Infrared behavior of the ghost-gluon scattering kernel

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Mauricio N. Ferreira

In collaboration with:
Arlene C. Aguilar,
Clara T. Figueiredo,
Joannis Papavassiliou.

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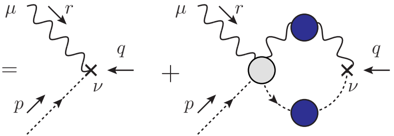
Motivation

The non-Abelian character of QCD makes the ghost sector of the theory non-trivial.

- It is necessary to understand the ghost sector to correctly describe the gauge sector.
- While the ghost propagator in Landau gauge is well understood from Lattice and continuum methods…
- …the ghost vertices are less known.
- The ghost-gluon scattering kernel has not been studied much, beyond perturbation theory.

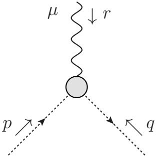
The **ghost-gluon scattering kernel** is given by

$H_{\nu\mu}(q,p,r) =$



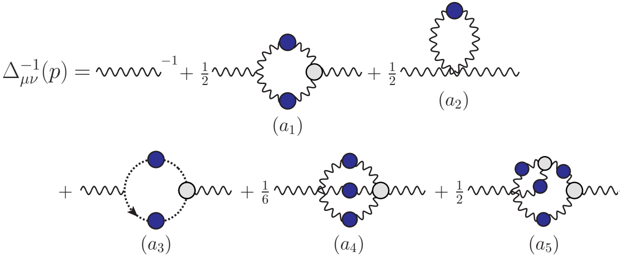
and is closely related to the **ghost-gluon vertex**,

$$\boxed{q^\nu H_{\nu\mu}(q,p,r) = \Gamma_\nu(q,p,r)\,.}$$

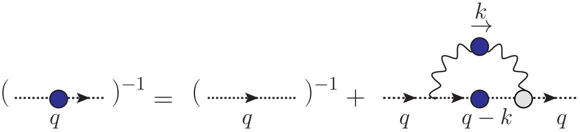


$= \Gamma_\mu(q,p,r)$

The ghost-gluon vertex is an ingredient in several Schwinger-Dyson Equations (SDEs), *e.g.* the **gluon propagator**



and the **ghost propagator** *



*A. C. Aguilar, D. Ibáñez and J. Papavassiliou, Phys. Rev. D **87** (2013) 11, 114020.
*D. Dudal, O. Oliveira and J. Rodriguez-Quintero, Phys. Rev. D **86**, 105005 (2012).

The **scattering kernel**, is a bit more unusual.
It appears in the Slavnov-Taylor identity (STI) for the **three-gluon vertex** [*],

$$\boxed{r^{\mu}\Gamma_{\alpha\mu\nu}(q,r,p) = F(r)[\Delta^{-1}(q)P_{\alpha}^{\mu}(q)H_{\mu\nu}(q,r,p) - \Delta^{-1}(p)P_{\nu}^{\mu}(p)H_{\mu\alpha}(p,r,q)]\,,}$$

where the gluon propagator, in **Landau gauge** is

$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q)\,, \qquad P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}\,,$$

and the ghost propagator is denoted by

$$D(q) = \frac{iF(q)}{q^2}\,.$$

- If $\Delta(q)$, $F(q)$ and $H_{\nu\mu}(q,p,r)$ are known, one can solve the STI for the three-gluon vertex[*].
- Indeed, that was one of our main interests in studying $H_{\nu\mu}(q,p,r)$[**].

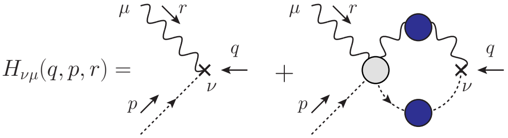
[*] J. S. Ball and T. W. Chiu, Phys. Rev. D **22**, 2550 (1980) Erratum: [Phys. Rev. D **23**, 3085 (1981)].
[**] A. C. Aguilar, C. T. Figueiredo, M. N. F. and J. Papavassiliou, in preparation.

Properties of the ghost-gluon scattering kernel

The most general tensor structure of the ghost-gluon scattering kernel is

$$H_{\nu\mu}(q,p,r) = g_{\mu\nu}A_1 + q_\mu q_\nu A_2 + r_\mu r_\nu A_3 + q_\mu r_\nu A_4 + r_\mu q_\nu A_5\,,$$

where $A_i \equiv A_i(q,p,r)$.

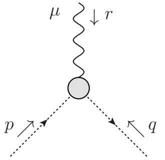


The scalar form factors $A_i(q,p,r)$ are the objects to compute.
At tree level, $A_1^{(0)}(q,p,r) = 1$, and the others are zero, so that

$$\boxed{H^{(0)}_{\nu\mu}(q,p,r) = g_{\nu\mu}\,.}$$

In its turn, the ghost-gluon vertex has the Lorentz structure

$$\Gamma_\mu(q,p,r) = q_\mu B_1(q,p,r) + r_\mu B_2(q,p,r)\,.$$



$$= \Gamma_\mu(q,p,r)$$

From, $q^\nu H_{\nu\mu}(q,p,r) = \Gamma_\mu(q,p,r)$, we can relate

$$\begin{aligned} B_1(q,p,r) =& A_1(q,p,r) + q^2 A_2(q,p,r) + q\cdot r A_4(q,p,r)\,,\\ B_2(q,p,r) =& q\cdot r A_3(q,p,r) + q^2 A_5(q,p,r)\,. \end{aligned}$$

At tree level, $B_1^{(0)}(q,p,r) = 1$ and $B_2^{(0)}(q,p,r) = 0$, so that

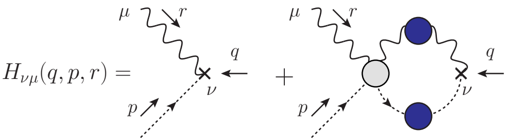
$$\boxed{\Gamma_\mu^{(0)} = q_\mu\,.}$$

The **Taylor theorem*** guarantee's that in **Landau gauge**,

$$H_{\nu\mu}(q,p,r) = g_{\nu\mu} + p^{\rho} I_{\rho\nu\mu}(q,p,r)\,,$$

i.e. the ghost momentum, p, factorizes.

This is an immediate consequence of the transversality of the gluon propagator.*



$$\boxed{\text{Then, for } p = 0 \Rightarrow H_{\nu\mu}(q,0,-q) = H^{(0)}_{\nu\mu}(q,0,-q) = g_{\nu\mu}\,.}$$

*J. C. Taylor, Nucl. Phys. B **33**, 436 (1971).

In particular, this implies that $H_{\nu\mu}(q,p,r)$ is finite in Landau gauge.

From $q^{\nu}H_{\nu\mu}(q,p,r) = \Gamma_{\mu}(q,p,r)$, the renormalization constant is the same for both, the scattering kernel and the vertex. Let

$$\Gamma_R^{\mu}(q,p,r) = Z_1 \Gamma^{\mu}(q,p,r)\,,$$

then, Z_1 is finite in Landau gauge.

Moreover, if we renormalize in the **Taylor scheme**[*],

$$\Gamma_R^{\mu}(q,0,-q) = q^{\mu}\,,$$

i.e. tree level for $p=0$, then

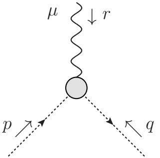
$$\boxed{Z_1 = 1\,.}$$

We will see that in the MOM scheme Z_1 would be only slightly different.

[*]P. Boucaud, F. De Soto, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene and J. Rodriguez-Quintero, Phys. Rev. D **79**, 014508 (2009).

Also in the Landau gauge, the "**classical**" form factor of the vertex has **ghost-anti-ghost symmetry**[*]

$$B_1(q,p,r) = B_1(p,q,r)\,.$$



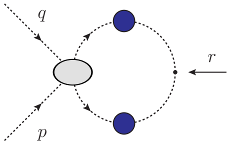
$$= \Gamma_\mu(q,p,r)$$

[*] A. C. Aguilar, D. Ibáñez and J. Papavassiliou, Phys. Rev. D **87** (2013) 11, 114020.
[*] R. Alkofer and L. von Smekal, Phys. Rept. **353**, 281 (2001).

The ghost-gluon scattering kernel satisfies the STI*

$$\boxed{r^\nu F^{-1}(r) H_{\mu\nu}(q,p,r) + p^\nu F^{-1}(p) H_{\mu\nu}(q,r,p) = -q_\mu F^{-1}(q) C(r,p,q)\,,}$$

where

$C(r,p,q) = 1 +$ 

Contracting with $P_\alpha^\mu(q)$ and rearranging one finds

$$\mathcal{R}(q^2,p^2,r^2) = \frac{F(r)[A_1(q,r,p) + p^2 A_3(q,r,p) + (q\cdot p)A_4(q,r,p)]}{F(p)[A_1(q,p,r) + r^2 A_3(q,p,r) + (q\cdot r)A_4(q,p,r)]} = 1\,.$$

*D. Binosi and J. Papavassiliou, JHEP **1103**, 121 (2011).

The truncation of an SDE may lead to the violation of the symmetries of the theory.

For instance,

$$\mathcal{R}(q^2, p^2, r^2) = 1 \,,$$

may cease to hold for a truncated $H_{\nu\mu}(q, p, r)$.

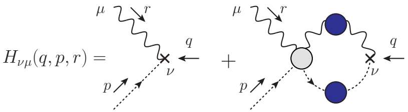
In this case, we may take

$$\boxed{\mathcal{R}(q^2, p^2, r^2) \equiv \frac{F(r)[A_1(q,r,p) + p^2 A_3(q,r,p) + (q\cdot p)A_4(q,r,p)]}{F(p)[A_1(q,p,r) + r^2 A_3(q,p,r) + (q\cdot r)A_4(q,p,r)]}}$$

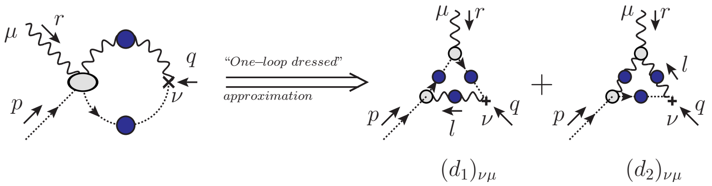
as a **measure of the violation of the STI.**

Truncation

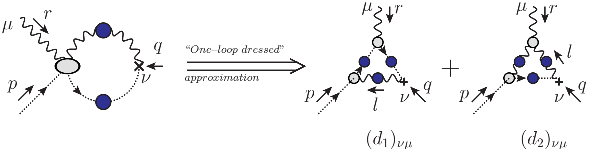
We truncate the SDE for $H_{\nu\mu}(q,p,r)$ at the "**one-loop dressed**" level*,



where



*A. C. Aguilar, C. T. Figueiredo, M. N. F. and J. Papavassiliou, in preparation.



Then,

$$H_{\nu\mu}(q,p,r) = g_{\nu\mu} + (d_1)_{\nu\mu} + (d_2)_{\nu\mu}\,,$$

where

$$(d_1)_{\nu\mu} = \frac{g^2 C_{\rm A}}{2} p_\rho \int_\ell \Delta_\nu^\rho(\ell) D(\ell+p) D(\ell-q) \Gamma_\mu(q-\ell,\ell+p,r) B_1(-\ell-p,p,\ell)\,,$$
$$(d_2)_{\nu\mu} = \frac{g^2 C_{\rm A}}{2} p_\rho \int_\ell \Delta_\nu^\alpha(\ell) \Delta^{\beta\rho}(\ell+r) D(\ell-q) \Gamma_{\mu\alpha\beta}(r,\ell,-\ell-r) B_1(q-\ell,p,\ell+r)\,,$$

and we use the notation

$$\int_\ell \equiv \frac{\mu^\epsilon}{(2\pi)^d} \int \mathrm{d}^d\ell\,.$$

Notice the overall factors of p_ρ in both diagrams, in agreement with the Taylor theorem.

The renormalization of the SDE is achieved by replacing

$$\begin{aligned}
\Delta_R(q) =& Z_A^{-1}\Delta(q)\,,\\
F_R(q) =& Z_c^{-1}F(q)\,,\\
\Gamma^{\mu}_R(q,p,r) =& Z_1\Gamma^{\mu}(q,p,r)\,,\\
\Gamma^{\mu\alpha\beta}_R(q,p,r) =& Z_3\Gamma^{\mu\alpha\beta}(q,p,r)\,,\\
g_R = Z_g^{-1}g =& Z_1^{-1}Z_A^{1/2}Z_c g\,,
\end{aligned}$$

and invoking the STI for the renormalization constants*

$$\boxed{\frac{Z_3}{Z_A} = \frac{Z_1}{Z_c}\,.}$$

Then one finds

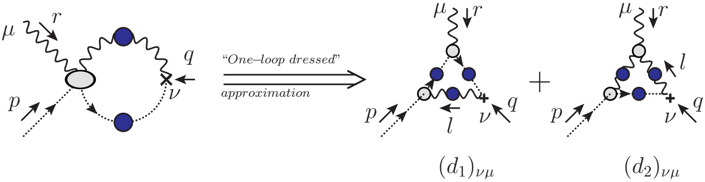
$$H^{\nu\mu}_R(q,p,r) = Z_1[g^{\nu\mu} + (d_1)^{\nu\mu}_R + (d_2)^{\nu\mu}_R]\,,$$

and may set $Z_1 = 1$, in the Taylor scheme.**

*J. C. Taylor, Nucl. Phys. B **33**, 436 (1971).

P. Boucaud, F. De Soto, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene and J. Rodriguez-Quintero, Phys. Rev. D **79, 014508 (2009).

The ghost-anti-ghost symmetry is violated as a consequence of truncation keeping only corrections to the ghost leg.



This is remedied by **averaging** over ghost and anti-ghost dressings*, which amounts to the substitutions

$$B_1(-\ell-p,p,\ell) \to \mathcal{V}_1(\ell,q,p,r) = \frac{1}{2}[B_1(-\ell-p,p,\ell)+B_1(q,\ell-q,-\ell)]\,,$$
$$B_1(q-\ell,p,\ell+r) \to \mathcal{V}_2(\ell,q,p,r) = \frac{1}{2}[B_1(q-\ell,p,\ell+r)+B_1(q,\ell-q,-\ell)]\,,$$

into $(d_1)_{\nu\mu}$ and $(d_2)_{\nu\mu}$, respectively.

* Similar procedures:
A. Blum, M. Q. Huber, M. Mitter and L. von Smekal, Phys. Rev. D **89**, 061703 (2014).
A. K. Cyrol, M. Q. Huber and L. von Smekal, Eur. Phys. J. C **75**, 102 (2015).

With the “symmetrized” version of the one-loop dressed truncation we preserve

- the Taylor theorem, effortlessly;
- and the ghost-anti-ghost symmetry, with only a slight adjustment.

The STI for $H_{\nu\mu}(q, p, r)$, however, is violated, as we shall see.

Inputs and additional approximations

We found by experience that the STI constraint

$$\mathcal{R}(q^2,p^2,r^2) \equiv \frac{F(r)[A_1(q,r,p)+p^2A_3(q,r,p)+(q\cdot p)A_4(q,r,p)]}{F(p)[A_1(q,p,r)+r^2A_3(q,p,r)+(q\cdot r)A_4(q,p,r)]} = 1\,,$$

is **violated by the truncated** $H_{\nu\mu}(q,p,r)$.

Moreover, **the violation is worse in the UV**.

- This is due to higher order corrections that should cancel with others that are missing.

This problem can be somewhat mitigated by constructing inputs that reduce continuously to tree level in the UV, then

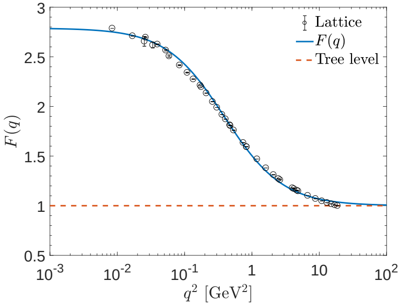
$$\boxed{\text{Tree level in the UV as input} \;\Rightarrow\; \text{one-loop in the UV as output.}}$$

For instance, for the ghost dressing function we use a fit to lattice data*,

$$F(q) = 1 + \frac{\sigma_1}{q^2 + \sigma_2}\,,$$

such that

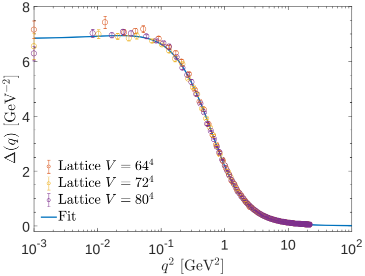
$$\lim_{q^2\to\infty} F(q) = 1\,.$$

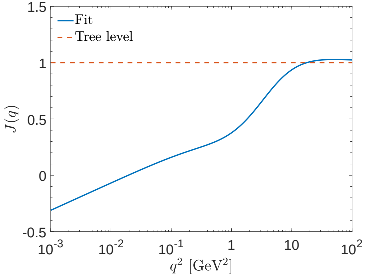


Throughout, we shall use $\mu = 4.3$ GeV and $\alpha_s(\mu) = 0.22$.

*I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, PoS LATTICE **2007**, 290 (2007).

Something on the same lines may be used for the gluon propagator.





Specifically,

$$\Delta^{-1}(q) = q^2 J(q) + m^2(q)\,,$$

where the kinetic term is given by

$$J(q) = 1 + \frac{C_{\rm A}\alpha_s}{4\pi}\left(\frac{\tau_1}{q^2+\tau_2}\right)\left[2\ln\left(\frac{q^2+\rho_l m^2(q)}{\mu^2}\right) + \frac{1}{6}\ln\left(\frac{q^2}{\mu^2}\right)\right]\,,$$

and the dynamical mass by

$$m^2(q) = \frac{m_0^2}{1+q^2/\rho_m^2}\,.$$

I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, PoS LATTICE **2007**, 290 (2007).

For the ghost-gluon vertex that appears as input, we do the following:

1. Run the SDE with bare vertices.
2. Keep only the tree level structure

$$\Gamma_\mu(q,p,r) \to q_\mu B_1(q,p,r)\,.$$

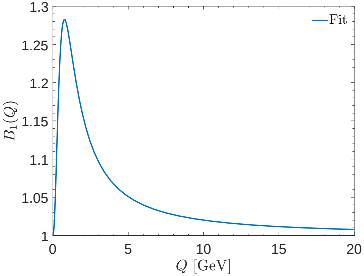
3. Fit the resulting $B_1(q,p,r)$ in the symmetric point,

$$q^2 = p^2 = r^2 = Q^2\,,$$
$$q.p = q.r = p.r = -\frac{Q^2}{2}\,,$$

$$B_1(Q) = 1 + \frac{\tau_1 Q^2}{(1+\tau_2 Q^2)^\lambda}\,,$$

which satisfies

$$\lim_{Q^2\to\infty} B_1(Q) = 1\,.$$



$$\boxed{r^{\mu}\Gamma_{\alpha\mu\nu}(q,r,p) = F(r)[\Delta^{-1}(q)P_{\alpha}^{\mu}(q)H_{\mu\nu}(q,r,p) - \Delta^{-1}(p)P_{\nu}^{\mu}(p)H_{\mu\alpha}(p,r,q)]\,,}$$

As for the three-gluon vertex, we keep only the tree level structure and use an "Abelianized" Ball-Chiu ansatz,

$$\begin{aligned}\overline{\Gamma}_{\mu\alpha\beta}(r,\ell,t) =& g_{\mu\alpha}(r-\ell)_{\beta}\overline{X}_1(r,\ell,t) + g_{\alpha\beta}(\ell-t)_{\mu}\overline{X}_1(\ell,t,r) \\ &+ g_{\beta\mu}(t-r)_{\alpha}\overline{X}_1(t,r,\ell)\,,\end{aligned}$$

where $t = -r-\ell$ and

$$\overline{X}_1(r,\ell,t) = \frac{1}{2}[J(r)+J(\ell)]\,.$$

This simplified ansatz preserves the **Bose symmetry** of $\Gamma_{\mu\alpha\beta}(r,\ell,t)$ and the feature of a **zero-crossing**, and…

- …reduces to tree level as long as $J(q)$ does.

Numerical results for $H_{\nu\mu}(q,p,r)$

In Euclidean space it is convenient to use spherical coordinates, such that

$$A_1(q,p,r) \to A_1(q^2,p^2,\theta)\,,$$

where

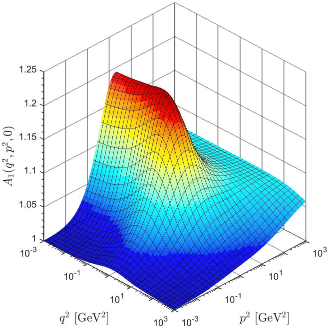
$$q\cdot p = |q||p|\cos\theta\,,$$

and similarly for the other form factors.

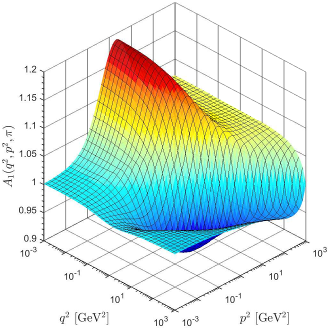
Form factor A_1, recalling $A_1^{(0)} = 1$ and

$$H_{\nu\mu}(q,p,r) = g_{\nu\mu}\boxed{A_1} + q_\mu q_\nu A_2 + r_\mu r_\nu A_3 + q_\mu r_\nu A_4 + q_\nu r_\mu A_5 \,.$$

$\theta = 0$



$\theta = \pi$

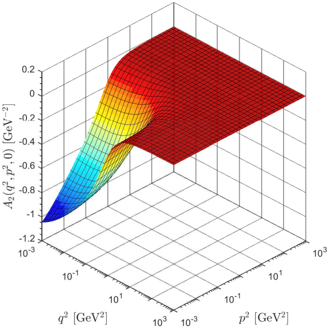


Differs significantly from tree level in the IR.

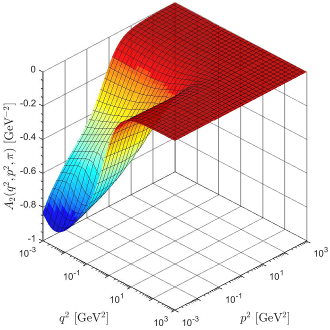
Form factor A_2, recalling $A_2^{(0)} = 0$ and

$$H_{\nu\mu}(q,p,r) = g_{\nu\mu}A_1 + q_\mu q_\nu \boxed{A_2} + r_\mu r_\nu A_3 + q_\mu r_\nu A_4 + q_\nu r_\mu A_5 \,.$$

$\theta = 0$



$\theta = \pi$

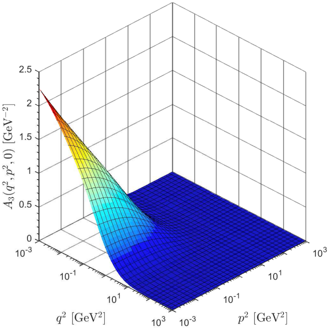


Differs significantly from tree level in the IR.

Form factor A_3, recalling $A_3^{(0)} = 0$ and

$$H_{\nu\mu}(q,p,r) = g_{\nu\mu}A_1 + q_\mu q_\nu A_2 + r_\mu r_\nu \boxed{A_3} + q_\mu r_\nu A_4 + q_\nu r_\mu A_5 \,.$$

$\theta = 0$



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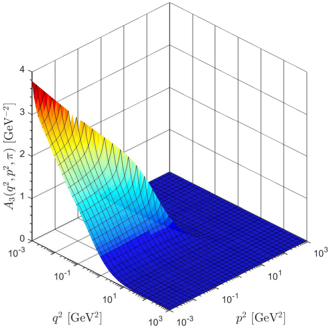
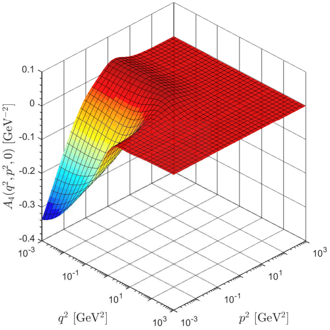


Differs significantly from tree level in the IR.

Form factor A_4, recalling $A_4^{(0)} = 0$ and

$$H_{\nu\mu}(q,p,r) = g_{\nu\mu}A_1 + q_\mu q_\nu A_2 + r_\mu r_\nu A_3 + q_\mu r_\nu \boxed{A_4} + q_\nu r_\mu A_5\,.$$

$\theta = 0$



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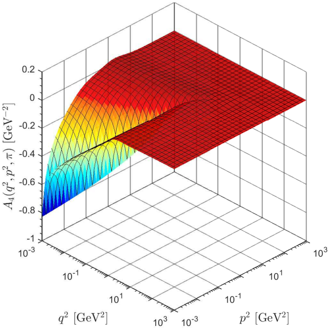
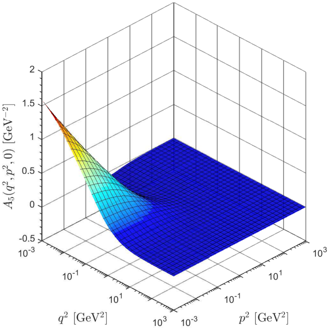


Differs significantly from tree level in the IR.

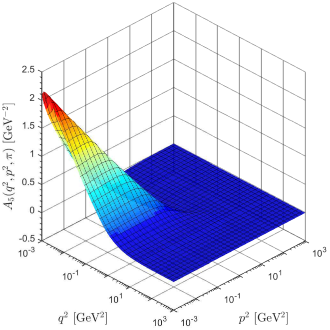
Form factor A_5, recalling $A_5^{(0)} = 0$ and

$$H_{\nu\mu}(q,p,r) = g_{\nu\mu}A_1 + q_\mu q_\nu A_2 + r_\mu r_\nu A_3 + q_\mu r_\nu A_4 + q_\nu r_\mu \boxed{A_5}\,.$$

$\theta = 0$



$\theta = \pi$



Differs significantly from tree level in the IR.

We can check the UV behavior by comparing the results with a one-loop calculation*.

At one-loop

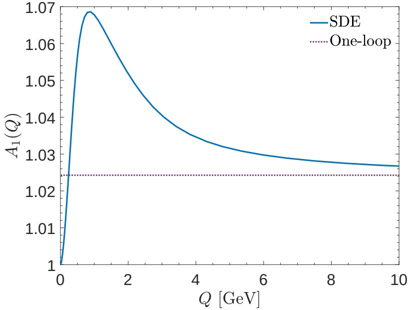
$$A_1^{(1)}(Q) = 1 + \frac{C_{\rm A}\alpha_s}{288\pi}\left[27 + \mathcal{I}\right] ,$$

where

$$\mathcal{I} \equiv \psi_1\left(\frac{1}{3}\right) - \psi_1\left(\frac{2}{3}\right) \approx 7.03 \, ,$$

and

$$\psi_1(z) = \frac{d^2}{dz^2}\ln[\Gamma(z)] \, .$$



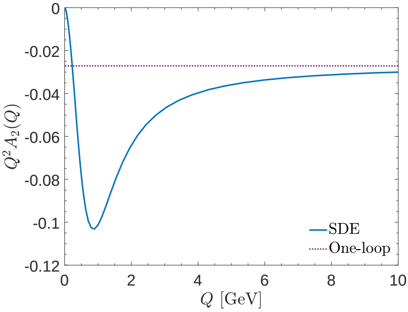
At $\mu = 4.3$ GeV, $A_1(\mu) = 1.033$, only 3% larger than tree level.

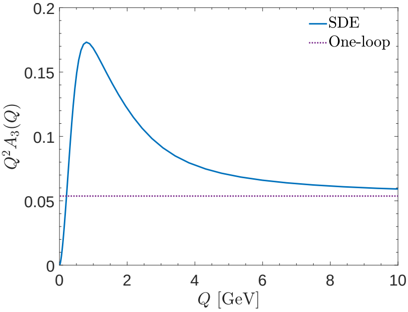
*Our perturbative results agree with those of:
A. I. Davydychev, P. Osland and O. V. Tarasov, Phys. Rev. D **54**, 4087 (1996).
Erratum: [Phys. Rev. D **59**, 109901 (1999)].

Similarly, for the other form factors.

$$Q^2 A_2^{(1)}(Q) = -\frac{C_\mathrm{A}\alpha_s}{144\pi}\left[12+\mathcal{I}\right] ,$$

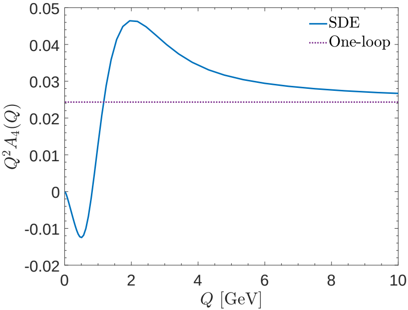
$$Q^2 A_3^{(1)}(Q) = \frac{C_\mathrm{A}\alpha_s}{96\pi}\left[4+3\mathcal{I}\right] .$$

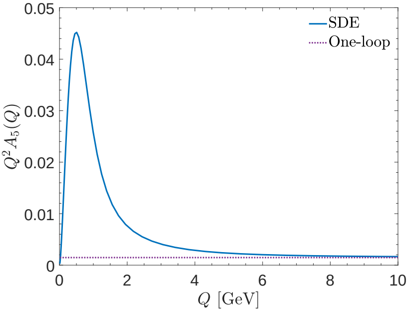




And finally,

$$Q^2 A_4^{(1)}(Q) = \frac{C_{\rm A}\alpha_s}{144\pi}\left[3 + 2\mathcal{I}\right] , \qquad Q^2 A_5^{(1)}(Q) = \frac{C_{\rm A}\alpha_s}{144\pi}\left[-6 + \mathcal{I}\right] .$$





The STI constraint

We now turn to the STI constraint over $H_{\nu\mu}(q,p,r)$.

We shall consider the ratio

$$\mathcal{R}(q^2,p^2,r^2) \equiv \frac{F(r)[A_1(q,r,p) + p^2 A_3(q,r,p) + (q\cdot p)A_4(q,r,p)]}{F(p)[A_1(q,p,r) + r^2 A_3(q,p,r) + (q\cdot r)A_4(q,p,r)]}$$

as a measure of the violation of the STI by the truncation.

Recall that the STI requires

$$\mathcal{R}(q^2,p^2,r^2) = 1\,.$$

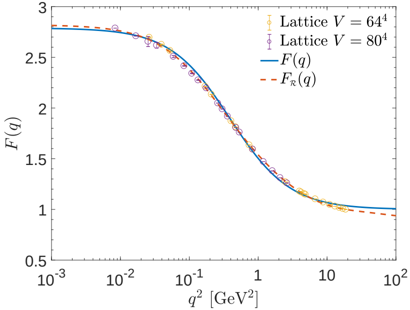
- To compute $H_{\nu\mu}(q,p,r)$ we used inputs that tend to tree level in the UV.
- Consequently our $H_{\nu\mu}(q,p,r)$ reduces to the one-loop level in the UV.
- To maintain the one-loop behavior for $\mathcal{R}(q^2,p^2,r^2)$, we shall use an $F(q)$ that reduces to one-loop in the UV as well.

We shall use*

$$F_{\mathcal{R}}(q) = 1 + \frac{9C_{\rm A}\alpha_s}{48\pi}\left(1 + De^{-\rho_4 q^2}\right) \times \ln\left(\frac{q^2 + \rho_3 M^2(q)}{\mu^2}\right),$$

where

$$M^2(q) = \frac{m^2}{1 + q^2/\rho_2^2}.$$



*A. C. Aguilar, J. C. Cardona, M. N. F. and J. Papavassiliou, Phys. Rev. D **98**, no. 1, 014002 (2018).

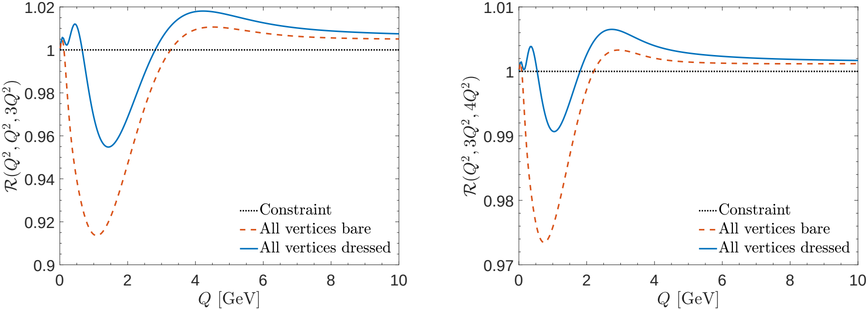
We can always rewrite,

$$\mathcal{R}(q^2, p^2, r^2) = \mathcal{R}(Q^2, aQ^2, bQ^2)\,,$$

i.e. $q^2 = Q^2$, $p^2 = aQ^2$ and $r^2 = bQ^2$.

We compare $H_{\nu\mu}$ with bare vertices vs. dressed vertices in the SDE.

$a = 1\,, \qquad b = 3\,.$ $\qquad\qquad$ $a = 3\,, \qquad b = 4\,.$



With dressed vertices, deviation from $\mathcal{R} = 1$ is less than 5%.

Implications for the ghost-gluon vertex

Once $H_{\nu\mu}(q,p,r)$ has been computed, we may obtain $\Gamma_{\mu}(q,p,r)$ from

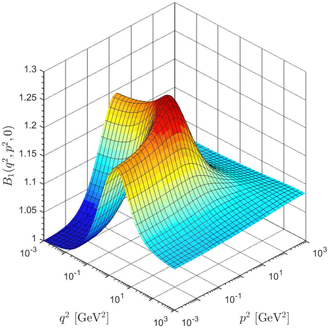
$$q^{\nu}H_{\nu\mu}(q,p,r) = \Gamma_{\mu}(q,p,r)\,.$$

For the vertex there are more nonperturbative results to compare.

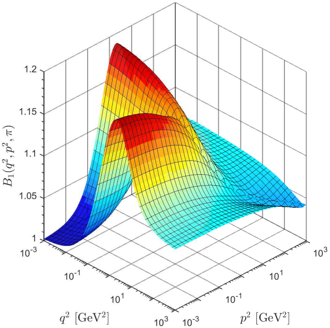
Form factor B_1, recalling $B_1^{(0)} = 1$ and

$$\Gamma_\mu(q,p,r) = q_\mu \boxed{B_1(q,p,r)} + r_\mu B_2(q,p,r)\,.$$

$\theta = 0$



$\theta = \pi$

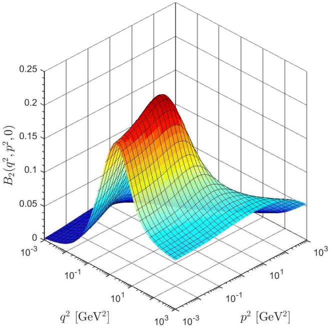


Notice the **ghost-anti-ghost symmetry.**

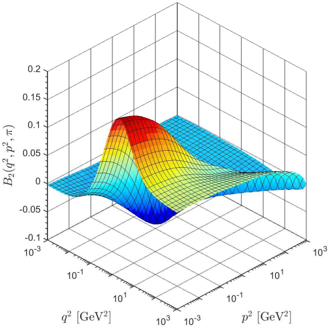
Form factor B_2, recalling $B_2^{(0)} = 0$ and

$$\Gamma_\mu(q,p,r) = q_\mu B_1(q,p,r) + r_\mu \boxed{B_2(q,p,r)}\,.$$

$\theta = 0$

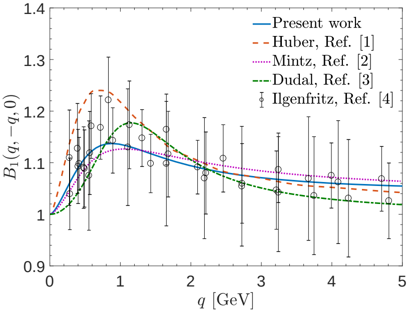


$\theta = \pi$



Differs significantly from tree level in the IR.

In the **soft gluon kinematics**, $r = 0$, we may compare B_1 to previous works.

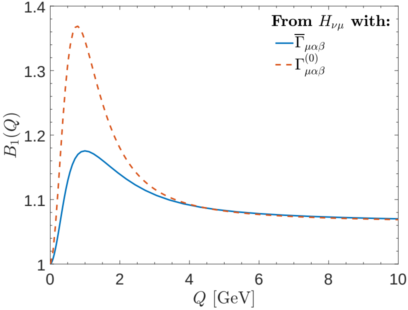


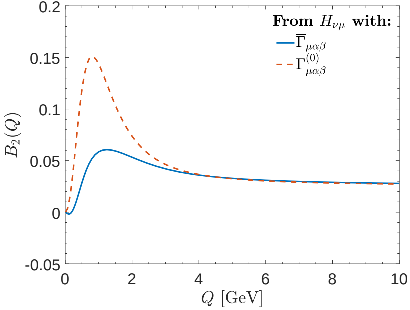
[1] M. Q. Huber and L. von Smekal, JHEP **1304**, 149 (2013).
[2] B. W. Mintz, L. F. Palhares, S. P. Sorella and A. D. Pereira, Phys. Rev. D **97**, no. 3, 034020 (2018).
[3] D. Dudal, O. Oliveira and J. Rodriguez-Quintero, Phys. Rev. D **86**, 105005 (2012).
[4] E.-M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, A. Schiller and I. L. Bogolubsky, Braz. J. Phys. **37**, 193 (2007).

The ansatz for the three-gluon vertex has a major effect on the ghost-gluon one.

This is seen by running the SDE again with bare $\Gamma_{\mu\alpha\beta}$ and comparing the results.

In the symmetric point we find:





Suppression of the amplitude on the 50% order around 1 GeV with dressed three-gluon vertex.

This suppression happens because our ansatz for the three-gluon vertex goes below one in the IR:

Recall

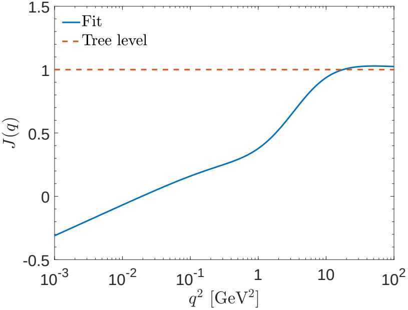
$$\overline{\Gamma}_{\mu\alpha\beta}(r,\ell,t) = g_{\mu\alpha}(r-\ell)_\beta \overline{X}_1(r,\ell,t) + g_{\alpha\beta}(\ell-t)_\mu \overline{X}_1(\ell,t,r) + g_{\beta\mu}(t-r)_\alpha \overline{X}_1(t,r,\ell)\,,$$

with

$$\overline{X}_1(r,\ell,t) = \frac{1}{2}[J(r)+J(\ell)]\,.$$

In the symmetric point,

$$\boxed{\overline{X}_1(Q) = J(Q)\,.}$$



- The exact amount of suppression is model dependent,
- but the lattice results for $\Gamma_{\mu\alpha\beta}$ do display the same behavior*.

The following projection has been simulated on the lattice,

$$L^{sym}(q,p,r) = \frac{-iW^{\alpha\mu\nu}(q,r,p)\Delta_{\alpha}^{\alpha'}(q)\Delta_{\mu}^{\mu'}(r)\Delta_{\nu}^{\nu'}(p)\Gamma_{\alpha'\mu'\nu'}(q,r,p)}{\Delta(q)\Delta(r)\Delta(p)W^{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,r,p)} \,,$$

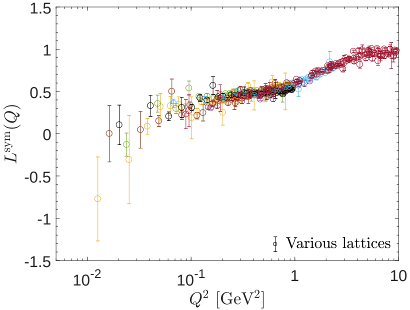
where the projector

$$W_{\alpha\mu\nu}(q,p,r) = \lambda^{tree}_{\alpha\mu\nu}(q,r,p) + \frac{\lambda^{s}_{\alpha\mu\nu}(q,r,p)}{2} \,,$$

is defined in terms of the transverse tensors

$$\lambda^{tree}_{\alpha\mu\nu}(q,r,p) = \Gamma^{(0)}_{\alpha'\mu'\nu'}(q,r,p)P_{\alpha}^{\alpha'}(q)P_{\mu}^{\mu'}(r)P_{\nu}^{\nu'}(p) \,,$$

$$\lambda^{s}_{\alpha\mu\nu}(q,r,p) = (r-p)_{\alpha}(p-q)_{\mu}(q-r)_{\nu}/r^2 \,,$$



*A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J. Papavassiliou, J. Rodriguez-Quintero and S. Zafeiropoulos, Phys. Lett. B **761**, 444 (2016).

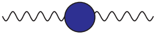
Conclusions

- A one-loop dressed SDE for the ghost-gluon scattering kernel can be devised that preserves its Landau gauge properties,
    - The Taylor theorem;
    - and the ghost-anti-ghost symmetry.
- However, this truncation does violate the STI that $H_{\nu\mu}(q,p,r)$ should satisfy.
- With a careful choice of inputs the problem can be minimized, albeit not entirely avoided.
- The resulting $H_{\nu\mu}(q,p,r)$ displays significant departure from its perturbative behavior in the IR.
- Moreover, $H_{\nu\mu}(q,p,r)$ suffers a suppression in the IR when the three gluon vertex is dressed, in accordance with the effect observed in several other quantities.

Backup

$H_{\nu\mu}(q,p,r)$ also plays a role in relating Background and Quantum Green's functions, by participating in some **Background-Quantum Identities***.

For instance

 $= (1+G)^{-2} \otimes$ 

$$\widehat{\Delta}(q) = [1+G(q)]^{-2}\Delta(q)$$

 $= (1+G)^{-1} \otimes$ 

$$\widetilde{\Delta}(q) = [1+G(q)]^{-1}\Delta(q)$$

where

$$1+G(q) = \frac{P^{\mu\nu}(q)}{(d-1)} \int_k H^{(0)}_{\rho\mu} D(p-k)\Delta^{\rho\sigma}(k) H_{\sigma\nu}(-p-k,p,k)\,.$$

*D. Binosi and J. Papavassiliou, Phys. Rev. D **66**, 025024 (2002).
*P. A. Grassi, T. Hurth and M. Steinhauser, Annals Phys. **288**, 197 (2001).

The form factors A_i can be obtained by projecting out the truncated SDE for $H_{\nu\mu}(qp, r)$,

$$\boxed{A_i(q,p,r) = \frac{\mathcal{T}_i^{\mu\nu}(q,r) H_{\nu\mu}(q,p,r)}{2h^2(q,r)}\,,}$$

where

$$\begin{aligned}
\mathcal{T}_1^{\mu\nu}(q,r) =& h(q,r)\left[h(q,r)g^{\mu\nu} + h^{\mu\nu}(q,r)\right]\,,\\
\mathcal{T}_2^{\mu\nu}(q,r) =& -h(q,r)r^2 g^{\mu\nu} - 2h(q,r)r^\mu r^\nu - 3r^2 h^{\mu\nu}(q,r)\,,\\
\mathcal{T}_3^{\mu\nu}(q,r) =& \mathcal{T}_2^{\mu\nu}(r,q)\,,\\
\mathcal{T}_4^{\mu\nu}(q,r) =& h(q,r)(r\cdot q)g^{\mu\nu} + 2h(q,r)q^\mu r^\nu + 3(r\cdot q)h^{\mu\nu}(q,r)\,,\\
\mathcal{T}_5^{\mu\nu}(q,r) =& \mathcal{T}_4^{\mu\nu}(r,q)\,,
\end{aligned}$$

and

$$\begin{aligned}
h(q,r) =& q^2 r^2 - (q\cdot r)^2\,,\\
h^{\mu\nu}(q,r) =& (q\cdot r)\left[q^\mu r^\nu + q^\nu r^\mu\right] - r^2 q^\mu q^\nu - q^2 r^\mu r^\nu\,.
\end{aligned}$$

More perturbative comparisons: Soft anti-ghost $H_{\nu\mu}(q,p,r)$

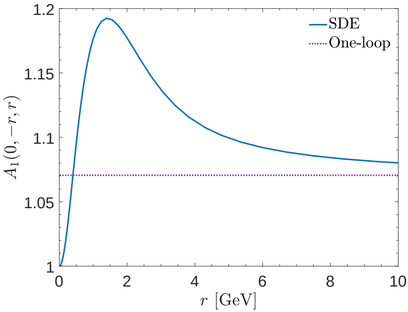
In the soft anti-ghost configuration, *i.e.* $q=0$,

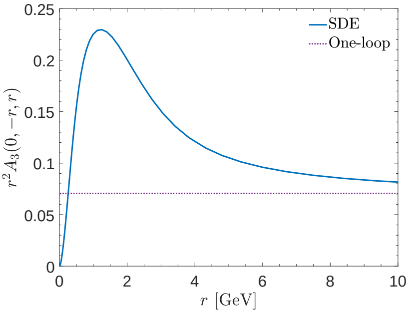
$$H^{(1)}_{\nu\mu}(0,-r,r) = A^{(1)}_1(0,-r,r)g_{\mu\nu} + A^{(1)}_3(0,-r,r)r_\mu r_\nu \,,$$

where

$$A^{(1)}_1(0,-r,r) = 1 + \frac{11 C_{\rm A}\alpha_s}{32\pi}\,; \qquad A^{(1)}_3(0,-r,r) = \frac{11 C_{\rm A}\alpha_s}{32\pi r^2}\,.$$

The **nonperturbative** results are in good agreement with the **perturbative** ones.





More perturbative comparisons: Soft gluon $H_{\nu\mu}(q,p,r)$

In the soft gluon configuration, *i.e.* $r=0$,

$$H^{(1)}_{\nu\mu}(q,-q,0) = A^{(1)}_1(q,-q,0)g_{\mu\nu} + A^{(1)}_2(q,-q,0)q_\mu q_\nu \,,$$

where, in this case

$$A^{(1)}_1(q,-q,0) = 1\,; \qquad A^{(1)}_2(q,-q,0) = -\frac{3C_{\rm A}\alpha_s}{16\pi q^2}\,.$$

The **nonperturbative** results are in good agreement with the **perturbative** ones.