Critical phenomena in the quantum Rabi model

Ground, excited-state and dynamical quantum phase transitions in a finite-component system

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Instituto de Física Fundamental

Outline

- **Introduction and finite-component phase transitions**
- **Quantum Rabi model (QRM)**
  - **Quantum phase transition (QPT)**
  - **Excited-state quantum phase transitions (ESQPTs)**
  - **Dynamical quantum phase transitions (DPTs)**
- **Single trapped-ion experiment**
- **Summary**

M.-J. Hwang, RP, M. B. Plenio, *Phys. Rev. Lett.* ***115****, 180404 (2015)*

RP, M.-J. Hwang, M. B. Plenio, *Phys. Rev. A* ***94****, 023835 (2016)*

RP, M.-J. Hwang, J. Casanova, M. B. Plenio, *Phys. Rev. Lett.* ***118****, 073001 (2017)*

RP, *Phys. Rev. B* ***102****, 220302(R) (2020)*

Introduction

Introduction

- **From statistical mechanics**

$$Z = \sum_k e^{-H(\sigma_k)/k_b T}$$

Partition function
kth microstate configuration σ_k
$H(\sigma_k)$ energy of the kth microstate

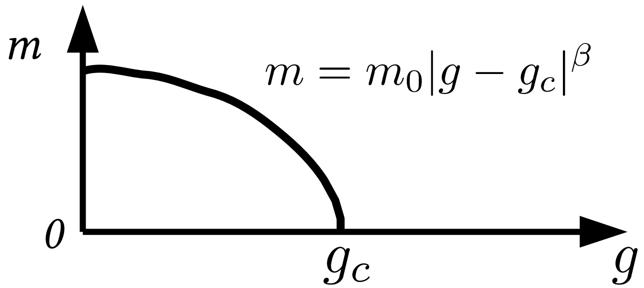
$$\langle A \rangle = \sum_k A_k e^{-H(\sigma_k)/k_b T} / Z$$

Macroscopic value of A

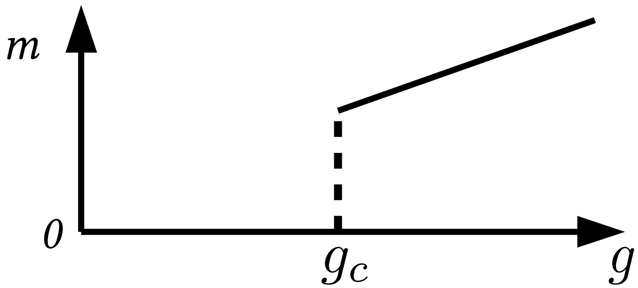
$$h = -k_b T \log Z$$

Helmholtz free energy

- **Different phases of matter can be characterized in terms of an order parameter:** $m = \dfrac{\partial h}{\partial g}$



continuous phase transitions



first-order phase transitions

Introduction

- **From statistical mechanics**

$Z = \sum_k e^{-H(\sigma_k)/k_b T}$	Partition function kth microstate configuration σ_k $H(\sigma_k)$ energy of the kth microstate
$\langle A \rangle = \sum_k A_k e^{-H(\sigma_k)/k_b T}/Z$	Macroscopic value of A
$h = -k_b T \log Z$	Helmholtz free energy

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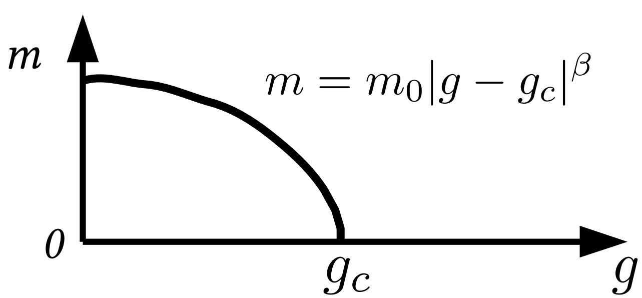
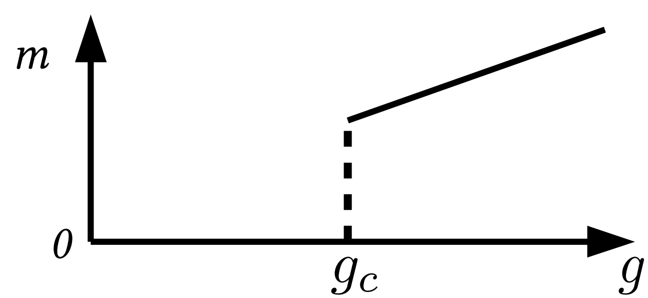


continuous phase transitions



first-order phase transitions

K. Huang, *Statistical mechanics*

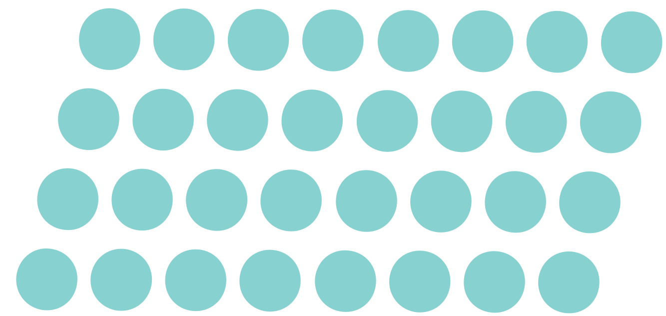
Introduction

- **Standard notion for phase transitions:**

$$Z = \sum_k e^{-H(\sigma_k)/k_b T} \longrightarrow \text{Sum over all microstates of the system}$$

Any finite system (finite number of particles) shows smooth and well-behaved properties

True singularities or criticality only in the thermodynamic limit



- **Quantum many-body systems:**

Dimension of the Hilbert space for a N spin-1/2 system

$$\dim\{\mathcal{H}\} = 2^N$$

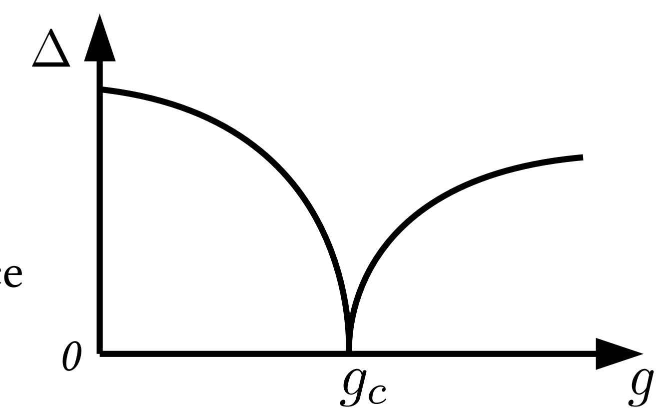
In the thermodynamic limit $N \to \infty$

$$\xi = \xi_0 |g - g_c|^{-\nu}$$

$$\Delta = \Delta_0 |g - g_c|^{z\nu}$$

$$m = m_0 |g - g_c|^{\beta}$$

The ground state is spanned by this infinitely large Hilbert space



S. Sachdev, *Quantum phase transitions*

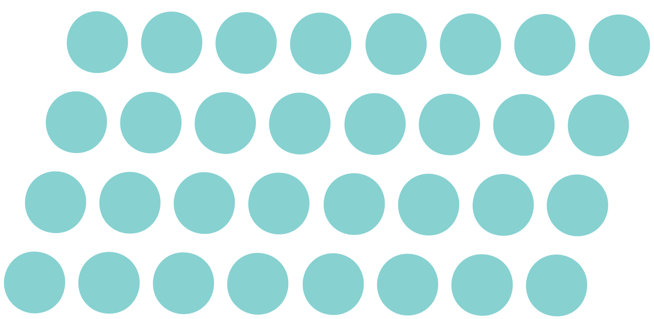
Finite-component systems

- **Is it possible to observe a phase transition in a finite-component quantum system?**

Quantum systems with an unbounded Hilbert space $\mathcal{H}$

Ground state must by spanned by an infinitely large number of states within $\mathcal{H}$

This can be achieved by tunning the system's parameters: Critical behavior appears in a limiting case $\eta \rightarrow \infty$



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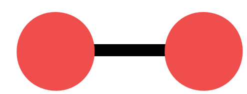

$$\dim\{\mathcal{H}\} = 2^N$$

$$\text{N} \rightarrow \infty$$

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$$m = m_0 |g - g_c|^{\beta}$$

$$\dim\{\mathcal{H}\} = \infty$$

$$\eta \rightarrow \infty$$

$$\xi \neq \xi_0 |g - g_c|^{-\nu}$$

$$\Delta = \Delta_0 |g - g_c|^{z\nu}$$

$$m = m_0 |g - g_c|^{\beta}$$

Finite-component phase transitions

- **Different models featuring finite-component phase transitions:**

Spin-1/2 interacting with a bosonic mode

Jaynes-Cummings model | Quantum Rabi model | Generalized finite-N Dicke models

- Rich phenomenology (QPT, dissipative phase transitions, ESQPTs, DPTs, etc.)
- Different universality classes (distinct critical exponents)

Theory: *Phys. Rev. Lett.* ***115***, *180404 (2015), Phys. Rev. Lett.* ***117***, *123602 (2016), Phys. Rev. A* ***94***, *023835 (2016), Phys. Rev. A* ***97***, *013825 (2018), ...*

Finite-Component Multicriticality at the Superradiant Quantum Phase Transition, H.-J. Zhu et al., *Phys. Rev. Lett.* ***125***, *050402 (2020)*

Experiment: *Nat. Comm.* ***12***, *1126 (2021)*
Applications: *Phys. Rev. Lett.* ***124***, *230602 (2020), Phys. Rev. Lett.* ***124***, *120504 (2020), Phys. Rev. Lett.* ***126***, *010502 (2021)*

Finite-component phase transitions

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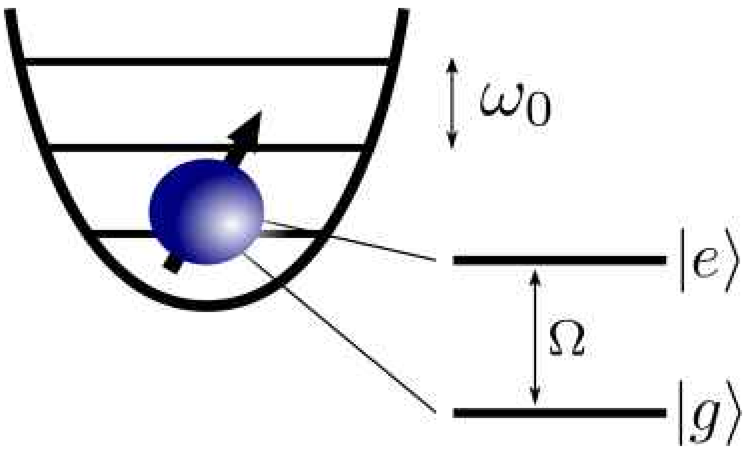
Criticality by tunning the system's parameters rather than scaling up the system components

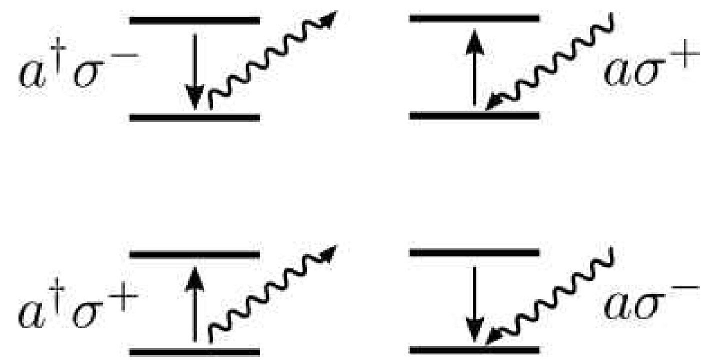
Quantum Rabi model

Quantum Rabi model

- **The quantum Rabi model (QRM):**

$$\mathrm{H}_{\mathrm{QRM}} = \frac{\Omega}{2}\sigma_z + \omega_0 a^\dagger a + \lambda \sigma_x (a + a^\dagger)$$





The QRM describes the coherent interaction of a spin with a single bosonic mode

The Hamiltonian exhibits a Z_2 parity symmetry: $[H_{\mathrm{QRM}}, \Pi] = 0$ with $\Pi = e^{i\pi(a^\dagger a + \sigma^+ \sigma^-)}$

$$\Pi|\phi\rangle = \pm|\phi\rangle$$

Single spin-1/2 version of the Dicke model

Quantum Rabi model

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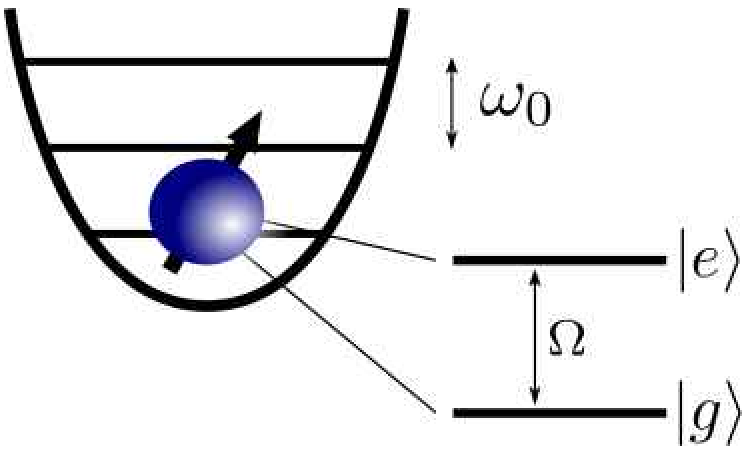
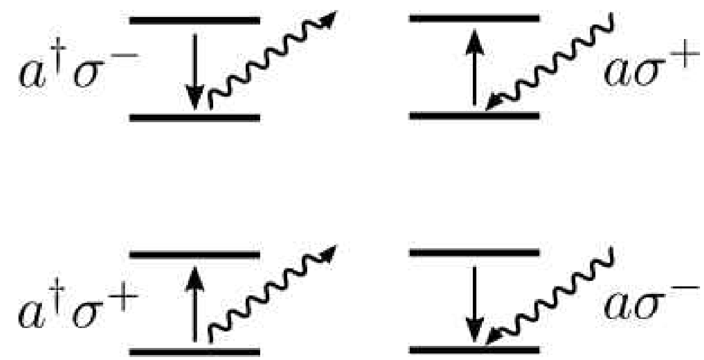
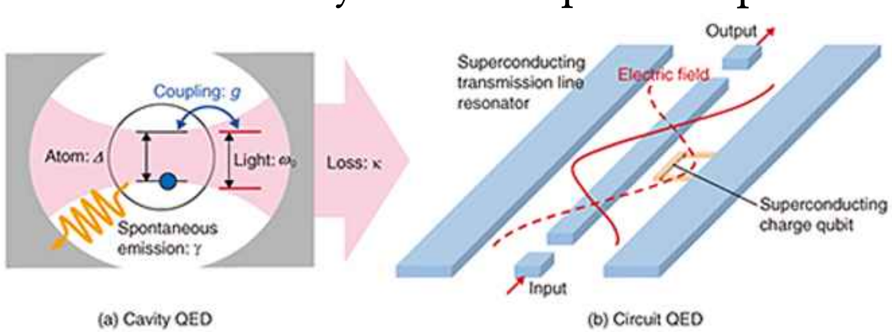
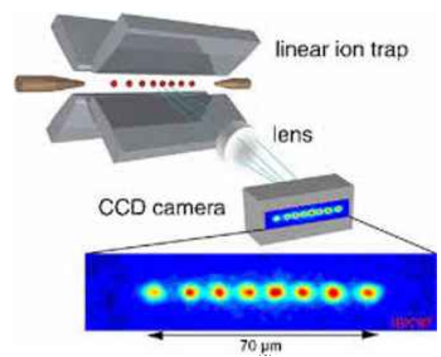
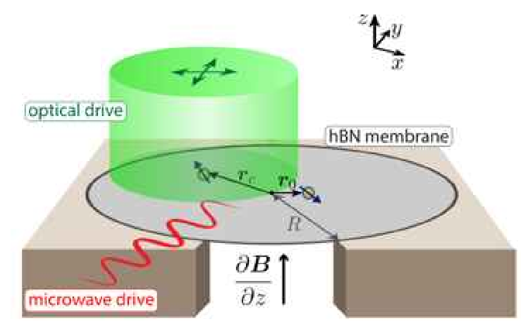




Relevant in many different quantum platforms:







P. Forn-Diaz *et al.*, *Phys. Rev. Lett.* ***105****, 237001 (2010)*
D. Lv *et al.*, *Phys. Rev X* ***8****, 021027 (2018)*
M. Abdi *et al.*, *Phys. Rev. Lett.* ***119****, 233602 (2017)*

Quantum Rabi model

- **In the weak coupling:** $\lambda\sqrt{\langle n\rangle+1} \ll |\Omega+\omega_0|$

$$\mathrm{H}_{\mathrm{QRM}} \approx H_{\mathrm{JCM}} = \frac{\Omega}{2}\sigma_z + \omega_0 a^\dagger a + \lambda(\sigma^+ a + \sigma^- a^\dagger)$$

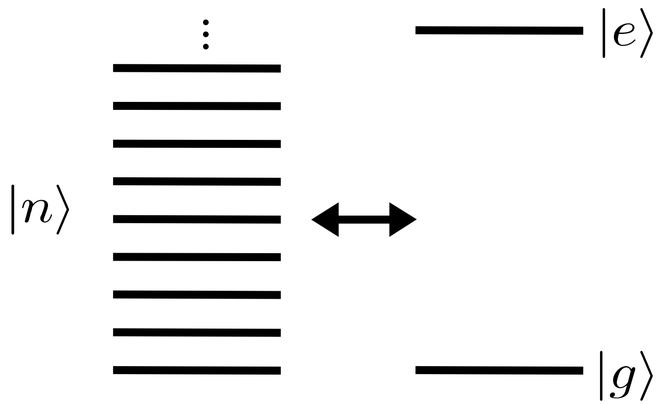
- **But more interesting physics happen in the strong coupling and beyond**

  In particular, we are interested in the parameter regime

$$\omega_0 \ll \lambda \ll \Omega$$

  In the limit $\eta \equiv \Omega/\omega_0 \to \infty$ and $\lambda/\omega_0 \to \infty$ keeping $\lambda/\sqrt{\omega_0\Omega}$ finite, we find a QPT in the QRM

  In this manner, the ground state of the QRM can explore the infinitely large Hilbert space



Low-energy effective description

- **Low-energy Hamiltonians in the limit** $\eta \equiv \Omega/\omega_0 \to \infty$ $\qquad g = 2\lambda/\sqrt{\Omega\omega_0}$

$$H_{\mathrm{np}} = \lim_{\eta\to\infty}\langle\downarrow|U^\dagger H_{\mathrm{QRM}}U|\downarrow\rangle = \omega_0 a^\dagger a - \frac{g^2\omega_0}{4}(a+a^\dagger)^2 - \frac{\Omega}{2} \qquad g \le g_c = 1$$

$\mathrm{U{=}e}^{\lambda/\Omega(a+a^\dagger)(\sigma^+-\sigma^-)}$ Schrieffer-Wolff transformation

For g>1 we need first to displace the mode and define new spin states

$$a \to a \pm \alpha_g(g) \qquad \alpha_g^2(g) = \frac{\Omega}{\omega_0}\frac{g^2-g^{-2}}{4}$$

$$|\downarrow^\pm\rangle = \mp\sqrt{\frac{1-g^{-2}}{2}}|\uparrow\rangle + \sqrt{\frac{1+g^{-2}}{2}}|\downarrow\rangle$$

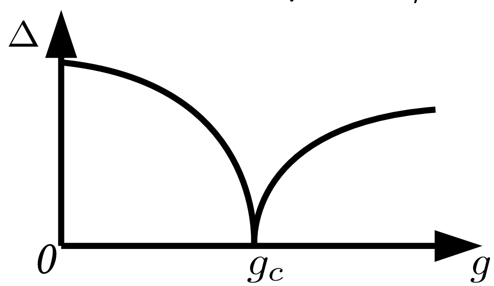
$$H_{\mathrm{sp}} = \omega_0 a^\dagger a - \frac{\omega_0}{4g^4}(a+a^\dagger)^2 - \frac{\Omega}{4}(g^2+g^{-2}) \qquad g > g_c = 1$$

- Both solutions are quadratic, so it can be diagonalized:

$$\Delta(g) = \begin{cases}\omega_0\sqrt{1-g^2} & 0 \le g \le 1 \\ \omega_0\sqrt{1-g^{-4}} & g > 1\end{cases} \longrightarrow \Delta(g) \propto |g-g_c|^{z\nu} \quad \text{with} \quad z\nu = 1/2$$

Quantum Rabi model

- **Phase diagram of the QRM in the limit** $\eta \equiv \Omega/\omega_0 \to \infty$



Normal phase

$0 \leq g \leq 1$

Z$_2$ symmetry holds

$$|\phi^0_{\mathrm{np}}\rangle = \mathcal{S}[r_{\mathrm{np}}(g)]|0\rangle|\downarrow\rangle$$

$$r_{\mathrm{np}}(g) = -1/4\log(1-g^2)$$

Superradiant phase

$g > 1$

Spontaneous symmetry breaking of **Z$_2$**

$$|\phi^0_{\mathrm{sp}}\rangle_\pm = \mathcal{D}[\pm\alpha_g(g)]\mathcal{S}[r_{\mathrm{sp}}(g)]|0\rangle|\downarrow^\pm\rangle$$

$$r_{\mathrm{sp}}(g) = -1/4\log(1-g^{-4})$$

- **The physics is very similar to the superradiant QPT in the Dicke model, where N atoms are collectively coupled to a bosonic mode**

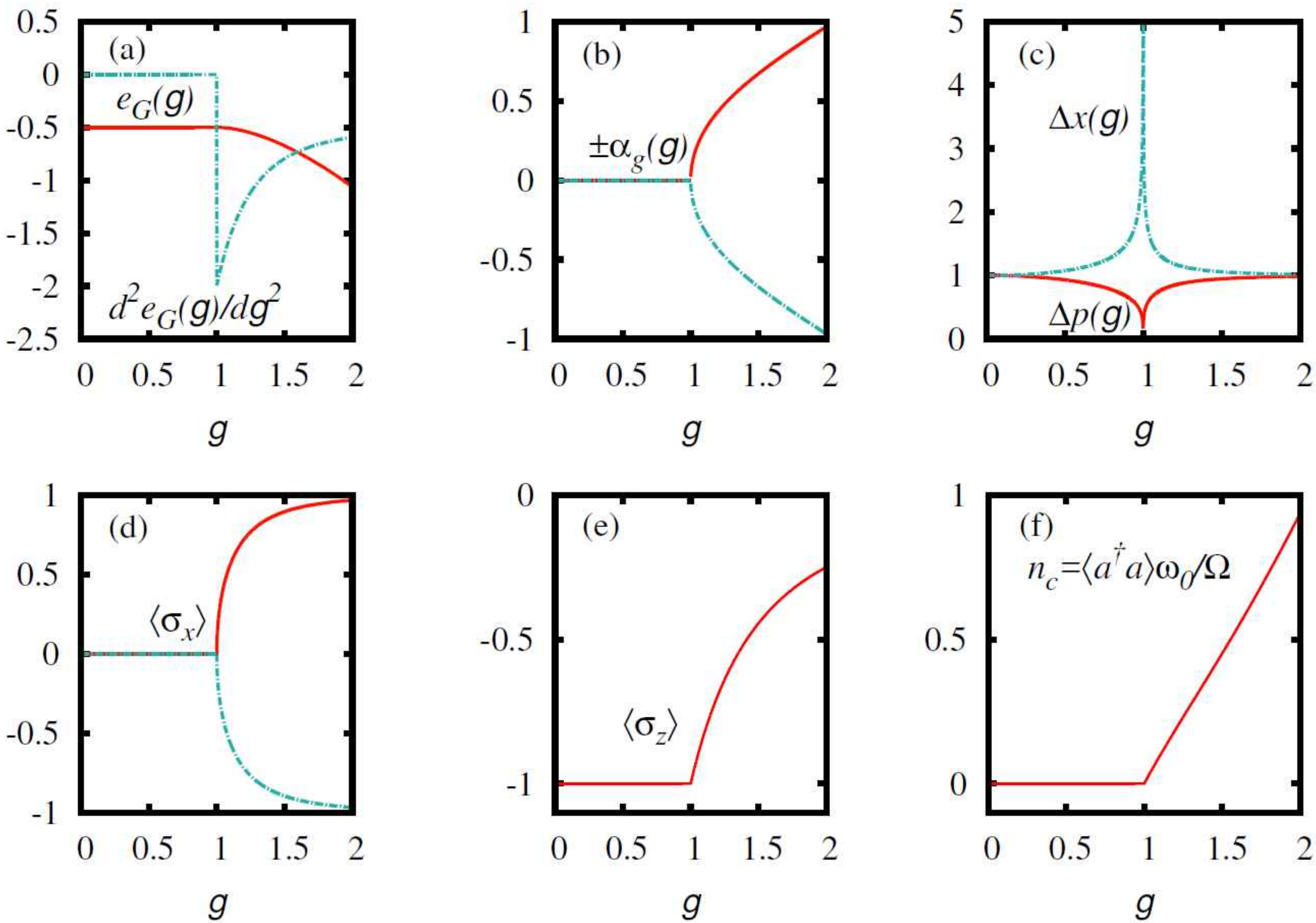
There is no notion of spatial dimension (correlation length is ill-defined)

Zero dimensional system *d=0*

Same critical exponents --> both belong to the same universality class

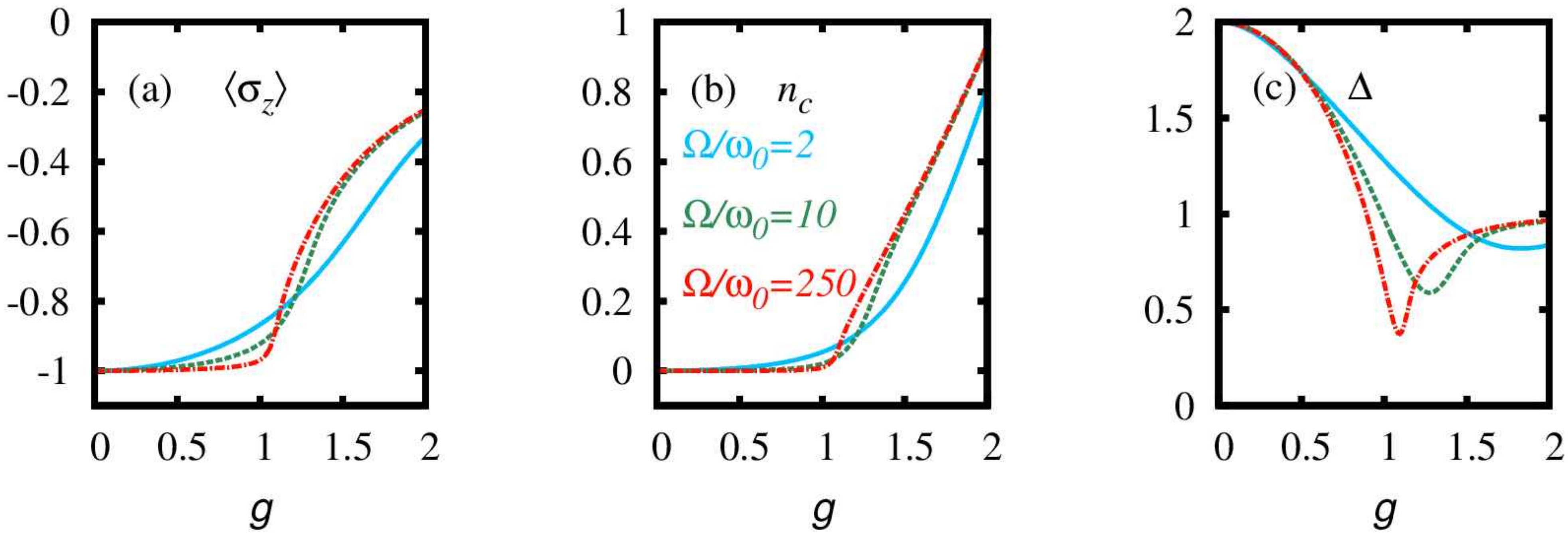
QPT in the QRM

- **Ground-state properties in the limit** $\eta \equiv \Omega/\omega_0 \to \infty$



QPT in the QRM

- **Ground-state properties for finite $\eta \equiv \Omega/\omega_0$**



- **Finite values of η similar effect as finite N in standard quantum many-body systems**

$\longrightarrow$ Finite-size scaling theory

Finite-η scaling

- **Recall finite-size scaling theory:**
For a standard many-body system in the thermodynamic limit
$$\langle A \rangle^{\infty} = a_0 |g - g_c|^{\alpha} \qquad N \to \infty$$
the finite-size scaling hypothesis (based on a coarse graining or renormalization group analysis) states that
$$\langle A \rangle^{N} = |g - g_c|^{\alpha} f(L/\xi) = |g - g_c|^{\alpha} f(N|g - g_c|^{\nu})$$
where *f(x)* is a scaling function that must satisfy
$$\lim_{x\to 0} f(x) \propto x^{-\alpha/\nu}$$
$$\lim_{x\to\infty} f(x) = a_0$$
which ensures that no true singularities happen for finite N

At the critical point it follows $\left.\langle A \rangle^{N}\right|_{g=g_c} \propto N^{-\alpha/\nu}$

- For finite-component systems, even if there is no correlation length one can apply this in terms of, e.g. $\eta \equiv \Omega/\omega_0$ for the QRM rather than N
$$\langle A \rangle^{\infty} = a_0 |g - g_c|^{\alpha} \Rightarrow \langle A \rangle^{\eta} = |g - g_c|^{\alpha} f(\eta |g - g_c|^{\nu})$$
$$\langle A \rangle^{\eta} |_{g=g_c} \propto \eta^{-\alpha/\nu}$$

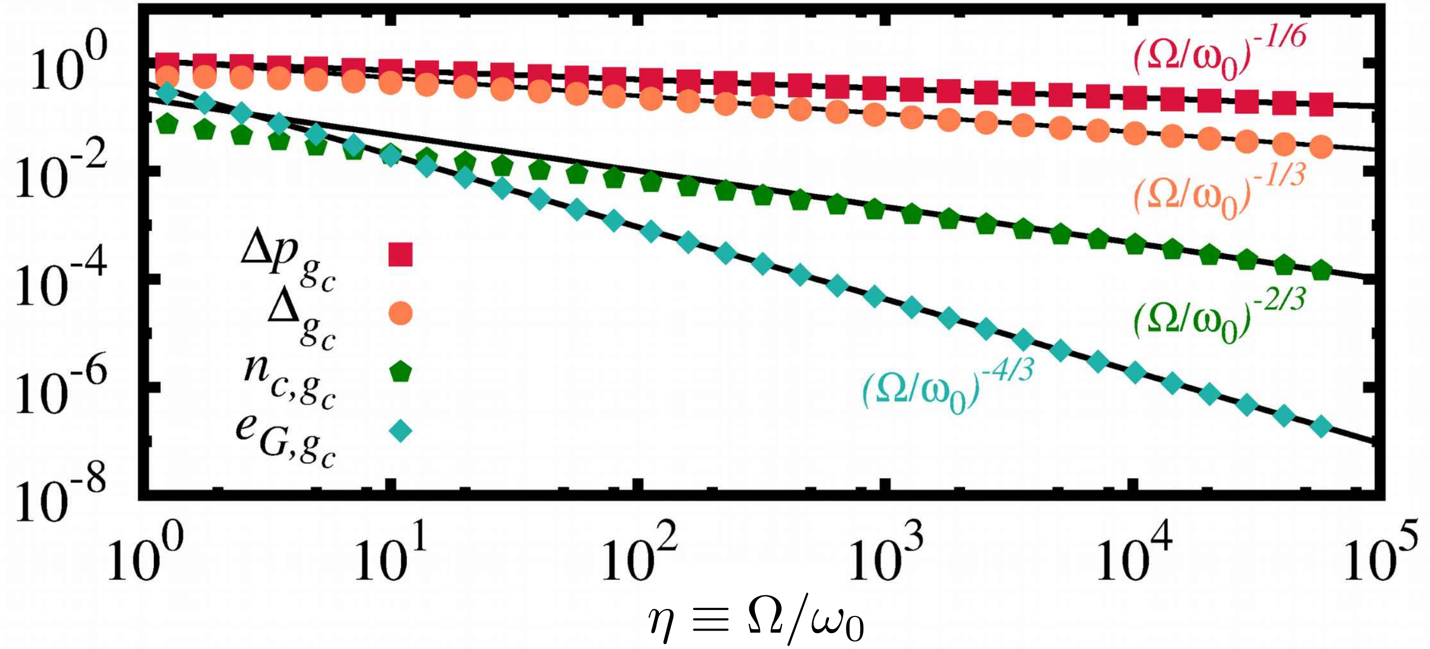
M. E. Fisher, M. N. Barber, *Phys. Rev. Lett.* ***28****, 1516 (1972);* R. Botet et al., *Phys. Rev. Lett.* ***49****, 478 (1983)*

Finite-η scaling

- **For the QRM we find** $\nu = 3/2$ **so that** $\langle A \rangle^\eta \,|_{g=g_c} \propto \eta^{-\alpha/\nu} = \eta^{-2\alpha/3}$

$$\Delta p^\infty \propto |g - g_c|^{1/4} \Rightarrow \Delta p^\eta \,|_{g=g_c} \propto \eta^{-1/6}$$

$$\Delta^\infty \propto |g - g_c|^{1/2} \Rightarrow \Delta^\eta \,|_{g=g_c} \propto \eta^{-1/3}$$

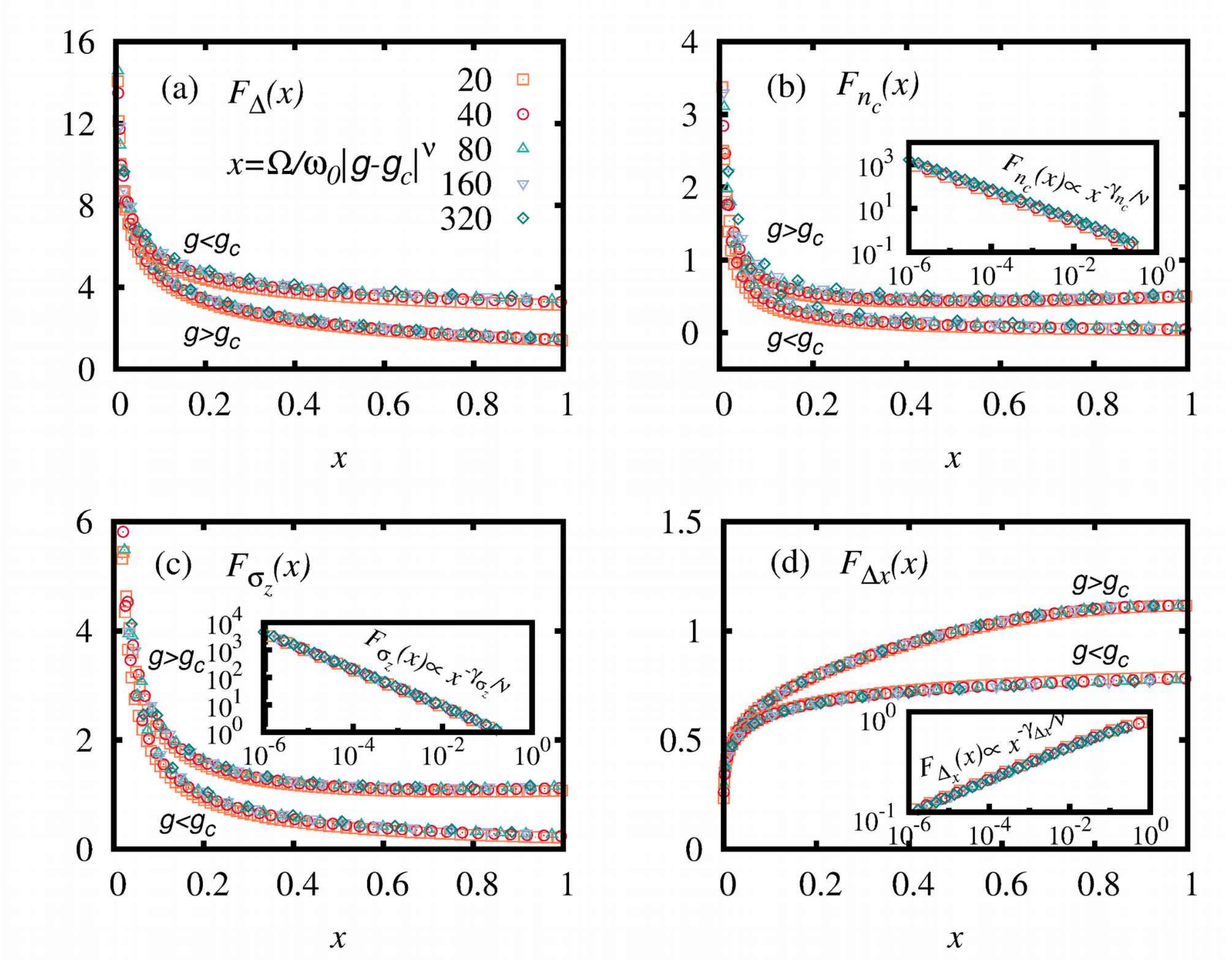


Finite-η scaling functions

- **In addition, finite-η scaling functions follow from**

$$\langle A \rangle^{\eta} = |g-g_c|^{\alpha} f(\eta|g-g_c|^{\nu}) \Rightarrow \langle A \rangle^{\eta}|g-g_c|^{-\alpha} = f(\eta|g-g_c|^{\nu}) = f(x)$$

Finite-η scaling functions depend only on the scaling variable x (η-independent function)



ESQPT

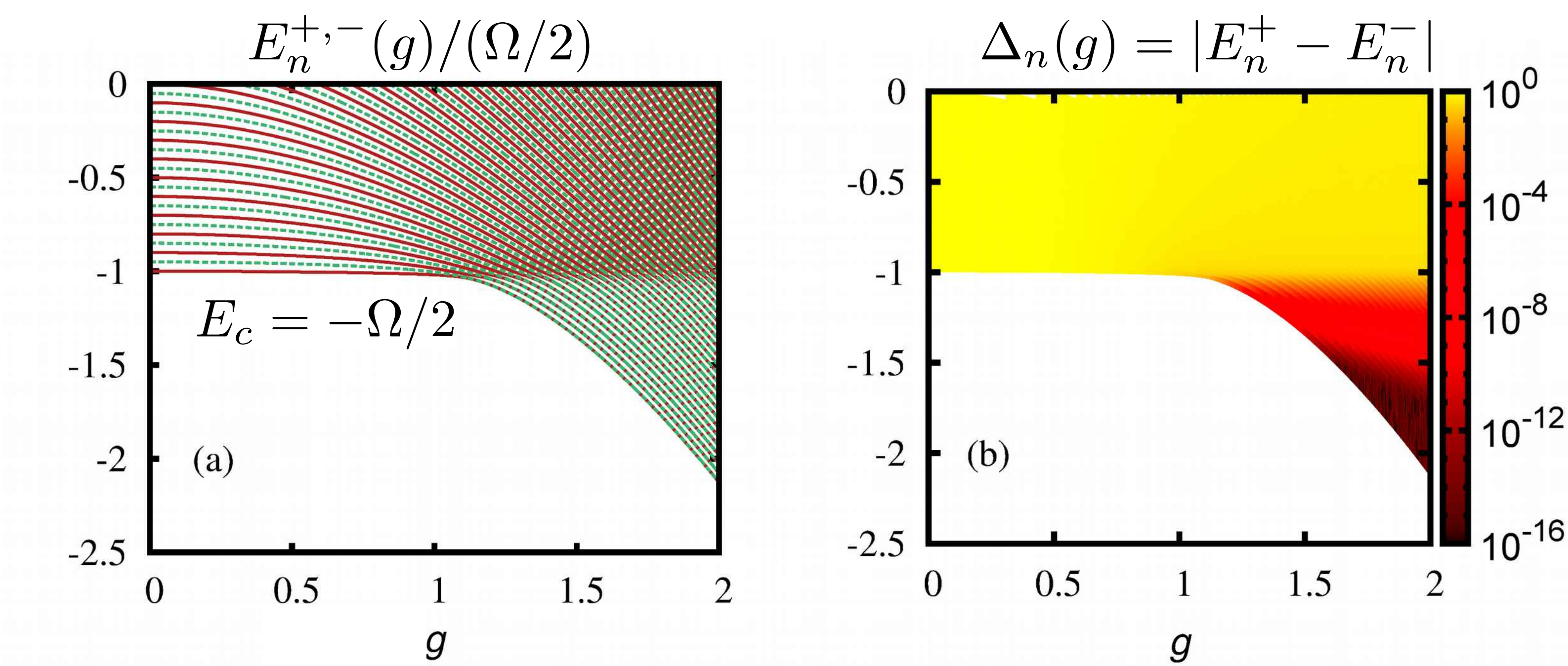
ESQPT in the QRM

- **Having one degree of freedom *f=1* and displaying a QPT, the QRM is a perfect candidate to exhibit an ESQPT**

$$\mathrm{H}_{\mathrm{QRM}} = \frac{\Omega}{2}\sigma_z + \omega_0 a^\dagger a + g\frac{\sqrt{\Omega\omega_0}}{2}\sigma_x(a + a^\dagger)$$

From previous works, we expect a density of states diverging logarithmically close to a critical value of the excitation energy E_c

$$\rho(E) \sim -\log|E - E_c|, \qquad \text{for} \quad g > g_c = 1, |E - E_c| \ll 1$$



P. Cejnar, P. Stransky, M. Macek, M. Kloc, *J. Phys. A: Math. Theor.* ***54***, *133001 (2021)*

ESQPT in the QRM

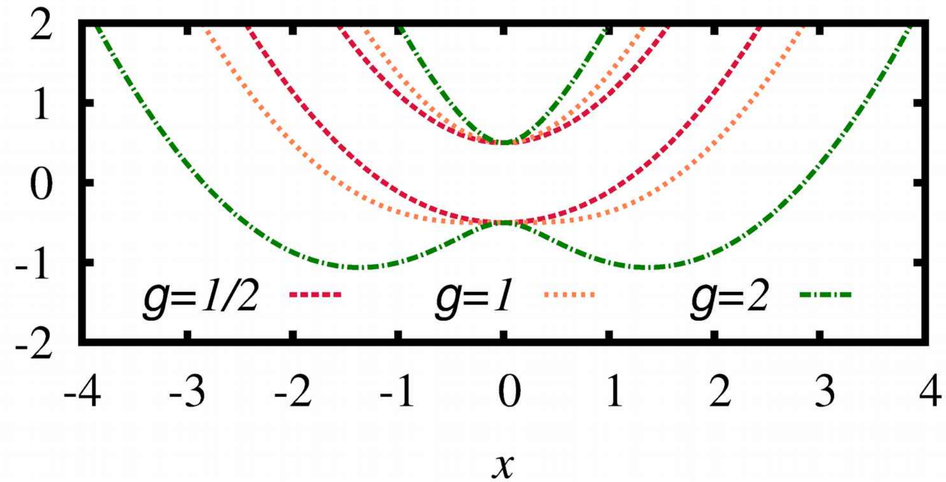
- **We rely on a semiclassical approximation to obtain the density of states**

$$H_{\text{QRM}} \to H_{\text{scl}}(p,x,g)/\Omega = \frac{p^2}{2} + \underbrace{\frac{x^2}{2} + \frac{1}{2}\sigma_z - \frac{g}{\sqrt{2}}x\sigma_x}_{V^{\pm}_{\text{eff}}(x)/\Omega = \frac{1}{2}x^2 \pm \frac{1}{2}\sqrt{1+2g^2x^2}}$$

$$x = (a + a^\dagger)/\sqrt{2\eta}$$
$$p = i(a^\dagger - a)/\sqrt{2\eta}$$

The effective potential $V^{-}_{\text{eff}}(x)$ shows the standard single- to double-well shape

$$x_{\min} = \begin{cases} 0, & g \leq 1 \\ \pm\frac{1}{\sqrt{2}}\sqrt{g^2 - g^{-2}}, & g > 1 \end{cases}$$



The separatrix takes place at $E_c = -\Omega/2$ for $g > 1$

ESQPT in the QRM: DoS

- **The semiclassical density of states (DoS) is given by**

$$\rho(E,g) = \frac{1}{2\pi}\int dxdp\ \delta\left[E - H_{\mathrm{scl}}(x,p,g)\right]$$

It is then possible to find solutions close to $\epsilon_c = -1$ with $\epsilon = E/|E_c| = 2E/\Omega$

- For g=1 (at the QPT) we find

$$\rho(\epsilon, g=1) = \frac{\Gamma(5/4)}{\Gamma(3/4)}\frac{2^{5/4}}{\omega_0\sqrt{\pi}}|\epsilon - \epsilon_c|^{-1/4} \qquad |\epsilon - \epsilon_c| \ll 1$$

- For g>1 and close to the critical energy $\epsilon_c = -1$

$$\rho(\epsilon, g>1) \sim \frac{-\log|\epsilon - \epsilon_c|}{\omega_0 \pi\sqrt{g^2 - 1}} + K \qquad |\epsilon - \epsilon_c| \ll 1$$

$\longrightarrow$ **Logarithmic divergence of the DoS at the ESQPT**

ESQPT in the QRM: DoS

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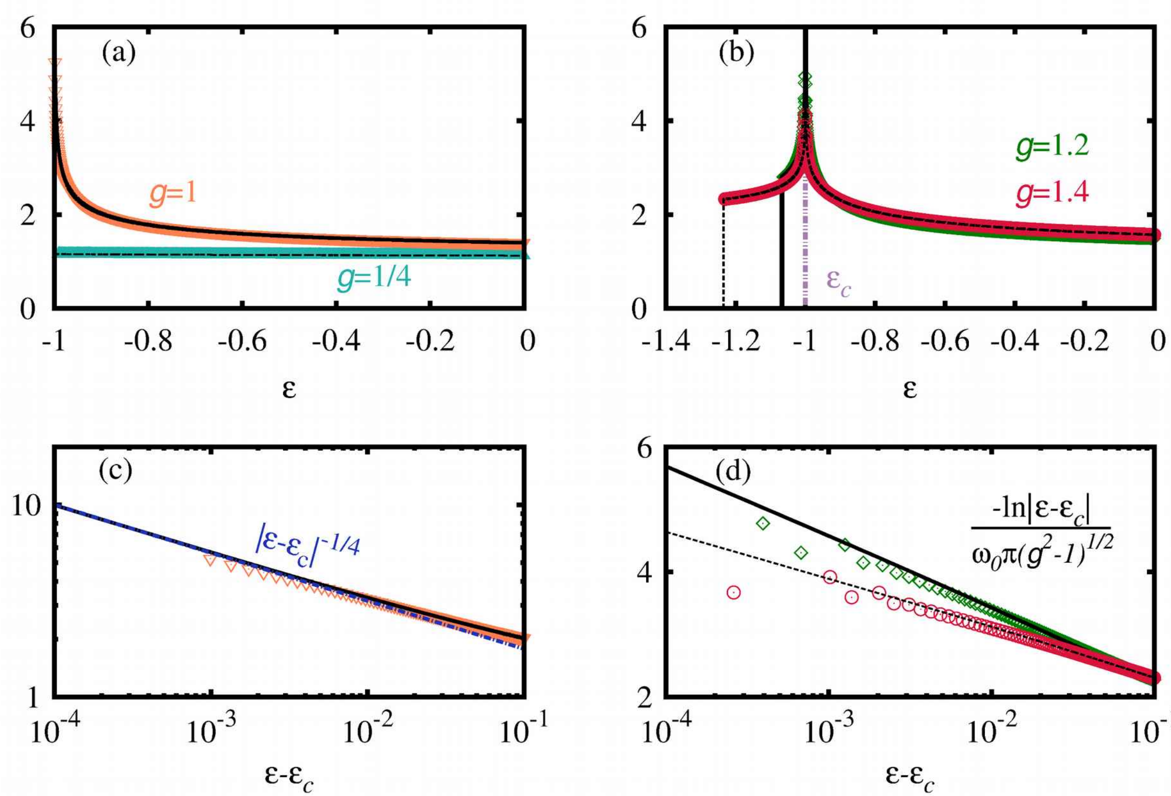
$$\rho(\epsilon, g=1) = \frac{\Gamma(5/4)}{\Gamma(3/4)}\frac{2^{5/4}}{\omega_0\sqrt{\pi}}|\epsilon-\epsilon_c|^{-1/4}$$

$$\rho(\epsilon, g>1) \sim \frac{-\log|\epsilon-\epsilon_c|}{\omega_0\pi\sqrt{g^2-1}} + K$$

Points correspond to the quantum averaged DoS for $\eta = 10^3$

$$\overline{\rho}(\epsilon, g) = N/\Delta\epsilon$$



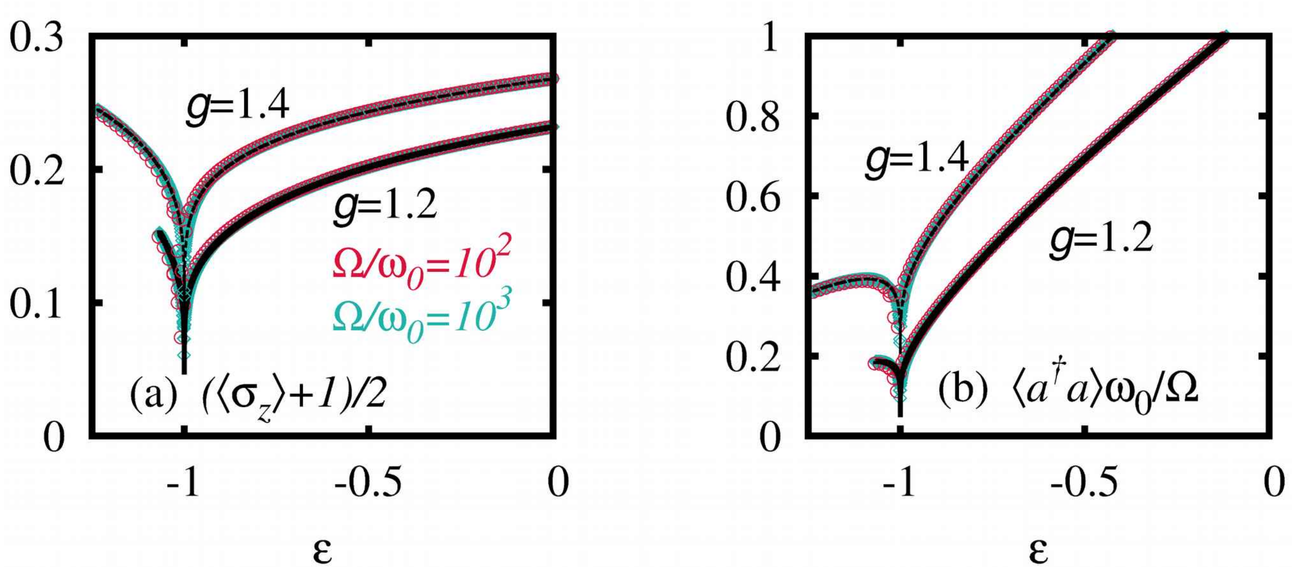
Signatures of the ESQPT

- **Apart fromt the DoS, relevant quantities display also a critical behavior**

$$\langle A\rangle(\epsilon,g)=\frac{1}{\rho(\epsilon,g)}\sum_{k,s=\pm}\langle\varphi_k^s|A|\varphi_k^s\rangle=-\frac{1}{\rho(\epsilon,g)}\frac{\partial}{\partial\beta}\mathcal{N}(\epsilon,g)\quad \text{if}\quad A=\partial_\beta H$$

$$\mathcal{N}(\epsilon,g)=\int_{-\infty}^{\epsilon}d\epsilon'\rho(\epsilon',g)$$

$$\langle\sigma_z\rangle(\epsilon,g)=-\frac{2}{\rho(\epsilon,g)}\partial_\Omega\mathcal{N}(\epsilon,g)\qquad \langle a^\dagger a\rangle(\epsilon,g)=-\frac{1}{\rho(\epsilon,g)}\partial_{\omega_0}\mathcal{N}(\epsilon,g)$$



DPTs

DPTs

- **Dynamics upon a sudden quench can reveal the so-called dynamical phase transitions**

Non-analytic behavior in the nonequilibrium dynamics rather than in eigenstates

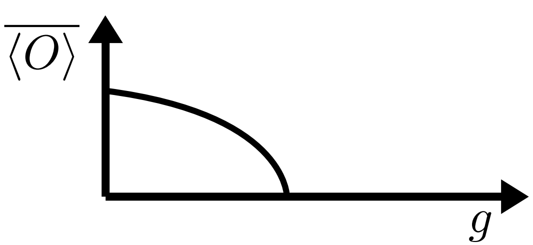
Two different types

DPT-I

- Time-averaged order parameter

$$|\psi(0)\rangle \to |\psi(t)\rangle = e^{-itH(g)}|\psi(0)\rangle$$

$$\overline{\langle O\rangle} = \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\ \langle O(t)\rangle$$

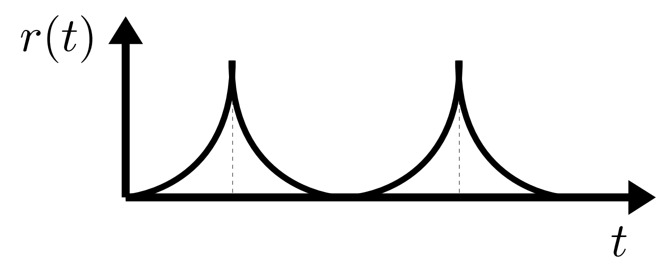


DPT-II

- Non-analyticities in the Loschmidt echo

$$L(t) = \langle\psi(0)|e^{itH(g_1)}e^{-itH(g_2)}|\psi(0)\rangle$$

$$r(t) = -\frac{1}{N}\log|L(t)|^2$$



M. Heyl, A. Polkovnikov, and S. Kehrein, *Phys. Rev. Lett.* ***110***, *135704 (2013)*

M. Heyl, *Rep. Prog. Phys.* ***81***, *054001 (2018)*

Observation of DPTs

Selected for a Viewpoint in *Physics*
PRL **119,** 080501 (2017) PHYSICAL REVIEW LETTERS week ending 25 AUGUST 2017

Direct Observation of Dynamical Quantum Phase Transitions in an Interacting Many-Body System

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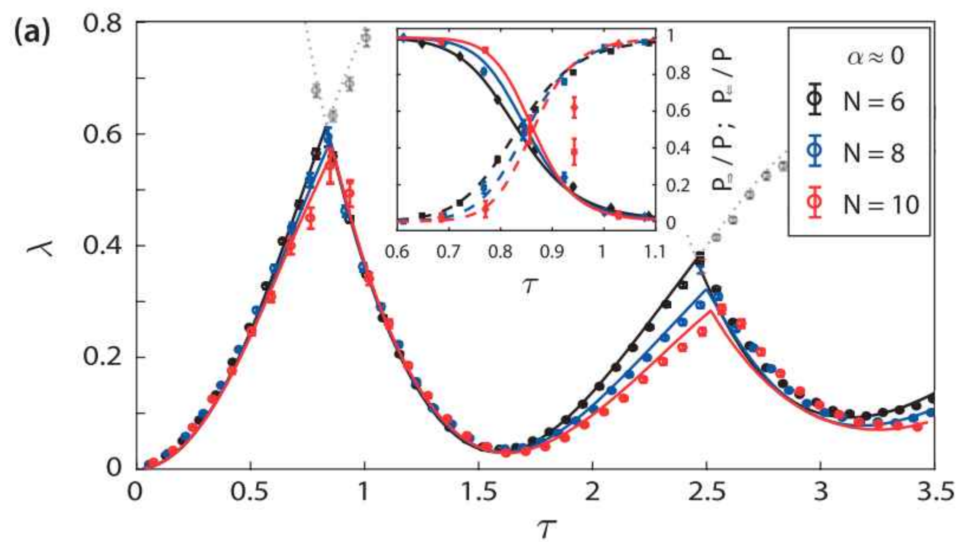
$$L(t) = \langle \psi(0) | e^{itH(g_1)} e^{-itH(g_2)} | \psi(0) \rangle$$

$$r(t) = -\frac{1}{N} \log |L(t)|^2$$



Observation of DPTs

LETTER

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Observation of a many-body dynamical phase transition with a 53-qubit quantum simulator

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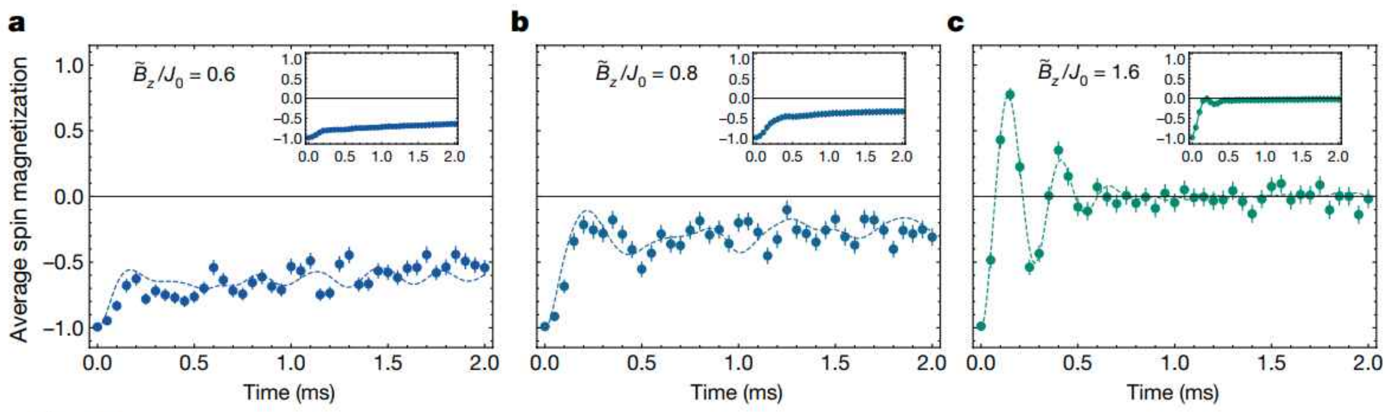


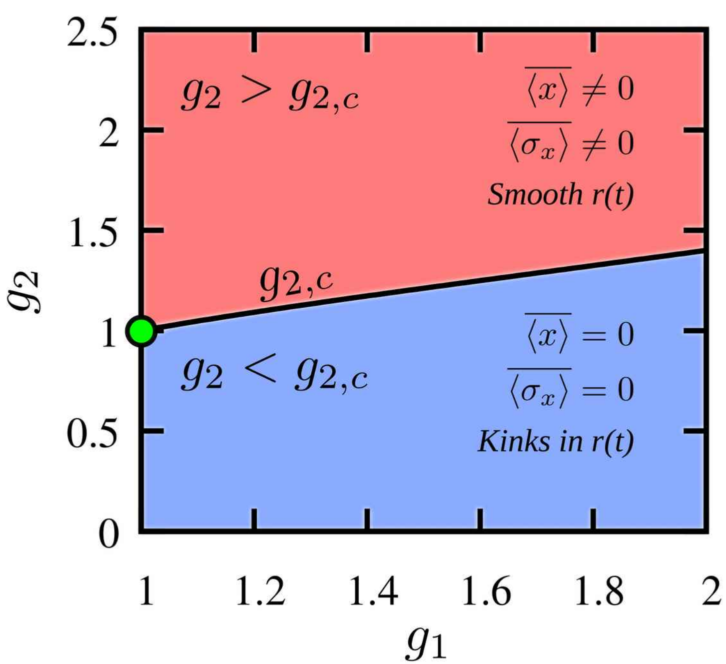
$$|\psi(0)\rangle \rightarrow |\psi(t)\rangle = e^{-itH(g)}|\psi(0)\rangle$$

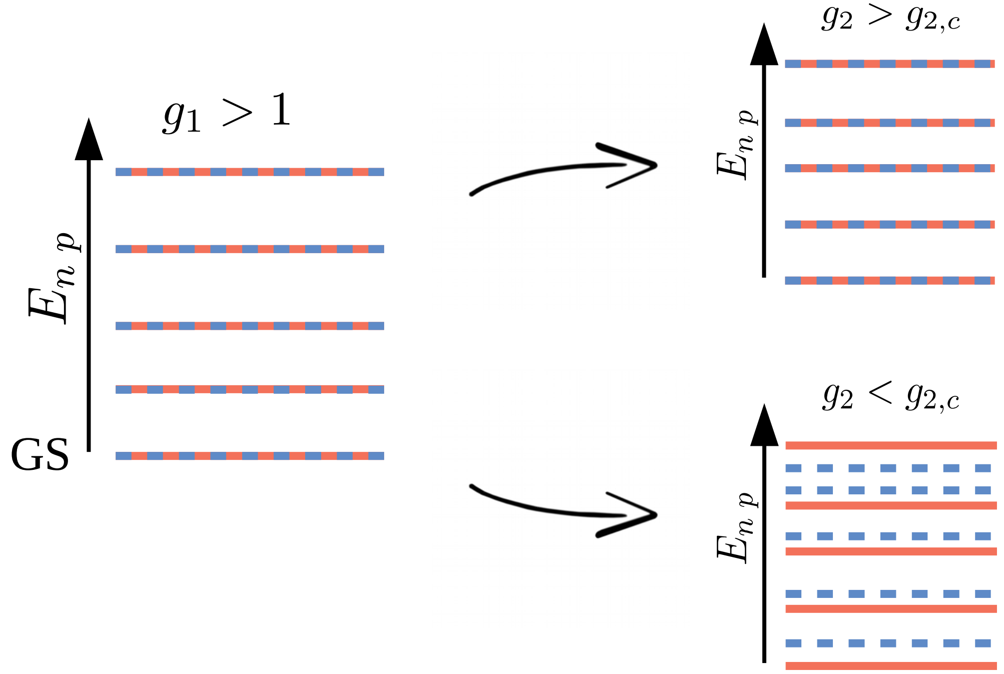
$$\overline{\langle O\rangle} = \lim_{T\rightarrow\infty}\frac{1}{T}\int_0^T dt\ \langle O(t)\rangle$$

DPTs in the QRM

$$H_{\mathrm{QRM}} = \frac{\Omega}{2}\sigma_z + \omega_0 a^\dagger a + g\frac{\sqrt{\Omega\omega_0}}{2}\sigma_x(a + a^\dagger)$$

- Start from a symmetry-breaking ground state $g_1 > 1$ and quench it to g_2
- In the limit $\eta \equiv \frac{\Omega}{\omega_0} \to \infty$ we find the non-equilibrium phase diagram $g_{2,c} = \frac{g_1(3+g_1^2)}{2(1+g_1^2)}$

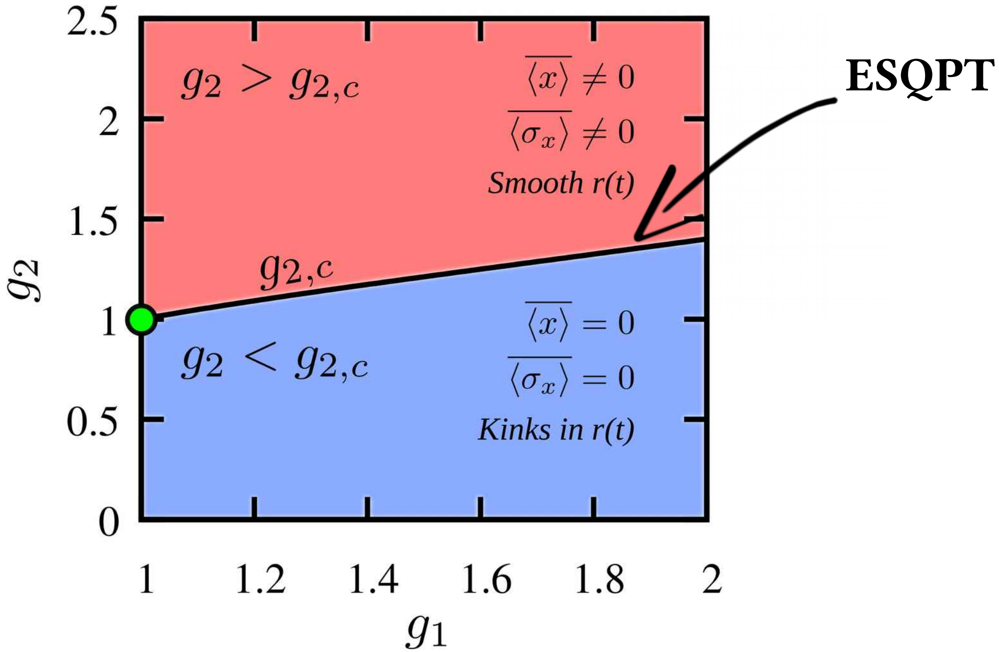


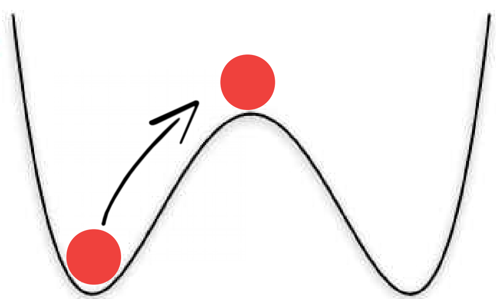


DPTs in the QRM

- **The critical point of the DPT coincides with the position of the ESQPT**

$$\langle\psi(0)|H(g_{2,c})|\psi(0)\rangle \equiv E_c \qquad |\psi(0)\rangle = |\varphi_0(g_1)\rangle \Rightarrow g_{2,c} = \frac{g_1(3+g_1^2)}{2(1+g_1^2)}$$





$$H(g_1) \rightarrow H(g_2 > g_{2,c})$$

Sufficient energy to go over the critical energy E_c

$$\langle O_{\mathrm{SB}}(t)\rangle = \langle\psi(t)|O_{\mathrm{SB}}|\psi(t)\rangle = \sum_{n,m} c_n c_m^* e^{-it(E_n^- - E_m^+)}\langle\varphi_m^+(g_2)|O_{\mathrm{SB}}|\varphi_n^-(g_2)\rangle + \mathrm{H.c.}$$

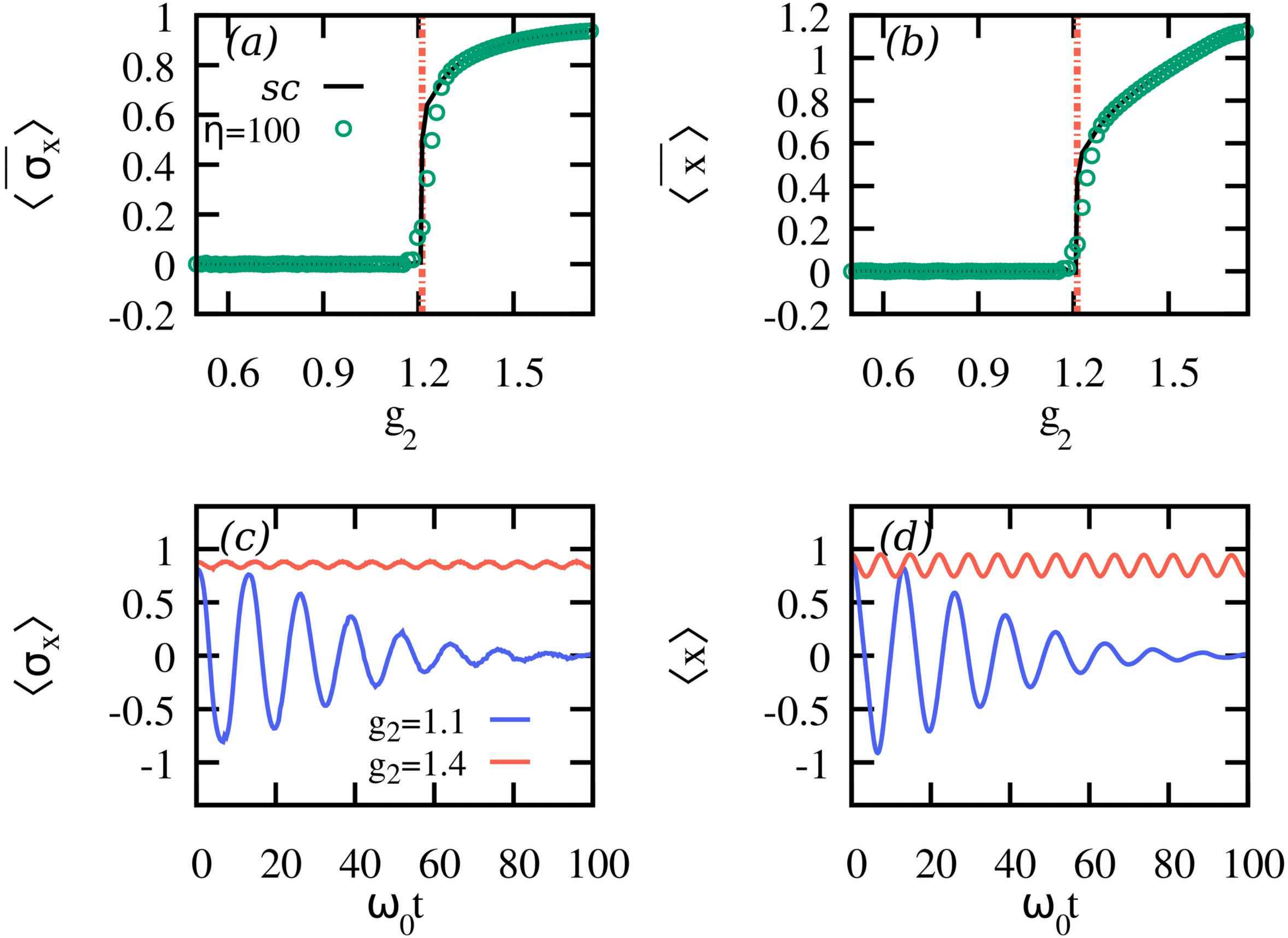
$$\overline{\langle O_{\mathrm{SB}}\rangle} = \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\langle O_{\mathrm{SB}}(t)\rangle$$

$$\overline{\langle O_{\mathrm{SB}}\rangle} = 0 \quad \text{if} \quad E_n^- \neq E_n^+$$

$$\overline{\langle O_{\mathrm{SB}}\rangle} \neq 0 \quad \text{if} \quad E_n^- = E_n^+$$

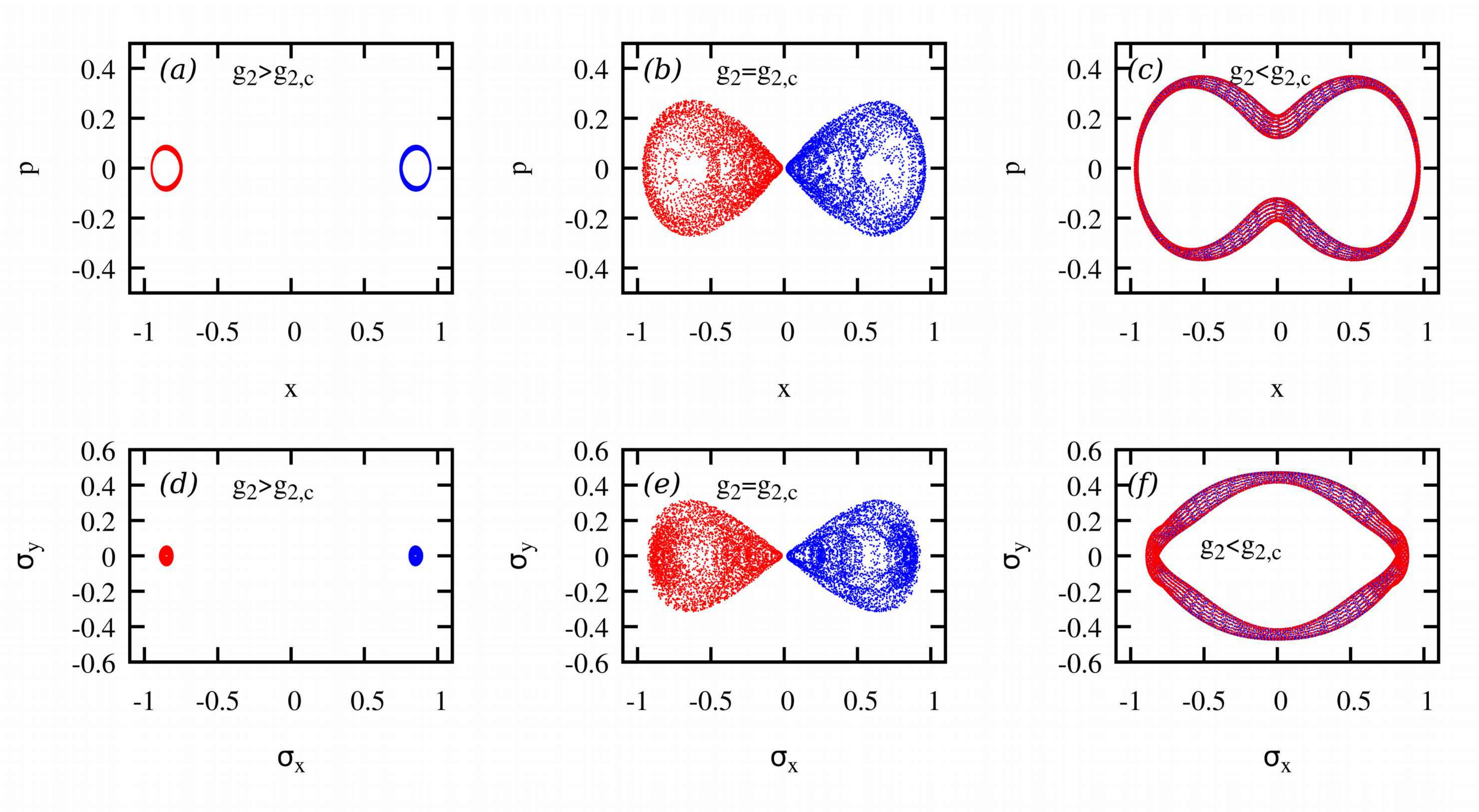
DPT-I

- Numerical results



DPT-I

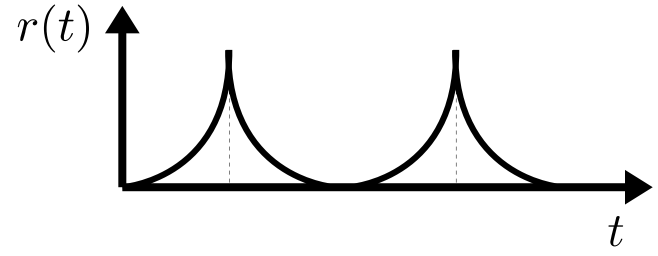
- Semiclassical approximation (Poincare sections)



DPT-II

- **The DPT-II refers to non-analyticities in the rate function at certain critical times**

$$r(t) = -\frac{1}{\eta}\log|L(t)|^2 \qquad r^{\infty}(t) \propto |t - t_c|^{\beta}$$



Choosing the ground state at g_1, $|\varphi_0^{+}(g_1)\rangle$, the Loschmidt echo must take into account the two-fold degenerate ground state:

$$|L(t)|^2 = \sum_{q=\pm} |\langle\varphi_0^{q}(g_1)|e^{-itH_{\mathrm{QRM}}(g_2)}|\varphi_0^{+}(g_1)\rangle|^2$$

In the QRM, the bosonic states are displaced by $|\alpha| \propto \sqrt{\eta}$ and therefore

$$|L(t)|^2 = P_{+}(t) + P_{-}(t), \quad \text{with} \quad P_{\pm}(t) \propto e^{-\eta f_{\pm}(t)}$$

$$r^{\infty}(t) = \lim_{\eta\to\infty} r(t) = \min_{q=\pm} f_q(t)$$

Non-analytic when $f_+(t)$ and $f_-(t)$ cross

DPT-II

- Numerical results for $r(t) = -\frac{1}{\eta}\log|L(t)|^2$

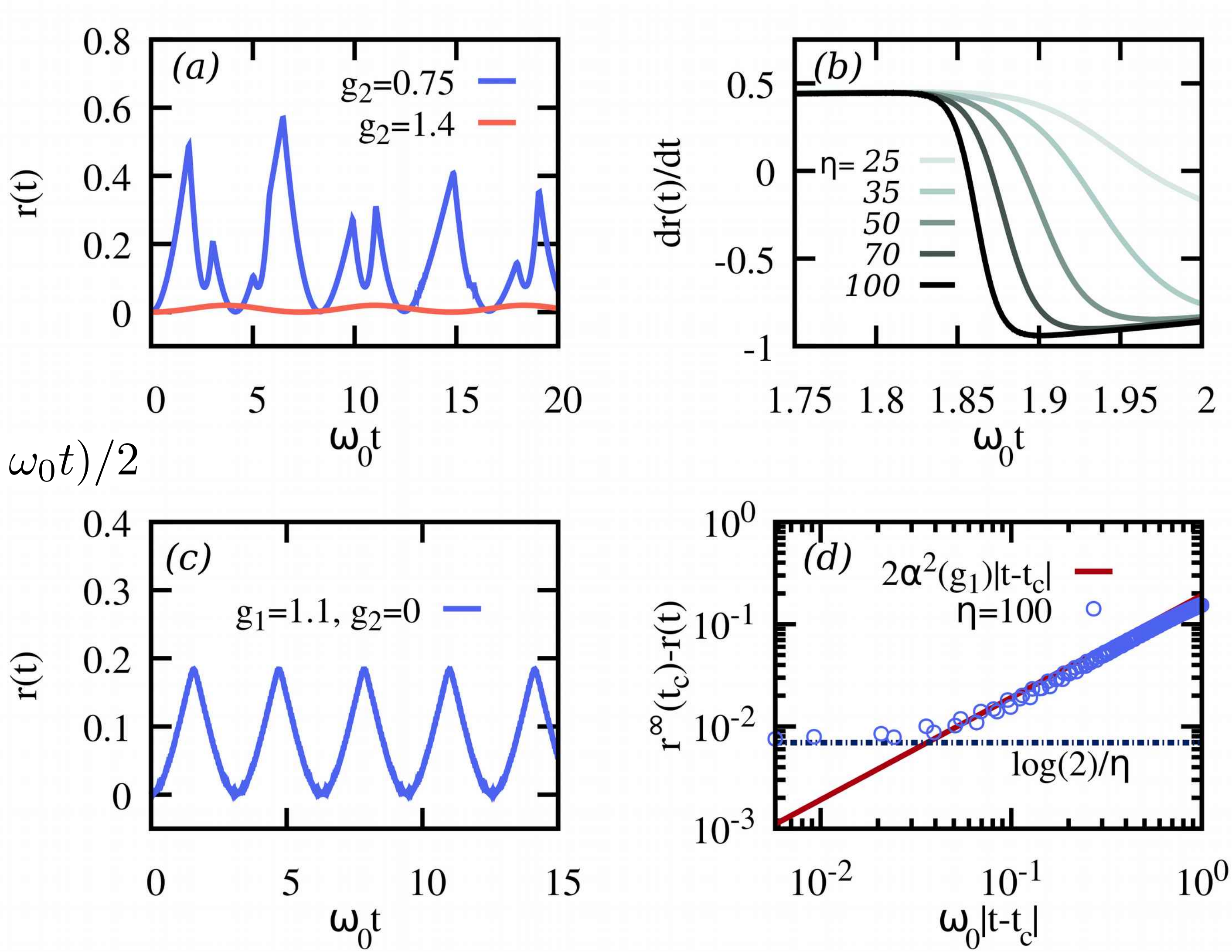
$$r^\infty(t) = \min f_\pm(t)$$

$$g_2 = 0$$

$$f_\pm(t) = (g_1^2 - g_1^{-2})(1 \pm \cos\omega_0 t)/2$$

$$r^\infty(t) \propto |t - t_c|$$

$$\omega_0 t_c = \pi/2 + n\pi$$

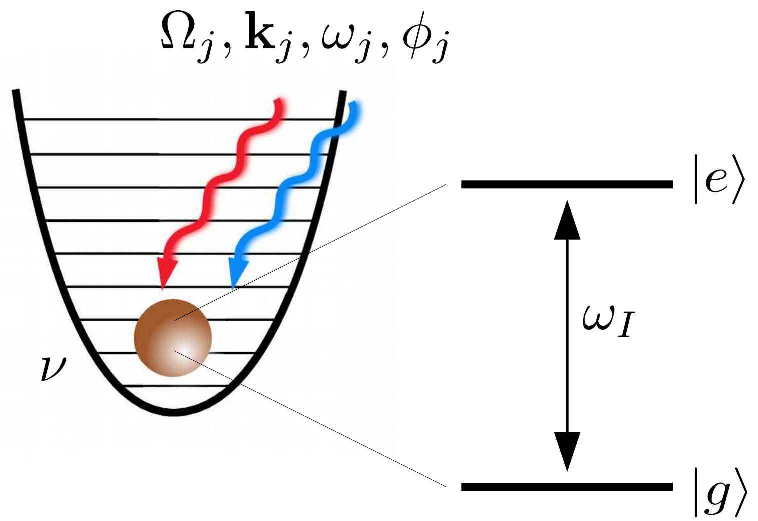


Single trapped-ion experiment

Single trapped-ion setup

- **It is possible to realize the QRM with tunable parameters using a single trapped ion**

$$H_{\mathrm{TI}} = \frac{\omega_I}{2}\sigma_z + \nu a^\dagger a + \sum_j \frac{\Omega_j}{2}\sigma_x \left[e^{i(\mathbf{k}_j\cdot\mathbf{r} - \omega_j t - \phi_j)} + \mathrm{H.c.} \right]$$



In the case of [40]Ca+ a qubit can be encoded in the states $|S_{1/2}, m_j = 1/2\rangle$, $|D_{5/2}, m_j = 3/2\rangle$ separated by an optical transition of 729nm.

$$\omega_I = 2\pi \cdot 4 \cdot 10^{14} \ \mathrm{Hz}$$

$$\omega_I \approx \omega_j \gg \nu \gg \Omega_j$$

$$\eta_j = \frac{k_j}{\sqrt{2m\nu}}$$ Lamb-Dicke parameter

$$x = (a + a^\dagger)/\sqrt{2m\nu}$$

$\longrightarrow$ **Coupling of ion's internal levels with its motion in the trap**

D. Leibfried, R. Blatt, C. Monroe, D. Wineland, *Rev. Mod. Phys.* ***75****, 281 (2003)*

H. Häffner, C. F. Roos, R. Blatt, *Phys. Rep.* ***469****, 155 (2008)*

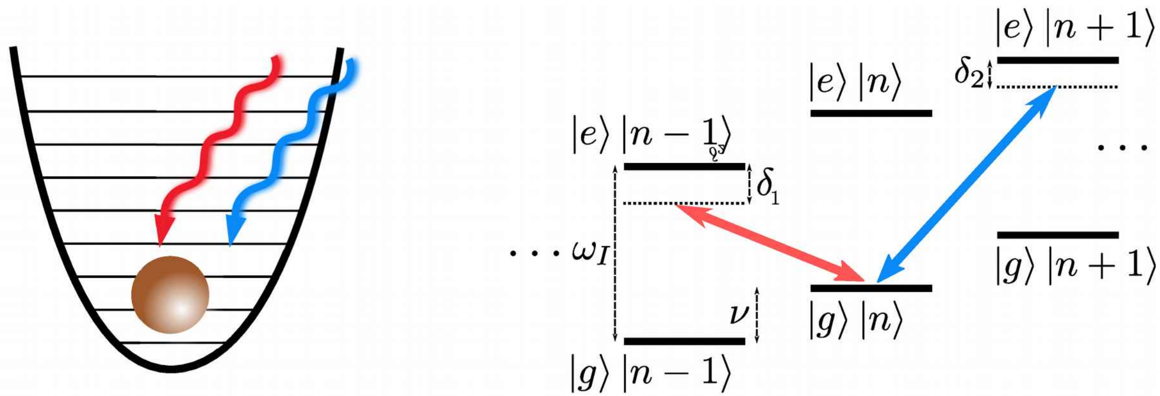
Generating interaction terms

- **Approximations to obtain interaction terms** $|\omega_I - \omega_j| \ll |\omega_I + \omega_j|$ Optical RWA

$\eta\sqrt{\langle (a + a^\dagger)^2 \rangle} \ll 1$ Lamb-Dicke approx.

$$H^I_{\mathrm{TI}} = \sum_j \frac{\Omega_j}{2} \left[\sigma^+ \left(I + i\eta(ae^{-i\nu t} + a^\dagger e^{i\nu t}) \right) e^{i(\omega_I - \omega_j)t} e^{-i\phi_j} + \mathrm{H.c.} \right]$$

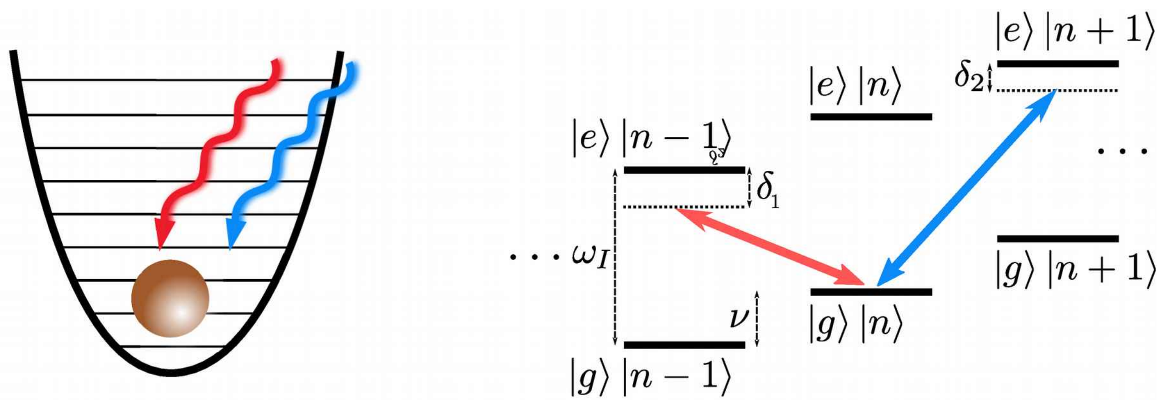
Tuning the frequency of the radiation to match $\omega_I - \omega_j = \pm\nu$ (red- and blue-sidebands) one obtains the desired interaction terms



$H_1 \propto \sigma^+ a + \sigma^- a^\dagger$ $H_2 \propto \sigma^- a + \sigma^+ a^\dagger$

Realization of a tunable QRM

- **Leaving a small detuning with respect to the red- and blue-sidebands one finds a tunable QRM**



$H_1 \propto \sigma^+ a + \sigma^- a^\dagger$ $\qquad$ $H_2 \propto \sigma^- a + \sigma^+ a^\dagger$

$$\boxed{H^I = \tilde{\omega}_0 a^\dagger a + \frac{\tilde{\Omega}}{2}\sigma_z - \tilde{\lambda}(a + a^\dagger)\sigma_x}$$

$$\tilde{\omega}_0 = \tfrac{1}{2}(\delta_1 - \delta_2) \qquad \tilde{\Omega} = \tfrac{1}{2}(\delta_1 + \delta_2) \qquad \tilde{\lambda} = \frac{\eta\Omega}{2}$$

It is possible to reach the demanding parameter regime to probe the critical QRM

J. S. Pedernales, I. Lizuain, S. Felicetti, G. Romero, L. Lamata, E. Solano, *Sci. Rep.* ***5****, 15472 (2015)*
RP, M.-J. Hwang, J. Casanova, M. B. Plenio, *Phys. Rev. Lett.* ***118****, 073001 (2017)*
M.-L. Cai et al., *Nat. Comm.* ***12****, 1126 (2021)*

Summary

- Finite-component systems can display rich critical phenomena

  Novel route toward quantum criticality

- Quantum Rabi model as an example:

    A ground-state quantum phase transition (QPT)

    Excited-state quantum phase transition (ESQPT)

    Dynamical quantum phase transitions (DPT)

- These critical phenomena can be readily realized in a single trapped-ion experiment

Thank you!

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