Exceptional Spectral Phase in a Dissipative Collective Spin Model

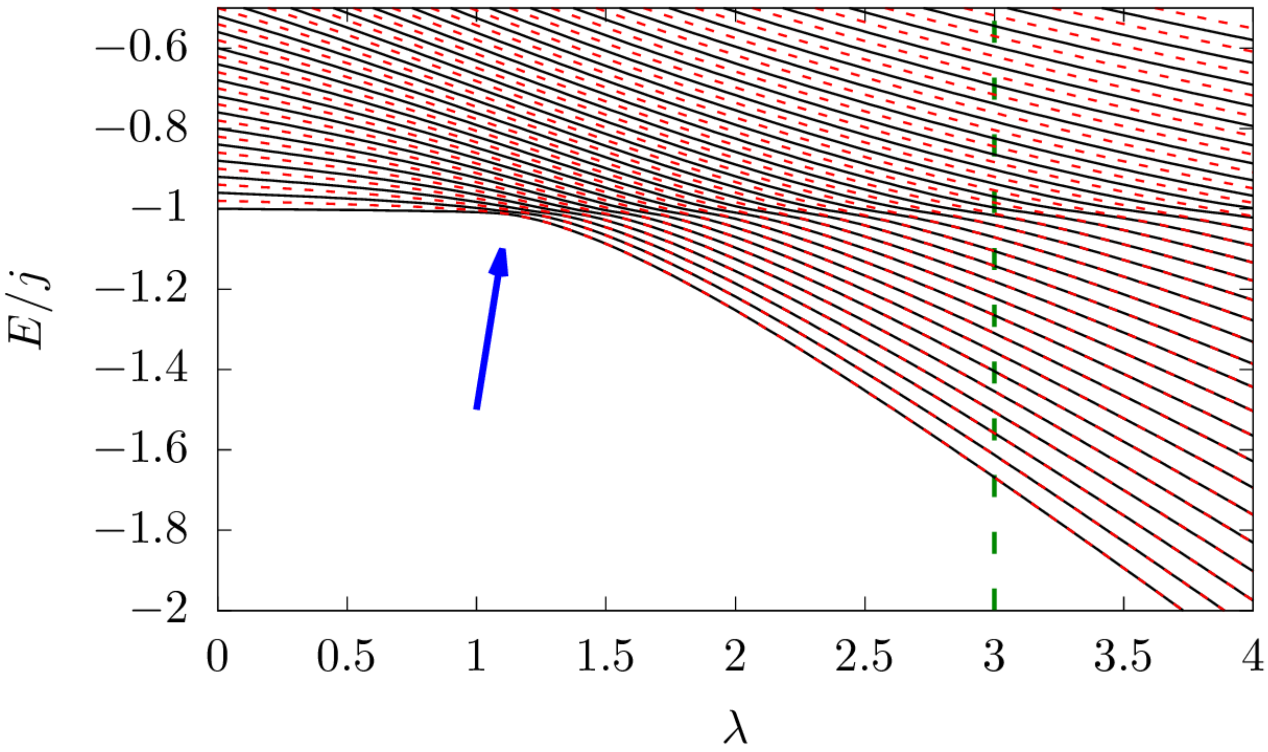
A. Relaño
A. Rubio-García, A. L. Corps, R. A. Molina,
F. Pérez-Bernal, J. E. García-Ramos & J. Dukelsky
arXiv:2202.09337

Granada, June 8th, 2022

Quantum phase transitions in closed collective models

A little excursion to the LMG model

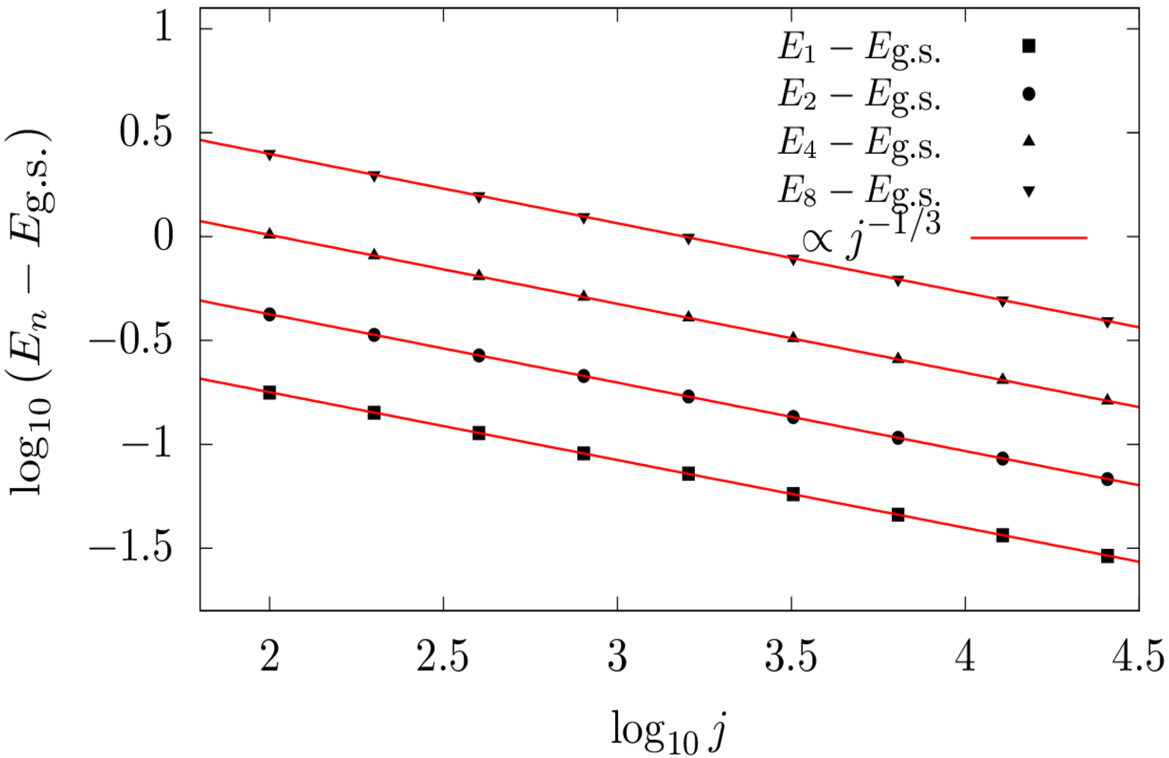
$$H - \frac{\lambda}{4N}\sum_{i,j=1}^{N}\sigma_i^x\sigma_j^x + \frac{h}{2}\sum_{i=1}^{N}\sigma_i^z = -\frac{\lambda}{N}\hat{J}_x^2 + h\hat{J}_z$$



Quantum phase transitions in closed collective models

A little excursion to the LMG model

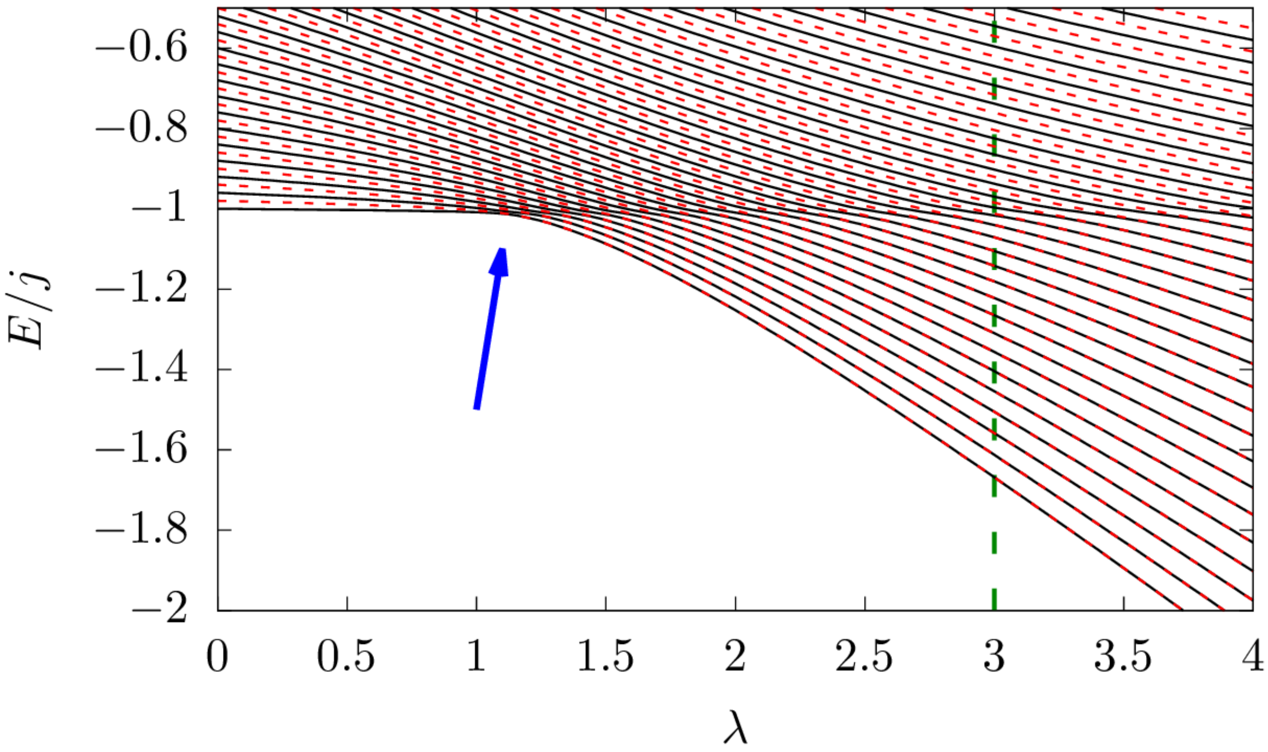
$$H - \frac{\lambda}{4N}\sum_{i,j=1}^{N}\sigma_i^x\sigma_j^x + \frac{h}{2}\sum_{i=1}^{N}\sigma_i^z = -\frac{\lambda}{N}\hat{J}_x^2 + h\hat{J}_z$$



Quantum phase transitions in closed collective models

A little excursion to the LMG model

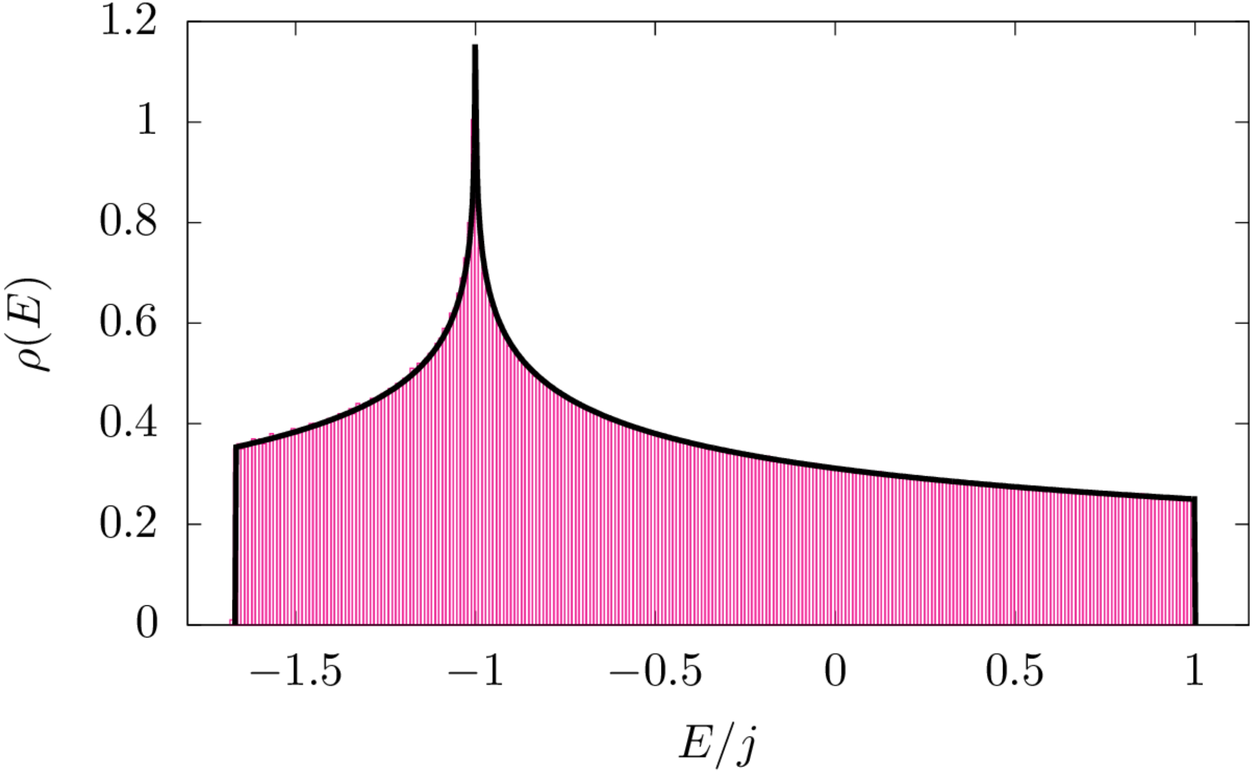
$$H - \frac{\lambda}{4N}\sum_{i,j=1}^{N}\sigma_i^x\sigma_j^x + \frac{h}{2}\sum_{i=1}^{N}\sigma_i^z = -\frac{\lambda}{N}\hat{J}_x^2 + h\hat{J}_z$$



Quantum phase transitions in closed collective models

A little excursion to the LMG model

$$H - \frac{\lambda}{4N}\sum_{i,j=1}^{N}\sigma_i^x\sigma_j^x + \frac{h}{2}\sum_{i=1}^{N}\sigma_i^z = -\frac{\lambda}{N}\hat{J}_x^2 + h\hat{J}_z$$



Quantum phase transitions in closed collective models

A little excursion to the LMG model
$$H - \frac{\lambda}{4N}\sum_{i,j=1}^{N}\sigma_i^x\sigma_j^x + \frac{h}{2}\sum_{i=1}^{N}\sigma_i^z = -\frac{\lambda}{N}\hat{J}_x^2 + h\hat{J}_z$$

In the thermodynamic limit, $j \to \infty$

- QPT at $\lambda = \lambda_c \longrightarrow \begin{cases} \text{Closing gap at } \lambda = \lambda_c \\ E_{0,+} = E_{0,-} \text{ if } \lambda > \lambda_c \\ E_{0,+} \neq E_{0,-} \text{ if } \lambda < \lambda_c \end{cases}$

- If $\lambda > \lambda_c$, ESQPT at $E = E_c \longrightarrow \begin{cases} \text{Diverging } \rho(E) \text{ at } E = E_c \\ E_{n,+} = E_{n,-} \text{ if } E < E_c \\ E_{n,+} \neq E_{n,-} \text{ if } E > E_c \end{cases}$

Precursors at finite sizes

Some physics of open collective models

- Lindblad master equation

$$\frac{\partial \rho}{\partial t} = \underbrace{-i\left[\hat{H},\rho\right]}_{\text{Unitary evolution}} + \underbrace{\sum_i \left(\hat{L}_i \rho \hat{L}_i^\dagger - \frac{1}{2}\left\{\hat{L}_i^\dagger \hat{L}_i, \rho\right\}\right)}_{\text{Gain, loss and decoherence}} \longrightarrow \rho(t) = \mathrm{e}^{\mathcal{L}t}\rho(0)$$

ρ can be treated as a vector

- Trace-preserving evolution
- Equilibrium state given by the eigenvector of $\mathcal{L}$ with eigenvalue equal to zero
- Time evolution given by the eigenvalues of $\mathcal{L}$

Closed versus open quantum systems

Closed systems

- Time evolution determined by eigenvalues and eigenvectors of an hermitian operator, $\hat{H}$
- Unitary evolution
- Effective equilibrium state depending on the initial state

Open systems

- Time evolution determined by eigenvalues and eigenvectors of a non-Hermitian operator, $\hat{\mathcal{L}}$
- Non-unitary evolution
- Real equilibrium state given by an eigenvector of $\hat{\mathcal{L}}$

Some physics of open collective models

Our model:

$$\hat{H} = -h\hat{J}_z$$

$$\hat{L}_0 = \sqrt{\frac{\Gamma_0}{j}}\hat{J}_z, \quad \hat{L}_+ = \sqrt{\frac{\Gamma}{j}\frac{1-p}{2}}\hat{J}_+, \quad \hat{L}_- = \sqrt{\frac{\Gamma}{j}\frac{1+p}{2}}\hat{J}_-$$

- Integrable model. Spectrum given by 2 quantum numbers, $\lambda_{N,M}$
- M given by a weak symmetry, $-2j \leq M \leq 2j$
- $\mathrm{Im}\,(\lambda_{N,M}) = M \longrightarrow$ Bands given by M, $0 \leq N \leq 2j - |M|$

Some physics of open collective models

Our model:

$$\hat{H} = -h\hat{J}_z$$

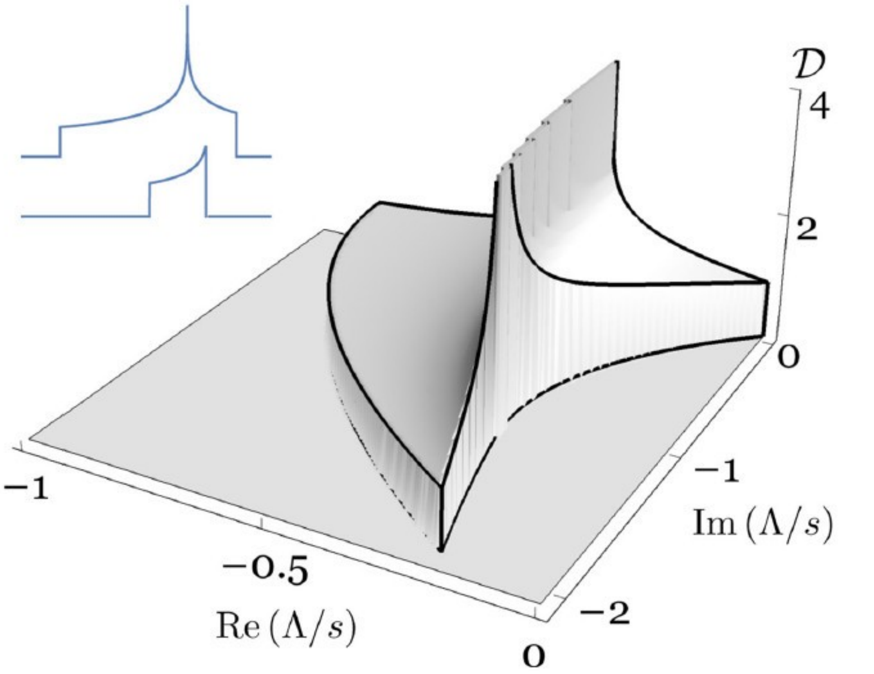
$$\hat{L}_0 = \sqrt{\frac{\Gamma_0}{j}}\hat{J}_z, \quad \hat{L}_+ = \sqrt{\frac{\Gamma}{j}\frac{1-p}{2}}\hat{J}_+, \quad \hat{L}_- = \sqrt{\frac{\Gamma}{j}\frac{1+p}{2}}\hat{J}_-$$

Density of $\mathcal{L}$ eigenvalues

$j = 17, \; p = 0.9, \; h = 1,$

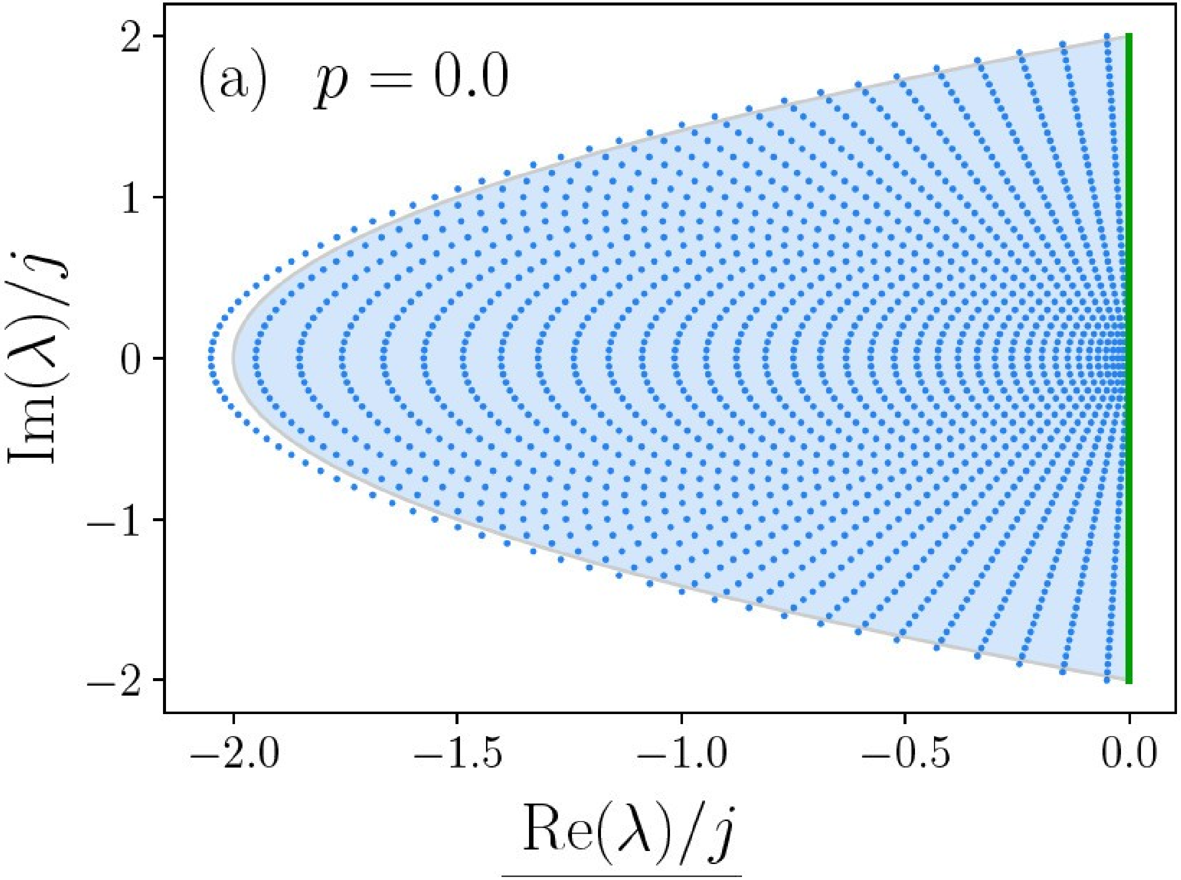
$\Gamma = 1.2, \; \Gamma_0 = 0.2$

Ribeiro & Prosen, PRL **122**, 010401 (2019)



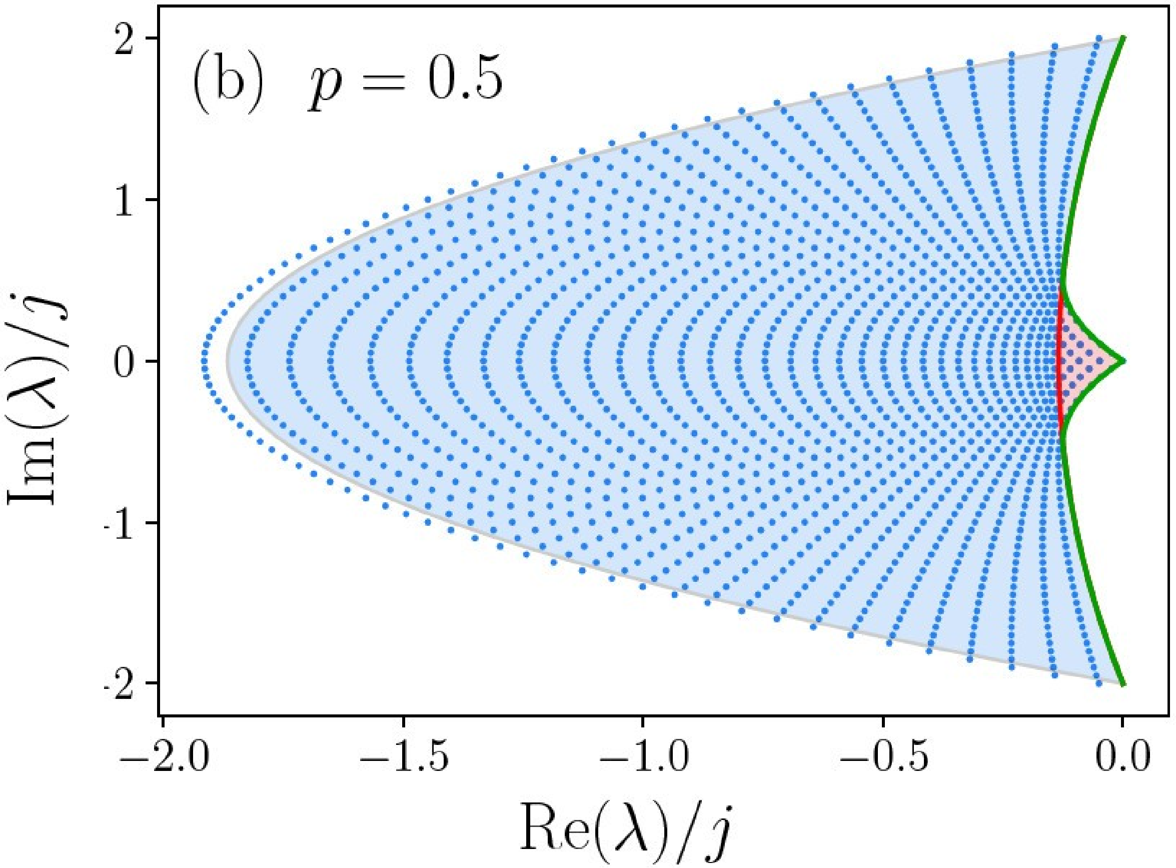
Studying the eigenvalues of $\mathcal{L}$

$j = 20,\ h = 1,\ \Gamma = 1,\ \Gamma_0 = 0$



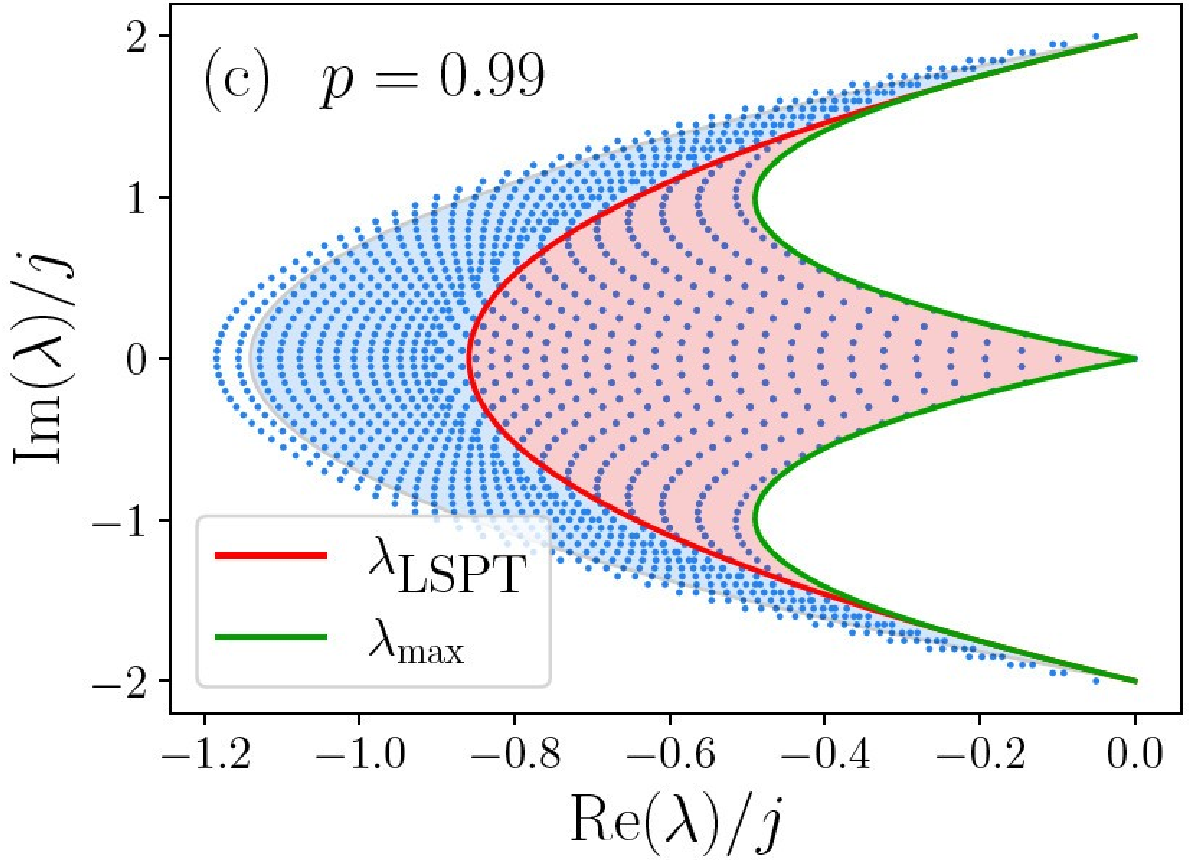
Studying the eigenvalues of $\mathcal{L}$

$j = 20,\ h = 1,\ \Gamma = 1,\ \Gamma_0 = 0$



Studying the eigenvalues of $\mathcal{L}$

$j = 20,\ h = 1,\ \Gamma = 1,\ \Gamma_0 = 0$

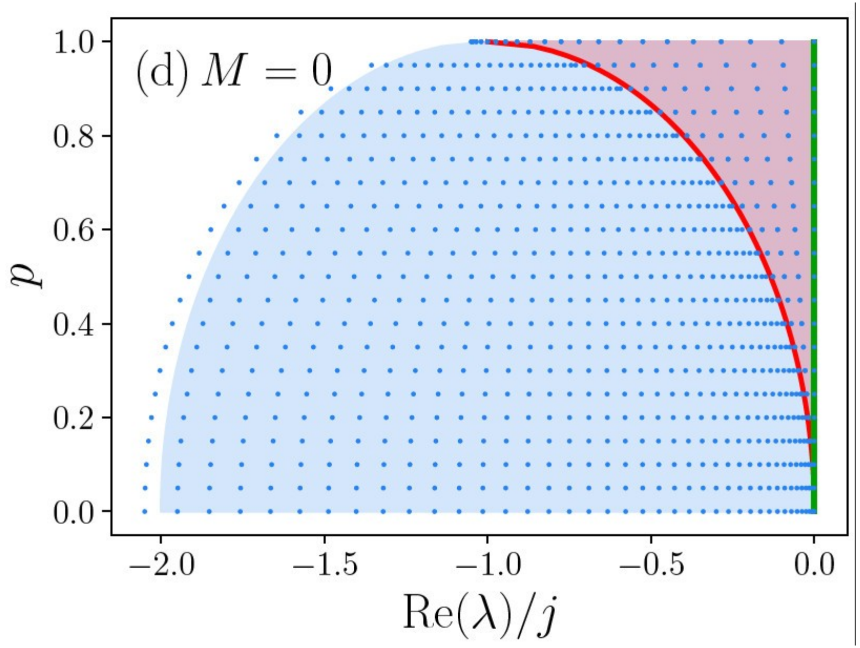


Studying the eigenvalues of $\mathcal{L}$

$$j = 20,\ h = 1,\ \Gamma = 1,\ \Gamma_0 = 0$$

For $p \neq 0,$ the spectrum seems to be splitted into two regions

- $\mathrm{Re}\,(\lambda_{N,M}) > \lambda_{c,M} \longrightarrow \lambda_{N,M} = \lambda_{N+1,M}$
- $\mathrm{Re}\,(\lambda_{N,M}) < \lambda_{c,M} \longrightarrow \lambda_{N,M} \neq \lambda_{N+1,M}$



A kind of ESQPTs in collective open systems?

The operator $\mathcal{L}$ is non-Hermitian, and this allows *weird* phenomena

Simple example with 2×2 $\mathcal{L}$ operators:

$$\mathcal{L}_1 = \begin{pmatrix} \lambda_1 & \kappa \\ 0 & \lambda_2 \end{pmatrix}, \; \lambda_1 \neq \lambda_2$$

$$\mathcal{L}_2 = \begin{pmatrix} \lambda & \kappa \\ 0 & \lambda \end{pmatrix}$$

- $\mathcal{L}_1$ is diagonalizable $\longrightarrow$ $\mathrm{e}^{\mathcal{L}_1 t} = \begin{pmatrix} \mathrm{e}^{\lambda_1 t} & \frac{\left(\mathrm{e}^{\lambda_1 t} - \mathrm{e}^{\lambda_2 t}\right)\kappa}{\lambda_1 - \lambda_2} \\ 0 & \mathrm{e}^{\lambda_2 t} \end{pmatrix}$
- $\mathcal{L}_2$ is *not* diagonalizable $\longrightarrow$ $\mathrm{e}^{\mathcal{L}_2 t} = \begin{pmatrix} \mathrm{e}^{\lambda t} & t\mathrm{e}^{\lambda t}\kappa \\ 0 & \mathrm{e}^{\lambda t} \end{pmatrix}$

A kind of ESQPTs in collective open systems?

As $\mathcal{L}$ is non-Hermitian, we have the following possibilities:

- Non-degenerate eigenvalues, $\lambda_i \neq \lambda_j, \; \forall i \neq j$

- Degenerate eigenvalues, $\exists \, \lambda_i = \lambda_j, \; i \neq j,$ with:

  $$\hat{\mathcal{L}} \left| v_i \right\rangle = \lambda_i \left| v_i \right\rangle, \; \hat{\mathcal{L}} \left| v_j \right\rangle = \lambda_i \left| v_j \right\rangle, \; \left| \left\langle v_i \right| v_j \right\rangle \right| \neq 1$$

- Exceptional points, $\exists \, \lambda_i = \lambda_j, \; i \neq j,$ with:

  $$\hat{\mathcal{L}} \left| v_i \right\rangle = \lambda_i \left| v_i \right\rangle, \; \hat{\mathcal{L}} \left| v_j \right\rangle = \lambda_i \left| v_j \right\rangle, \; \left| \left\langle v_i \right| v_j \right\rangle \right| = 1$$

Exceptional points

$\mathcal{L}_2$ is a paradigmatic example of an **exceptional point**:

We have two degenerate eigenvalues sharing a single eigenvector

What happens in our system when $0 < p < 1$?

Exceptional points

$\mathcal{L}_2$ is a paradigmatic example of an **exceptional point**:

We have two degenerate eigenvalues sharing a single eigenvector

What happens in our system when $0 < p < 1$?

- We diagonalize $\mathcal{L}$ for $M = 0$ with huge precision
- We compute the distance between consecutive eigenvectors

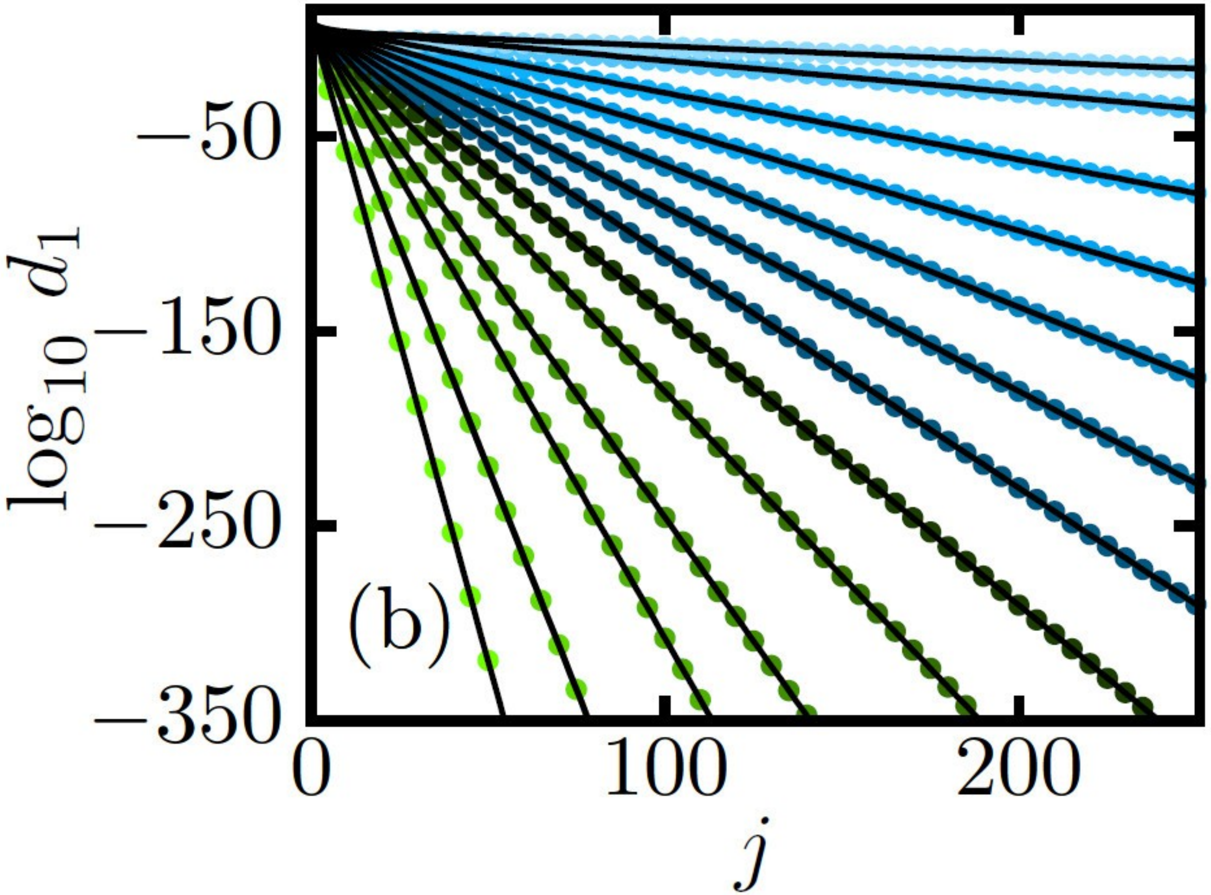
$$d_N = 1 - |\langle N+1, 0|\, N, 0\rangle|$$

- We study how this distance changes as j increases

The idea is to study possible finite-size precursors of a phase transition

Results

Eigenvectors pair closest to the steady state. Several values of p



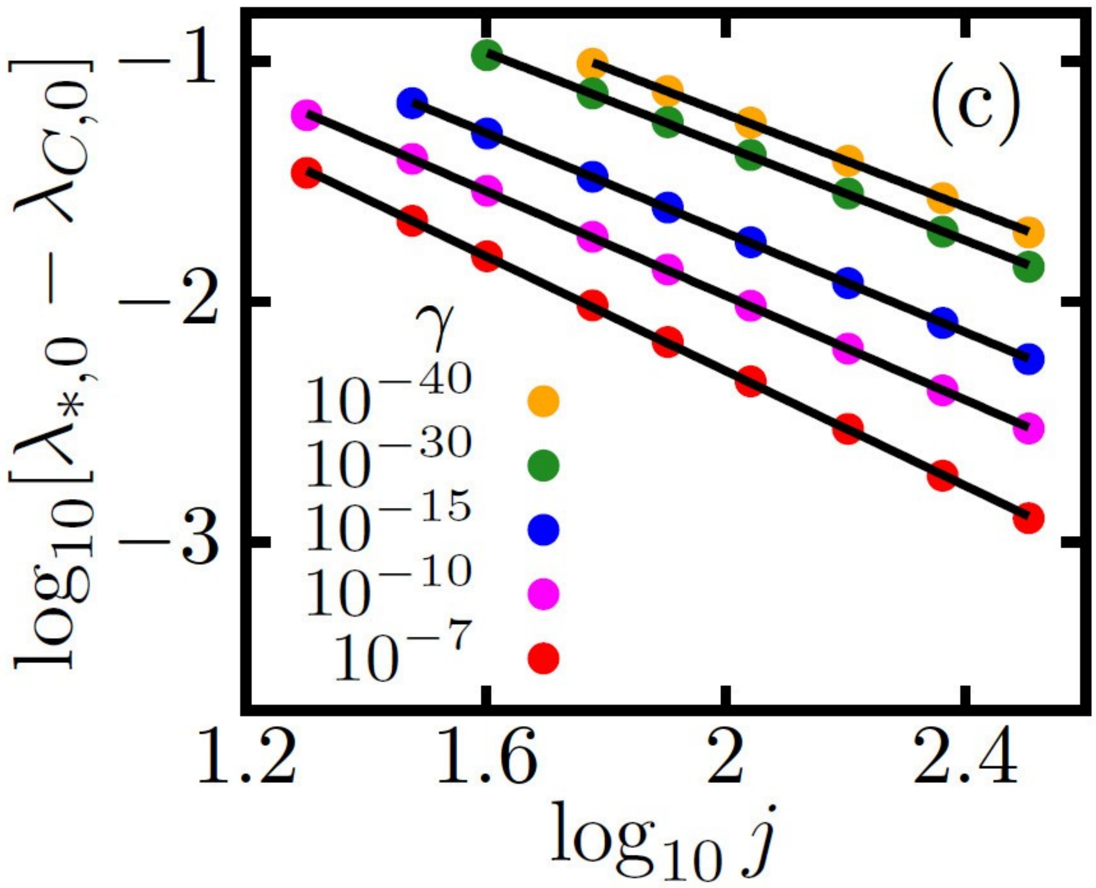
Results

Seeking the critical eigenvalue

- Chosing a bound: if $d_N < \gamma$, with $\gamma \ll 1$, coallesced eigenvectors
- Finite-size precursor: eigenvalue with larger real part with $d_N < \gamma$
  - We call it $\lambda_{*,0}(\gamma, j)$
- Evolution of this precursor with the system size
  - Does it happen that $\lim_{j\to\infty} \lambda_{*,0}(j, \gamma) \to \lambda_{c,0}, \ \forall\gamma \ll 1$?

Results

Seeking the critical eigenvalue



A first conclusion

If $0 < p < 1$ a Liouvillian Spectral Phase Transition occurs at $\lambda_{c,M}$
In the thermodynamic limit, $j \to \infty$:

- If $\mathrm{Re}(\lambda_{N,M}) > \lambda_c(M),$ all the eigenvalues are exceptional points
  - The corresponding time evolution is not exponential, but has linear correction terms
- If $\mathrm{Re}(\lambda_{N,M}) < \lambda_c(M),$ there are no exceptional points
  - The corresponding time evolution is exponential

What happens if p = 0?

In this case, we can calculate exactly the spectrum!

$$\lambda_{N,M} = ihM + \frac{\Gamma - \Gamma_0}{2j} M^2 - \frac{\Gamma}{2j} N(N+1)$$

In the thermodynamic limit, $j \to \infty$:

- There are an infinite number of steady states, $\lambda_{N,0} = 0$
- There are an infinite number of eigenvalues with zero real part, and non-zero imaginary part
  - This is a boundary time crystal!

The physics of our model at $\mathbf{p = 0}$

We can calculate exactly the expectation values for certain observables!

$$\left\langle \frac{\hat{J}_z(t)}{j} \right\rangle = \left\langle \frac{\hat{J}_z(0)}{j} \right\rangle \exp\left(-\frac{\Gamma t}{j} \right) \longrightarrow \lim_{j \to \infty} \left\langle \frac{\hat{J}_z(t)}{j} \right\rangle = \left\langle \frac{\hat{J}_z(0)}{j} \right\rangle$$

When $j \to \infty$, any initial value of $\hat{J}_z/j$ is a steady state!

The physics of our model at $\mathbf{p = 0}$

We can calculate exactly the expectation values for certain observables!

$$\left\langle \frac{\hat{J}_x(t)}{j} \right\rangle = \left\langle \frac{\hat{J}_x(0)}{j} \right\rangle \exp\left(-\frac{\Gamma + \Gamma_0}{2j} t\right) \cos(ht) \longrightarrow$$

$$\longrightarrow \lim_{j \to \infty} \left\langle \frac{\hat{J}_x(t)}{j} \right\rangle = \left\langle \frac{\hat{J}_x(0)}{j} \right\rangle \cos(ht)$$

When $j \to \infty$, $\hat{J}_x/j$ remains oscillating forever!

The integrated conclusion

- At $p_c = 0$, our system experiments a dissipative phase transition, which implies the existence of an infinite number of steady states, and induces the formation of a boundary time crystal
- At $p \neq 0$, our system experiments a LSPT, and its spectrum is split into two liouvillian spectral phases:
  - A phase full of exceptional points, close to the steady state
  - A phase without exceptional points far away the steady state

Some open questions

ESQPTs and the corresponding degeneracies in closed collective systems imply equilibrium and non-equilibrium consequences

↓

See Angel's talk this afternoon!

But, besides slowing down relaxation dynamics to the steady state, which are the consequences of having an exceptional phase?

More work is needed!