# Un qubit acoplado a un baño térmico

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# 1. El sistema abierto S

Un qubit :

- El estado excitado es $|2\rangle$ y el estado base es $|1\rangle$
- Operadores de transición del qubit

$$\sigma_+ = |2\rangle\langle 1| \;, \qquad \sigma_- = |1\rangle\langle 2|$$

- Operadores de momento angular:

$$\begin{aligned} S_x &= \frac{\hbar}{2}(\sigma_- + \sigma_+) = \frac{\hbar}{2}\sigma_x \\ S_y &= i\frac{\hbar}{2}(\sigma_- - \sigma_+) = \frac{\hbar}{2}\sigma_y \\ S_z &= \frac{\hbar}{2}(|2\rangle\langle 2| - |1\rangle\langle 1|) = \frac{\hbar}{2}\sigma_z \end{aligned}$$

- El hamiltoniano del qubit

$$H_S = \frac{\hbar\omega_q}{2}\sigma_z$$

## 2. Los baños térmicos $B_1$ y $B_2$

$2$ reservorios independientes de osciladores armónicos desacoplados:

- Los operadores de creación y aniquilación del oscilador $k$ del reservorio $j$ son $a_{jk}^{\dagger}$ y $a_{jk}$:

$$[a_{jk}, a_{j'k'}^{\dagger}] = \delta_{jj'}\delta_{kk'}$$

- El hamiltoniano del reservorio $j$:

$$H_{B_j} = \sum_k \hbar\omega_{jk} a_{jk}^{\dagger} a_{jk}$$

- El estado del reservorio $j$ es

$$\hat{\rho}_{B_j}(0) = \prod_k \frac{1}{Z_{jk}} e^{-\beta_{jk} a_{jk}^{\dagger} a_{jk}}$$

con

$$\beta_{jk} = \frac{\hbar\omega_{jk}}{k_B T}\ , \quad Z_{jk} = N(\omega_{jk}, T) + 1\ , \quad N(\omega, T) = \frac{1}{e^{\hbar\omega/(k_B T)} - 1}$$

# 3. Interacción qubit-baños térmicos

- Interacción qubit-$B_1$

$$-\hbar\sigma_x E_1 \quad \text{con} \quad E_1 = \sum_k (g_{1k} a_{1k}^\dagger + g_{1k}^* a_{1k})$$

- Interacción qubit-$B_2$

$$-\hbar|2\rangle\langle 2|E_2 \quad \text{con} \quad E_2 = \sum_{k,l} g_{2kl} a_{2k}^\dagger a_{2l}$$

- La interacción qubit-reservorios

$$-\hbar\sigma_x E_1 - \hbar|2\rangle\langle 2|E_2$$

# 4. Hamiltoniano del sistema completo

- El hamiltoniano del qubit + baño $1$ + baño $2$

$$
\begin{aligned}
H &= H_q + H_{B_1} + H_{B_2} - \hbar\sigma_x E_1 - \hbar|2\rangle\langle 2|E_2 \\
&= \frac{\hbar\omega_q}{2}\sigma_z + \sum_{j=1,2}\sum_k \hbar\omega_{jk} a_{jk}^\dagger a_{jk} \\
&\quad -\hbar\sigma_x \sum_k (g_{1k} a_{1k}^\dagger + g_{1k}^* a_{1k}) \; - \hbar|2\rangle\langle 2| \sum_{k,l} g_{2kl} a_{2k}^\dagger a_{2l}
\end{aligned}
$$

- El estado del reservorio $j$ es

$$
\hat{\rho}_{B_j}(0) = \prod_k \frac{1}{Z_{jk}} e^{-\beta_{jk} a_{jk}^\dagger a_{jk}}
$$

con

$$
\beta_{jk} = \frac{\hbar\omega_{jk}}{k_B T} \; , \quad Z_{jk} = N(\omega_{jk}, T) + 1 \; , \quad N(\omega, T) = \frac{1}{e^{\hbar\omega/(k_B T)} - 1}
$$

# 5. Paso 1 - Reexpresión del hamiltoniano

- Valor esperado del operador de la interacción qubit-$B_1$

$$\mathrm{Tr}_{B_1}\left[E_1\rho_{B_1}(0)\right] = \mathrm{Tr}_{B_1}\left[\sum_k (g_{1k}a_{1k}^\dagger + g_{1k}^* a_{1k})\prod_k \frac{1}{Z_{jk}} e^{-\beta_{jk}a_{jk}^\dagger a_{jk}}\right] = 0$$

- Valor esperado del operador de la interacción qubit-$B_2$

$$\begin{aligned}\delta_2 &= \mathrm{Tr}_{B_2}\left[E_2\rho_{B_2}(0)\right] = \mathrm{Tr}_{B_2}\left[\sum_{k,l} g_{2kl}a_{2k}^\dagger a_{2l}\prod_k \frac{1}{Z_{jk}} e^{-\beta_{jk}a_{jk}^\dagger a_{jk}}\right] \\ &= \sum_k g_{2kk}N(\omega_{jk},T)\end{aligned}$$

- Reexpresión del hamiltoniano del

$$\begin{aligned}H &= \frac{\hbar\omega_q}{2}\sigma_z + H_{B_1} + H_{B_2} - \hbar\sigma_x E_1 - \hbar|2\rangle\langle 2|E_2 \\ &= \frac{\hbar\omega_q}{2}\sigma_z - \hbar\delta_2|2\rangle\langle 2| + H_{B_1} + H_{B_2} - \hbar\sigma_x E_1 - \hbar|2\rangle\langle 2|(E_2 - \delta_2)\end{aligned}$$

# 6. Paso 2 - Pasar a un esquema de interacción (IP)

- Hamiltoniano del sistema completo

$$\begin{aligned} H &= \frac{\hbar\omega_q}{2}\sigma_z - \hbar\delta_2|2\rangle\langle 2| + H_{B_1} + H_{B_2} - \hbar\sigma_x E_1 - \hbar|2\rangle\langle 2|(E_2 - \delta_2) \\ &= H_q + H_B - \hbar\sigma_x E_1 + V \end{aligned}$$

- Ecuación de evolución

$$\frac{d}{dt}\rho_{SB}(t) = -\frac{i}{\hbar}\Big[H,\ \rho_{SB}(t)\Big]$$

- Pasamos al IP definido por

$$U_I(t) = exp\Big[-\frac{i}{\hbar}(H_q + H_B)t\Big]$$

- Si $A(t)$ es un operador en esquema de Schrödinger (SP), entonces

$$A_I(t) = U_I^\dagger(t)A(t)U_I(t)$$

# 7. Paso 3 - Integrar la ecuación de von Neumann

- Ecuación de von Neumann en el IP

$$\frac{d}{dt}(\rho_{SB})_I(t) = -\frac{i}{\hbar}\Big[V_I(t),\ (\rho_{SB})_I(t)\Big] \tag{1}$$

- Integramos (1)

$$(\rho_{SB})_I(t) = (\rho_{SB})_I(0) - \frac{i}{\hbar}\int_0^t dt'\Big[V_I(t'),\ (\rho_{SB})_I(t')\Big] \tag{2}$$

- Substituimos (2) en el lado derecho de (1)

$$\begin{aligned} \frac{d}{dt}(\rho_{SB})_I(t) \quad = \quad & -\frac{i}{\hbar}\Big[V_I(t),\ (\rho_{SB})_I(0)\Big] \\ & -\frac{1}{\hbar^2}\int_0^t dt'\Bigg[V_I(t),\ \Big[V_I(t'),\ (\rho_{SB})_I(t')\Big]\Bigg] \end{aligned} \tag{3}$$

# 8. Paso 4 - Trazar sobre los grados de libertad de los baños

- Trazamos (3) sobre los grados de libertad de los baños

$$\begin{aligned}
\frac{d}{dt}(\rho_q)_I(t) &= \frac{d}{dt}\mathsf{Tr}_{B_1+B_2}\Big[(\rho_{SB})_I(t)\Big] \\
&= -\frac{i}{\hbar}\mathsf{Tr}_{B_1+B_2}\Big[V_I(t),\ (\rho_{SB})_I(0)\Big] \\
&\quad -\frac{1}{\hbar^2}\int_0^t dt'\mathsf{Tr}_{B_1+B_2}\Big[V_I(t),\ \Big[V_I(t'),\ (\rho_{SB})_I(t')\Big]\Big] \\
&= -\frac{1}{\hbar^2}\int_0^t dt'\mathsf{Tr}_{B_1+B_2}\Big[V_I(t),\ \Big[V_I(t'),\ (\rho_{SB})_I(t')\Big]\Big]
\end{aligned}$$

- Se usó que

$$\mathsf{Tr}_{B_1+B_2}\Big[V_I(t),\ (\rho_{SB})_I(0)\Big] = 0$$

# 9. Paso 5 - Aproximación de Born o de acoplamiento débil

- Suponemos que

$$\rho_{SB}(t) = \rho_q(t) \otimes \rho_{B_1}(0) \otimes \rho_{B_2}(0) = \rho_q(t) \otimes \rho_B(0) \ .$$

- La ecuación de evolución queda

$$\begin{aligned} & \frac{d}{dt}(\rho_q)_I(t) \\ = \quad & -\frac{1}{\hbar^2}\int_0^t dt' \mathrm{Tr}_B \left[ V_I(t),\ \left[ V_I(t'),\ (\rho_q)_I(t') \otimes \rho_B(0) \right] \right] \end{aligned}$$

# 10. Paso 6 - Expansión del integrando - parte 1

$$\begin{aligned}
& -\frac{1}{\hbar^2}\mathrm{Tr}_B\left[V_I(t),\ \left[V_I(t'),\ (\rho_q)_I(t')\otimes\rho_B(0)\right]\right] \\
= & -\frac{1}{\hbar^2}\mathrm{Tr}_B\left[V_I(t)V_I(t')(\rho_q)_I(t')\rho_B(0) - V_I(t)(\rho_q)_I(t')\rho_B(0)V_I(t')\right. \\
& \left. -V_I(t')(\rho_q)_I(t')\rho_B(0)V_I(t) + (\rho_q)_I(t')\rho_B(0)V_I(t')V_I(t)\right] \\
= & -\frac{1}{\hbar^2}\mathrm{Tr}_B\left[V_I(t)V_I(t')(\rho_q)_I(t')\rho_B(0) - V_I(t)(\rho_q)_I(t')\rho_B(0)V_I(t')\right] \\
& +h.c.
\end{aligned}$$

Recordamos que

$$V = -\hbar\sigma_x E_1 - \hbar|2\rangle\langle 2|(E_2-\delta_2)$$

# 11. Paso 6 - Expansión del integrando - parte 2

$$
\begin{aligned}
& -\frac{1}{\hbar^2}\mathsf{Tr}_B\left[V_I(t)V_I(t')(\rho_q)_I(t')\rho_B(0)\right] \\
= \; & -\mathsf{Tr}_B\Big[(\sigma_x)_I(t)(E_1)_I(t)(\sigma_x)_I(t')(E_1)_I(t')(\rho_q)_I(t')\rho_B(0) \\
& +(\sigma_x)_I(t)(E_1)_I(t)(|2\rangle\langle 2|)_I(t')(E_2-\delta_2)_I(t')(\rho_q)_I(t')\rho_B(0) \\
& +(|2\rangle\langle 2|)_I(t)(E_2-\delta_2)_I(t)(\sigma_x)_I(t')(E_1)_I(t')(\rho_q)_I(t')\rho_B(0) \\
& +(|2\rangle\langle 2|)_I(t)(E_2-\delta_2)_I(t)(|2\rangle\langle 2|)_I(t')(E_2-\delta_2)_I(t')(\rho_q)_I(t')\rho_B(0)\Big] \\
= \; & -(\sigma_x)_I(t)(\sigma_x)_I(t')(\rho_q)_I(t')\mathsf{Tr}_B\left[(E_1)_I(t)(E_1)_I(t')\rho_B(0)\right] \\
& -(\sigma_x)_I(t)(|2\rangle\langle 2|)_I(t')(\rho_q)_I(t')\mathsf{Tr}_B\left[(E_1)_I(t)(E_2-\delta_2)_I(t')\rho_B(0)\right] \\
& -(|2\rangle\langle 2|)_I(t)(\sigma_x)_I(t')(\rho_q)_I(t')\mathsf{Tr}_B\left[(E_2-\delta_2)_I(t)(E_1)_I(t')\rho_B(0)\right] \\
& -(|2\rangle\langle 2|)_I(t)(|2\rangle\langle 2|)_I(t')(\rho_q)_I(t')\times \\
& \qquad \times\mathsf{Tr}_B\left[(E_2-\delta_2)_I(t)(E_2-\delta_2)_I(t')\rho_B(0)\right]
\end{aligned}
$$

# 12. Paso 7 - Funciones de autocorrelación de los baños

Las funciones de autocorrelación son

$$
\begin{aligned}
c_{11}(t,t') &= \mathrm{Tr}_B\Big[(E_1)_I(t)(E_1)_I(t')\rho_B(0)\Big] \\
c_{12}(t,t') &= \mathrm{Tr}_B\Big[(E_1)_I(t)(E_2-\delta_2)_I(t')\rho_B(0)\Big] \;=\; 0 \\
c_{21}(t,t') &= \mathrm{Tr}_B\Big[(E_2-\delta_2)_I(t)(E_1)_I(t')\rho_B(0)\Big] \;=\; 0 \\
c_{22}(t,t') &= \mathrm{Tr}_B\Big[(E_2-\delta_2)_I(t)(E_2-\delta_2)_I(t')\rho_B(0)\Big]
\end{aligned}
$$

Son homogéneas porque $\rho_B(0) = \rho_{B_1(0} \otimes \rho_{B_2}(0)$ es un estado estacionario de $H_B = H_{B_1} + H_{B_2}$:

$$
\begin{aligned}
c_{11}(t,t') &= \mathrm{Tr}_B\Big[(E_1)_I(t-t')(E_1)_I(0)\rho_B(0)\Big] = C_1(t-t') \\
c_{22}(t,t') &= \mathrm{Tr}_B\Big[(E_2-\delta_2)_I(t-t')(E_2-\delta_2)_I(0)\rho_B(0)\Big] = C_2(t-t')
\end{aligned}
$$

# 13. Paso 6 - Expansión del integrando - parte 3

La ecuación de evolución queda

$$\begin{aligned}
&\frac{d}{dt}(\rho_q)_I(t) \\
= \; & -\frac{1}{\hbar^2}\int_0^t dt' \mathrm{Tr}_B\left[V_I(t),\ \left[V_I(t'),\ (\rho_q)_I(t')\otimes\rho_B(0)\right]\right] \\
= \; & -\int_0^t dt' \Big\{ C_1(t-t')(\sigma_x)_I(t)(\sigma_x)_I(t')(\rho_q)_I(t') \\
& -C_1(t'-t)(\sigma_x)_I(t)(\rho_q)_I(t')(\sigma_x)_I(t') \\
& +C_2(t-t')(|2\rangle\langle 2|)_I(t)(|2\rangle\langle 2|)_I(t')(\rho_q)_I(t') \\
& -C_2(t'-t)(|2\rangle\langle 2|)_I(t)(\rho_q)_I(t')(|2\rangle\langle 2|)_I(t') \Big\} + h.c.
\end{aligned}$$

# 14. Ecuación en la aproximación de Born

Haciendo el cambio de variable $\tau = t - t'$

$$
\begin{aligned}
&\frac{d}{dt}(\rho_q)_I(t) \\
= \quad & -\int_0^t d\tau \Big\{ C_1(\tau)(\sigma_x)_I(t)(\sigma_x)_I(t-\tau)(\rho_q)_I(t-\tau) \\
& -C_1(\tau)^*(\sigma_x)_I(t)(\rho_q)_I(t-\tau)(\sigma_x)_I(t-\tau) \\
& +C_2(\tau)(|2\rangle\langle 2|)_I(t)(|2\rangle\langle 2|)_I(t-\tau)(\rho_q)_I(t-\tau) \\
& -C_2(\tau)^*(|2\rangle\langle 2|)_I(t)(\rho_q)_I(t-\tau)(|2\rangle\langle 2|)_I(t-\tau) \Big\} + h.c.
\end{aligned}
$$

# 15. Paso 8 - Aproximación de Markov

- Las funciones de autocorrelación decaen a cero en una escala de tiempo $\tau_B$
$$|C_j(\tau)| \simeq 0 \quad \text{si} \quad \tau \geq \tau_B.$$
- La escala de tiempo de evolución $\tau_q$ de $(\rho_q)_I(t)$ satisface
$$\tau_B \ll \tau_q$$

Entonces

- Consideramos $t > \tau_B$ y aproximamos
$$(\rho_q)_I(t-\tau) = (\rho_q)_I(t)$$
- Extendemos el intervalo de integración a $+\infty$

# 16. Paso 9 - Ecuación de Born-Markov

$$
\begin{aligned}
& \frac{d}{dt}(\rho_q)_I(t) \\
= \quad & -\int_0^{+\infty} d\tau \Big\{ C_1(\tau)(\sigma_x)_I(t)(\sigma_x)_I(t-\tau)(\rho_q)_I(t) \\
& \quad -C_1(\tau)^*(\sigma_x)_I(t)(\rho_q)_I(t)(\sigma_x)_I(t-\tau) \\
& \quad +C_2(\tau)(|2\rangle\langle 2|)_I(t)(|2\rangle\langle 2|)_I(t-\tau)(\rho_q)_I(t) \\
& \quad -C_2(\tau)^*(|2\rangle\langle 2|)_I(t)(\rho_q)_I(t)(|2\rangle\langle 2|)_I(t-\tau) \Big\} + h.c.
\end{aligned}
$$

# 17. Paso 10 - Operadores del qubit en el IP - parte 1

Recordamos que

- El IP está definido por

$$U_I(t) \quad = \quad exp\Big[-\frac{i}{\hbar}(H_q + H_B)t\Big]$$

donde

$$H_q \quad = \quad \frac{\hbar\omega_q}{2}\sigma_z - \hbar\delta_2|2\rangle\langle 2| = \frac{\hbar(\omega_q - \delta_2)}{2}\sigma_z - \frac{\hbar\delta_2}{2}\mathbb{I}$$

- Si $A(t)$ es un operador en SP, entonces

$$A_I(t) = U_I^\dagger(t)A(t)U_I(t)$$

Usando la fórmula Baker-Campbell-Hausdorff

$$\begin{aligned} (\sigma_x)_I(t) \quad &= \quad e^{-i(\omega_q-\delta_2)t}\sigma_- + e^{i(\omega_q-\delta_2)t}\sigma_+ \\ (|2\rangle\langle 2|)_I(t) \quad &= \quad |2\rangle\langle 2| \end{aligned}$$

# 18. Paso 10 - Operadores del qubit en el IP - parte 2

$$
\begin{aligned}
& \frac{d}{dt}(\rho_q)_I(t) \\
= \quad & -\int_0^{+\infty} d\tau \Big\{ C_1(\tau)\Big[\sigma_-\sigma_+ e^{-i(\omega_q-\delta_2)\tau} + \sigma_+\sigma_- e^{i(\omega_q-\delta_2)\tau}\Big](\rho_q)_I(t) \\
& \quad -C_1(\tau)^*\sigma_-(\rho_q)_I(t)\Big[e^{-i(\omega_q-\delta_2)(2t-\tau)}\sigma_- + e^{-i(\omega_q-\delta_2)\tau}\sigma_+\Big] \\
& \quad -C_1(\tau)^*\sigma_+(\rho_q)_I(t)\Big[e^{i(\omega_q-\delta_2)\tau}\sigma_- + e^{i(\omega_q-\delta_2)(2t-\tau)}\sigma_+\Big] \\
& \quad +C_2(\tau)|2\rangle\langle 2|(\rho_q)_I(t) - C_2(\tau)^*|2\rangle\langle 2|(\rho_q)_I(t)|2\rangle\langle 2| \Big\} + h.c.
\end{aligned}
$$

# 19. Paso 11 - Aproximación de la onda rotante

Suponemos que la escala de tiempo de evolución $\tau_q$ de $(\rho_q)_I(t)$ satisface

$$\frac{2\pi}{2(\omega_q - \delta_2)} \ll \tau_q$$

Entonces podemos promediar la ecuación en un intervalo de longitud $2\pi/[2(\omega_q - \delta_2)]$

$$\begin{aligned}
& \frac{d}{dt}(\rho_q)_I(t) \\
= \quad & -\int_0^{+\infty} d\tau \Big\{ \\
& C_1(\tau)e^{-i(\omega_q-\delta_2)\tau}\sigma_-\sigma_+(\rho_q)_I(t) - C_1(\tau)^* e^{i(\omega_q-\delta_2)\tau}\sigma_+(\rho_q)_I(t)\sigma_- \\
& +C_1(\tau)e^{i(\omega_q-\delta_2)\tau}\sigma_+\sigma_-(\rho_q)_I(t) - C_1(\tau)^* e^{-i(\omega_q-\delta_2)\tau}\sigma_-(\rho_q)_I(t)\sigma_+ \\
& \quad +C_2(\tau)|2\rangle\langle 2|(\rho_q)_I(t) - C_2(\tau)^*|2\rangle\langle 2|(\rho_q)_I(t)|2\rangle\langle 2| \Big\} + h.c.
\end{aligned}$$

# 20. Paso 12 - Transformada de Fourier de las funciones de autocorrelación

Definimos

$$\Gamma_1(\omega) = \int_0^{+\infty} d\tau C_1(\tau) e^{i\omega\tau} \ , \quad \Gamma_2 = \int_0^{+\infty} d\tau C_2(\tau)$$

Entonces la ecuación toma la forma

$$\begin{aligned} & \frac{d}{dt}(\rho_q)_I(t) \\ = \ & \Gamma_1[-(\omega_q - \delta_2)]^* \sigma_+ (\rho_q)_I(t) \sigma_- - \Gamma_1[-(\omega_q - \delta_2)] \sigma_- \sigma_+ (\rho_q)_I(t) \\ & + \Gamma_1(\omega_q - \delta_2)^* \sigma_- (\rho_q)_I(t) \sigma_+ - \Gamma_1(\omega_q - \delta_2) \sigma_+ \sigma_- (\rho_q)_I(t) \\ & + \Gamma_2^* |2\rangle\langle 2| (\rho_q)_I(t) |2\rangle\langle 2| - \Gamma_2 |2\rangle\langle 2| (\rho_q)_I(t) + h.c. \end{aligned}$$

# 21. Paso 13 - Las funciones de autocorrelación - parte 1

Recordamos que

$$\begin{aligned} C_1(\tau) &= \mathrm{Tr}_{B_1}\Big[(E_1)_I(\tau)(E_1)_I(0)\rho_{B_1}(0)\Big] \\ C_2(\tau) &= \mathrm{Tr}_{B_2}\Big[(E_2-\delta_2)_I(\tau)(E_2-\delta_2)_I(0)\rho_{B_2}(0)\Big] \end{aligned}$$

donde

$$\begin{aligned} E_1 &= \sum_k (g_{1k}a_{1k}^\dagger + g_{1k}^* a_{1k}) \ , \qquad E_2 = \sum_{k,l} g_{2kl}a_{2k}^\dagger a_{2l} \\ U_I(t) &= exp\Big[-\frac{i}{\hbar}(H_q + H_B)t\Big] \ , \quad A_I(t) = U_I^\dagger(t)A(t)U_I(t) \end{aligned}$$

La fórmula Baker-Campbell-Hausdorff nos dice que

$$e^{\alpha a^\dagger a} a e^{-\alpha a^\dagger a} = e^{-\alpha} a \ , \quad e^{\alpha a^\dagger a} a^\dagger e^{-\alpha a^\dagger a} = e^{\alpha} a^\dagger$$

# 22. Paso 13 - Las funciones de autocorrelación - parte 2

Expresión para las funciones de autocorrelación

$$C_1(\tau) = \sum_k |g_{1k}|^2 \Big\{ e^{-i\omega_{1k}\tau} \Big[ N(\omega_{1k}, T) + 1 \Big] + e^{i\omega_{1k}\tau} N(\omega_{1k}, T) \Big\}$$

$$C_2(\tau) = \sum_{k,l} g_{2kl}^2 N(\omega_{2k,T}) \Big[ N(\omega_{2l}, T) + 1 \Big] e^{i(\omega_{2k} - \omega_{2l})\tau}$$

Tomamos el límite continuo para los baños

$$\begin{aligned} \sum_k (\cdot) &\rightarrow \int_0^{+\infty} d\omega \rho_{Dj}(\omega)(\cdot) \\ \omega_{jk} &\rightarrow \omega \\ g_{1k} = g_1(\omega_{1k}) &\rightarrow g_1(\omega) \\ g_{2kl} = g_2(\omega_{2k}, \omega_{2l}) &\rightarrow g_2(\omega, \omega') \end{aligned}$$

donde $\rho_{Dj}(\omega)$ es la densidad de estados: $\rho_{Dj}(\omega) d\omega$ es el número de osciladores con frecuencias en $[\omega, \omega + d\omega]$.

# 23. Paso 13 - Las funciones de autocorrelación - parte 3

Expresión para las funciones de autocorrelación

$$C_1(\tau) = \sum_k |g_{1k}|^2 \Big\{ e^{-i\omega_{1k}\tau} \Big[ N(\omega_{1k},T) + 1 \Big] + e^{i\omega_{1k}\tau} N(\omega_{1k},T) \Big\}$$

$$= \int_0^{+\infty} d\omega \rho_{D1}(\omega) |g_1(\omega)|^2 \Big\{ e^{-i\omega\tau} \Big[ N(\omega,T) + 1 \Big] + e^{i\omega\tau} N(\omega,T) \Big\}$$

$$C_2(\tau) = \sum_{k,l} g_{2kl}^2 N(\omega_{2k},T) \Big[ N(\omega_{2l},T) + 1 \Big] e^{i(\omega_{2k}-\omega_{2l})\tau}$$

$$= \int_0^{+\infty} d\omega \rho_{D2}(\omega) \int_0^{+\infty} d\omega' \rho_{D2}(\omega') g_2(\omega,\omega')^2 N(\omega,T) \times$$

$$\times \Big[ N(\omega',T) + 1 \Big] e^{i(\omega-\omega')\tau}$$

Expresión para la frecuencia

$$\delta_2 = \sum_k g_{2kk} N(\omega_{jk},T) = \int_0^{+\infty} d\omega \rho_{D2}(\omega) g_2(\omega,\omega) N(\omega,T)$$

# 24. Paso 14 - TF de funciones de autocorrelación - parte 1

Recordamos que

$$\Gamma_1(\omega) = \int_0^{+\infty} d\tau C_1(\tau) e^{i\omega\tau} \ , \quad \Gamma_2 = \int_0^{+\infty} d\tau C_2(\tau)$$

y que

$$\int_0^{+\infty} d\tau e^{-i\omega\tau} = \pi\delta(\omega) - i\text{P.V.}\frac{1}{\omega}$$

Substituimos en las $\Gamma_j$ las funciones de autocorrelación

$$\begin{aligned} & C_1(\tau) \\ = \ & \int_0^{+\infty} d\omega \rho_{D1}(\omega) |g_1(\omega)|^2 \Big\{ e^{-i\omega\tau} \Big[ N(\omega, T) + 1 \Big] + e^{i\omega\tau} N(\omega, T) \Big\} \\ & C_2(\tau) \\ = \ & \int_0^{+\infty} d\omega \rho_{D2}(\omega) \int_0^{+\infty} d\omega' \rho_{D2}(\omega') g_2(\omega, \omega')^2 N(\omega, T) \times \\ & \times \Big[ N(\omega', T) + 1 \Big] e^{i(\omega - \omega')\tau} \end{aligned}$$

# 25. Paso 14 - TF de funciones de autocorrelación - parte 2

Se obtiene

$$\begin{aligned}\Gamma_1(\omega') &= \int_0^{+\infty} d\omega \rho_{D1}(\omega)|g_1(\omega)|^2 \Bigg\{ \\ & \Big[N(\omega,T)+1\Big]\Big[\pi\delta(\omega-\omega') - i\mathsf{PV}\frac{1}{\omega-\omega'}\Big] \\ & +N(\omega,T)\Big[\pi\delta(\omega+\omega') + i\mathsf{PV}\frac{1}{\omega+\omega'}\Big]\Bigg\}\end{aligned}$$

Entonces

$$\begin{aligned} & \Gamma_1[-(\omega_q-\delta_2)] \\ = \; & i\mathsf{PV}\int_0^{+\infty} d\omega \rho_{D1}(\omega)|g_1(\omega)|^2 \left\{ -\frac{N(\omega,T)+1}{\omega+(\omega_q-\delta_2)} + \frac{N(\omega,T)}{\omega-(\omega_q-\delta_2)} \right\} \\ & +\pi\rho_{D1}(\omega_q-\delta_2)|g_1(\omega_q-\delta_2)|^2 N(\omega_q-\delta_2,T) \qquad (4)\end{aligned}$$

# 26. Paso 14 - TF de funciones de autocorrelación - parte 3

Como

$$\begin{aligned}\Gamma_1(\omega') &= \int_0^{+\infty} d\omega \rho_{D1}(\omega)|g_1(\omega)|^2 \Bigg\{ \\ &\Big[N(\omega,T)+1\Big]\Big[\pi\delta(\omega-\omega') - i\text{PV}\frac{1}{\omega-\omega'}\Big] \\ &+N(\omega,T)\Big[\pi\delta(\omega+\omega') + i\text{PV}\frac{1}{\omega+\omega'}\Big]\Bigg\}\end{aligned}$$

se obtiene

$$\begin{aligned}&\Gamma_1(\omega_q-\delta_2) \\ =\ &i\text{PV}\int_0^{+\infty} d\omega \rho_{D1}(\omega)|g_1(\omega)|^2 \left\{ -\frac{N(\omega,T)+1}{\omega-(\omega_q-\delta_2)} + \frac{N(\omega,T)}{\omega+(\omega_q-\delta_2)} \right\} \\ &+\pi\rho_{D1}(\omega_q-\delta_2)|g_1(\omega_q-\delta_2)|^2\Big[N(\omega_q-\delta_2,T)+1\Big]\end{aligned}$$

# 27. Paso 14 - TF de funciones de autocorrelación - parte 4

Entonces la ecuación toma la forma

$$
\begin{aligned}
& \rightarrow \Gamma_1(\omega_q - \delta_2) \\
= \; & i\mathsf{PV}\int_0^{+\infty} d\omega \rho_{D1}(\omega)|g_1(\omega)|^2 \left\{ -\frac{N(\omega,T)+1}{\omega-(\omega_q-\delta_2)} + \frac{N(\omega,T)}{\omega+(\omega_q-\delta_2)} \right\} \\
& +\pi\rho_{D1}(\omega_q-\delta_2)|g_1(\omega_q-\delta_2)|^2 \Big[ N(\omega_q-\delta_2,T)+1 \Big] \\
= \; & \frac{\gamma_0}{2}\Big[ N(\omega_q-\delta_2,T)+1 \Big] - i\Delta_0
\end{aligned}
$$

$$
\begin{aligned}
& \rightarrow \Gamma_1[-(\omega_q - \delta_2)] \\
= \; & i\mathsf{PV}\int_0^{+\infty} d\omega \rho_{D1}(\omega)|g_1(\omega)|^2 \left\{ -\frac{N(\omega,T)+1}{\omega+(\omega_q-\delta_2)} + \frac{N(\omega,T)}{\omega-(\omega_q-\delta_2)} \right\} \\
& +\pi\rho_{D1}(\omega_q-\delta_2)|g_1(\omega_q-\delta_2)|^2 N(\omega_q-\delta_2,T) \\
= \; & \frac{\gamma_0}{2} N(\omega_q-\delta_2,T) - i\Delta_1
\end{aligned}
$$

# 28. Paso 14 - TF de funciones de autocorrelación - parte 5

Entonces la ecuación toma la forma

$$
\begin{aligned}
& \rightarrow \Gamma_1(\omega_q - \delta_2) \\
= \quad & i\mathsf{PV}\int_0^{+\infty} d\omega \rho_{D1}(\omega)|g_1(\omega)|^2 \left\{ -\frac{N(\omega,T)+1}{\omega-(\omega_q-\delta_2)} + \frac{N(\omega,T)}{\omega+(\omega_q-\delta_2)} \right\} \\
& +\pi\rho_{D1}(\omega_q-\delta_2)|g_1(\omega_q-\delta_2)|^2\Big[N(\omega_q-\delta_2,T)+1\Big] \\
= \quad & \frac{\gamma_0}{2}\Big[N(\omega_q-\delta_2,T)+1\Big] - i\Delta_0
\end{aligned}
$$

$$
\begin{aligned}
& \rightarrow \Gamma_1[-(\omega_q - \delta_2)] \\
= \quad & i\mathsf{PV}\int_0^{+\infty} d\omega \rho_{D1}(\omega)|g_1(\omega)|^2 \left\{ -\frac{N(\omega,T)+1}{\omega+(\omega_q-\delta_2)} + \frac{N(\omega,T)}{\omega-(\omega_q-\delta_2)} \right\} \\
& +\pi\rho_{D1}(\omega_q-\delta_2)|g_1(\omega_q-\delta_2)|^2 N(\omega_q-\delta_2,T) \\
= \quad & \frac{\gamma_0}{2} N(\omega_q-\delta_2,T) - i\Delta_1
\end{aligned}
$$

# 29. Paso 14 - TF de funciones de autocorrelación - parte 6

Recordamos que

$$\Gamma_2 = \int_0^{+\infty} d\tau C_2(\tau)$$

y que

$$\int_0^{+\infty} d\tau e^{-i\omega\tau} = \pi\delta(\omega) - i\text{P.V.}\frac{1}{\omega}$$

Substituimos en $\Gamma_{2j}$

$$\begin{aligned} & \to C_2(\tau) \\ = \quad & \int_0^{+\infty} d\omega \rho_{D2}(\omega) \int_0^{+\infty} d\omega' \rho_{D2}(\omega') g_2(\omega,\omega')^2 N(\omega,T) \times \\ & \times \Big[ N(\omega',T) + 1 \Big] e^{i(\omega-\omega')\tau} \end{aligned}$$

# 30. Paso 14 - TF de funciones de autocorrelación - parte 7

$$
\begin{aligned}
& \Gamma_2 \\
= \quad & \pi \int_0^{+\infty} d\omega \rho_{D2}(\omega)^2 g_2(\omega,\omega)^2 N(\omega,T) \Big[ N(\omega,T) + 1 \Big] \\
& -i\text{P.V.} \int_0^{+\infty} d\omega \rho_{D2}(\omega) \int_0^{+\infty} d\omega' \rho_{D2}(\omega') g_2(\omega,\omega')^2 N(\omega,T) \times \\
& \times \frac{N(\omega',T)+1}{\omega'-\omega} \\
= \quad & \gamma_2 - i\Delta_2
\end{aligned}
$$

# 31. Paso 15 - Substitución en la ecuación maestra - parte 1

Tenemos

$$
\begin{aligned}
& \frac{d}{dt}(\rho_q)_I(t) \\
= \; & \Gamma_1[-(\omega_q-\delta_2)]^*\sigma_+(\rho_q)_I(t)\sigma_- - \Gamma_1[-(\omega_q-\delta_2)]\sigma_-\sigma_+(\rho_q)_I(t) \\
& +\Gamma_1(\omega_q-\delta_2)^*\sigma_-(\rho_q)_I(t)\sigma_+ - \Gamma_1(\omega_q-\delta_2)\sigma_+\sigma_-(\rho_q)_I(t) \\
& +\Gamma_2^*|2\rangle\langle 2|(\rho_q)_I(t)|2\rangle\langle 2| - \Gamma_2|2\rangle\langle 2|(\rho_q)_I(t) + h.c. \\
= \; & \Big[\frac{\gamma_0}{2}N(\omega_q-\delta_2,T)+i\Delta_1\Big]\sigma_+(\rho_q)_I(t)\sigma_- \\
& -\Big[\frac{\gamma_0}{2}N(\omega_q-\delta_2,T)-i\Delta_1\Big]\sigma_-\sigma_+(\rho_q)_I(t) \\
& +\Bigg\{\frac{\gamma_0}{2}\Big[N(\omega_q-\delta_2,T)+1\Big]+i\Delta_0\Bigg\}\sigma_-(\rho_q)_I(t)\sigma_+ \\
& -\Bigg\{\frac{\gamma_0}{2}\Big[N(\omega_q-\delta_2,T)+1\Big]-i\Delta_0\Bigg\}\sigma_+\sigma_-(\rho_q)_I(t) \\
& +\Big(\gamma_2+i\Delta_2\Big)|2\rangle\langle 2|(\rho_q)_I(t)|2\rangle\langle 2| - \Big(\gamma_2-i\Delta_2\Big)|2\rangle\langle 2|(\rho_q)_I(t) + h.c.
\end{aligned}
$$

# 32. Paso 15 - Substitución en la ecuación maestra - parte 2

si $N_0 = N(\omega_q - \delta_2, T)$

$$
\begin{aligned}
= \quad & \left[\frac{\gamma_0}{2} N_0 + i\Delta_1\right]\sigma_+ (\rho_q)_I(t)\sigma_- + \left[\frac{\gamma_0}{2} N_0 - i\Delta_1\right]\sigma_+ (\rho_q)_I(t)\sigma_- \\
& - \left[\frac{\gamma_0}{2} N_0 - i\Delta_1\right]\sigma_-\sigma_+ (\rho_q)_I(t) - \left[\frac{\gamma_0}{2} N_0 + i\Delta_1\right](\rho_q)_I(t)\sigma_-\sigma_+ \\
& + \left[\frac{\gamma_0}{2}\Big(N_0 + 1\Big) + i\Delta_0\right]\sigma_-(\rho_q)_I(t)\sigma_+ + \Big(\gamma_2 + i\Delta_2\Big)|2\rangle\langle 2|(\rho_q)_I(t)|2\rangle\langle 2| \\
& + \left[\frac{\gamma_0}{2}\Big(N_0 + 1\Big) - i\Delta_0\right]\sigma_-(\rho_q)_I(t)\sigma_+ + \Big(\gamma_2 - i\Delta_2\Big)|2\rangle\langle 2|(\rho_q)_I(t)|2\rangle\langle 2| \\
& - \left[\frac{\gamma_0}{2}\Big(N_0 + 1\Big) - i\Delta_0\right]\sigma_+\sigma_-(\rho_q)_I(t) - \Big(\gamma_2 - i\Delta_2\Big)|2\rangle\langle 2|(\rho_q)_I(t) \\
& - \left[\frac{\gamma_0}{2}\Big(N_0 + 1\Big) + i\Delta_0\right](\rho_q)_I(t)\sigma_+\sigma_- - \Big(\gamma_2 + i\Delta_2\Big)(\rho_q)_I(t)|2\rangle\langle 2|
\end{aligned}
$$

# 33. Paso 15 - Substitución en la ecuación maestra - parte 3

si $N_0 = N(\omega_q - \delta_2, T)$

$$
\begin{aligned}
= \quad & \gamma_0 N_0 \sigma_+ (\rho_q)_I(t) \sigma_- - \frac{\gamma_0}{2} N_0 \Big\{ \sigma_- \sigma_+ ,\ (\rho_q)_I(t) \Big\} \\
& + i\Delta_1 \Big[ \sigma_- \sigma_+ ,\ (\rho_q)_I(t) \Big] + i\Delta_2 \Big[ |2\rangle\langle 2| ,\ (\rho_q)_I(t) \Big] \\
& + 2\gamma_2 |2\rangle\langle 2| (\rho_q)_I(t) |2\rangle\langle 2| - \gamma_2 \Big\{ |2\rangle\langle 2| ,\ (\rho_q)_I(t) \Big\} \\
& + i\Delta_0 \Big[ \sigma_+ \sigma_- ,\ (\rho_q)_I(t) \Big] \\
& + \gamma_0 \Big( N_0 + 1 \Big) \sigma_- (\rho_q)_I(t) \sigma_+ - \frac{\gamma_0}{2} \Big( N_0 + 1 \Big) \Big\{ \sigma_+ \sigma_- ,\ (\rho_q)_I(t) \Big\} \\
= \quad & \gamma_0 N_0 \left( \sigma_+ (\rho_q)_I(t) \sigma_- - \frac{1}{2} \Big\{ \sigma_- \sigma_+ ,\ (\rho_q)_I(t) \Big\} \right) \\
& + i\Delta_1 \Big[ |1\rangle\langle 1| ,\ (\rho_q)_I(t) \Big] + i(\Delta_0 + \Delta_2) \Big[ |2\rangle\langle 2| ,\ (\rho_q)_I(t) \Big] \\
& + 2\gamma_2 \left( |2\rangle\langle 2| (\rho_q)_I(t) |2\rangle\langle 2| - \frac{1}{2} \Big\{ |2\rangle\langle 2| ,\ (\rho_q)_I(t) \Big\} \right) \\
& + \gamma_0 \Big( N_0 + 1 \Big) \left( \sigma_- (\rho_q)_I(t) \sigma_+ - \frac{1}{2} \Big\{ \sigma_+ \sigma_- ,\ (\rho_q)_I(t) \Big\} \right)
\end{aligned}
$$

# 34. Paso 15 - Substitución en la ecuación maestra - parte 3

si $N_0 = N(\omega_q - \delta_2, T)$

$$
\begin{aligned}
&= \mathcal{D}\Big[(\rho_q)_I(t)\Big] + i\Delta_1\Big[|1\rangle\langle 1|,\ (\rho_q)_I(t)\Big] + i(\Delta_0 + \Delta_2)\Big[|2\rangle\langle 2|,\ (\rho_q)_I(t)\Big] \\
&= \mathcal{D}\Big[(\rho_q)_I(t)\Big] + i\frac{\Delta_1}{2}\Big[\mathbb{I} - \sigma_z,\ (\rho_q)_I(t)\Big] + i\frac{(\Delta_0 + \Delta_2)}{2}\Big[\mathbb{I} + \sigma_z,\ (\rho_q)_I(t)\Big] \\
&= \mathcal{D}\Big[(\rho_q)_I(t)\Big] + \frac{i}{\hbar}\left[\frac{\hbar(\Delta_0 - \Delta_1 + \Delta_2)}{2}\sigma_z,\ (\rho_q)_I(t)\right]
\end{aligned}
$$

donde el disipador está dado por

$$
\begin{aligned}
\mathcal{D}(\rho) \quad = \quad & \gamma_0 N_0\left(\sigma_+\rho\sigma_- - \frac{1}{2}\Big\{\sigma_-\sigma_+,\ \rho\Big\}\right) \\
& +\gamma_0\Big(N_0 + 1\Big)\left(\sigma_-\rho\sigma_+ - \frac{1}{2}\Big\{\sigma_+\sigma_-,\ \rho\Big\}\right) \\
& +2\gamma_2\left(|2\rangle\langle 2|\rho|2\rangle\langle 2| - \frac{1}{2}\Big\{|2\rangle\langle 2|,\ \rho\Big\}\right)
\end{aligned}
$$

# 35. Paso 16 - Regreso al esquema de Schrödinger - parte 3

Recordamos que

- Pasamos al IP definido por

$$U_I(t) = exp\Big[-\frac{i}{\hbar}(H_q + H_B)t\Big]$$

donde

$$H_q = \frac{\hbar\omega_q}{2}\sigma_z - \hbar\delta_2|2\rangle\langle 2| = \frac{\hbar(\omega_q - \delta_2)}{2}\sigma_z - \frac{\hbar\delta_2}{2}\mathbb{I}$$

- Usando la fórmula Baker-Campbell-Hausdorff

$$\begin{aligned}(\sigma_x)_I(t) &= e^{-i(\omega_q-\delta_2)t}\sigma_- + e^{i(\omega_q-\delta_2)t}\sigma_+ \\ (|2\rangle\langle 2|)_I(t) &= |2\rangle\langle 2|\end{aligned}$$

# 36. Ecuación maestra de Born-Markov-RWA en la forma de Lindblad

$$\frac{d}{dt}\rho_q(t) \quad = \quad -\frac{i}{\hbar}\left[\frac{\hbar\omega_q'}{2}\sigma_z,\ \rho_q(t)\right] + \mathcal{D}\Big[\rho_q(t)\Big]$$

donde

$$\begin{aligned}
\omega_q' &= \omega_q - \Big(\delta_2 + \Delta_0 - \Delta_1 + \Delta_2\Big) \ , \\
N_0 &= N(\omega_q - \delta_2, T) \ , \\
\mathcal{D}(\rho) &= \gamma_0 N_0 \left(\sigma_+\rho\sigma_- - \frac{1}{2}\Big\{\sigma_-\sigma_+,\ \rho\Big\}\right) \\
&\quad + \gamma_0\Big(N_0 + 1\Big)\left(\sigma_-\rho\sigma_+ - \frac{1}{2}\Big\{\sigma_+\sigma_-,\ \rho\Big\}\right) \\
&\quad + 2\gamma_2\left(|2\rangle\langle 2|\rho|2\rangle\langle 2| - \frac{1}{2}\Big\{|2\rangle\langle 2|,\ \rho\Big\}\right)
\end{aligned}$$

# 37. Ecuaciones para los elementos de matriz

$$\begin{aligned}
\frac{d}{dt}\rho_{22}(t) &= -\gamma_0(N_0+1)\rho_{22}(t) + \gamma_0 N_0 \rho_{11}(t) \\
\frac{d}{dt}\rho_{11}(t) &= \gamma_0(N_0+1)\rho_{22}(t) - \gamma_0 N_0 \rho_{11}(t) \\
\frac{d}{dt}\rho_{21}(t) &= -\left[\gamma_0\left(N_0+\frac{1}{2}\right) + \gamma_2 + i\omega_q'\right]\rho_{21}(t) \\
\frac{d}{dt}\rho_{12}(t) &= -\left[\gamma_0\left(N_0+\frac{1}{2}\right) + \gamma_2 - i\omega_q'\right]\rho_{12}(t)
\end{aligned}$$

donde

$$\rho_{jk}(t) = \langle j|\rho_q(t)|k\rangle$$

Nota: para que sea válida la rwa se necesita

$$\gamma_0(N_0+1),\ \gamma_0(N_0+1/2)+\gamma_2 \ll \omega_q'$$

# 38. Solución para los elementos de matriz

$$\begin{aligned}
\rho_{22}(t) &= \frac{N_0}{2N_0+1} + e^{-\gamma_0(2N_0+1)t}\left[\rho_{22}(0) - \frac{N_0}{2N_0+1}\right] \\
\rho_{11}(t) &= \frac{N_0+1}{2N_0+1} - e^{-\gamma_0(2N_0+1)t}\left[\rho_{22}(0) - \frac{N_0}{2N_0+1}\right] \\
\rho_{21}(t) &= \rho_{21}(0)exp\left\{-\left[\gamma_0\left(N_0+\frac{1}{2}\right) + \gamma_2 + i\omega_q'\right]t\right\} \\
\rho_{12}(t) &= \rho_{21}(t)^*
\end{aligned}$$

Si $t \to +\infty$, entonces el qubit se termaliza

$$\rho_q(t) \quad \to \quad \frac{1}{Z}\text{exp}\left[-\beta\frac{\hbar(\omega_q-\delta_2)}{2}\sigma_z\right]$$

con

$$\beta = \frac{1}{k_B T}\ , \quad Z = 2\text{cosh}\left[\beta\frac{\hbar}{2}\left(\omega_s-\delta_2\right)\right]$$

# 39. Evolución del vector de Bloch

El vector de Bloch es

$$
\begin{aligned}
\mathbf{r}_B(t) &= \Big( \langle\sigma_x\rangle(t)\ ,\ \langle\sigma_z\rangle(t)\ , \langle\sigma_z\rangle(t) \Big) \\
&= 2|\rho_{12}(0)|e^{-[\gamma_0(N_0+1/2)+\gamma_2]t}\Big(\cos(\omega_q' t+\phi_0),\ \text{sen}(\omega_q' t+\phi_0),\ 0\Big) \\
&\quad +2\left[\rho_{22}(0)-\frac{N_0}{2N_0+1}\right]e^{-\gamma_0(2N_0+1)t}(0,0,1) \\
&\quad +\frac{1}{2N_0+1}(0,0,-1)
\end{aligned}
$$

donde

$$\phi_0 = \arg\left[\rho_{12}(0)\right]$$

y

$$\rho_q(t) = \frac{1}{2}\left[\mathbb{I}_2 + \mathbf{r}_B(t)\cdot\bar{\sigma}\right]$$

# 40. Evolución del vector de Bloch - Gráfica 1

Qubit en el estado excitado

$$\rho_q(0) = |2\rangle\langle 2| \ , \quad \mathbf{r}_B(0) = (0,0,1) \ , \quad [\rho_q(0)] = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Grado de mezcla $S_L(t) = 1 - \mathrm{Tr}\left[\rho_q^2(t)\right]$

Tomamos

$$\frac{\gamma_0}{\omega_q'} = \frac{1}{20}, \ \gamma_2 = 0, \ N_0 = 0 \ (rojo), \ 0.5(azul), \ 1 \ (verde)$$

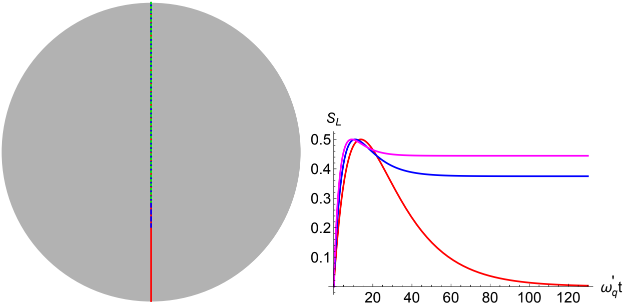


Figure: Evolución del vector de Bloch y grado de mezcla

# 41. Evolución del vector de Bloch - Gráfica 2

Qubit en eigenestado de $\sigma_x$

$$\rho_q(0) = |\psi\rangle\langle\psi| \ , \quad |\psi\rangle = \frac{1}{\sqrt{2}} \left(|2\rangle + |1\rangle\right) \ , \quad \mathbf{r}_B(0) = (1,0,0) \ ,$$

$$[\rho_q(0)] = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Tomamos

$$\frac{\gamma_0}{\omega'_q} = \frac{1}{20}, \ \gamma_2 = 0, \ N_0 = 0 \ (rojo), \ 0.5(azul), \ 1 \ (verde)$$

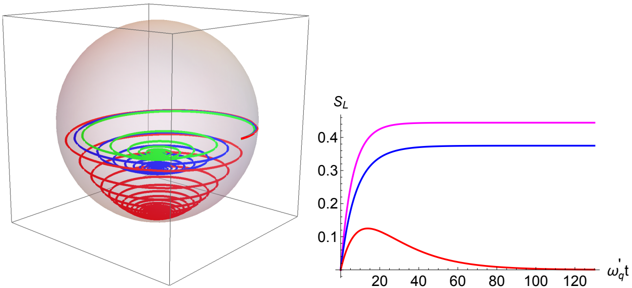


Figure: Evolución del vector de Bloch y grado de mezcla

# 41. Decoherencia del qubit

- Pasamos al IP

$$(\rho_q)_I(t) = e^{\frac{i}{\hbar}H_q t}\rho_q(t)e^{-\frac{i}{\hbar}H_q t} \qquad \text{con} \quad H_q = \frac{\hbar\omega'_q}{2}\sigma_z$$

- En el IP se tiene

$$\langle 2|(\rho_q)_I(t)|2\rangle = \langle 2|\rho_q(t)|2\rangle \ , \quad \langle 1|(\rho_q)_I(t)|1\rangle = \langle 1|\rho_q(t)|1\rangle \ ,$$

$$\langle 2\ |(\rho_q)_I(t)|1\rangle = e^{i\omega'_q t}\langle 2|\rho(t)|1\rangle = \rho_{21}(0)e^{-[\gamma_0(N_0+1/2)+\gamma_2]t}$$

- La función de decoherencia es

$$\Gamma(t) = -\left[\gamma_0\left(N_0 + \frac{1}{2}\right) + \gamma_2\right]t = \Gamma_{vac}(t) + \Gamma_{th}(t)$$

con

$$\Gamma_{vac}(t) = -\left(\frac{\gamma_0}{2} + \gamma_2\right)t, \qquad \Gamma_{th}(t) = -\frac{\gamma_0}{2}N_0 t$$

- El tiempo de decoherencia es

$$\tau_c = \frac{1}{\gamma_0\left(N_0 + \frac{1}{2}\right) + \gamma_2}$$