Jacobi-type transitions in finite systems

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Outline

I. Background

II. Three questions:

A. Classical nature of the Jacobi-type transitions (JTTs) in finite systems.

B. Identification of the Jacobi-type transitions.

C. The relation between the JTTs and excited-state quantum phase transitions (ESQPTs).

III. Summary

Background

a) The interacting boson model (IBM) involves rich QPTs with the precusors well confirmed in nuclei. Two examples: the U(5)-O(6) and U(5)-SU(3) QPTs.

b) The ESQPTs associated with the U(5)-O(6) and the U(5)-SU(3) QPTs are shown to occur in the IBM with the control parameter $1>\eta>\eta_c$

$$H=(1-\eta)\,H_{U(5)}+\eta\,H_{II}$$

I)The ESQPTs associated with the U(5)-O(6) QPT were earlist idenfied and fully discussed.

S. Heinze, P. Cejnar, J. Jolie, M. Macek, PRC 73 (2006)014306
M. Macek, P. Cejnar, J. Jolie, S. Heinze, PRC 73 (2006)014307
P.Cejnar, M. Macek, S.Heinze, J.Jolie, J. Dobeš, JPA 39 (2006) L515
M. Caprio, P.Cejnar, F. Iachello, Ann. Phys. 323 (2008) 1106
...

II)Signatures of the same type of ESQPT (U(2)-SO(3)) were found in the nonrigid molecules.

F. Pérez-Bernal, F. Iachello, PRA 77 (2008)032115
D. Larese, F. Pérez-Bernal, F. Iachello, J. Mol. Struct. 1051 (2013)301

III)The ESQPTs associated with U(5)-SU(3) QPT were seldom discussed.

M. Macek, P. Stránský, A. Leviatan, P. Cejnar, PRC 99 (2019)064323
Y. Zhang, Y. Zuo, F. Pan, J. P. Draayer, PRC 93 (2016) 044302

c) The Jacobi-type transitional phenomina associated with the U(5)-SU(3) QPT were found to also occur in the IBM with the control parameter $1>\eta>\eta_{c.}$

Y. Zhang, F. Iachello, PRC 95 (2017) 061304(R)

I) Signatures of the Jacobi-type transitions were found in the transitional nuclei.

What is the Jacobi-type transition

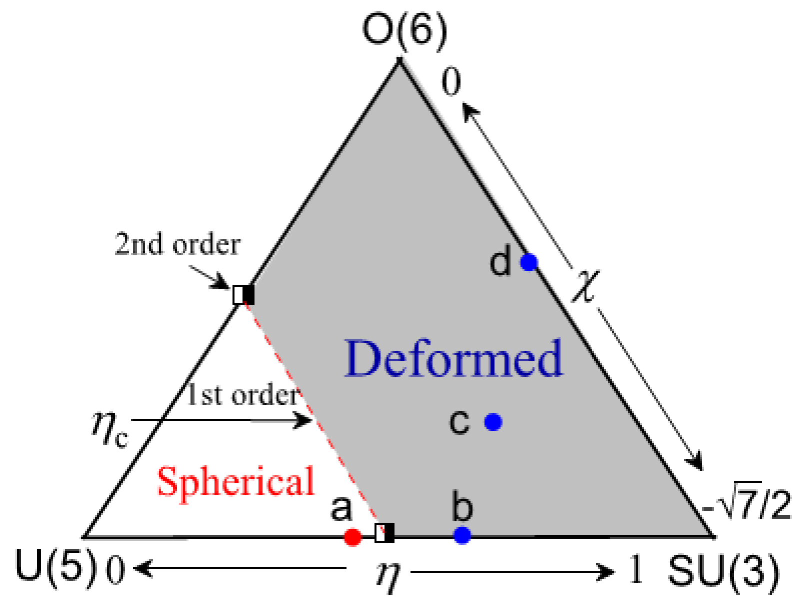
The consistent-Q Hamiltonian:

D.D. Warner, R.F. Casten, PRC 28 (1983)1798

$$\hat{H}(\eta,\ \chi) = \left[(1-\eta)\hat{n}_d - \frac{\eta}{4N}\hat{Q}^{\chi}\cdot\hat{Q}^{\chi}\right]$$

$$\hat{n}_d = d^{\dagger}\cdot\tilde{d} \qquad \hat{Q}^{\chi} = (d^{\dagger}s + s^{\dagger}\tilde{d})^{(2)} + \chi(d^{\dagger}\tilde{d})^{(2)}$$

$$\eta \in [0,1] \qquad \chi \in [-\sqrt{7}/2, 0]$$



The classical potential:

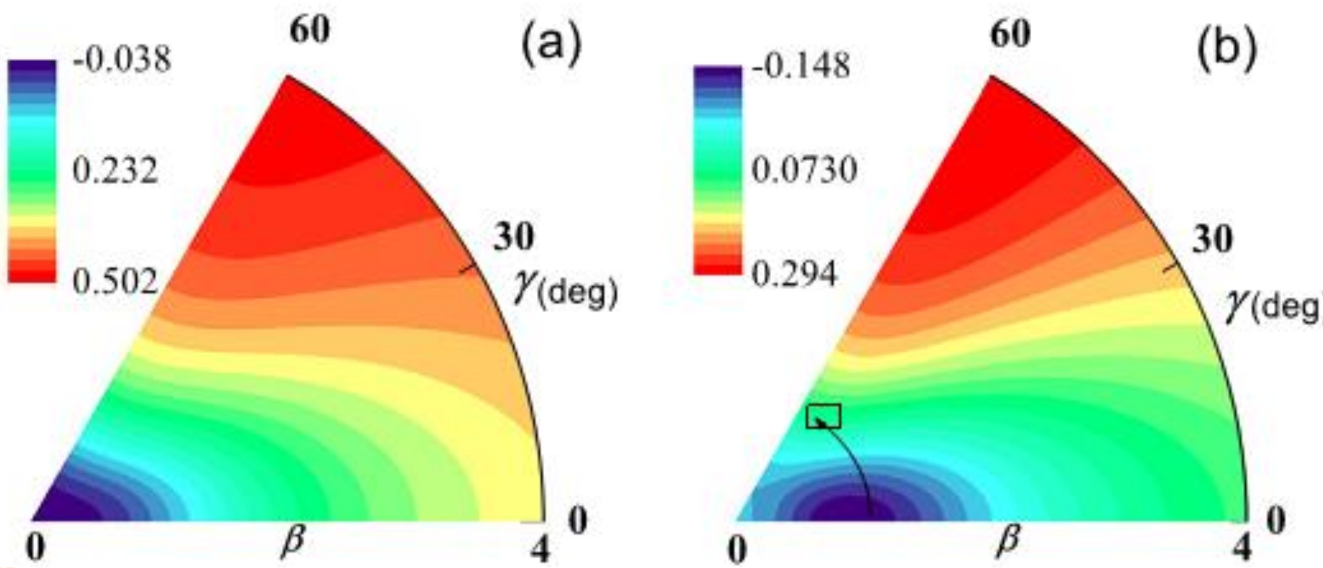
$$|\beta,\gamma,N\rangle = [s^{\dagger} + \beta\cos\gamma\ d_0^{\dagger} + \frac{1}{\sqrt{2}}\beta\sin\gamma(d_2^{\dagger} + d_{-2}^{\dagger})]^N|0\rangle$$

$$\begin{aligned} V(\beta,\gamma) &= \frac{1}{N}\frac{\langle\beta,\gamma,N|\hat{H}(\eta,\ \chi)|\beta,\gamma,N\rangle}{\langle\beta,\gamma,N|\beta,\gamma,N\rangle} \\ &= \frac{\beta^2}{1+\beta^2}[(1-\eta)-(\chi^2+1)\frac{\eta}{4N}] \\ &\quad -\frac{5\eta}{4N(1+\beta^2)} - \frac{\eta(N-1)}{4N(1+\beta^2)^2} \\ &\quad \times[4\beta^2 - 4\sqrt{\frac{2}{7}}\chi\beta^3\cos3\gamma + \frac{2}{7}\chi^2\beta^4] \end{aligned}$$

	(η , χ)
(a)	(0.45, −1.32)
(b)	**(0.65, −1.32)***
(c)	(0.70, −0.80)
(d)	(1.00, −0.50)

Countours of the potential V(β,γ)

N=15



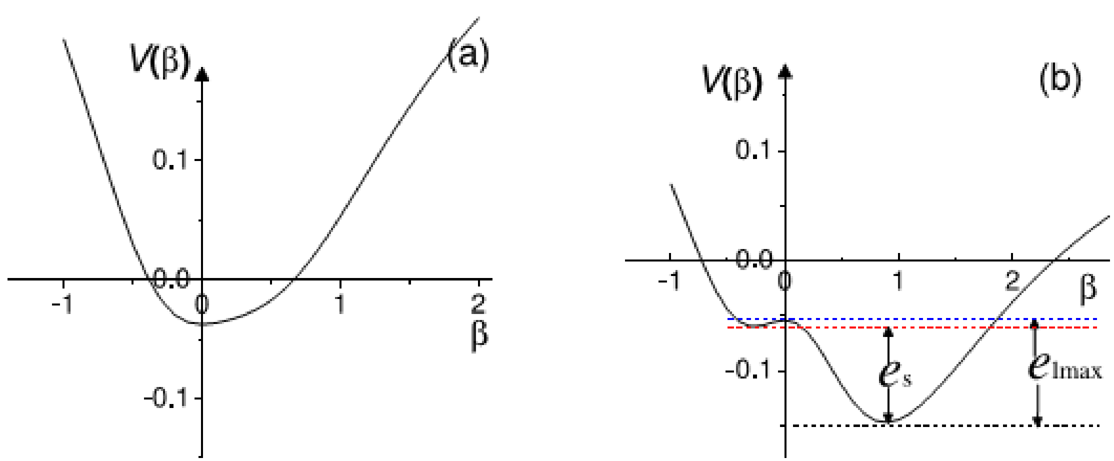
γ = 0

γ = 0

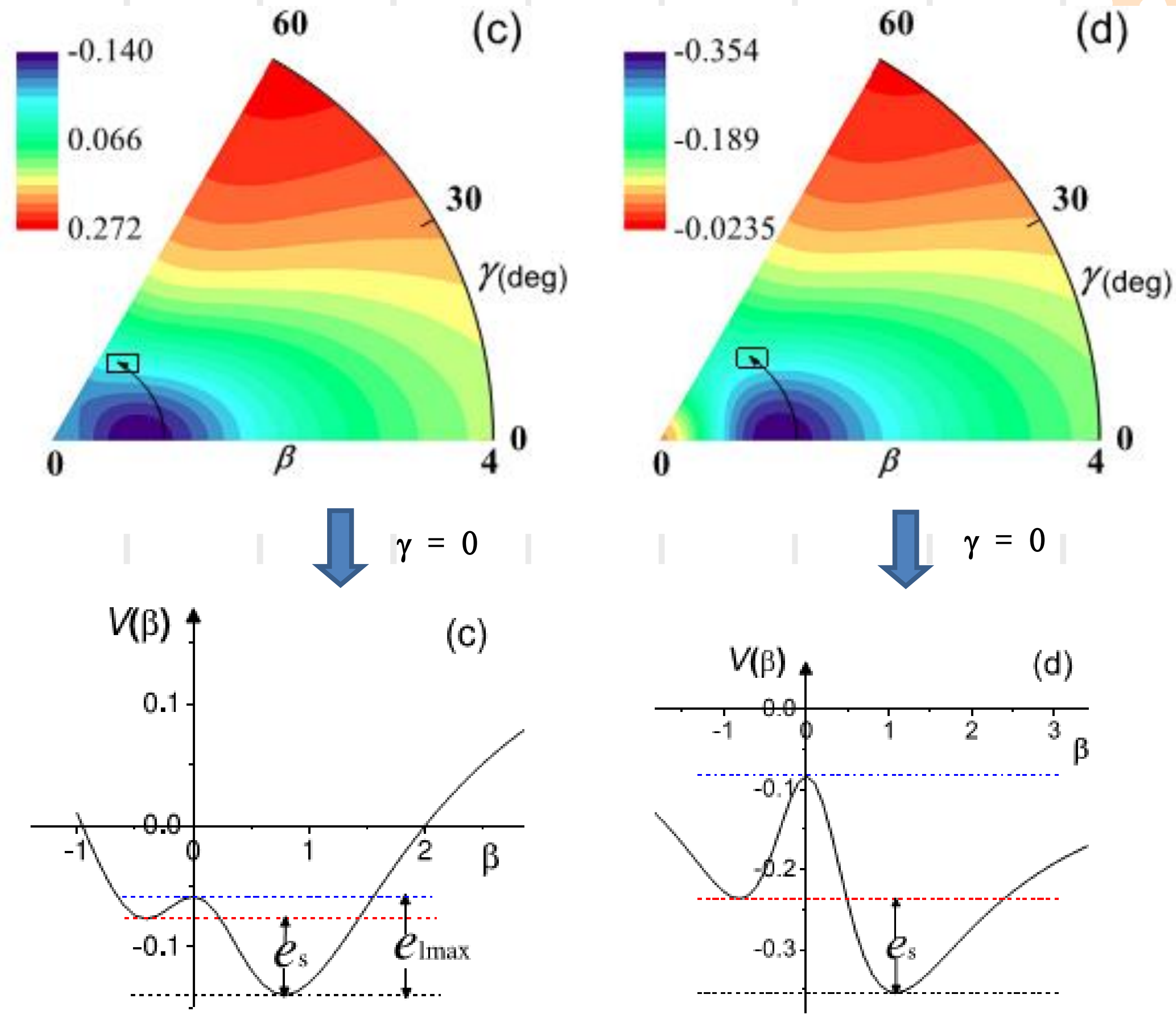
$V(\beta, \gamma = 60)$

||

$V(-\beta, \gamma = 0)$



after involving the O(6) component



Learn from $V(\beta,\gamma)$:

1) The states (vibrational or rotational) in a transitional system may change their characteristics once they are excited out of the low-energy well below the saddle point.

2) The γ rigidity inside the low-energy well (deformed) is due to the SU(3) dynamical symmetry (DS), while the γ softness outside the well originates from either the U(5) or O(6) DSs or even their mixing.

The Jacobi-type transition: A transitional system as a function of the angular momentum may change its shape from axial to triaxial (γ-rigid to γ-soft) near the saddle point.

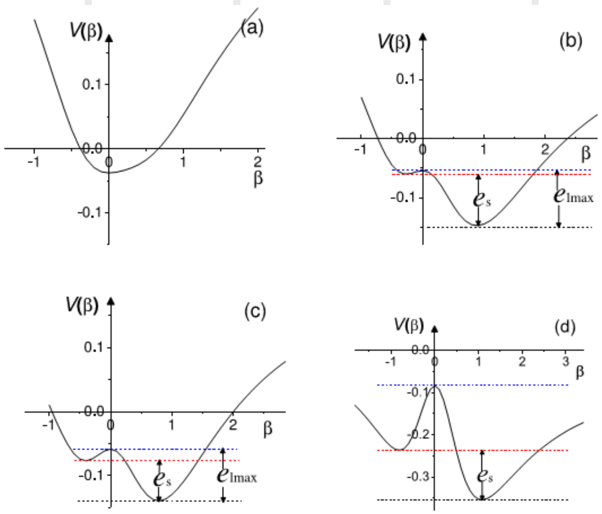
Y. Zhang, F. Iachello, PRC 95 (2017) 061304(**R**)

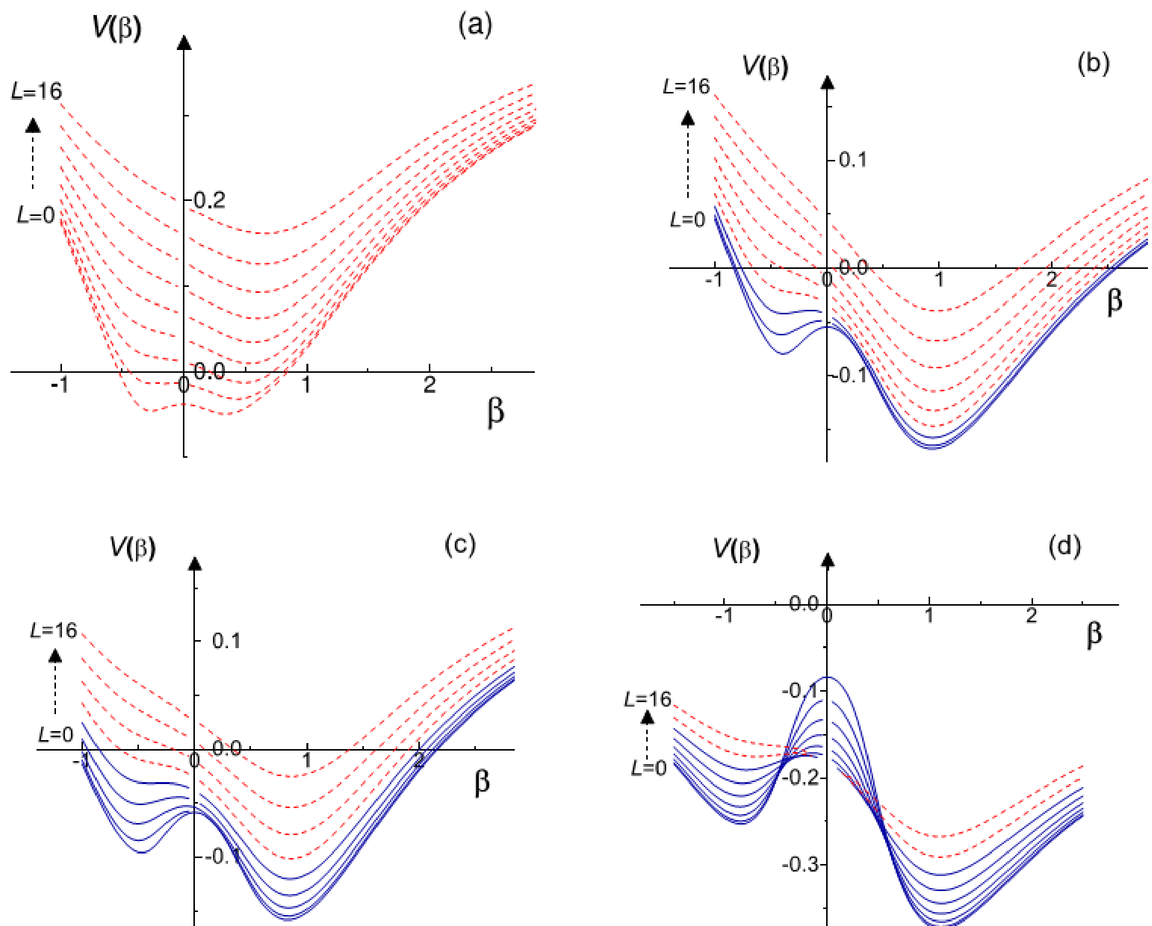
The angular momentum projection of V(β)

$$K = M = 0$$

$$P^{L}_{MK} = \frac{(2L+1)}{8\pi^2}\int D^{L*}_{MK}(\Omega)R(\Omega)d\Omega$$

$$V(\beta,\gamma)_g = \frac{\langle\beta,\gamma,N|\hat{H}(\eta,\ \chi)P^{L}_{00}|\beta,\gamma,N\rangle}{\langle\beta,\gamma,N|P^{L}_{00}|\beta,\gamma,N\rangle}$$
$$= \frac{\int\ d\theta\sin\theta d^{L}_{00}(\theta)\langle\beta,\gamma,N|\hat{H}(\eta,\ \chi)e^{-i\theta L_y}|\beta,\gamma,N\rangle}{\int\ d\theta\sin\theta d^{L}_{00}(\theta)\langle\beta,\gamma,N|e^{-i\theta L_y}|\beta,\gamma,N\rangle}$$





Y. Zhang, C. Lin, C. Xiu, X. D. Song
Ann. Phys. 424(2021)168380

Cranking the IBM Hamiltonian

H. Schaaser, D. M. Bink, NPA 452(1986)1
P. Cejnar, PRC 65(2002)044312

$$F(N,\ a)=\frac{1}{N}\langle N,\ a|\hat{H}(\eta,\chi)-\omega\hat{L}_x|N,\ a\rangle$$

$$|N,\ a\rangle=\frac{1}{\sqrt{N!(1+\sum_u a_u^2)^N}}[s^\dagger+a_u d_u^\dagger]^N|0\rangle$$

$$\bar{L}\equiv\langle N,a|\hat{L}_x|N,\ a\rangle=2Na_1\frac{(\sqrt{6}a_0+2a_2)}{(1+\sum_v a_v^2)}$$

$$\langle N,\ a|\hat{Q}_u^\chi|N,\ a\rangle=\frac{N}{(1+\sum_v a_v^2)}[a_u+\tilde{a}_u+\chi(a\tilde{a})_u^{(2)}]$$

$$\begin{aligned}a_0&=\beta\cos\gamma,\\ a_{\pm1}&=(\beta/\sqrt{2})\sin\gamma\sin\delta,\\ a_{\pm2}&=(\beta/\sqrt{2})\sin\gamma\cos\delta\,.\end{aligned}$$

$$\Gamma=\mathrm{arccot}(\langle\hat{Q}_0^\chi\rangle/\sqrt{2}\langle\hat{Q}_2^\chi\rangle)$$

$$\begin{aligned}F(\beta,\gamma,\delta)|_{N\to\infty}&=\frac{1}{N}\langle N,\beta,\gamma,\delta|\hat{H}(\eta,\ \chi)-\omega\hat{L}_x|N,\beta,\gamma,\delta\rangle\\
&=\frac{(1-\eta)\beta^2}{1+\beta^2}-\frac{\eta\beta^2}{(1+\beta^2)^2}(1-\sin^2\gamma\sin^2\delta)\\
&+\sqrt{\frac{2}{7}}\frac{\chi\eta\beta^3}{(1+\beta^2)^2}\Big[\cos3\gamma+\sin^2\delta\\
&\times\Big(\frac{7}{4}\sin\gamma\sin2\gamma-\frac{\sqrt{3}}{2}\sin^3\gamma\cos\delta\Big)\Big]\\
&-\frac{1}{14}\frac{\chi^2\eta\beta^4}{(1+\beta^2)^2}\Big[1-\Big(\frac{1}{4}\sin^22\gamma\\
&+3\sin^4\gamma\cos^2\delta-\sqrt{3}\sin^2\gamma\sin2\gamma\cos\delta\Big)\sin^2\delta\Big]\\
&-\omega\frac{\beta}{1+\beta^2}\Big[2\cos\gamma-\sqrt{\frac{2}{7}}\chi\beta\Big(\cos2\gamma\\
&+\frac{1}{2}\sin^2\gamma\sin^2\delta\Big)\Big]\,.\end{aligned}$$

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The cranking function $F(\gamma, \delta)_{\beta^0}$ at b, c, d

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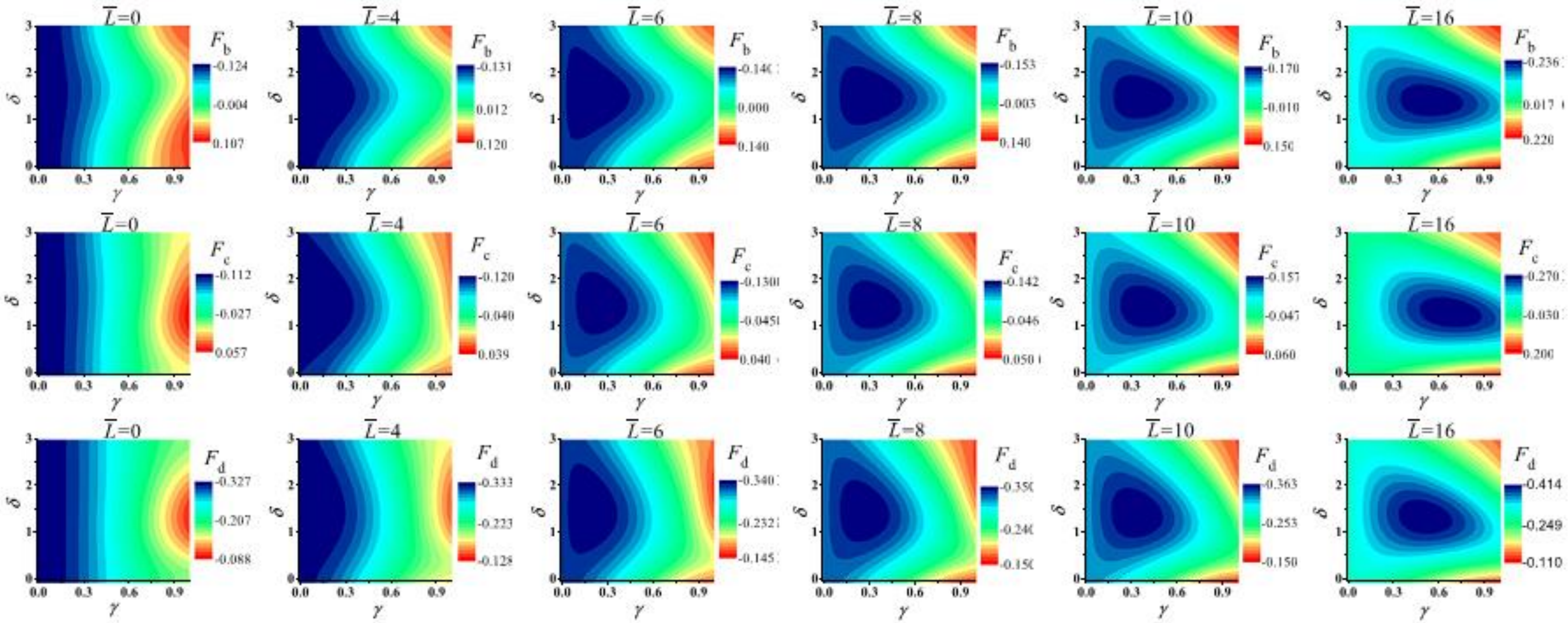
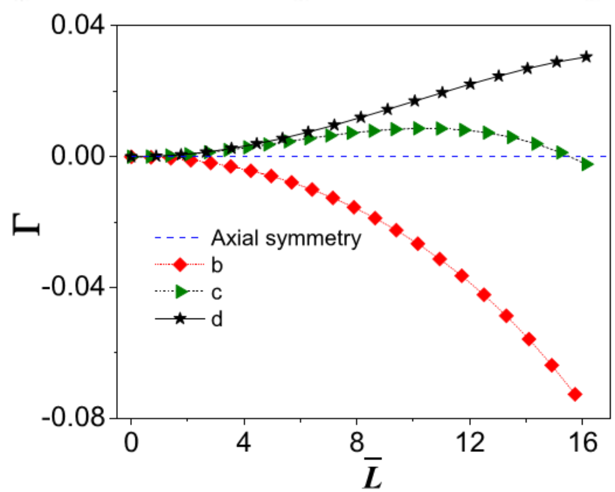


$$\Gamma = \mathrm{arccot}(\langle \hat{Q}_0^{\chi} \rangle / \sqrt{2} \langle \hat{Q}_2^{\chi} \rangle)$$



Axial symmetry of a rotating system will be clearly broken at "critical" angular momentum.

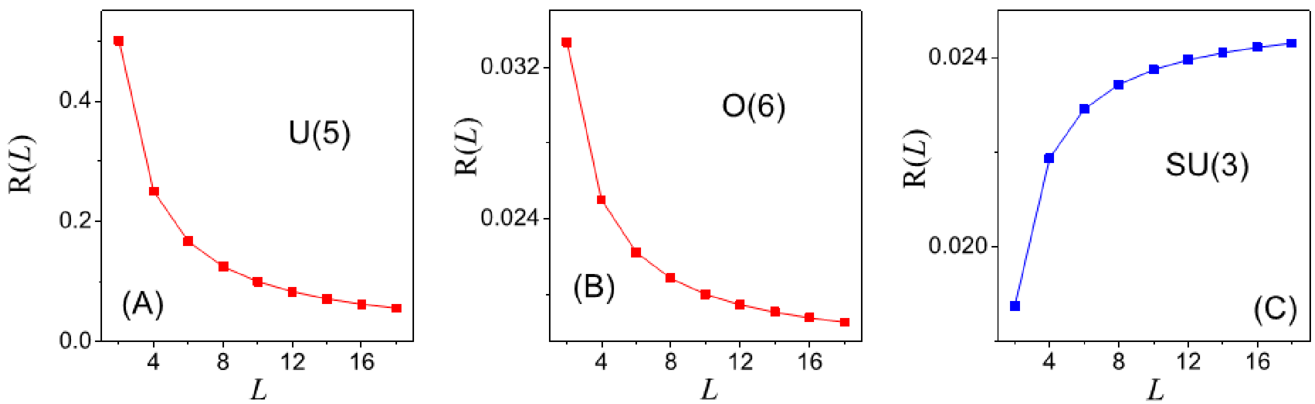
How to identify the Jacobi-type transitions

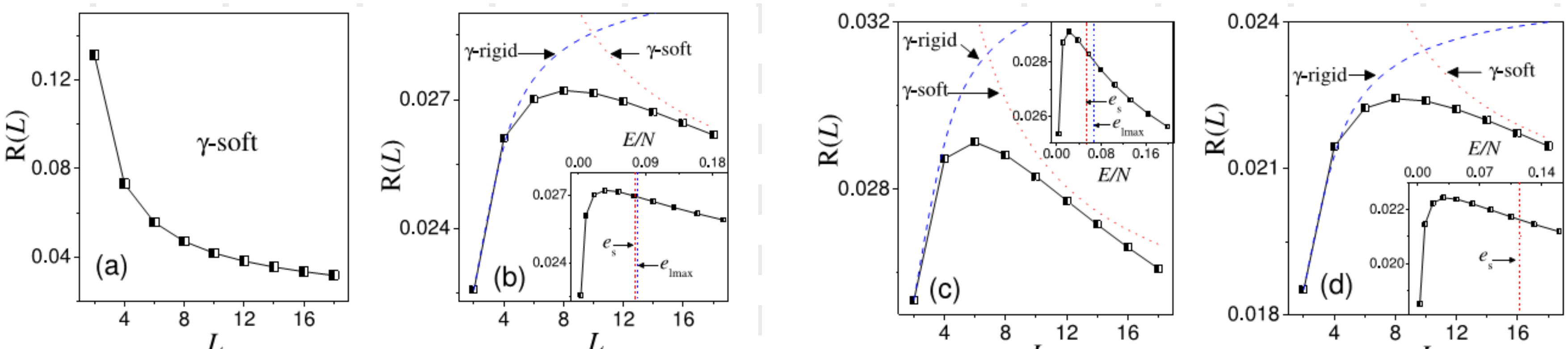
$$R(L) = \frac{E_\gamma(L \to L-2)}{L}$$

Regan et al., PRL 90 (2003) 152502

Y. Zhang, F. Iachello, PRC 95 (2017) 061304(R)

$R \propto \frac{1}{L}$ $\qquad$ $R \propto (1+\frac{2}{L})$ $\qquad$ $R \propto (4-\frac{2}{L})$





Finite N effects on the Jacobi-type transitions

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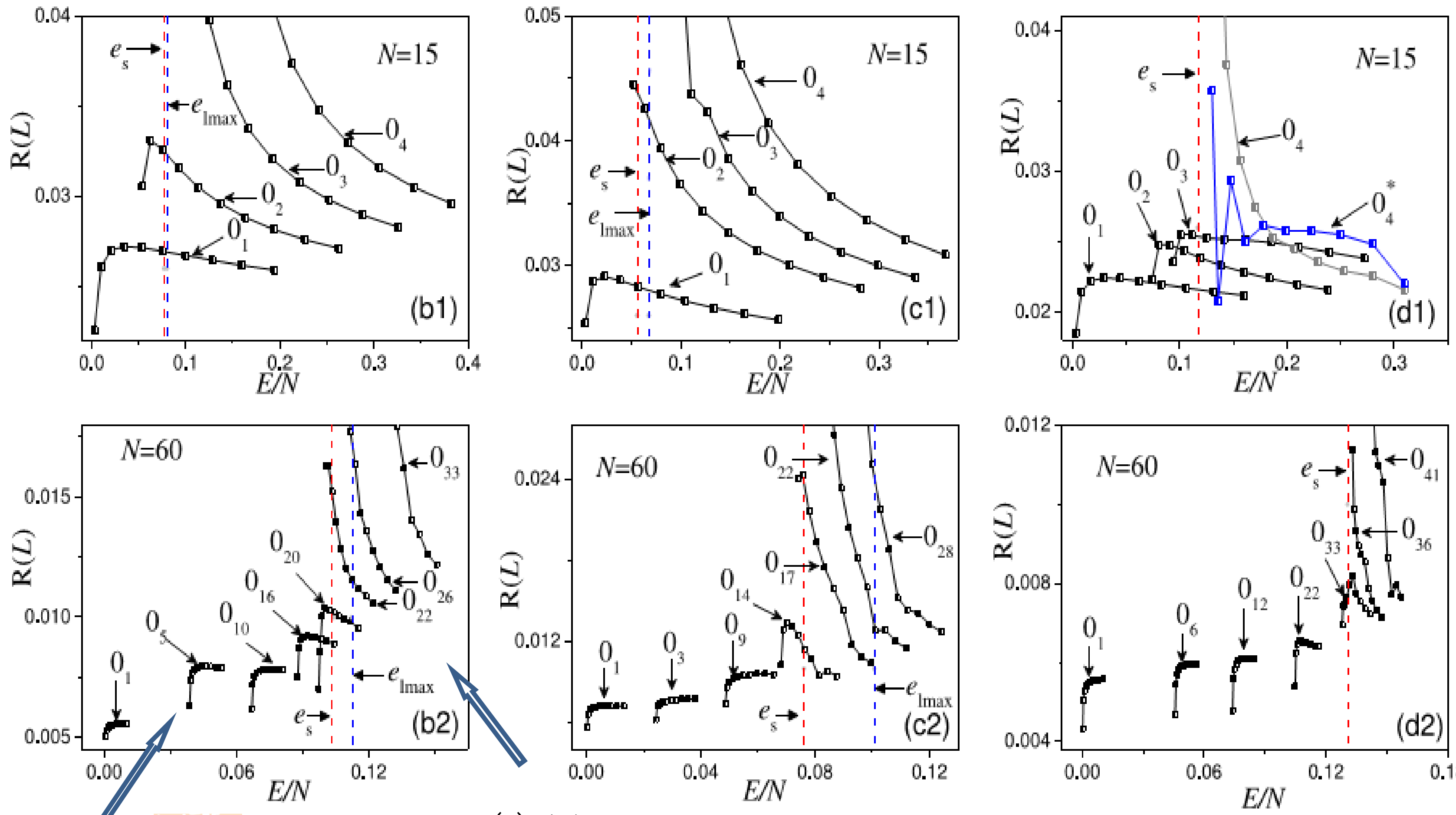
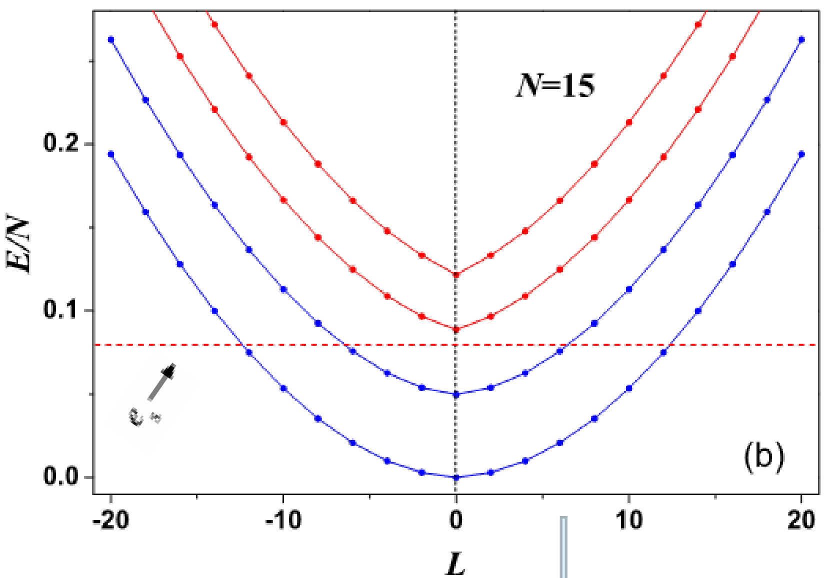


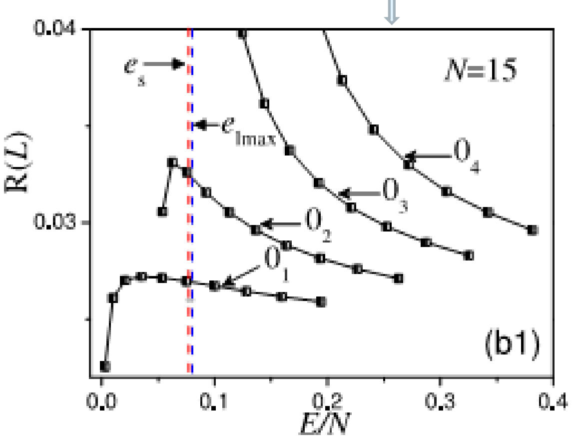
SU(3)-like
$E(L) \approx AL(L+1)$

U(5)-like
$E(L) \approx BL$

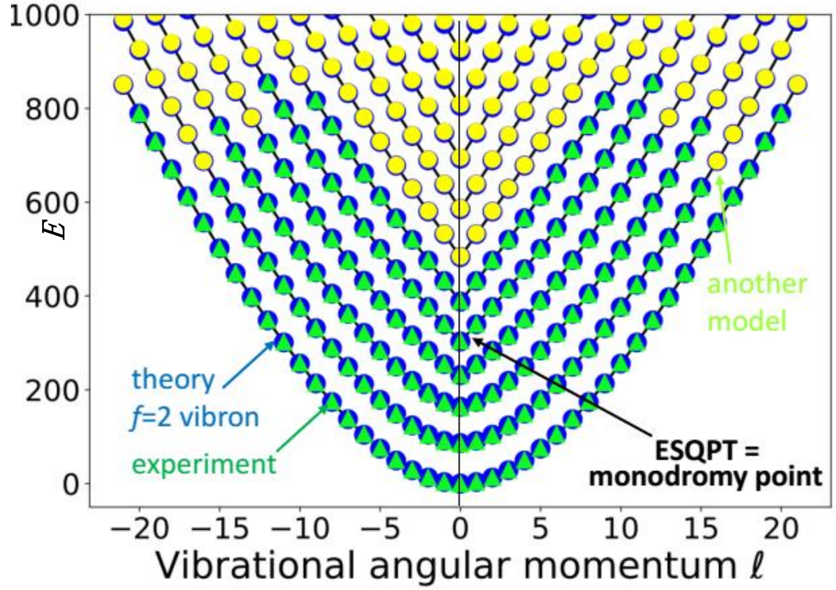
"Monodromy" in the ESQPTs

Spectrum of the lowest four bands at the point b





Vibrational spectrum of molecule NCNCS



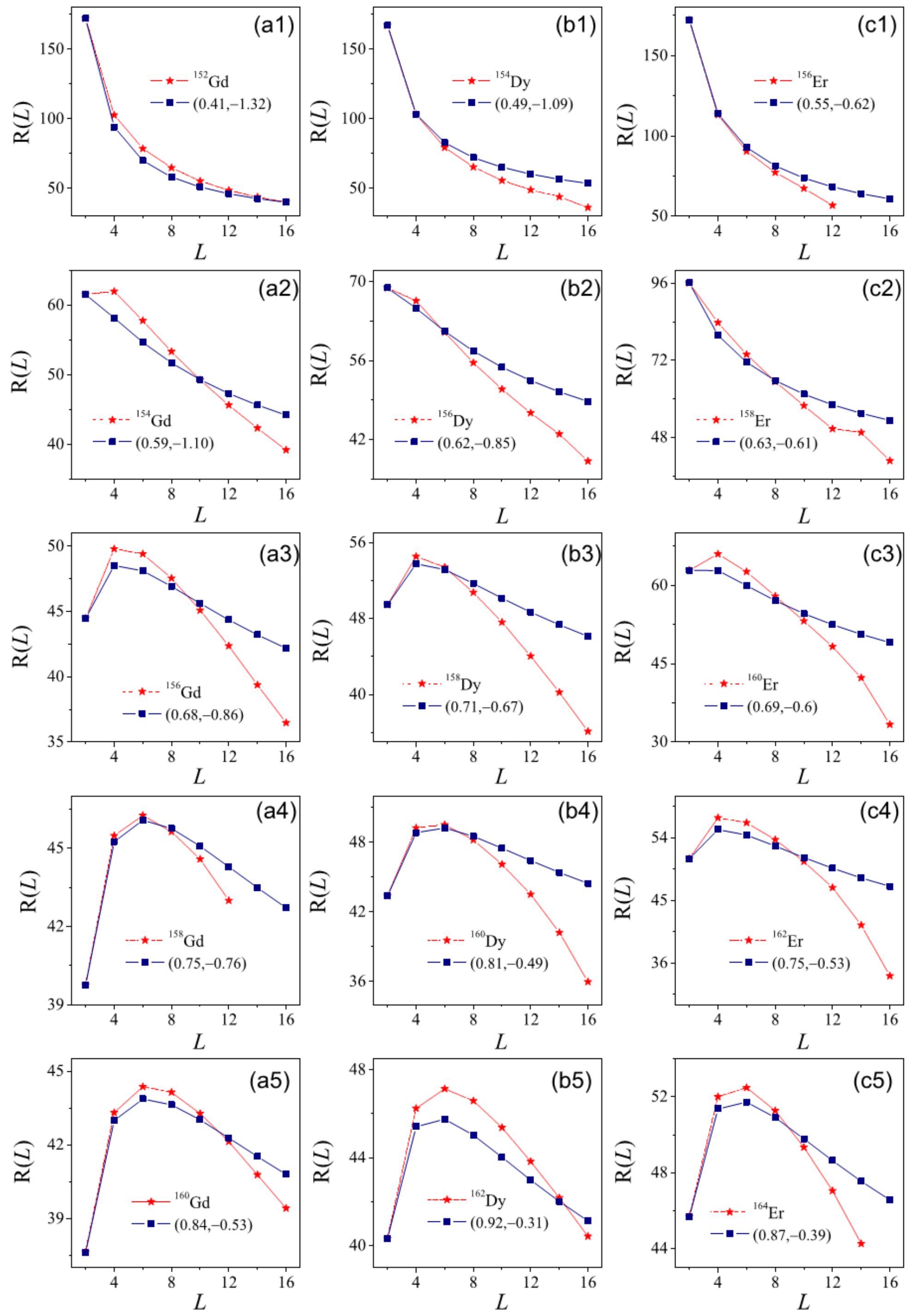
D. Larese, F. Iachello, J. Mol. Struct. 1006 (2011) 611

D. Larese, F. Pérez-Bernal, F. Iachello, J. Mol. Struct. 1051 (2013) 310

The transitional features predicted in the IBM are well confirmed in nuclei

Y.Zhang, C. Lin, C. Xiu, X.D. Song
Ann.Phys.424(2021)168380

The IBM Parameters taken from:
McCutchan, Zamfir, Casten, PRC 69(2004)064306



Classification of the ESQPTs (L=0) around the stationary points

(I)P. Stránský, M. Macek, P. Cejnar, Ann. Phys. 345 (2014)73
(II)P. Stránský, M. Macek, A. Leviatan, P. Cejnar, Ann. Phys. 356 (2015)57
(III)M. Macek, P. Stránský, A. Leviatan, P. Cejnar, PRC 99 (2019)064323

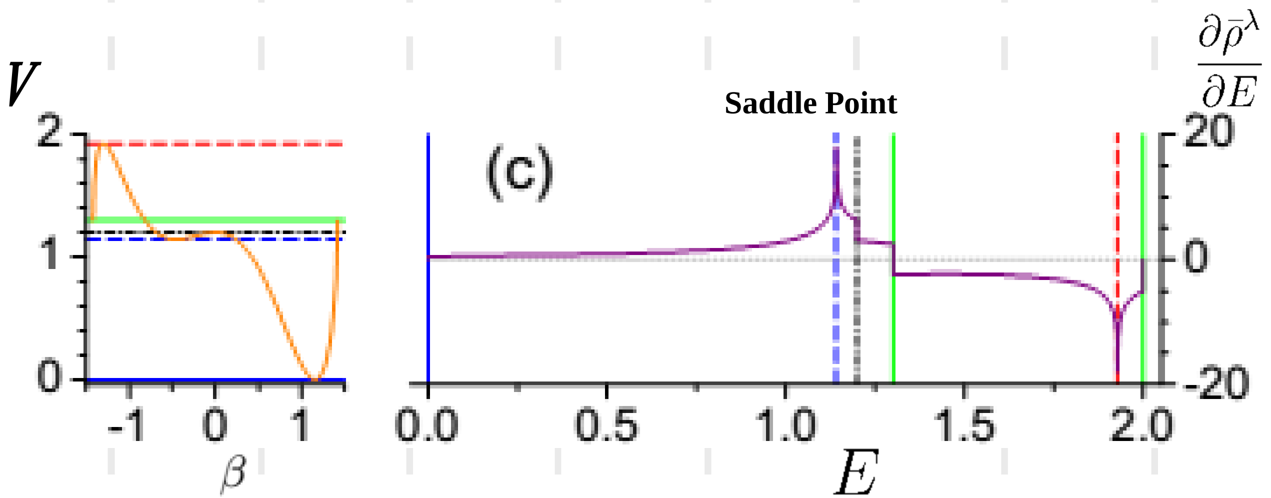
$$\hat{H}_1(\beta_0',\zeta) = \frac{2(1-\zeta^2)}{N}\,\hat{n}_d(\hat{n}_d-1) + \frac{1}{N}\left(\hat{D}^\dagger(\beta_0',\zeta)\cdot\tilde{\hat{D}}(\beta_0',\zeta)\right)$$

$$\hat{H}_2(\beta_0',\xi) = \hat{H}_1(\beta_0',\zeta=1) + \frac{\xi}{N}\left(\hat{S}^\dagger(\beta_0')\hat{S}(\beta_0')\right)$$

$$\hat{S}^\dagger(\beta_0') = (\hat{d}^\dagger\cdot\hat{d}^\dagger) - \beta_0'^2(\hat{s}^\dagger\hat{s}^\dagger)\,,$$

$$\hat{D}^\dagger_\mu(\beta_0',\zeta) = \sqrt{2}\beta_0'\left[\hat{s}^\dagger\hat{d}^\dagger\right]^{(2)}_\mu + \sqrt{7}\zeta\left[\hat{d}^\dagger\hat{d}^\dagger\right]^{(2)}_\mu$$

$$\bar{\rho}^\lambda(E) = \sum_i \bar{\delta}(E-E_i^\lambda)$$



Taken from Fig.3 in (III)

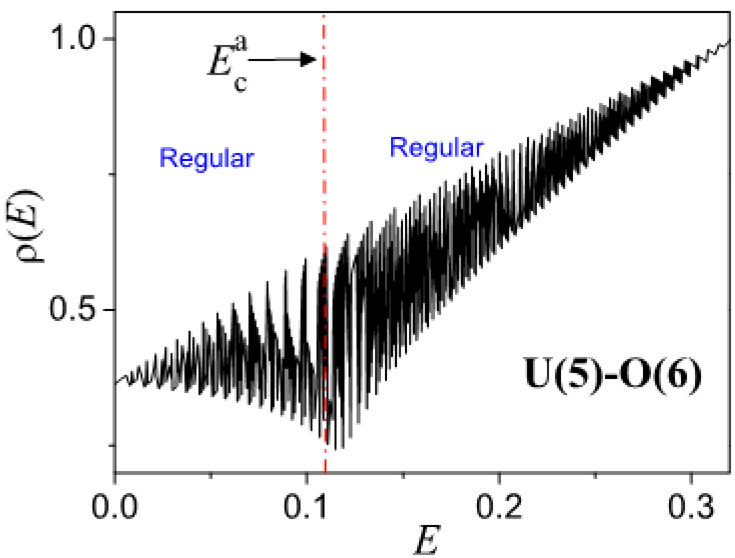
The Effective order parameter for the ESQPT (L=0) in the IBM (N=100)

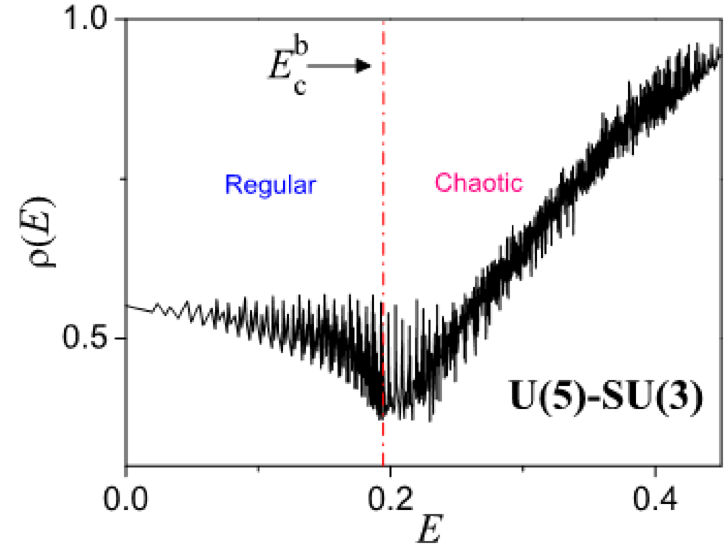
$$\hat{H}(\eta,\ \chi) = \epsilon\left[(1-\eta)\hat{n}_d - \frac{\eta}{4N}\hat{Q}^{\chi}\cdot\hat{Q}^{\chi}\right]$$

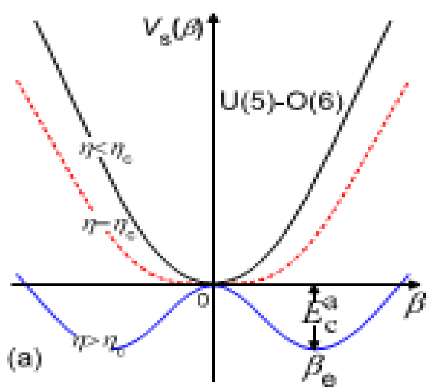
Y. Zhang, Y. Zuo, F. Pan, J. P. Draayer, PRC 93 (2016) 044302
W. T. Dong, Y. Zhang, B. C. He, *et al.*, JPG 424 (2021) 168380

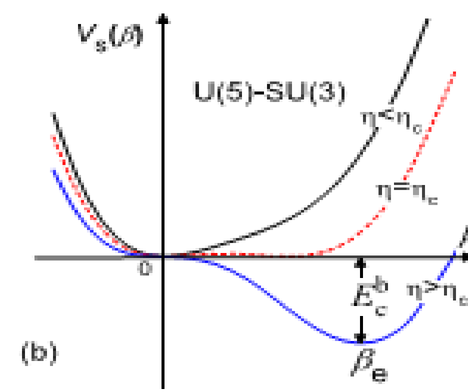
$$\rho(E) \propto \langle H_{\mathrm{U}(5)}\rangle \propto B(E0;0_n \to 0_n)$$

$$P_{\omega}(S) = \alpha(1+\omega)S^{\omega}\exp(-\alpha S^{(1+\omega)})$$

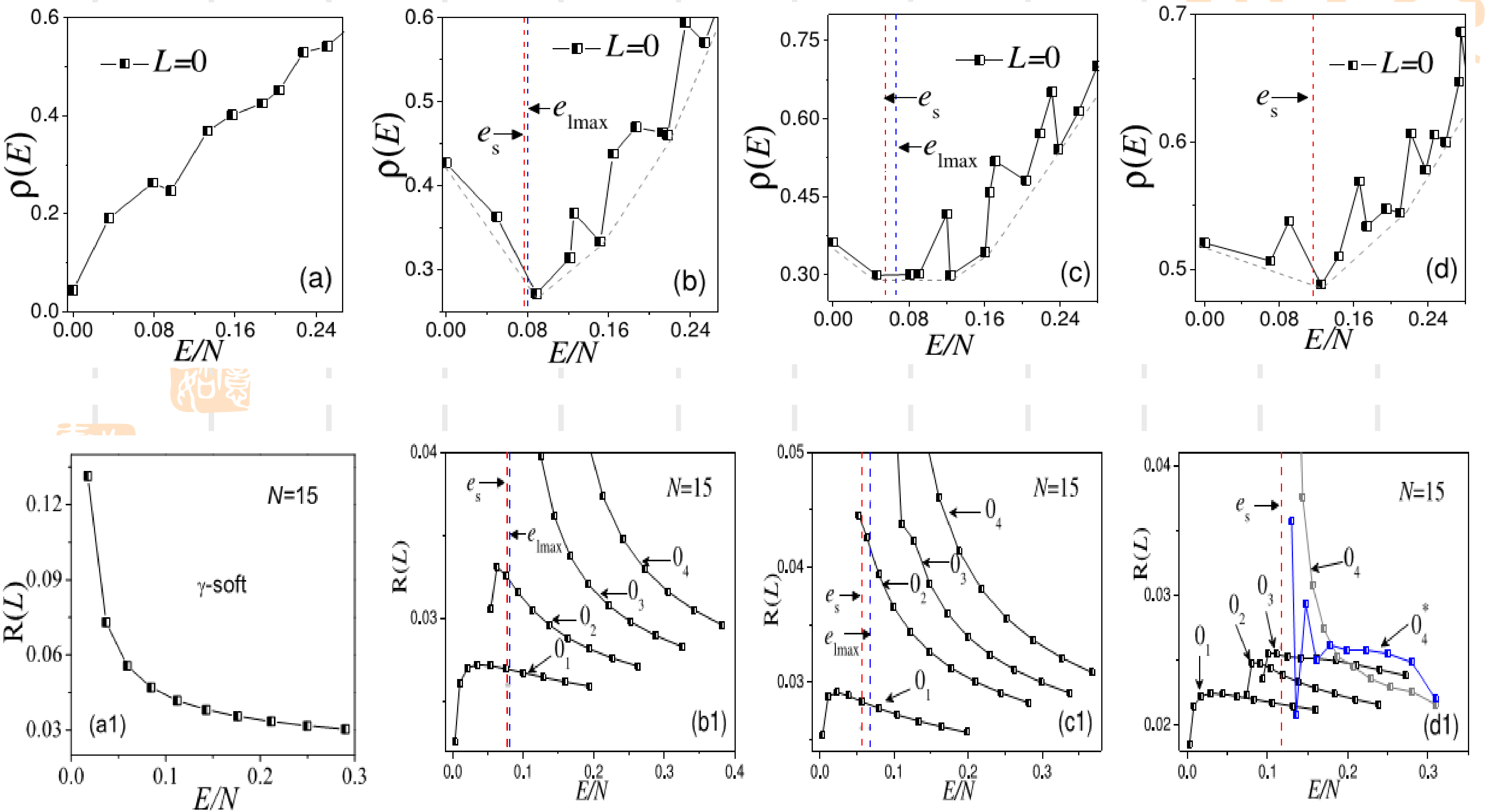








Finite-N precursors of the ESQPTs around the saddle points



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Summary:

1. Classically, the Jacobi-type transitions (JTTs) originate from the γ softness (triaxiality) near the saddle point therefore being considered as a manifest of the saddle point in rotational states. But the γ softness may appear earlier than the saddle point energy due to the involvement of the O(6) DS.

2 The precursors of the ESQPTs can be also observed around the saddle point in a finite system and can thus be regarded as one of the saddle point effects in vibrational states.

Their relation: JTTs $\xrightarrow{\text{rotation}}$ Saddle point $\xrightarrow{\text{vibration}}$ ESQPTs

Remain to answer:

1 Are there any singularities in the Jacobi-type transitions in the large-N limit.
2 Are there any other observables (like $R(L)$) that are sensitive to the Jacobi-type transitions.
3 How about the Jacobi-type transitions in odd nuclei.

Thanks for your attention!