An Effective Field Theory View of Heavy Quarkonium Hybrids

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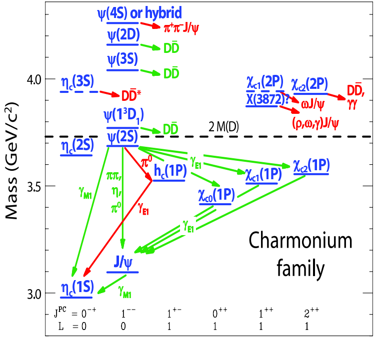
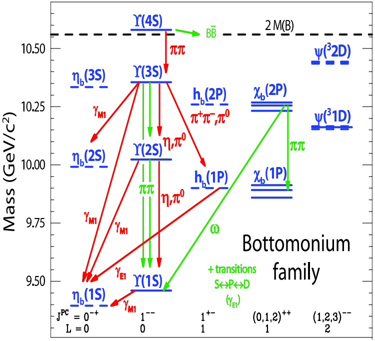
The Charmonium and bottomonium systems were discovered in the 1970s and 1980s
Experimentally clear spectrum of narrow states below the open-flavor threshold





E. Eichten *et al.*, Rev. Mod. Phys. 80 (2008) 1161.
(for a review see: N. Brambilla *et al.*, Eur. Phys. J. C71 (2011) 1534.)

- Heavy quarkonia are bound states made of a heavy quark and its antiquark ($c\bar{c}$ charmonium and $b\bar{b}$ bottomonium).
- They can be classified in terms of the quantum numbers of a nonrelativistic bound state → Reminds positronium [(e^+e^-)-bound state] in QED.
- Heavy quarkonium is a very well established multiscale system which can serve as an ideal laboratory for testing all regimes of QCD.

The nonrelativistic expansion

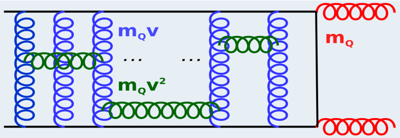
☞ Heavy quarkonium is a nonrelativistic system:

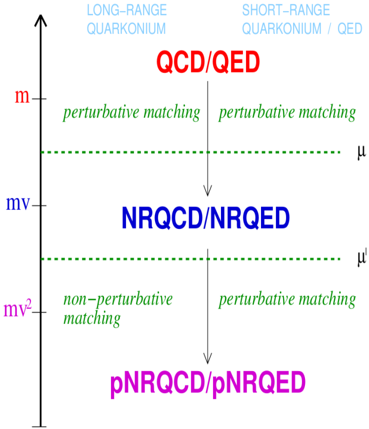
$$v_c \sim 0.55 \quad v_b \sim 0.32 \quad (v_{\rm light} \sim 1.0)$$

☞ Heavy quarkonium is a multiscale system:

$$m_Q \gg p \sim 1/r \sim m_Q v \gg E \sim m_Q v^2$$

☞ Scales are entangled in full QCD:





☞ Systematic expansions in the small heavy-quark velocity v may be implemented at the Lagrangian level by constructing suitable effective field theories (EFTs):

- Expanding QCD in p/m_Q, E/m_Q leads to **NRQCD.**
  Caswell and Lepage PLB 167 (1986) 437; Bodwin, Braaten and Lepage PRD 51 (1995) 1125.
- Expanding NRQCD in E/p leads to **pNRQCD.**
  Brambilla, Pineda, Soto and Vairo NPB 566 (2000) 275.

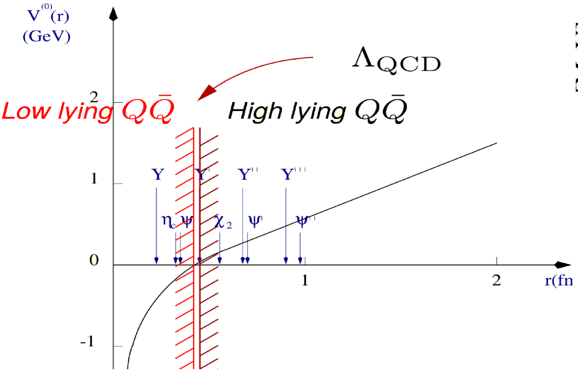
There is another scale in QCD: Λ_{QCD}

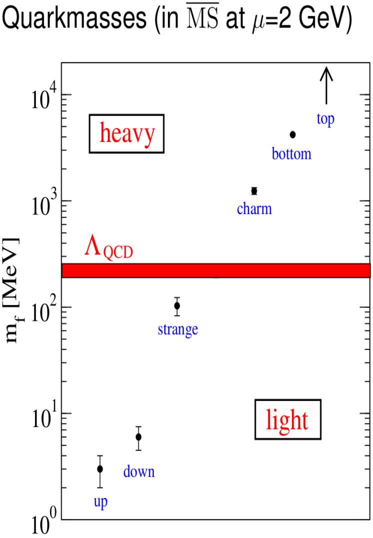
☞ The matching of QCD to NRQCD

- $m_Q \gg \Lambda_{QCD} \rightarrow$ Perturbative matching.

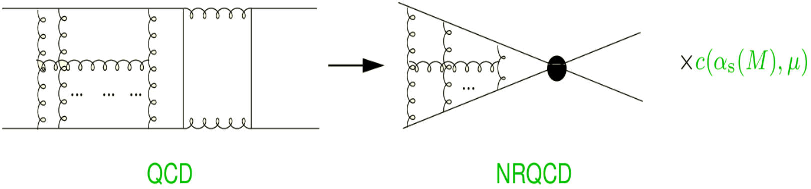
☞ The matching of NRQCD to pNRQCD

- $p \sim 1/r \gg \Lambda_{QCD} \rightarrow$ Perturbative matching.
- $p \sim 1/r \gtrsim \Lambda_{QCD} \rightarrow$ Nonperturbative matching.





☞ Physics at the scale m_Q: Quarkonium annihilation and production.



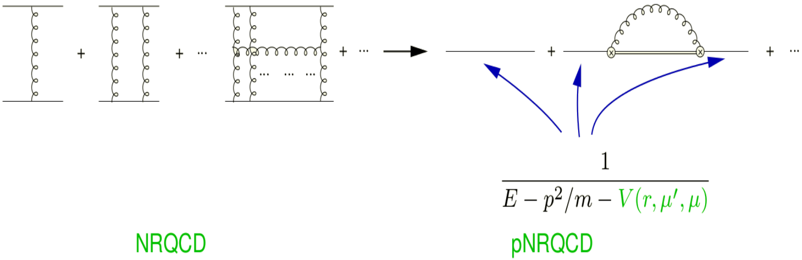
☞ The effective Lagrangian is organized as an expansion in $1/m_Q$ and $\alpha_s(m_Q)$:

$$\mathcal{L}_{\text{NRQCD}} = \sum_n \frac{c_n(\alpha_s(m_Q), \mu)}{m_Q^n} \times \mathcal{O}_n(\mu, m_Q v, m_Q v^2, \ldots)$$

- $\mathcal{L}_{\text{NRQCD}}$ is made of all low-energy operators $\mathcal{O}_n$ that may be built from the effective degrees of freedom and are consistent with the symmetries of $\mathcal{L}_{\text{QCD}}$.
- The Wilson coefficients c_n encode the high energy physics. They are calculated by imposing that $\mathcal{L}_{\text{NRQCD}}$ and $\mathcal{L}_{\text{QCD}}$ describe the same physics at $\mu = m_Q$.

EFTs: pNRQCD at weak coupling (I)

☞ Physics at the scale $m_Q v$: Quarkonium formation.



☞ The effective Lagrangian is organized as an expansion in $1/m_Q$, $\alpha_s(m_Q)$ and $1/p \sim r$:

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r \sum_n \sum_k \frac{c_n(\alpha_s(m_Q),\mu)}{m_Q^n} \times V_{n,k}(r,\mu',\mu) r^k \times \mathcal{O}_k(\mu', m_Q v^2, \ldots)$$

where a multipole expansion of the gluon field has been performed.

☞ The Wilson coefficients of pNRQCD depends on the distance r (and scales μ, μ'):

- $V_{n,0}$ are the potentials in the Schrödinger equation.
- $V_{n,k\neq 0}$ are the couplings with the low-energy degrees of freedom, which provide corrections to the potential picture.

EFTs: pNRQCD at weak coupling (II)

For reviews see:

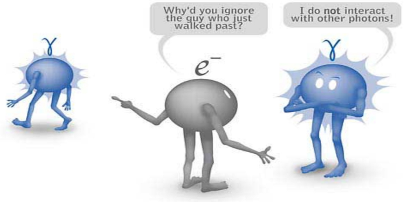
N. Brambilla, A. Pineda, J. Soto and A. Vairo, Rev. Mod. Phys. 77 (2005) 1423.
A. Pineda, Prog. Part. Nucl. Phys. 67 (2012) 735.

- Provides a QM description from FT: the matching coefficients are the interaction potentials and the leading order dynamical equation is of the Schrödinger type.
- The degrees of freedom in pNRQCD (at weak coupling) are color singlet and octet quark-antiquark fields and ultrasoft gluon fields.
- Account for non-potential terms as well. Singlet to Octet transitions via ultrasoft gluons provide loop corrections to the leading potential picture.

pNRQCD is today the theory used to address Quarkonium bound state properties

- Conventional meson spectrum: higher order perturbative corrections in v and α_s.
- Inclusive and semi-inclusive decays, E1 and M1 transitions, EM line-shapes.
- Precise extraction of Standard Model parameters: m_c, m_b, α_s, ...
- Doubly- and triply-heavy baryons.
- **Exotic states such as heavy quark-gluon hybrids.**
- Properties of Quarkonium systems at finite temperature.

Quantum Electrodynamics (QED)

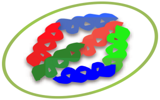


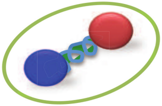
- Theory of the electroweak interaction.
- d.o.f: electrons and photons.
- No Photon self-interactions.

Quantum Chromodynamics (QCD)

- Theory of the strong interaction.
- d.o.f.: quarks and gluons.
- **GLUON SELF-INTERACTIONS.**

Origin of confinement, DCSB, ...? How does glue manifest itself in low energy regime?





☞ Possible clues looking at hadrons with explicit gluonic d.o.f. **Same role played by gluons and quarks in making matter!!**

☞ LHCb@CERN, GlueX@JLab12 and PANDA@FAIR are producing a rich environment of gluons in order to promote the formation of glueballs and quark-gluon hybrids.

☞ Hybrid mesons with a heavy-quark pair are the most amenable to theoretical treatment.

Heavy quarkonium hybrids: *bound-state systems formed by a heavy quark, a heavy antiquark, and excited glue degrees of freedom.*

☞ Since $m_Q \gg \Lambda_{\rm QCD}$, one can distinguish between *slow* and *fast* degrees of freedom:

$$\begin{array}{lcl} Q\bar{Q}\text{-pair (color octet)} & \to & \text{Slow} \\ \text{gluons} & \to & \text{Fast} \end{array}$$

It entails no restriction on the strength of the coupling between the slow and the fast degrees of freedom.

☞ In the static limit, $m_Q, m_{\bar{Q}} \to \infty$, the quark and the antiquark serve as color source and sink at distance r.

☞ The gluonic field arranges itself in configurations described by the quantum numbers fixed by the symmetry of the system.

☞ The gluonic dynamics is collective and nonperturbative

$\Downarrow$

Gluonic static energies have been extracted from Lattice QCD

☞ Solve the Schrödinger equation for the color octet $Q\bar{Q}$-pair with an effective potential given by the gluonic static energies.

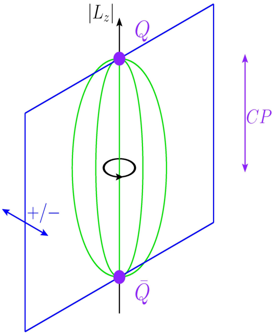
In static NRQCD: The gluonic excitations between static quarks have the same symmetries as the diatomic molecule.

☞ The static energies correspond to the irreducible representations of $D_{\infty h}$ (symmetry group of a cylinder).

☞ Representations labeled by Λ_η^ϵ:

- Λ: rotational quantum number $|\hat{r}\cdot\vec{K}| = 0, 1, 2, \ldots$ corresponds to $\Lambda = \Sigma, \Pi, \Delta, \ldots$
- η: eigenvalue of CP: $g \equiv +1$ and $u \equiv -1$.
- ϵ: eigenvalue of reflections. Only displayed in Σ states (others are degenerated).

☞ There can be more than one state for each irreducible representation of $D_{\infty h}$, usually denoted by Π_u, Π_u', Π_u'', ...

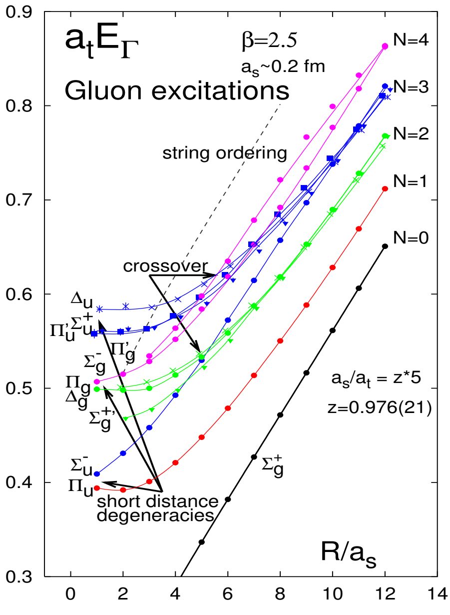


☞ *In the $r \to 0$ limit extra symmetries for the gluonic excitations between static quarks appear: $D_{\infty h} \to O(3) \times C$:*

- *Several Λ_η^ϵ representations are contained in one K^{PC} representation.*
- *Static energies in these multiplets have same $r \to 0$ limit.*

They are Nonperturbative quantities!!

- $E^{(0)}_{\Sigma_g^+}(r)$ is the ground state potential that generates the standard Quarkonium states.
- The rest of the static energies correspond to gluonic excitations that generate hybrids.
- The two lowest hybrid static energies are $E^{(0)}_{\Pi_u}(r)$ and $E^{(0)}_{\Sigma_u^-}(r)$.
  $\rightarrow$ Nearly degenerate at short distances.
- Good agreement was found between quenched and unquenched computations.



Juge et al. PRL 90 (2003) 161601

Hybrid static energies in Lattice QCD (II)

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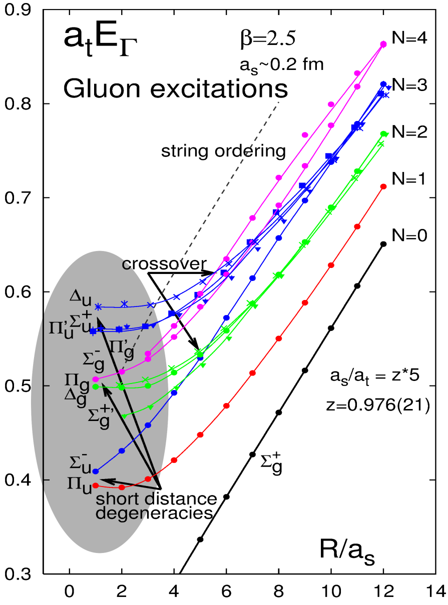


Juge et al. PRL 90 (2003) 161601

- The lowest static energies are given by:

$$E_n^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle X_n,\ T/2 | X_n,\ -T/2\rangle$$

- Any state $|X_n\rangle$ can be written as an expansion:

$$|X_n\rangle = c_n\ |\underline{n};\ \boldsymbol{x}_1,\ \boldsymbol{x}_2\rangle^{(0)} + c_{n'}\ |\underline{n}';\ \boldsymbol{x}_1,\ \boldsymbol{x}_2\rangle^{(0)} + \dots$$

- The static states $|\underline{n};\ \boldsymbol{x}_1,\ \boldsymbol{x}_2\rangle^{(0)}$ form a complete basis and fulfill:

$${}^{(0)}\langle \underline{n};\ \boldsymbol{x}_1,\ \boldsymbol{x}_2,\ T/2|\ \underline{n};\ \boldsymbol{x}_1,\ \boldsymbol{x}_2,\ -T/2\rangle^{(0)} = \mathcal{N}\ \exp\left[-iE_n^{(0)}(r)\ T\right]$$

$$\langle X_n,\ T/2|X_n,\ -T/2\rangle = \mathcal{N}|c_n|^2 \exp\left[-iE_n^{(0)}(r)\ T\right] + \mathcal{N}|c_{n'}|^2 \exp\left[-iE_{n'}^{(0)}(r)\ T\right] + \dots$$

- The Hamiltonian of NRQCD up to $1/m_Q$ for the one-quark-one-antiquark sector:

$$H_{\rm NRQCD} = H^{(0)} + \frac{1}{m_Q} H^{(1,0)} + \frac{1}{m_{\bar{Q}}} H^{(0,1)} + \dots$$

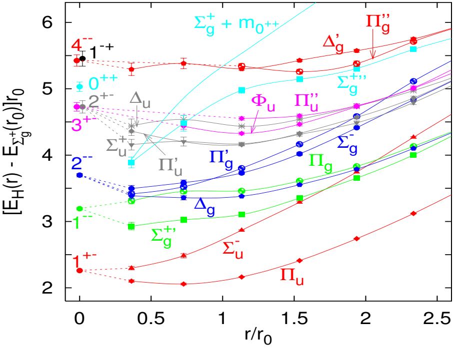
$$H^{(0)} = \int d^3x\, \frac{1}{2}\left(\boldsymbol{E}^a \cdot \boldsymbol{E}^a + \boldsymbol{B}^a \cdot \boldsymbol{B}^a\right) - \sum_{j=1}^{n_f} \int d^3\boldsymbol{x}\, \bar{q}_j\, i\boldsymbol{D}\cdot\gamma\, q_j$$

$$H^{(1,0)} = -\frac{1}{2}\int d^3x\, \psi^\dagger \left(\boldsymbol{D}^2 + g\, c_F\, \boldsymbol{\sigma}\cdot\boldsymbol{B}\right)\psi$$

$$H^{(0,1)} = \frac{1}{2}\int d^3x\, \chi^\dagger \left(\boldsymbol{D}^2 + g\, c_F\, \boldsymbol{\sigma}\cdot\boldsymbol{B}\right)\chi$$

Hybrid static energies in NRQCD (II)

G.S. Bali *et al.* PRD 69 (2004) 094001



Λ_η^σ	K^{PC}	P_n^a
Σ_u^-	1^{+-}	$\hat{\boldsymbol{r}}\cdot\boldsymbol{B},\ \hat{\boldsymbol{r}}\cdot(\boldsymbol{D}\times\boldsymbol{E})$
Π_u	1^{+-}	$\hat{\boldsymbol{r}}\times\boldsymbol{B},\ \hat{\boldsymbol{r}}\times(\boldsymbol{D}\times\boldsymbol{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{\boldsymbol{r}}\cdot\boldsymbol{E},\ \hat{\boldsymbol{r}}\cdot(\boldsymbol{D}\times\boldsymbol{B})$
Π_g	1^{--}	$\hat{\boldsymbol{r}}\times\boldsymbol{E},\ \hat{\boldsymbol{r}}\times(\boldsymbol{D}\times\boldsymbol{B})$
Σ_g^-	2^{--}	$(\hat{\boldsymbol{r}}\cdot\boldsymbol{D})(\hat{\boldsymbol{r}}\cdot\boldsymbol{B})$
Π_g'	2^{--}	$\hat{\boldsymbol{r}}\times((\hat{\boldsymbol{r}}\cdot\boldsymbol{D})\boldsymbol{B}+\boldsymbol{D}(\hat{\boldsymbol{r}}\cdot\boldsymbol{B}))$
Δ_g	2^{--}	$(\hat{\boldsymbol{r}}\times\boldsymbol{D})^i(\hat{\boldsymbol{r}}\times\boldsymbol{B})^j+(\hat{\boldsymbol{r}}\times\boldsymbol{D})^j(\hat{\boldsymbol{r}}\times\boldsymbol{B})^i$
Σ_u^+	2^{+-}	$(\hat{\boldsymbol{r}}\cdot\boldsymbol{D})(\hat{\boldsymbol{r}}\cdot\boldsymbol{E})$
Π_u'	2^{+-}	$\hat{\boldsymbol{r}}\times((\hat{\boldsymbol{r}}\cdot\boldsymbol{D})\boldsymbol{E}+\boldsymbol{D}(\hat{\boldsymbol{r}}\cdot\boldsymbol{E}))$
Δ_u	2^{+-}	$(\hat{\boldsymbol{r}}\times\boldsymbol{D})^i(\hat{\boldsymbol{r}}\times\boldsymbol{E})^j+(\hat{\boldsymbol{r}}\times\boldsymbol{D})^j(\hat{\boldsymbol{r}}\times\boldsymbol{E})^i$

Brambilla et al. NPB 566 (2000) 275

- A convenient choice for the $|X_n\rangle$ states gives the static energies in terms of Wilson loops, so we define:

$$|X_n\rangle=\chi(\boldsymbol{x}_2)\,\phi(\boldsymbol{x}_2,\boldsymbol{R})T^aP_n^a(\boldsymbol{R})\phi(\boldsymbol{R},\boldsymbol{x}_1)\,\psi^\dagger(\boldsymbol{x}_1)|\mathrm{vac}\rangle$$

- The large time correlator of these states is given by a static Wilson loop with insertions of P_n^a in the strings at the center-of-mass.

- For $E^{(0)}_{\Sigma^+}(r)$: Insert a color-neutral gluonic operator with $K^{PC}=0^{++}$.

Hybrid static energies in pNRQCD (I)

☞ Matching between NRQCD and pNRQCD for the static (gluelump) eigenstates:

$$|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} \stackrel{\sim}{=} \left(O^{a\,\dagger}(\mathbf{r},\mathbf{R})\,\hat{n}_i\;G^a_{n,\,i}(\mathbf{R})+\mathcal{O}(r)\right)|0\rangle$$

☞ Matching between NRQCD and pNRQCD for the $|X_n\rangle$ state:

$$|X_n\rangle \stackrel{\sim}{=} \left(Z_n(r)\,O^{a\,\dagger}(\mathbf{r},\mathbf{R})P^a_n(\hat{\mathbf{r}},\mathbf{R})+\mathcal{O}(r)\right)|0\rangle$$

☞ The large time correlators are then given by

$$\langle X_n,T/2|X_n,-T/2\rangle=\mathcal{N}e^{-iV_o(r)T}\langle 0|P^a_n(T/2)\phi^{ab}_{\rm adj}(T/2,-T/2)P^b_n(-T/2)|0\rangle+\mathcal{O}\left(r^2\right)$$

☞ The gluonic correlator can only be evaluated nonperturbatively. We can argue that

$$\langle 0|P^a_n(T/2)\phi(T/2,-T/2)^{\rm adj}_{ab}P^b_n(-T/2)|0\rangle=|c_n|^2e^{-i\Lambda_H T}+|c_{n'}|^2e^{-i\Lambda_{H'}T}+\ldots$$

☞ Matching between NRQCD and pNRQCD for the static energy:

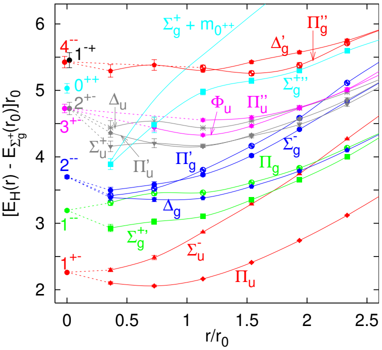
$$\boxed{E^{(0)}_n(r)=\lim_{T\to\infty}\frac{i}{T}\log\langle X_n,T/2|X_n,-T/2\rangle=V_o(r)+\Lambda_H+\mathcal{O}\left(r^2\right)}$$

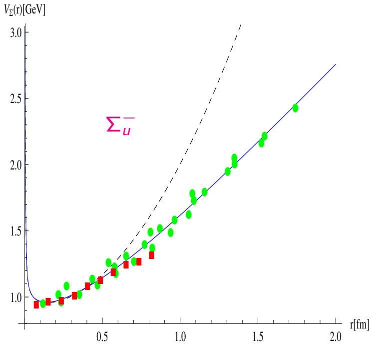
M. Berwein, N. Brambilla, J. Tarrús-Castellà and A. Vairo, Phys. Rev. D92 (2015) 114019.

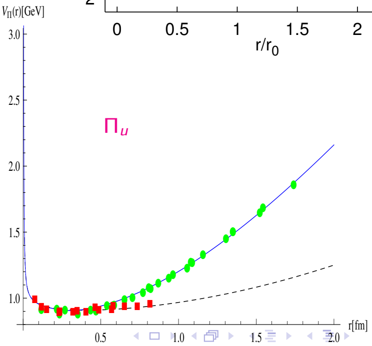
☞ We consider only states of the lowest lying symmetry multiplet, i.e., the Σ_u^- and Π_u states.

☞ They are generated from a gluelump with quantum numbers $K^{PC} = 1^{+-} \to \boldsymbol{r} \cdot \boldsymbol{B}^a$ and $\boldsymbol{r} \times \boldsymbol{B}^a$.

$$E_n^{(0)}(r) = +\frac{\alpha_s}{6r} + \Lambda_H + b_n r^2 + c_n$$







☞ The $1/m_Q$ pNRQCD Hamiltonian in the octet sector:

$$\mathcal{H}^{(1)} = \int d^3R\, d^3r\, O^{a\dagger}(\boldsymbol{r},\boldsymbol{R}) \left[-\frac{\boldsymbol{\nabla}_r^2 \delta^{ab}}{m_Q} + \dots \right] O^b(\boldsymbol{r},\boldsymbol{R})$$

☞ Schrödinger-like equation:

$$\sum_{n=\Sigma,\Pi^\pm} \hat{\boldsymbol{n}}'^*(\theta,\phi) \left[-\frac{\boldsymbol{\nabla}_r^2}{m_Q} + E_n^{(0)}(r) \right] \hat{\boldsymbol{n}}(\theta,\phi) \Psi_n^{(N)}(\boldsymbol{r}) = \mathcal{E}_N \Psi_{n'}^{(N)}(\boldsymbol{r})$$

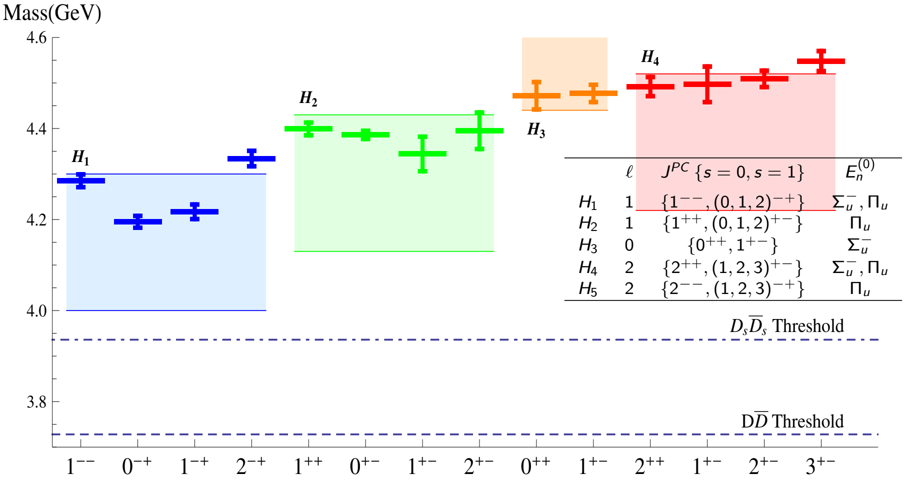
☞ The final expressions are:

$$\left[-\frac{1}{m_Q r^2} \partial_r r^2 \partial_r + \frac{1}{m_Q r^2} \begin{pmatrix} \ell(\ell+1)+2 & 2\sqrt{\ell(\ell+1)} \\ 2\sqrt{\ell(\ell+1)} & \ell(\ell+1) \end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)} \end{pmatrix} \right] \begin{pmatrix} \psi_\Sigma^{(N)} \\ \psi_{-\Pi}^{(N)} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \psi_\Sigma^{(N)} \\ \psi_{-\Pi}^{(N)} \end{pmatrix}$$

$$\left[-\frac{1}{m_Q r^2} \partial_r r^2 \partial_r + \frac{\ell(\ell+1)}{m_Q r^2} + E_\Pi^{(0)} \right] \psi_{+\Pi}^{(N)} = \mathcal{E}_N \psi_{+\Pi}^{(N)}$$

Hybrid static energies in pNRQCD (IV)

Masses computed by the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 400 MeV. L. Liu *et al.*, JHEP 07 (2012) 126.



	ℓ	$J^{PC}\ \{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Overall agreement with the mass gaps between multiplets.

Error bands take into account the uncertainty on the gluelump mass: $0.87 \pm 0.15\,\mathrm{GeV}$

☞ The hybrid EFT Lagrangian reads

$$L_{\rm HEFT} = \int d^3R\, d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \left[i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}^2_r}{M} P^{i}_{\kappa\lambda'} \right] \Psi_{\kappa\lambda'}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \ldots$$

where the dots stand for operators producing transitions to standard quarkonium and transitions between hybrid states with different κ.

☞ The potential $V_{\kappa\lambda\lambda'}$ can be organized as an expansion in $1/m_Q$ and a sum of spin-dependent and -independent contributions:

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q} + \frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m_Q^2} + \ldots$$

$$V^{(1)}_{\kappa\lambda\lambda'}(r) = V^{(1)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(1)}_{\kappa\lambda\lambda'\,SI}(r)$$
$$V^{(2)}_{\kappa\lambda\lambda'}(r) = V^{(2)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(2)}_{\kappa\lambda\lambda'\,SI}(r)$$

☞ The spin-dependent potentials:

$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SK}(r) \left(P^{i\dagger}_{1\lambda} \boldsymbol{K}^{ij}_{1} P^{j}_{1\lambda'} \right) \cdot \boldsymbol{S}$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,LSa}(r) \left(P^{i\dagger}_{1\lambda} \boldsymbol{L}_{Q\bar{Q}} P^{i}_{1\lambda'} \right) \cdot \boldsymbol{S} + V_{1\,LSb} P^{i\dagger}_{1\lambda}(r) \left(\boldsymbol{L}^{i}_{Q\bar{Q}} \boldsymbol{S}^{j} + \boldsymbol{S}^{i} \boldsymbol{L}^{j}_{Q\bar{Q}} \right) P^{j}_{1\lambda'}$$
$$+ V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)\boldsymbol{S}_{12}\delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r) P^{i\dagger}_{1\lambda} P^{j}_{1\lambda'} \left(\boldsymbol{S}^{i}_{1}\boldsymbol{S}^{j}_{2} + \boldsymbol{S}^{i}_{2}\boldsymbol{S}^{j}_{1} \right)$$

☞ The hybrid EFT Lagrangian reads

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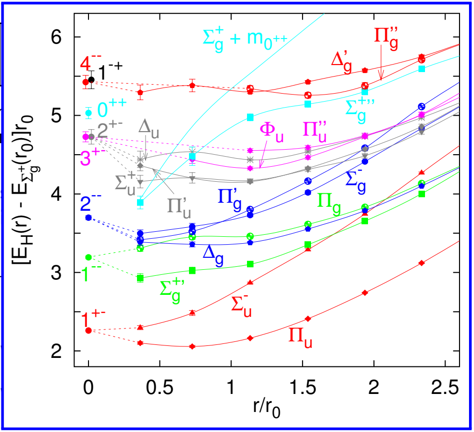
spin-dependent and -independent contribut

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} +$$

$$V^{(1)}_{\kappa\lambda\lambda'}(r) = V^{(1)}_{\kappa\lambda\lambda'\,SD}(r) +$$

$$V^{(2)}_{\kappa\lambda\lambda'}(r) = V^{(2)}_{\kappa\lambda\lambda'\,SD}(r) +$$



☞ The spin-dependent potentials:

$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SK}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{K}^{ij}_1 P^j_{1\lambda'}\right)\cdot \boldsymbol{S}$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,LSa}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{L}_{Q\bar{Q}}P^i_{1\lambda'}\right)\cdot\boldsymbol{S} + V_{1\,LSb}P^{i\dagger}_{1\lambda}(r)\left(\boldsymbol{L}^i_{Q\bar{Q}}\boldsymbol{S}^j + \boldsymbol{S}^i\boldsymbol{L}^j_{Q\bar{Q}}\right)P^j_{1\lambda'}$$

$$+ V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)\boldsymbol{S}_{12}\delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r)P^{i\dagger}_{1\lambda}P^j_{1\lambda'}\left(\boldsymbol{S}^i_1\boldsymbol{S}^j_2 + \boldsymbol{S}^i_2\boldsymbol{S}^j_1\right)$$

☞ The hybrid EFT Lagrangian reads

$$L_{\rm HEFT} = \int d^3R\, d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \left[i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}_r^2}{M} P^{i}_{\kappa\lambda'} \right] \Psi_{\kappa\lambda'}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \ldots$$

where the dots stand for operators producing transitions to standard quarkonium and transitions between hybrid states with different κ.

☞ The potential $V_{\kappa\lambda\lambda'}$ can be organized as an expansion in $1/m_Q$ and a sum of spin-dependent and -independent contributions:

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q} + \frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m_Q^2} + \ldots$$

$$V^{(1)}_{\kappa\lambda\lambda'}(r) = V^{(1)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(1)}_{\kappa\lambda\lambda'\,SI}(r)$$
$$V^{(2)}_{\kappa\lambda\lambda'}(r) = V^{(2)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(2)}_{\kappa\lambda\lambda'\,SI}(r)$$

☞ The spin-dependent potentials:

$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SK}(r)\left(P^{i\dagger}_{1\lambda} \boldsymbol{K}^{ij}_{1} P^{j}_{1\lambda'}\right)\cdot \boldsymbol{S}$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,LSa}(r)\left(P^{i\dagger}_{1\lambda} \boldsymbol{L}_{Q\bar{Q}} P^{i}_{1\lambda'}\right)\cdot \boldsymbol{S} + V_{1\,LSb} P^{i\dagger}_{1\lambda}(r)\left(\boldsymbol{L}^{i}_{Q\bar{Q}}\boldsymbol{S}^{j} + \boldsymbol{S}^{i}\boldsymbol{L}^{j}_{Q\bar{Q}}\right) P^{j}_{1\lambda'}$$
$$+ V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)\boldsymbol{S}_{12}\delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r) P^{i\dagger}_{1\lambda} P^{j}_{1\lambda'}\left(\boldsymbol{S}^{i}_{1}\boldsymbol{S}^{j}_{2} + \boldsymbol{S}^{i}_{2}\boldsymbol{S}^{j}_{1}\right)$$

☞ The hybrid EFT Lagrangian reads

$$L_{\rm HEFT}=\int d^3R\,d^3r\sum_{\kappa}\sum_{\lambda\lambda'}\Psi^{\dagger}_{\kappa\lambda}(t,\,\boldsymbol{r},\,\boldsymbol{R})\left[i\partial_t-V_{\kappa\lambda\lambda'}(r)+P^{i\dagger}_{\kappa\lambda}\frac{\boldsymbol{\nabla}^2_r}{M}P^{i}_{\kappa\lambda'}\right]\Psi_{\kappa\lambda'}(t,\,\boldsymbol{r},\,\boldsymbol{R})\ldots$$

where the dots stand for operators producing transitions to standard quarkonium and transitions between hybrid states with different κ.

☞ The potential $V_{\kappa\lambda\lambda'}$ can be organized as an expansion in $1/m_Q$ and a sum of spin-dependent and -independent contributions:

$$V_{\kappa\lambda\lambda'}(r)=V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'}+\frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q}+\frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m_Q^2}+\ldots$$

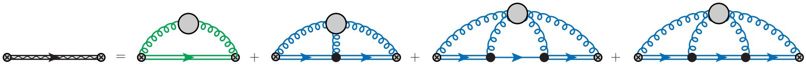
$$V^{(1)}_{\kappa\lambda\lambda'}(r)=V^{(1)}_{\kappa\lambda\lambda'\,SD}(r)+V^{(1)}_{\kappa\lambda\lambda'\,SI}(r)$$
$$V^{(2)}_{\kappa\lambda\lambda'}(r)=V^{(2)}_{\kappa\lambda\lambda'\,SD}(r)+V^{(2)}_{\kappa\lambda\lambda'\,SI}(r)$$

☞ The spin-dependent potentials:

$$V^{(1)}_{1\lambda\lambda'\,SD}(r)=V_{1\,SK}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{K}^{ij}_{1}P^{j}_{1\lambda'}\right)\cdot\boldsymbol{S}$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r)=V_{1\,LSa}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{L}_{Q\bar{Q}}P^{i}_{1\lambda'}\right)\cdot\boldsymbol{S}+V_{1\,LSb}P^{i\dagger}_{1\lambda}(r)\left(\boldsymbol{L}^{i}_{Q\bar{Q}}\boldsymbol{S}^{j}+\boldsymbol{S}^{i}\boldsymbol{L}^{j}_{Q\bar{Q}}\right)P^{j}_{1\lambda'}$$
$$+V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'}+V_{1\,S_{12}a}(r)\boldsymbol{S}_{12}\delta_{\lambda\lambda'}+V_{1\,S_{12}b}(r)P^{i\dagger}_{1\lambda}P^{j}_{1\lambda'}\left(\boldsymbol{S}^{i}_{1}\boldsymbol{S}^{j}_{2}+\boldsymbol{S}^{i}_{2}\boldsymbol{S}^{j}_{1}\right)$$

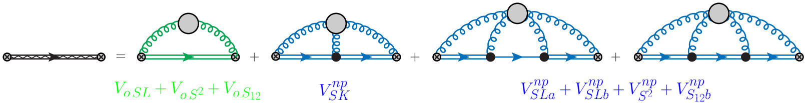
☞ Matching diagrams in position space (at leading order in a multipole expansion for small quark-antiquark distances):



The single, double and curly lines represent $Q\bar{Q}$-singlet, -octet and gluon fields

$$V_{1\,SK} = V^{np}_{SK}$$
$$V_{1\,SLa} = V^{np}_{SLa} + V_{o\,SL}$$
$$V_{1\,SLb} = V^{np}_{SLb}$$
$$V_{1\,S^2} = V^{np}_{S^2} + V_{o\,S^2}$$
$$V_{1\,S_{12}a} = V_{o\,S_{12}} + V^{np}_{S_{12}a}$$
$$V_{1\,S_{12}b} = V^{np}_{S_{12}b}$$

☞ Matching diagrams in position space (at leading order in a multipole expansion for small quark-antiquark distances):



The single, double and curly lines represent $Q\bar{Q}$-singlet, -octet and gluon fields

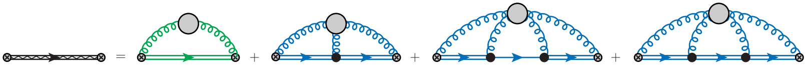
$$
\begin{aligned}
V_{1\,SK} &= V^{np}_{SK} \\
V_{1\,SLa} &= V^{np}_{SLa} + V_{o\,SL} \\
V_{1\,SLb} &= V^{np}_{SLb} \\
V_{1\,S^2} &= V^{np}_{S^2} + V_{o\,S^2} \\
V_{1\,S_{12}a} &= V_{o\,S_{12}} + V^{np}_{S_{12}a} \\
V_{1\,S_{12}b} &= V^{np}_{S_{12}b}
\end{aligned}
$$

☞ The perturbative part is given by the octet quark-antiquark spin-dependent potential:

$$
\begin{aligned}
V_{o\,LS}(r) &= \left(C_F - \frac{C_A}{2}\right)\left(\frac{c_s}{2} + c_F\right)\frac{\alpha_s(\nu)}{r^3} \\
V_{o\,S^2}(r) &= \left[\frac{4\pi}{3}\left(C_F - \frac{C_A}{2}\right) c_F^2 \alpha_s(\nu) + T_F \left(f_8(^1S_0) - f_8(^3S_1)\right)\right]\delta^3(\mathbf{r}) \\
V_{o\,S_{12}}(r) &= \left(C_F - \frac{C_A}{2}\right)\frac{\alpha_s(\nu)}{4r^3}
\end{aligned}
$$

Matching of the Spin-dependent operators for $\kappa = 1^{+-}$

☞ Matching diagrams in position space (at leading order in a multipole expansion for small quark-antiquark distances):



The single, double and curly lines represent $Q\bar{Q}$-singlet, -octet and gluon fields

$$
\begin{aligned}
V_{1\,SK} &= V^{np}_{SK} \\
V_{1\,SLa} &= V^{np}_{SLa} + V_{o\,SL} \\
V_{1\,SLb} &= V^{np}_{SLb} \\
V_{1\,S^2} &= V^{np}_{S^2} + V_{o\,S^2} \\
V_{1\,S_{12}a} &= V_{o\,S_{12}} + V^{np}_{S_{12}a} \\
V_{1\,S_{12}b} &= V^{np}_{S_{12}b}
\end{aligned}
$$

☞ The nonperturbative part is given in terms of gluon correlators $\widetilde{U}$:

$$
\begin{aligned}
V^{np}_{SK} &= c_F\, \widetilde{U}^K_B \\
V^{np}_{SLa} &= -\frac{3c_F}{2}\, \widetilde{U}^o_{Ba} + c_s \left(\widetilde{U}^s_{Ea} - \frac{N_c^2 - 4}{2N_c^2}\, \widetilde{U}^o_{Ea} \right) \\
V^{np}_{SLb} &= -\frac{3c_F}{2}\, \widetilde{U}^o_{Bb} + c_s \left(\widetilde{U}^s_{Eb} - \frac{N_c^2 - 4}{2N_c^2}\, \widetilde{U}^o_{Eb} \right) \\
V^{np}_{S^2} &= -c_F^2 \left(\widetilde{U}^s_{Ba} + \frac{2(N_c^2 - 1)}{N_c^2}\, \widetilde{U}^o_{Ba} \right) \\
V^{np}_{S_{12}b} &= -2c_F^2 \left(\widetilde{U}^s_{Bb} + \frac{2(N_c^2 - 1)}{N_c^2}\, \widetilde{U}^o_{Bb} \right) \;;\quad V^{np}_{S_{12}a} = 0
\end{aligned}
$$

☞ Two-point gluon correlator functions:

$$\widetilde{U}_B^K = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \frac{1}{48T_F} \int_{-T/2}^{T/2} dt \left[\langle 0|\boldsymbol{G}^{\dagger}(T/2)\cdot(g\boldsymbol{B}_{\rm adj}(t)\times\boldsymbol{G}(-T/2))\,|0\rangle\right]$$

$$\widetilde{U}_{Ba}^s + 4\widetilde{U}_{Bb}^s = \lim_{T\to\infty} \frac{e^{i\Lambda T}}{iT} \frac{N_c}{3T_F} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \langle 0|(\boldsymbol{G}^{a\dagger}(T/2)\cdot g\boldsymbol{B}^a(t))(g\boldsymbol{B}^a(t')\cdot\boldsymbol{G}^a(-T/2))|0\rangle$$

$$3\widetilde{U}_{Ba}^s + 2\widetilde{U}_{Bb}^s = \lim_{T\to\infty} \frac{e^{i\Lambda T}}{iT} \frac{N_c}{3T_F} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \langle 0|\boldsymbol{G}^{a\dagger}(T/2)\cdot\left((g\boldsymbol{B}^a(t)\cdot g\boldsymbol{B}^a(t'))\boldsymbol{G}^a(-T/2)\right)|0\rangle$$

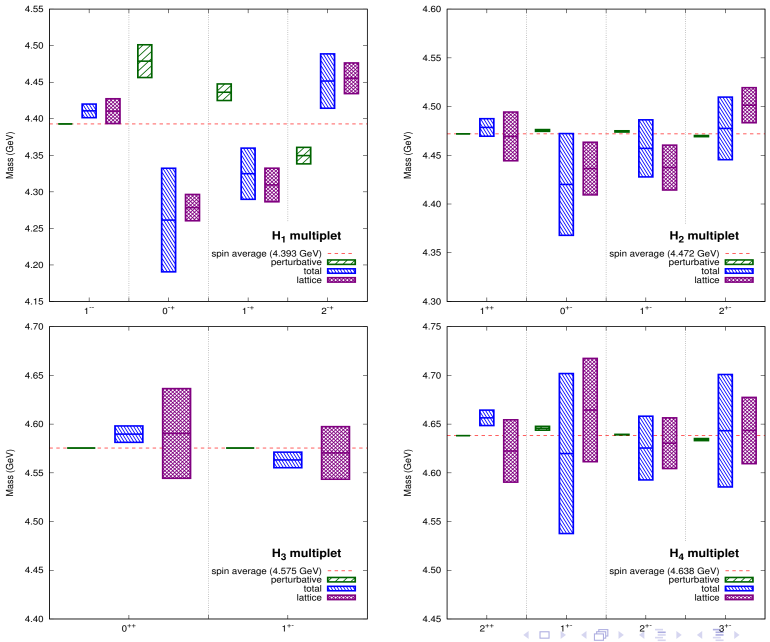
$$\widetilde{U}_{Ba}^o + 4\widetilde{U}_{Bb}^o = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \frac{1}{18T_F^2} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \langle 0|(\boldsymbol{G}^{\dagger}(T/2)\cdot g\boldsymbol{B}_{\rm adj}(t))(g\boldsymbol{B}_{\rm adj}(t')\cdot\boldsymbol{G}^a(-T/2))|0\rangle$$

$$3\widetilde{U}_{Ba}^o + 2\widetilde{U}_{Bb}^o = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \frac{1}{18T_F^2} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \langle 0|\boldsymbol{G}^{\dagger}(T/2)\cdot\left((g\boldsymbol{B}_{\rm adj}(t)\cdot g\boldsymbol{B}_{\rm adj}(t'))\boldsymbol{G}(-T/2)\right)|0\rangle$$

☞ Observations:

- $\widetilde{U}_{Ea}^o$ and $\widetilde{U}_{Eb}^o$ are defined by replacing $\boldsymbol{B}$ for $\boldsymbol{E}$.
- The gluon correlators $\widetilde{U}$ are independent of r and the heavy quark flavor.

Charmonium hybrids



Features of the numerical calculation

☞ Parameters used:

$$m_c^{RS}(\nu_f = 1.0\,\text{GeV}) = 1.477\,\text{GeV} \qquad m_b^{RS}(\nu_f = 1.0\,\text{GeV}) = 4.863\,\text{GeV}$$

$$\alpha_s^{n_f=3,\text{4-loops}}(2.6\,\text{GeV}) = 0.26 \qquad \Lambda_{\text{QCD}} = 0.5\,\text{GeV}$$

☞ Fitted values:

	Cheung 2016 ($m_\pi = 240\,\text{MeV}$)	Liu 2012 ($m_\pi = 400\,\text{MeV}$)
$V_{SK}^{np}/\Lambda_{\text{QCD}}^2$	$+0.86$	$+0.80$
$V_{SLa}^{np}/\Lambda_{\text{QCD}}^3$	-0.87	-0.56
$V_{SLb}^{np}/\Lambda_{\text{QCD}}^3$	-0.56	-0.68
$V_{S^2}^{np}/\Lambda_{\text{QCD}}^3$	$+0.11$	$+0.11$
$V_{S_{12}b}^{np}/\Lambda_{\text{QCD}}^3$	-0.43	-0.59

☞ Lattice data is weighted as

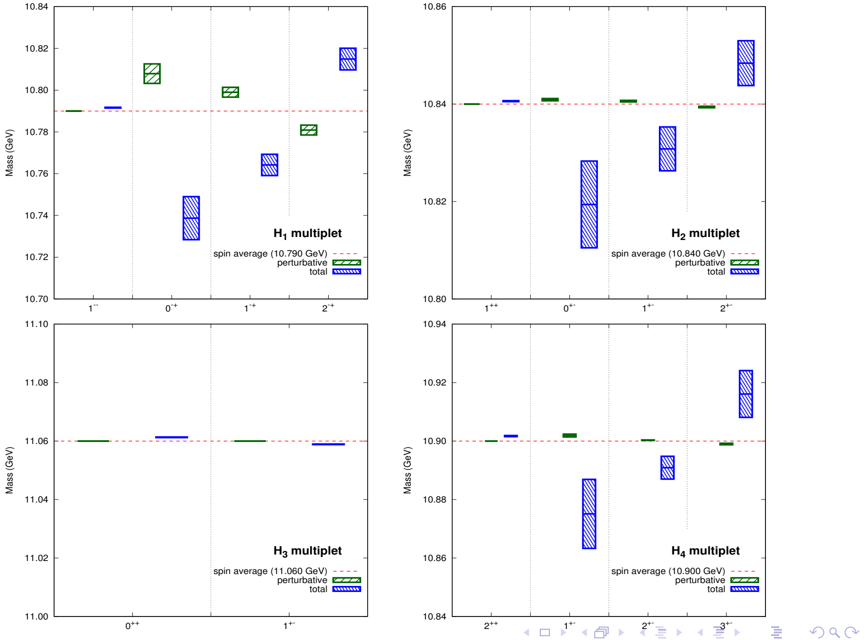
$$(\Delta^2_{\text{lattice}} + \Delta^2_{\text{high-order}})^{-1/2}$$

where Δ_{lattice} is lattice uncertainty and $\Delta_{\text{high-order}} = (m_{\text{lattice}} - m_{\text{spin-average}}) \times \frac{\Lambda_{\text{QCD}}}{m_Q}$.

☞ Theoretical uncertainty:

$$(\Delta^2_{\text{pert.}} + \Delta^2_{\text{nonpert.}} + \Delta^2_{\text{fit}})^{-1/2}$$

where $\Delta_{\text{pert.}} = \mathcal{O}(m\alpha_s^5)$, $\Delta_{\text{nonpert.}} = \mathcal{O}(\Lambda_{\text{QCD}}(\Lambda_{\text{QCD}}/m_Q)^3)$, and Δ_{fit} is the statistical error of the fit.



Mass (GeV)
10.84
10.82
10.80
10.78
10.76
10.74
10.72
10.70
1--
0-+
1-+
2-+
H1 multiplet
spin average (10.790 GeV)
perturbative
total
10.86
10.84
10.82
10.80
1++
0+-
1+-
2+-
H2 multiplet
spin average (10.840 GeV)
perturbative
total
11.10
11.08
11.06
11.04
11.02
11.00
0++
1+-
H3 multiplet
spin average (11.060 GeV)
perturbative
total
10.94
10.92
10.90
10.88
10.86
10.84
2++
1+-
2+-
3+-
H4 multiplet
spin average (10.900 GeV)
perturbative
total

Conclusions

- Quarkonium hybrids can be studied in a model independent way combining EFT with lattice inputs.
- In this framework we have obtained the $1/m_Q$ and $1/m_Q^2$ spin-dependent contributions to the spectrum.
- The matching coefficients have been characterized in terms of two-point gluonic correlators.
- The hybrid EFT provides constrains to cross check lattice determinations of the charmonium hybrid spectrum.
- We have obtained the fine and hyperfine structure of the hybrid spectrum in the bottom sector, where direct lattice predictions are still sparse.