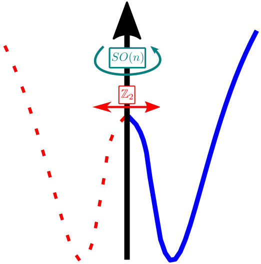
# Effects of excited state quantum phase transitions over the out-of-time-order correlators in systems with a $U(n)$ dynamical algebra

Jamil Khalouf-Rivera[a], Miguel Carvajal[a], Francisco Pérez-Bernal[a], José Enrique García-Ramos[a], Lea F. Santos[b], Qian Wang[c]

[a] Universidad de Huelva. CEAFMC
[b] Yeshiva University, USA
[c] Zhejiang Normal University, China



# Outline

- Algebraic models:
  - $U(2)$ Lipkin-Meshkov-Glick model.
  - $U(3)$ 2D limit of the vibron model
  - $U(4)$ Vibron model
  - $U(6)$ Interacting boson model
- Difference between Parity and $SO(n-1)$ symmetries
- Out-of-time-order correlators

# U(2): Lipkin-Meshkov-Glick model A. Frank et al. *SyG editores* (2005)

$$U(2) = \langle t^\dagger t,\ t^\dagger s,\ s^\dagger t,\ s^\dagger s \rangle \text{ with } \hat{\Pi} t^\dagger \hat{\Pi}^{-1} = -1 \text{ and } \hat{\Pi} s^\dagger \hat{\Pi}^{-1} = 1$$

$$= \left\langle \underbrace{t^\dagger t + s^\dagger s}_{\hat{N}},\ \underbrace{\frac{1}{2}\left(s^\dagger s - t^\dagger t\right)}_{\frac{1}{2}(\hat{N}-2\hat{n})=\hat{J}_x},\ \underbrace{\frac{1}{2}\left(t^\dagger s + s^\dagger t\right)}_{\hat{J}_y},\ \underbrace{\frac{i}{2}\left(t^\dagger s - s^\dagger t\right)}_{\hat{J}_z} \right\rangle$$

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$$\begin{array}{ccc} & & U(1) \\ & & \hat{n} \sim \hat{J}_x \\ U(2) & \nearrow & \\ [N] & \searrow & \\ & & SO(2) \\ & & \hat{J}_z \end{array}$$

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## model Hamiltonian

$$\hat{H} = (1-\xi)\hat{n} + \frac{\xi}{N-1}\left(N^2 - \hat{J}_z^2\right)$$

$$\left[\hat{H}, \hat{\Pi}\right] = 0$$

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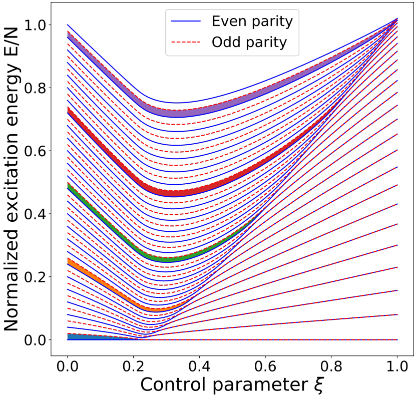

# U(3): 2D Vibron Model F. Pérez-Bernal et al. *Phys. Rev. A* **77** 032115 (2008)

$$U(3) = \left\langle \sigma^\dagger\sigma,\ \sigma^\dagger\tau_i,\ \tau_i^\dagger\sigma,\ \tau_i^\dagger\tau_j \text{ with } i,j = x,y \right\rangle$$

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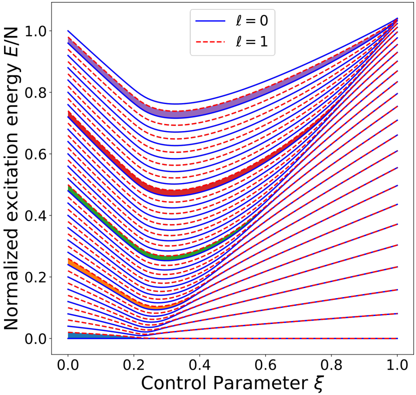

# U(4): Vibron Model

F. Iachello et al. *J. Chem. Phys.* **74** 4872 (1981)

$$U(4) = \left\langle \sigma^\dagger \tilde{\sigma},\ \hat{n} = \left[\pi^\dagger \times \tilde{\pi}\right]^{(0)},\ \hat{J}_\mu = \left[\pi^\dagger \times \tilde{\pi}\right]^{(1)}_\mu,\ \hat{Q}_\nu = \left[\pi^\dagger \times \tilde{\pi}\right]^{(2)}_\nu,\right.$$

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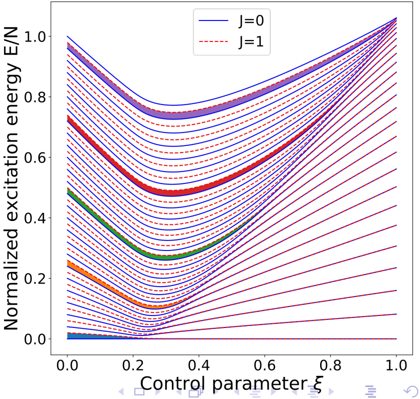


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$$U(6) = \left\langle \hat{n} = d^\dagger \cdot \tilde{d},\ s^\dagger \cdot \tilde{s},\ \cdots \right\rangle,\ \hat{\mathcal{C}}_2\,[SO(5)] = 4\left[\left|\left[d^\dagger \times \tilde{d}\right]^{(1)}\right|^2 + \left|\left[d^\dagger \times \tilde{d}\right]^{(3)}\right|^2\right]$$

$$\hat{\mathcal{C}}_2\,[SO(6)] = 2\left[N(N+4) - \left|\tilde{d}\cdot\tilde{d} - \tilde{s}\cdot\tilde{s}\right|^2\right]$$

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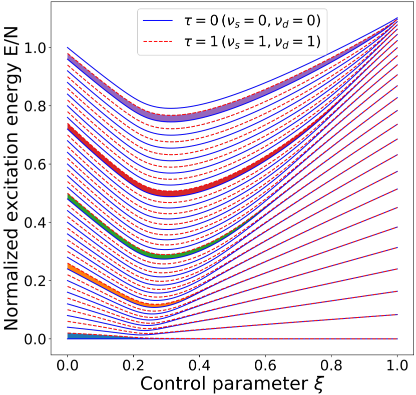


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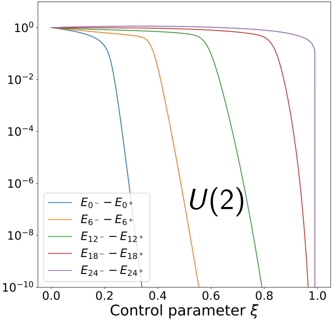
# Degeneration I: $N = 50$ M.A. Caprio et al. *Annals of Physics* **323** 5 (2008)



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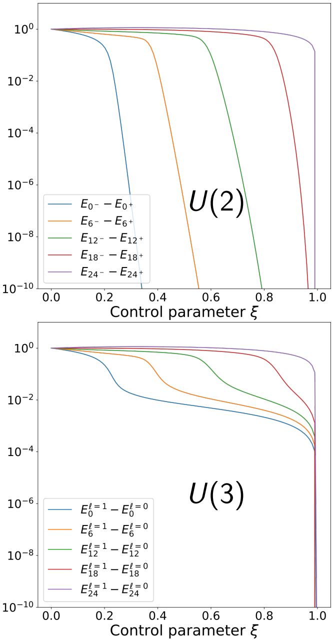

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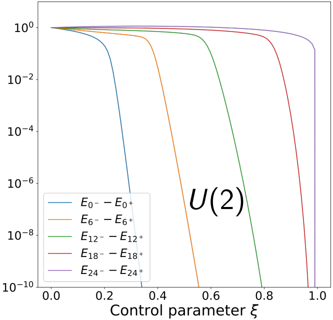


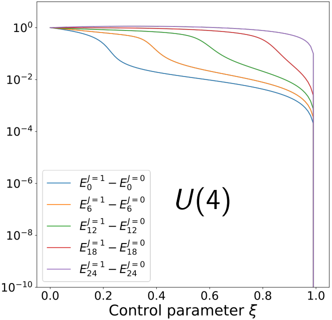


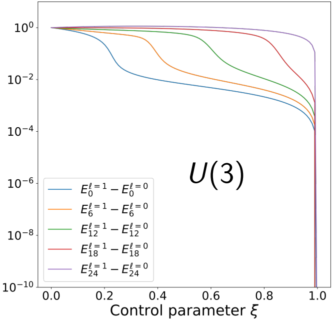

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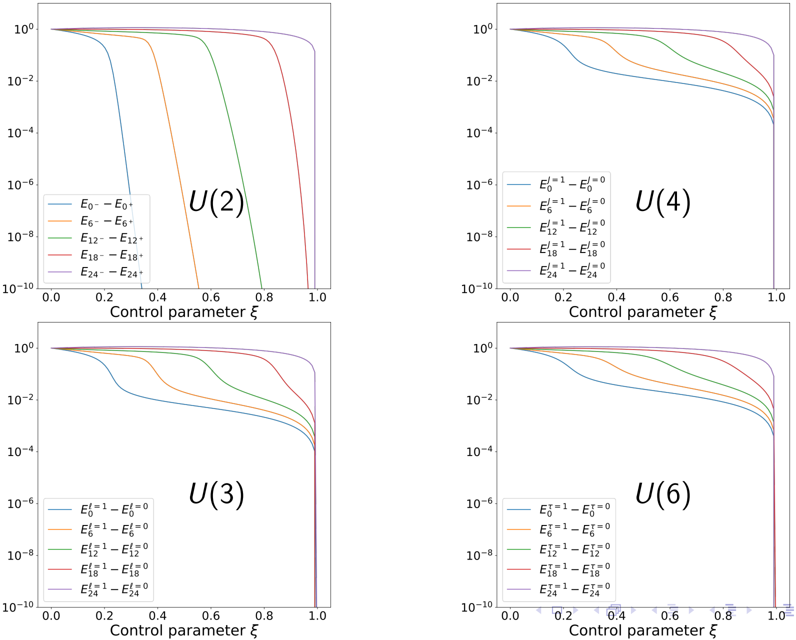
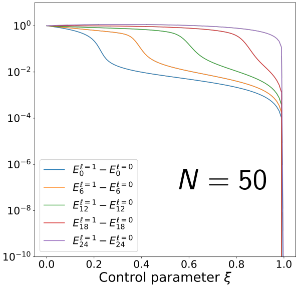


# Degeneration II: 2DVM, scaling exponent

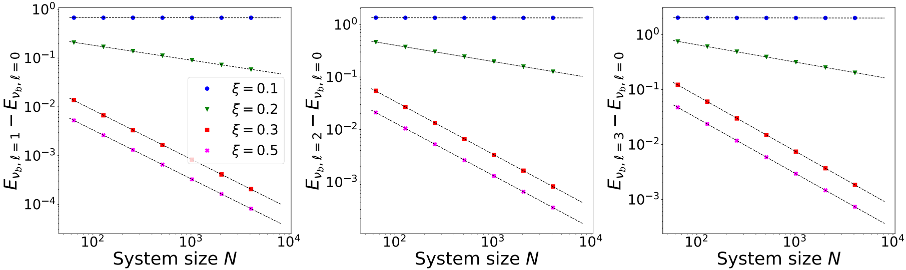



## Power Law

$$\left|E_{\nu_b=0,\ell} - E_{\nu_b=0,\ell=0}\right| \propto N^a$$

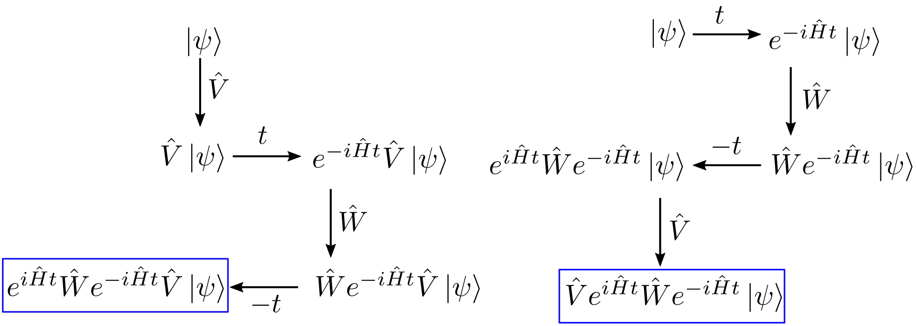
P. Pérez-Fernández et al. *Phys. Rev. A* **83** 062125 (2011)

| $\ell$ | $\xi = 0.1$ | $\xi = 0.2$ | $\xi = 0.3$ | $\xi = 0.5$ |
|---|---|---|---|---|
| 1 | $-7.2(1.7)10^{-4}$ | $-0.3071(22)$ | $-1.0093(24)$ | $-1.0025(6)$ |
| 2 | $-1.6(4)10^{-3}$ | $-0.3118(17)$ | $-1.0086(21)$ | $-1.0025(6)$ |
| 3 | $-2.4(6)10^{-3}$ | $-0.3150(14)$ | $-1.0075(16)$ | $-1.0024(6)$ |



# Out-of-time-order correlator

B. Swingle *Nature Phys.* **14** 988-990 (2018)



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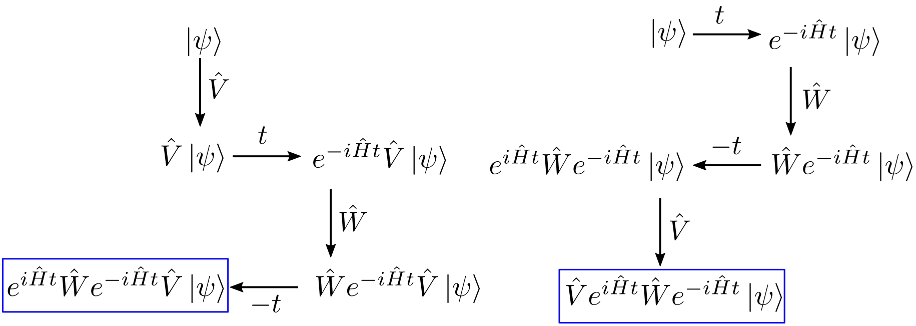


$$F^{\psi}_{\hat{V},\hat{W}}(t) = \langle\psi|\,\hat{W}^{\dagger}(t)\,\hat{V}^{\dagger}\hat{W}(t)\,\hat{V}\,|\psi\rangle$$

# Out-of-time-order correlator

B. Swingle *Nature Phys.* **14** 988-990 (2018)

$$|\psi\rangle \xrightarrow{\hat{V}} \hat{V}|\psi\rangle \xrightarrow{t} e^{-i\hat{H}t}\hat{V}|\psi\rangle \xrightarrow{\hat{W}} \hat{W}e^{-i\hat{H}t}\hat{V}|\psi\rangle \xrightarrow{-t} \boxed{e^{i\hat{H}t}\hat{W}e^{-i\hat{H}t}\hat{V}|\psi\rangle}$$

$$|\psi\rangle \xrightarrow{t} e^{-i\hat{H}t}|\psi\rangle \xrightarrow{\hat{W}} \hat{W}e^{-i\hat{H}t}|\psi\rangle \xrightarrow{-t} e^{i\hat{H}t}\hat{W}e^{-i\hat{H}t}|\psi\rangle \xrightarrow{\hat{V}} \boxed{\hat{V}e^{i\hat{H}t}\hat{W}e^{-i\hat{H}t}|\psi\rangle}$$

$$F^{\psi}_{\hat{V},\hat{W}}(t) = \langle\psi|\,\hat{W}^{\dagger}(t)\,\hat{V}^{\dagger}\hat{W}(t)\,\hat{V}\,|\psi\rangle$$

- LMG: Q. Wang et al. *Phys. Rev. A* **100** 062113 (2019)
  - $\hat{V} = \hat{W} = \hat{J}_z$
- 2DVM:
  - $\hat{V} = \hat{D}_-$
  - $\hat{W} = \hat{D}_+$

# Stationary value of $F(t)$ in the LMG model ($U(2)$)

$$F^{\psi_n^+}_{\hat{J}_z,\hat{J}_z}(t) = \sum_{n_1^-,n_2^+,n_3^-} e^{i(E_n^+ + E_{n_2}^+ - E_{n_1}^- - E_{n_3}^-)t} \left[\hat{J}_z\right]_{n^+,n_1^-} \left[\hat{J}_z\right]_{n_1^-,n_2^+} \left[\hat{J}_z\right]_{n_2^+,n_3^-} \left[\hat{J}_z\right]_{n_3^-,n^+}$$

$$\boxed{\overline{F^{\psi_n^+}_{\hat{J}_z,\hat{J}_z}} \neq 0 \iff E_n^+ + E_{n_2}^+ - E_{n_1}^- - E_{n_3}^- = 0}$$

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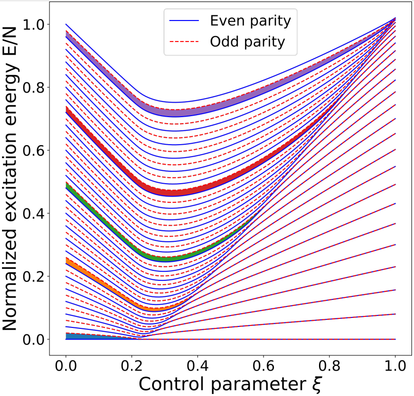
$$\overline{F^{\psi_n^+}_{\hat{J}_z,\hat{J}_z}} \neq 0 \iff E_n^+ + E_{n_2}^+ - E_{n_1}^- - E_{n_3}^- = 0$$

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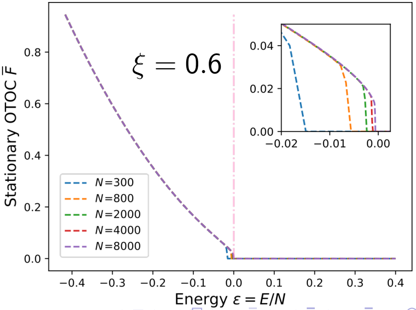


# Stationary value of $F(t)$ in the 2DVM ($U(3)$)

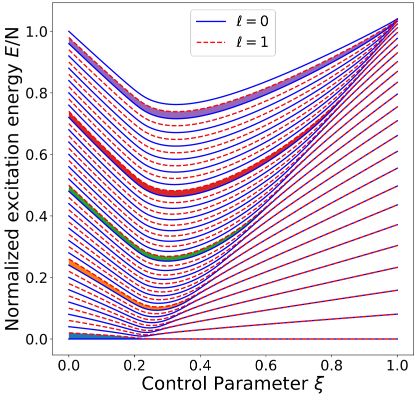
$$F^{\psi_j^\ell}_{\hat{D}_-\hat{D}_+}(t) = \sum_{j_1,j_2,j_3} e^{i(E_{j,\ell}-E_{j_1,\ell+1}+E_{j_2,\ell}-E_{j_3,\ell-1})t} \left[\hat{D}_-\right]^{j,\ell}_{j_1\ell+1} \left[\hat{D}_+\right]^{j_1,\ell+1}_{j_2,\ell} \left[\hat{D}_+\right]^{j_1,\ell}_{j_3,\ell-1} \left[\hat{D}_-\right]^{j_3,\ell-1}_{j,\ell}$$

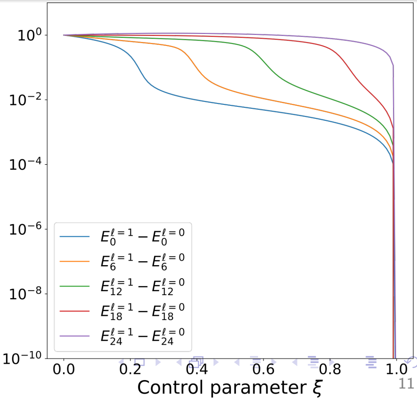
$$\boxed{\overline{F^{\psi_j^\ell}_{\hat{D}_-\hat{D}_+}} \neq 0 \iff E_{j,\ell} - E_{j_1,\ell+1} + E_{j_2,\ell} - E_{j_3,\ell-1} = 0}$$

# Stationary value of $F(t)$ in the 2DVM ($U(3)$)

$$F^{\psi_j^\ell}_{\hat{D}_-\hat{D}_+}(t) = \sum_{j_1,j_2,j_3} e^{i(E_{j,\ell}-E_{j_1,\ell+1}+E_{j_2,\ell}-E_{j_3,\ell-1})t}\left[\hat{D}_-\right]^{j,\ell}_{j_1\ell+1}\left[\hat{D}_+\right]^{j_1,\ell+1}_{j_2,\ell}\left[\hat{D}_+\right]^{j_1,\ell}_{j_3,\ell-1}\left[\hat{D}_-\right]^{j_3,\ell-1}_{j,\ell}$$

$$\boxed{\overline{F^{\psi_j^\ell}_{\hat{D}_-\hat{D}_+}} \neq 0 \iff E_{j,\ell} - E_{j_1,\ell+1} + E_{j_2,\ell} - E_{j_3,\ell-1} = 0}$$

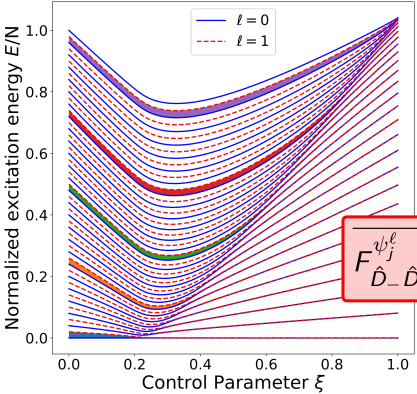


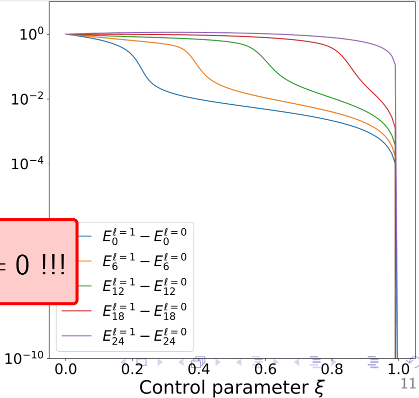
# Stationary value of $F(t)$ in the 2DVM ($U(3)$)

$$F^{\psi_j^\ell}_{\hat{D}_-\hat{D}_+}(t) = \sum_{j_1,j_2,j_3} e^{i(E_{j,\ell}-E_{j_1,\ell+1}+E_{j_2,\ell}-E_{j_3,\ell-1})t}\left[\hat{D}_-\right]^{j,\ell}_{j_1\ell+1}\left[\hat{D}_+\right]^{j_1,\ell+1}_{j_2,\ell}\left[\hat{D}_+\right]^{j_1,\ell}_{j_3,\ell-1}\left[\hat{D}_-\right]^{j_3,\ell-1}_{j,\ell}$$

$$\overline{F^{\psi_j^\ell}_{\hat{D}_-\hat{D}_+}} \neq 0 \Longleftrightarrow E_{j,\ell} - E_{j_1,\ell+1} + E_{j_2,\ell} - E_{j_3,\ell-1} = 0$$




$$\overline{F^{\psi_j^\ell}_{\hat{D}_-\hat{D}_+}} = 0 \text{ !!!}$$



# Conclusions

- The $\mathbb{Z}_2$ symmetry of the $U(2)$ algebraic model and the $SO(n-1)$ of $U(n)$ models present different phenomenologies
- In the 2DVM, VM and IBM, the degeneration in the $SO(n-1)$ symmetry labels is only achieved in the thermodynamic ($N \to \infty$) limit
- This issue has strong implications in the system dynamics
- The stationary value of the OTOC is a good order parameter only in the LMG model

Thanks for your attention!