Ghost suppression in gluon mass dynamics

A. C. Aguilar, D. Binosi, C. T. F., and J. Papavassiliou,
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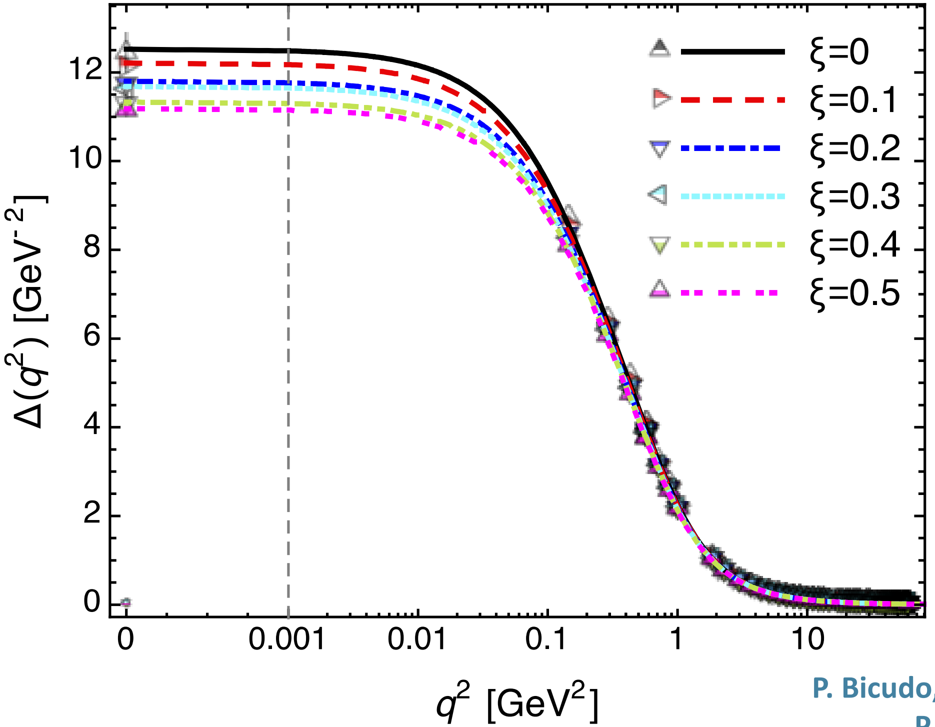
State University of Campinas - Brazil

Sevilla, November 9th, 2018 - NPQCD2018 Workshop

Lattice simulations

- Lattice simulations reveal that the gluon propagator saturates in the deep IR



P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira, and P. J. Silva, Phys. Rev. D92, 114514 (2015).

- Saturation can be explained by **dynamical gluon mass generation**

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2) \Rightarrow \Delta^{-1}(0) = m^2(0)$$

Schwinger Mechanism

J. S. Schwinger, Phys. Rev.125, 397 (1962);
Phys.Rev.128, 2425 (1962).

- Propagator in the Landau gauge:

$$\Delta_{\mu\nu} = -i\left[P_{\mu\nu}(q)\Delta(q^2)\right] \qquad P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

- Self-energy: $\Pi_{\mu\nu}(q) = P_{\mu\nu}(q) q^2 \mathbf{\Pi}(q^2)$

$$\Rightarrow \quad \Delta^{-1}(q^2) = q^2[1 + \mathbf{\Pi}(q^2)]$$

- If the vaccum polarization has a pole in $q^2 = 0$ with positive residue m^2,

$$\Delta^{-1}(q^2) = q^2 + m^2 \quad \Rightarrow \quad \Delta^{-1}(0) = m^2$$

- Dynamical gluon mass generation requires the existence of vertices containing poles of nonpertubative origin.

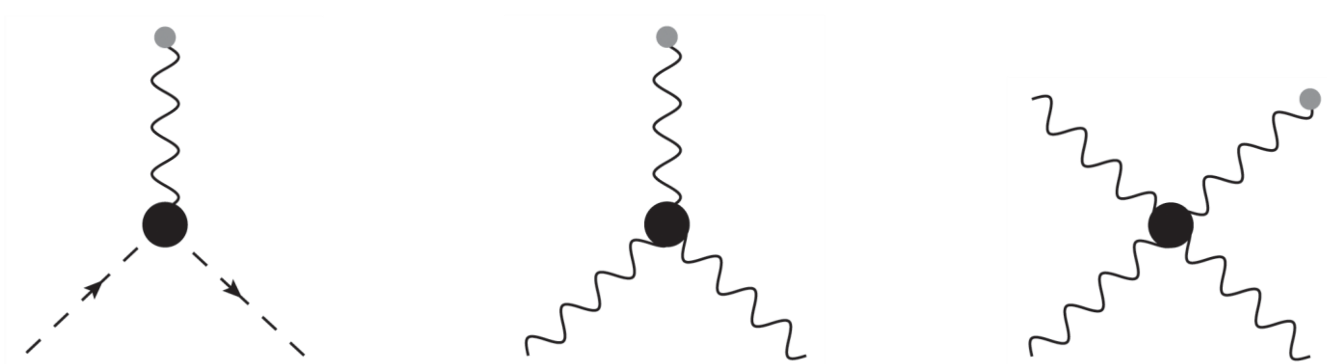
PT-BFM: New Green's Functions

- Three propagators can appear:

$$\widetilde{\Delta}(q) = [\mathbf{1} + \mathbf{G(q)}]\Delta(q)$$

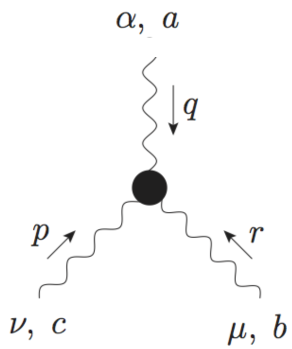
$$\widehat{\Delta}(q) = [\mathbf{1} + \mathbf{G(q)}]^{\mathbf{2}}\Delta(q)$$

- New vertices



Slavnov-Taylor Identities (STI)

- Convencional three-gluon vertex



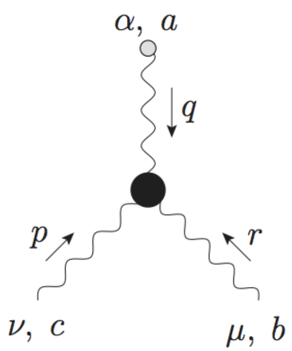
$F(q^2)$: ghost dressing function

$D(q^2) = F(q^2)/q^2$: ghost propagator

$H_{\mu\nu}(q,r,p)$: ghost-gluon scattering kernel

$$q^\alpha \Gamma_{\alpha\mu\nu}(q,r,p) = iF(q)\left[\Delta^{-1}_{\sigma\nu}(r)H^\sigma_\mu(q,r,p) - \Delta^{-1}_{\sigma\mu}(p)H^\sigma_\nu(q,r,p)\right]$$

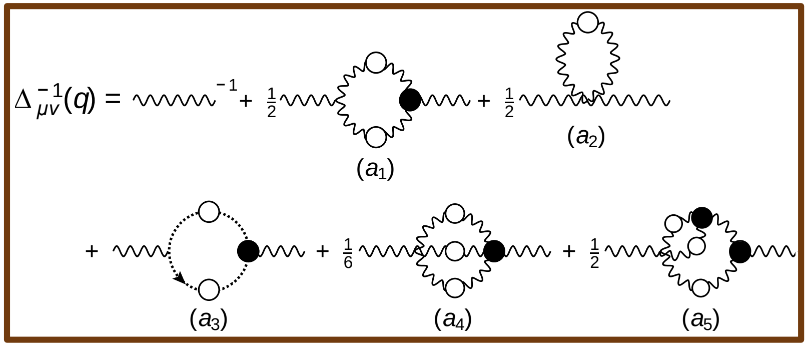
- BQQ vertex:

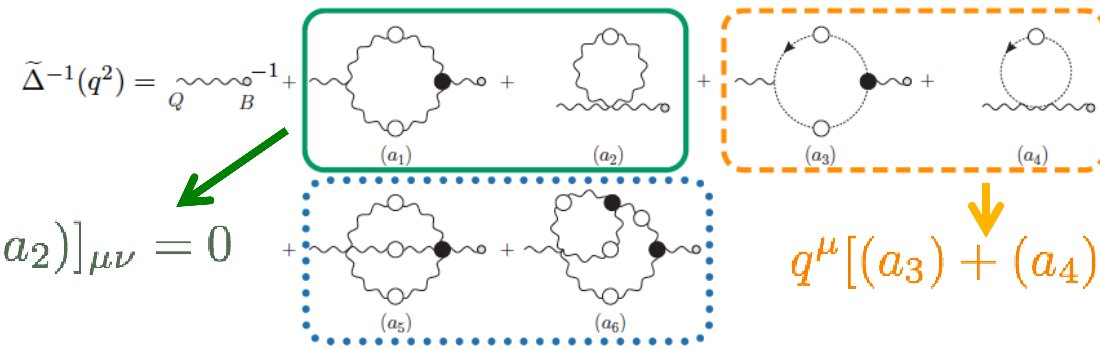


$$q^\alpha \widetilde{\Gamma}_{\alpha\mu\nu}(q,r,p) = i\Delta^{-1}_{\mu\nu}(r) - i\Delta^{-1}_{\mu\nu}(p)$$

Abelian-like Identity (Ward-Takahashi)

Gluon SDE in the PT–BFM framework





$$q^{\mu}[(a_1) + (a_2)]_{\mu\nu} = 0$$

$$q^{\mu}[(a_3) + (a_4)]_{\mu\nu} = 0$$

$$q^{\mu}[(a_5) + (a_6)]_{\mu\nu} = 0$$

$$\Delta^{-1}(q^2) P_{\mu\nu}(q) = \frac{q^2 P_{\mu\nu}(q) + i \sum_{i=1}^{6} (a_i)_{\mu\nu}}{1 + G(q^2)}$$

A. C. Aguilar and J. Papavassiliou, JHEP 0612, 012 (2006)

D. Binosi and J. Papavassiliou, Phys.Rev. D77, 061702 (2008); JHEP 0811:063 (2008).

From Takahashi to Ward identities

- Takahashi Identities

QED : $q^{\mu}\Gamma_{\mu}(q,r,p) = S^{-1}(r) - S^{-1}(p)$

Yang-Mills BFM: $q^{\mu}\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$

- Taylor expansion around $q=0$, in the absence of $1/q^2$ poles in the vertex:

$$q^{\mu}\Gamma_{\mu}(q,r,p) = q^{\mu}\Gamma_{\mu}(0,r,-r) + \mathcal{O}(q^2) = q^{\mu}\left\{\frac{\partial}{\partial q^{\mu}}\mathcal{D}^{-1}(q+r)\right\}_{q=0} + \mathcal{O}(q^2)$$

QED : $\Gamma_{\mu}(0,r,-r) = \partial S^{-1}(r)/\partial r^{\mu}$

Yang-Mills BFM: $\widetilde{\Gamma}_{\mu\alpha\beta}(0,r,-r) = -i\dfrac{\partial\Delta^{-1}_{\alpha\beta}(r)}{\partial r^{\mu}}$

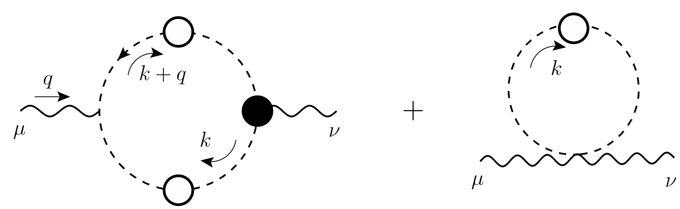
Seagull Identity

- In dimensional regularization, for any function that satisfies

$$\int_k f(k^2) = \frac{1}{(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \int_0^\infty dy y^{\frac{d}{2}-1} f(y) = \text{finite}, \quad \text{for } 0 < d < d^*$$

$$\Rightarrow \quad \int_k k^2 \frac{\partial f(k^2)}{\partial k^2} + \frac{d}{2}\int_k f(k^2) = 0$$

- Example: Scalar QED $\Pi_{\mu\nu}(q) =$



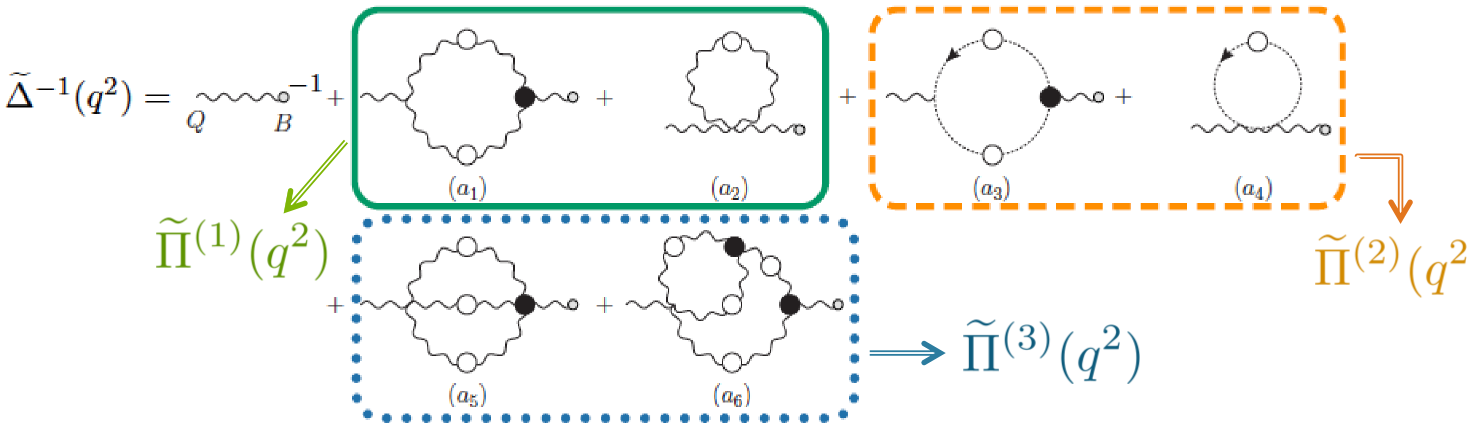
$$\int_k \frac{k^2}{(k^2+m^2)^2} = (4\pi)^{\frac{d}{2}} \left(\frac{d}{2}\right) \Gamma\left(1-\frac{d}{2}\right)(m^2)^{\frac{d}{2}-1}$$

$$\int_k \frac{1}{k^2+m^2} = (4\pi)^{\frac{d}{2}} \Gamma\left(1-\frac{d}{2}\right)(m^2)^{\frac{d}{2}-1}$$

$$\Pi(0) \sim -\int_k \frac{k^2}{(k^2+m^2)^2} + \frac{d}{2}\int_k \frac{1}{k^2+m^2} = 0 \quad \Rightarrow \quad f(k^2) = \frac{1}{k^2+m^2}$$

A. C. Aguilar, D. Binosi, C. T. F. and J. Papavassiliou, Phys. Rev. D94, no. 4, 045002 (2016).

Gluon propagator at the origin



$$\widetilde{\Delta}^{-1}(q^2) = q^2 + i\left[\widetilde{\Pi}^{(1)}(q^2) + \widetilde{\Pi}^{(2)}(q^2) + \widetilde{\Pi}^{(3)}(q^2)\right]$$

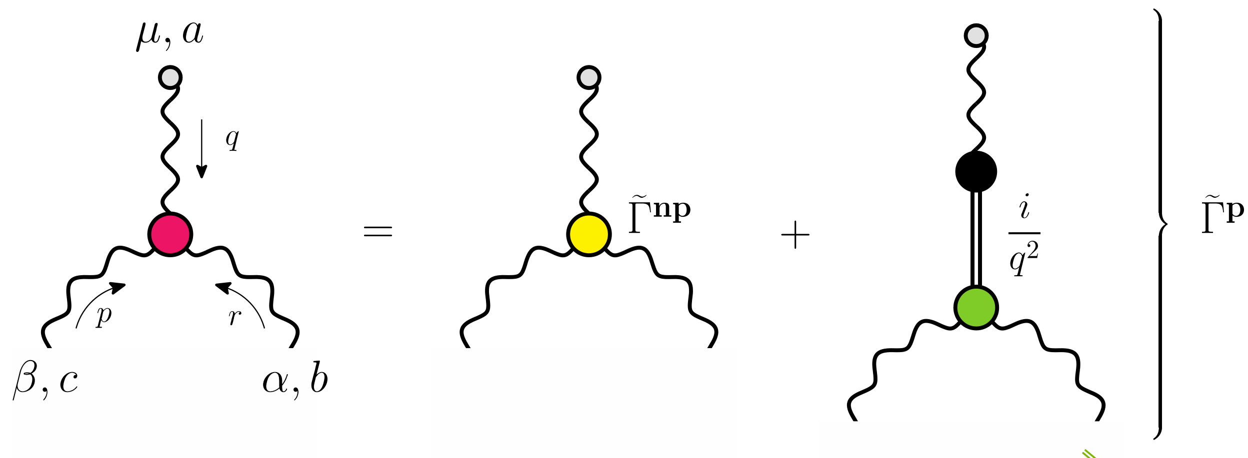
Ward Identities (no poles) + Seagull Identity → $\widetilde{\Delta}^{-1}(0) = 0$

A. C. Aguilar, D. Binosi, C. T. F. and J. Papavassiliou, Phys. Rev. D94, no. 4, 045002 (2016).

Vertices with massless poles

- We can add the possibility of poles in the B leg of the vértices (which will be responsible for the implementation of the Schwinger mechanism)

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p)$$

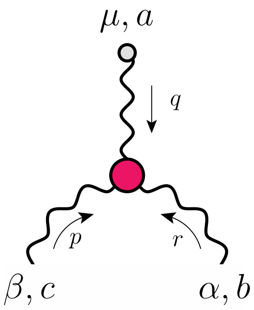


$$\widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p) = \frac{q_\mu}{q^2}\widetilde{C}_{\alpha\beta}(q,r,p)$$

Generates a BSE

- The poles correspond to massless bound-state (colored) excitations.

Ward identities in the presence of poles



$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \frac{q_\mu}{q^2}\widetilde{C}_{\alpha\beta}(q,r,p)$$

$$q^\mu \widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

$$q^\mu \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{C}_{\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p),$$

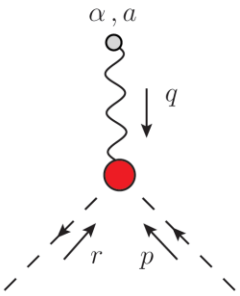
$$q^\mu \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) + \widetilde{C}_{\alpha\beta}(0,r,-r) + q^\mu \left\{ \frac{\partial}{\partial q^\mu}\widetilde{C}_{\alpha\beta}(q,r,p) \right\}_{q=0} = -iq^\mu \frac{\partial \Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu}$$

$$\widetilde{C}_{\alpha\beta}(0,r,-r) = 0$$

$$\widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial \Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu} - \left\{ \frac{\partial}{\partial q^\mu}\widetilde{C}_{\alpha\beta}(q,r,p) \right\}_{q=0}$$

$$\widetilde{C}_{\alpha\beta}(q,r,p) = \widetilde{C}_{gl}(q,r,p) g_{\alpha\beta} + \cdots$$

Ward identities in the presence of poles



$$\widetilde{\Gamma}_\mu(q,r,p) = \widetilde{\Gamma}^{\rm np}_\mu(q,r,p) + \frac{q_\mu}{q^2}\widetilde{C}_{{\rm gh}(q,r,p)}$$

$$q^\mu \widetilde{\Gamma}_\mu(q,r,p) = iD^{-1}(r^2) - iD^{-1}(p^2)$$

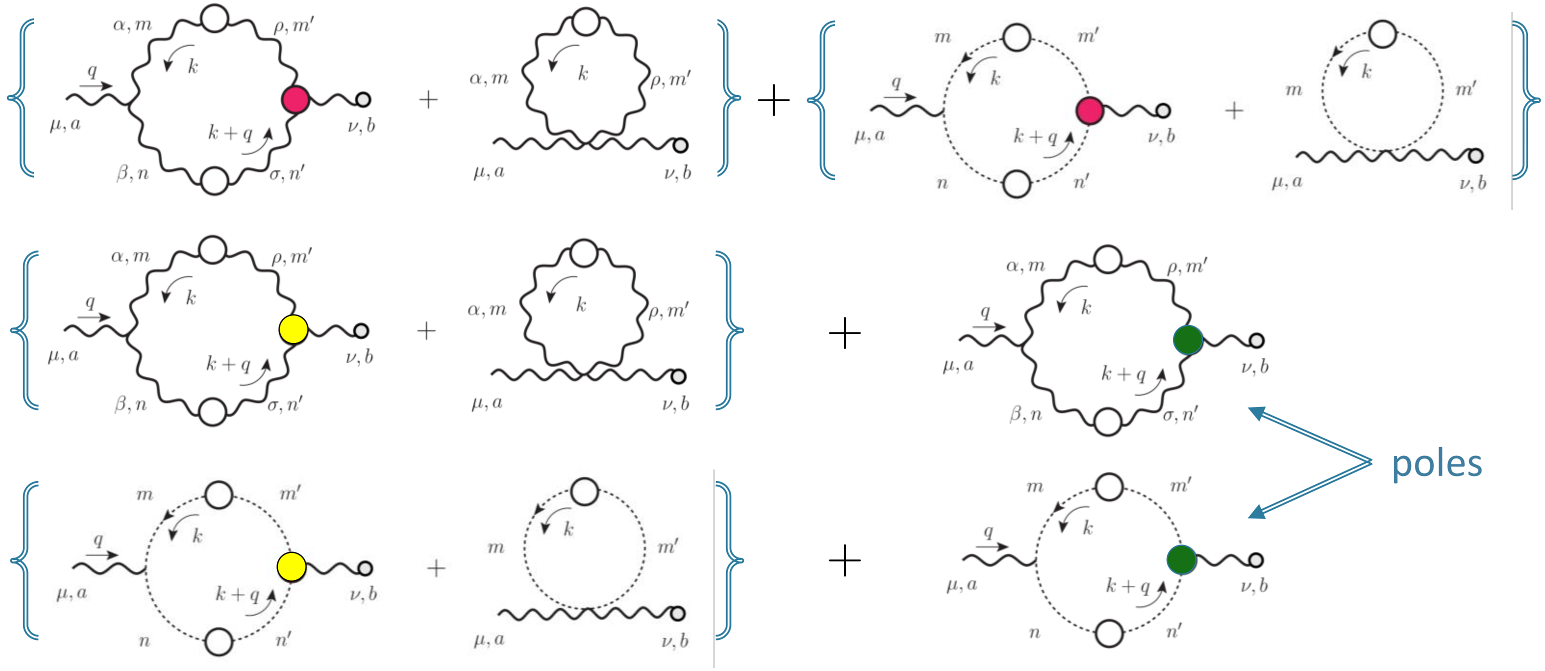
$$q^\mu \widetilde{\Gamma}^{\rm np}_\mu(q,r,p) + \widetilde{C}_{\rm gh}(q,r,p) = iD^{-1}(r^2) - iD^{-1}(p^2)$$

$$\widetilde{C}_{\rm gh}(0,r,-r) = 0$$

$$\widetilde{\Gamma}^{\rm np}_\mu(0,r,-r) = -i\frac{\partial}{\partial r^\mu}D^{-1}(r^2) - \left\{\frac{\partial}{\partial q^\mu}\widetilde{C}_{\rm gh}(q,r,-r-q)\right\}_{q=0}$$

Evading the seagull cancelation

$$\widetilde{\Delta}^{-1}(0) = \lim_{q\to 0}$$



Triggers seagull identity as before

Vanishes identically

$$\widetilde{C}'_i(k^2) = \lim_{q\to 0}\left\{\frac{\partial \widetilde{C}_i(q,k,-k-q)}{\partial (k+q)^2}\right\}$$

$$\Delta^{-1}(0) = \frac{3}{2}g^2 C_A F(0)\left\{\int_k k^2\Delta^2(k^2)\left[1-\frac{3}{2}g^2C_AY(k^2)\right]\widetilde{C}'_{gl}(k^2) - \frac{1}{3}\int_k k^2 D^2(k^2)\widetilde{C}'_{gh}(k^2)\right\}$$

Relation with the Gluon Mass Function

- The infrared saturation of the gluon propagator suggests the physical parametrization

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

- Using the modified STI:

$$q^\mu \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{C}_{\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p),$$

$$\Downarrow$$

$$\widetilde{C}_{\alpha\beta}(q,r,p) = m^2(p^2)P_{\alpha\beta}(p) - m^2(r^2)P_{\alpha\beta}(r)$$

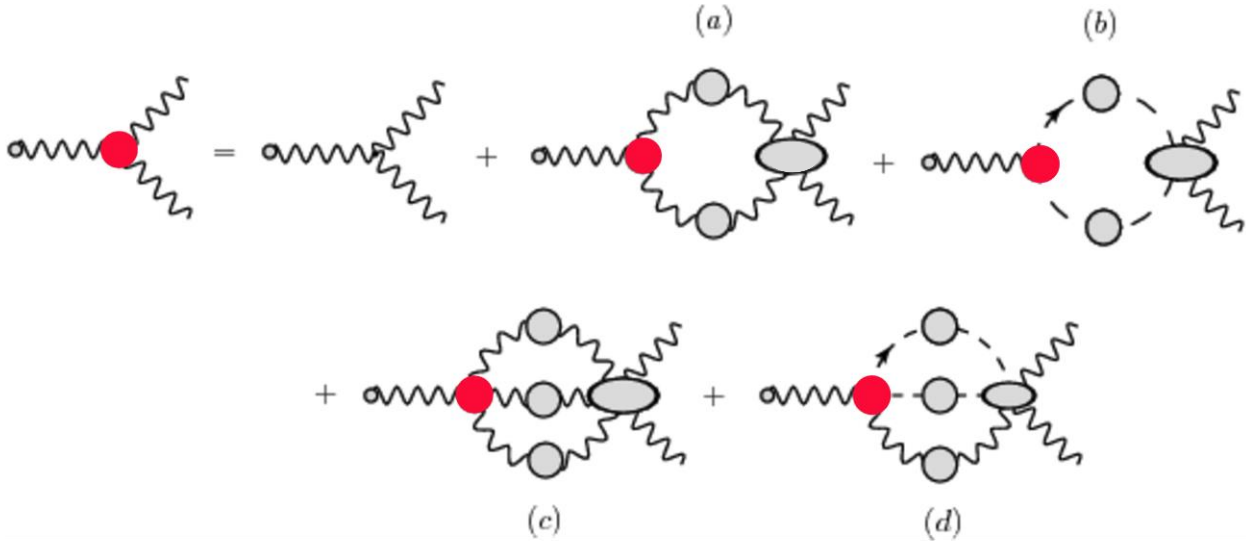
- Focusing on $g_{\alpha\beta}$

$$\widetilde{C}_{\rm gl}(q,r,p) = m^2(r^2) - m^2(p^2) \quad \xrightarrow{q \to 0} \quad \widetilde{C}'_{\rm gl}(r^2) = \frac{\mathrm{d}m^2(r^2)}{\mathrm{d}r^2}$$

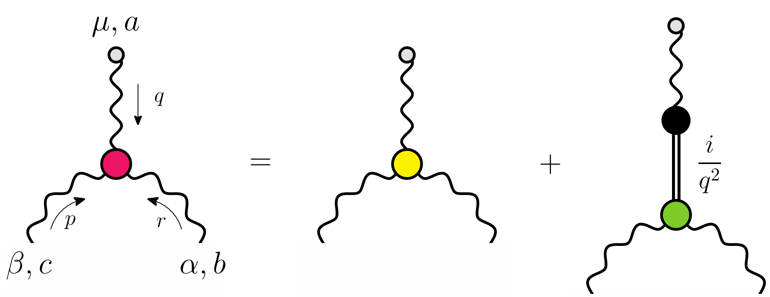
$$m^2(q^2) = \Delta^{-1}(0) + \int_0^{q^2} \mathrm{d}y\, \widetilde{C}'_{\rm gl}(y)$$

Dynamical formation of massless poles

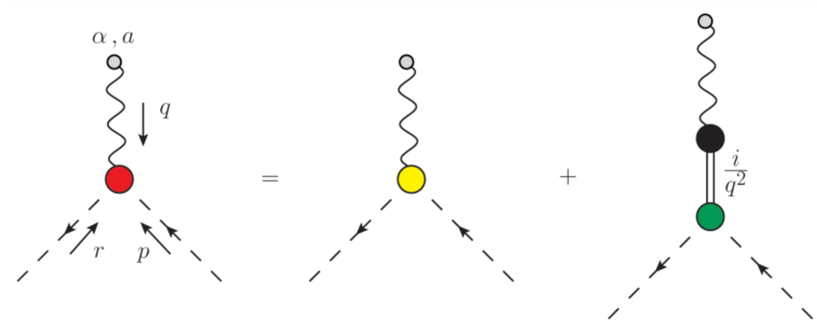
- Bethe-Salpeter equation for the three-gluon vertex:



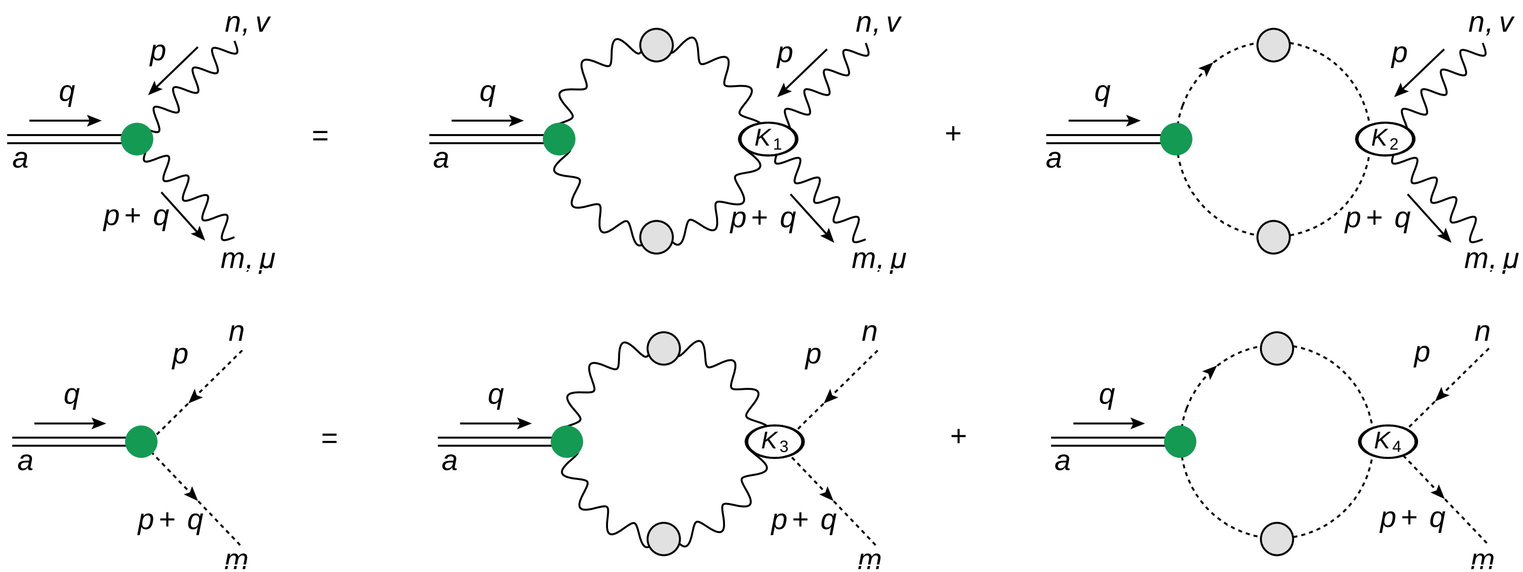
- In diagrams (a) and (b), we make the replacements:



and



Coupled system

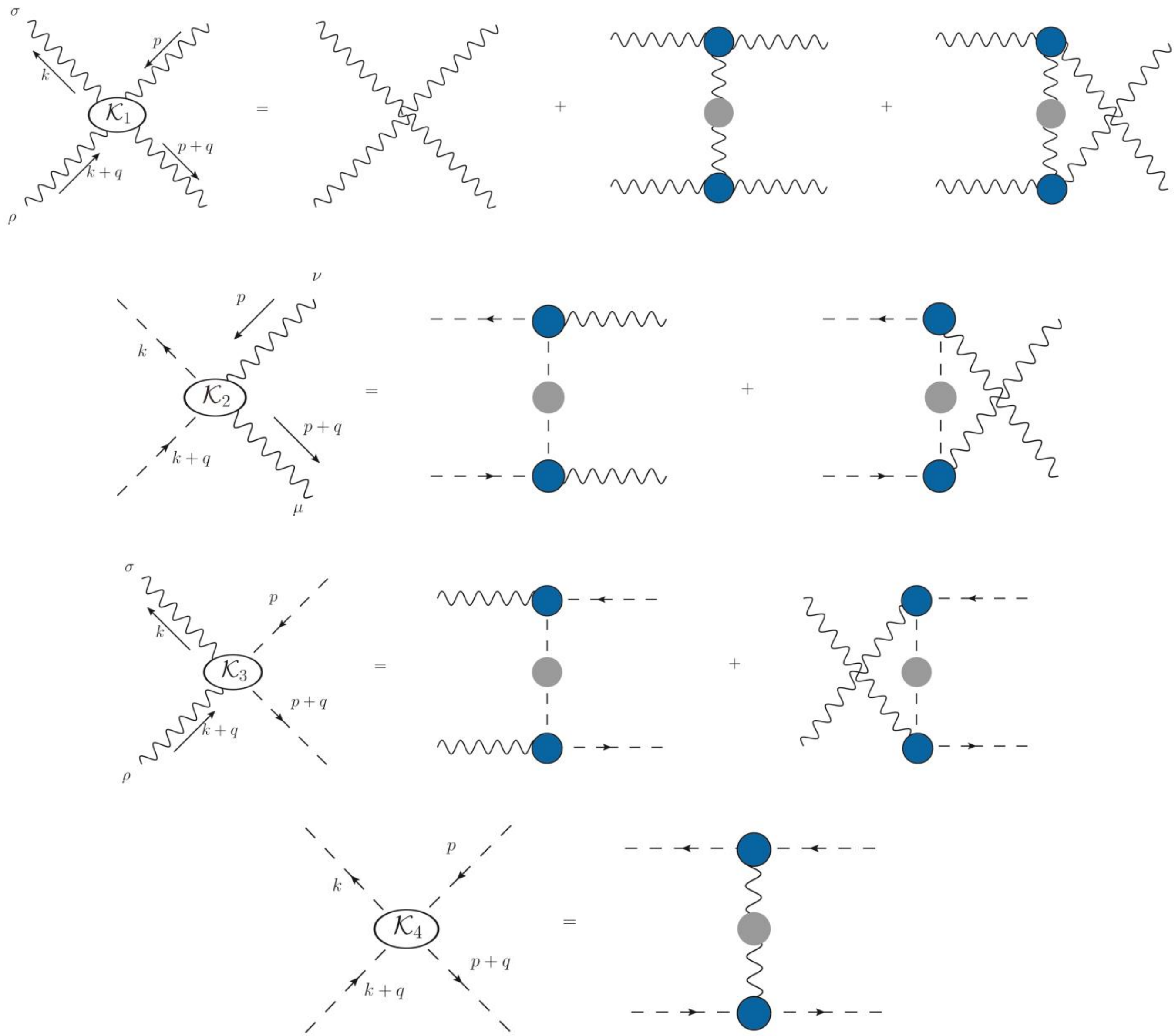


$$C'_{gl} g_{\mu\nu} = \int_k C'_{gl} g_{\gamma\delta} \Delta^{\gamma\rho}(k) \Delta^{\delta\sigma}(k+q) \mathcal{K}_{1\rho\mu\nu\sigma} + \int_k C'_{gh} D(k) D(k+q) \mathcal{K}_{2\mu\nu}$$

$$C'_{gh} = \int_k C'_{gl} g_{\gamma\delta} \Delta^{\gamma\rho}(k) \Delta^{\delta\sigma}(k+q) \mathcal{K}_{3\rho\sigma} + \int_k C'_{gh} D(k) D(k+q) \mathcal{K}_4$$

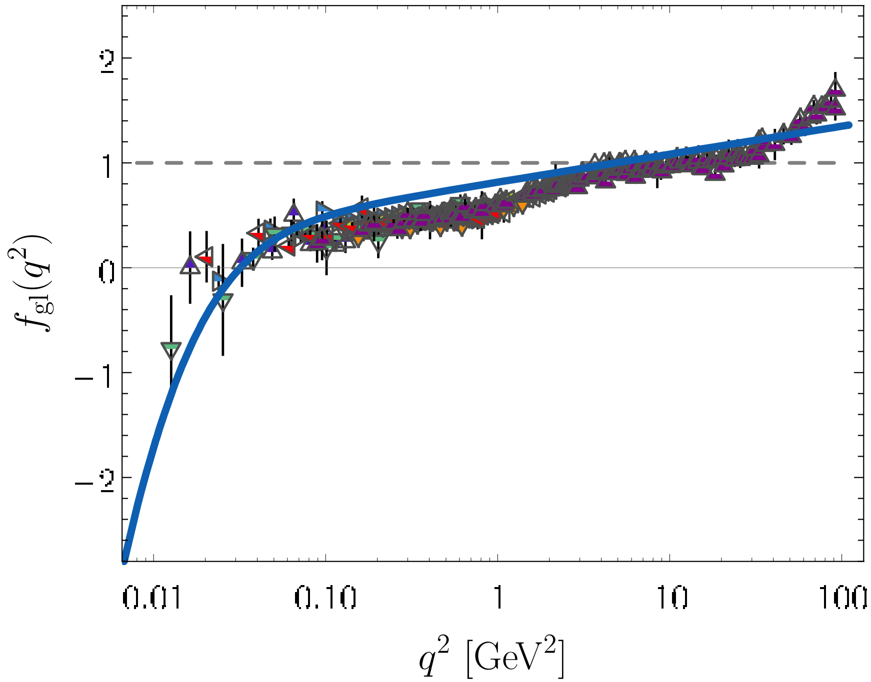
A. C. Aguilar, D. Binosi, C. T. F., and J. Papavassiliou, Eur. Phys. J. C 78, no. 3, 181 (2018) .

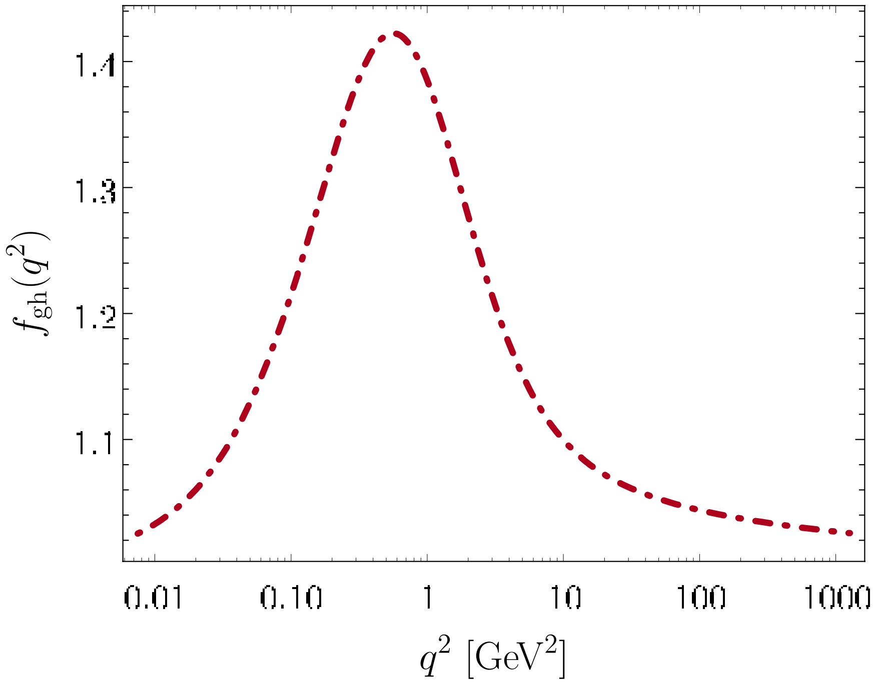
Approximation for the kernels



Form Factors

$$\Gamma_{\mu\alpha\beta}(q,r,p) = f_{gl}(r)\Gamma^{(0)}_{\mu\alpha\beta}(q,r,p)$$
$$\Gamma_{\mu}(q,r,p) = f_{gh}(r)\Gamma^{(0)}_{\mu}(q,r,p)$$

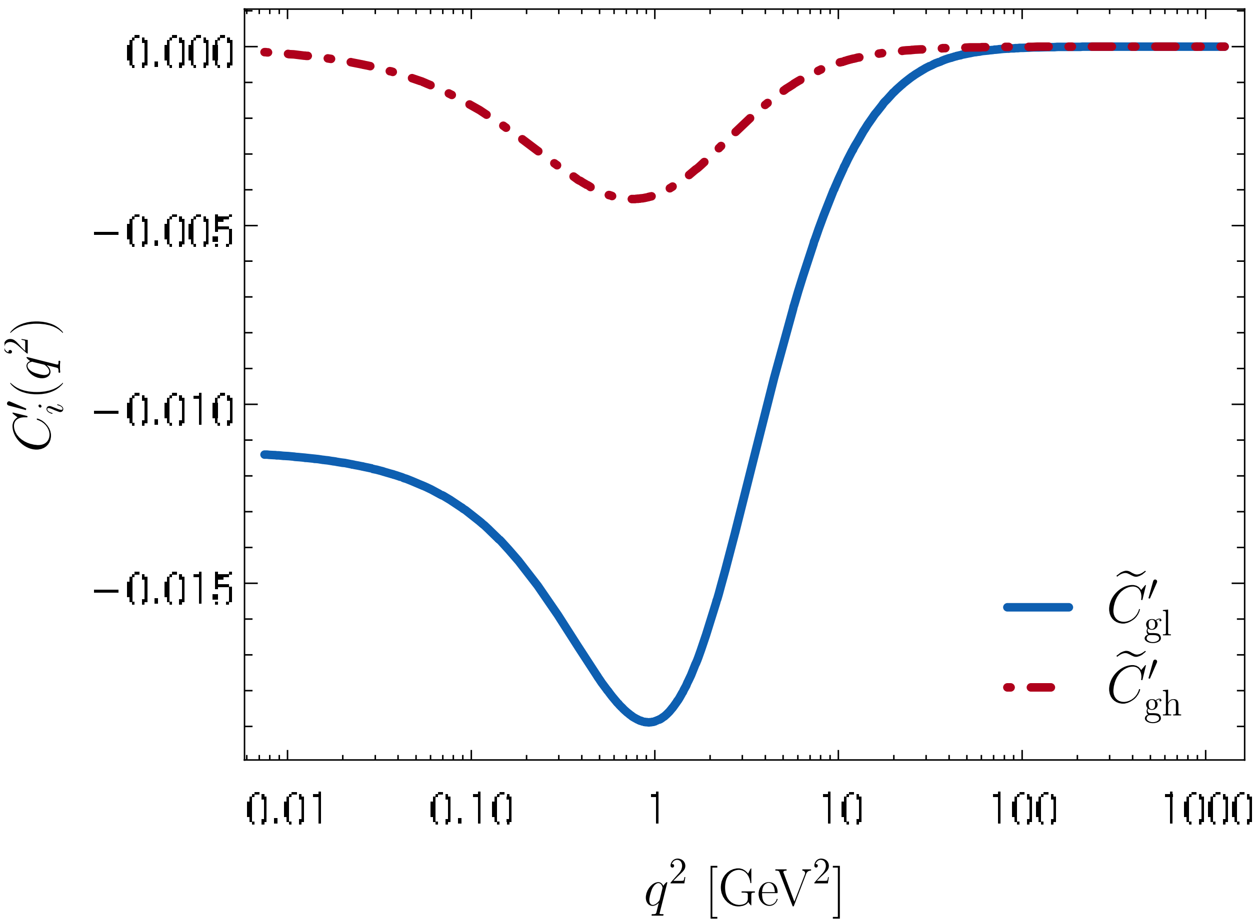




$q^2 = p^2 = r^2; \quad q\cdot p = q\cdot r = p\cdot r = -q^2/2$

A. Athenodorou, et al, Phys. Lett. B761, 444 (2016).
P. Boucaud, et al, Phys. Rev. D95, 114503 (2017).
D. Binosi and J. Papavassiliou, Phys. Rev. D97, 054029 (2018)

Numerical Solution

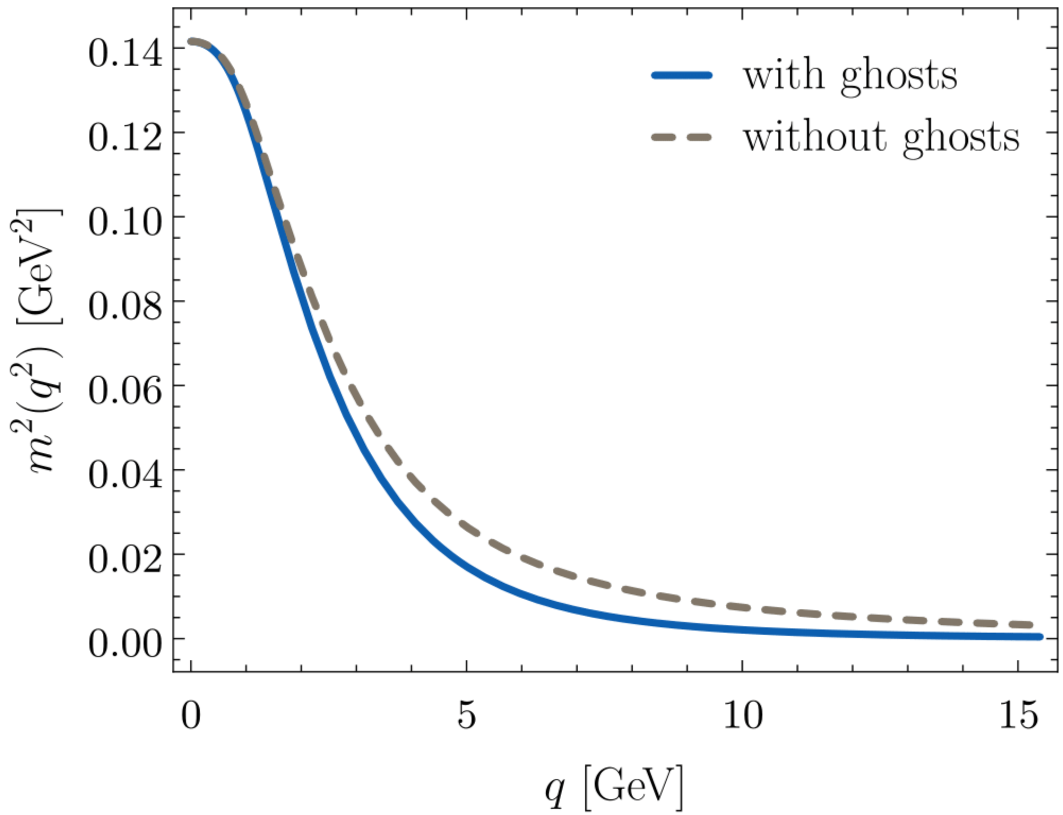


$\alpha_s = 0.43$

A. C. Aguilar, D. Binosi, C. T. F., and J. Papavassiliou, Eur. Phys. J. C78, no. 3, 181 (2018) .

Gluon Mass Function

- The ghost sector affects slightly the running of the gluon mass.



$\alpha_s = 0.43$

$\alpha_s = 0.45$

$$m^2(q^2) = m^2(0)/[1 + (q^2/m_1^2)^{1+p}]$$

$m_1 = 0.36\,\text{GeV}\,, \quad p = 0.1$

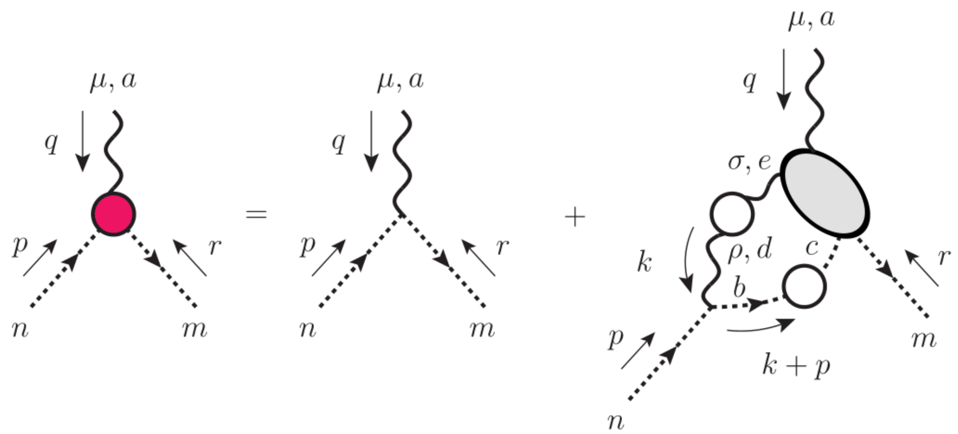
$m_1 = 0.37\,\text{GeV}\,, \quad p = 0.24$

A. C. Aguilar, D. Binosi, C. T. F., and J. Papavassiliou, Eur. Phys. J. C78, no. 3, 181 (2018) .

Ghost-gluon vertex

Taylor's Theorem

- SDE for the ghost-gluon vertex:



$$g f^{amn}\Gamma_\mu(q,r,p) = -g f^{amn} r_\mu + g f^{dbn}\int_k (k+p)_\rho \Delta^{\rho\sigma}(k) D(k+p) \mathcal{Q}^{damb}_{\sigma\mu}(-k,q,r,k+p)$$

- In the Landau gauge, $(k+p)_\rho \Delta^{\rho\sigma}(k) = p_\rho \Delta^{\rho\sigma}(k)$

- When $p \to 0$ (Taylor's limit)

$$\Gamma_\mu(q,-q,0) = q_\mu$$

Taylor's Theorem

- Tensorial decomposition for the conventional ghost-gluon vertex:

$$\Gamma_\mu(q,r,p) = A(q,r,p)q_\mu + B(q,r,p)p_\mu$$

- In the limit $p \to 0$, $\Gamma_\mu(q,-q,0) = A(q,-q,0)q_\mu$

$$\Rightarrow \quad A(q,-q,0) = 1$$

- If instead one expresses the vertex in terms of q and r,

$$\Gamma_\mu(q,r,p) = A_1(q,r,p)q_\mu - B_1(q,r,p)r_\mu$$

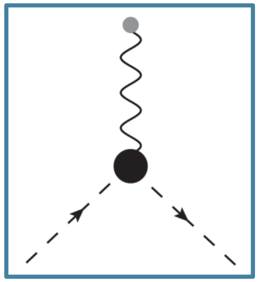
$$\Downarrow$$

$$A(q,r,p) = A_1(q,r,p) + B_1(q,r,p)$$

$$\Downarrow$$

$$A_1(q,-q,0) + B_1(q,-q,0) = 1$$

Taylor's Theorem in PT-BFM



- STI for $\widetilde{\Gamma}_\mu(q,r,p)$

$$q^\mu \widetilde{\Gamma}_\mu(q,r,p) = iD^{-1}(r^2) - iD^{-1}(p^2)$$

$$\Downarrow \; p \to 0$$

$$q^\mu \widetilde{\Gamma}_\mu(q,-q,0) = q^2 F^{-1}(q^2)$$

- Tensorial decomposition

$$\widetilde{\Gamma}_\mu(q,r,p) = \widetilde{A}(q,r,p)q_\mu + \widetilde{B}(q,r,p)p_\mu$$

$$\Downarrow \; p \to 0$$

$$\widetilde{\Gamma}_\mu(q,-q,0) = \widetilde{A}(q,-q,0)q_\mu$$

- Finally,

$$\widetilde{A}(q,-q,0) = F^{-1}(q^2)$$

A. C. Aguilar, D. Binosi, C. T. F., and J. Papavassiliou, Eur. Phys. J. C78, no. 3, 181 (2018) .

Ghost-gluon Vertex

$$\widetilde{\Gamma}_\mu(q,r,p) \quad = \widetilde{A}^{\rm np}(q,r,p)q_\mu + \widetilde{B}^{\rm np}(q,r,p)p_\mu + \frac{q_\mu}{q^2}\widetilde{C}_{\rm gh}(q,r,p)$$

$$\Rightarrow \quad \mathcal{R}(q,r,p) := i\,\frac{D^{-1}(r^2) - D^{-1}(p^2)}{r^2 - p^2} = \frac{r^2F^{-1}(r^2) - p^2F^{-1}(p^2)}{r^2 - p^2}$$

$$\widetilde{A}^{\rm np}(q,r,p) = \mathcal{R}(q,r,p) + \boxed{f_A(q,r,p)}$$

$$\widetilde{B}^{\rm np}(q,r,p) = 2\,\mathcal{R}(q,r,p) + \boxed{f_B(q,r,p)}$$

→ purely nonpertubative

- Taylor's theorem:

$$\widetilde{C}_{\rm gh}(q,-q,0) + q^2\widetilde{A}^{\rm np}(q,-q,0) = q^2F^{-1}(q^2)$$

$$\mathcal{R}(q,-q,0) = F^{-1}(q^2) \quad \Rightarrow \quad \widetilde{C}_{\rm gh}(q,-q,0) = -q^2 f_A(q,-q,0)$$

Special Case

$$f_A(q,r,p) = f(q,r,p) = \frac{1}{2} f_B(q,r,p)$$

⇓

$$\widetilde{C}_{\rm gh}(q,r,p) = -(r^2 - p^2) f(q,r,p)$$

- Ansatz: $\widetilde{C}_{\rm gh}(q,r,p) = r^2 h(r^2) - p^2 h(p^2)$

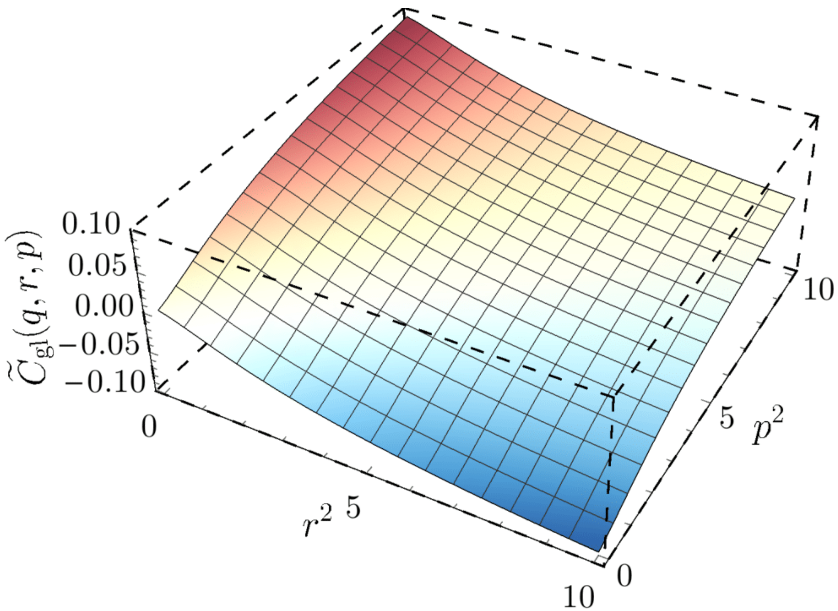
⇒ $$\widetilde{C}'_{\rm gh}(r^2) = [r^2 h(r^2)]'$$

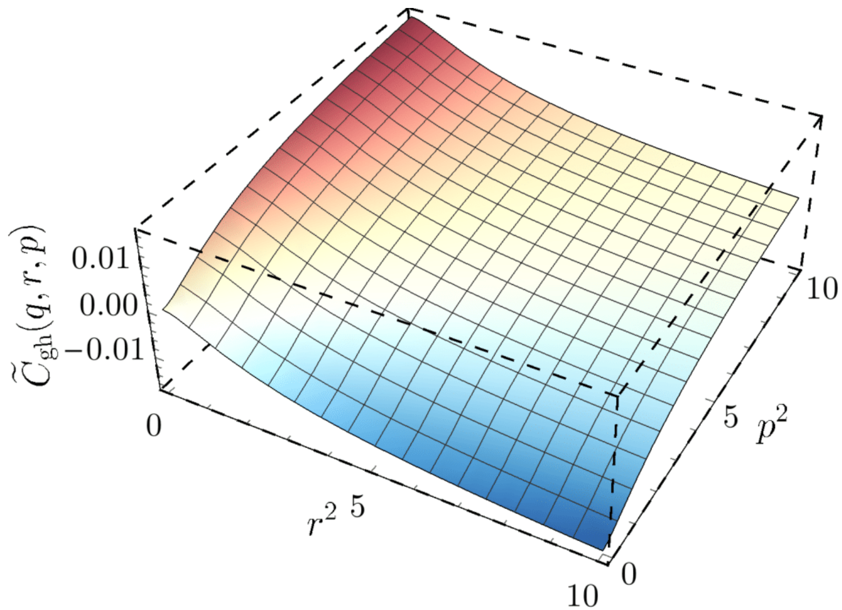
$$\widetilde{C}'_i(k^2) = \lim_{q \to 0} \left\{ \frac{\partial \widetilde{C}_i(q,k,-k-q)}{\partial (k+q)^2} \right\}$$

$$r^2 h(r^2) = c + \int_0^{r^2} {\rm d}y \, \widetilde{C}'_{\rm gh}(y)$$ → c = 0 (Taylor limit)

$$\widetilde{C}_{\rm gh}(q,r,p) = \int_0^{r^2} {\rm d}y \, \widetilde{C}'_{\rm gh}(y) - \int_0^{p^2} {\rm d}y \, \widetilde{C}'_{\rm gh}(y)$$

Special Case





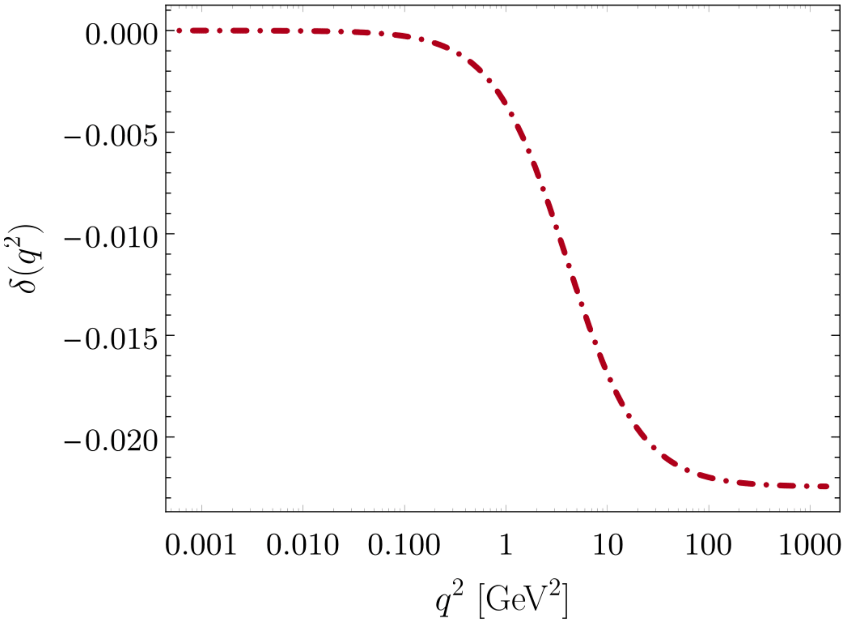
$$\widetilde{C}_{\rm gl}(q,r,p) = m^2(r^2) - m^2(p^2)$$

$$\widetilde{C}_{\rm gh}(q,r,p) = r^2 h(r^2) - p^2 h(p^2)$$

Special Case

$$F_{\text{eff}}^{-1}(q^2) := F^{-1}(q^2)[1 + \underbrace{h(q^2)F(q^2)}_{\delta(q^2)}]$$

$$\delta(q^2) = D(q^2) \int_0^{q^2} dy\, \widetilde{C}'_{\text{gh}}(y)$$



Conclusions

- By solving the system of BSEs which describes the dynamics of massless poles formation in the three-gluon and ghost-gluon vertices, we found non-trivial solutions for both $\widetilde{C}'_{\rm gl}$ and $\widetilde{C}'_{\rm gh}$. This fact indicates that the dynamics of QCD is indeed sufficiently strong to generate such poles.

- The contribution associated to the ghost-gluon vertex pole is suppressed in comparison to the one coming from the three-gluon vertex. Specifically, when all quantities entering into the kernels of the BSE system have been renormalized (MOM) at μ = 4.3 GeV, the relative size between the two is approximately $\widetilde{C}'_{\rm gh}/\widetilde{C}'_{\rm gl} \simeq 1/5$.

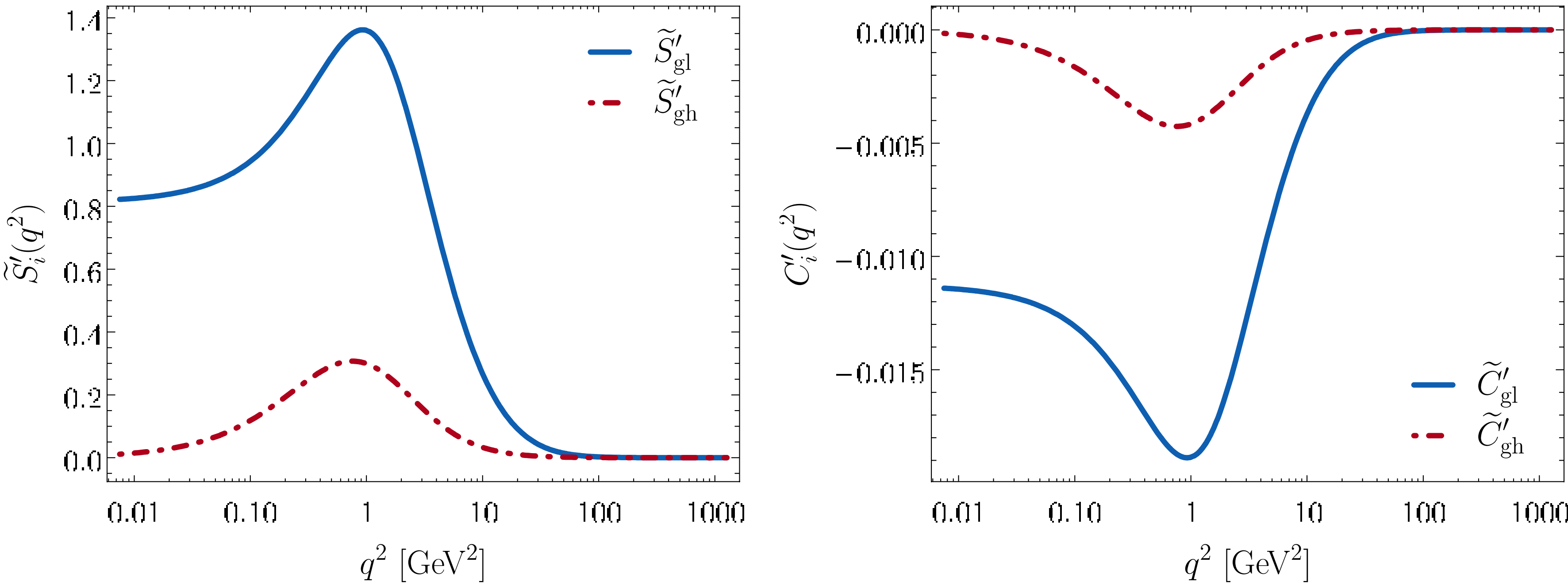
- The main effect of the presence of ghosts is a small but discernible difference in the running of the dynamical gluon mass.

Obrigada!

Backup Slides

Numerical Solution

$$C_i' = -|c|S_i', \quad |c| = \frac{\Delta^{-1}(0)}{\int_0^\infty \mathrm{d}y\, S_{gl}'(y)} \quad \Rightarrow \quad m^2(q^2) = -\int_{q^2}^\infty \mathrm{d}y\, C_{gl}'(y)$$



$\alpha_s = 0.43$

A. C. Aguilar, D. Binosi, C. T. F., and J. Papavassiliou, (2017), arXiv:1712.06926v1 [hep-ph]

Ghost-gluon Vertex

- Contracting $q^{\mu}\widetilde{\Gamma}_{\mu}$

$$q^2 f_A(q,r,p) + (p\cdot q) f_B(q,r,p) + \widetilde{C}_{\rm gh}(q,r,p) = 0$$

$$\Downarrow \; q \to 0$$

$$2\widetilde{C}'_{\rm gh}(r^2) = f_B(0,r,-r) \longrightarrow \boxed{\text{Linear terms}}$$

- Connects explicitly $\widetilde{C}'_{\rm gh}(r^2)$ with the function that quantifies the necessary deviation of $\widetilde{B}^{\rm np}(q,r,p)$ from the expression that would saturate the STI identically.

BFM: Conceitos Básicos

- No esquema de quantização BFM, divide-se o campo de gauge:

$$A_\mu^a \to \widetilde{A}_\mu^a + A_\mu^a$$

$\widetilde{A}_\mu^a \to$ campo de background; $A_\mu^a \to$ campo quântico

- No gerador funcional integra-se somente sobre A_μ^a
- A condição de fixação de gauge *G*(*A*) é escolhida como $G(A) = D^\mu A_\mu^a$ onde $D_\mu = \partial_\mu - i\widetilde{A}_\mu^a t^a$
- A Lagrangiana, então, é invariante sob as transformações:

$$\widetilde{A}_\mu^a \to \widetilde{A}_\mu^a + D_\mu \theta^a \qquad A_\mu^a \to A_\mu^a - f^{abc} A_\mu^a \theta^b$$

- $\widetilde{A}_\mu^a$ carrega a transformação de gauge local, $\partial^\mu \theta^a$
- A_μ^a se transforma como campo de matéria na representação adjunta.

B. S. DeWitt, Phys. Rev. 162 (1967) 195—1239
G. ’t Hooft, In *Karpacz 1975, Proceedings, Acta Universitatis Wratislaviensis No.368, 1976, 345-369
L. F. Abbott, Nucl. Phys. B185 (1981) 189.

Derivação Identidade de Seagull

- Considere a classe de funções vetoriais

$$\mathcal{F}_\mu(k) = f(k^2)k_\mu \, .$$

$$\mathcal{F}_\mu(-k) = -\mathcal{F}_\mu(k) \quad \Rightarrow \quad \int_k \mathcal{F}_\mu(k) = 0$$

- Assumamos que $f(k^2)$ se anula rapidamente quando $k^2 \to \infty$, de forma que a integral (em coordenadas esféricas)

$$\int_k f(k^2) = \frac{1}{(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \int_0^\infty dy y^{\frac{d}{2}-1} f(y)$$

convirja para valores positivos de *d* abaixo de certo valor *d**.

- Invariância translacional: $\int_k \mathcal{F}_\mu(k+q) = 0$

- Expansão de Taylor:

$$\begin{aligned}\mathcal{F}_\mu(q+k) &= \mathcal{F}_\mu(k) + q^\nu \left\{ \frac{\partial}{\partial q^\nu} \mathcal{F}_\mu(q+k) \right\}_{q=0} + \mathcal{O}(q^2) \\ &= \mathcal{F}_\mu(k) + q^\nu \frac{\partial \mathcal{F}_\mu(k)}{\partial k^\nu} + \mathcal{O}(q^2)\end{aligned}$$

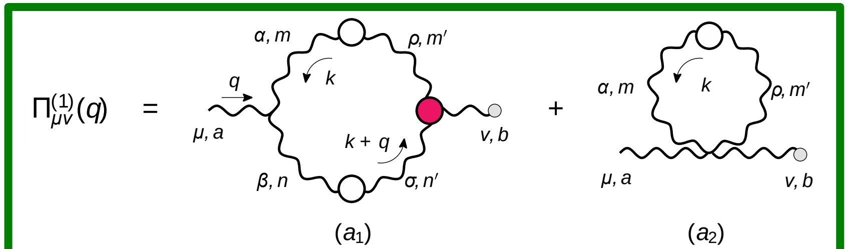
$$\int_k \frac{\partial \mathcal{F}_\mu(k)}{\partial k^\mu} = 0$$

A. C. Aguilar, D. Binosi, C. T. F. and J. Papavassiliou, arXiv:1604.08456 [hep-ph].

Autoenergia do gluon na origem

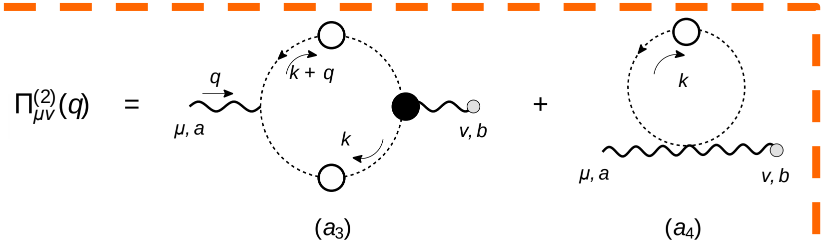
- Identidade de Seagull: $\int_k \frac{\partial \mathcal{F}_\mu(k)}{\partial k^\mu} = 0$, com $\mathcal{F}_\mu(k) = f(k^2)k_\mu$.

$$\int_k k^2 \frac{\partial \Delta(k^2)}{\partial k^2} + \frac{d}{2}\int_k \Delta(k) = 0$$



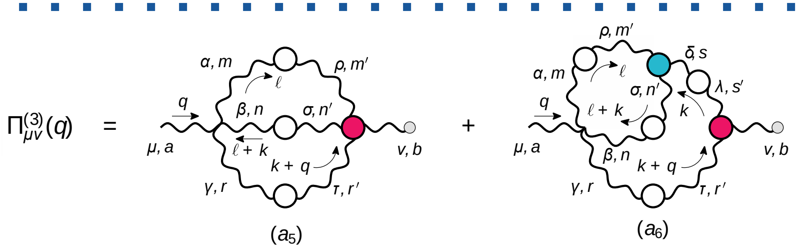
$$d\,\widetilde{\Pi}^{(1)}(0) = -g^2 C_A(d-1)\int_k \frac{\partial \mathcal{F}^{(1)}_\mu(k)}{\partial k^\mu} = 0$$

$$\mathcal{F}^{(1)}_\mu(k) = \Delta(k^2)k_\mu$$



$$d\,\widetilde{\Pi}^{(2)}(0) = g^2 C_A \int_k \frac{\partial \mathcal{F}^{(2)}_\mu(k)}{\partial k^\mu} = 0;$$

$$\mathcal{F}^{(2)}_\mu(k) = D(k^2)k_\mu$$



$$d\,\widetilde{\Pi}^{(3)}(0) = -i(d-1)g^4 C_A^2 \int_k \frac{\partial \mathcal{F}^{(3)}_\mu(k)}{\partial k^\mu};$$

$$\mathcal{F}^{(3)}_\mu(k) = \Delta(k^2)Y(k^2)k_\mu$$

$$Y^{\alpha\beta}_\delta(k) = \int_\ell \Delta^{\alpha\rho}(\ell)\Delta^{\beta\sigma}(k+\ell)\Gamma_{\sigma\rho\delta}(-k-\ell,\ell,k)$$

A. C. Aguilar, D. Binosi, C. T. F. and J. Papavassiliou, arXiv:1604.08456 [hep-ph].

Autoenergia do gluon na origem

- Somando as contribuições:

$$\widetilde{\Delta}^{-1}(q^2) = q^2 + i\left[\widetilde{\Pi}^{(1)}(q^2) + \widetilde{\Pi}^{(2)}(q^2) + \widetilde{\Pi}^{(3)}(q^2)\right] \Rightarrow \widetilde{\Delta}^{-1}(0) = 0$$

- Se a função $1 + G(0)$ é finita para todo ξ, temos, na ausência de polos nos vértices,

$$\Delta^{-1}(0) = \frac{\widetilde{\Delta}^{-1}(0)}{1 + G(0)} = 0$$

- No gauge de Landau, $F^{-1}(0) = 1 + G(0)$.

$$F^{-1}(q^2) = 1 + G(q^2) + L(q^2) + \xi K(q^2)$$

Fatores de forma do vértice de 3 gluons

- Vértice BQQ: $$\widetilde{\Gamma}^{\mu\alpha\beta}(q,r,p) = \sum_{i=1}^{14} A_i(q^2,r^2,q\cdot r)\, b_i^{\mu\alpha\beta}$$

$$b_1^{\mu\alpha\beta} = q^\mu g^{\alpha\beta}; \quad b_2^{\mu\alpha\beta} = q^\mu q^\alpha q^\beta; \quad b_3^{\mu\alpha\beta} = q^\mu q^\alpha r^\beta; \quad b_4^{\mu\alpha\beta} = q^\mu r^\alpha q^\beta; \quad b_5^{\mu\alpha\beta} = q^\mu r^\alpha r^\beta,$$
$$b_6^{\mu\alpha\beta} = r^\mu g^{\alpha\beta}; \quad b_7^{\mu\alpha\beta} = r^\mu q^\alpha q^\beta; \quad b_8^{\mu\alpha\beta} = r^\mu q^\alpha r^\beta; \quad b_9^{\mu\alpha\beta} = r^\mu r^\alpha q^\beta; \quad b_{10}^{\mu\alpha\beta} = r^\mu r^\alpha r^\beta,$$
$$b_{11}^{\mu\alpha\beta} = q^\alpha g^{\beta\mu}; \quad b_{12}^{\mu\alpha\beta} = q^\beta g^{\alpha\mu}; \quad b_{13}^{\mu\alpha\beta} = r^\alpha g^{\beta\mu}; \quad b_{14}^{\mu\alpha\beta} = r^\beta g^{\alpha\mu}.$$

- Em $q = ,0$

$$\widetilde{\Gamma}^{\mu\alpha\beta}(0,r,-r) = A_6(r^2) r^\mu g^{\alpha\beta} + A_{10}(r^2) r^\mu r^\alpha r^\beta + A_{13} r^\alpha g^{\beta\mu} + A_{14} r^\beta g^{\alpha\mu}$$

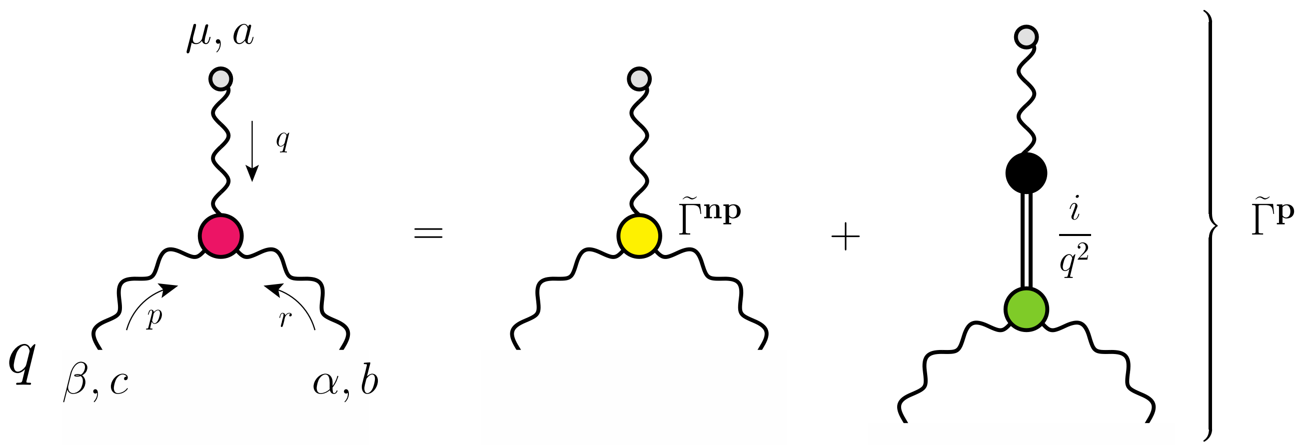
$$\widetilde{\Gamma}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial \Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu}$$

$$A_6(r^2) = 2\frac{\partial \Delta^{-1}(r^2)}{\partial r^2}, \quad A_{10}(r^2) = -2\frac{\partial}{\partial r^2}\left(\frac{\Delta^{-1}(r^2)}{r^2}\right), \quad A_{13} = A_{14} = \frac{1}{\xi} - \frac{\Delta^{-1}(r^2)}{r^2}$$

Vértice com polos não massivos

- Adicionando a possibilidade de polos ao vértice:

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p)$$



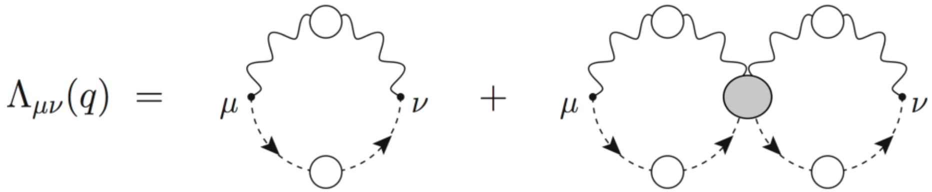
$$\widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p) = \frac{q_\mu}{q^2}\widetilde{C}_{\alpha\beta}(q,r,p)$$

Gera uma BSE

$$A_i = A_i^{\mathbf{np}} + A_i^{\mathbf{p}}, \quad i = 1, ..., 5;$$

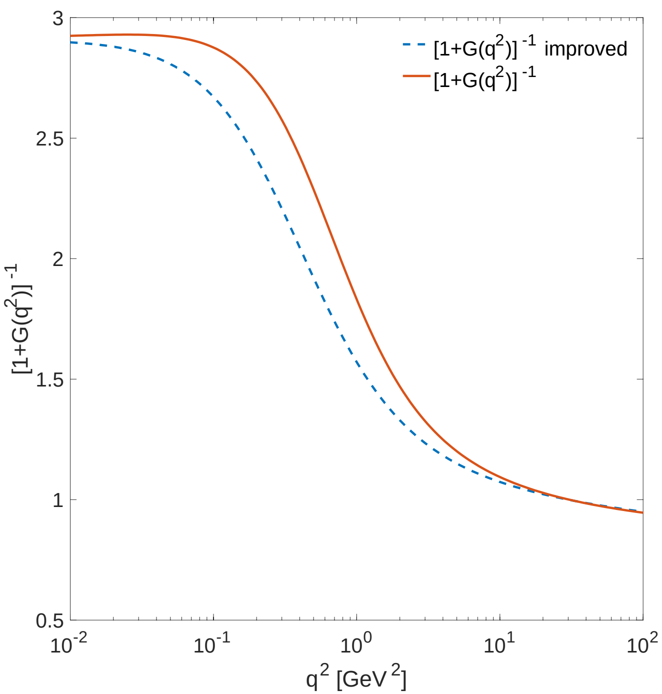
$$A_i = A_i^{\mathbf{np}}, \quad i = 6, ..., 14$$

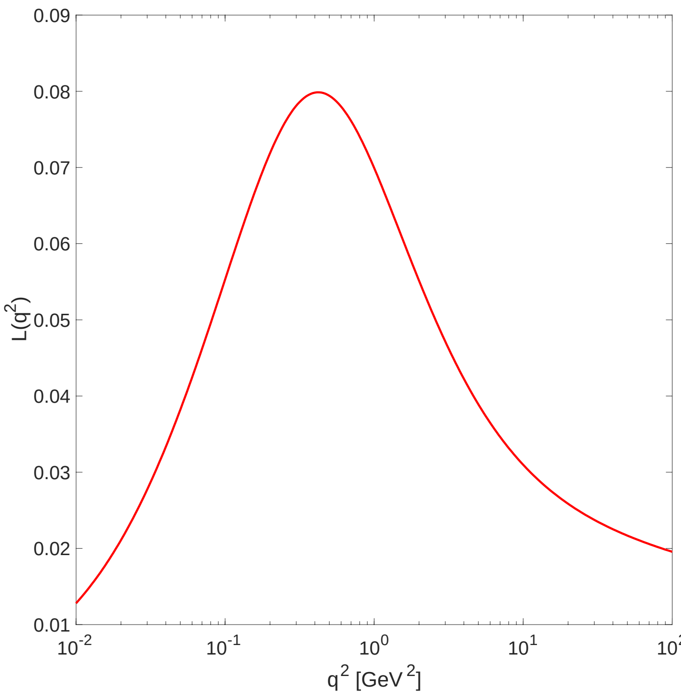
Funções G e L



$$\Lambda_{\mu\nu}(q) = g_{\mu\nu}G(q^2) + \frac{q_\mu q_\nu}{q^2}L(q^2)$$

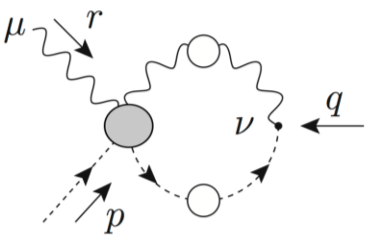
$$F^{-1}(q^2) = 1 + G(q^2) + L(q^2)\,.$$





Kernel de espalhamento

$$H_{\nu\mu}(q,p,r) \;=\; g_{\mu\nu} \;+$$



$$\Lambda_{\mu\nu}(q) = -ig^2 C_A \int_k D(q-k)\Delta_\mu^\sigma(k) H_{\nu\sigma}(-q, q-k, k)$$

$$-iH_{\mu\nu}(k,p) = A_1 g_{\mu\nu} + A_2 p_\mu p_\nu + A_3 k_\mu k_\nu + A_4 p_\mu k_\nu + A_5 k_\mu p_\nu$$

$$ip^\nu H_{\mu\nu}(k,p) = \Gamma_\mu(k,p)$$