Critical end point (CEP) in the presence of a chiral chemical potential (μ_5)

Zhu-Fang Cui（崔著钫）

In collaboration with Shu-Sheng Xu, Ya Lu, Craig Roberts, *et al.*

Phys. Rev. D 91, 034017 (2015); Phys. Rev. D 91, 056003 (2015);

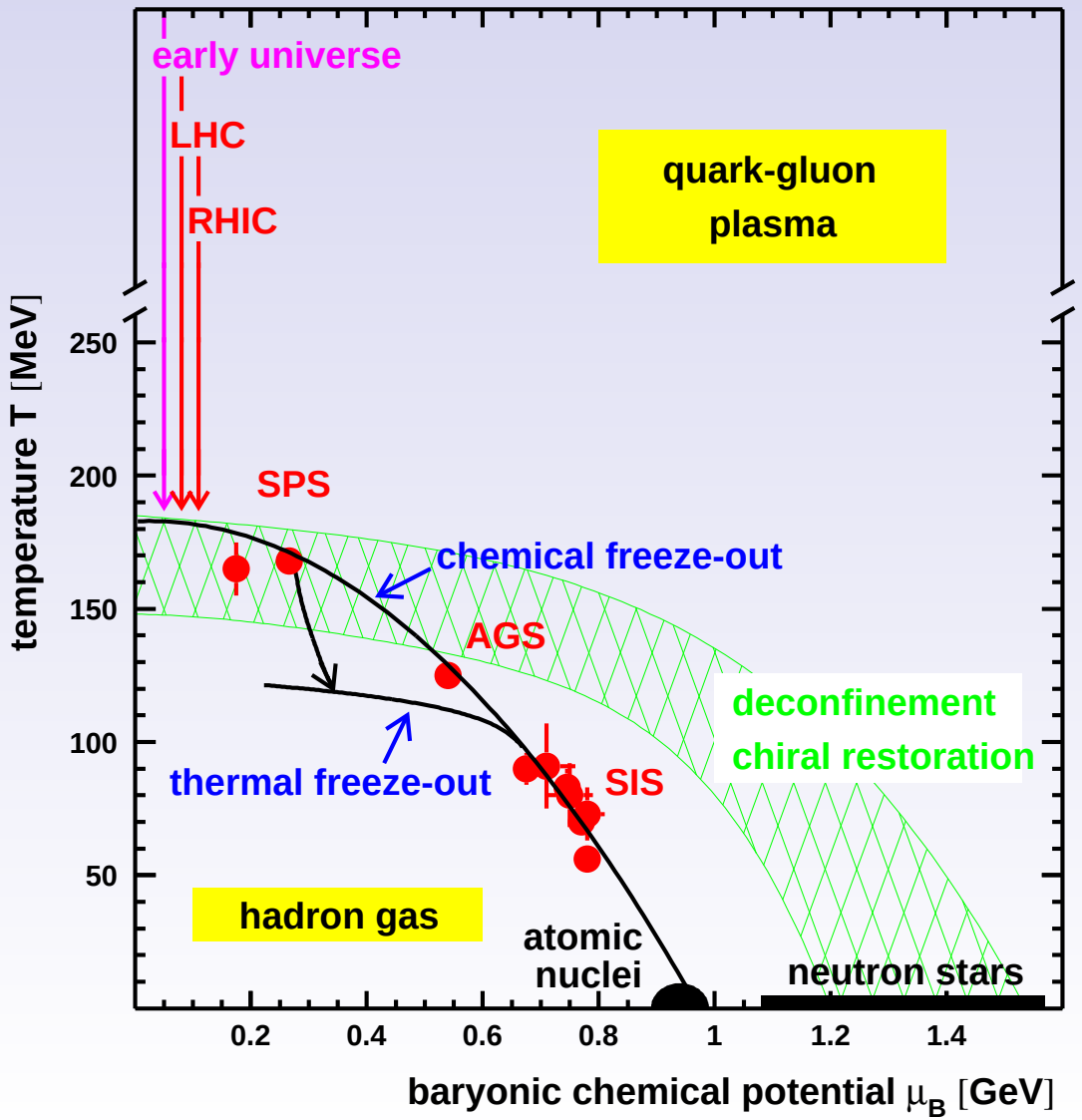
Phys. Rev. D 93, 074037 (2016); Phys. Rev. D 94, 071503(R) (2016).

Nanjing University, China

NPQCD2018, Sevilla. November 8, 2018

Outline

Conjectured QCD phase diagram and CEP



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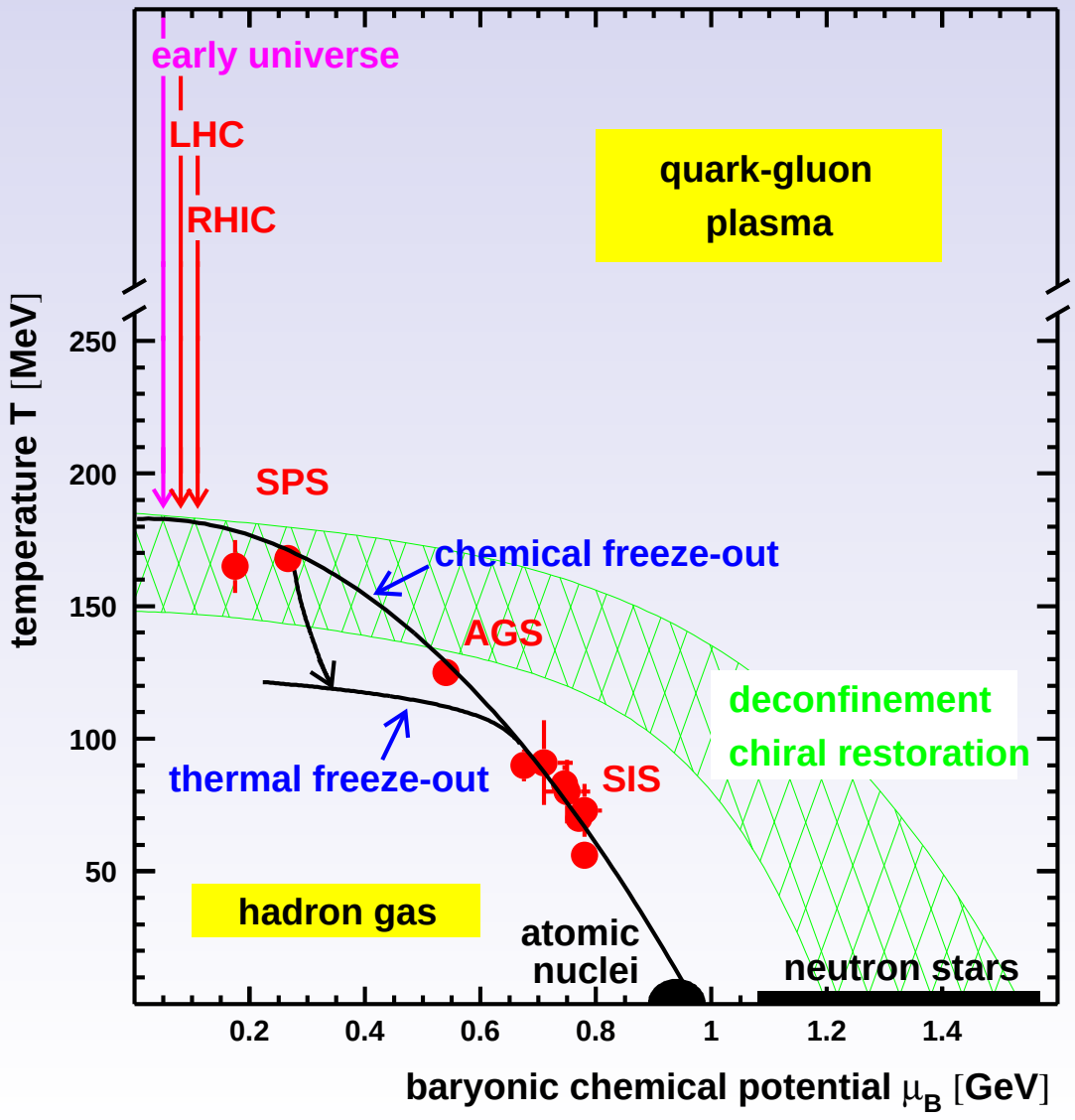

Questions:

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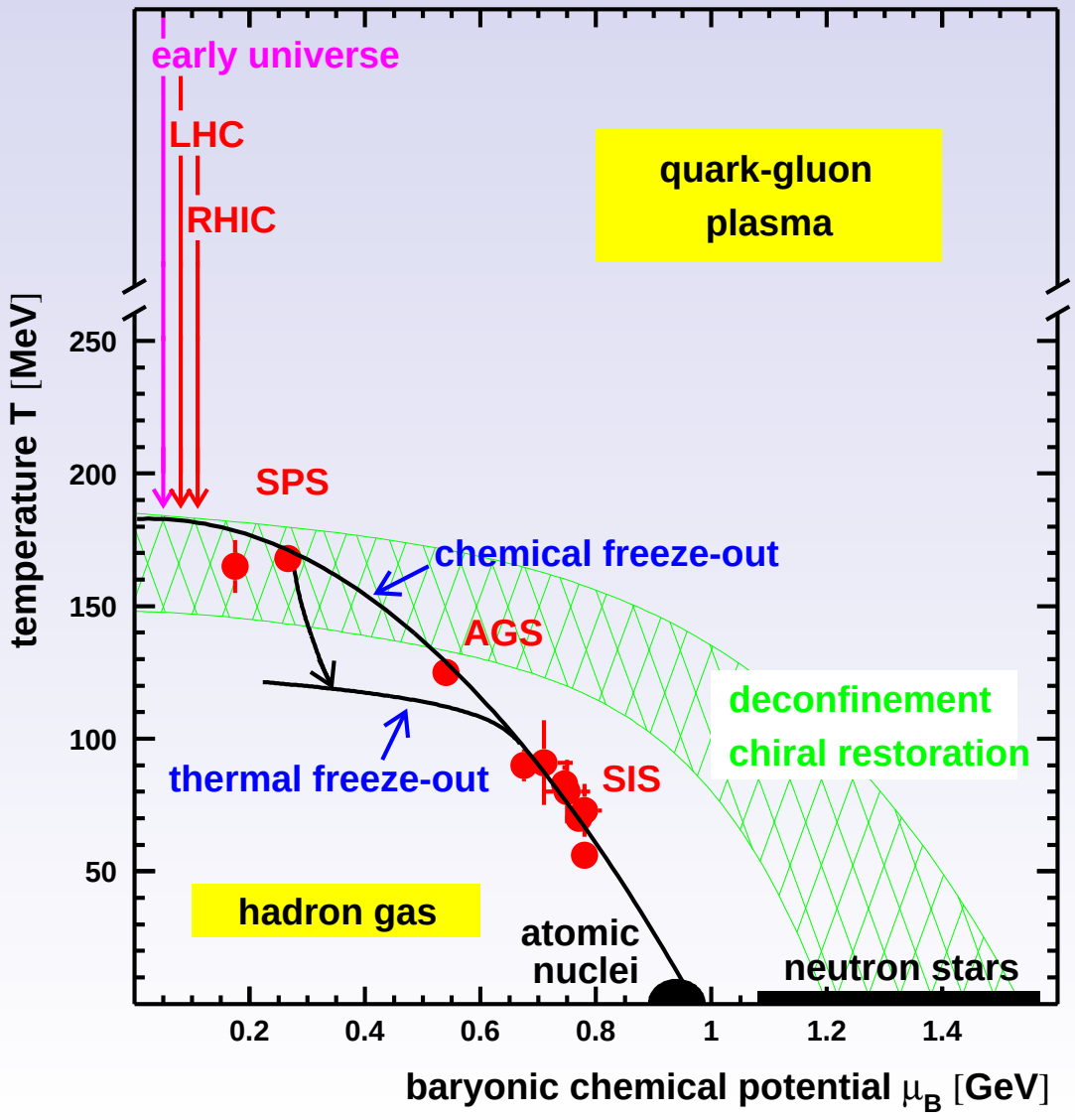

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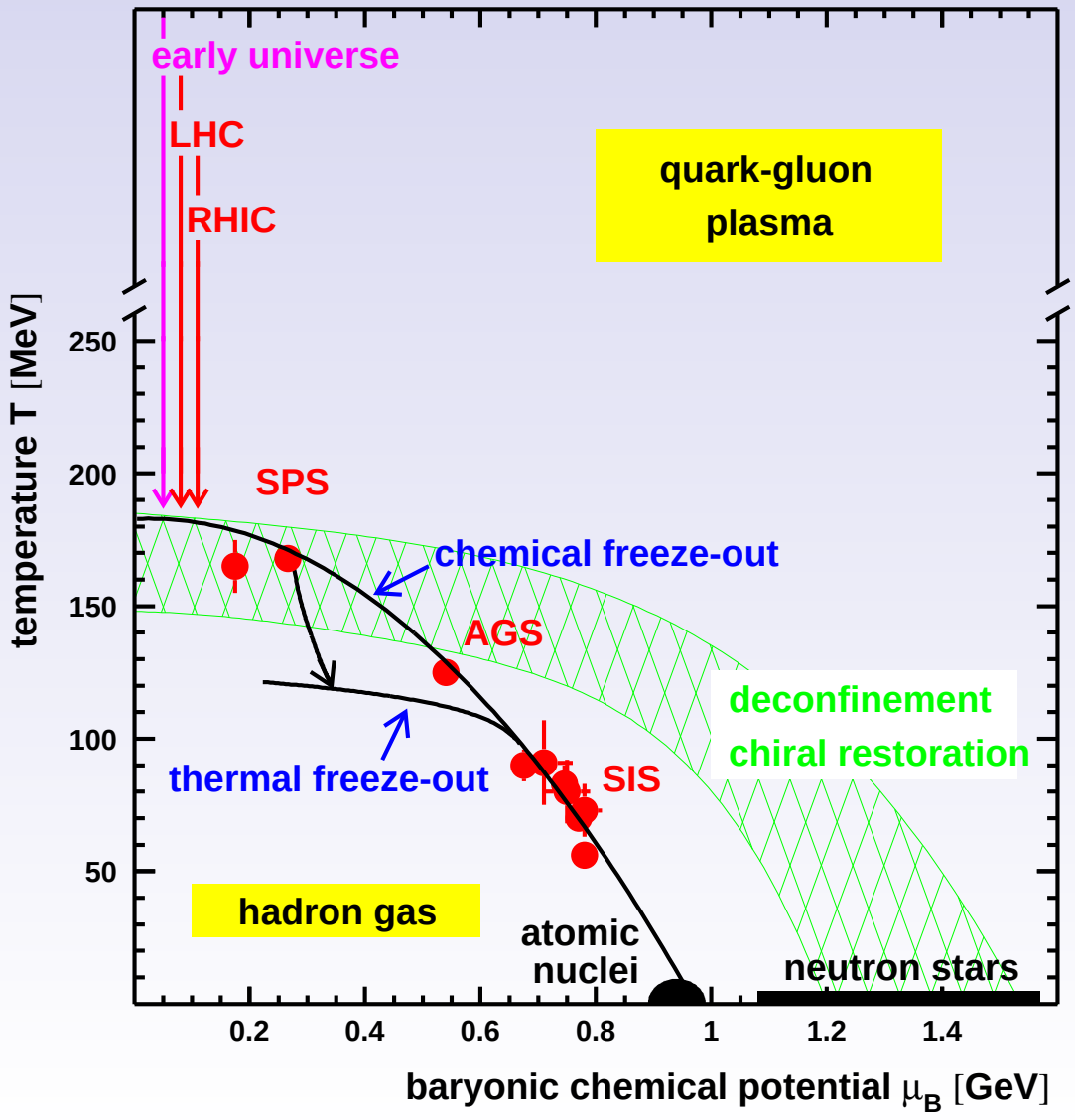

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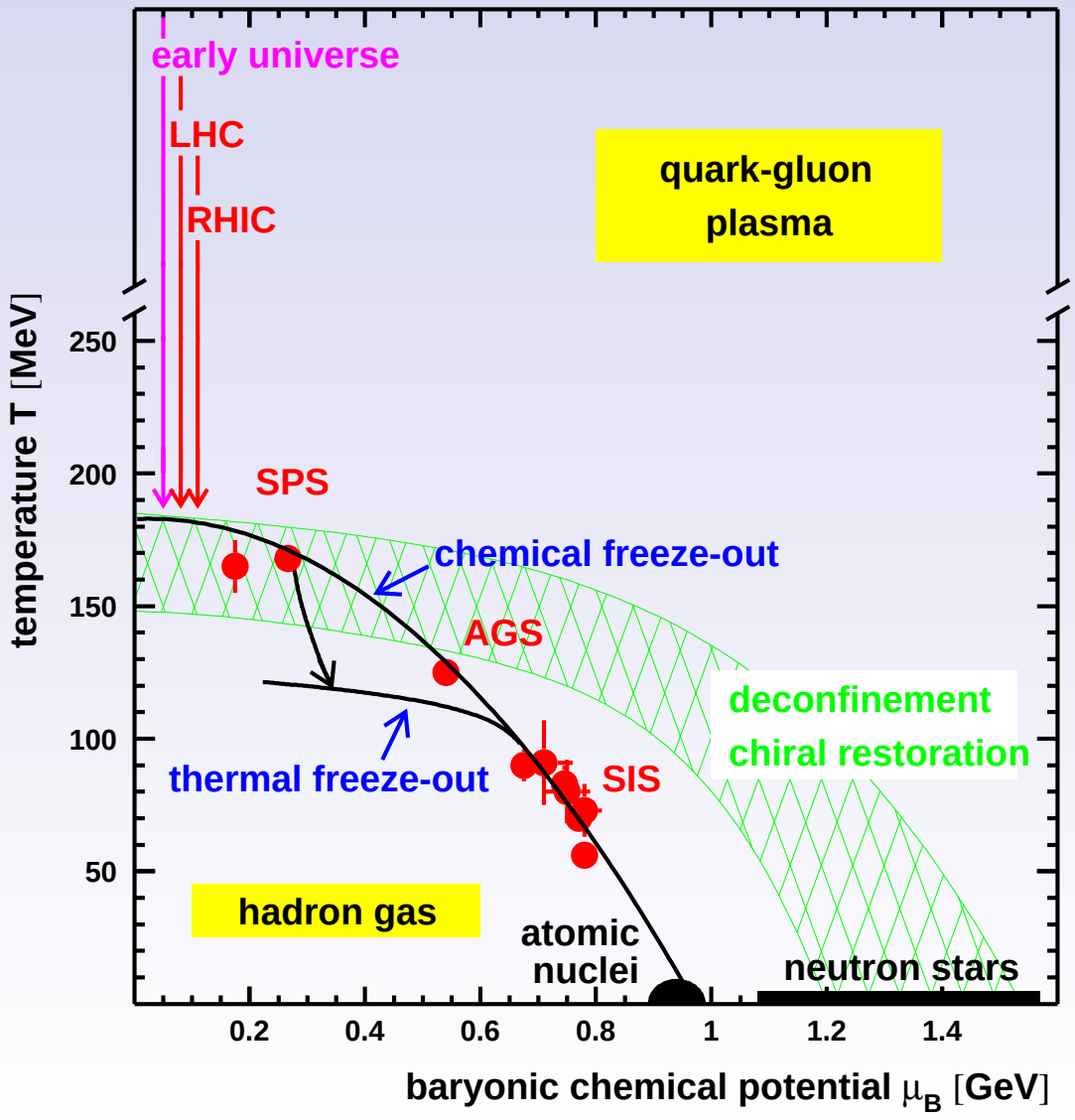

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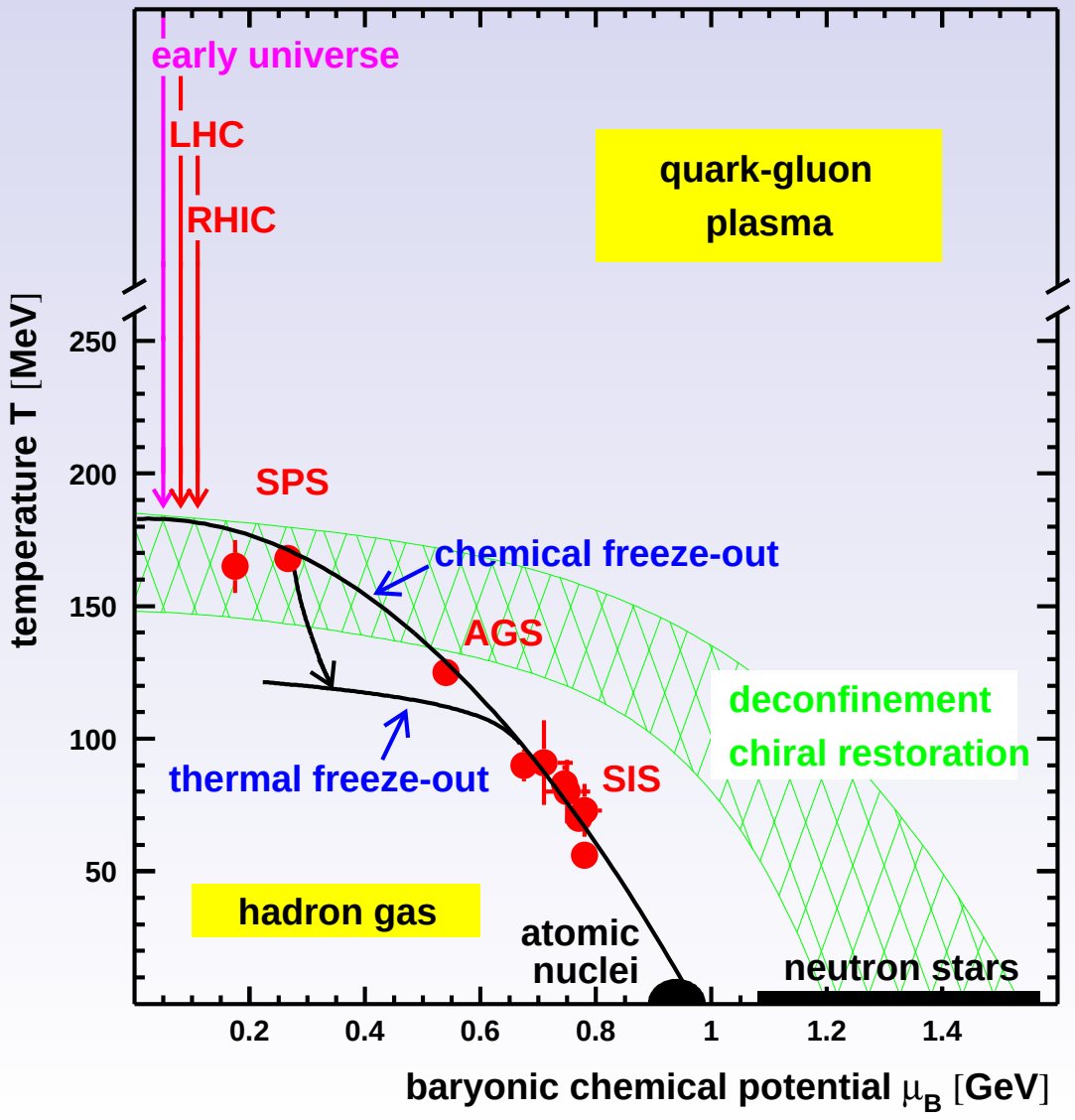

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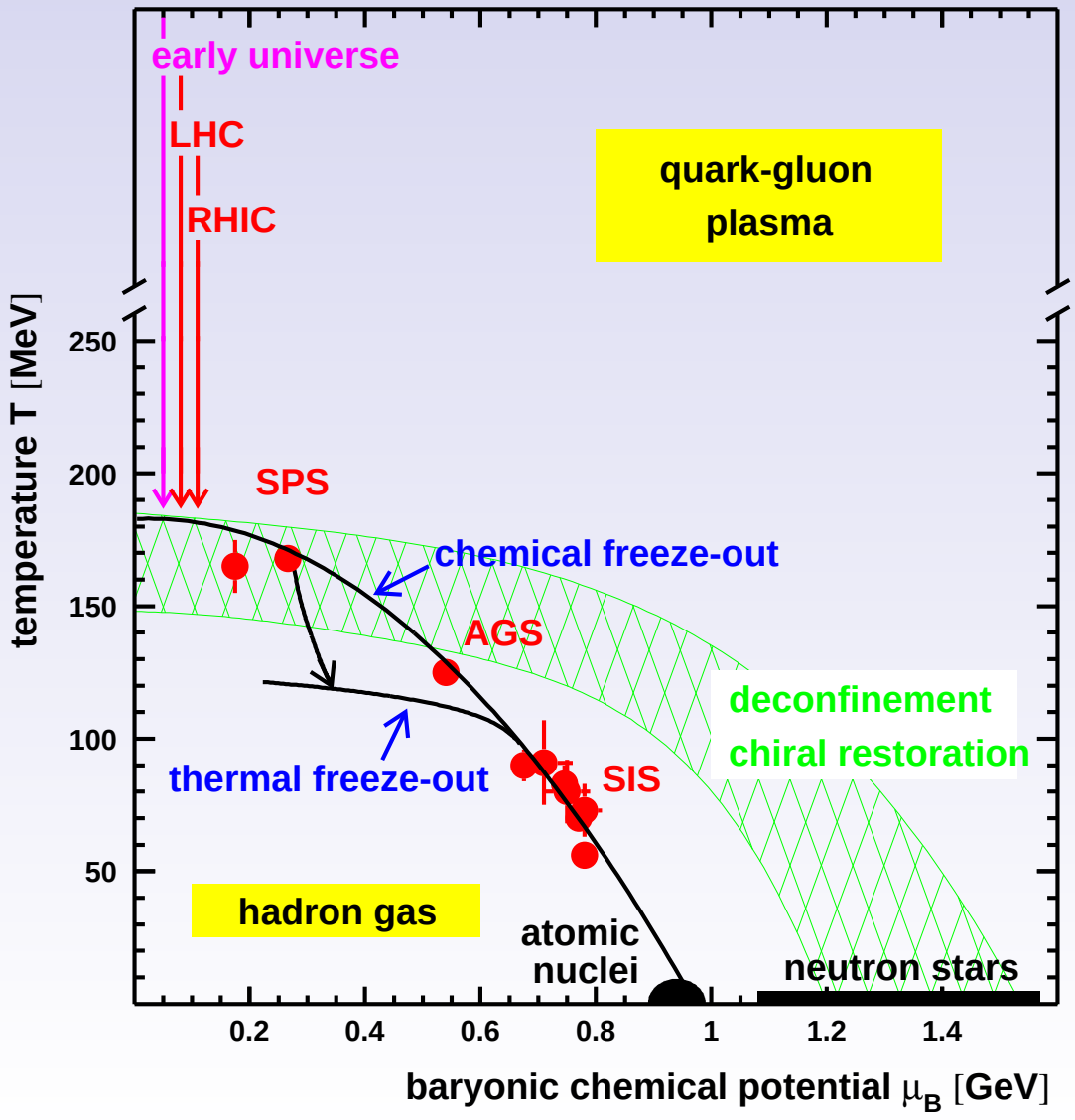

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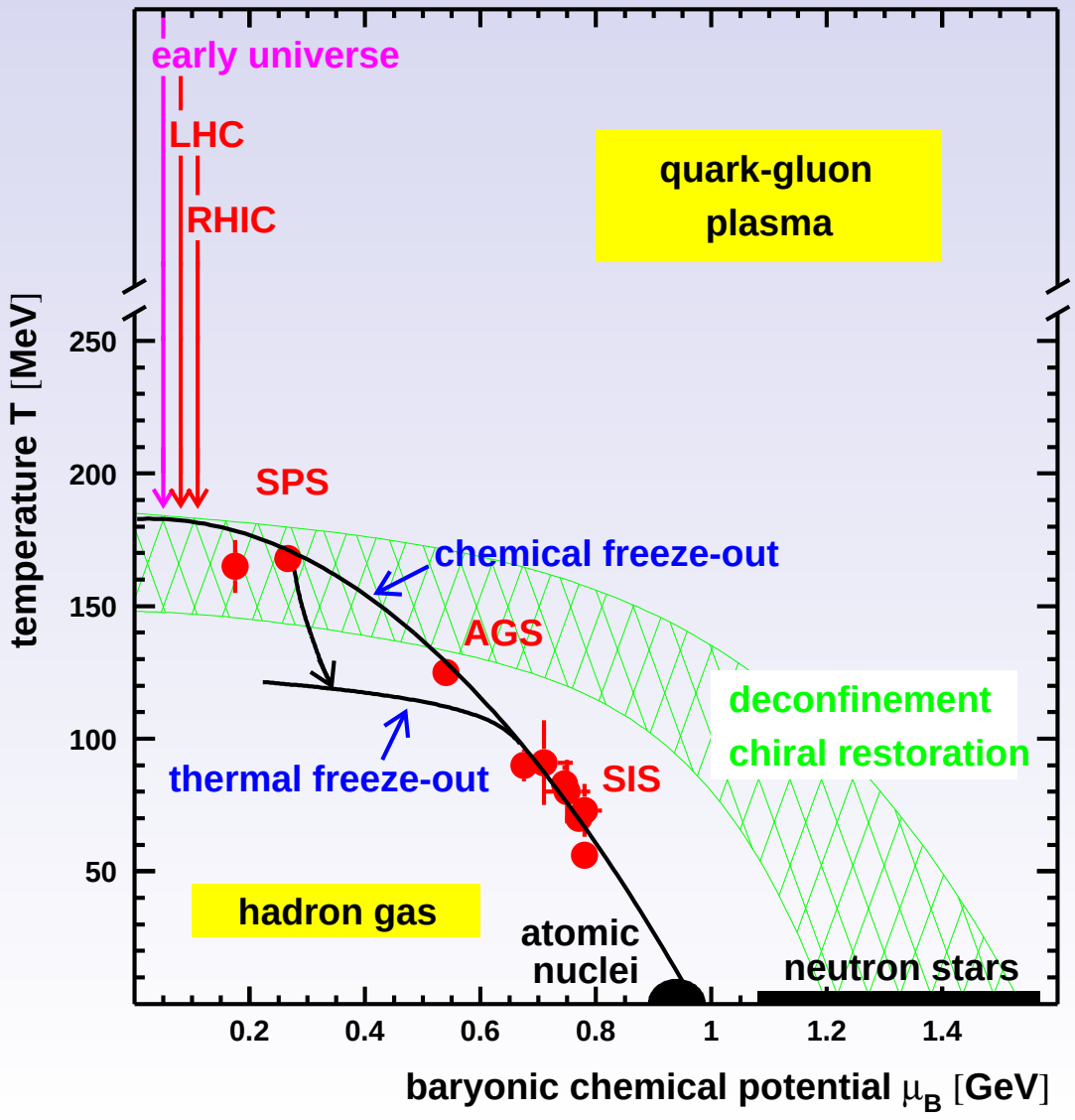

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- CEP? Yes? Where?
- No point is for sure!

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What is chiral chemical potential μ_5?

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μ_5 and CEP/CEP$_5$ within chiral models

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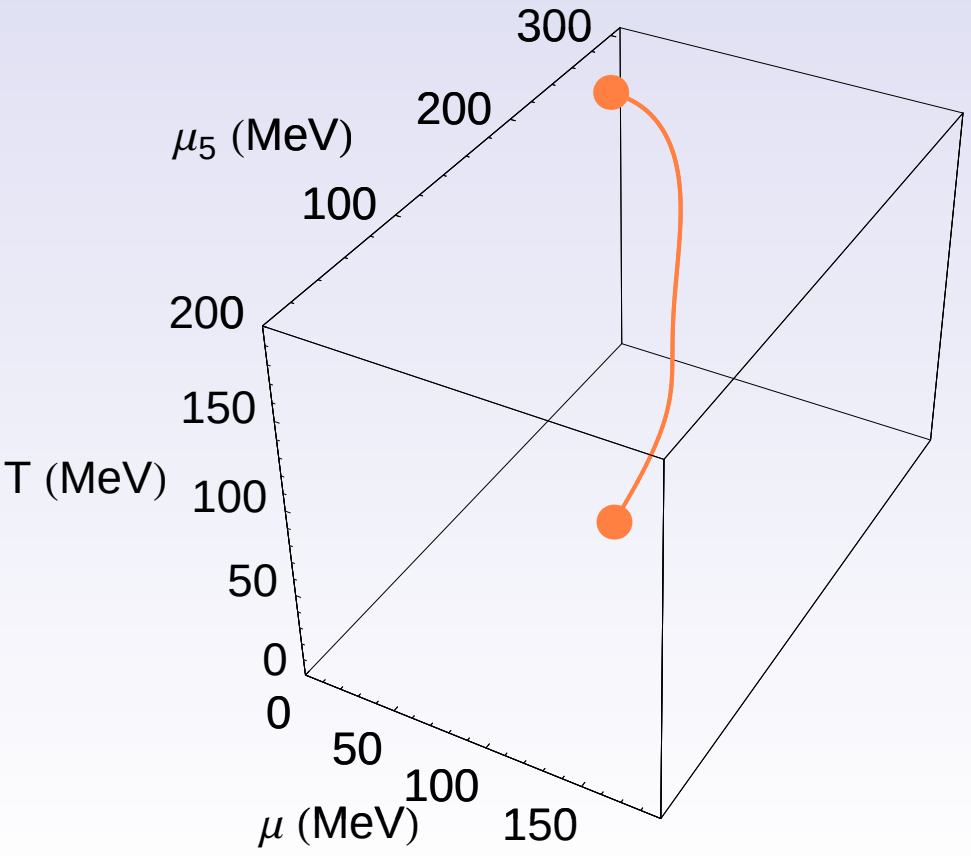

Evolution of the CEP in the $T-\mu-\mu_5$ space, for the PNJL model, taken from Phys. Rev. D 84, 014011 (2011).

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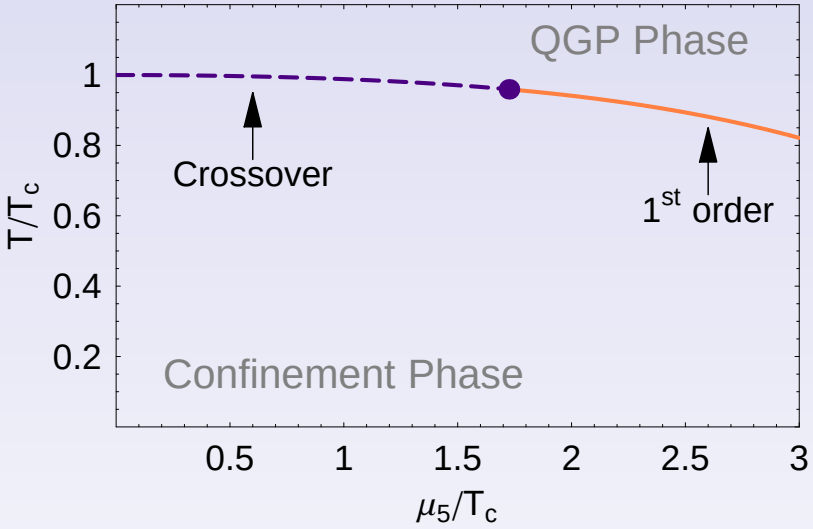

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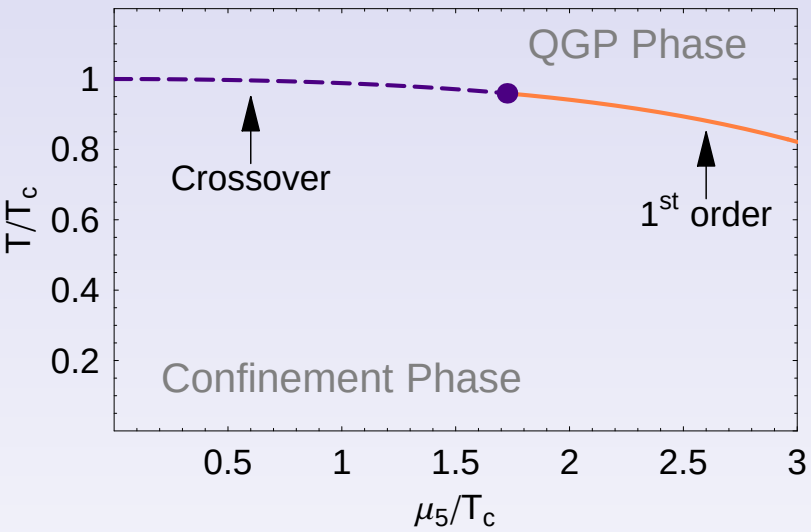

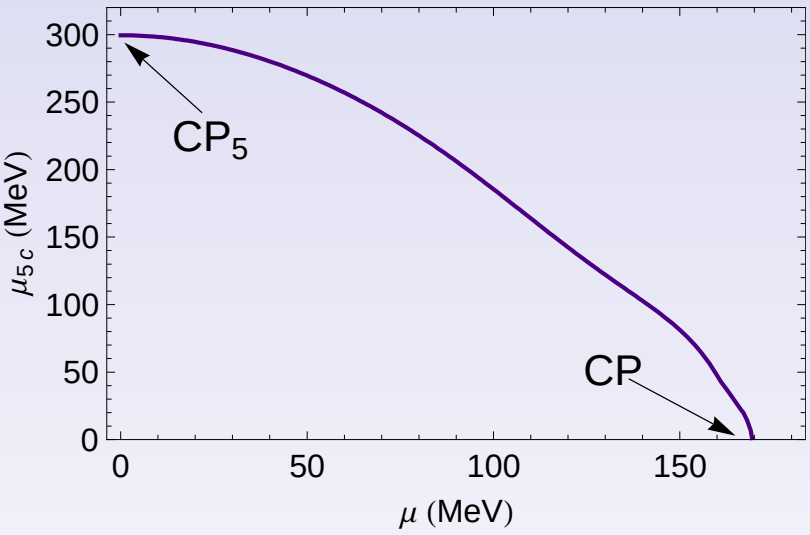

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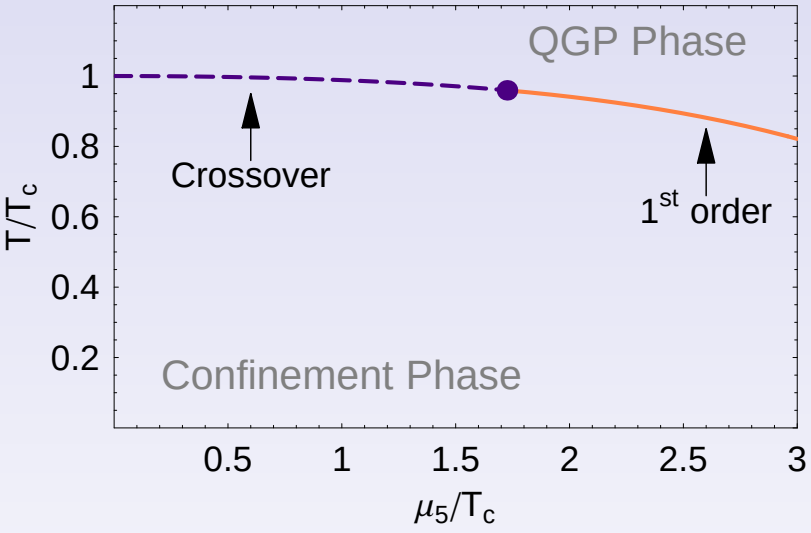

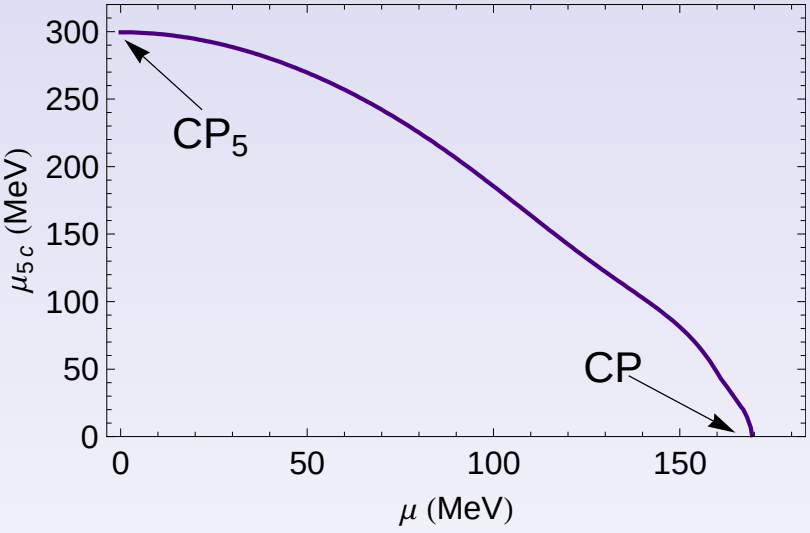

Therefore—

- A CEP$_5$ (CP$_5$) in the $T-\mu_5$ plane is found;

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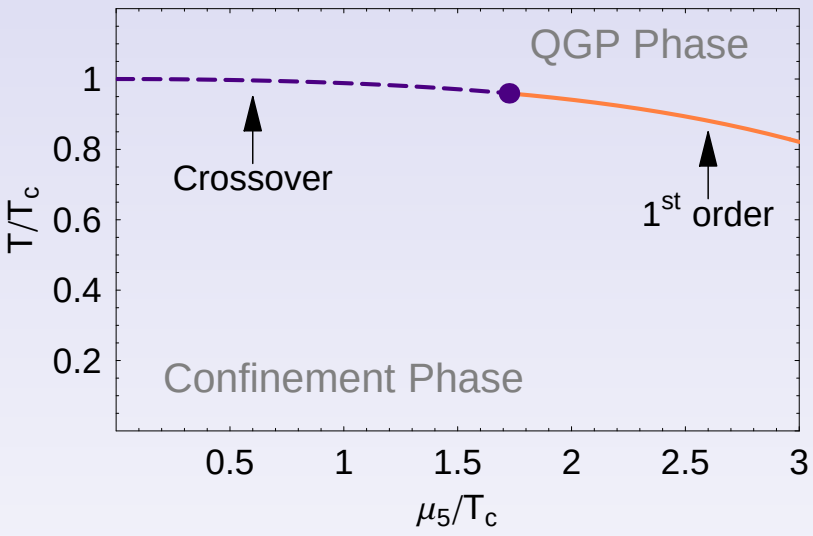

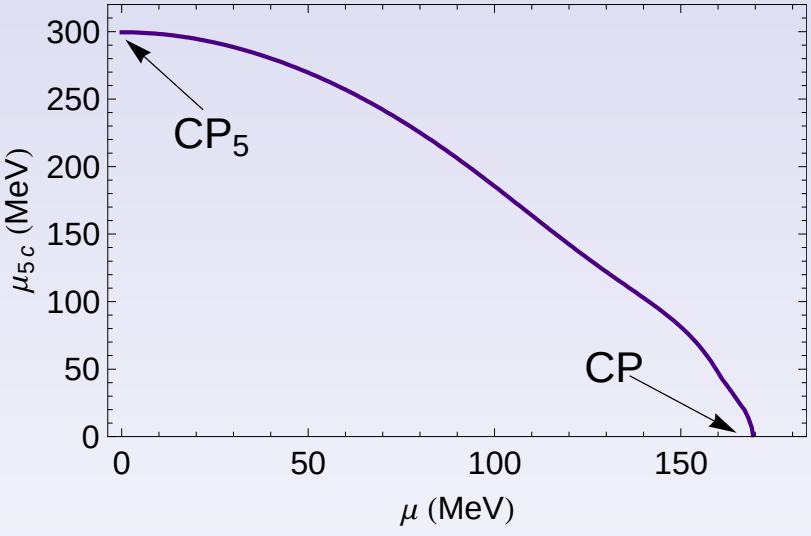

Therefore—

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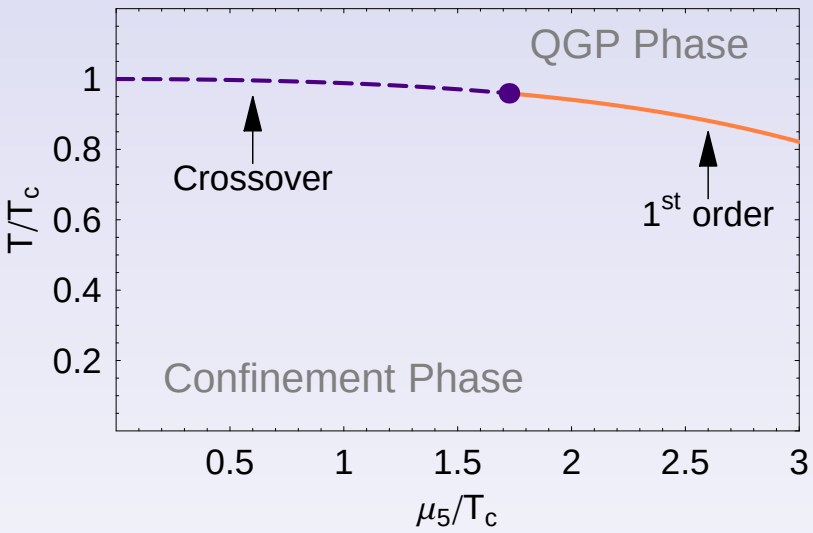
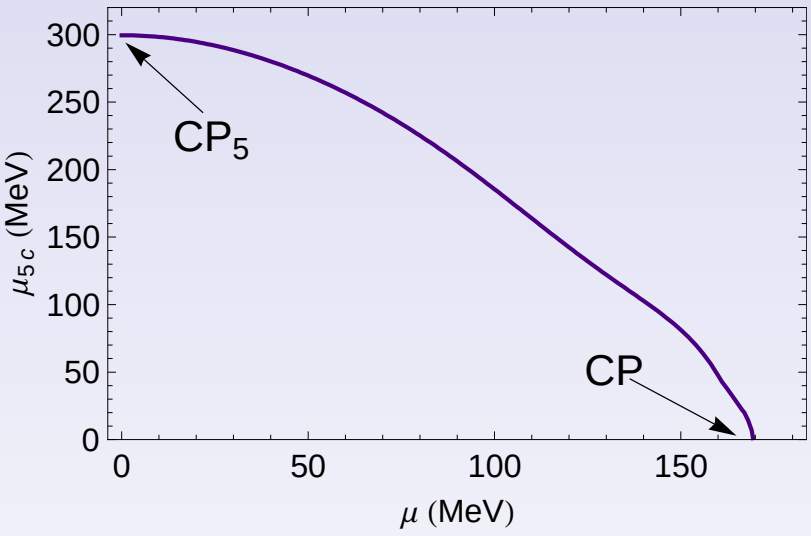
- CEP$_5$ is helpful for detecting CEP in the $T-\mu$ plane.

μ_5 and CEP/CEP$_5$ within chiral models

BUT, LQCD calculations say, sorry!





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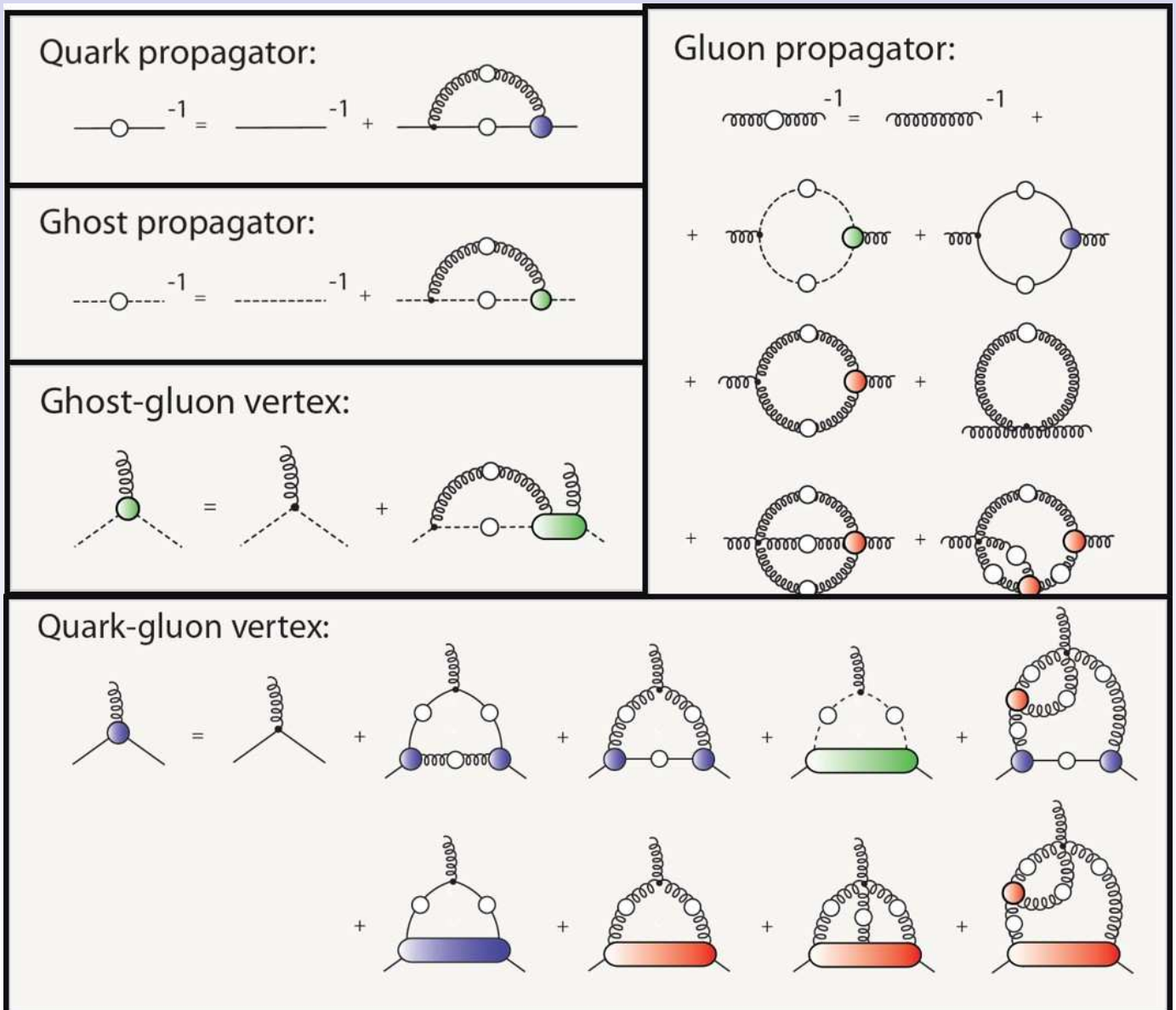
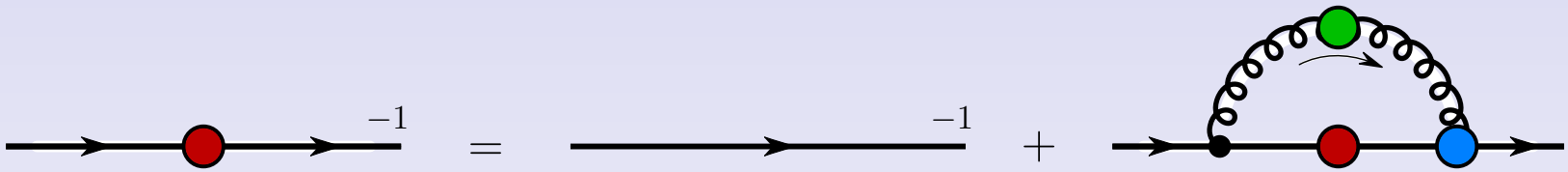
Basics of Dyson-Schwinger Equations (DSEs)



Calculations within DSE

The quark DSE reads,



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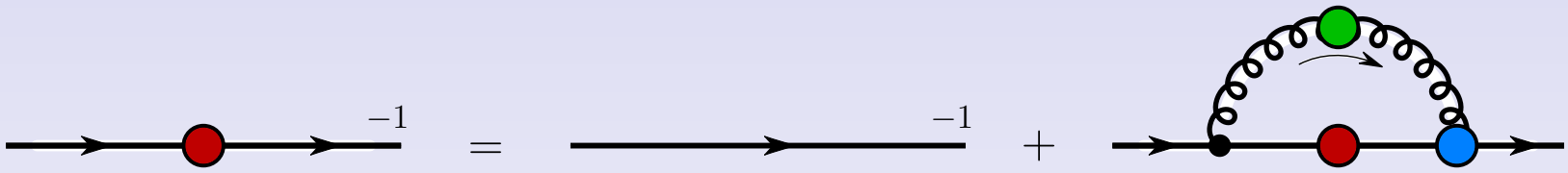

where

$$S_0(p_n,\mu_5)^{-1} = i\ \vec{p} + m + i\gamma_4\tilde{\omega}_n - \mu_5\gamma_4\gamma_5$$

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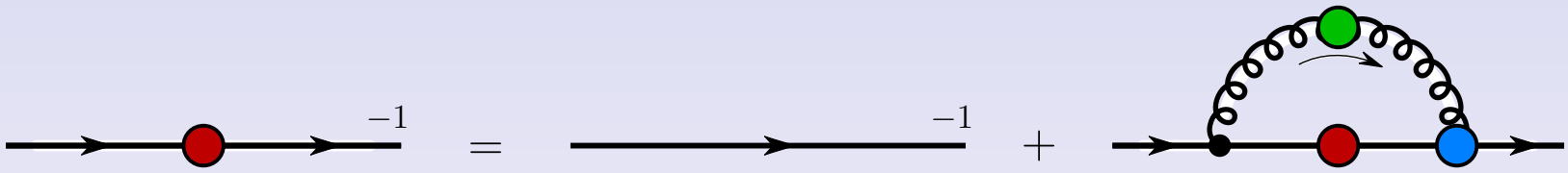

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$$\begin{aligned} S(p_n,\mu_5)^{-1} = {} & i\ \not{\vec{p}}A + B + i\gamma_4\tilde{\omega}_n C + \not{\vec{p}}\gamma_4\tilde{\omega}_n D \\ & + (i\ \not{\vec{p}}A_5 + B_5 + i\gamma_4\tilde{\omega}_n C_5 + \not{\vec{p}}\gamma_4\tilde{\omega}_n D_5)\gamma_5 \end{aligned}$$

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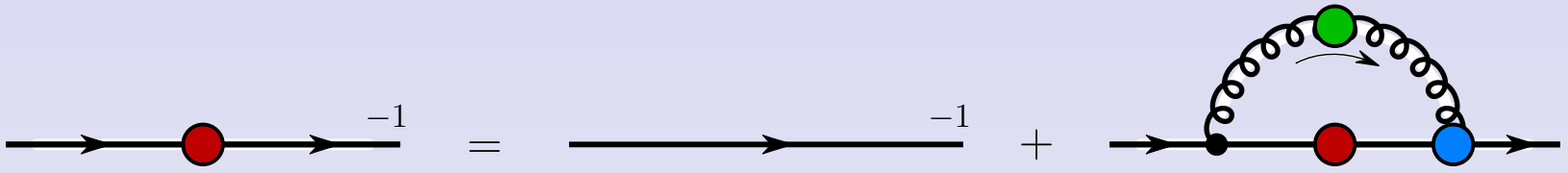

For the vertex, we take $\Gamma_\nu(p_n, q_l) = \gamma_\nu$.

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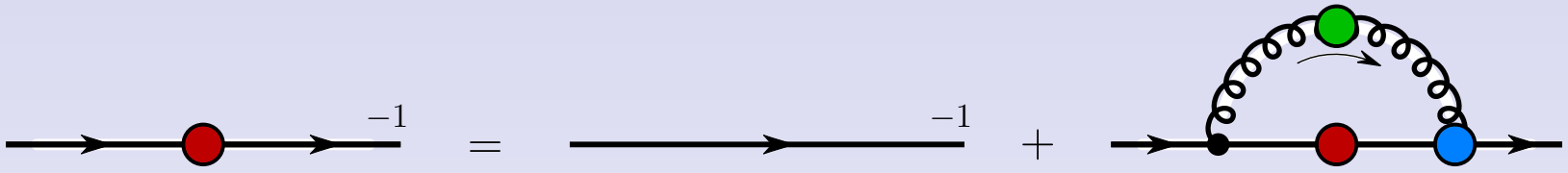

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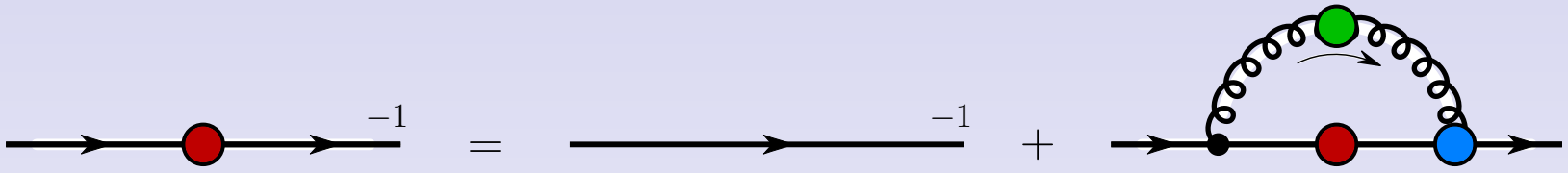

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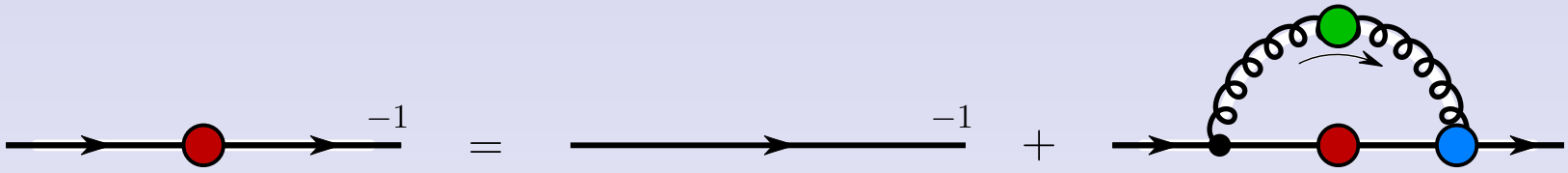

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- a Gaussian model Phys. Rev. Lett. 106, 172301 (2011).

$$g^2 D_{\mu\nu}(k_{nl}) = (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k_{nl}^2}) D_0 \frac{4\pi^2}{\sigma^6} k_{nl}^2 e^{-k_{nl}^2/\sigma^2}.$$

How we locate the CEP

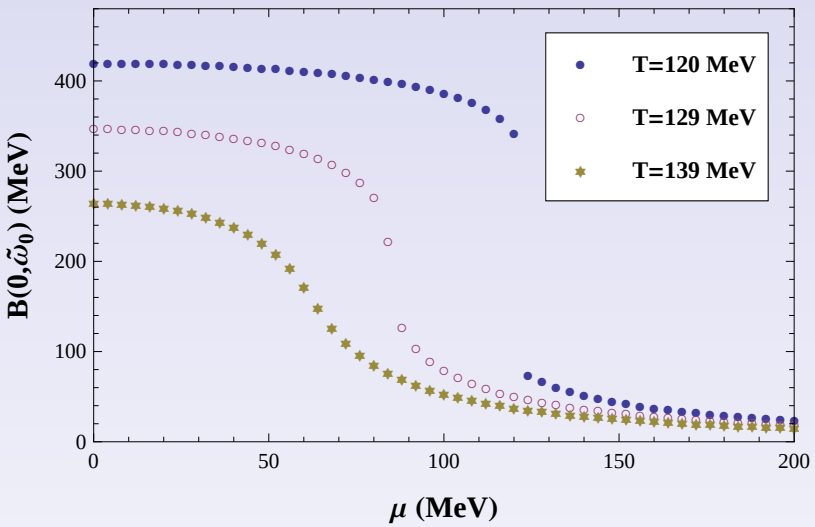

Fig: $B(0, \tilde{\omega}_0^2)$ as a function of μ for three different T.

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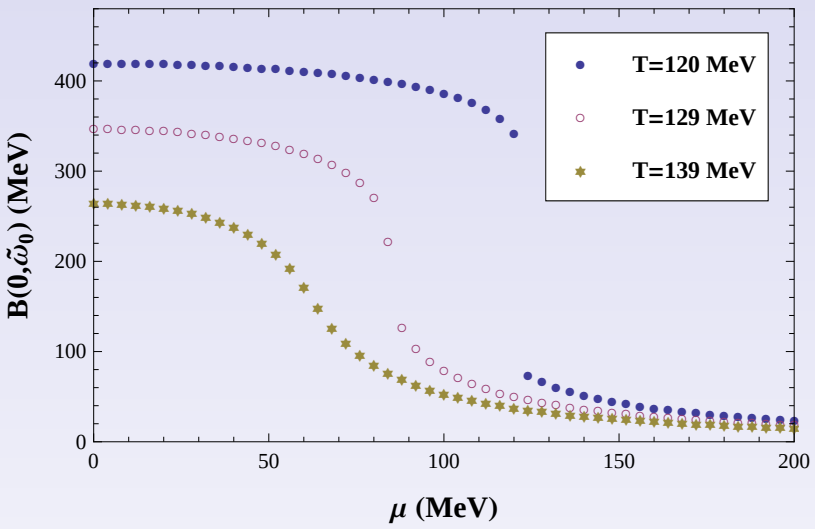

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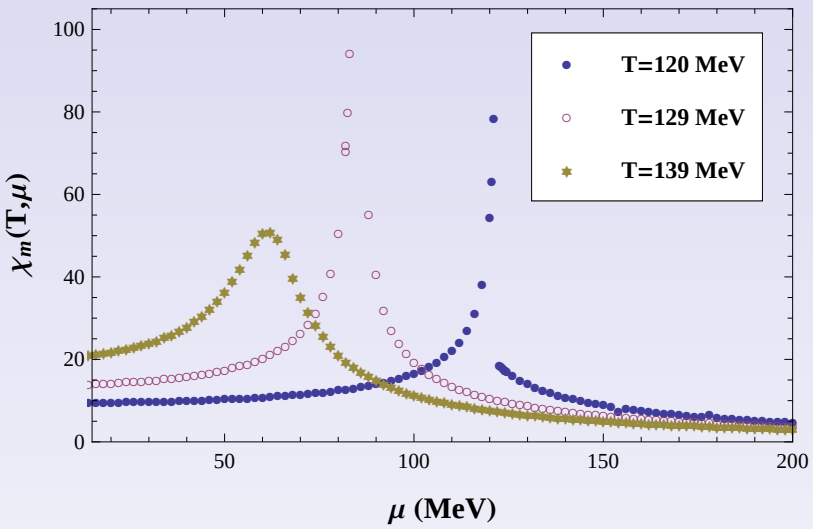

Fig: $\chi_m(T,\mu)=\partial B(0,\tilde{\omega}_0^2)/\partial m$ as a function of μ for three different T.

Results within DSE & the "separable model"

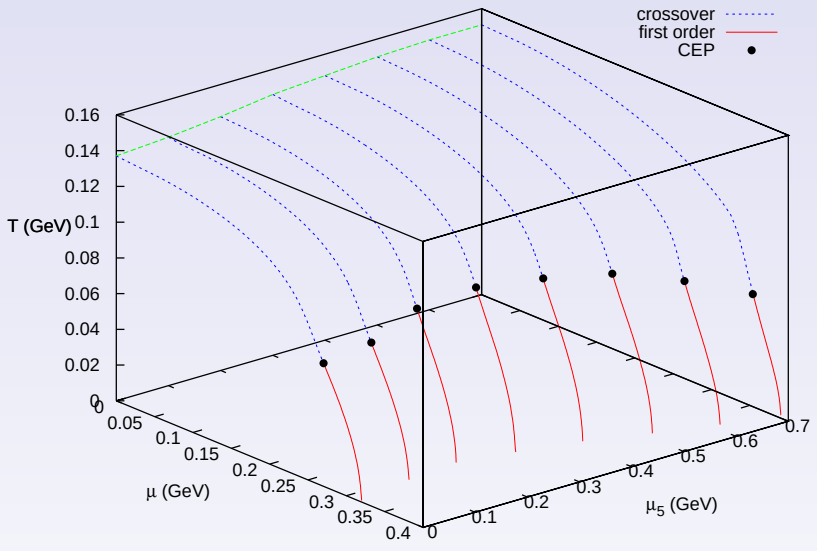

Fig: 3D phase diagram when μ_5 is introduced.

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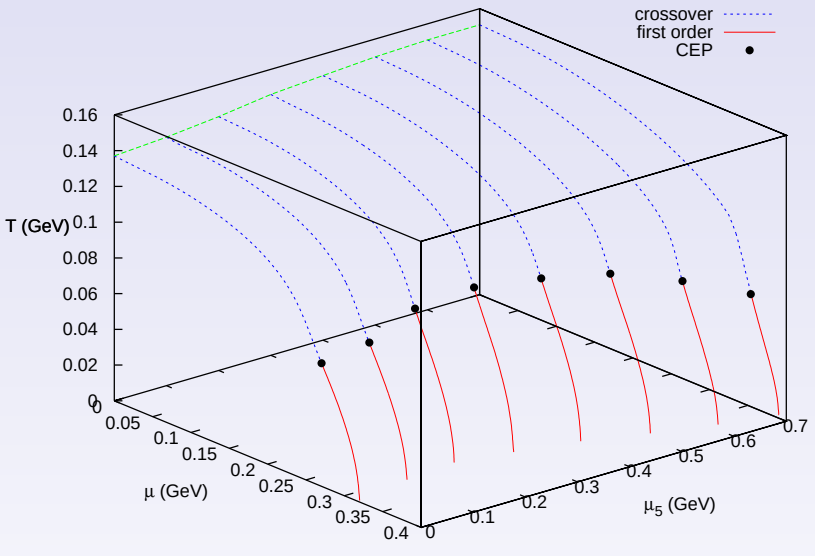

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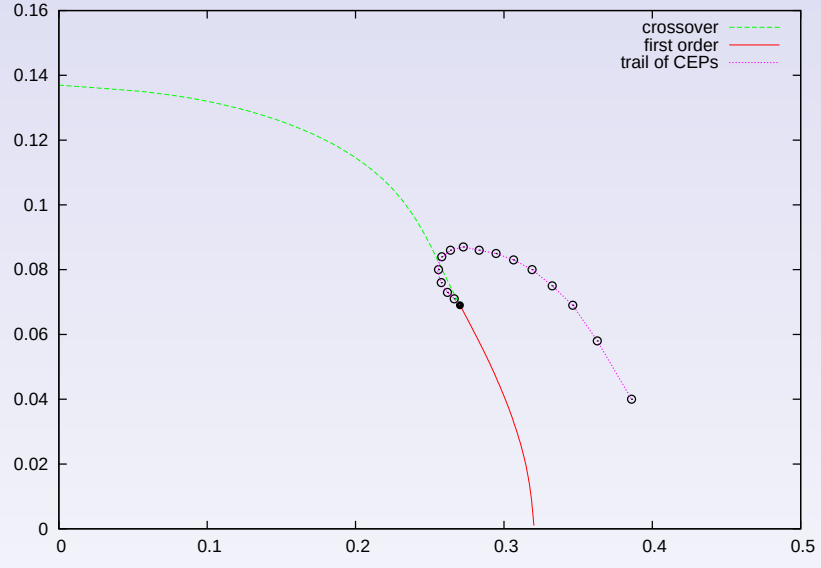
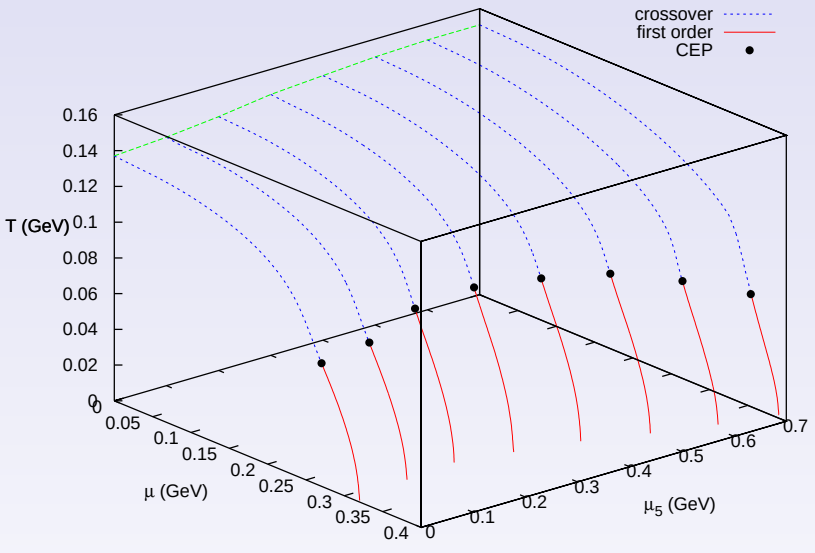


Fig: The chiral phase diagram and the trail of the CEPs in the $\mu - T$ plane.

Results within DSE & the "separable model"

NO CEP_5 (in $T-\mu_5$ plane)$\Leftrightarrow$ Agree with LQCD!



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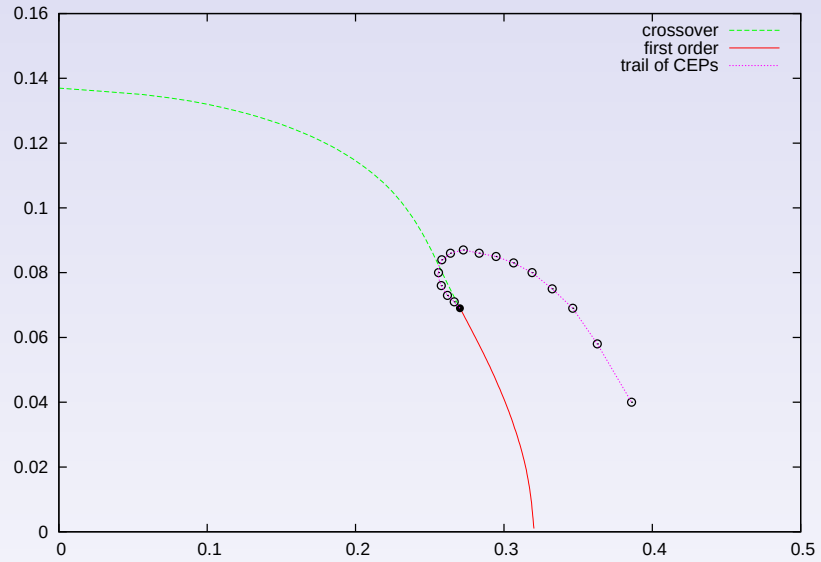

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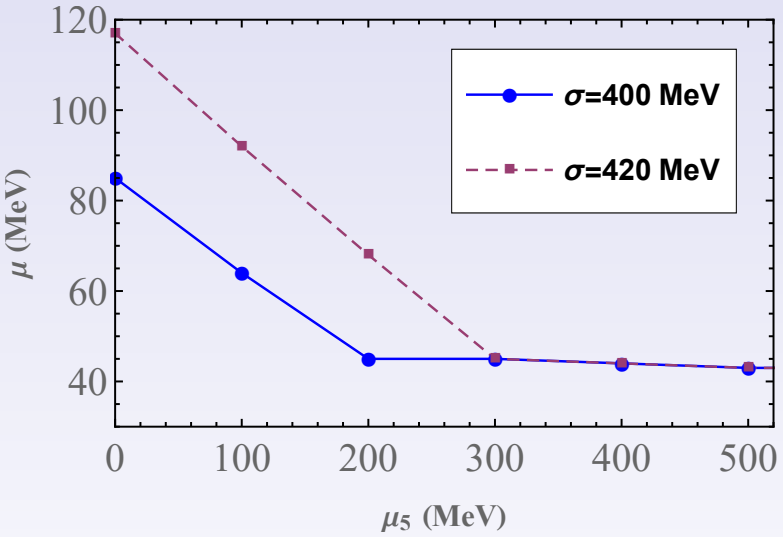

Fig: DSE predictions for the μ location of the CEP.

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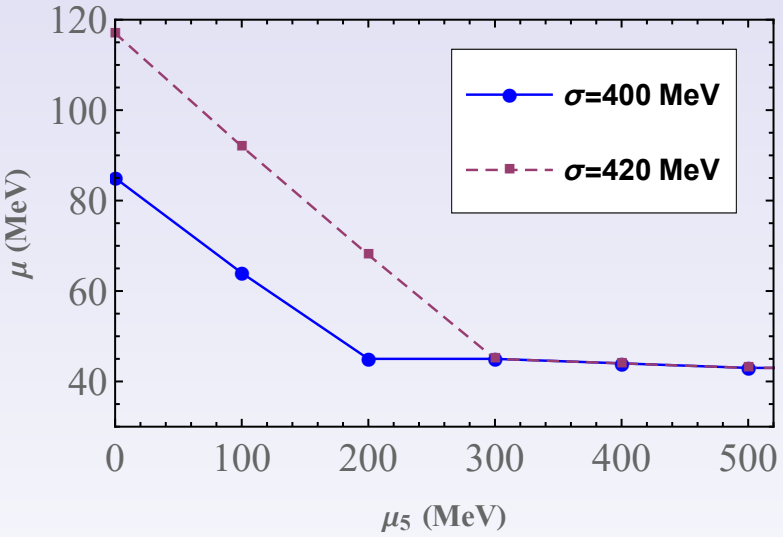

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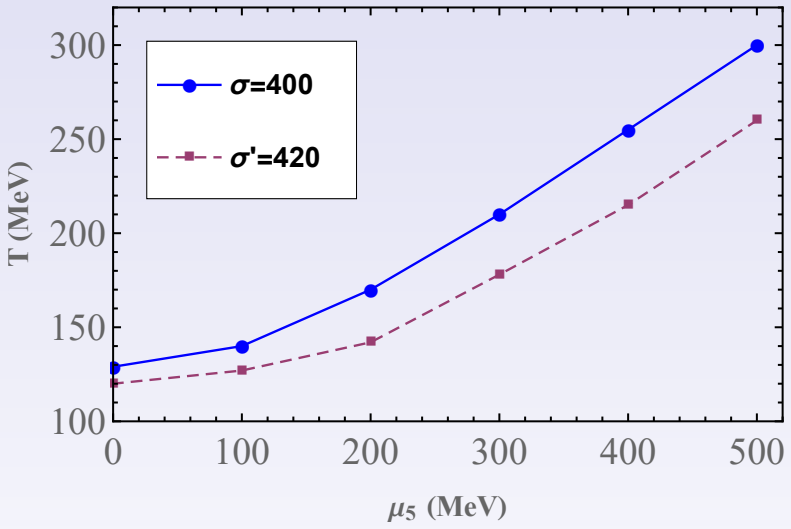
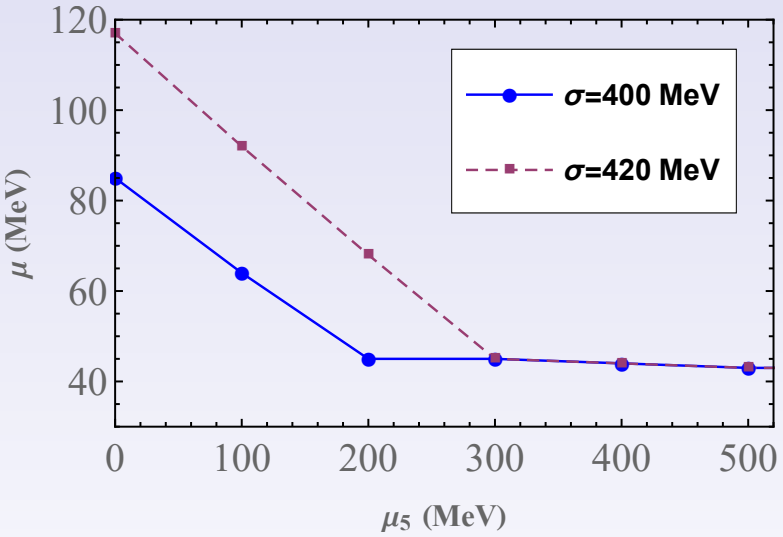


Fig: DSE predictions for the T location of the CEP.

Results within DSE & a Gaussian model

Qualitatively agree with "separable model"!



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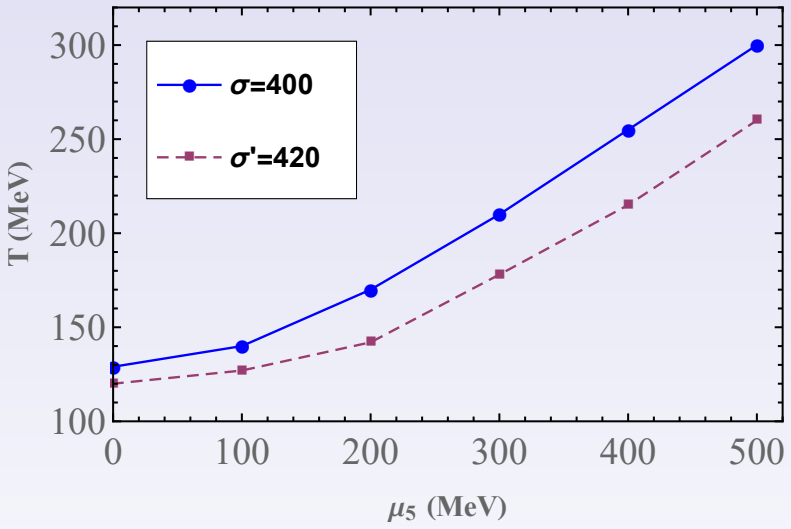

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How to understand & reconcile the contradiction

The effective potential for the PNJL model is

$$\Omega = \mathcal{U}(\Phi;T) + \frac{(M-m)^2}{4G} - 2N_c \sum_{s=\pm1} \int \frac{d^3\vec{p}}{(2\pi)^3}\, \omega_s$$

$$-\frac{2}{\beta} \sum_{s=\pm1} \int \frac{d^3\vec{p}}{(2\pi)^3} \ln[\mathcal{F}_+ \mathcal{F}_-]$$

where

$$\omega_s = \sqrt{(s|\vec{p}| - \mu_5)^2 + M^2}$$

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Question:

The 2nd integral is finite, is a cutoff necessary?
Keywords: Stefan-Boltzmann limit; high-momentum modes.

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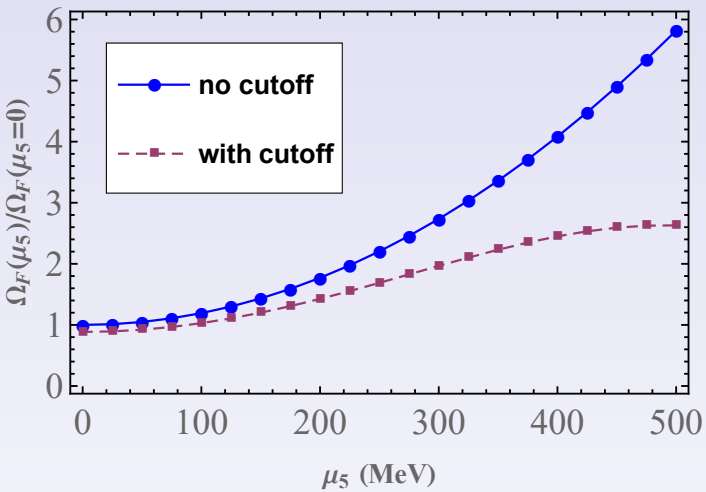

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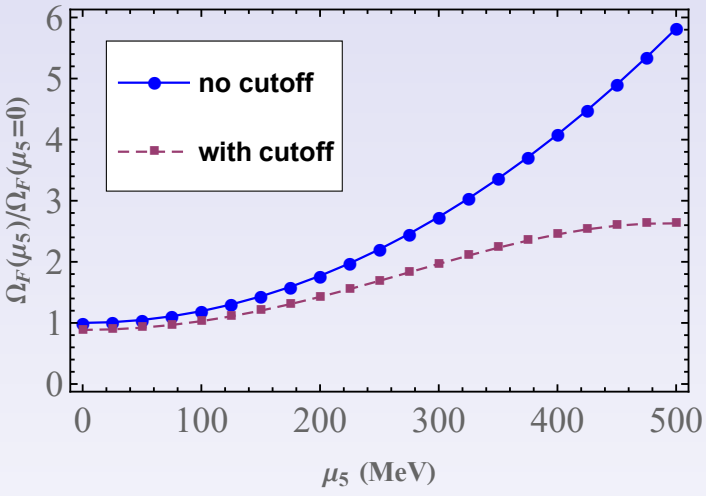

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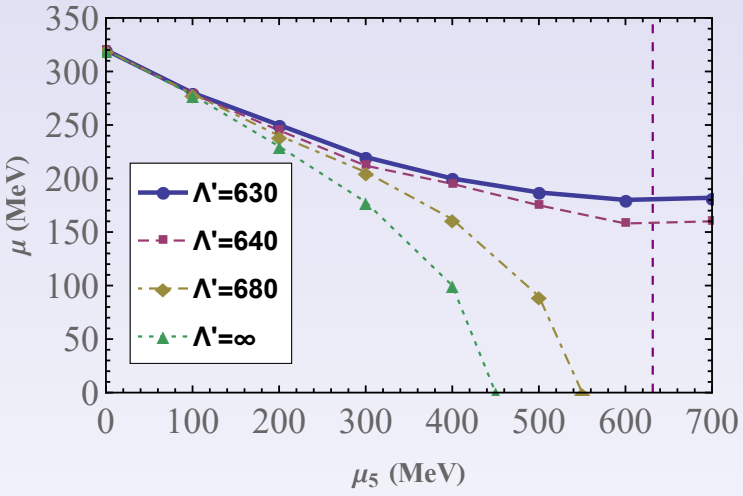

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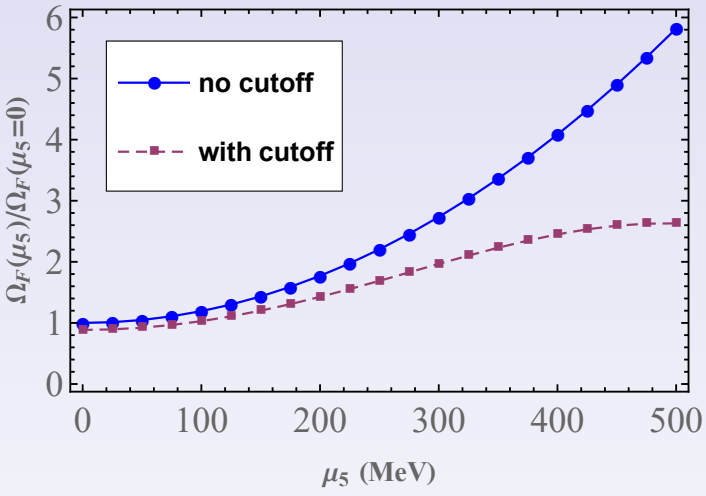
Problem solved!



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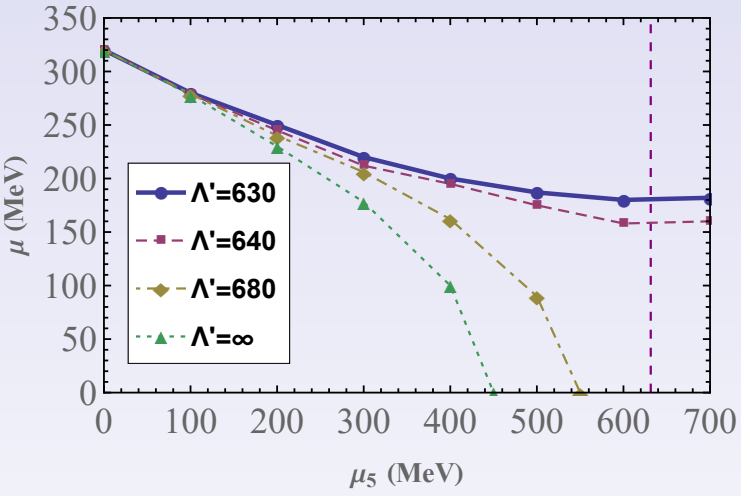

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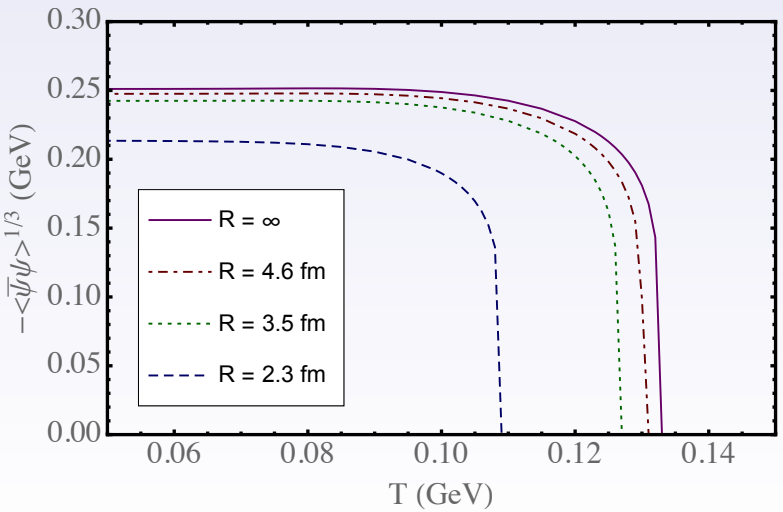

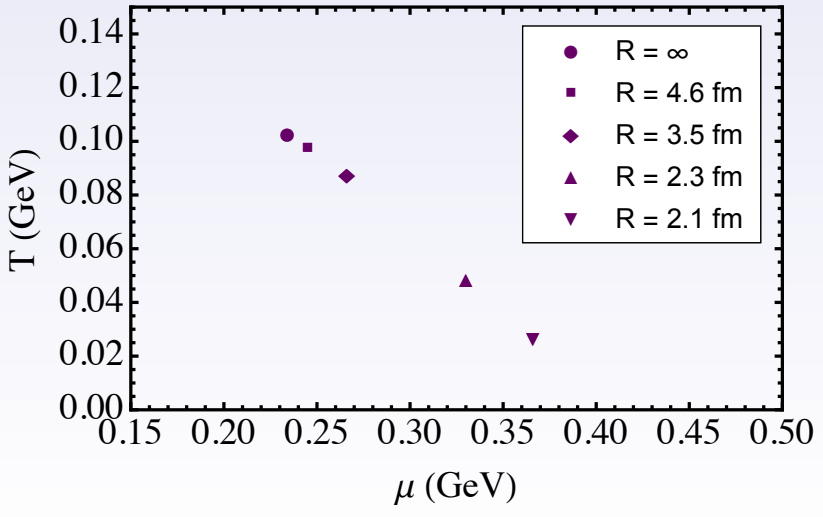

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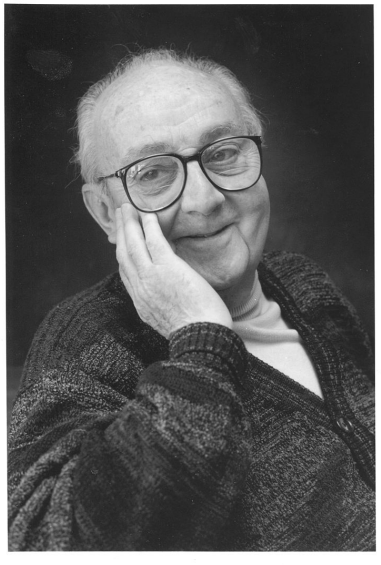

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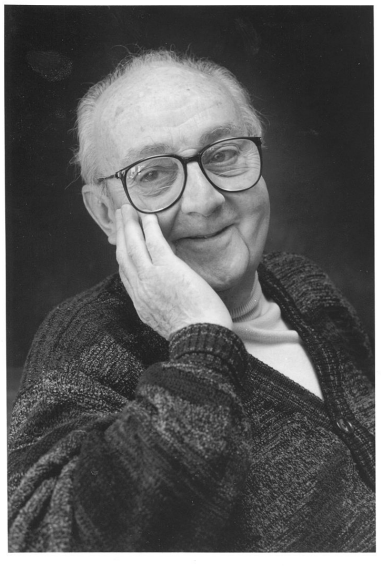
All models are wrong but some are useful.



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Chiral models are DANGEROUS!

Thank you!

Welcome for comments!

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