 \]

\[ \text{Pl}(A) = \sum_{A \cap B \neq \emptyset} m(B) = 1 - \text{Bel}(\sim A) \]

\[ \text{Pl}(\{a,b\}) = m(a) + m(\{a,b\}) + m(\{a,b,c\}) + m(\{a,b,c,d,e,f\}) = 0.1 + 0.2 + 0.1 = 0.6 \]
Security: Objectives and Sub-objectives

1.1. Protection against unauthorized logical access
- Policies exist to ensure systems are protected against unauthorized logical access.
- System security policies pertaining to logical access are communicated to authorized users.
- Procedures exist to protect against unauthorized logical access.

1.2. Protection against unauthorized physical access
- Policies exist to ensure systems are protected against unauthorized physical access.
- System security policies pertaining to physical access are communicated to authorized users.
- Procedures exist to protect against unauthorized physical access.

1.3. Protection against infection by computer viruses
- Policies exist to ensure systems are protected against virus infection.
- System security policies pertaining to virus infection are communicated to authorized users.
- Procedures exist to protect against virus infection.
Decision Theoretical Approach for Cost-Benefit Analysis

The management will buy the assurance service when:

\[ F + \sum_{i=1}^{5} m_{A_i}(\sim a_i)L_i \leq \sum_{i=1}^{5} m_{NA_i}(\sim a_i)L_i \]

For the most conservative case (\( \rho = 0 \)), the management will buy the assurance service when:

\[ F + \sum_{i=1}^{5} \{1 - m_{A_i}(a_i)\}L_i \leq \sum_{i=1}^{5} \{1 - m_{NA_i}(a_i)\}L_i \]

or

\[ F \leq \sum_{i=1}^{5} \{ m_{A_i}(a_i) - m_{NA_i}(a_i) \}L_i \]

Decision Theoretical Approach for Cost-Benefit Analysis

The break-even dollar amount for a company to obtain the assurance service is determined by:

- The management’s own beliefs on its system reliability in terms of the five principles before obtaining the assurance service;
- The management’s own beliefs on its system reliability in terms of the five principles after obtaining the assurance service;
- The degree of loss associated with each principle if it is not met;
- Management’s attitude towards resolving the ambiguity (\( \rho \));
- The audit fee for the assurance service.
Conclusion and Discussion

The approach proposed in the paper
- uses Dempster-Shafer theory of belief functions for modeling uncertainties associated with the evidence gathered in the process.
- allows the decision makers to incorporate relevant relationships and interdependencies among assertions and items of evidence.

The cost-benefit analysis shows
- the break-even point for a company to obtain the Trust Service.
- the factors influencing a company’s purchase decision of the assurance services.

Future research
- Develop an analytical model of the assurance services
- Empirical work is needed to demonstrate the value of evidential reasoning approach to Trust Services.

Prior Publications & Applications

Current Research Applications


- **SysTrust Assurance Services**: Chan Li and R. Srivastava


Other Applications


- **Decision Making under Ambiguity** (Srivastava, *Archives of Control Sciences*, Vol. 6 (XLII), 1997, No. 1-2, pp. 5-27

Papers on belief functions are available from the following website:

http://www.eycarat.ku.edu/CARAT/research/belief.htm
Thank You!