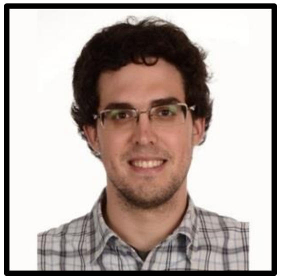
Excited-state quantum phase transitions in spin-orbit coupled Bose gases

ESQPT2021 Seminar Series

Josep Cabedo Bru

Physics Department, Universitat Autònoma de Barcelona

- Quantum and Atom Optics group (QAOS)
  - Group Leaders: Prof. Jordi Mompart and Prof. Veronica Ahufinger.
  - Focus of research: Ultracold atoms, Light-matter interaction, Photonic simulators and Conical refraction.
  - Webpage: http://grupsderecerca.uab.cat/qaos/



Dr. Joan Claramunt

Lancaster University

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Grup d'Òptica UAB

UAB
Universitat Autònoma de Barcelona

J. Cabedo, J. Claramunt, A. Celi, *arXiv:2101.08253v2* (2021)

Outline

I. **Introduction**

- Quantum many-body physics with spinor condensates
- The Raman-dressed condensate: synthetic spin-orbit (SO) coupling
- The stripe phase of the SO coupled gas

II. **Excited-state quantum phase transitions in spin-orbit coupled Bose gases**

- Tunable spin-changing collisions from synthetic SO coupling
- Three-mode model: effective spin Hamiltonian
- Adiabatic quenches through *excited-state quantum phase transitions* (ESQPTs)
- Preparation of the ferromagnetic stripe phase in an excited state

III. **Conclusion and outlook**

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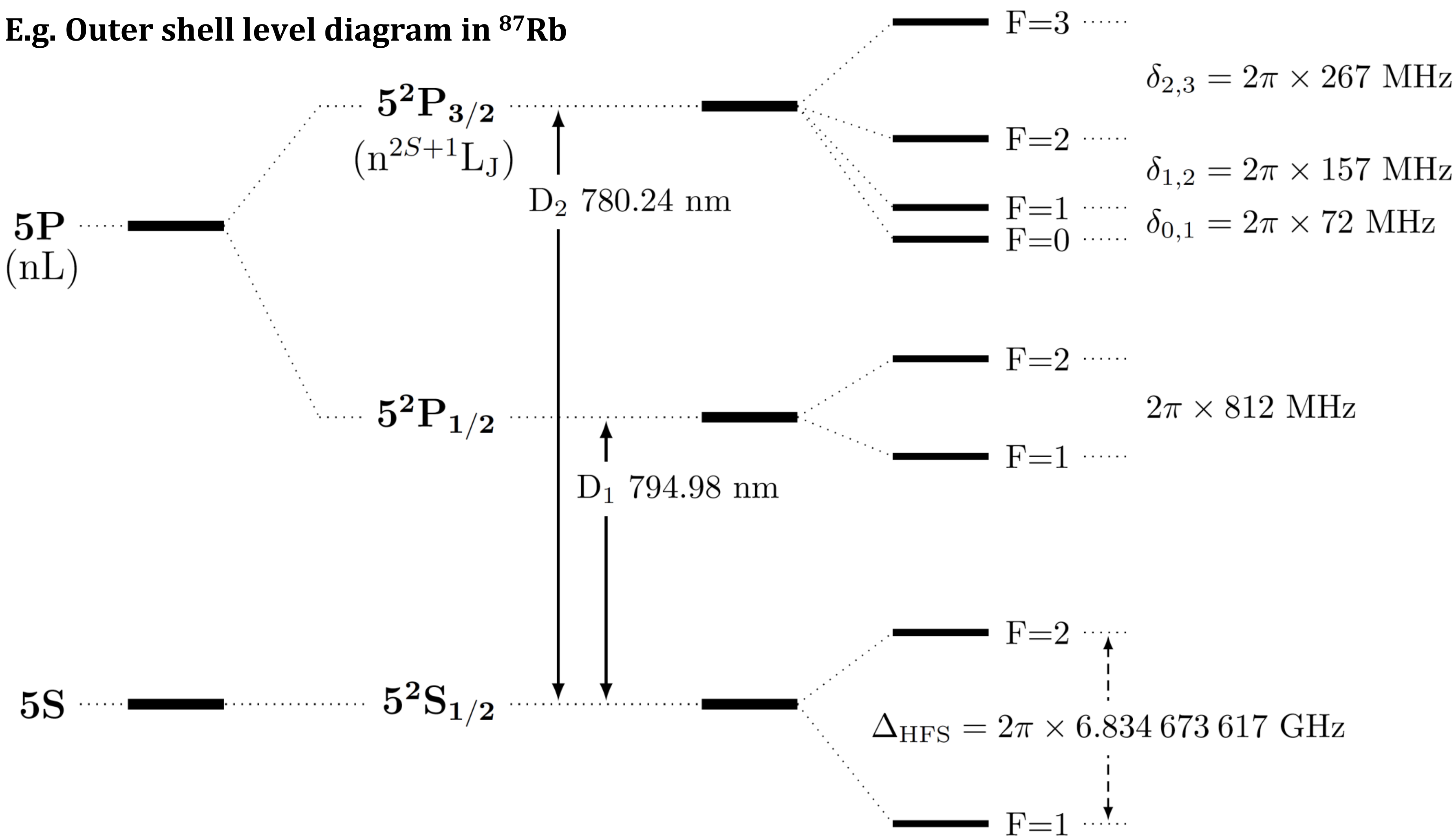
Many body physics with spinor gases

Spinor BECs: several internal atomic states within a hyperfine state manifold involved

E.g. Outer shell level diagram in ^{87}Rb



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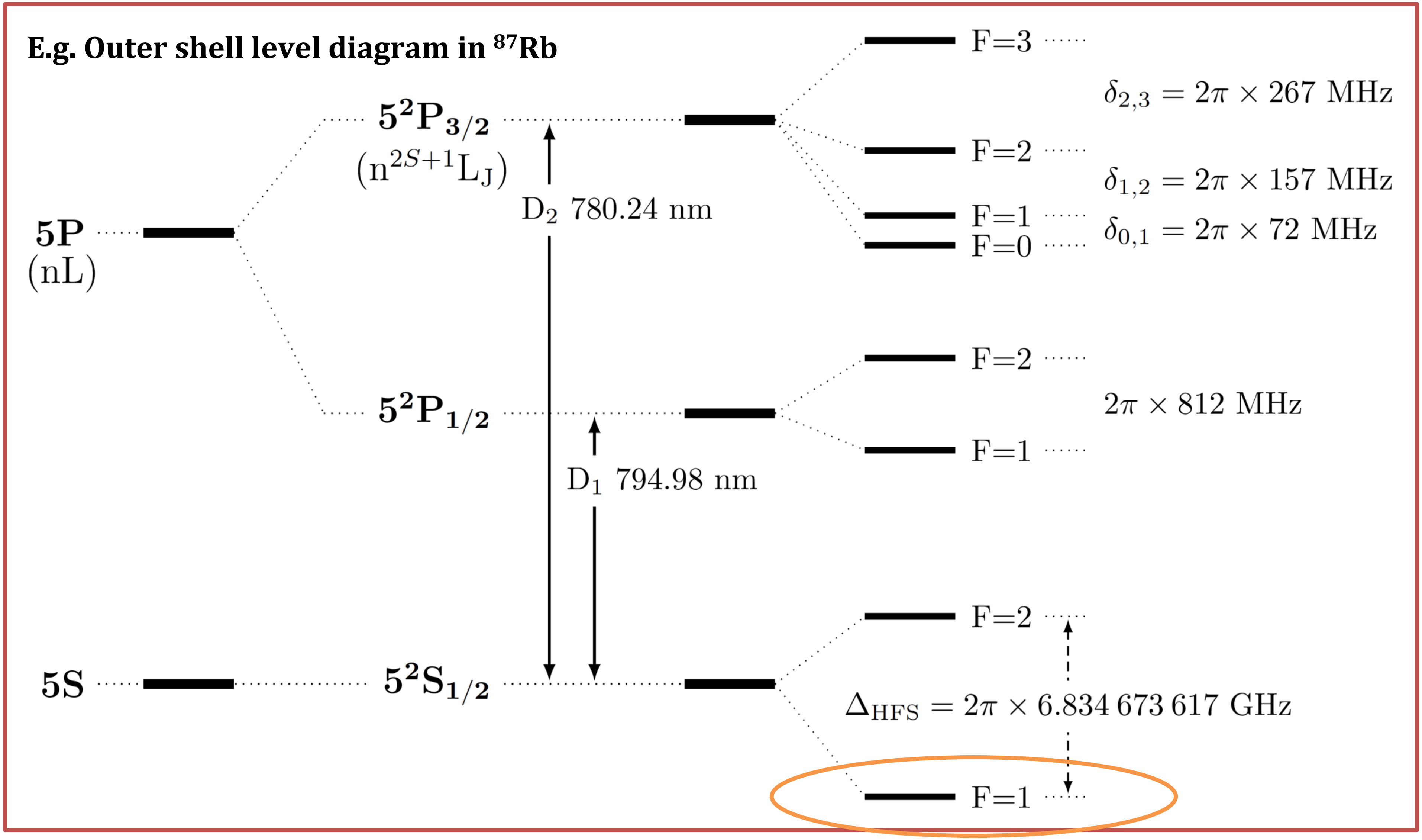

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- Atoms can exchange spin via s-wave collisions: spin-mixing dynamics

$$\hat{H}_S = \frac{4\pi\hbar^2(a_0 + 2a_2)}{3m} \int dx^3 \sum_{ij} \psi_i^\dagger \psi_j^\dagger \psi_i \psi_j$$

$$\hat{H}_A = \frac{4\pi\hbar^2(a_2 - a_0)}{3m} \int dx^3 \Big(\psi_1^\dagger \psi_1^\dagger \psi_1 \psi_1 + \psi_{-1}^\dagger \psi_{-1}^\dagger \psi_{-1} \psi_{-1} + 2\psi_1^\dagger \psi_0^\dagger \psi_1 \psi_0 + 2\psi_{-1}^\dagger \psi_0^\dagger \psi_{-1} \psi_0$$
$$- 2\psi_1^\dagger \psi_{-1}^\dagger \psi_1 \psi_{-1} + 2\psi_1^\dagger \psi_{-1}^\dagger \psi_0 \psi_0 + 2\psi_0^\dagger \psi_0^\dagger \psi_1 \psi_{-1} \Big)$$

C. K. Law, H. Pu, and N. P. Bigelow Phys. Rev. Lett. **81**, 5257 (1998)

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- **Single-spatial-mode approximation:** $\hat{H}_A$ as a perturbation. Motional degrees of freedom condensate into the same spatial mode for the different spin states.

$$\begin{cases} \hat{\psi}_j^\dagger \sim \phi(\boldsymbol{r})\hat{b}_j^\dagger \\ \hat{L}_j = \sum_{\mu,\nu} \hat{b}_\mu^\dagger \left(\hat{F}_j\right)_{\mu\nu} \hat{b}_\nu \end{cases} \Rightarrow \quad \boxed{\hat{H}_{\text{eff}} \simeq \hat{H}_A \propto \frac{\hat{L}^2}{2N}}$$

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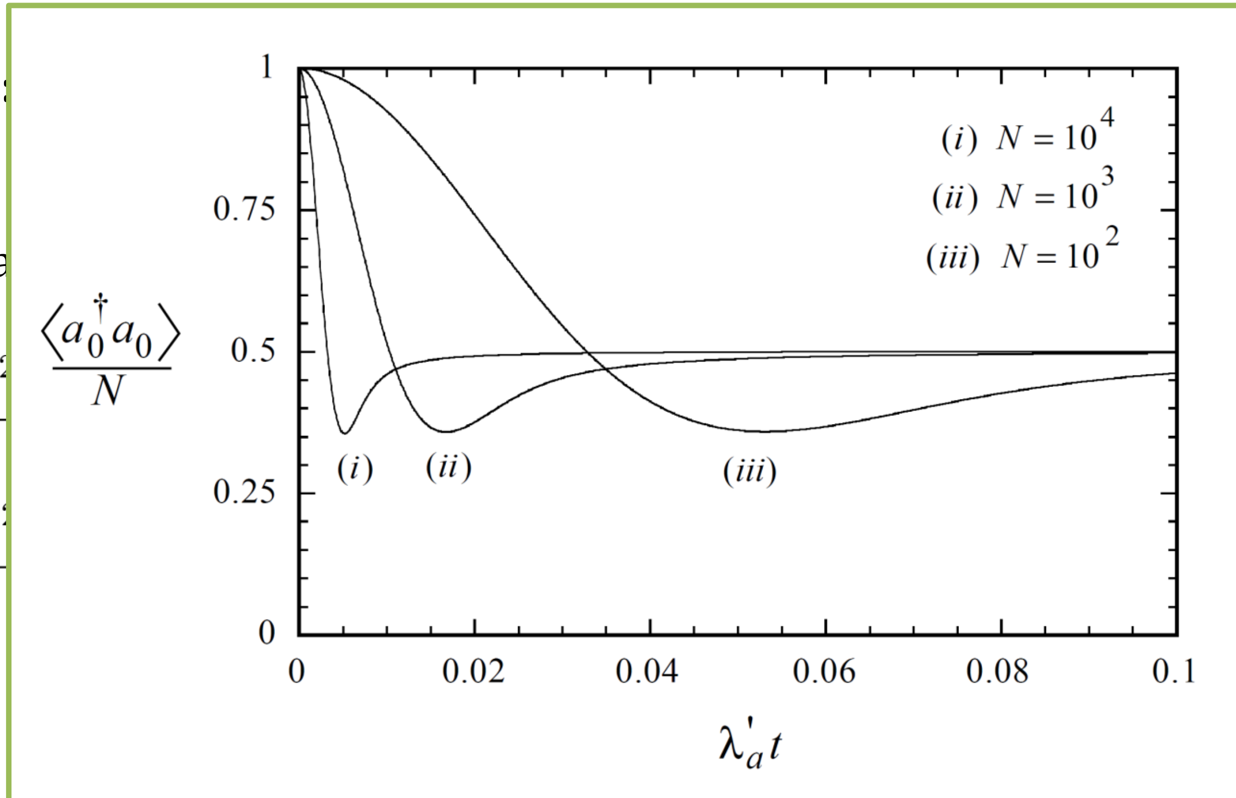

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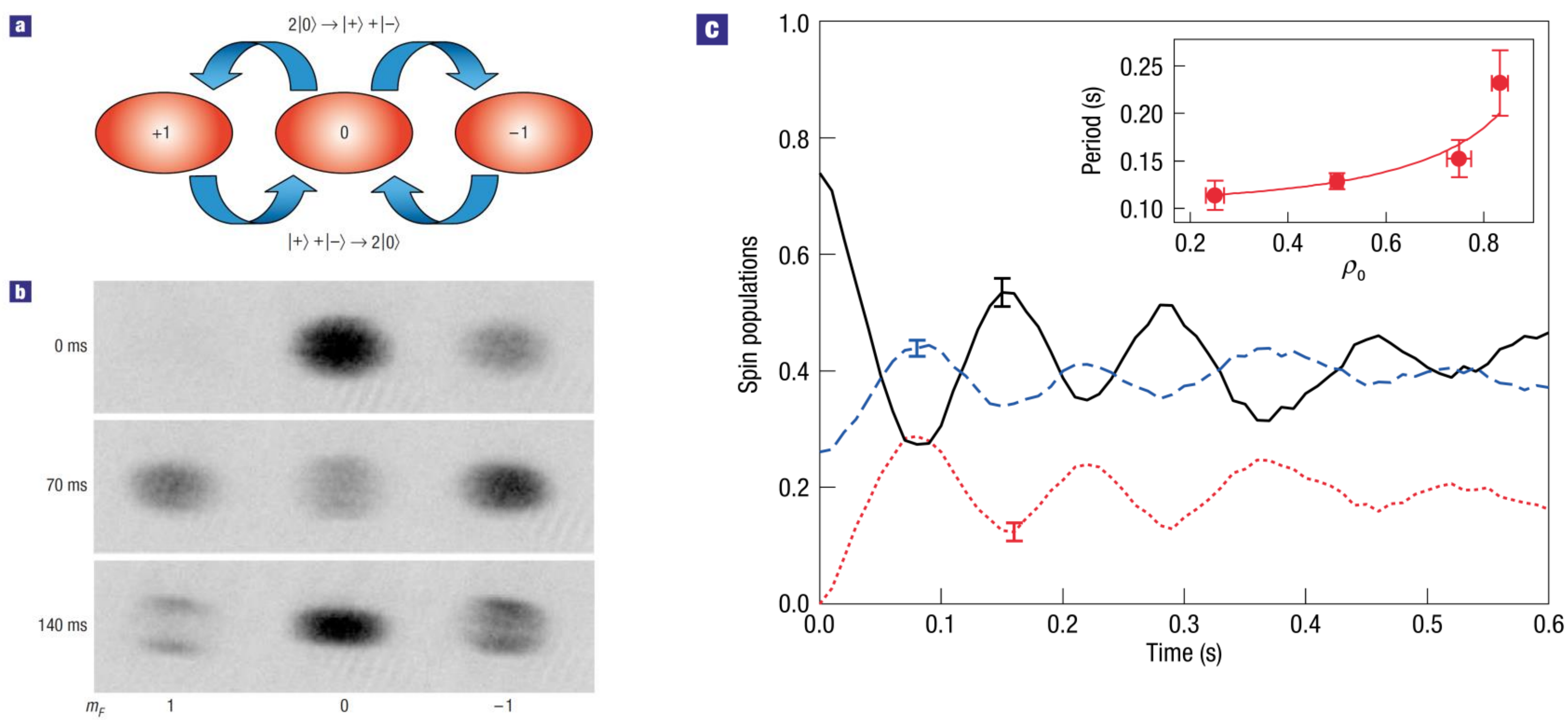

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Chang, M.-S., Q. Qin, W. Zhang, L. You, and M. Chapman, 2005, *Nat. Phys.* **1**, 111 (2005)

Many body physics with spinor gases

Why spinor BECs?

- Ultracold atoms: **long coherence** times + **tunability**

- **Simple framework**: orbital and spin degrees of freedom can decouple → "All for all" many-body spin Hamiltonian

- Rich **interplay** between a **linear** and a **non-linear** contributions

$$\hat{H} = c_2 \frac{\hat{L}^2}{2N} + q\hat{L}_{zz}$$

Z. Zhang and L.-M. Duan *Phys. Rev. Lett.* **111**, 180401 (2013)

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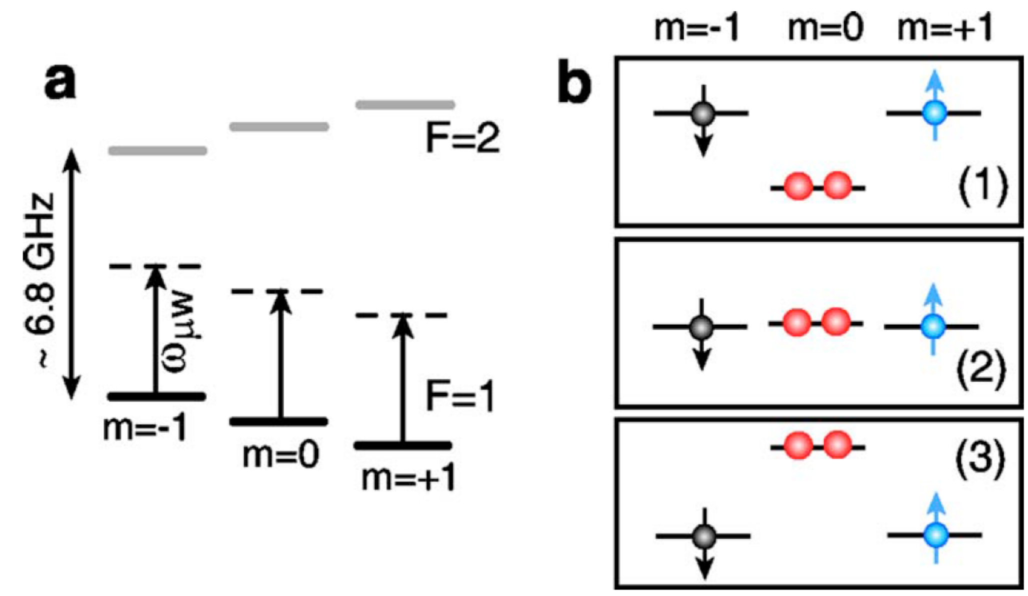

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Microwave drive near hyperfine transition

Gerbier, F., A. Widera, S. Folling, O. Mandel, and I. Bloch, *Phys. Rev. A* **73**, 041602(R) (2006)

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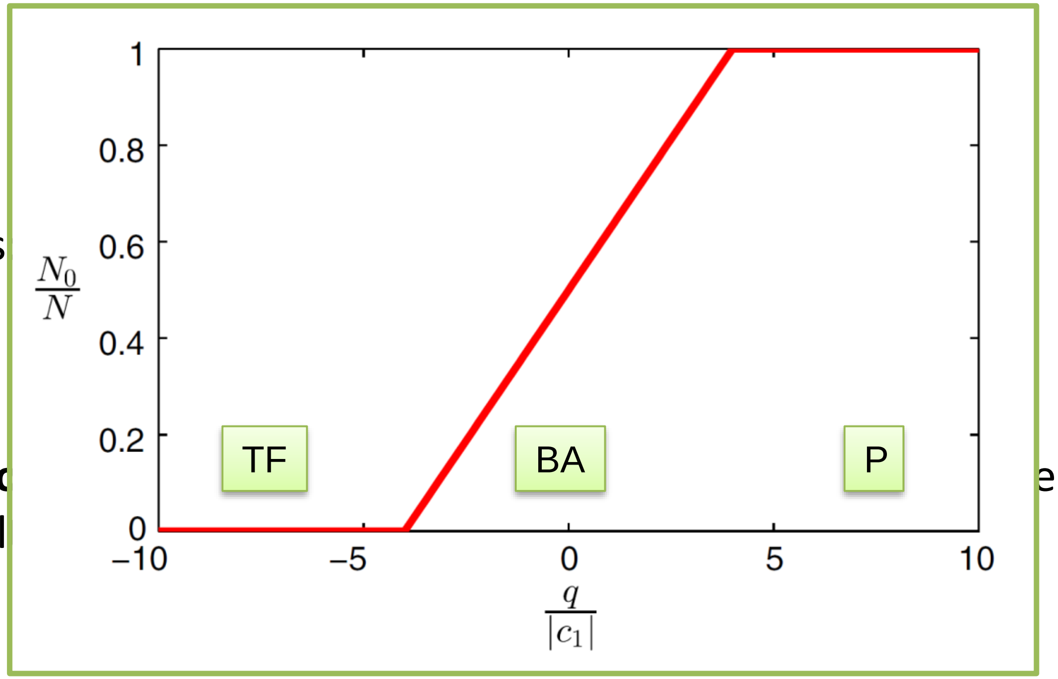

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➢ Coherent spin oscillations, spontaneous magnetization and symmetry breaking, parametric amplification and spin squeezing...

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- Macroscopic squeezing and entanglement generation

L.-M. Duan, A. Sørensen, J. I. Cirac, and P. Zoller, *Phys.Rev. Lett.***85**, 3991 (2000)

B. Lücke, M. Scherer, J. Kruse, L. Pezzé, F. Deuret-zbacher, P. Hyllus, O. Topic, J. Peise, W. Ertmer, J. Arlt,L. Santos, A. Smerzi, and C. Klempt, *Science* **334**, 773 (2011)

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- Dynamical and ESQPTs

T. Tian, H.-X. Yang, L.-Y. Qiu, H.-Y. Liang, Y.-B. Yang,Y. Xu, and L.-M. Duan, Phys. Rev. Lett.124, 043001(2020)

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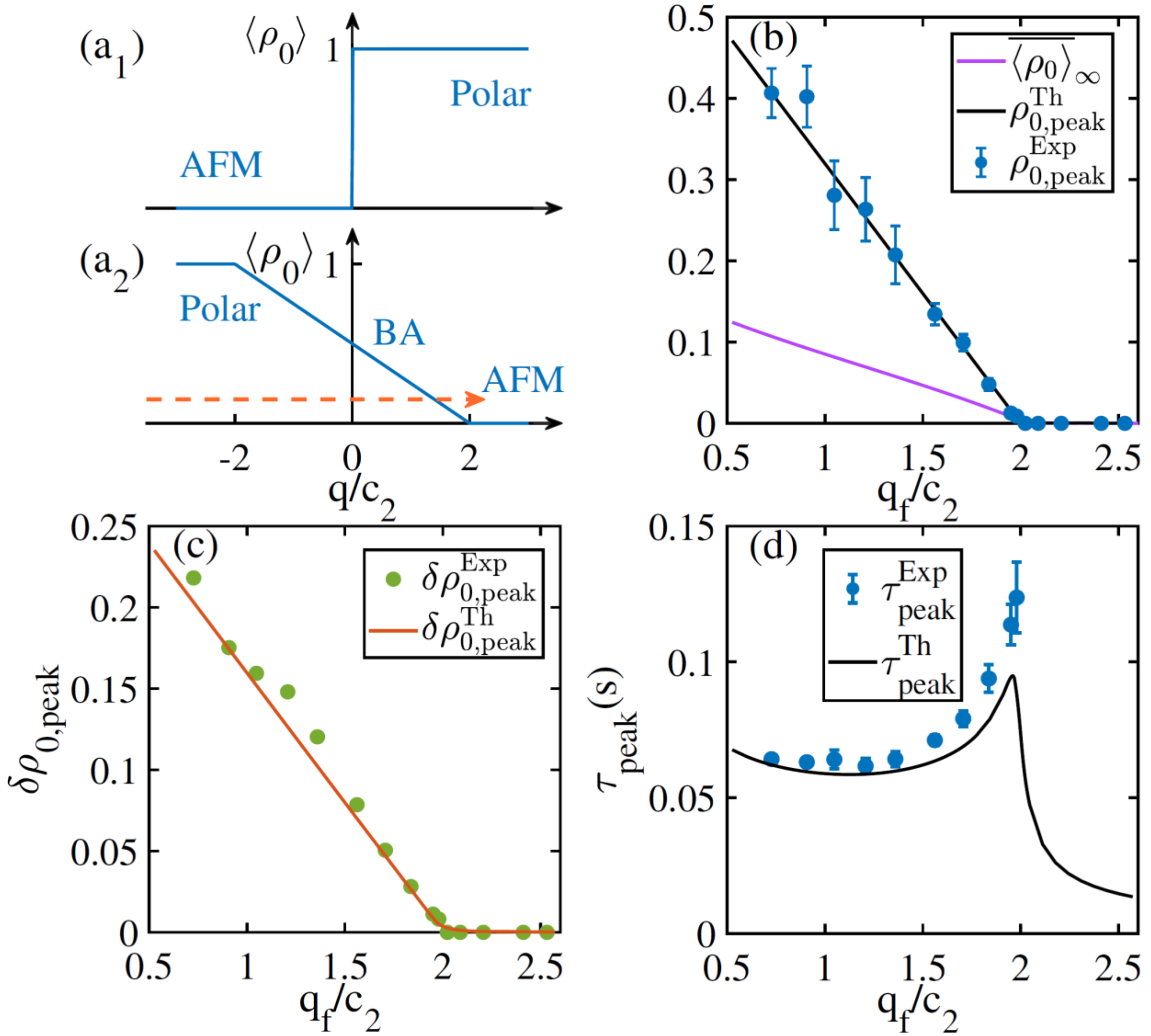

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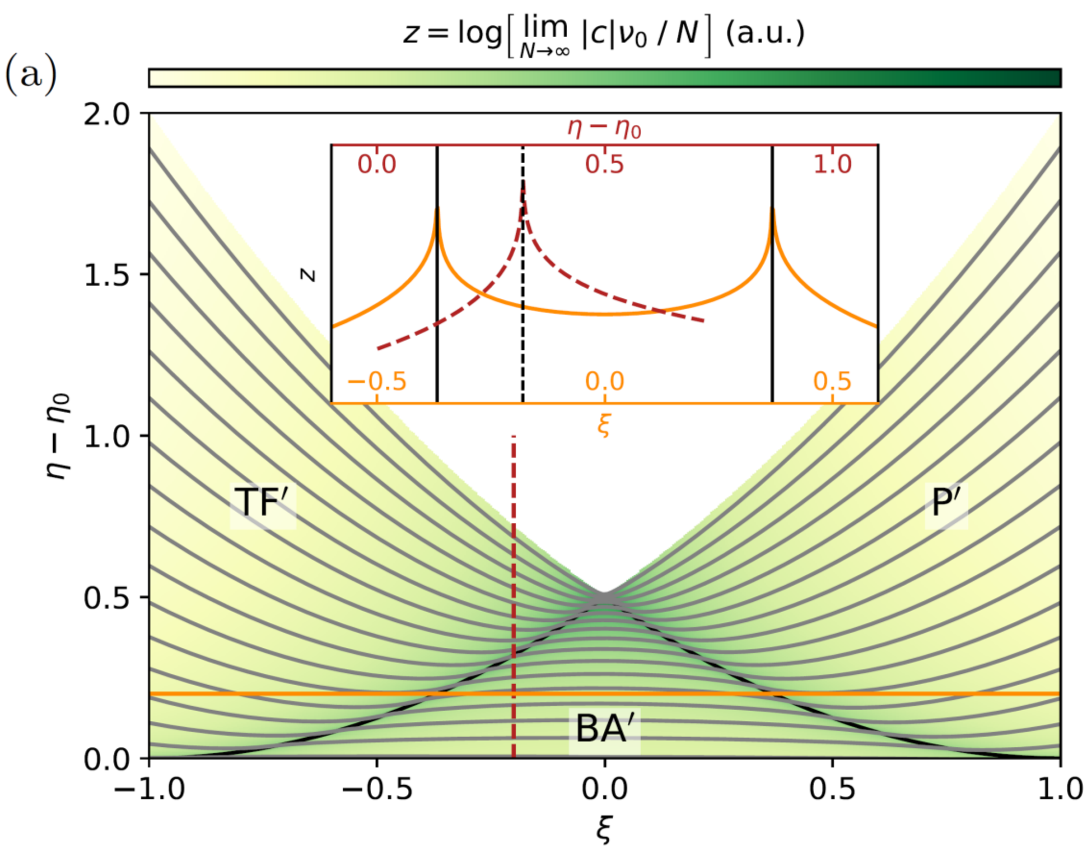
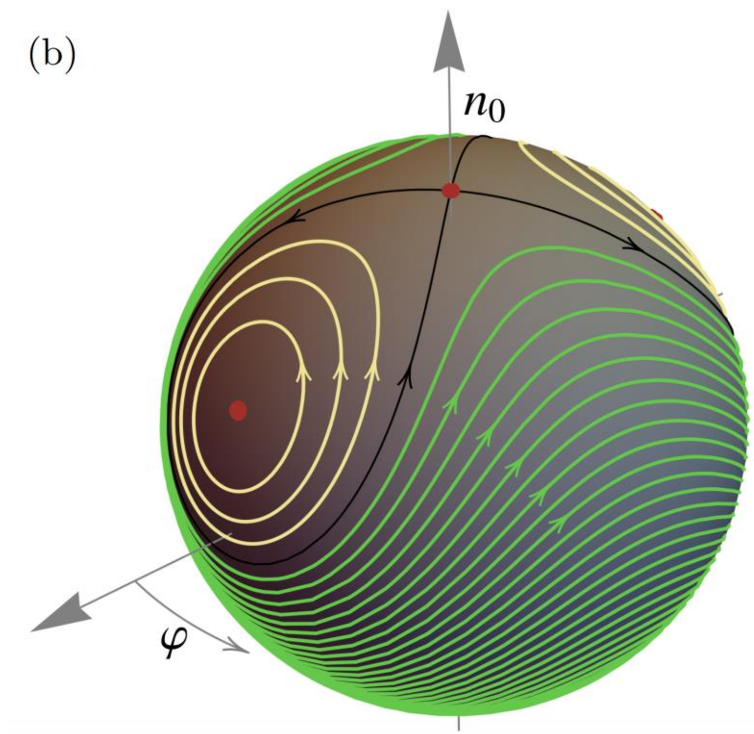
- Topological order parameter that distinguishes between excited-state phases across the spectrum: winding number of classical phase-space trajectories





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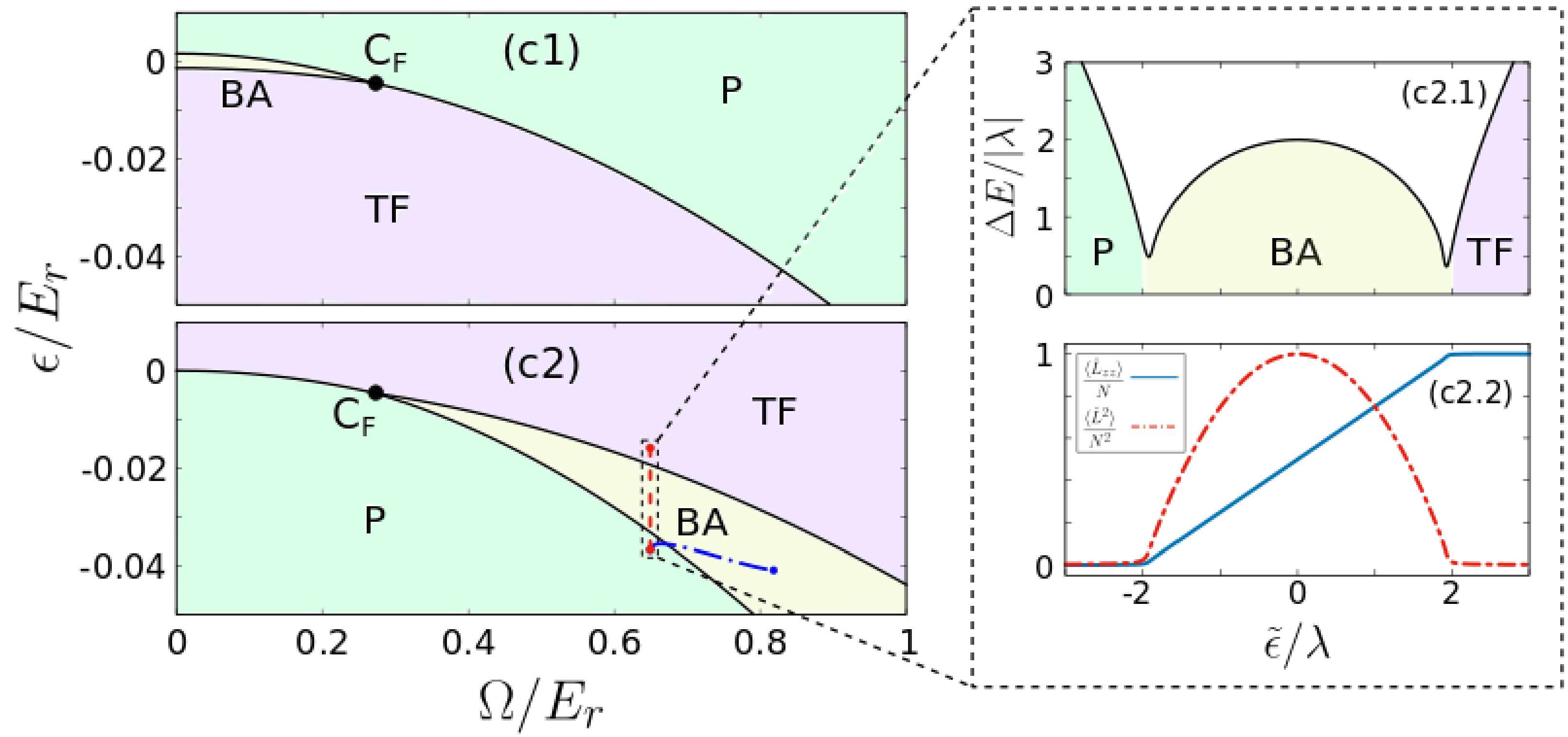
Our work

- Spin-orbit coupled BECs as a flexible framework to explore collective spin physics.

- Exploit an **ESQPT** of the spin model to prepare the elusive **ferromagnetic stripe phase (FS)**



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The Raman-dressed condensate: synthetic spin-orbit (SO) coupling

Spin-orbit coupling

Mechanism:

- Charged particles experience Zeeman shift

$$\Delta E = -\mu \cdot \mathbf{B} \qquad \mu = \frac{g_s e}{2m_e}\mathbf{S}$$

- External electric field $\mathbf{E}_0 = E_0 \mathbf{e}_z$

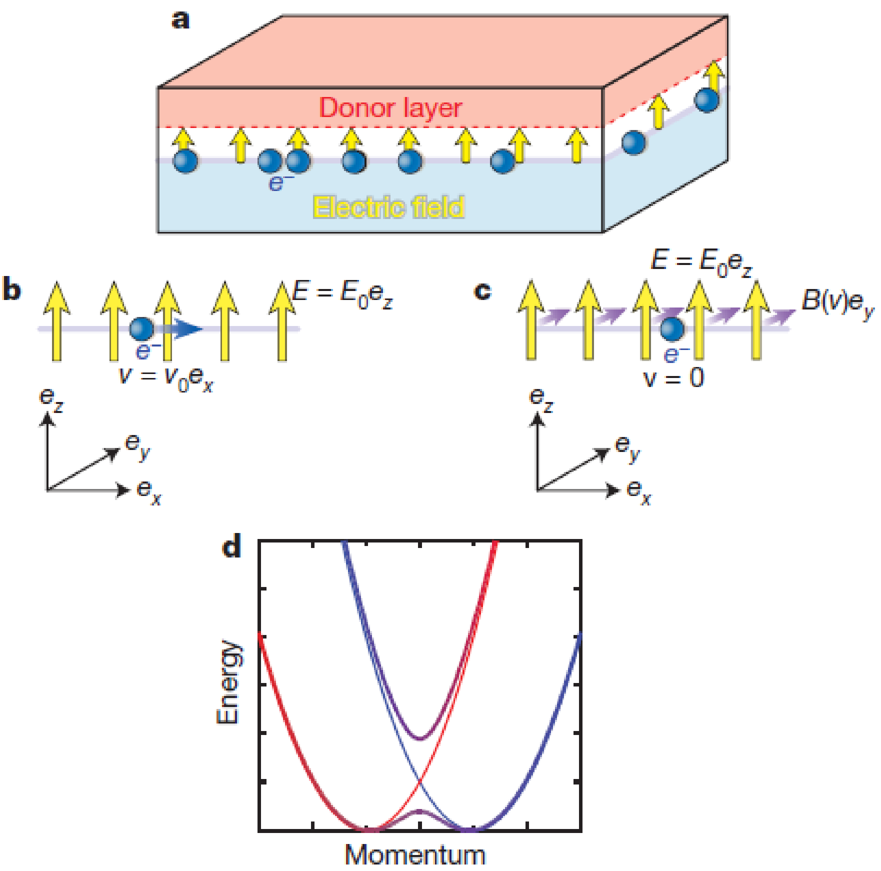
→ Magnetic field in the particle rest frame

$$\mathbf{B}_{\mathrm{SO}} = \frac{E_0 \hbar}{mc^2} \times (k_x \mathbf{e}_y - k_y \mathbf{e}_x)$$

- Coupling of a particle's spin to its momentum

→ **Spin-orbit coupling:**

$$\hat{H}_{\mathrm{SOC}} = -\mu \cdot \mathbf{B}_{\mathrm{SO}} \propto k_x S_y - k_y S_x$$



V. Galitski and Ian B. Spielman, Nature **494**, 49-54 (2013)

The Raman-dressed condensate: synthetic spin-orbit (SO) coupling

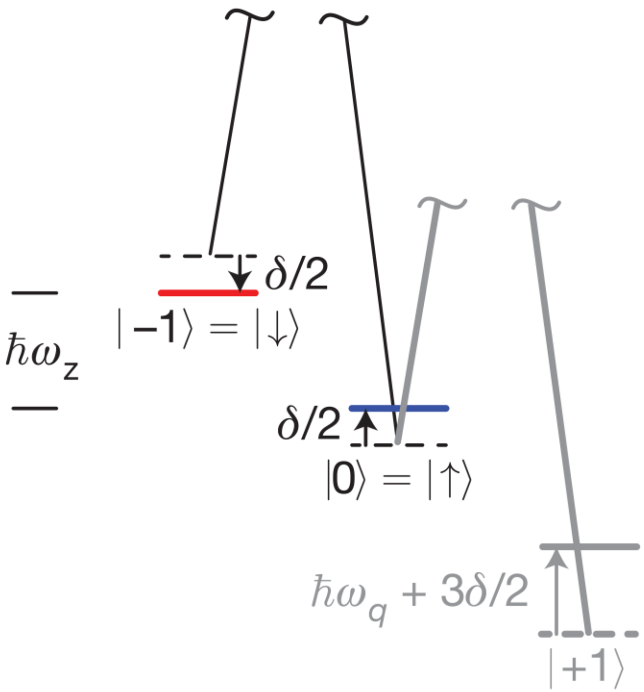
Spin-orbit coupling in ultracold atom gases

- Engineered in neutral ultracold atoms by Raman dressing

Y. –J. Lin, K. Jiménez-García, I.B. Spielman, *Nature* **471**, 83 (2011)

- Coupled Zeeman states as effective spin DOF

- Raman coupling: large recoil momentum $\hbar k_\mathrm{r}$ $\quad E_\mathrm{r} = \dfrac{\hbar^2 k_\mathrm{r}^2}{2m}$



- Dressed system:

$$\hat{\mathcal{H}} = \frac{\mathbf{p}^2}{2m} + \frac{\hbar\omega_\mathrm{z}}{2}\sigma_z + \frac{\hbar\Omega}{2}\sigma_x \cos(2k_r x - \omega t) - \frac{\hbar\Omega}{2}\sigma_y \sin(2k_r x - \omega t)$$

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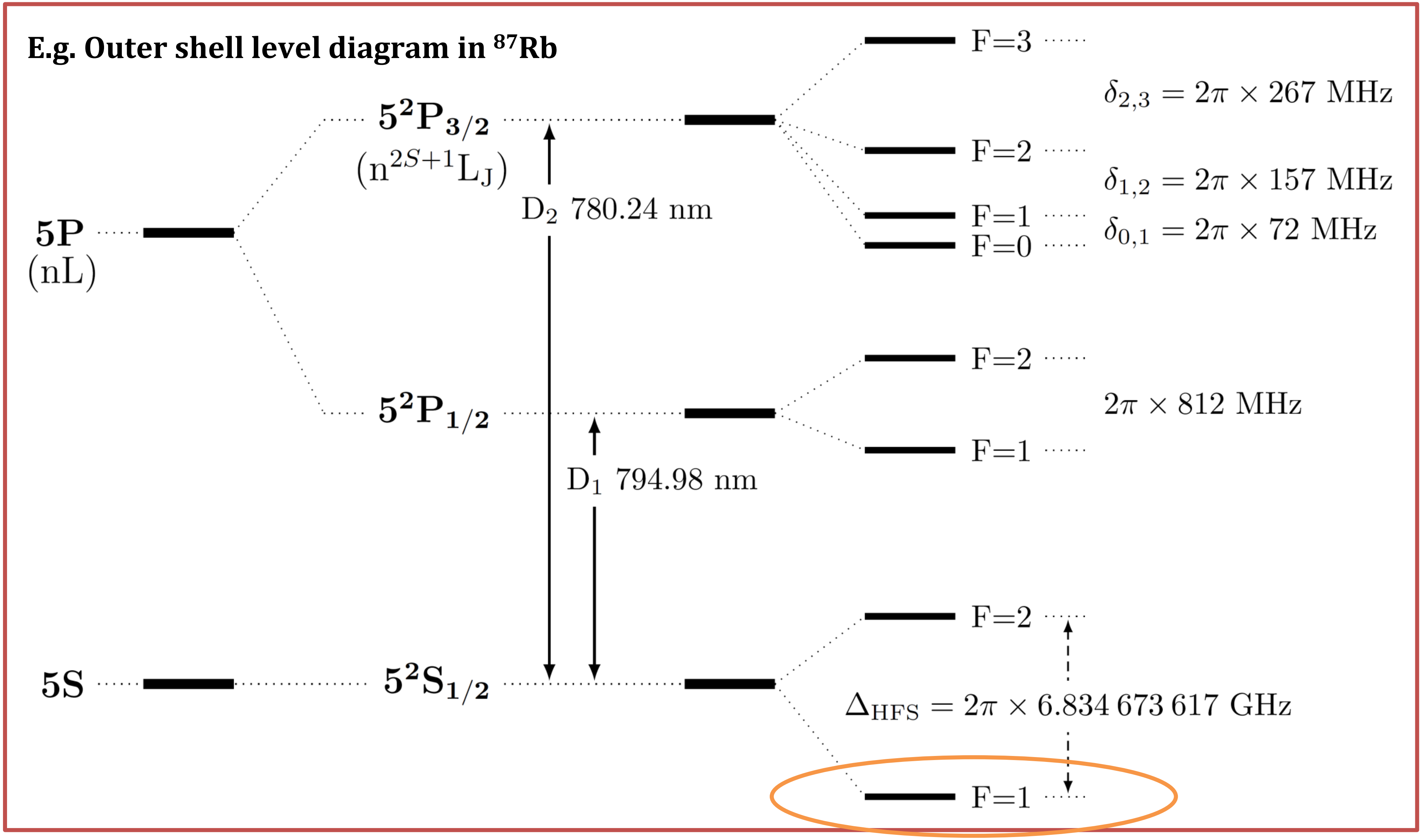
Spin-orbit coupling in ultracold atom gases

E.g. Outer shell level diagram in ^{87}Rb



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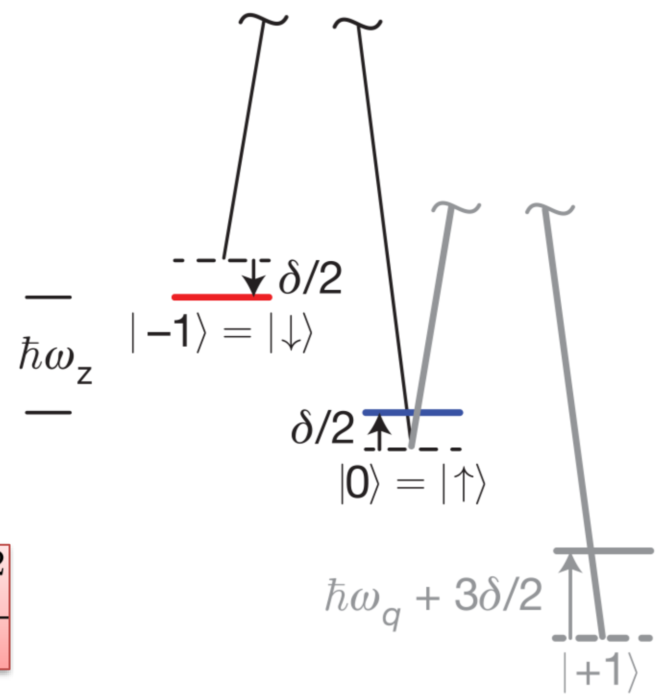

- Dressed system spatially modulated with a short wavelength:

$$\hat{\mathcal{H}} = \frac{\mathbf{p}^2}{2m} + \frac{\hbar\omega_{\mathrm{z}}}{2}\sigma_z + \frac{\hbar\Omega}{2}\sigma_x \cos(2k_r x - \omega t) - \frac{\hbar\Omega}{2}\sigma_y \sin(2k_r x - \omega t)$$

Position dependent transverse "magnetic field"

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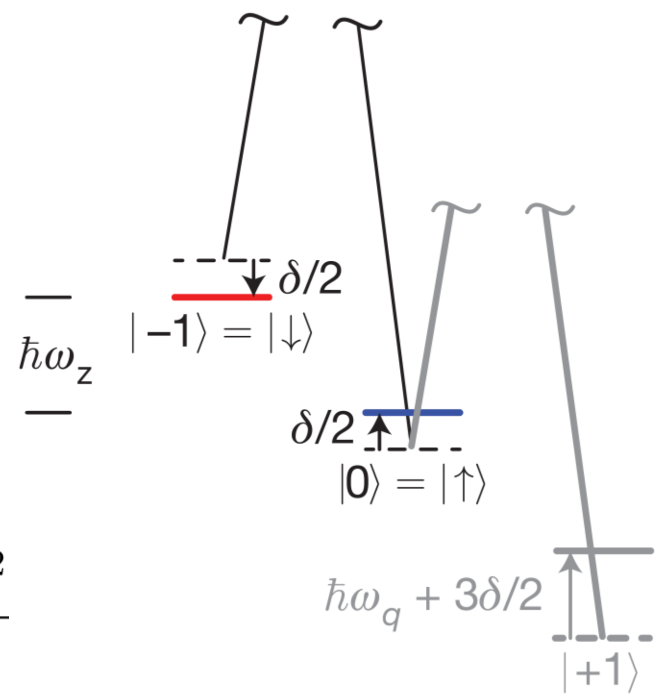


- In the frame co-rotating and co-moving with the Raman beams:

$$\hat{\mathcal{H}} = \frac{\mathbf{p}^2}{2m} - \gamma_\mathrm{RD} p_z S_z + \frac{\hbar\delta}{2}\sigma_z + \frac{\hbar\Omega}{2}\sigma_x$$

- Effective magnetic fields: Raman detuning $\hbar\delta = -g\mu_\mathrm{B} B_z$ Rabi coupling $\hbar\Omega = -g\mu_\mathrm{B} B_x$

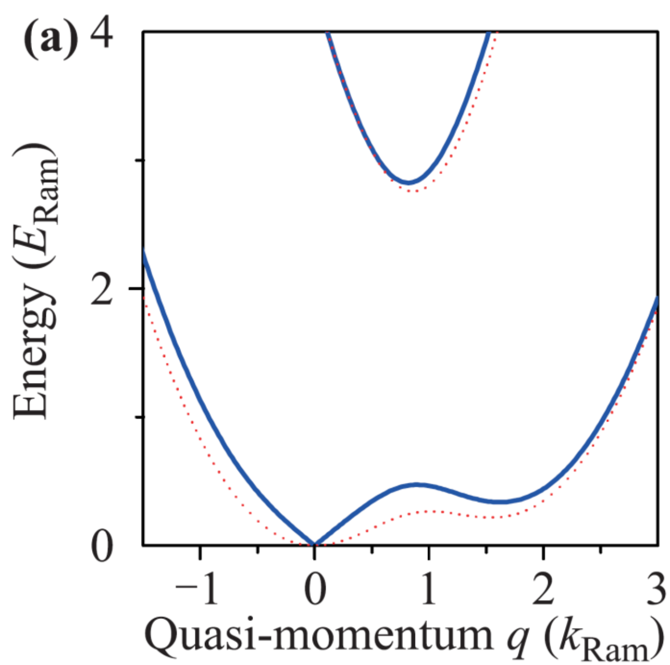
- Effective SOC : $\gamma_\mathrm{RD} = \frac{2k_\mathrm{r}}{m}$

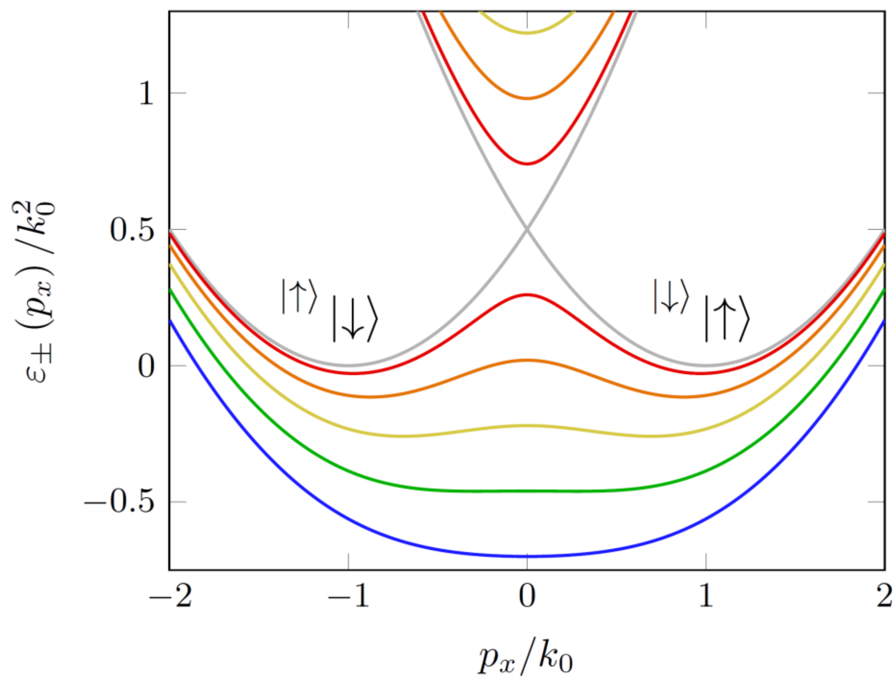
The Raman-dressed condensate: synthetic spin-orbit (SO) coupling

Spin-orbit coupling in ultracold atom gases

- Two tunable single-particle dispersion bands

- Rich interplay between single-particle bands and interatomic interactions





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The Raman-dressed condensate: synthetic spin-orbit (SO) coupling

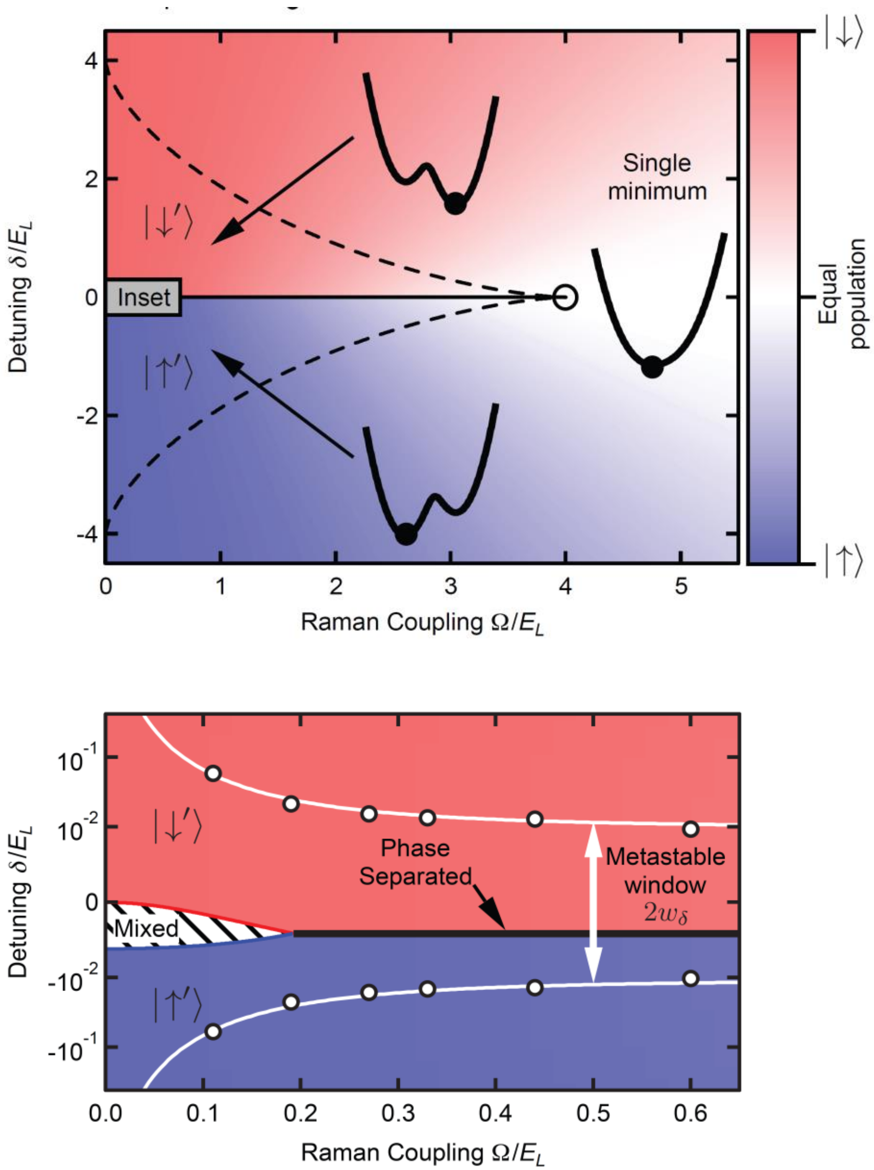
Spin-orbit coupling in ultracold atom gases

- Three many-body ground state phases

  - <u>Plane wave phase</u>: two minima, spontaneously condenses into a non-zero quasimomentum state. **Time reversal symmetry breaking**.

  - <u>Conventional BEC</u>: single minimum at the origin with tunable effective mass.

  - <u>Stripe phase</u>: **spin interactions** stabilize the simultaneous occupation of the two minima. **Continuous translation symmetry breaking.**



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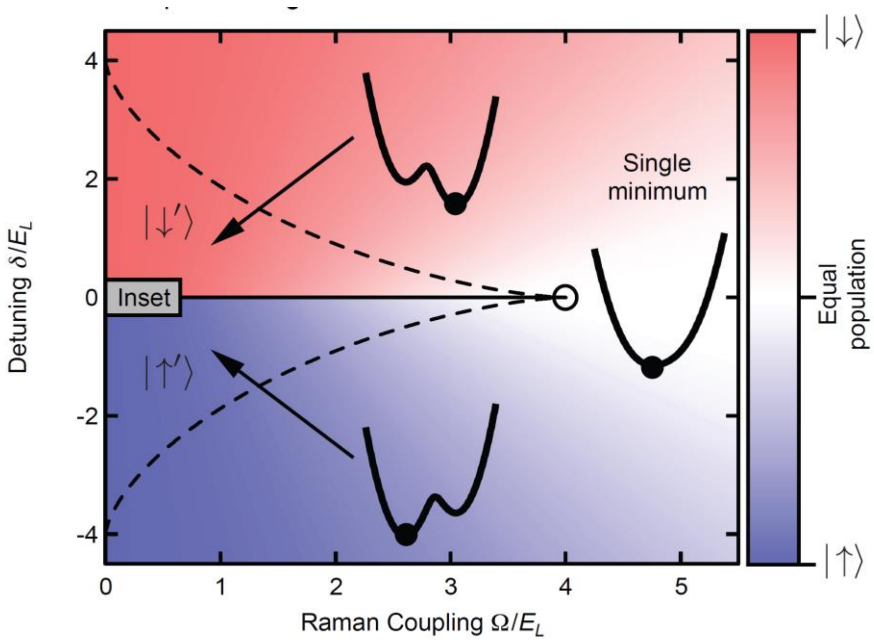

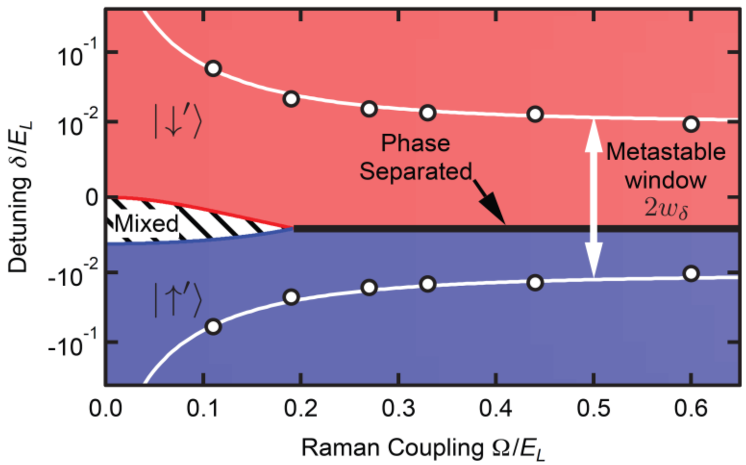

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Stripe phase of the SO coupled gas

"Supersolid-like" stripe phase

- Two-minima regime: two dressed boson fields

$$\tilde{\psi}_\uparrow = U_{\uparrow\uparrow}\psi_\uparrow + U_{\uparrow\downarrow}\psi_\uparrow$$

$$\tilde{\psi}_\downarrow = U_{\downarrow\uparrow}\psi_\uparrow + U_{\downarrow\downarrow}\psi_\uparrow$$

- Phase understood from dressed spin interactions

$$\hat{H}_{\text{int}} = \int d\boldsymbol{r} \left(\frac{\tilde{g}_{nn}}{2}\tilde{n}^2 + \frac{\tilde{g}_{ss}}{2}\tilde{s}_z^2 + \tilde{g}_{ns}\tilde{n}\tilde{s}_z \right)$$

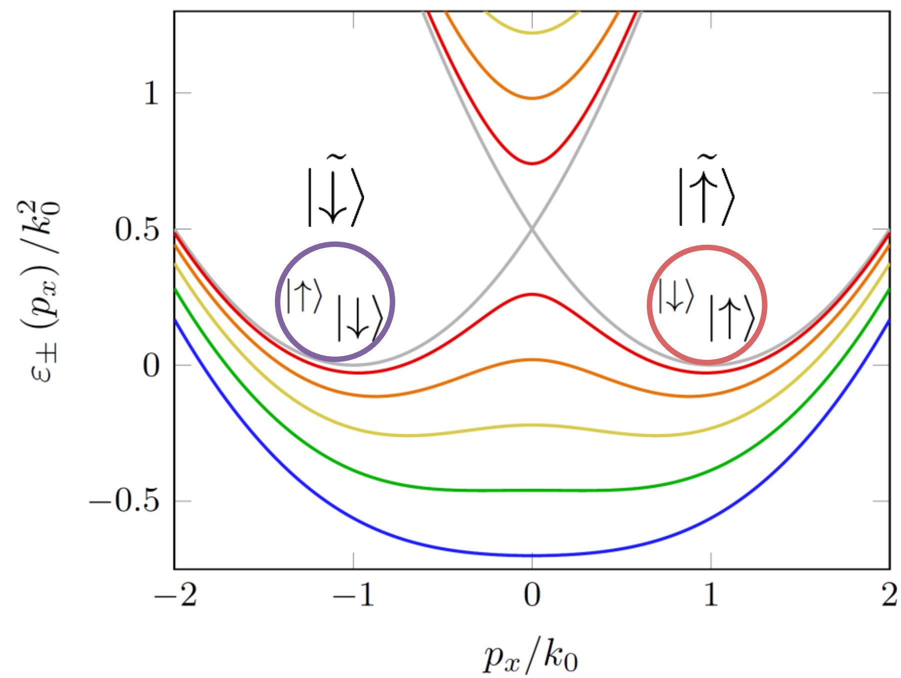
$$\tilde{g}_{ss} = -g_{nn}\frac{\Omega^2}{32} + g_{ss}\left(1 - \frac{\Omega^2}{16}\right)$$

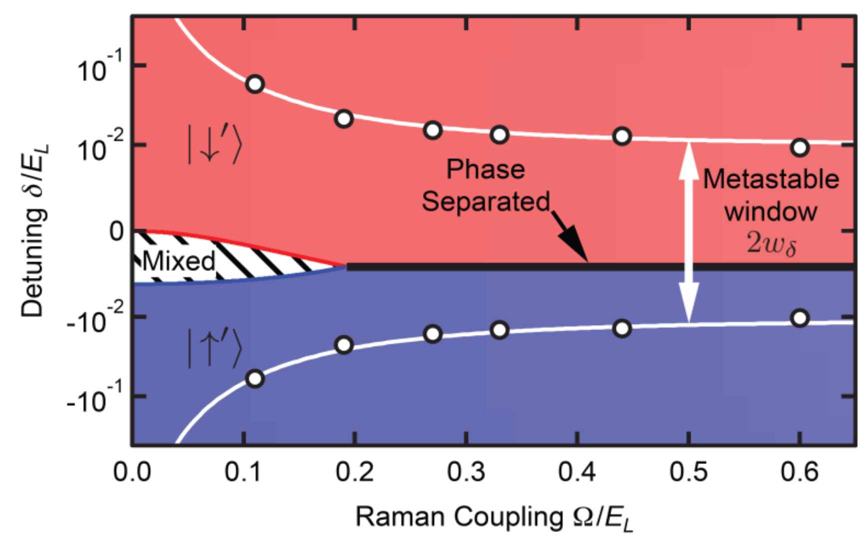
➢ **Miscibility favoured for** $\tilde{g}_{ss} > 0$

- **SO coupling:** dressed states momentum-shifted!

➡ Spatial density modulation $\propto \Omega$

spontaneous breaking of U(1) and translation symmetries





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- Common spinor BECs, such as ^{23}Na and ^{87}Rb, have near symmetric interactions

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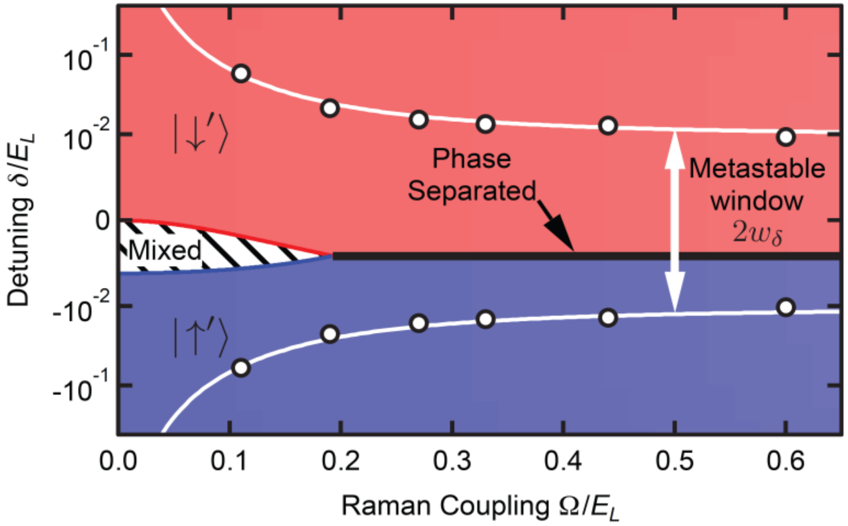

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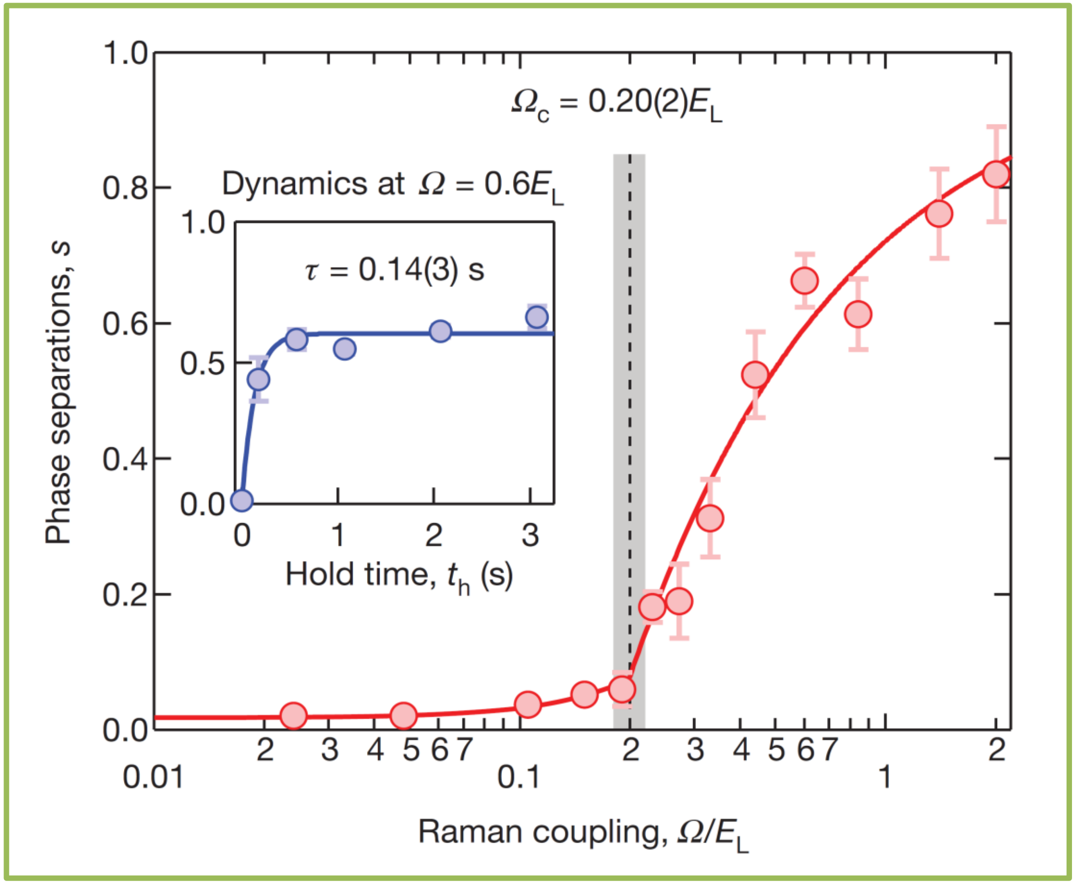

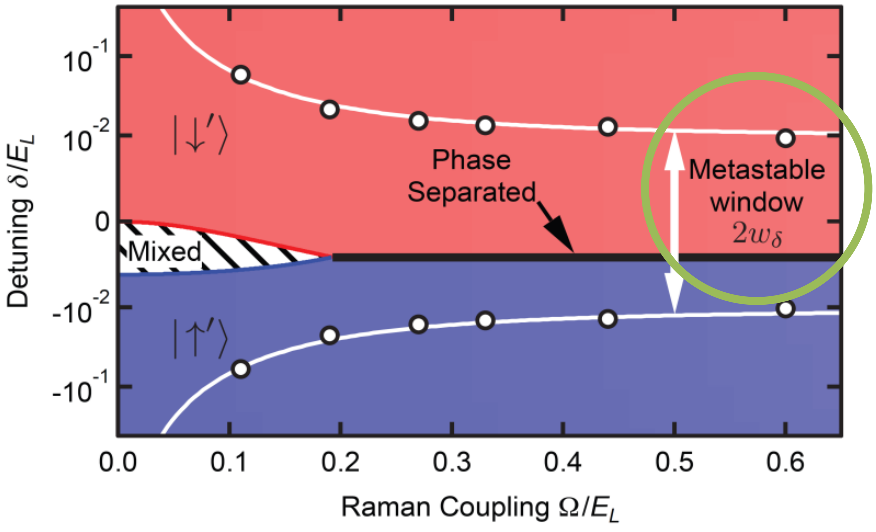

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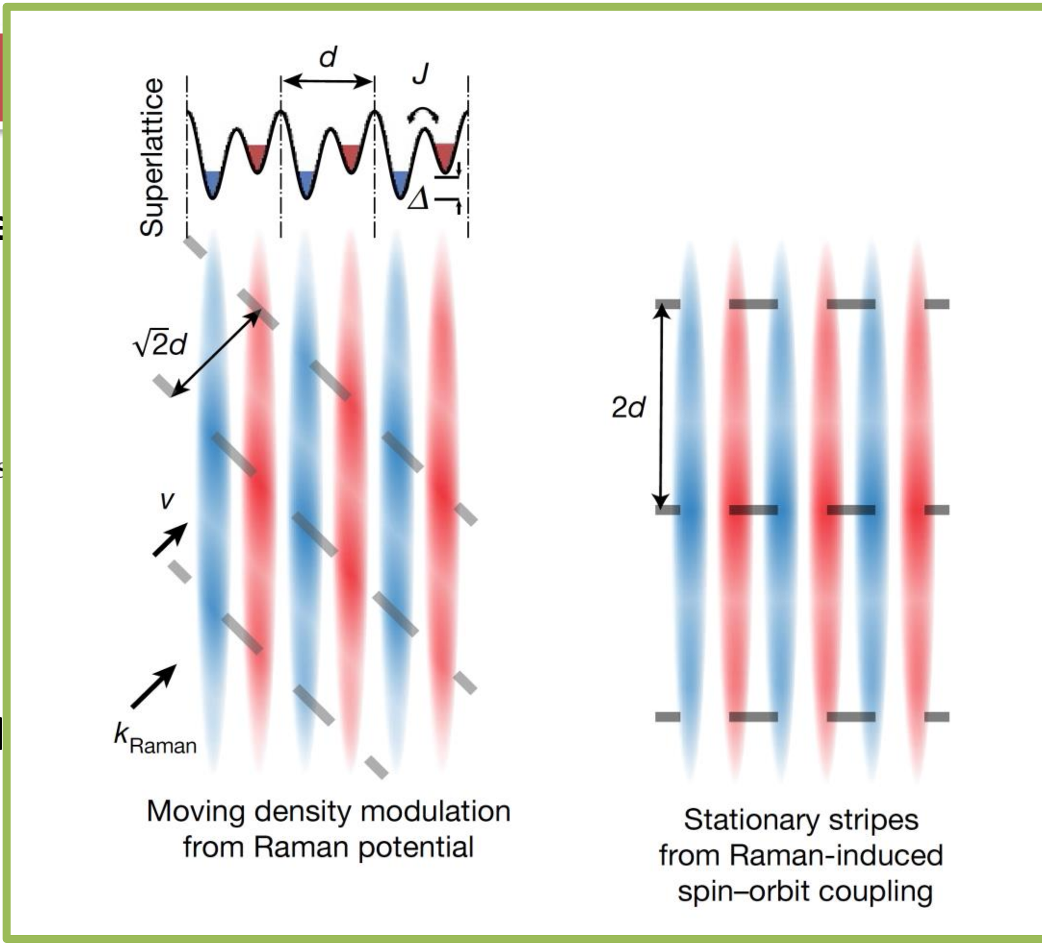

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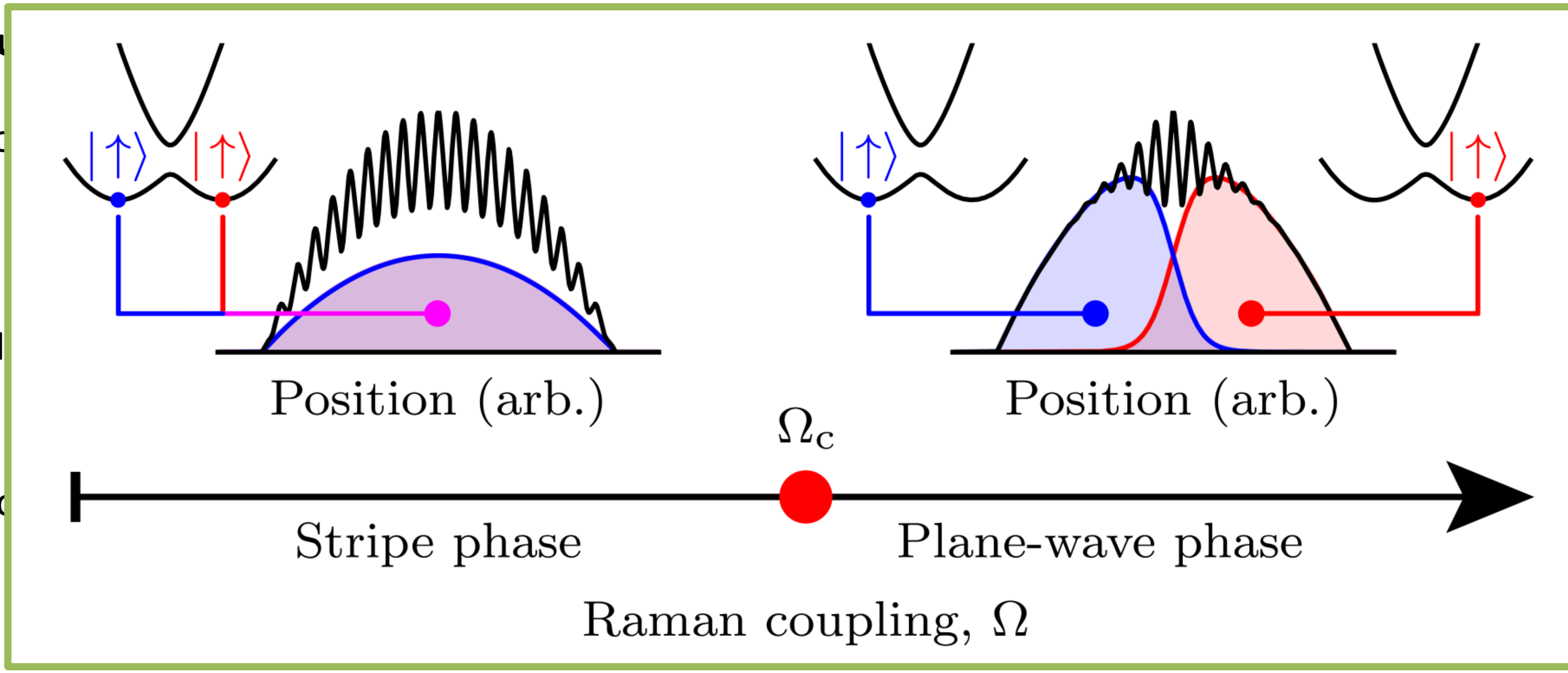

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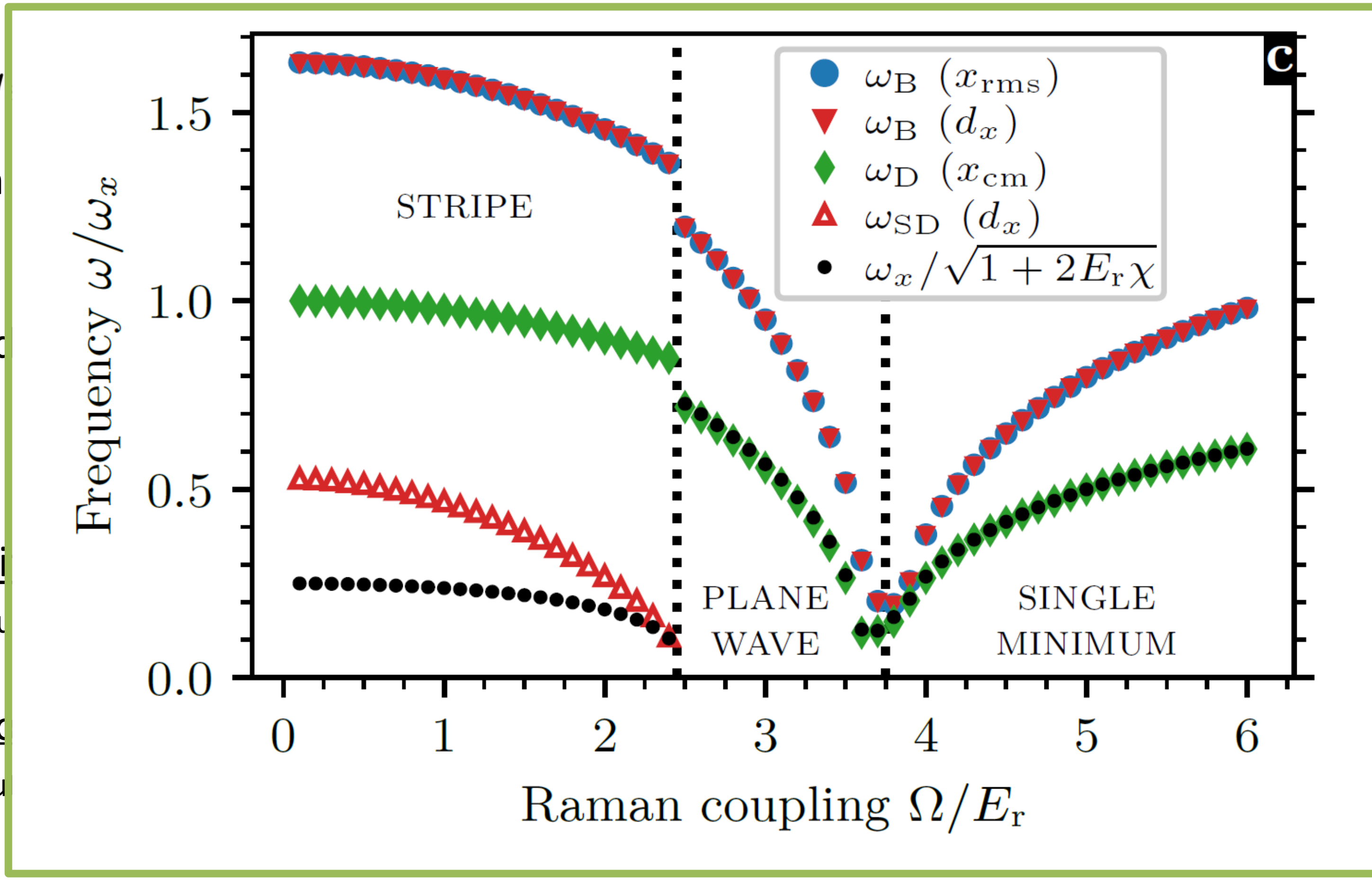

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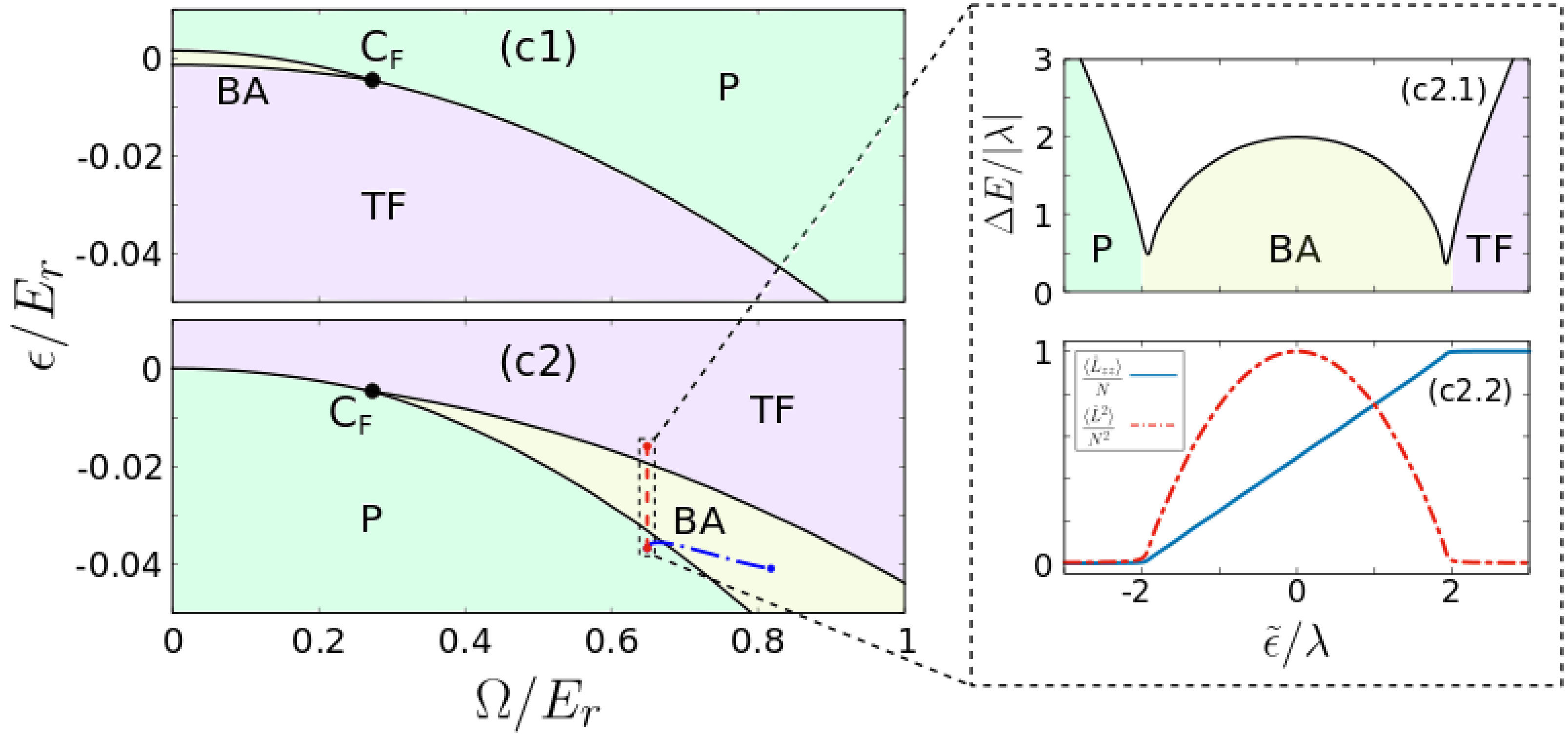
Stripe phase of the SO coupled gas

Stripe phase of the SO coupled gas as an excited phase?



J. Cabedo, J. Claramunt, A. Celi, *arXiv:2101.08253v2* (2021)

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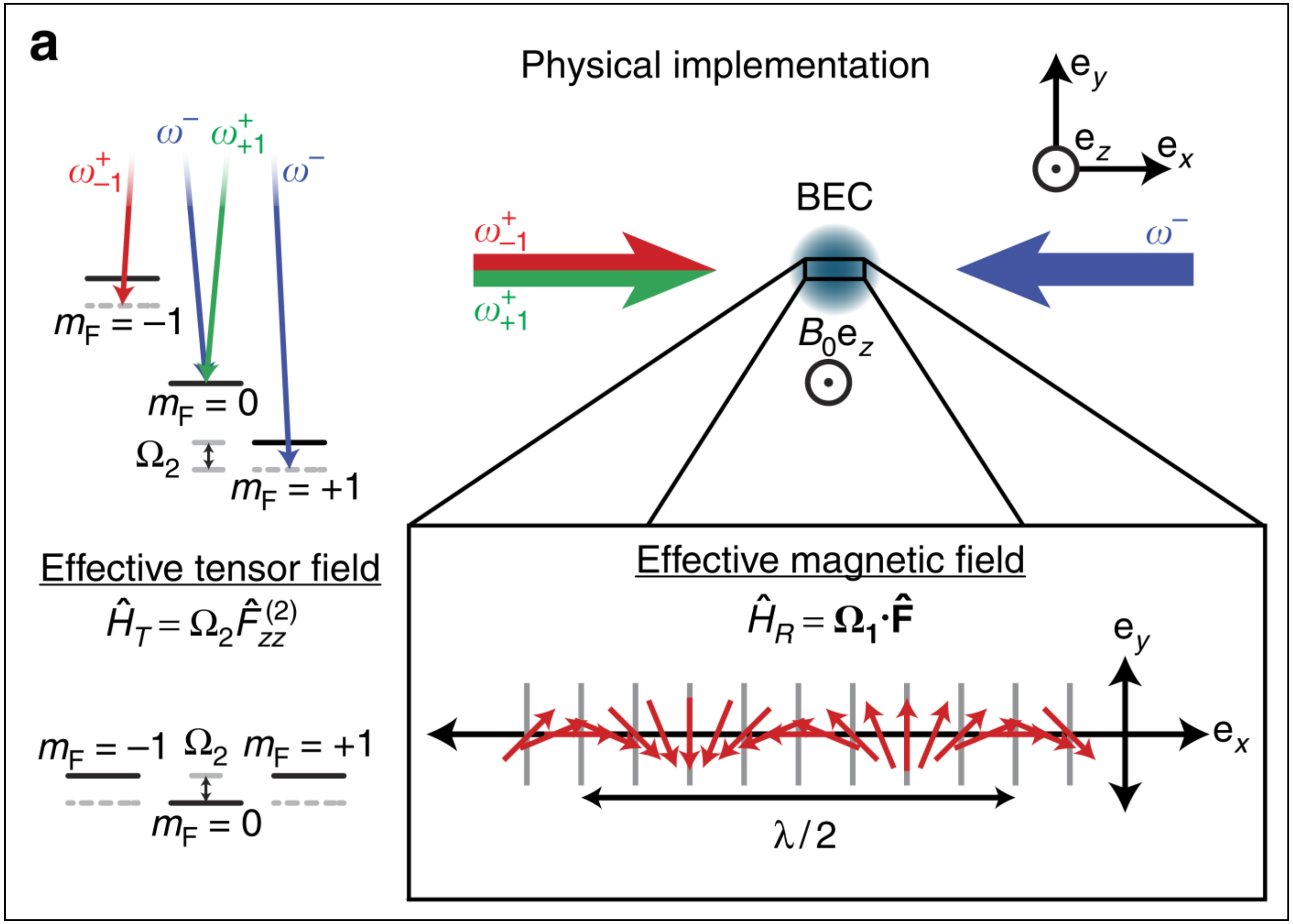
System: Raman-dressed spin-1 Bose gas (e.g. ^{87}Rb, $^{39\text{-}41}$K)



D. L. Campbell, R. M. Price, A. Putra, A. Valdés-Curiel,D. Trypogeorgos, and I. B. Spielman. *Nat. Commun.* **7**, 10897 (2016)

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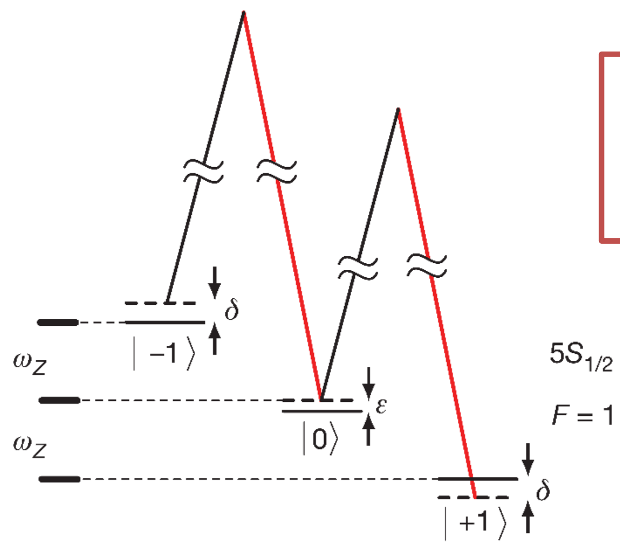

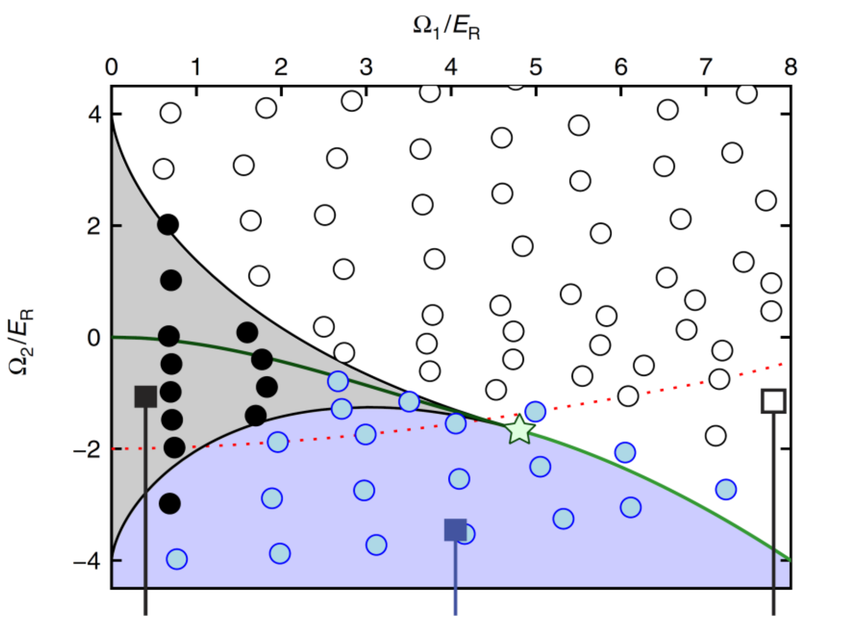

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Raman-dressed spin-1 Bose gas (e.g. ^{87}Rb, $^{39\text{-}41}$K)

$$\hat{H} = d\boldsymbol{r}\Big[\hat{\psi}^{\dagger}\Big(\hat{\mathcal{H}}_{\mathrm{k}} + V_{\mathrm{t}}\Big)\,\hat{\psi} + \frac{g_0}{2}(\hat{\psi}^{\dagger}\hat{\psi})^2 + \frac{g_2}{2}\sum_j(\hat{\psi}^{\dagger}\hat{F}_j\hat{\psi})^2\Big]$$

- In the frame co-rotating with the Raman beams:

$$\hat{\mathcal{H}}_{\mathrm{k}} = \frac{\hbar^2}{2m}\left(\boldsymbol{k} - 2k_r\hat{F}_z\boldsymbol{e}_z\right)^2 + \frac{\Omega}{\sqrt{2}}\hat{F}_x + \delta\hat{F}_z + \epsilon\hat{F}_z^2$$

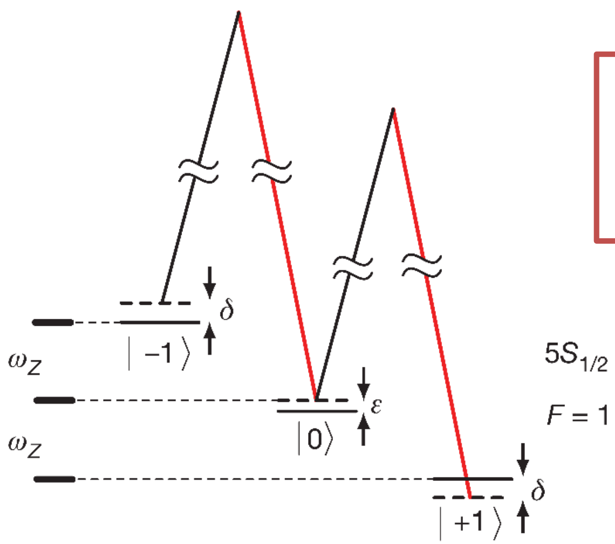
- Energy & momentum transfer $E_r = \frac{\hbar^2 k_r^2}{2m} \quad \hbar k_r$

- Effective magnetic fields $\delta = -g\mu_{\mathrm{B}}B_z \quad \epsilon = \hbar\epsilon_{\mathrm{B}}B_z^2$

$$\Omega = -g\mu_{\mathrm{B}}B_x$$

- Synthetic SO coupling $\hat{H}_{SO} = \gamma\boldsymbol{S}_z\cdot\boldsymbol{p}\,,\ \gamma = \frac{2k_r}{m}$

Tunable spin-changing collisions from synthetic SO coupling



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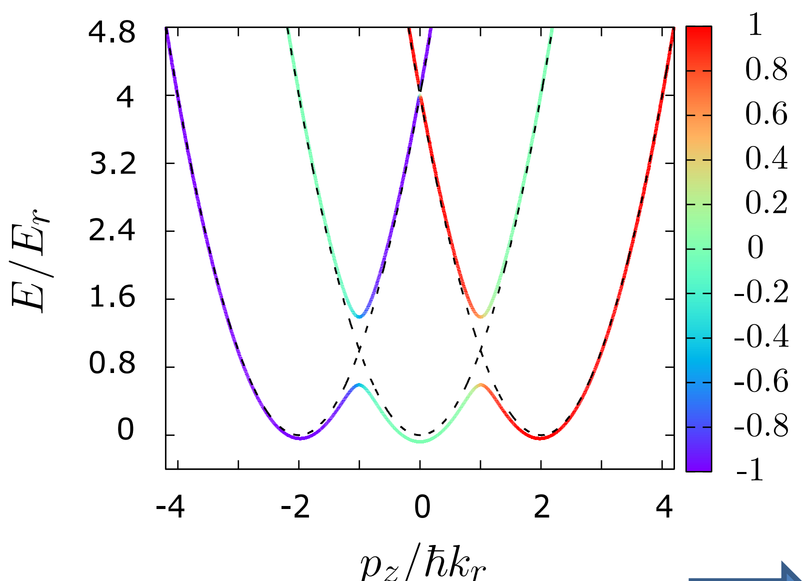
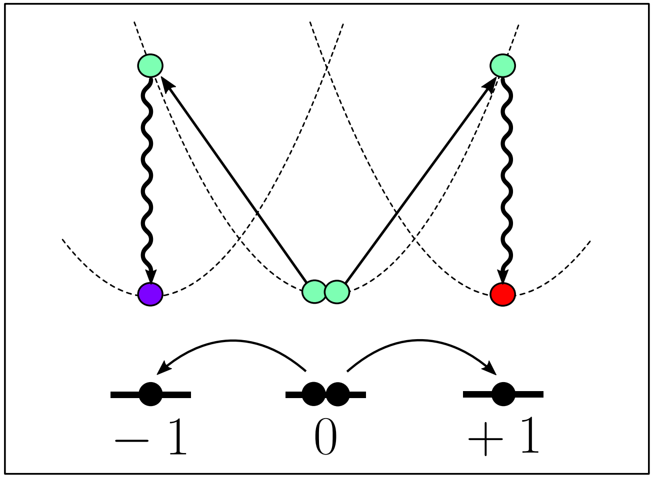
- Triple-well s.p. dispersion band at weak couplings $\Omega < 4E_r$





Dressing-induced **spin-changing collisions**

Tunable spin-changing collisions from synthetic SO coupling

Raman-dressed spin-1 Bose gas (e.g. ^{87}Rb, $^{39\text{-}41}$K)

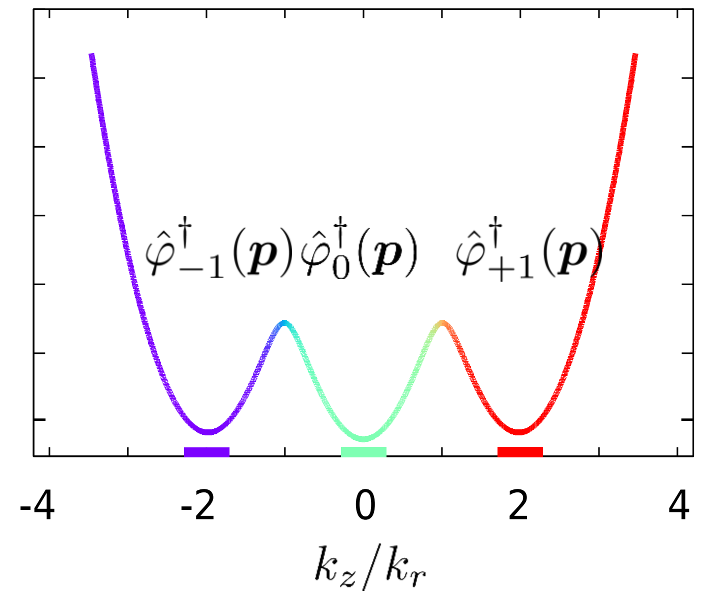
Low-energy effective theory

- Weakly-coupled, weakly-trapped, weakly-interacting regime

$$\Omega < E_r \qquad \hbar\omega_t, g_0 n, \delta, \epsilon \ll E_r \qquad V_t = \frac{1}{2} m\omega^2 \boldsymbol{r}^2$$

- Truncate field operator to the lowest-band contributions $\hat{\varphi}_j^\dagger(\boldsymbol{p})$ around $\boldsymbol{k}_j \sim j2k_r$

Cut-off in the spread $\boldsymbol{p}$ around $\hbar\boldsymbol{k}_j$: $|p_z| < \Lambda \ll \hbar k_r$



- Pseudo-spinor field $\hat{\boldsymbol{\varphi}} = (\hat{\varphi}_{-1}, \hat{\varphi}_0, \hat{\varphi}_1)^T$

$$\left[\hat{\varphi}_i(\boldsymbol{p}), \hat{\varphi}_j^\dagger(\boldsymbol{p}')\right] = \delta(\boldsymbol{p}-\boldsymbol{p}')\delta_{ij}$$

Tunable spin-changing collisions from synthetic SO coupling

Raman-dressed spin-1 Bose gas (e.g. ^{87}Rb, $^{39-41}$K)

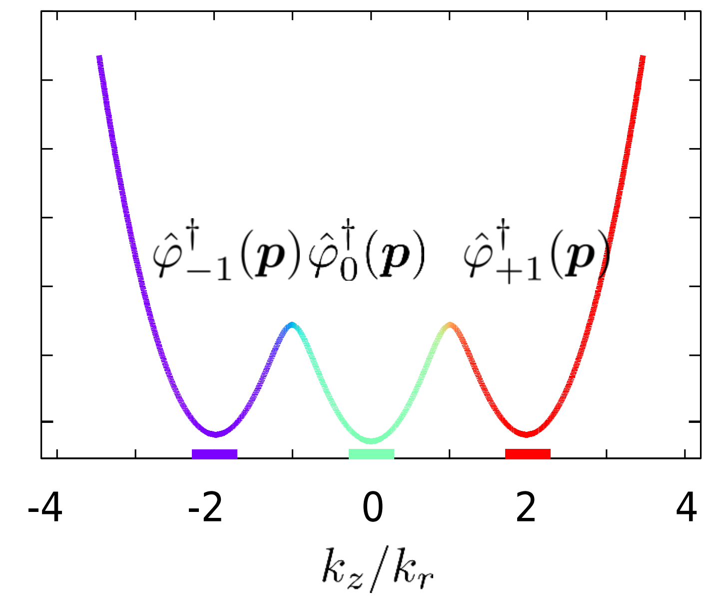
Low-energy effective theory $\Omega < E_r$ $\hbar\omega_{\rm t}, g_0 n, \delta, \epsilon \ll E_r$

- Perturbation theory to 2nd order in $\Omega/4E_r$ (and dropping terms $\propto \Lambda\Omega^2,\ g_2\Omega^2, \omega_{\rm t}^2\Omega^2$)

$$\hat{H} \simeq \hat{H}_{\rm S} + \hat{H}_{\rm A}$$

$$\hat{H}_{\rm S} = \int d\boldsymbol{r} \left[\sum_i \hat{\varphi}_i^\dagger \left(\frac{\boldsymbol{p}^2}{2m} + V_{\rm t} \right) \hat{\varphi}_i + \frac{g_0}{2} \sum_{ij} \hat{\varphi}_i^\dagger \hat{\varphi}_j^\dagger \hat{\varphi}_j \hat{\varphi}_i \right],$$

$$\hat{H}_{\rm A} = \int d\boldsymbol{r} \Bigg[\frac{g_2}{2} \sum_j (\hat{\boldsymbol{\varphi}}^\dagger \hat{F}_j \hat{\boldsymbol{\varphi}})^2 + \tilde{g}_2 \left(\hat{\varphi}_1^\dagger \hat{\varphi}_1 + \hat{\varphi}_{-1}^\dagger \hat{\varphi}_{-1} \right) \hat{\varphi}_0^\dagger \hat{\varphi}_0$$
$$+ \tilde{g}_2 \left(\hat{\varphi}_1^\dagger \hat{\varphi}_{-1}^\dagger \hat{\varphi}_0 \hat{\varphi}_0 + {\rm H.c} \right) + \hat{\boldsymbol{\varphi}}^\dagger \left(\delta \hat{F}_z + \tilde{\epsilon} \hat{F}_z^2 \right) \hat{\boldsymbol{\varphi}} \Bigg]$$



$$\tilde{\epsilon} = \epsilon + \frac{\Omega^2}{16E_r}$$

$$\tilde{g}_2 \simeq g_0 \frac{\Omega^2}{16E_r^2}$$

Tunable spin-changing collisions from synthetic SO coupling

Raman-dressed spin-1 Bose gas (e.g. ^{87}Rb, $^{39\text{-}41}$K)

Low-energy effective theory $\quad \Omega < E_r \quad\quad \hbar\omega_{\rm t}, g_0 n, \delta, \epsilon \ll E_r$

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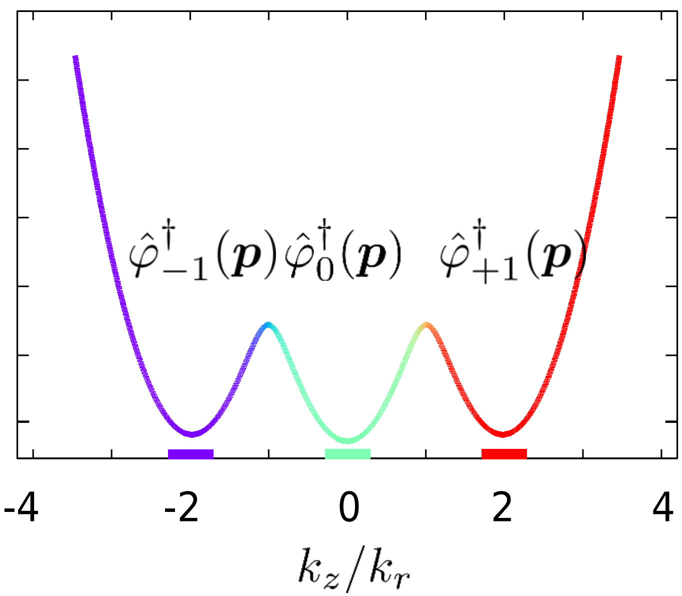
$$\hat{H}_{\rm S} = \int d\boldsymbol{r} \left[\sum_i \hat{\varphi}_i^\dagger \left(\frac{\boldsymbol{p}^2}{2m} + V_{\rm t} \right) \hat{\varphi}_i + \frac{g_0}{2} \sum_{ij} \hat{\varphi}_i^\dagger \hat{\varphi}_j^\dagger \hat{\varphi}_j \hat{\varphi}_i \right],$$

$$\hat{H}_{\rm A} = \int d\boldsymbol{r} \left[\frac{g_2}{2} \sum_j (\hat{\boldsymbol{\varphi}}^\dagger \hat{F}_j \hat{\boldsymbol{\varphi}})^2 + \tilde{g}_2 \left(\hat{\varphi}_1^\dagger \hat{\varphi}_1 + \hat{\varphi}_{-1}^\dagger \hat{\varphi}_{-1} \right) \hat{\varphi}_0^\dagger \hat{\varphi}_0 \right.$$
$$\left. + \tilde{g}_2 \left(\hat{\varphi}_1^\dagger \hat{\varphi}_{-1}^\dagger \hat{\varphi}_0 \hat{\varphi}_0 + \text{H.c} \right) + \hat{\boldsymbol{\varphi}}^\dagger \left(\delta \hat{F}_z + \tilde{\epsilon} \hat{F}_z^2 \right) \hat{\boldsymbol{\varphi}} \right]$$



$$\tilde{\epsilon} = \epsilon + \frac{\Omega^2}{16E_r}$$

$$\tilde{g}_2 \simeq g_0 \frac{\Omega^2}{16E_r^2}$$

Tunable spin-changing collisions from synthetic SO coupling

Raman-dressed spin-1 Bose gas (e.g. ^{87}Rb, $^{39-41}$K)

Low-energy effective theory $\Omega < E_r$ $\hbar\omega_{\rm t}, g_0 n, \delta, \epsilon \ll E_r$

- Perturbation theory to 2nd order in $\Omega/4E_r$ (and dropping terms $\propto \Lambda\Omega^2,\ g_2\Omega^2, \omega_{\rm t}^2\Omega^2$)

$$\hat{H} \simeq \hat{H}_{\rm S} + \hat{H}_{\rm A}$$

$$\hat{H}_{\rm S} = \int d\boldsymbol{r} \left[\sum_i \hat{\varphi}_i^\dagger \left(\frac{\boldsymbol{p}^2}{2m} + V_{\rm t} \right) \hat{\varphi}_i + \frac{g_0}{2} \sum_{ij} \hat{\varphi}_i^\dagger \hat{\varphi}_j^\dagger \hat{\varphi}_j \hat{\varphi}_i \right],$$

Nonsymmetric

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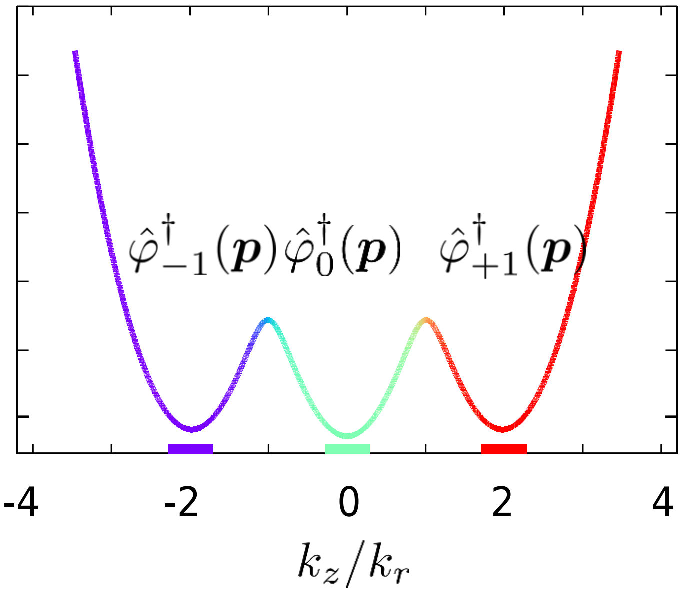

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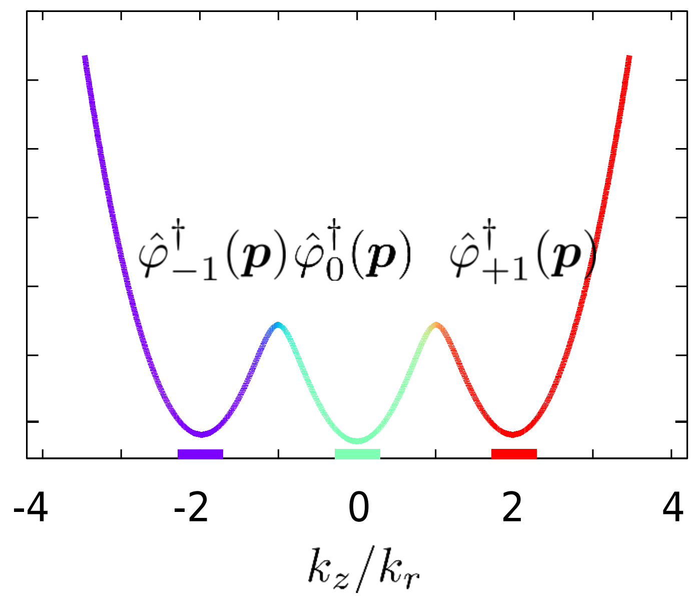
Raman-induced spin-changing collisions



$$\tilde{\epsilon} = \epsilon + \frac{\Omega^2}{16E_r}$$

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III. **Conclusion and outlook**

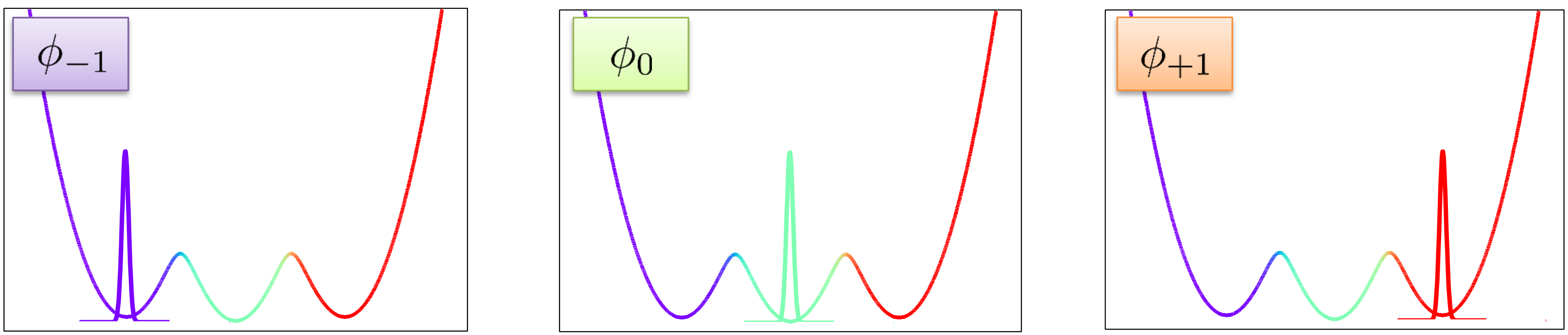
Three-mode model: effective spin Hamiltonian

$$\hat{H} \simeq \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{A}}$$

- Treat $\hat{H}_{\mathrm{A}}$ as a perturbation: $\tilde{g}_2, g_2 \ll g_0$ and $\tilde{g}_2 n, g_2 n \ll \hbar\omega$

- Truncate the field operators to 3 eigenmodes of $\hat{H}_{\mathrm{S}}$

$$\hat{\varphi}_i^\dagger(\boldsymbol{r}) \sim \phi_i(\boldsymbol{r})\hat{b}_i^\dagger$$



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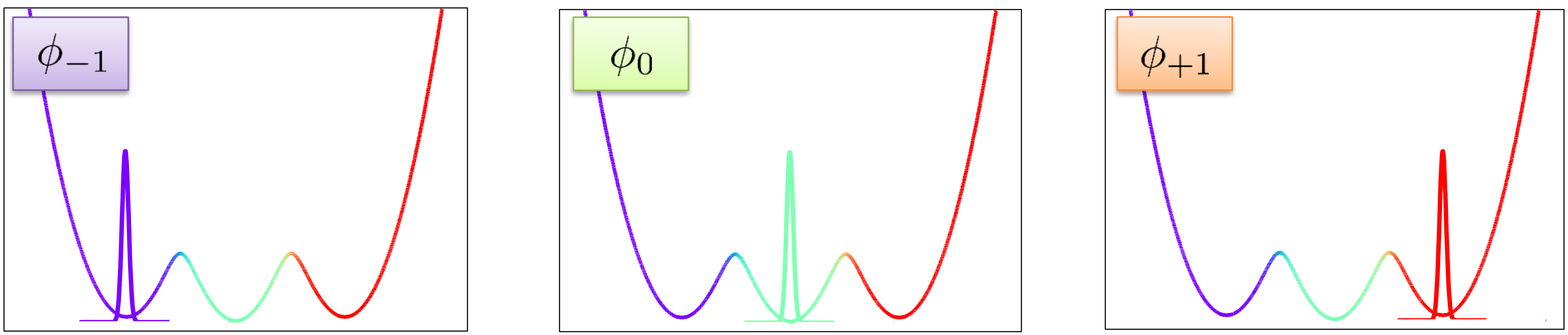

- Collective pseudospin operators $\hat{L}_{x,y,z} = \sum_{\mu\nu} \hat{b}_\mu^\dagger (\hat{F}_{x,y,z})_{\mu\nu} \hat{b}_\nu$ $\hat{L}_{zz} = \sum_{\mu\nu} \hat{b}_\mu^\dagger (\hat{F}_z^2)_{\mu\nu} \hat{b}_\nu$

$$\hat{H}_\mathrm{eff} \simeq H_A \simeq \frac{\tilde{g}_2 n}{2N}\left(\hat{L}_x^2 + \hat{L}_y^2\right) + \frac{g_2 n}{2N}\hat{L}^2 + \delta\hat{L}_z + \tilde{\epsilon}\hat{L}_{zz}$$

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$$\hat{H}_{ss} = \frac{\tilde{g}_2 n}{2N}\left(\hat{L}_x^2 + \hat{L}_y^2\right) + \frac{g_2 n}{2N}\hat{L}^2$$

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- Block-orthogonal in magnetization subspaces $\left[\hat{H}_{\rm eff}, \hat{L}_z\right] = 0$ $\quad \hat{H}_{\rm eff} = \sum_m \hat{H}_m$

$$\Rightarrow \quad \boxed{\hat{H}_0 = \lambda \frac{\hat{L}^2}{2N} + \tilde{\epsilon}\hat{L}_{zz}} \qquad \lambda = (\tilde{g}_2 + g_2)n$$

C. K. Law et al, *Phys. Rev. Lett.* **81**, 5257 (1998)

Three-mode model: effective spin Hamiltonian

$$\hat{H}_0 = \lambda \frac{\hat{L}^2}{2N} + \tilde{\epsilon} \hat{L}_{zz}$$

- “Synthetic” gas with tunable spin-spin interactions

$$\lambda = (\tilde{g}_2 + g_2) n \simeq \left(g_2 + g_0 \frac{\Omega^2}{16 E_r^2} \right) n$$

- Entanglement generation, nonequilibrium dynamics, dynamical and ESQPTs…

- Additional correlation spin-momentum due to SO coupling

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Adiabatic quenches through ESQPTs

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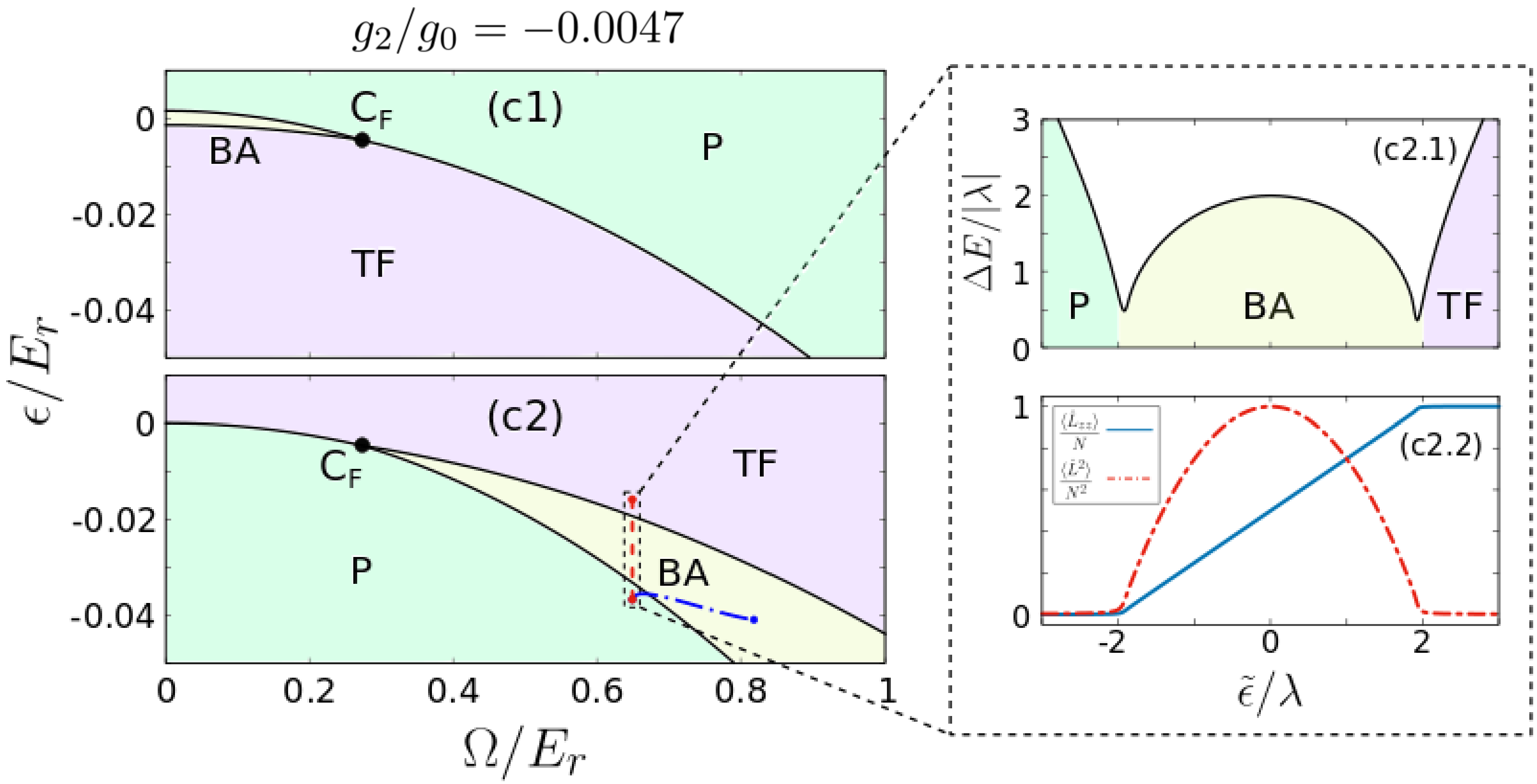

J. Cabedo, J. Claramunt, A. Celi, *arXiv:2101.08253v2* (2021)

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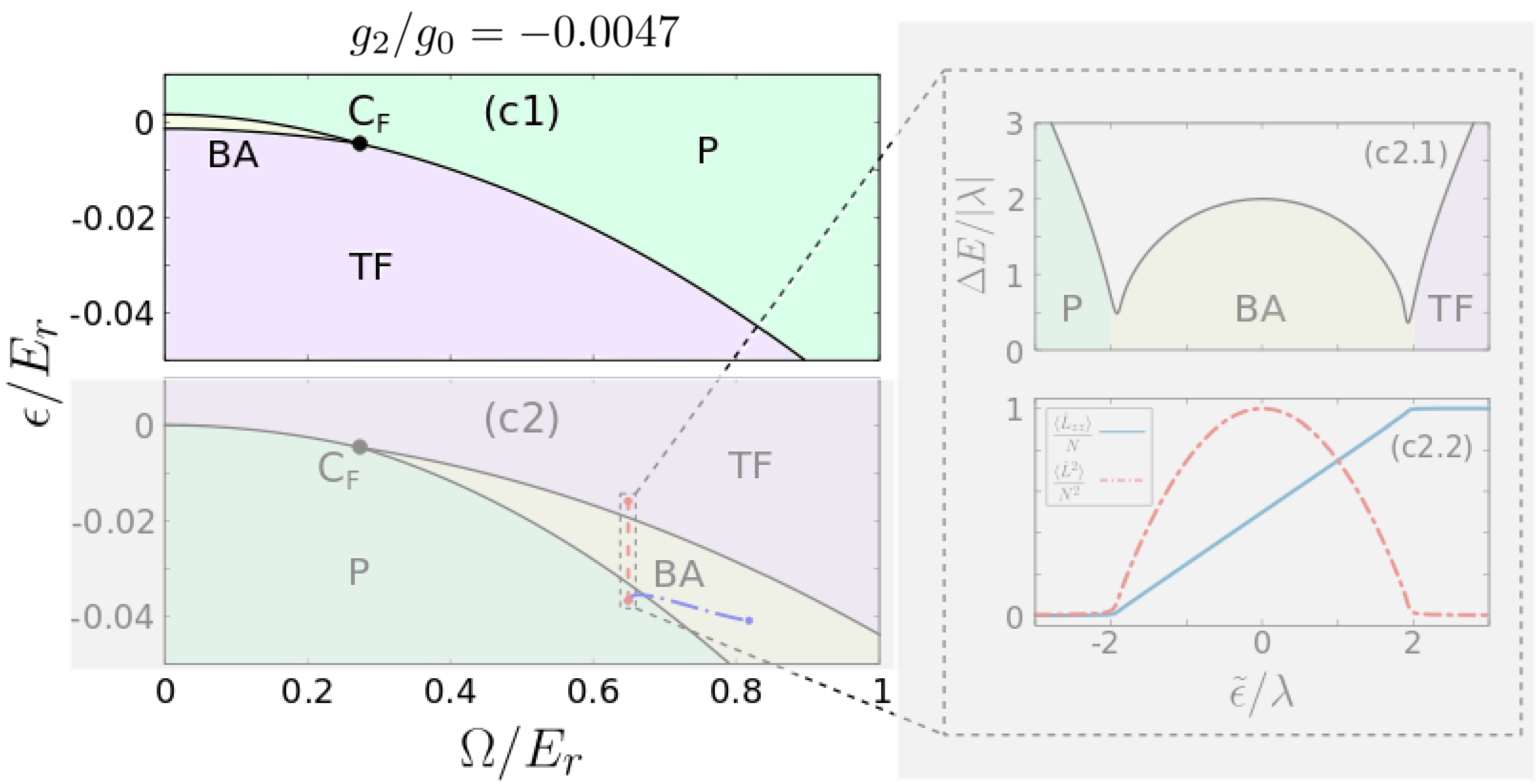

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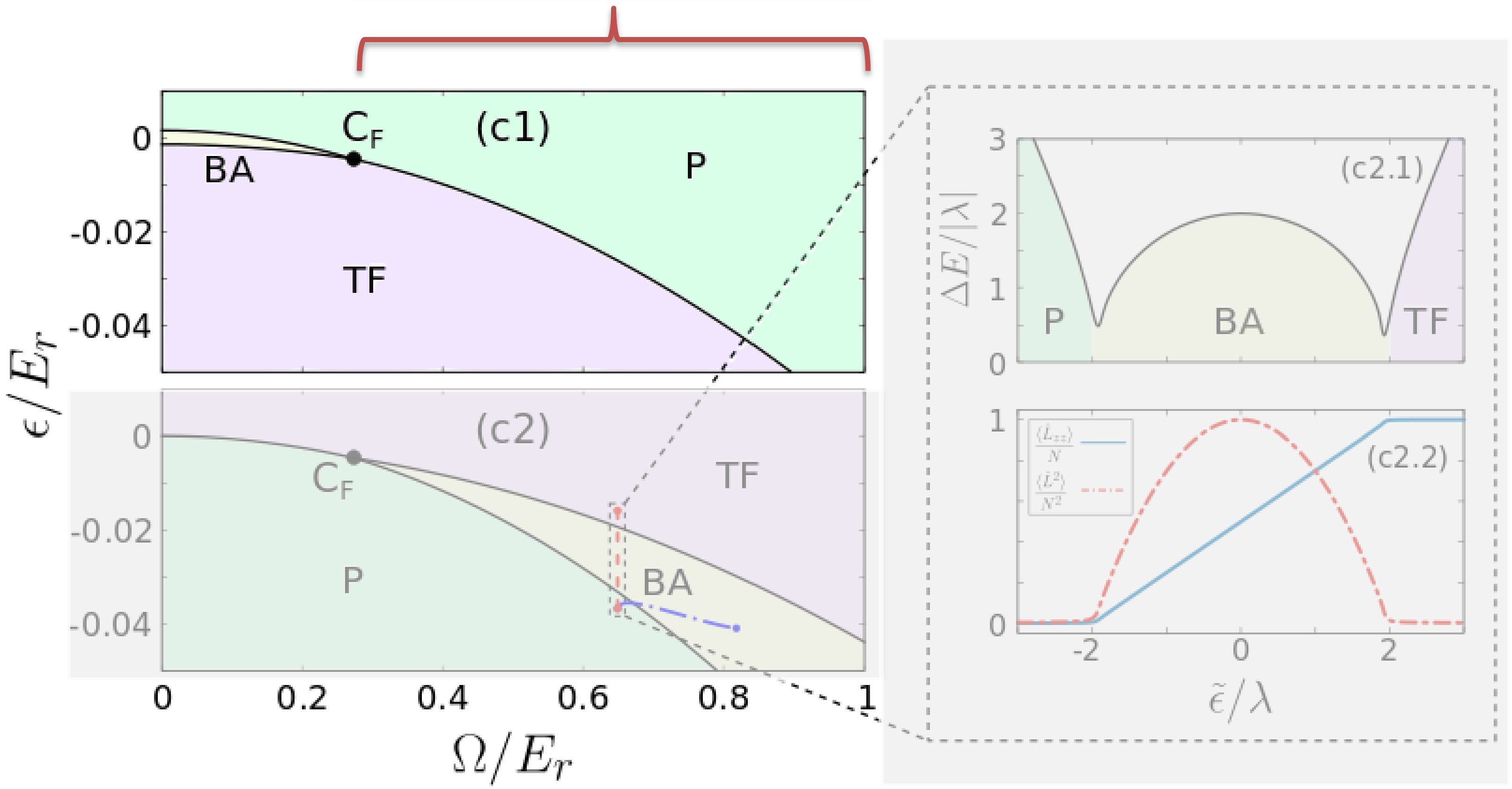

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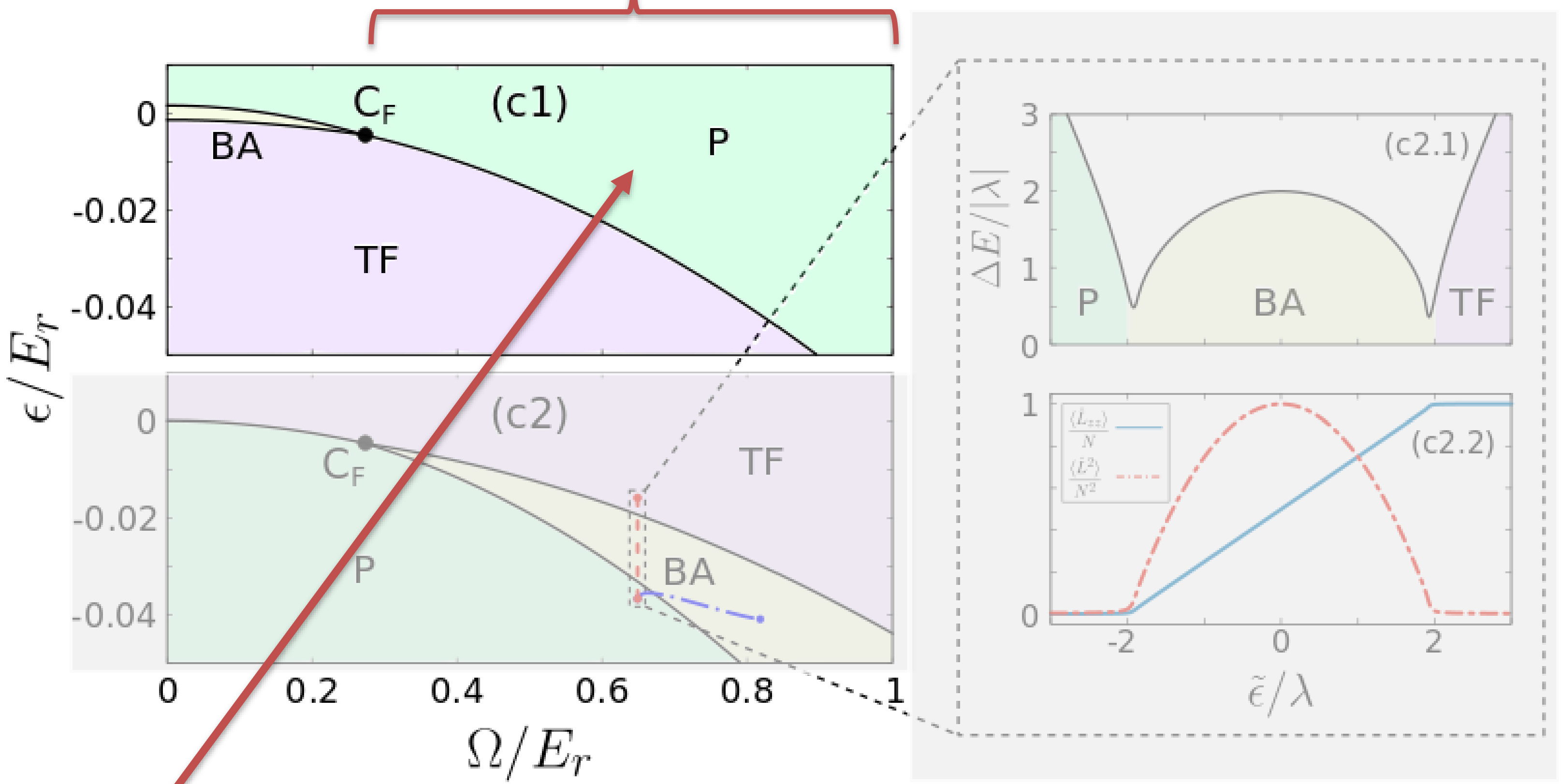

Polar (P) phase $\quad |\psi_{gs}\rangle = \frac{1}{\sqrt{N!}} (\hat{b}_0^\dagger)^N |0\rangle , \quad \tilde{\epsilon}(\Omega) > 0$

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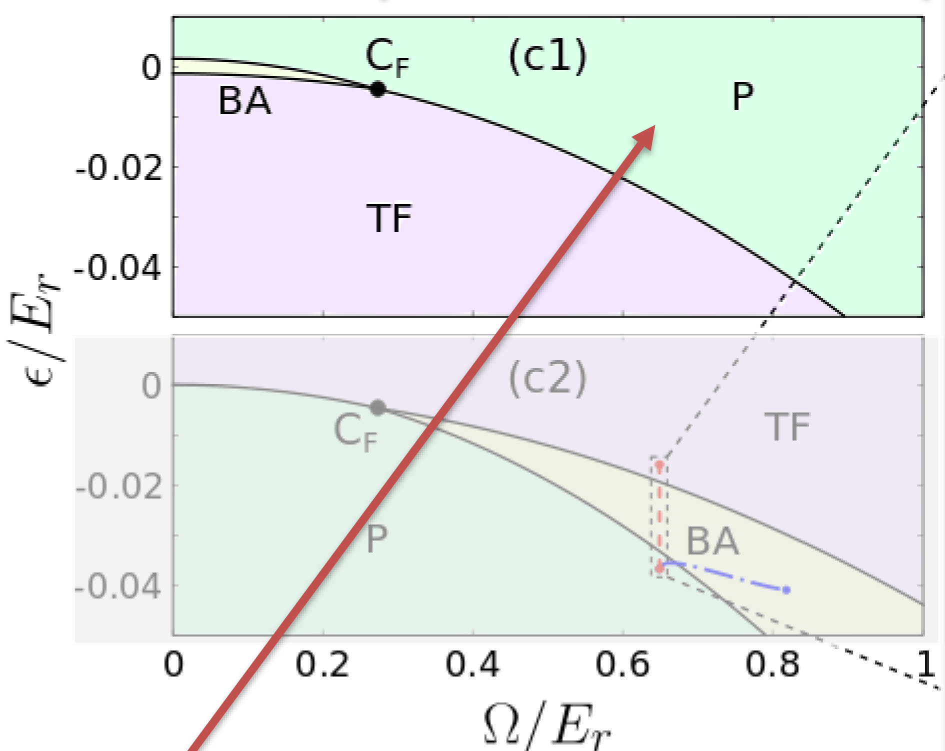

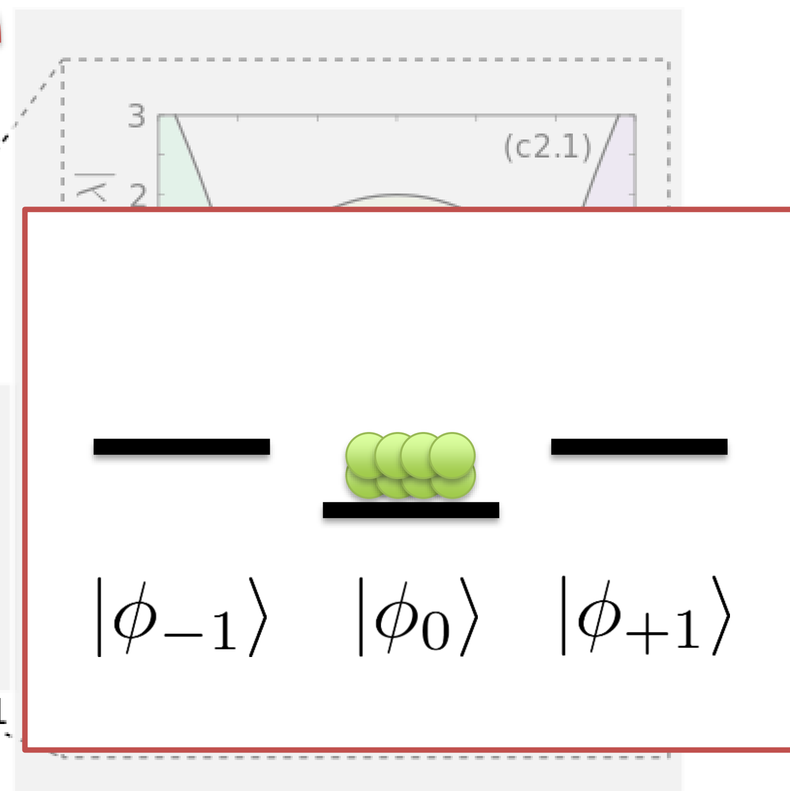

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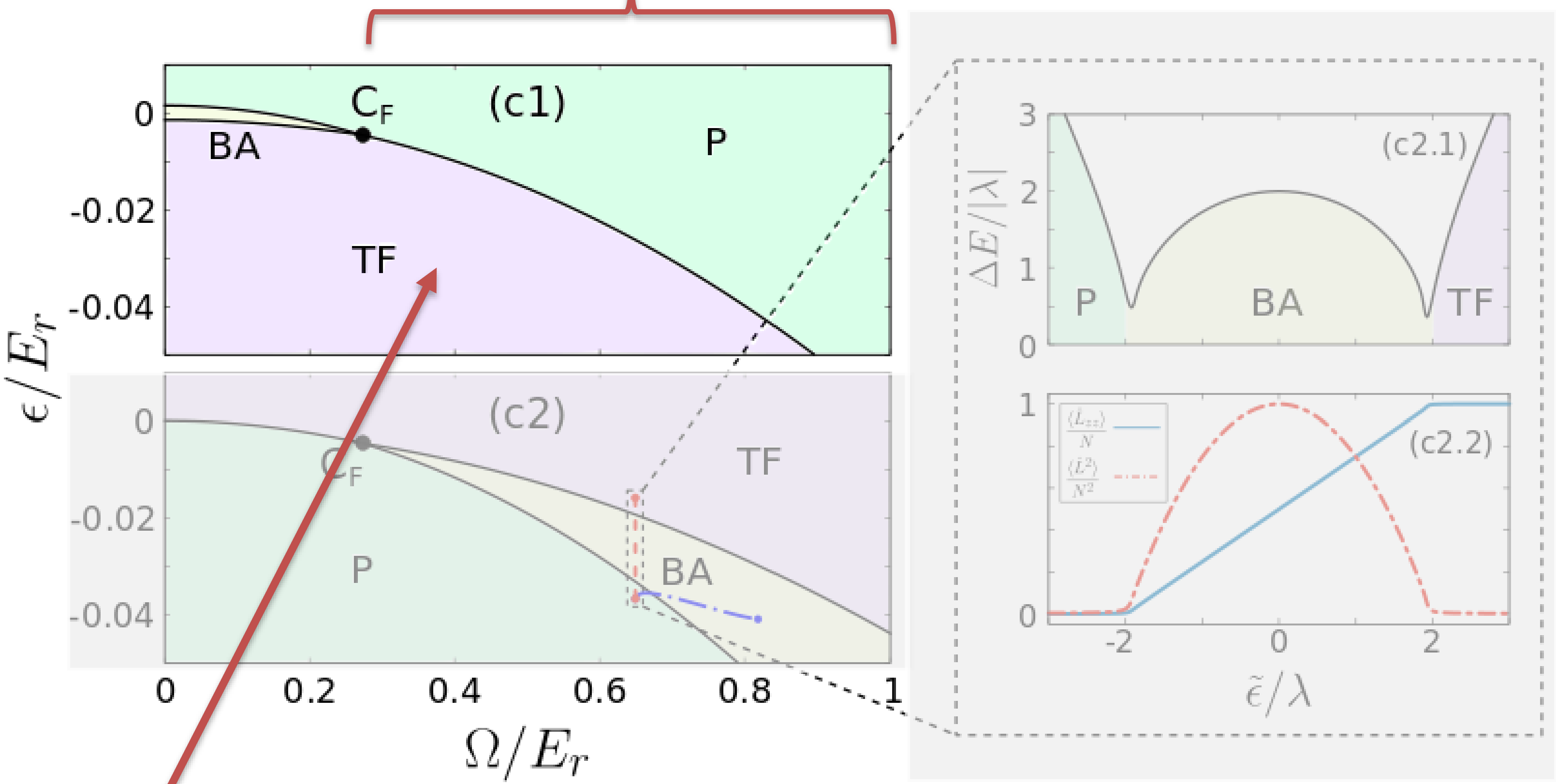

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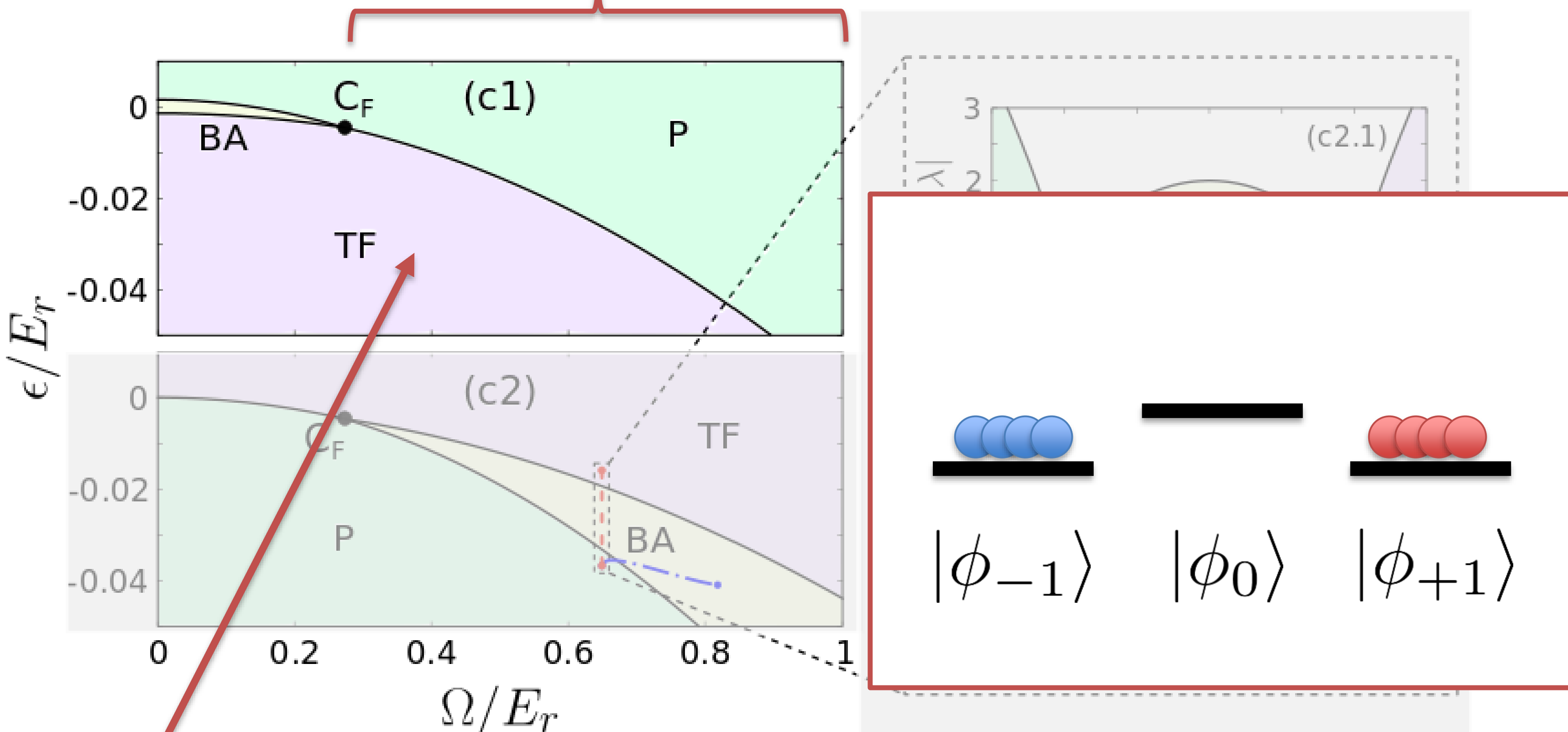

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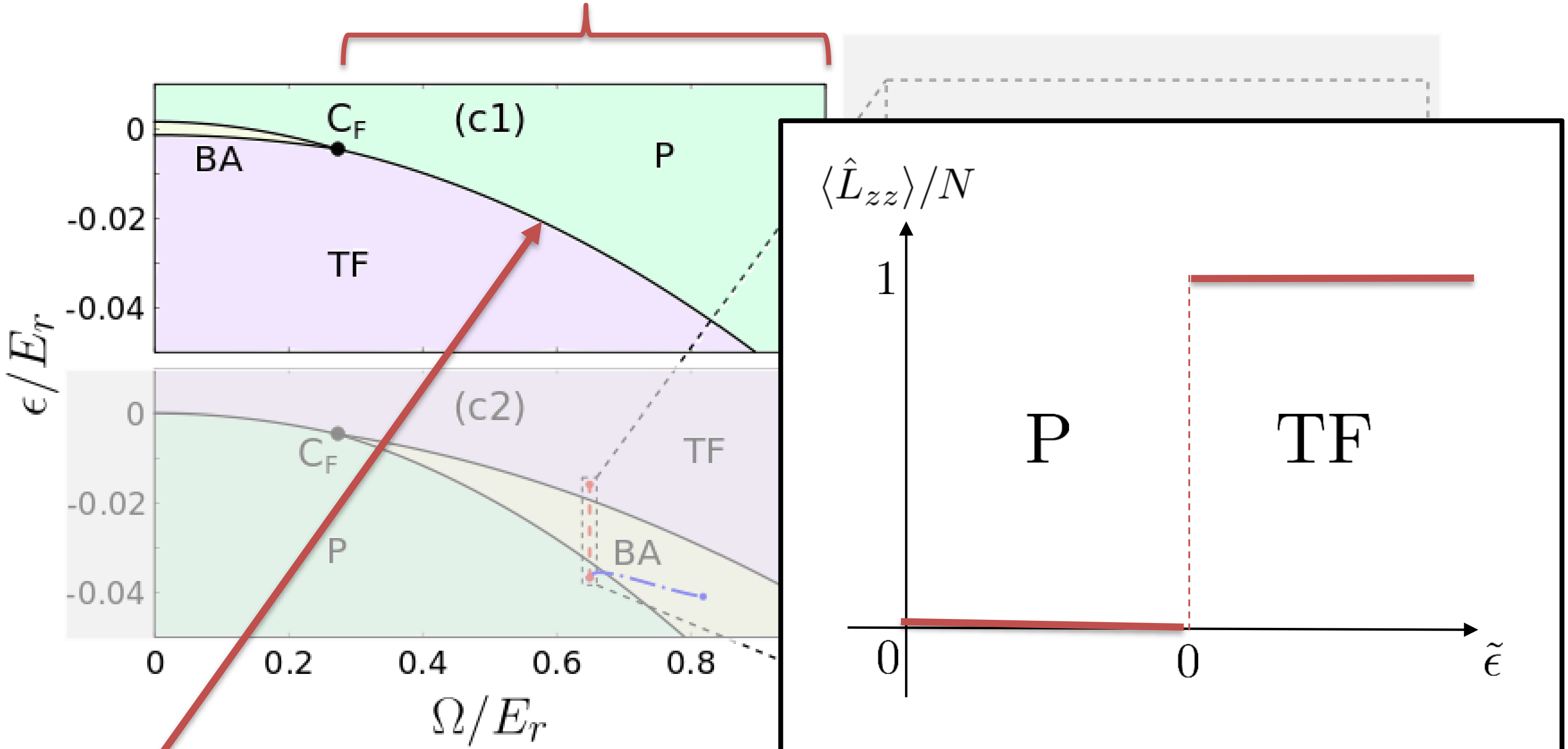

First order phase transition at $\epsilon = -\frac{\Omega^2}{16E_r}$

- Essentially, magnetic phases favored by $\hat{H}_{\text{mf}}$

Adiabatic quenches through ESQPTs

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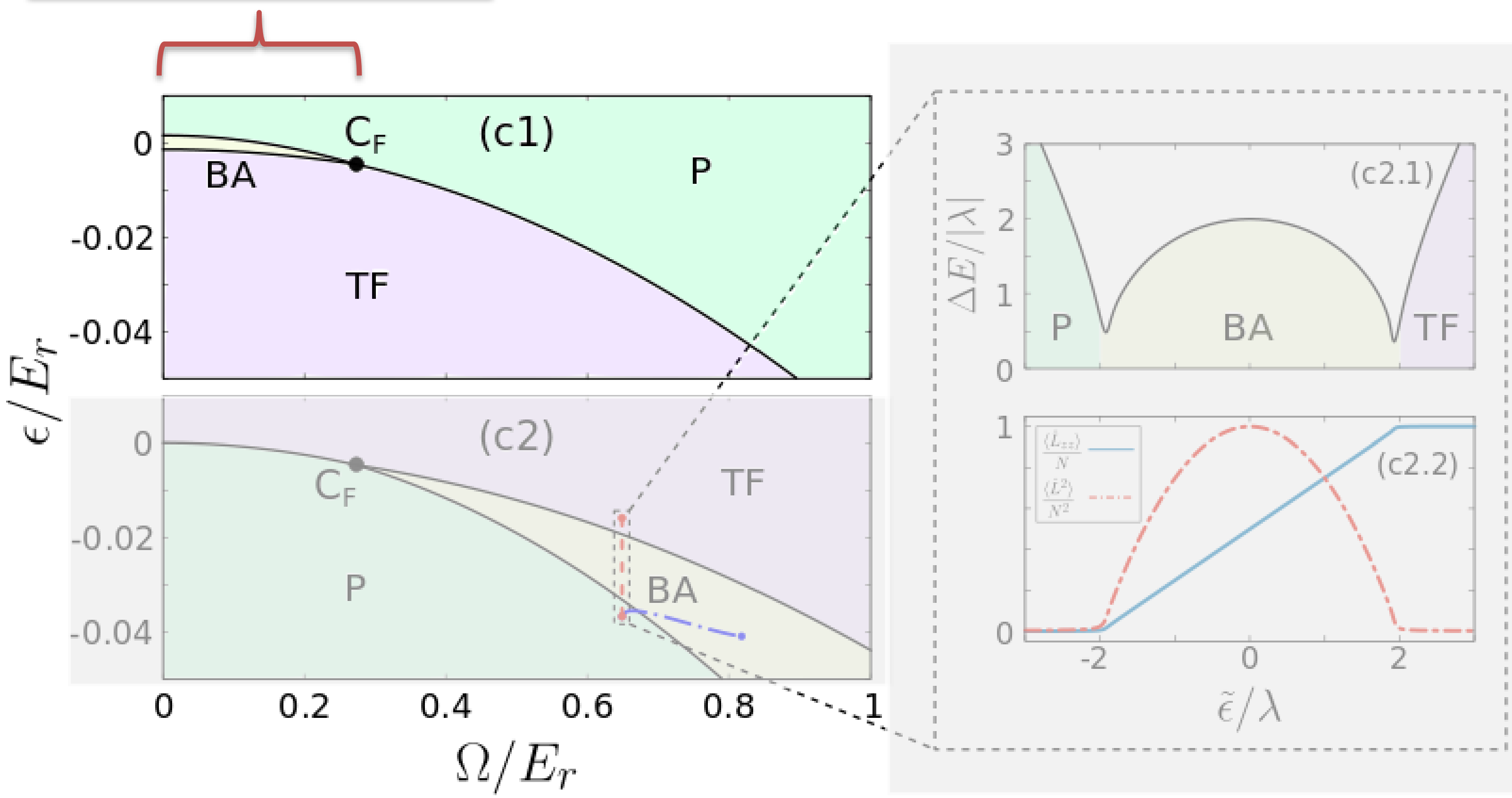
$$\Omega_c = 4E_r\sqrt{|g_2|/g_0}\,, \;\; \lambda = 0$$



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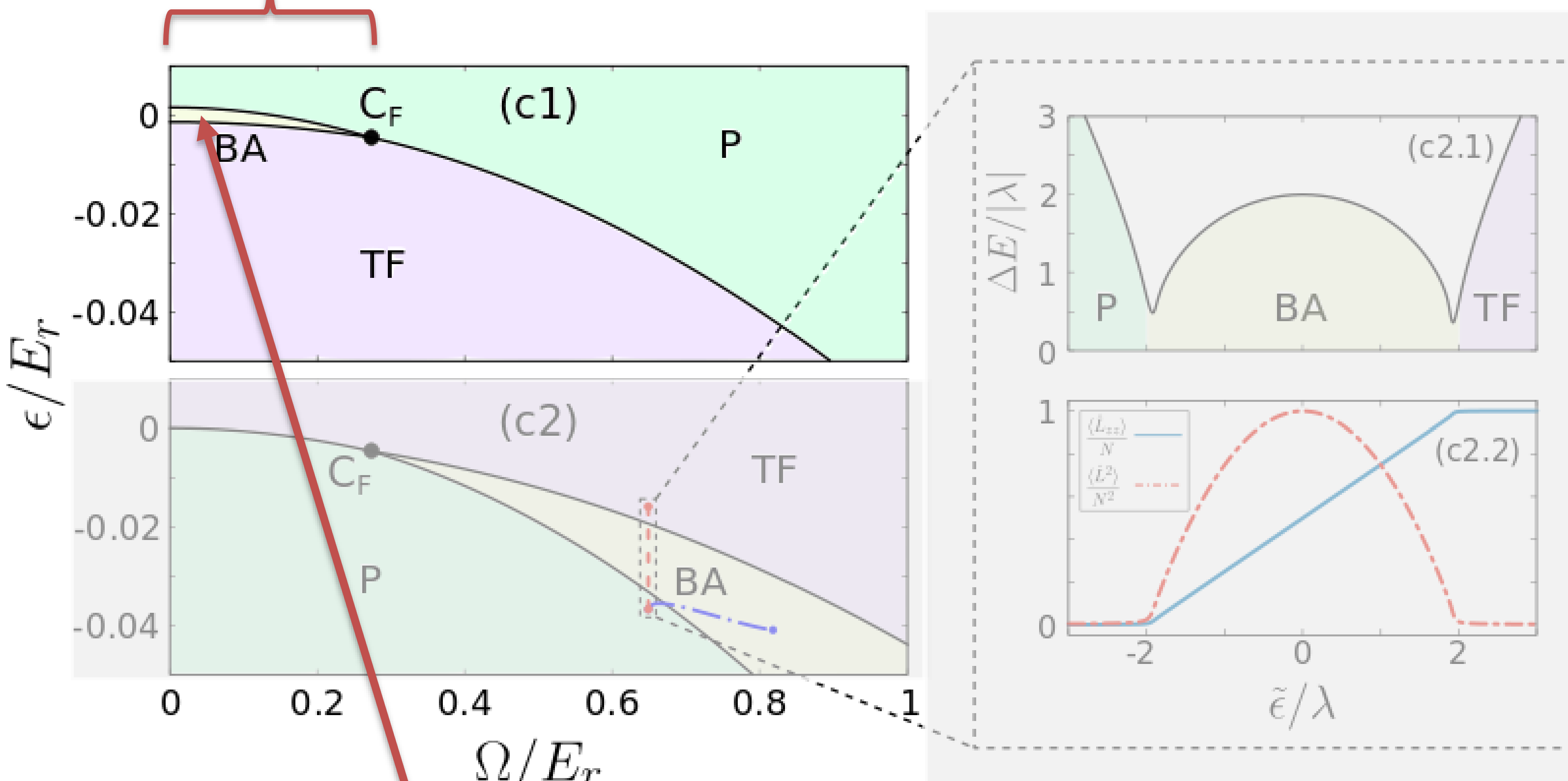

Broken-axisymmetry (BA) phase $\quad |\psi_{gs}\rangle = \int d\phi \left(\frac{e^{-i\phi}\hat{b}_+^\dagger + \sqrt{2}\hat{b}_0^\dagger + e^{i\phi}\hat{b}_+^\dagger}{2} \right)^{\otimes N} |0\rangle$

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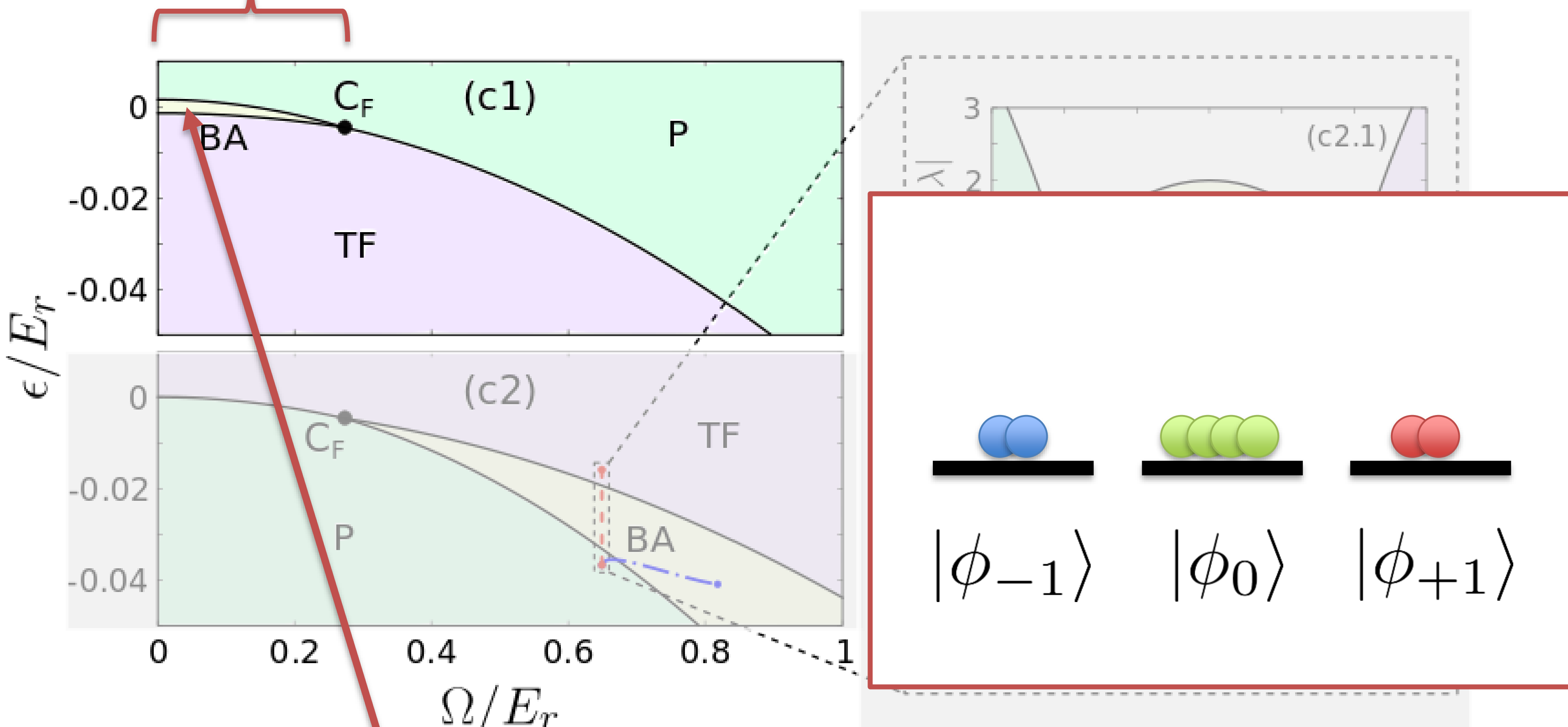

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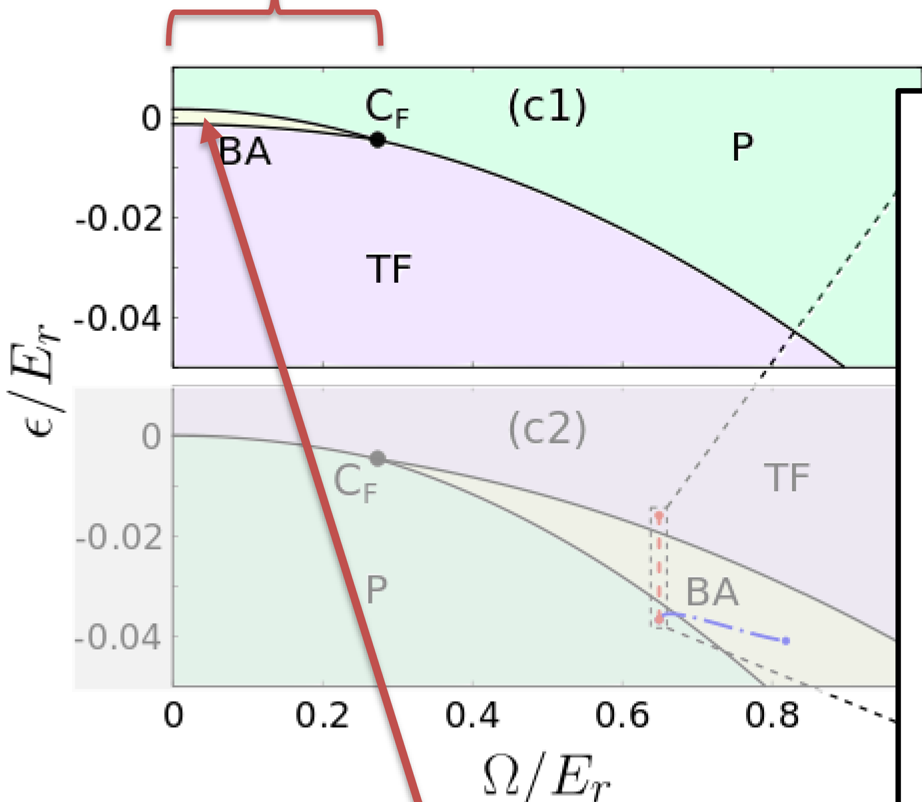
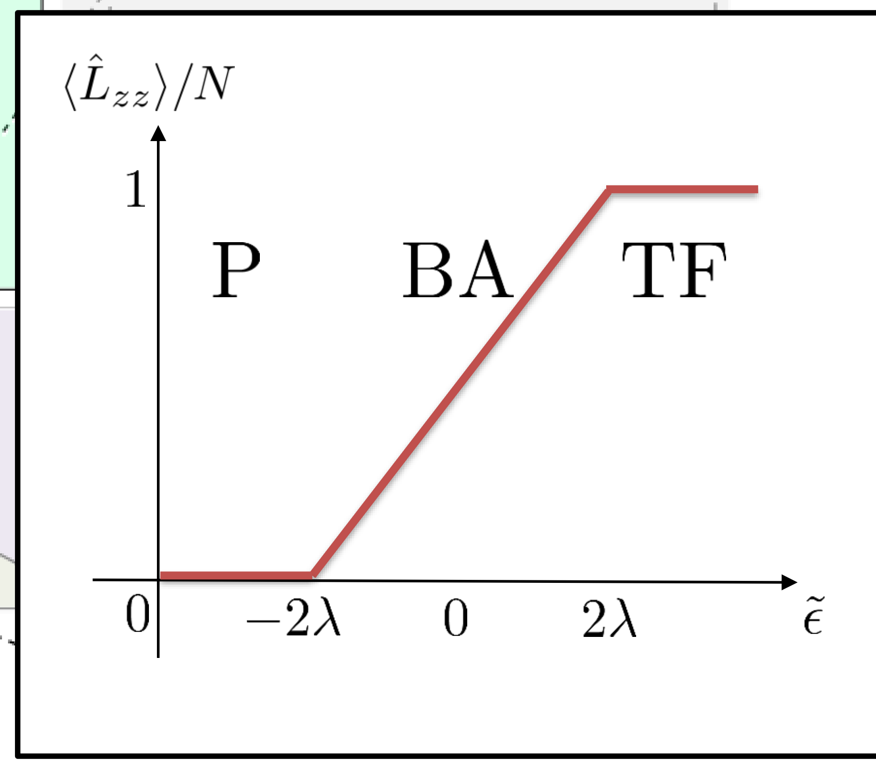




Second order phase transitions at $\tilde{\epsilon}/\lambda = \pm 2$

- BA phase favored by $\hat{H}_{ss}$

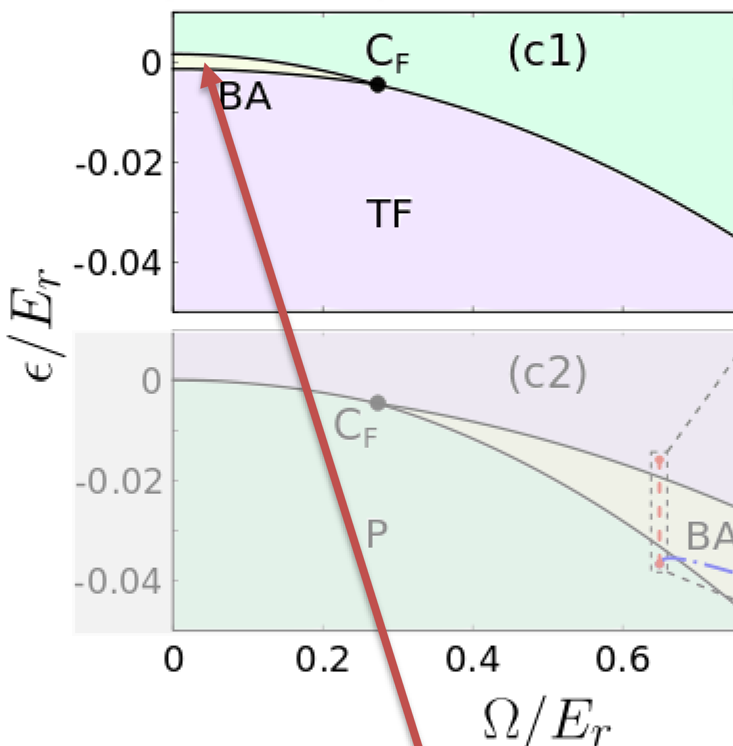
Adiabatic quenches through ESQPTs

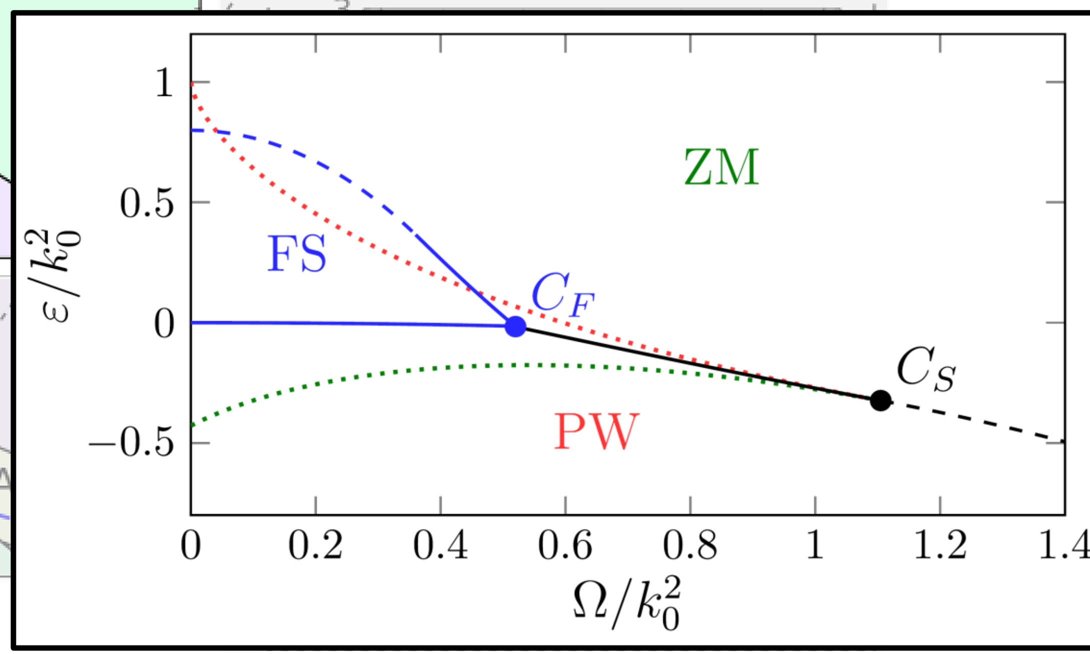
$$\hat{H}_0 = \lambda \frac{\hat{L}^2}{2N} + \tilde{\epsilon}\hat{L}_{zz} \qquad \lambda = \left(g_2 + g_0 \frac{\Omega^2}{16E_r^2}\right) n \quad \tilde{\epsilon} = \epsilon + \frac{\Omega^2}{16E_r}$$

- Phase diagram for the ground state (top) and the highest excited state (bottom)

Ferromagnetic $\lambda < 0$

$$\Omega_c = 4E_r\sqrt{|g_2|/g_0}\,, \;\; \lambda = 0$$





G. I. Martone, *et al., Phys. Rev. Lett.* **117**,125301 (2016)

BA in correspondence to the **ferromagnetic stripe** (FS) phase

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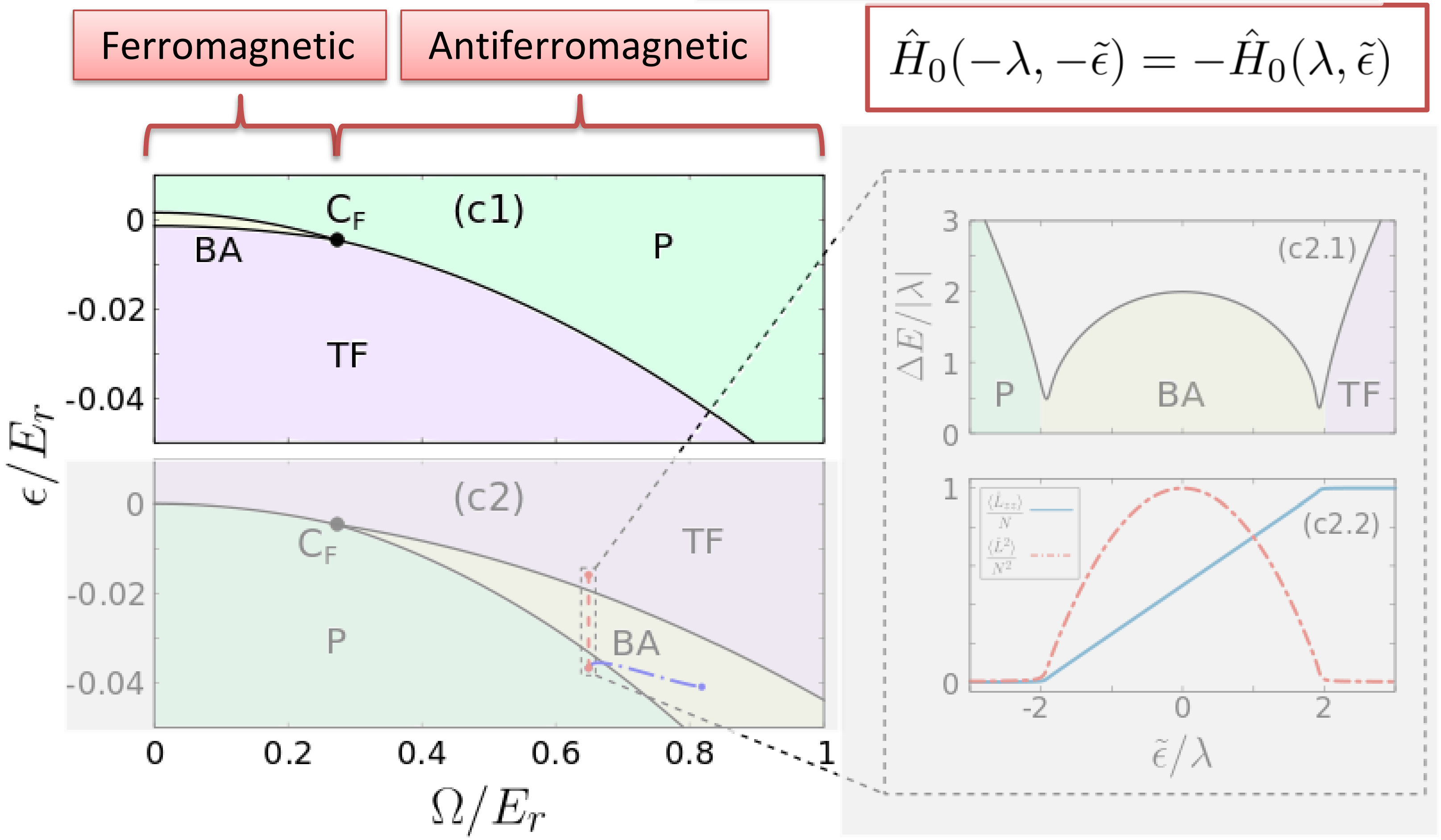

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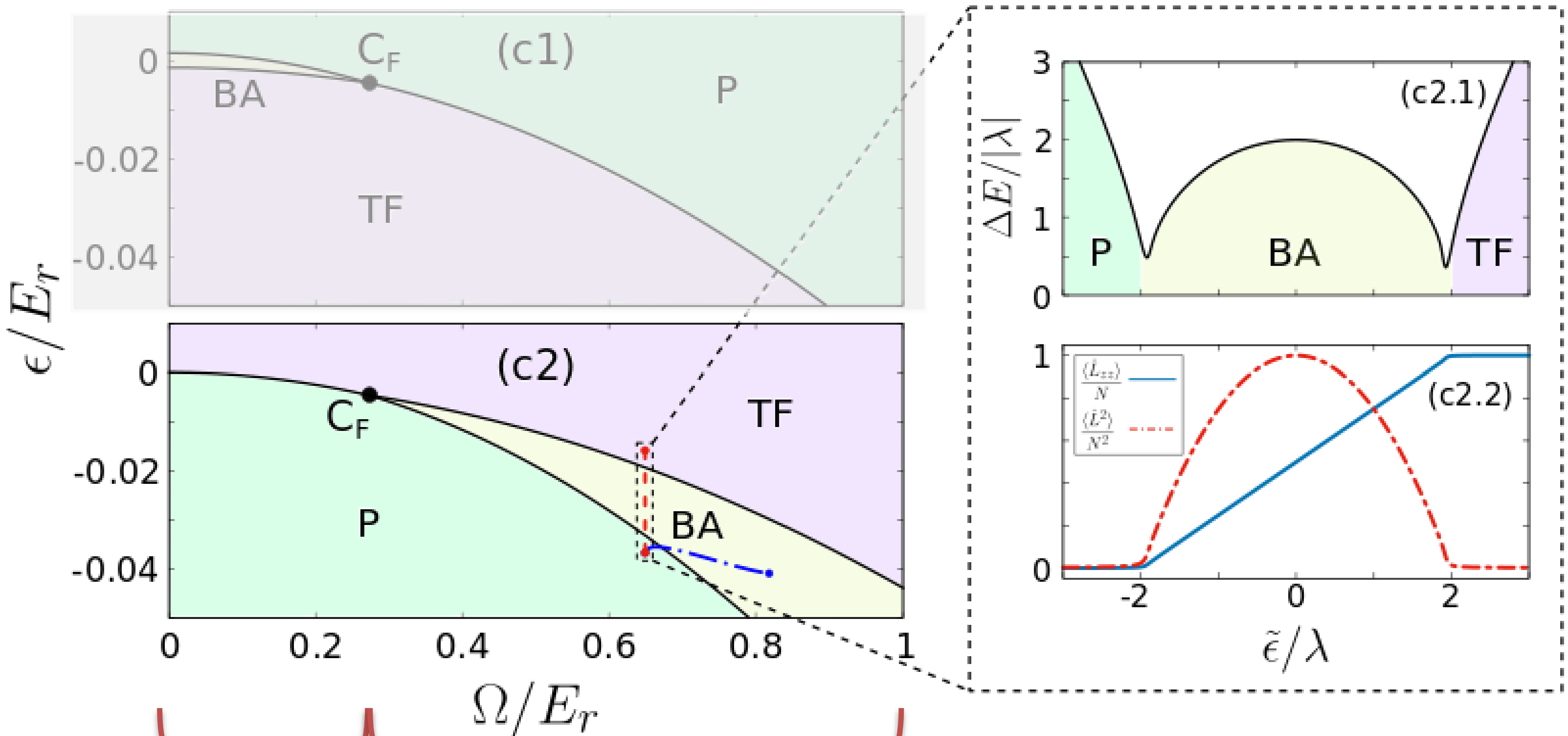

Antiferromagnetic

Ferromagnetic

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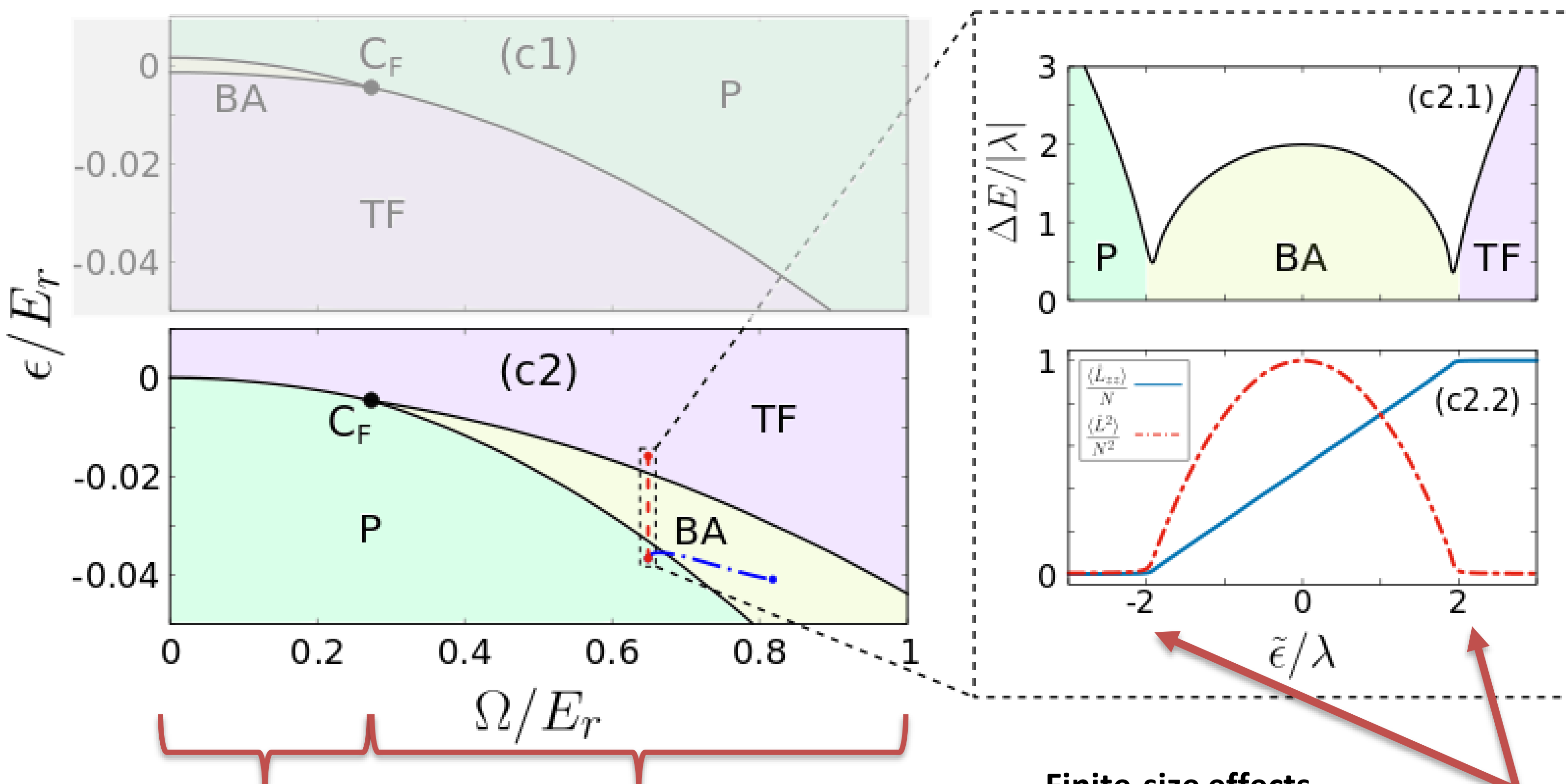


Antiferromagnetic

Ferromagnetic

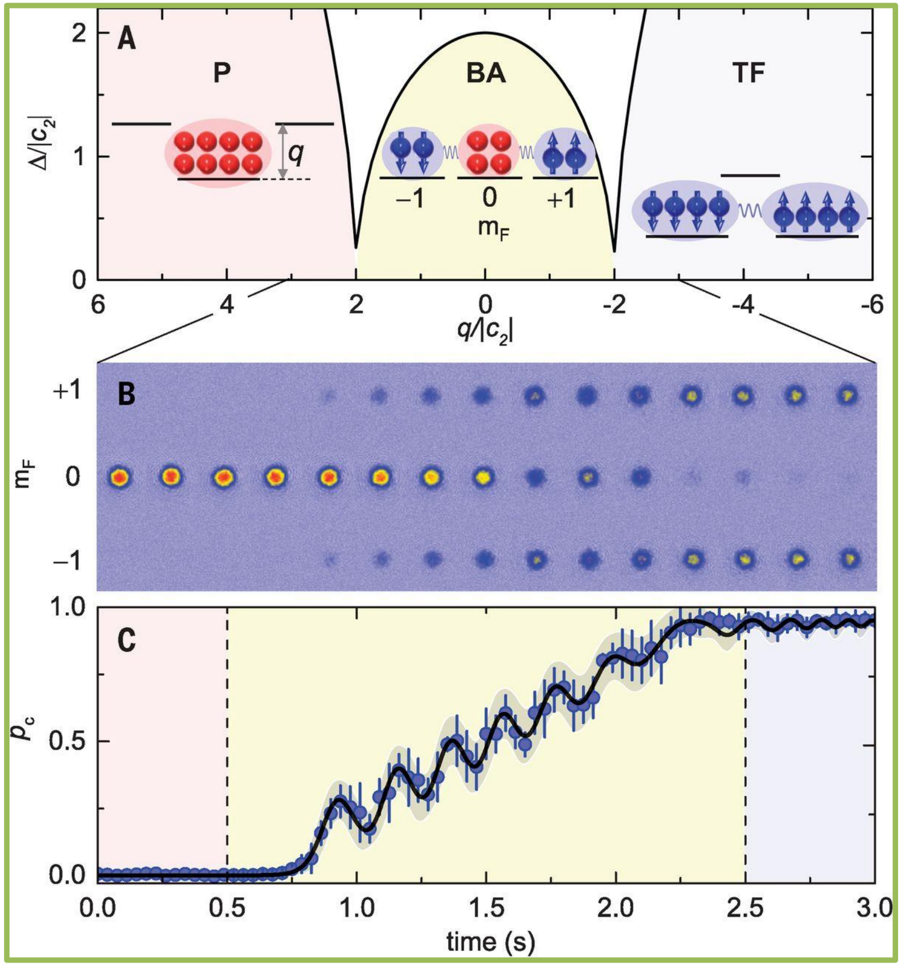
Finite-size effects
+
interactions

$\Delta E \propto \lambda N^{-1/3}$

Adiabatic quenches through ESQPTs

- Both BA and TF states are highly entangled

- In spinor gases, they can be accessed via adiabatic quenches exploiting the gap



X.-Y. Luo, Y.-Q. Zou, L.-N. Wu, Q. Liu, M.-F. Han,M. K. Tey, and L. You, *Science*, **355** 620 (2017)

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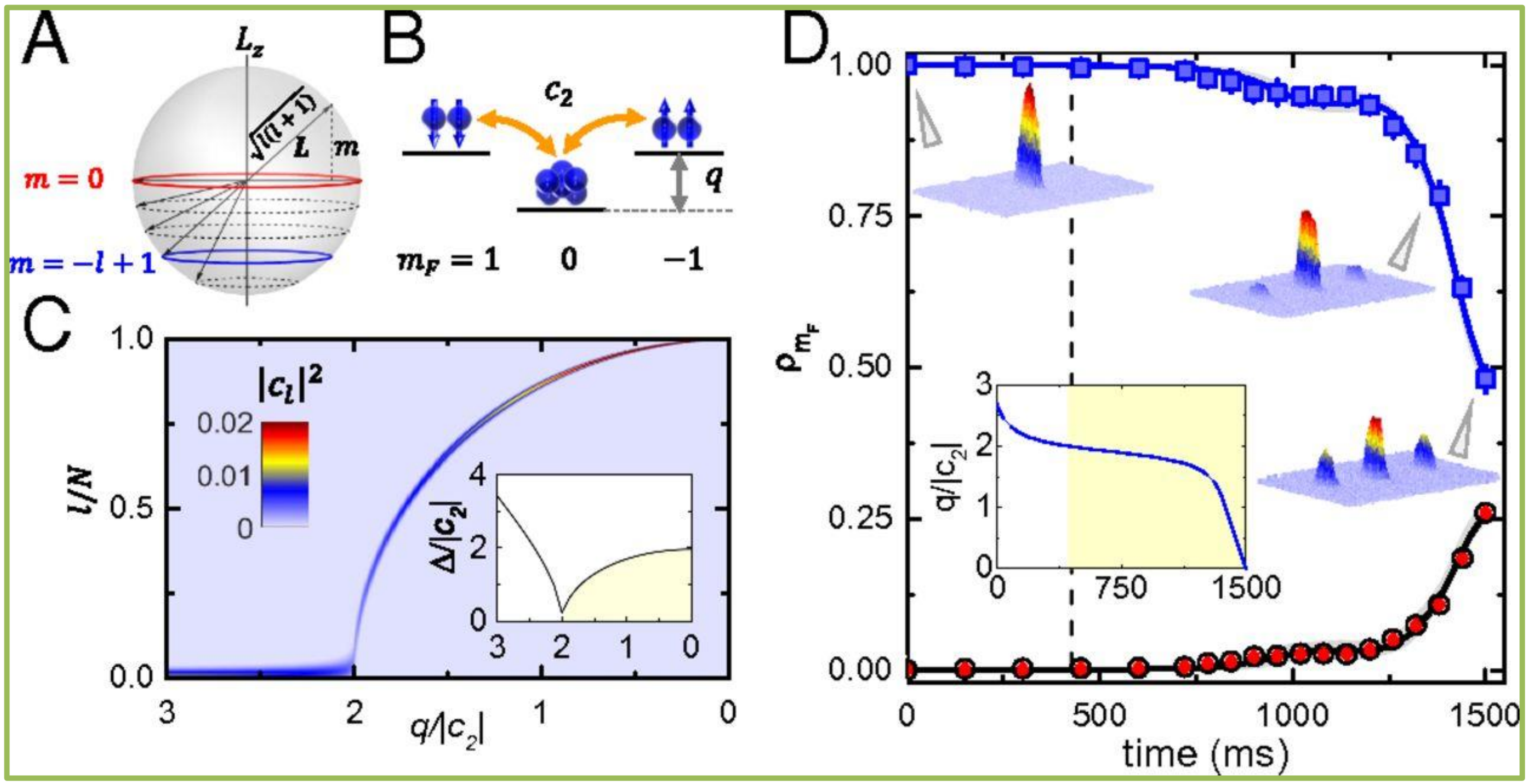
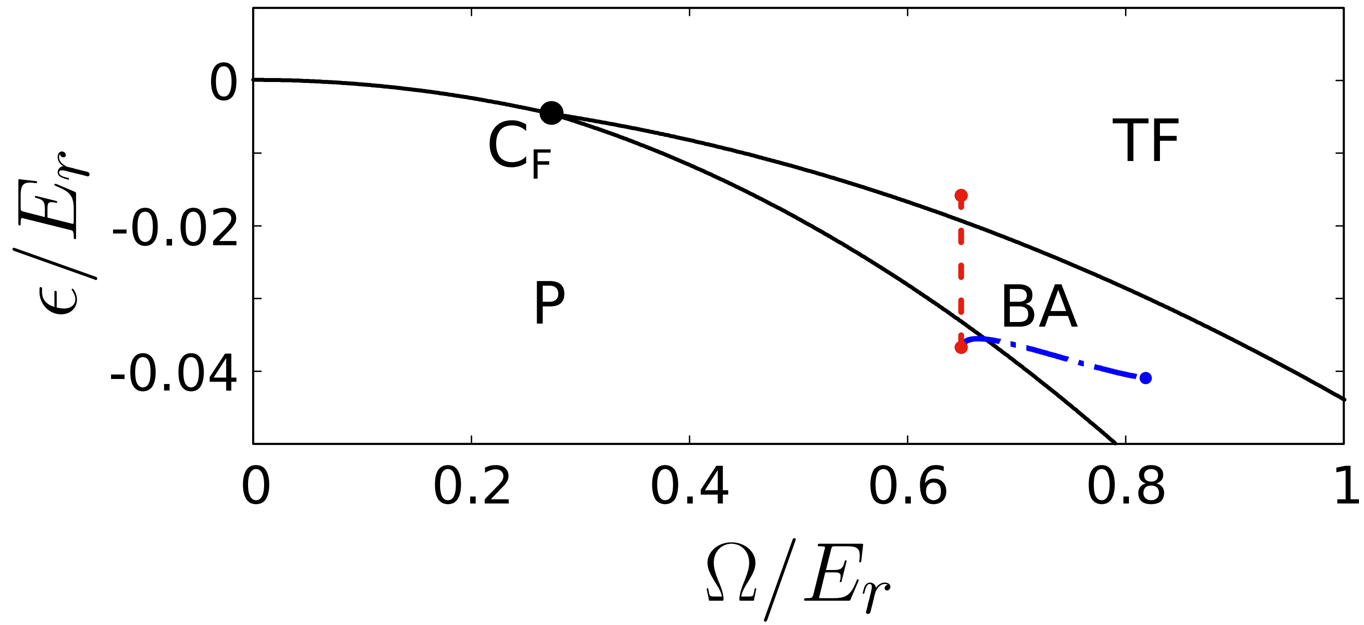


Y.-Q. Zou, L.-N. Wu, Q. Liu, X.-Y. Luo, S.-F. Guo, J.-H.Cao, M. K. Tey, and L. You, *PNAS* **115**, 6381 (2018)

Adiabatic quenches through ESQPTs

- Both BA and TF states are highly entangled
- In spinor gases, they can be accessed via adiabatic quenches exploiting the gap
- The same states of the effective Hamiltonian can be accessed in highest excited diagram

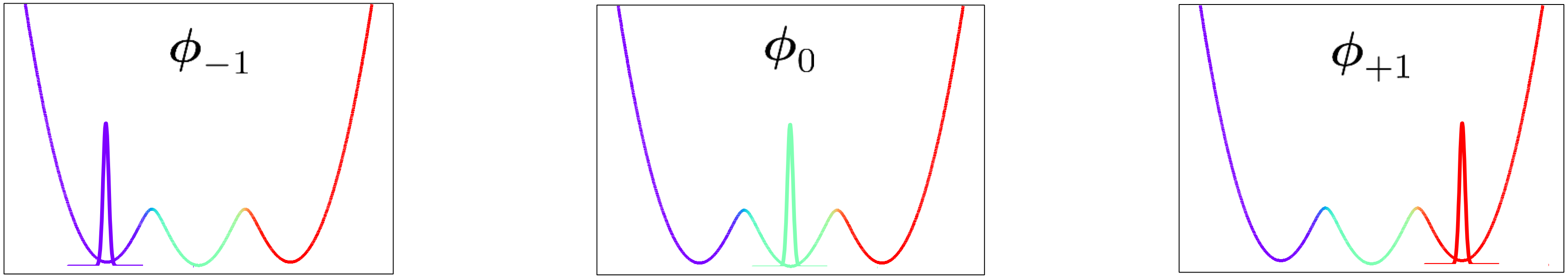


Adiabatic quenches through ESQPTs

- Numerical results: GPE of the dressed gas

$$i\hbar\dot{\psi}_j = \delta\mathcal{E}/\delta\psi_j^* \qquad \mathcal{E} = \psi^*\left(\hat{\mathcal{H}}_{\mathrm{k}} + V_{\mathrm{t}}\right)\psi + \frac{g_0}{2}|\psi|^4 + \frac{g_2}{2}\sum_j(\psi^*\hat{F}_j\psi)^2$$

- The three self-consistent modes calculated via imaginary time evolution



- Mean-field computations performed using the XMDS2 library

G. R. Dennis, J. J. Hope, and M. T. Johnsson, *Comput. Phys. Commun.***184**, 201–208 (2013)

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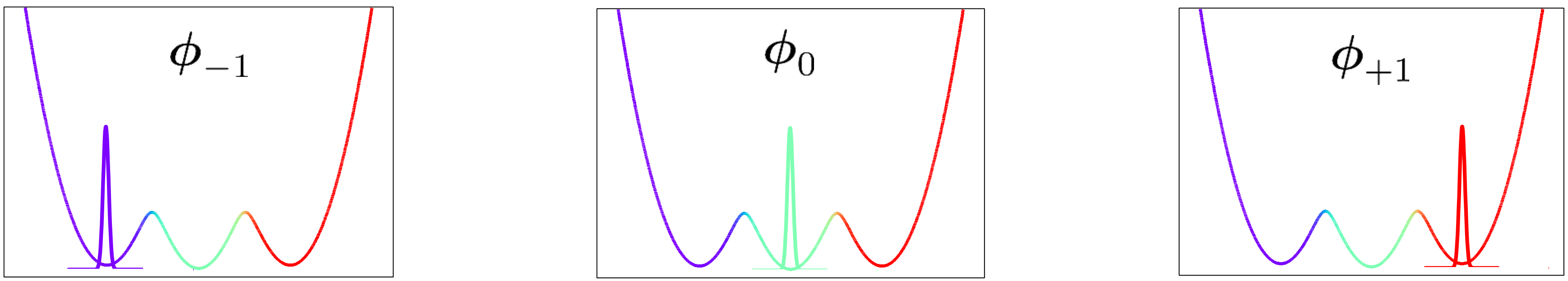

- **Problem**: in the protocol, initial Fock state $|\psi(0)\rangle = \frac{1}{\sqrt{N!}}(\hat{b}_0^\dagger)^N|0\rangle$

Dynamics dominated by quantum fluctuations! **GPE**

C. Klempt, *et al.*, *Phys. Rev. Lett*. **104**, 195303 (2010) B. Evrard, A. Qu, J. Dalibard, and F. Gerbier, *arXiv:2101.06716 (2021)*

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Adiabatic quenches through ESQPTs

- **Alternative**: coherent initial state with a small fraction of atoms in the edge modes.

$$|\psi(0)\rangle = \frac{1}{\sqrt{N!}}(\alpha e^{-i\theta_s/2}\hat{b}^\dagger_{-1} + \sqrt{1-2\alpha^2}\hat{b}^\dagger_0 + \alpha e^{-i\theta_s/2}\hat{b}^\dagger_1)^N\,|0\rangle$$

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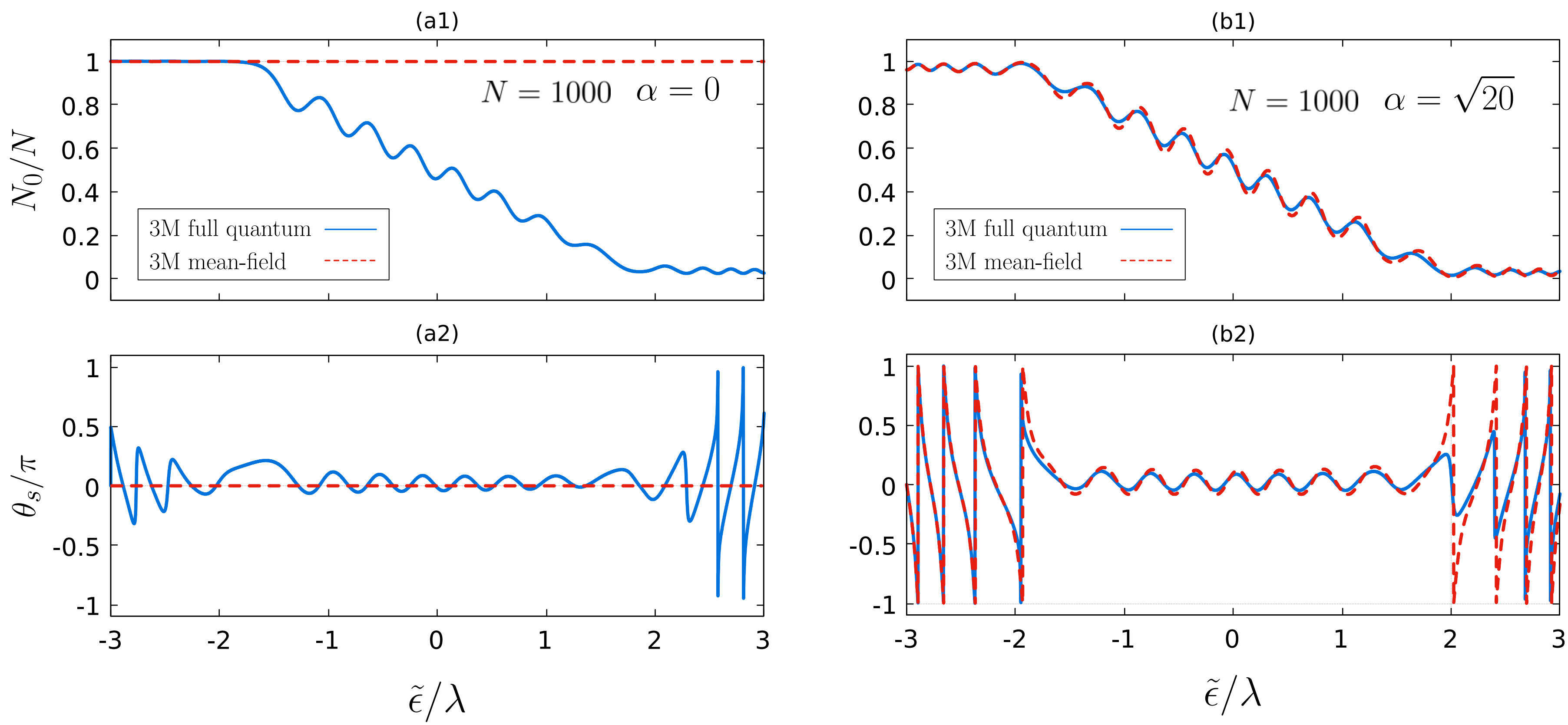

Adiabatic quenches through ESQPTs

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- **Numerical results**: realistic parameters for ^{87}Rb gases

$$E_r/\hbar = 2\pi \cdot 3680\,\mathrm{Hz}$$
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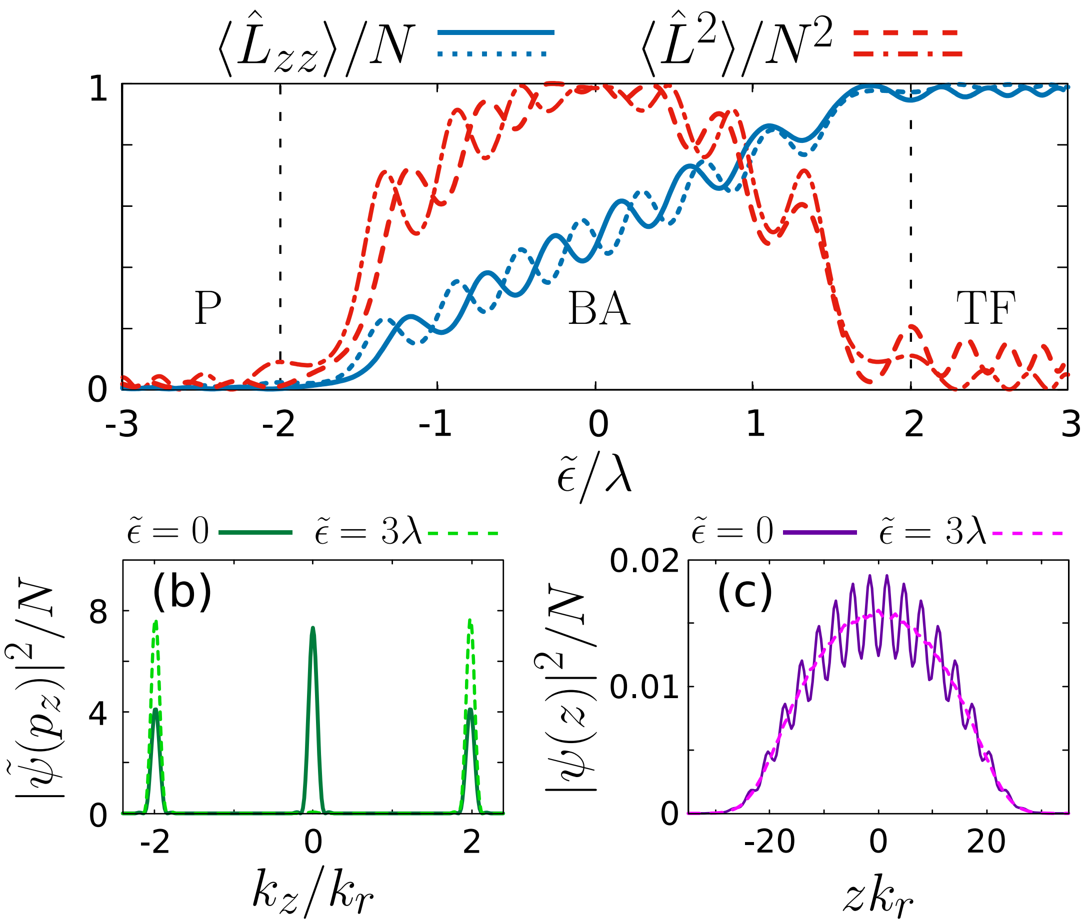

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$$N = 10^4$$

$$\alpha = \sqrt{50}$$

$$\Omega = 0.65 E_r$$

$$\hbar\omega_{\rm t} = 0.038 E_r/\hbar \simeq 2\pi \cdot 140\,\mathrm{Hz}$$

$$\lambda/\hbar \sim 2\pi \cdot 13\,\mathrm{Hz}$$

$$\tau_d = 8h/\lambda \sim 600\ \mathrm{ms}$$

Adiabati

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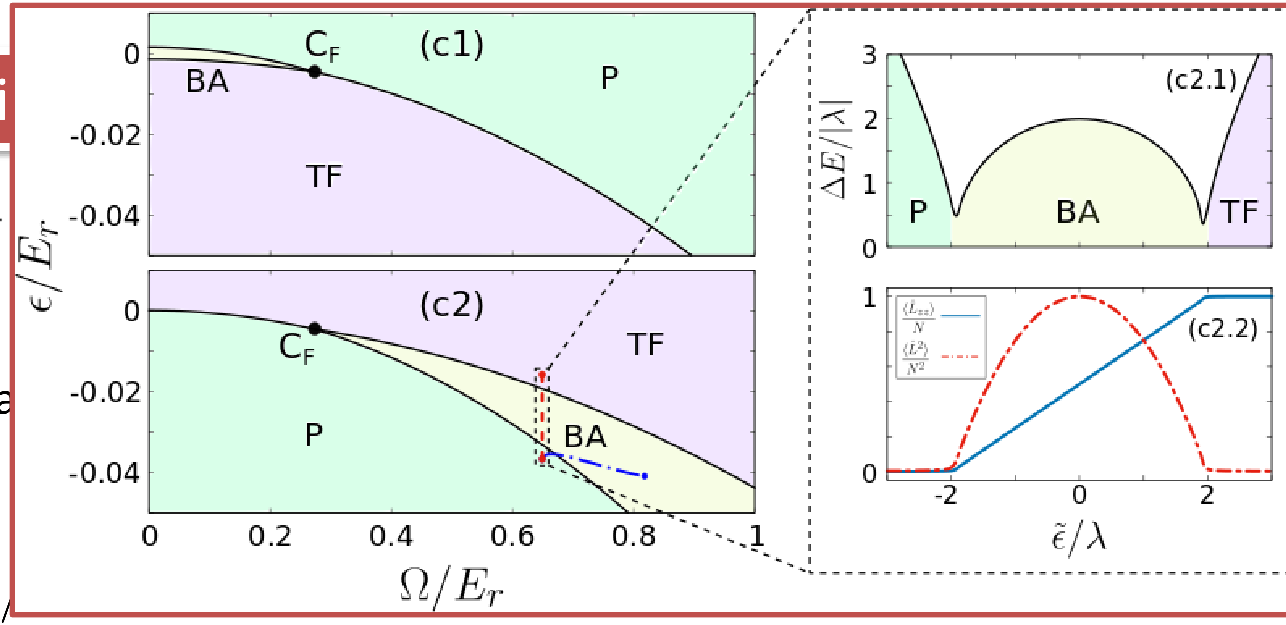

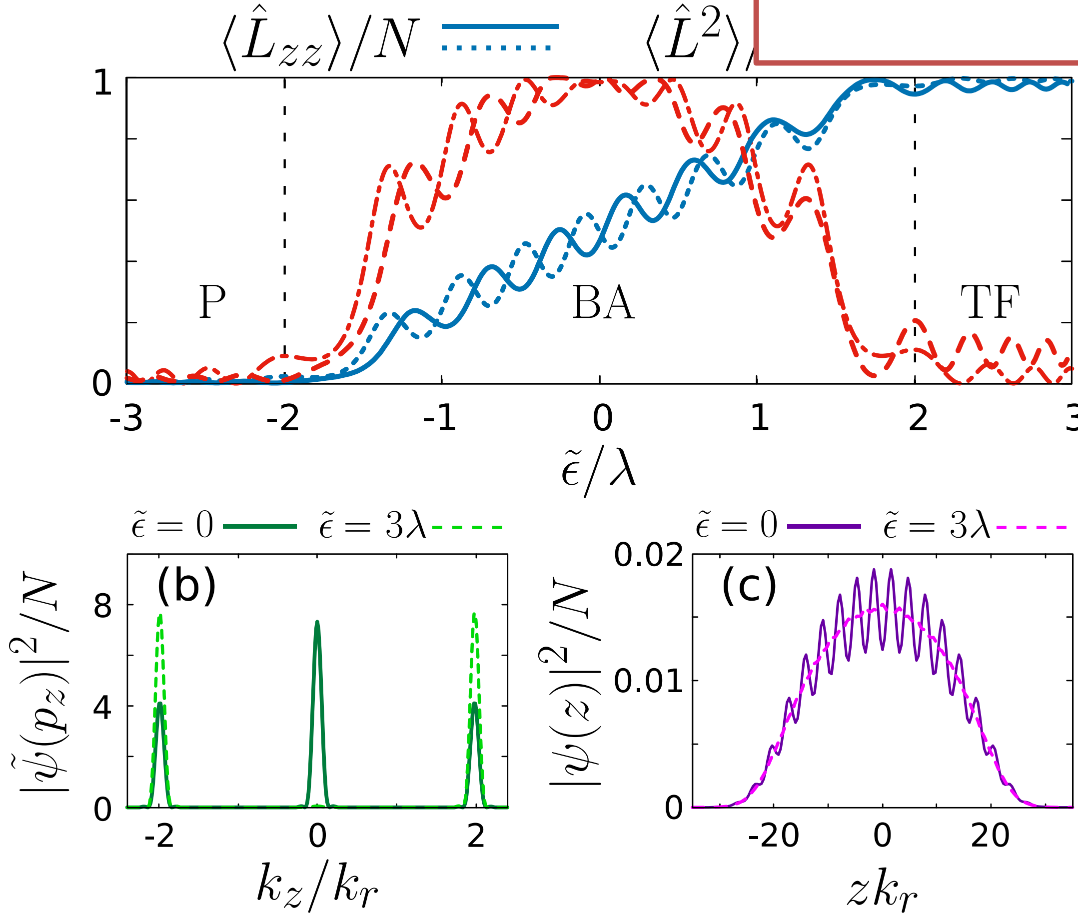

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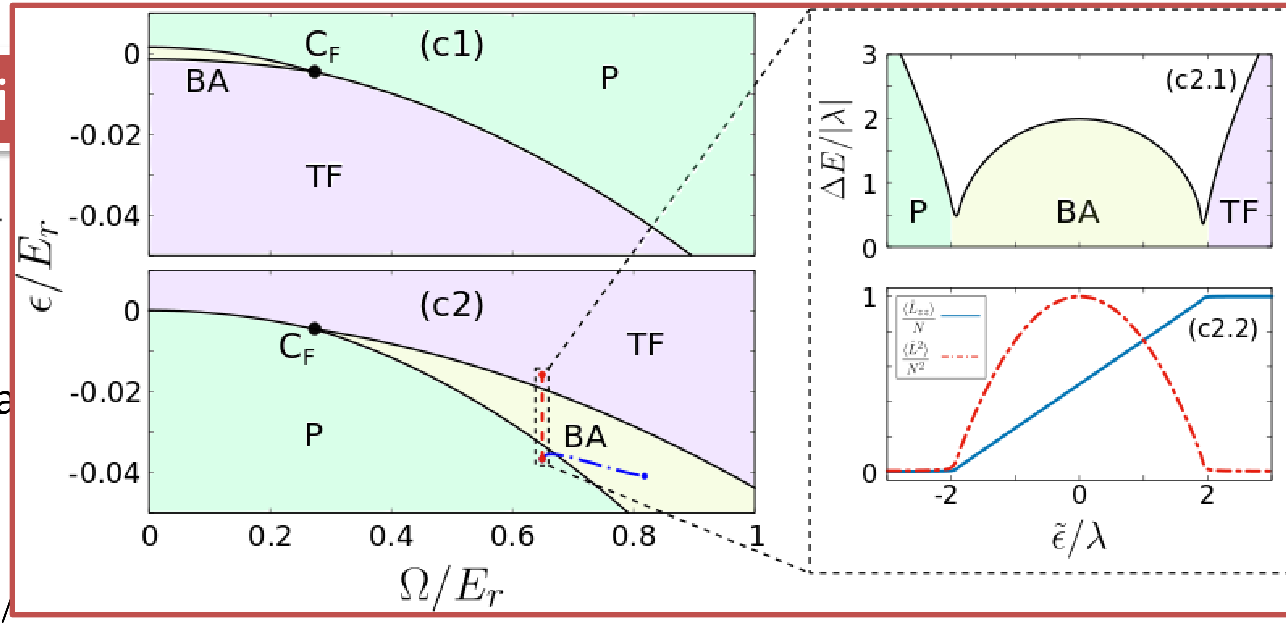

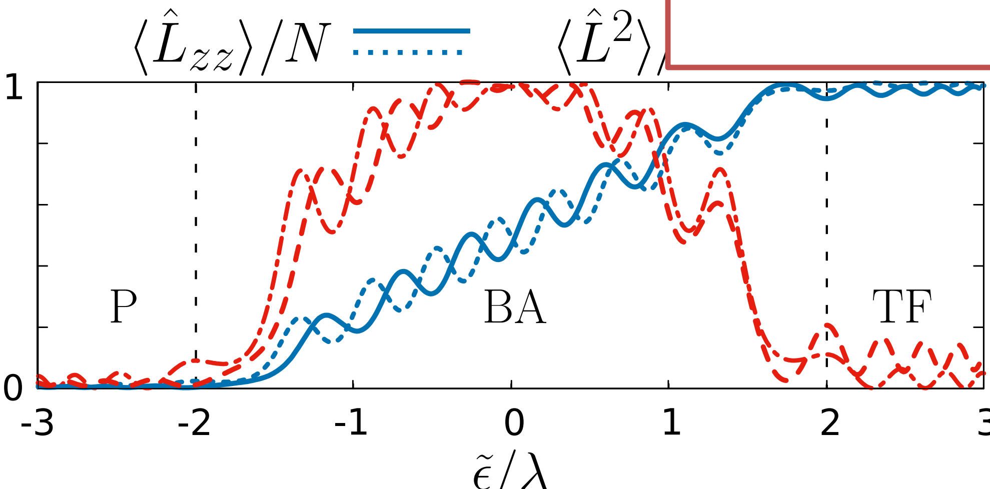

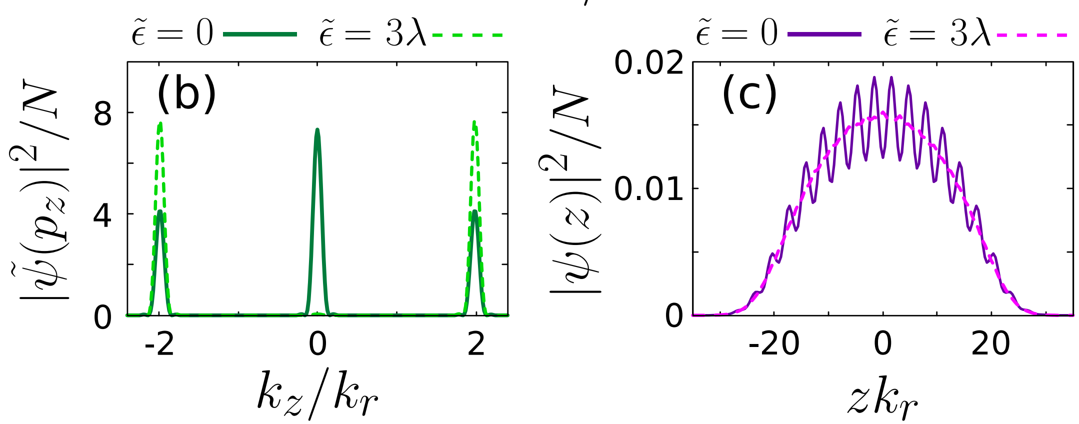

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R. P. Anderson, *Phys.Rev. Research* **2**, 013149 (2020)

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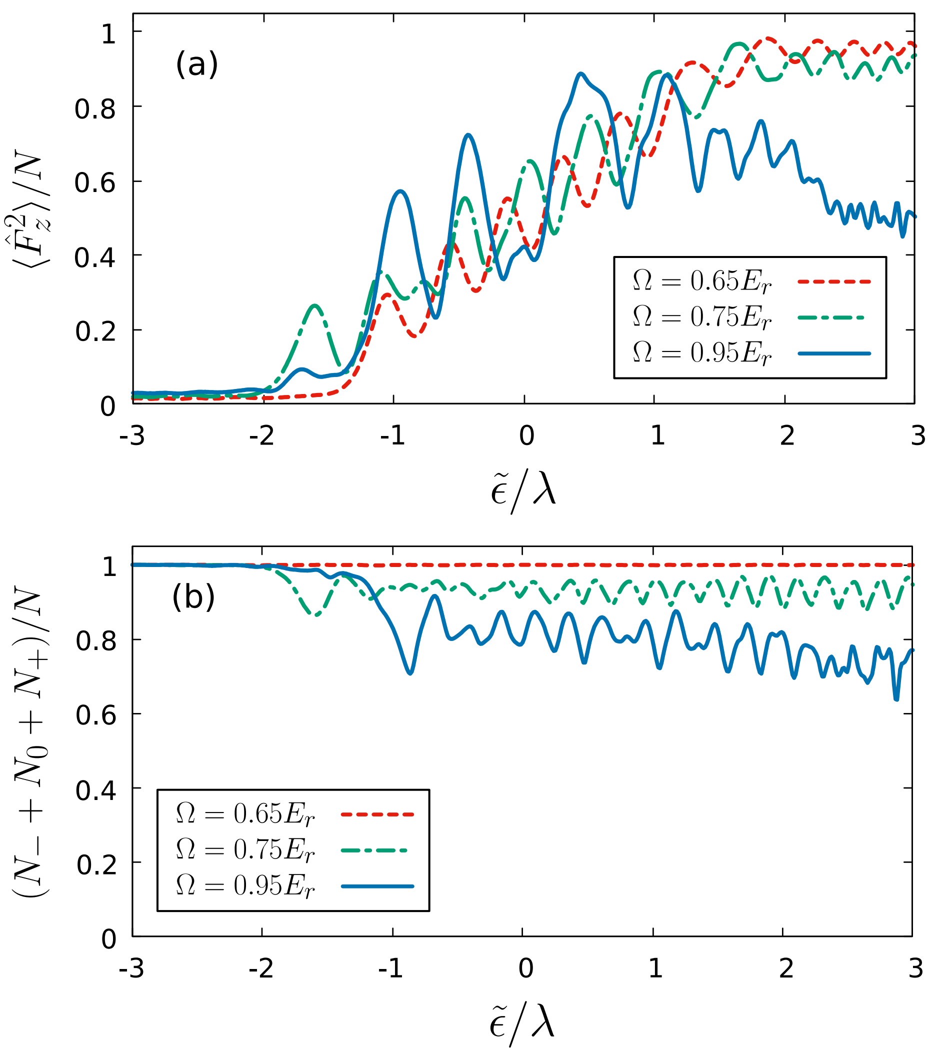

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$$\lambda/\hbar \sim 2\pi\cdot 13\,\mathrm{Hz},\ 18\,Hz,\ 30\,\mathrm{Hz}$$
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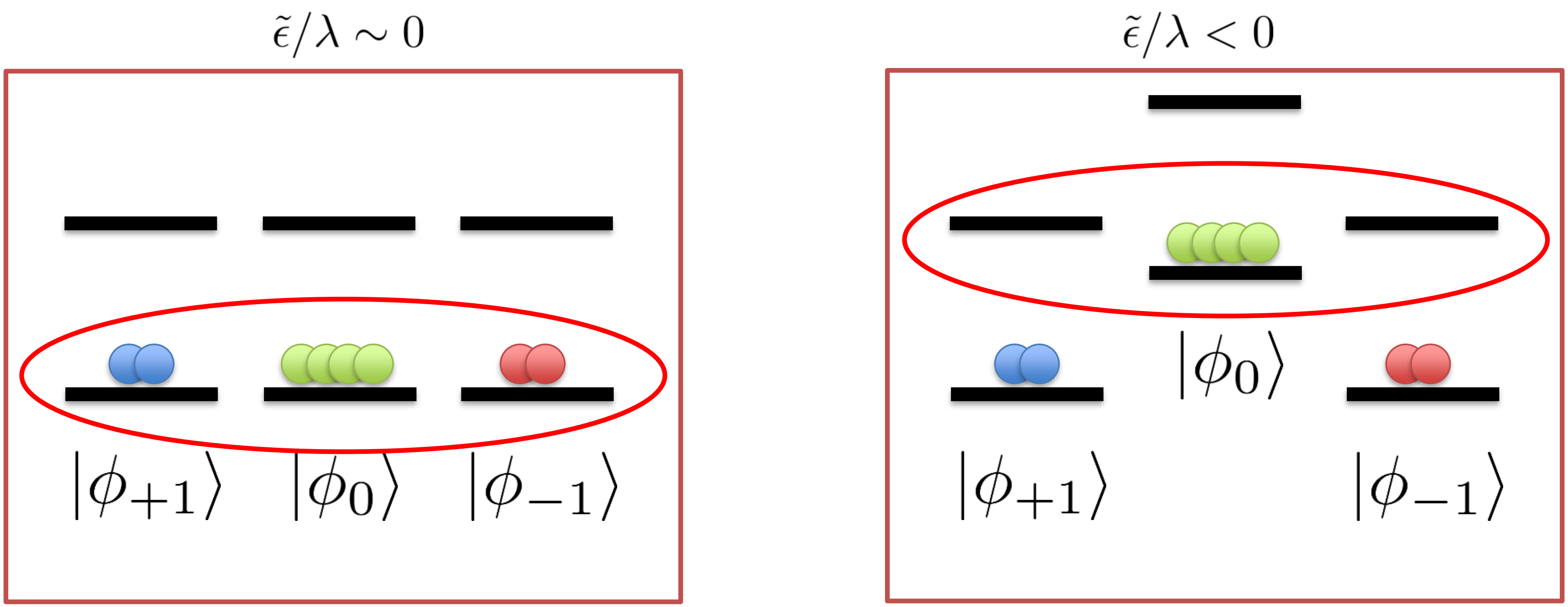
III. **Conclusion and outlook**

Preparation of the ferromagnetic stripe phase in an excited state

- **Tunability**

$$\lambda = \left(g_2 + g_0 \frac{\Omega^2}{16E_r^2}\right) n \qquad \tilde{\epsilon} = \epsilon + \frac{\Omega^2}{16E_r}$$



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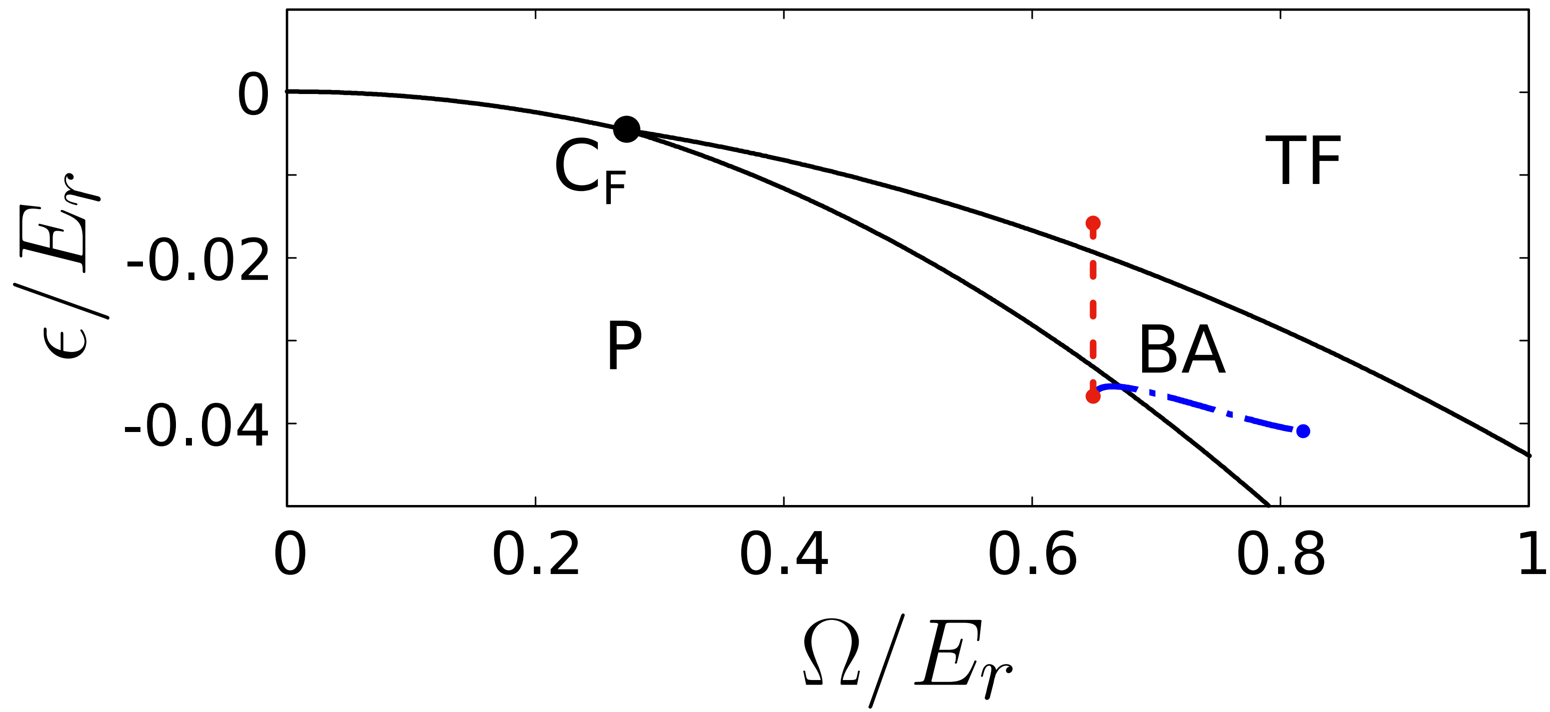

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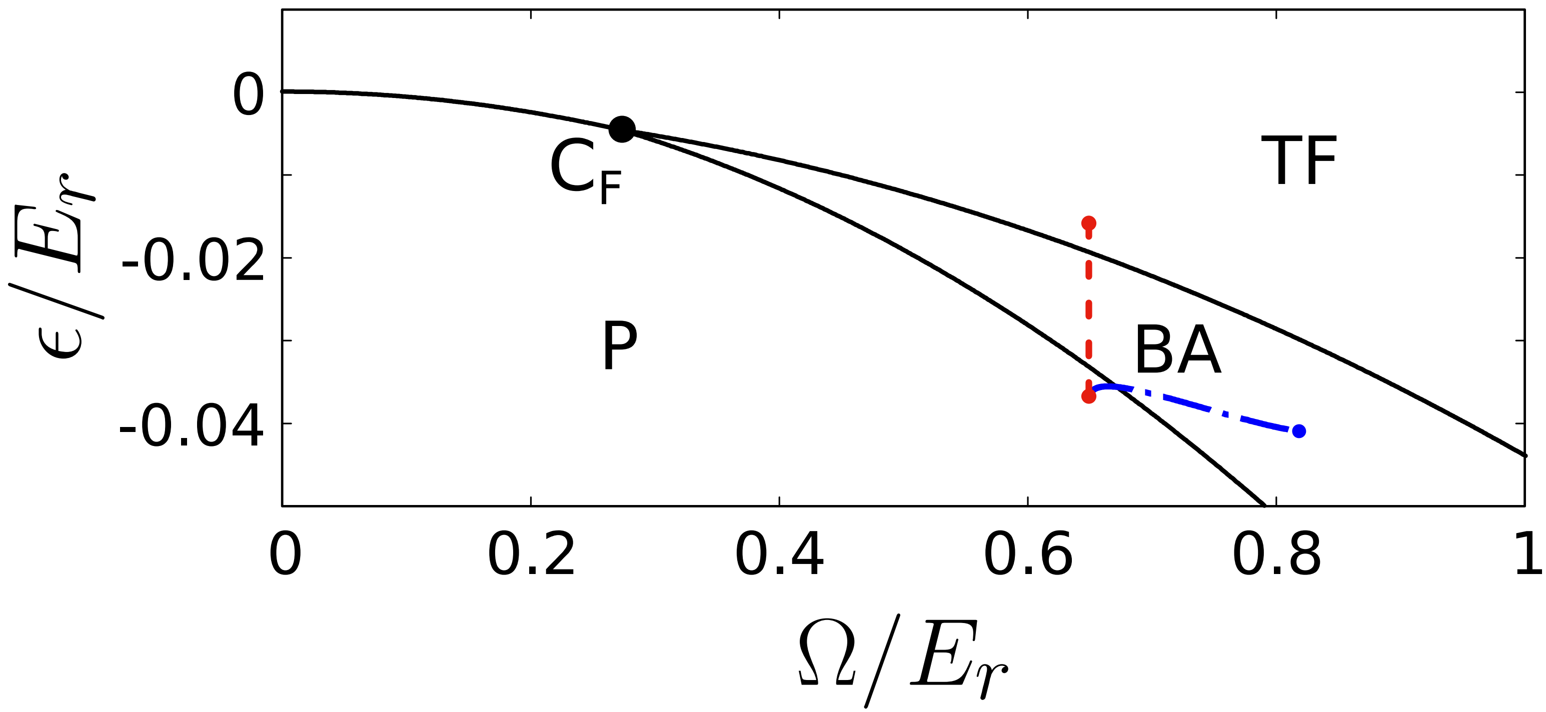

- Higher **robustness** against noise + larger density modulations ✓

Preparation of the ferromagnetic stripe phase in an excited state

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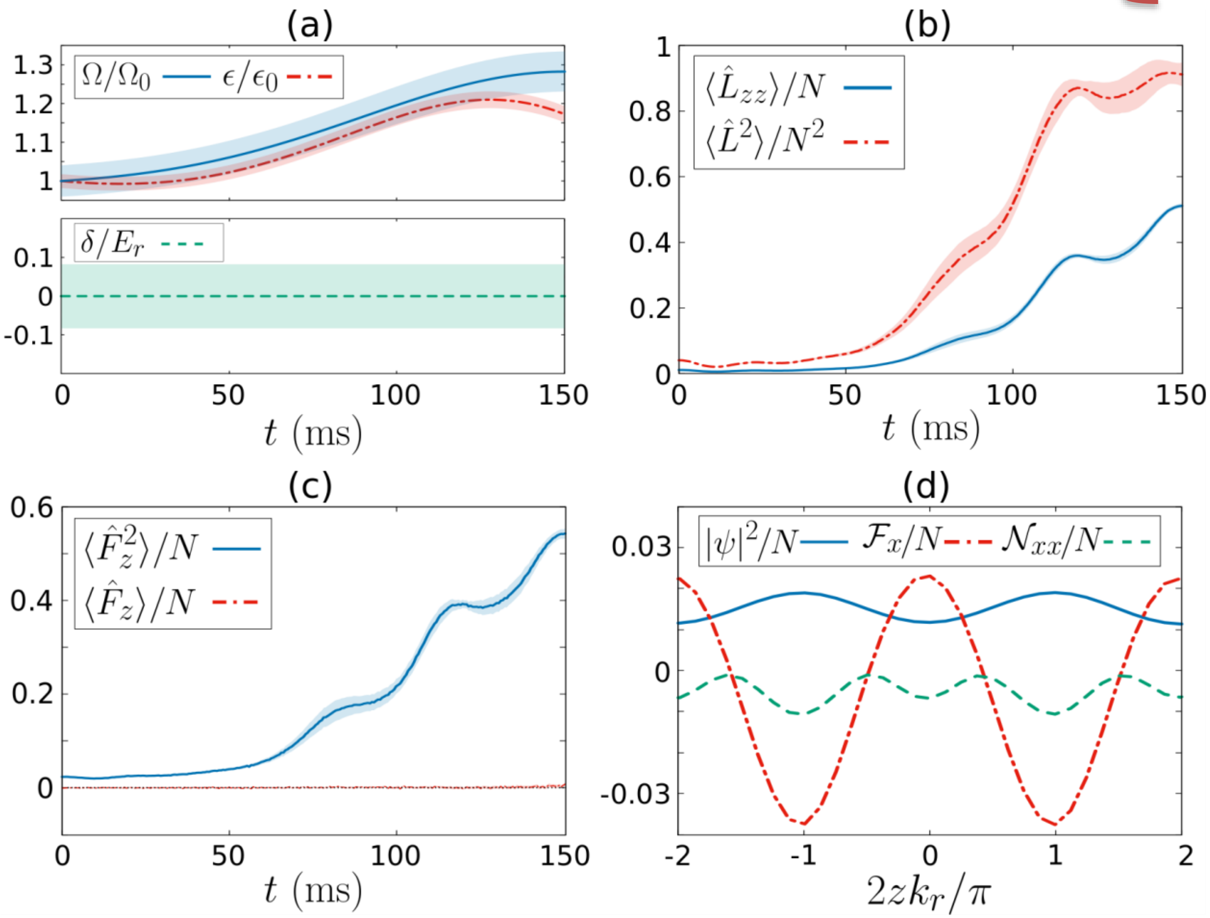

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$$\Delta\Omega = 125\ \mathrm{Hz}$$

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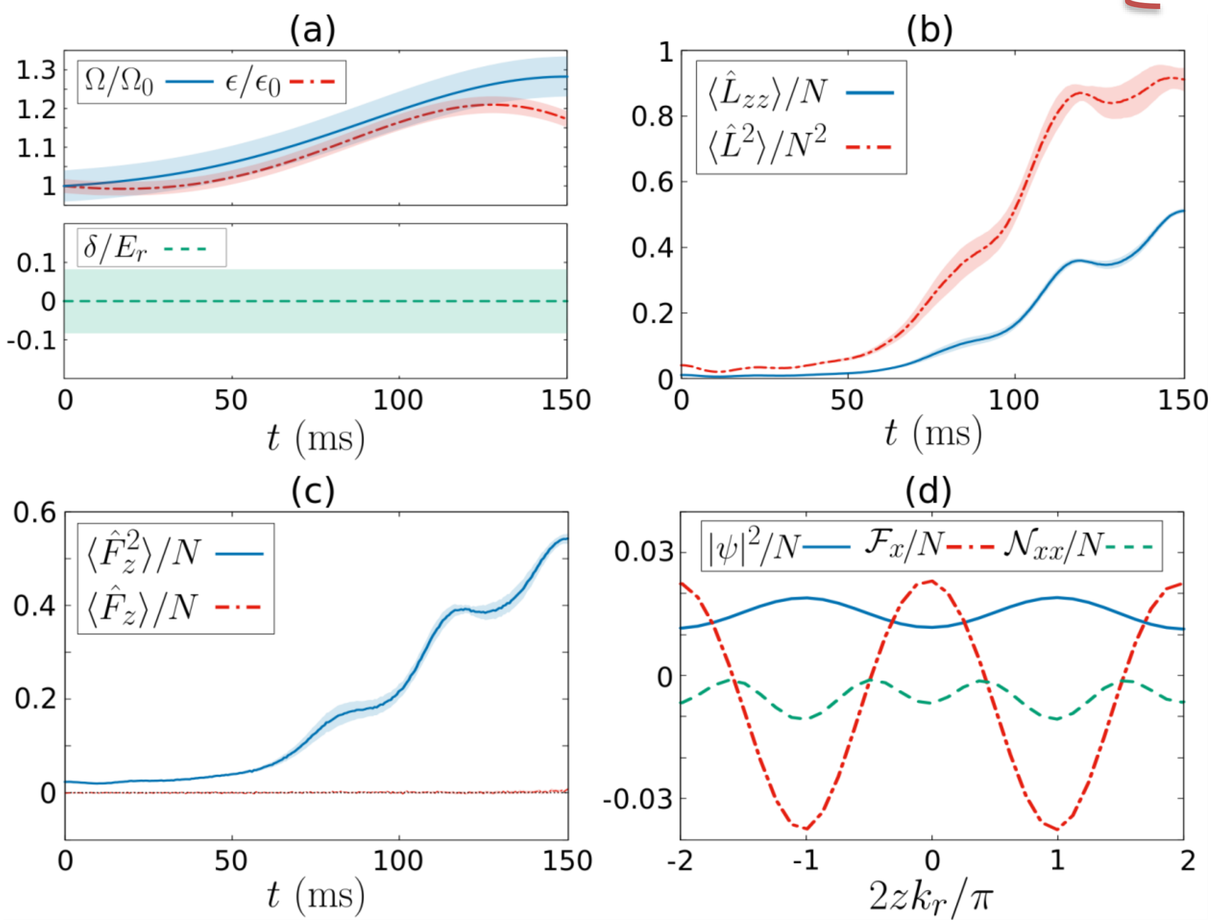

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~ 0.5 mG

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$\sim \pm 5\%$

D. Campbell *et al.* *Nat. Commun.* **7**, 10897 (2016)
R. P. Anderson, *et al., Phys.Rev. Research* **2**, 013149 (2020)

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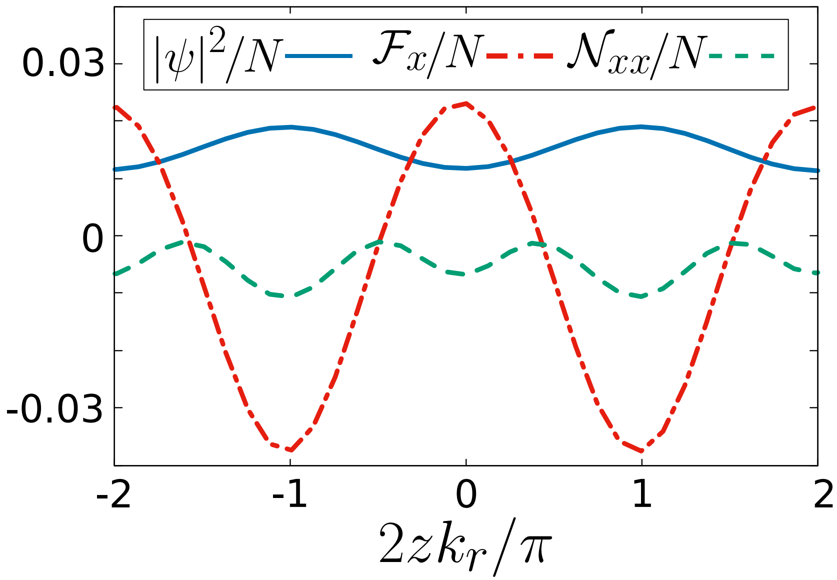
Ferromagnetic stripe state



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$\tau_d = 150$ ms ✓

$\Delta\epsilon = 2.5$ Hz, $\Delta\delta = 300$ Hz $\}$ ~ 0.5 mG ✓

$\Delta\Omega = 125$ Hz $\sim \pm 5\%$

Modulated densities	Periodicity
$|\psi|\,,\ \mathcal{F}_x = \psi^*\hat{F}_x\psi$	$2\pi/|\boldsymbol{k}_1|$ ✓
$\mathcal{N}_{xx} = \psi^*(2/3 - \hat{F}_x^2)\psi$	$2\pi/|\boldsymbol{k}_1|$ & $\pi/|\boldsymbol{k}_1|$ ✓

D. Campbell *et al.* *Nat. Commun.* **7**, 10897 (2016)

G. I. Martone, *et al., Phys. Rev. Lett.* **117**,125301 (2016)

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Conclusion and outlook

- **Ultracold atoms**: ideal platform to study many-body physics in a controlled environment.

- Non-equilibrium dynamics and **ESQPTs** have been studied with **spinor condensates**. These protocols rely on the nature of spin-spin interactions within the condensate.

- In a certain regime, the **Raman-dressed SO coupled BEC** is equivalent to an artificial spinor BEC with **tunable** nonsymmetric **spin interactions**.

- Through this equivalence, the **super-solid-like stripe phase** of the SO coupled gas is well understood.

- Guided by the mapping, we design and benchmark a robust experimental **preparation** of the elusive phase **through** an **ESQPT**.

- The work exemplifies **ESQPTs as a tool** to engineer quantum many-body states. It also suggests new directions for exploring nonequilibrium experiments, and the generation of macroscopic entanglement in momentum space.

Thank you for your attention!

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