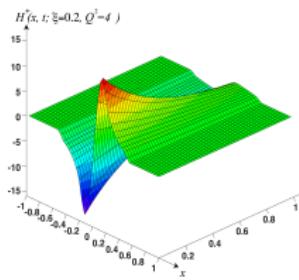
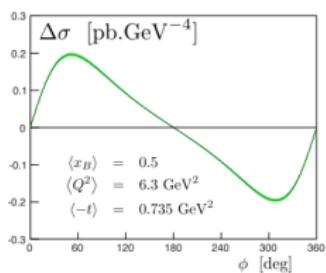
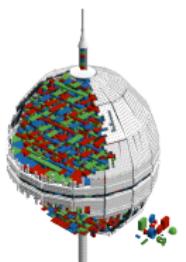


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NPQCD 2018, Sevilla | Hervé MOUTARDE

Nov. 8th, 2018

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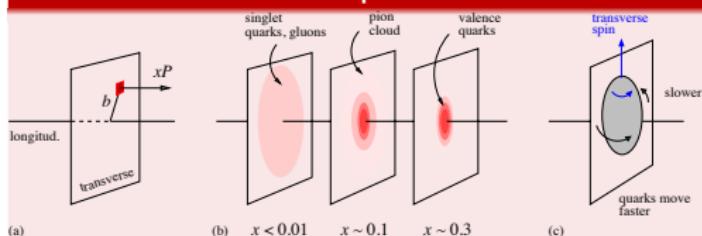
- Probabilistic interpretation of Fourier transform of $GPD(x, \xi = 0, t)$ in transverse plane.

$$\rho(x, b_\perp, \lambda, \lambda_N) = \frac{1}{2} \left[H(x, 0, b_\perp^2) + \frac{b_\perp^j \epsilon_{ji} S_\perp^i}{M} \frac{\partial E}{\partial b_\perp^2}(x, 0, b_\perp^2) + \lambda \lambda_N \tilde{H}(x, 0, b_\perp^2) \right]$$

- Notations : quark helicity λ , nucleon longitudinal polarization λ_N and nucleon transverse spin S_\perp .

Burkardt, Phys. Rev. D62, 071503 (2000)

Can we obtain this picture from exclusive measurements?



Weiss, AIP Conf.
Proc. 1149,
150 (2009)

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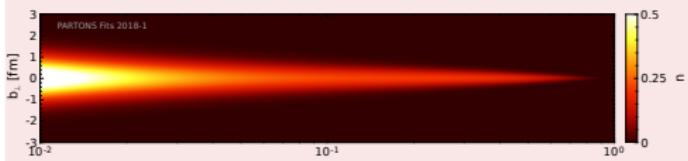
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- Notations : quark helicity λ , nucleon longitudinal polarization λ_N and nucleon transverse spin S_\perp .

Burkardt, Phys. Rev. D62, 071503 (2000)

Not quite, but close!



Moutarde et al.,
Eur. Phys. J. C78,
890 (2018)

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- Most general structure of matrix element of energy momentum tensor between nucleon states:

$$\begin{aligned} \left\langle N, P + \frac{\Delta}{2} \right| T^{\mu\nu} \left| N, P - \frac{\Delta}{2} \right\rangle &= \bar{u} \left(P + \frac{\Delta}{2} \right) \left[\color{blue}{A(t)} \gamma^{(\mu} P^{\nu)} \right. \\ &\quad \left. + \color{blue}{B(t)} P^{(\mu} i\sigma^{\nu)\lambda} \frac{\Delta_{\lambda}}{2M} + \frac{\color{blue}{C(t)}}{M} (\Delta^{\mu} \Delta^{\nu} - \Delta^2 \eta^{\mu\nu}) \right] u \left(P - \frac{\Delta}{2} \right) \end{aligned}$$

with $t = \Delta^2$.

- Key observation: link between GPDs and gravitational form factors

$$\int dx x H^q(x, \xi, t) = A^q(t) + 4\xi^2 C^q(t)$$

$$\int dx x E^q(x, \xi, t) = B^q(t) - 4\xi^2 C^q(t)$$

Ji, Phys. Rev. Lett. **78**, 610 (1997)

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■ Spin sum rule:

$$\int dx x (\textcolor{blue}{H}^q(x, \xi, 0) + \textcolor{blue}{E}^q(x, \xi, 0)) = \textcolor{blue}{A}^q(0) + \textcolor{blue}{B}^q(0) = 2J^q$$

Ji, Phys. Rev. Lett. **78**, 610 (1997)

■ Shear and pressure of a hadron considered as a continuous medium:

$$\langle N | T^{ij}(\vec{r}) | N \rangle_N = s(r) \left(\frac{r^i r^j}{\vec{r}^2} - \frac{1}{3} \delta^{ij} \right) + p(r) \delta^{ij}$$

Polyakov and Shubaev, hep-ph/0207153

■ Energy density, tangential and radial pressures of a hadron considered as a continuous medium.

Lorcé *et al.*, arXiv:1810.09837 [hep-ph]

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- ### ■ Matrix element in the Breit frame ($a = q, g$):

$$\begin{aligned} \left\langle \frac{\Delta}{2} |T_a^{\mu\nu}(0)| - \frac{\Delta}{2} \right\rangle &= M \left\{ \eta^{\mu 0} \eta^{\nu 0} \left[A_a(t) + \frac{t}{4M^2} B_a(t) \right] \right. \\ &\quad \left. + \eta^{\mu\nu} \left[\bar{C}_a(t) - \frac{t}{M^2} C_a(t) \right] + \frac{\Delta^\mu \Delta^\nu}{M^2} C_a(t) \right\} \end{aligned}$$

- #### ■ Anisotropic fluid in relativistic hydrodynamics:

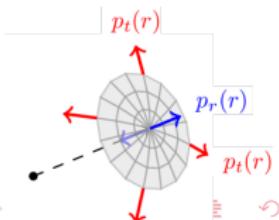
$$\Theta^{\mu\nu}(\vec{r}) = [\varepsilon(r) + p_t(r)] u^\mu u^\nu - p_t(r) \eta^{\mu\nu} + [p_r(r) - p_t(r)] \chi^\mu \chi^\nu$$

where u^μ and $\chi^\mu = x^\mu/r$.

- Define **isotropic pressure** and **pressure anisotropy**:

$$p(r) = \frac{p_r(r) + 2 p_t(r)}{3}$$

$$s(r) = p_r(r) - p_t(r)$$



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- Write dictionary between quantum and fluid pictures:

$$\frac{\varepsilon_a(r)}{M} = \int \frac{d^3 \vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} \left\{ A_a(t) + \bar{C}_a(t) + \frac{t}{4M^2} [B_a(t) - 4C_a(t)] \right\}$$

$$\frac{p_{r,a}(r)}{M} = \int \frac{d^3 \vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} \left\{ -\bar{C}_a(t) - \frac{4}{r^2} \frac{t^{-1/2}}{M^2} \frac{d}{dt} (t^{3/2} C_a(t)) \right\}$$

$$\frac{p_{t,a}(r)}{M} = \int \frac{d^3 \vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} \left\{ -\bar{C}_a(t) + \frac{4}{r^2} \frac{t^{-1/2}}{M^2} \frac{d}{dt} \left[t \frac{d}{dt} (t^{3/2} C_a(t)) \right] \right\}$$

$$\frac{p_a(r)}{M} = \int \frac{d^3 \vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} \left\{ -\bar{C}_a(t) + \frac{2}{3} \frac{t}{M^2} C_a(t) \right\}$$

$$\frac{s_a(r)}{M} = \int \frac{d^3 \vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} \left\{ -\frac{4}{r^2} \frac{t^{-1/2}}{M^2} \frac{d^2}{dt^2} (t^{5/2} C_a(t)) \right\}$$

Mechanical properties of hadrons. From the nucleon to compact stars (3/3).

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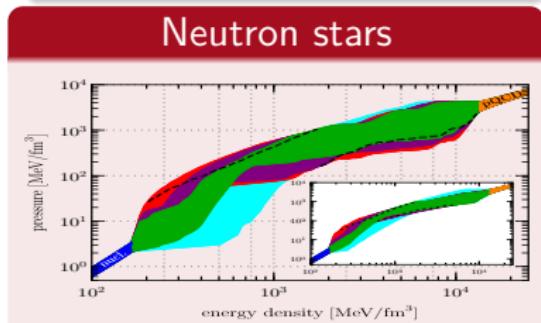
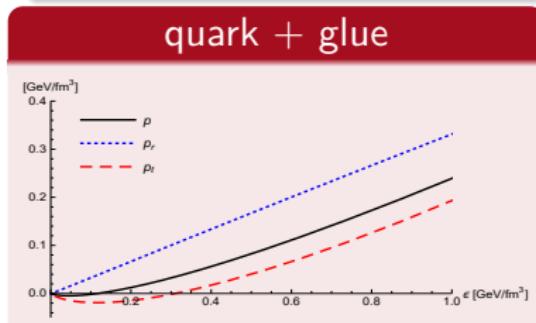
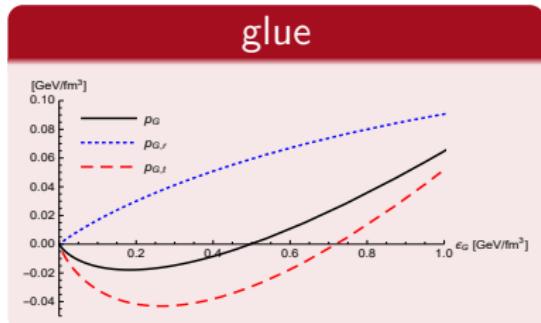
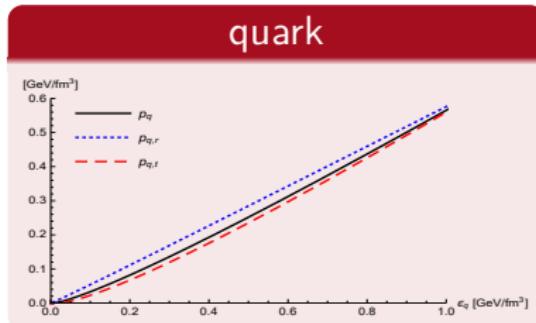
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■ Evaluate orders of magnitude with naive multiple model:



Lorcé *et al.*,
arXiv:1810.09837 [hep-ph]

Annala *et al.*, Phys. Rev. Lett. **120**, 172703 (2018)

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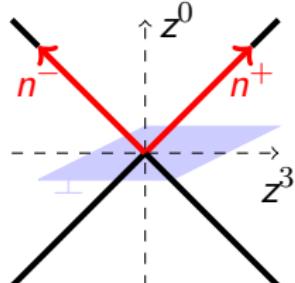
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$$H_\pi^q(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \right| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \left| \pi, P - \frac{\Delta}{2} \right\rangle_{z^+=0, z_\perp=0}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

- Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
- Ji, Phys. Rev. Lett. **78**, 610 (1997)
- Radyushkin, Phys. Lett. **B380**, 417 (1996)

■ PDF forward limit

$$H^q(x, 0, 0) = q(x)$$

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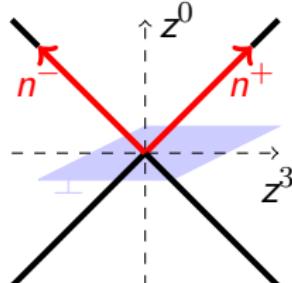
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$$H_\pi^q(x, \xi, t) =$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \right| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \left| \pi, P - \frac{\Delta}{2} \right\rangle_{z^+=0, z_\perp=0}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

- Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
- Ji, Phys. Rev. Lett. **78**, 610 (1997)
- Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF forward limit
- Form factor sum rule

$$\int_{-1}^{+1} dx H_\pi^q(x, \xi, t) = F_1^q(t)$$

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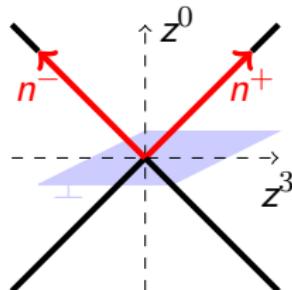
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$$H_\pi^q(x, \xi, t) =$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \right| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \left| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

- Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
- Ji, Phys. Rev. Lett. **78**, 610 (1997)
- Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF forward limit
- Form factor sum rule
- H^q is an even function of ξ from time-reversal invariance.

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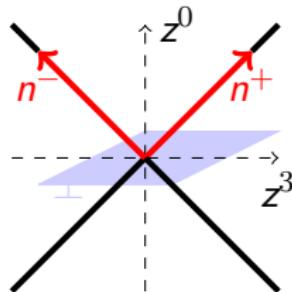
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$$H_\pi^q(x, \xi, t) =$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \right| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \left| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

- Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
- Ji, Phys. Rev. Lett. **78**, 610 (1997)
- Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF forward limit
- Form factor sum rule
- H^q is an even function of ξ from time-reversal invariance.
- H^q is real from hermiticity and time-reversal invariance.

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■ Polynomiality

$$\int_{-1}^{+1} dx x^n H^q(x, \xi, t) = \text{polynomial in } \xi$$

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$$H^q(x, \xi, t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

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■ H^q has support $x \in [-1, +1]$.

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■ H^q has support $x \in [-1, +1]$.

Relativistic quantum mechanics

■ Soft pion theorem (pion target)

$$H^q(x, \xi = 1, t = 0) = \frac{1}{2} \phi_\pi^q \left(\frac{1+x}{2} \right)$$

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■ H^q has support $x \in [-1, +1]$.

Relativistic quantum mechanics

■ Soft pion theorem (pion target)

Dynamical chiral symmetry breaking

How can we implement *a priori* these theoretical constraints?

- In the following, focus on **polynomiality** and **positivity**.
- Do not discuss the reduction to form factors or PDFs.

Dispersion relations and the lines $x = \pm\xi$.

Relation between $\text{Re}\mathcal{H}(\xi)$ and $\mathcal{H}(x = \pm\xi, \xi)$ at leading order.

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- Write dispersion relation **at fixed t and Q^2** :

$$\text{Re}\mathcal{H}(\xi) = \int_1^\infty \frac{d\omega}{\pi} \text{Im}C(\omega) \left\{ \int_{-1}^{+1} dx \left[\frac{1}{\omega\xi - x} - \frac{1}{\omega\xi + x} \right] H(x, \frac{x}{\omega}) + \mathcal{I}(\omega) \right\} .$$

Diehl and Ivanov, Eur. Phys. J. **C52**, 919 (2007)

- At **leading order** in α_s (no kinematic corrections):

$$\text{Im}C(\omega) \propto \pi \left[\delta(\omega - 1) - \delta(\omega + 1) \right] .$$

- Dispersion relation simplifies to:

$$\text{Re}\mathcal{H}(\xi) \propto \int_{-1}^{+1} dx \left[\frac{1}{\omega\xi - x} - \frac{1}{\omega\xi + x} \right] H(x, x) + \mathcal{I} ,$$

$$\text{Im}\mathcal{H}(\xi) \propto H(\xi, \xi) - H(-\xi, \xi) .$$

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- GPD fits **only in the small x_B region** with a **flexible** parameterization (kinematic simplifications).
- Global fits of CFFs in the sea and valence regions.
- Some GPD models with non-flexible parameterizations adjusted to experimental DVCS or DVMP data.

Kumerički *et al.*, Eur. Phys. J. **A52**, 157 (2016)

The situation can be improved!

- GPD parameterizations satisfying *a priori* all theoretical constraints on GPDs.
- Computing framework to go beyond leading order and leading twist analysis.

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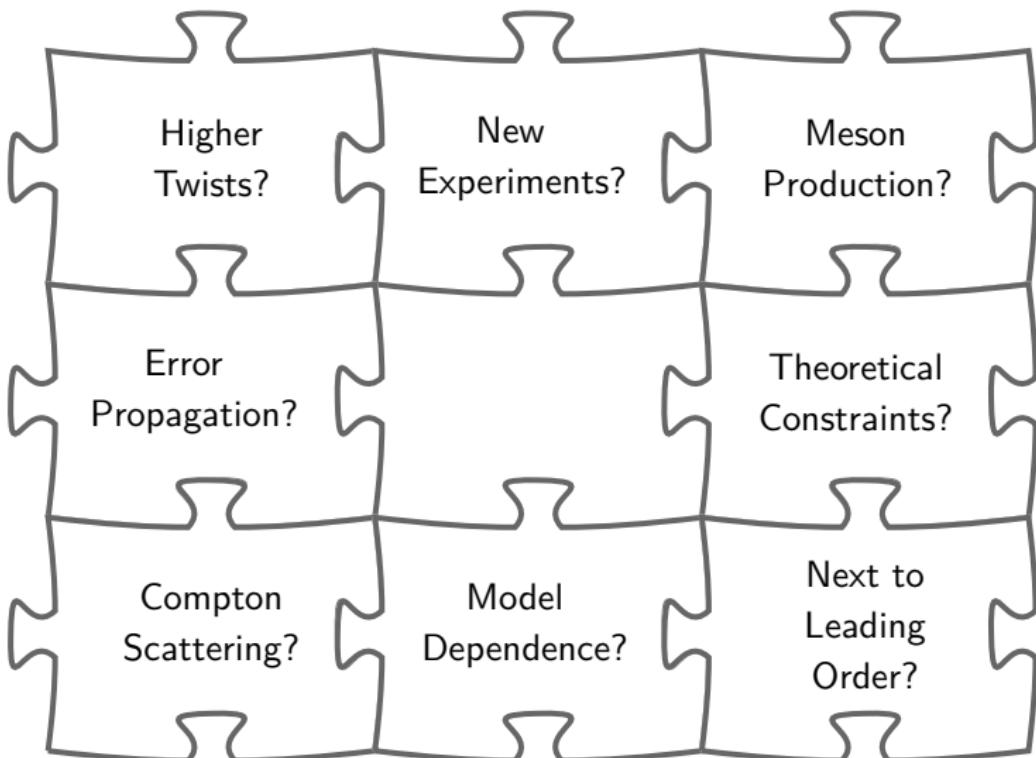
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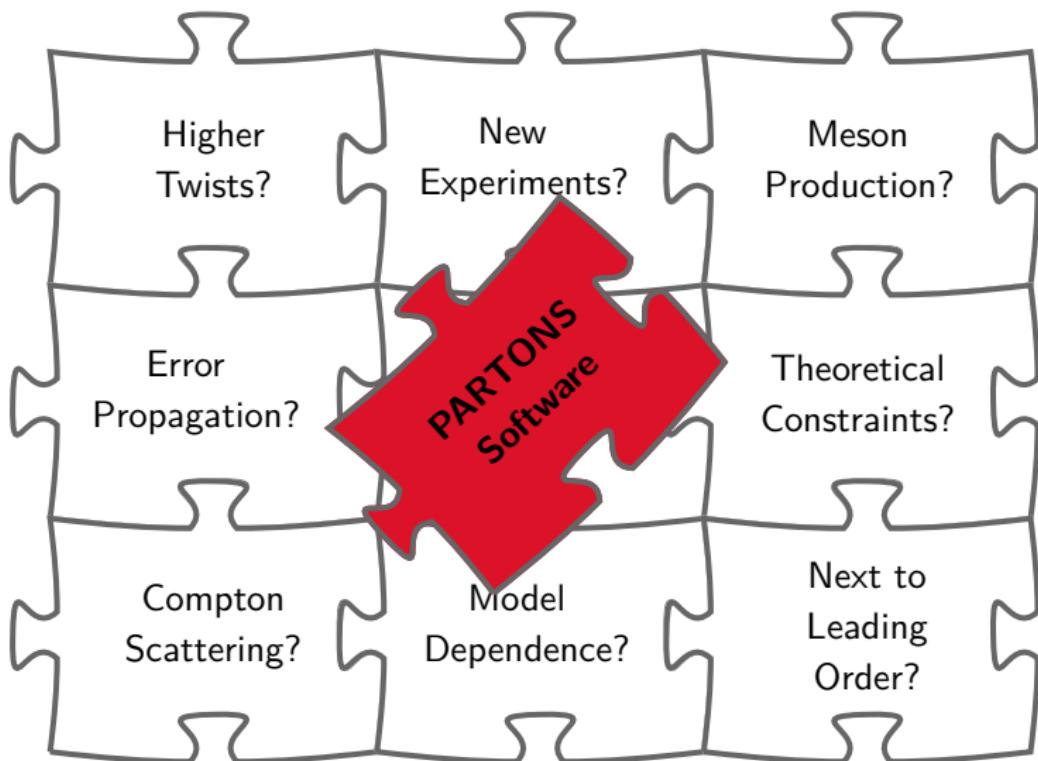
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Polynomiality.

Abstract formulation: the range of the Radon transform.

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- Write **polynomiality condition**:

$$\int_{-1}^1 dx x^m H^q(x, \xi, t) = \sum_{\substack{i=0 \\ \text{even}}}^m (2\xi)^i C_{mi}^q(t) + (2\xi)^{m+1} C_{mm+1}^q(t).$$

- Assume the existence of $D^q(z, t)$ such that:

$$\int_{-1}^{+1} dz z^m D(z, t) = C_{mm+1}^q(t).$$

- $H^q(x, \xi, t) - D(x/\xi, t)$ satisfies polynomiality at order m :

$$\int_{-1}^1 dx x^m \left(H^q(x, \xi, t) - D(x/\xi, t) \right) = \sum_{\substack{i=0 \\ \text{even}}}^m (2\xi)^i C_{mi}^q(t).$$

- **Ludwig-Helgason** condition: there exists F_D such that:

$$H(x, \xi, t) = D(x/\xi, t) + \int_{\Omega_{DD}} d\beta d\alpha F_D(\beta, \alpha, t) \delta(x - \beta - \alpha \xi).$$

The Radon transform.

Definition and properties.

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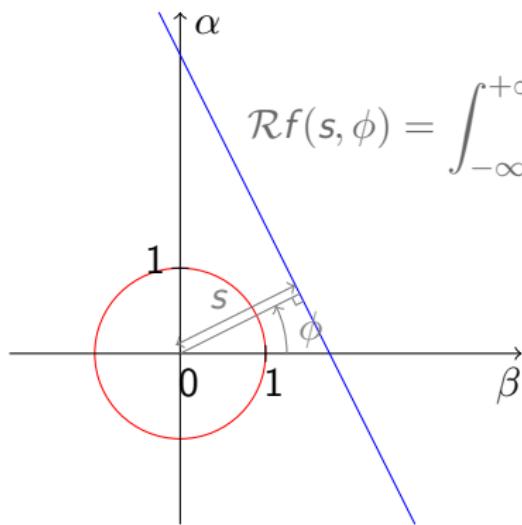
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For $s > 0$ and $\phi \in [0, 2\pi]$:

$$\mathcal{R}f(s, \phi) = \int_{-\infty}^{+\infty} d\beta d\alpha f(\beta, \alpha) \delta(s - \beta \cos \phi - \alpha \sin \phi)$$

and:

$$\mathcal{R}f(-s, \phi) = \mathcal{R}f(s, \phi \pm \pi)$$

Relation to GPDs:

$$x = \frac{s}{\cos \phi} \text{ and } \xi = \tan \phi$$

Relation between GPD H and DD F_D

$$\sqrt{1 + \xi^2} \left[H(x, \xi) - D\left(\frac{x}{\xi}\right) \right] = \mathcal{R}F_D(s, \phi) .$$

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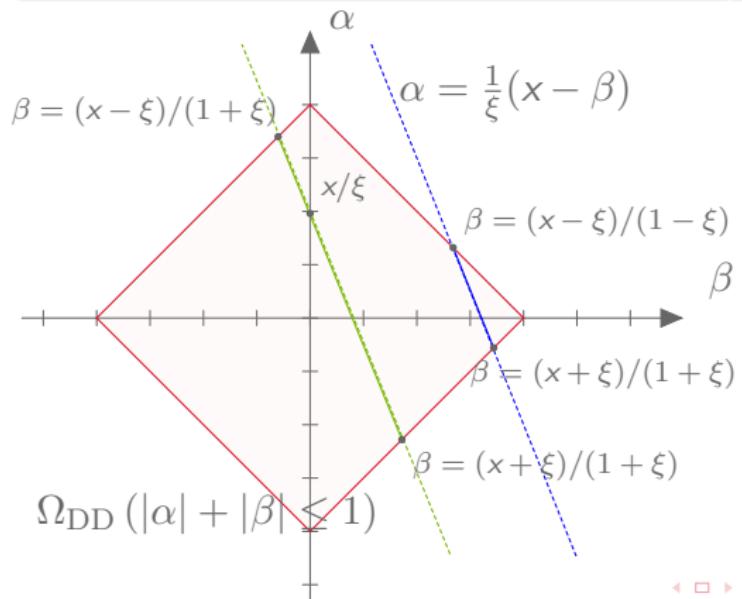
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DGLAP and ERBL regions

$$(x, \xi) \in \text{DGLAP} \Leftrightarrow |s| \geq |\sin \phi| ,$$

$$(x, \xi) \in \text{ERBL} \Leftrightarrow |s| \leq |\sin \phi| .$$



Each point (β, α) with $\beta \neq 0$ contributes to **both** DGLAP and ERBL regions.

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Theorem (simple case)

Let f be a compactly-supported summable function defined on \mathbb{R}^2 and $\mathcal{R}f$ its Radon transform.

Let $(s_0, \omega_0) \in \mathbb{R} \times S^1$ and U_0 an open neighborhood of ω_0 s.t.:

for all $s > s_0$ and $\omega \in U_0$ $\mathcal{R}f(s, \omega) = 0$.

Then $f(\mathbf{x}) = 0$ on the half-plane $\langle \mathbf{x} | \omega_0 \rangle > s_0$ of \mathbb{R}^2 .

Theorem (Boman and Todd Quinto, 1987)

Assume $(s_0, \omega_0) \in \mathbb{R} \times S^{n-1}$ and $f \in \mathcal{E}'(\mathbb{R})$. Let $\mu(\mathbf{x}, \omega)$ be a strictly positive real analytic function on $\mathbb{R}^n \times S^{n-1}$ that is even in ω . Let U_0 be an open neighborhood of ω_0 . Finally assume $R_\mu(s, \omega) = 0$ for $s > s_0$ and $\omega \in U_0$. Then $f = 0$ on the half space $\langle \mathbf{x} | \omega_0 \rangle > s_0$.

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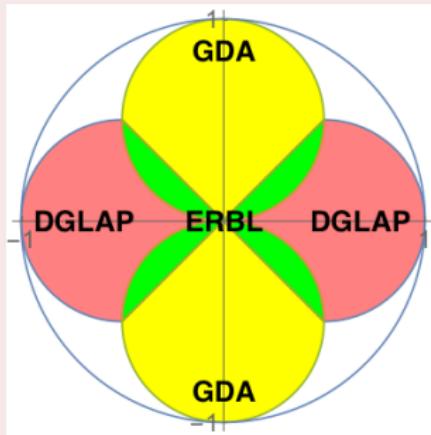
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Theorem (simple case)

for all $s > s_0$ and $\omega \in U_0$ $\mathcal{R}f(s, \omega) = 0$.

Then $f(\mathbb{N}) = 0$ on the half-plane $\langle \mathbb{N} | \omega_0 \rangle > s_0$ of \mathbb{R}^2 .

DGLAP and ERBL regions in polar coordinates



Theorem (simple case)

for all $s > s_0$ and $\omega \in U_0$ $\mathcal{R}f(s, \omega) = 0$.

Then $f(\mathbb{N}) = 0$ on the half-plane $\langle \mathbb{N} | \omega_0 \rangle > s_0$ of \mathbb{R}^2 .

Consider a GPD H being zero on the DGLAP region.

- Take $\xi_0 = \tan \phi_0 \in [0, 1]$, $x_0 \in]\xi_0, +\infty[$ and s_0 s.t.
 $x_0 \cos \phi_0 > s_0 > \sin \phi_0$.
- $\exists \epsilon > 0$ s.t. $s_0 > \sin \phi$ for $|\phi - \phi_0| < \epsilon$.
- Hyp: the underlying DD f has a zero Radon transform for all $\phi \in]\phi_0 - \epsilon, \phi_0 + \epsilon[$ and $s > s_0$ (DGLAP region).
- Then $f(\beta, \alpha) = 0$ for all (β, α) s.t.
 $\beta \cos \phi_0 + \alpha \sin \phi_0 = s > s_0$.
- At last select $s = x_0 \cos \phi_0$ to get $\beta + \alpha \xi_0 = x_0$.
- Cannot constrain the line $\beta = 0$. \square

▶ Proof.

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- Decompose an hadronic state $|H; P, \lambda\rangle$ in a Fock basis:

$$|H; P, \lambda\rangle = \sum_{N, \beta} \int [dx d\mathbf{k}_\perp]_N \psi_N^{(\beta, \lambda)}(x_1, \mathbf{k}_{\perp 1}, \dots, x_N, \mathbf{k}_{\perp N}) |\beta, k_1, \dots, k_N\rangle$$

- Derive an expression for the pion GPD in the DGLAP region $\xi \leq x \leq 1$:

$$H^q(x, \xi, t) \propto \sum_{\beta, j} \int [\bar{d}x \bar{d}\mathbf{k}_\perp]_N \delta_{j,q} \delta(x - \bar{x}_j) (\psi_N^{(\beta, \lambda)})^*(\hat{x}', \hat{\mathbf{k}}'_{\perp}) \psi_N^{(\beta, \lambda)}(\tilde{x}, \tilde{\mathbf{k}}_{\perp})$$

with $\tilde{x}, \tilde{\mathbf{k}}_{\perp}$ (resp. $\hat{x}', \hat{\mathbf{k}}'_{\perp}$) generically denoting incoming (resp. outgoing) parton kinematics.

Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)

- Similar expression in the ERBL region $-\xi \leq x \leq \xi$, but with overlap of N - and $(N+2)$ -body LFWFs.

Implementing Lorentz covariance.

Extend an overlap in the DGLAP region to the whole GPD domain.

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For any model of LFWF, one has to address the following three questions:

- 1 Does the extension exist?
- 2 If it exists, is it unique?
- 3 How can we compute this extension?

Modeling strategy

- 1 Ensure positivity by modeling the DGLAP region as an overlap of LFWFs.
- 2 Ensure polynomiality by inverting the Radon transform to identify an underlying DD.

Chouika *et al.*, Eur. Phys. J. **C77**, 906 (2017)

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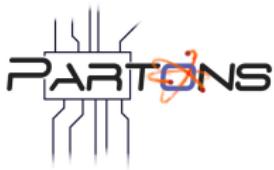
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And if the input data are inconsistent?

- Instead of solving $g = \mathcal{R}f$, find f such that $\|g - \mathcal{R}f\|_2$ is **minimum**.
- The solution **always exists**.
- The input data are **inconsistent** if $\|g - \mathcal{R}f\|_2 > 0$.

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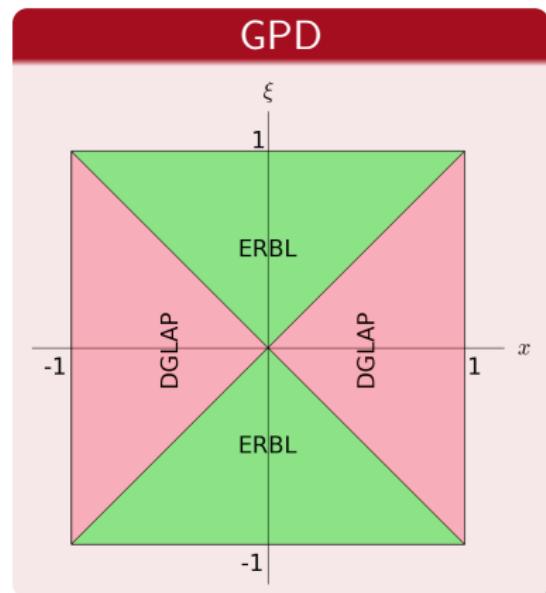
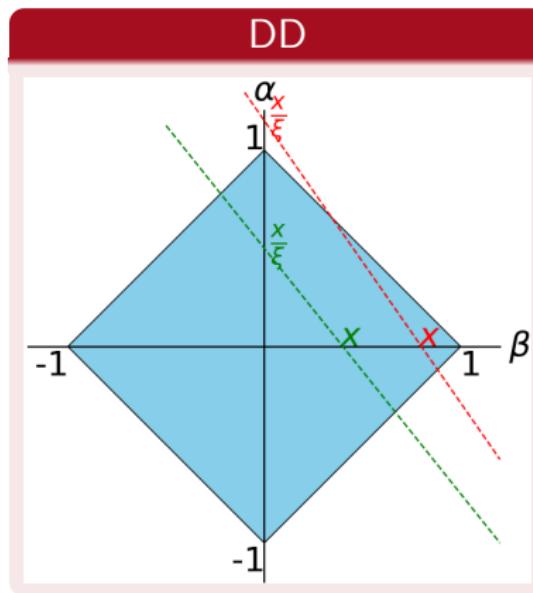
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How can we get a DD from a GPD in the DGLAP region?
 ■ Restrict to quark GPDs ($\beta > 0$).

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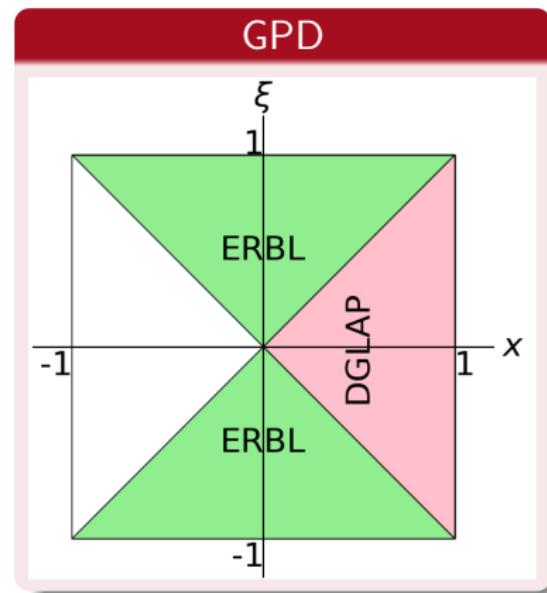
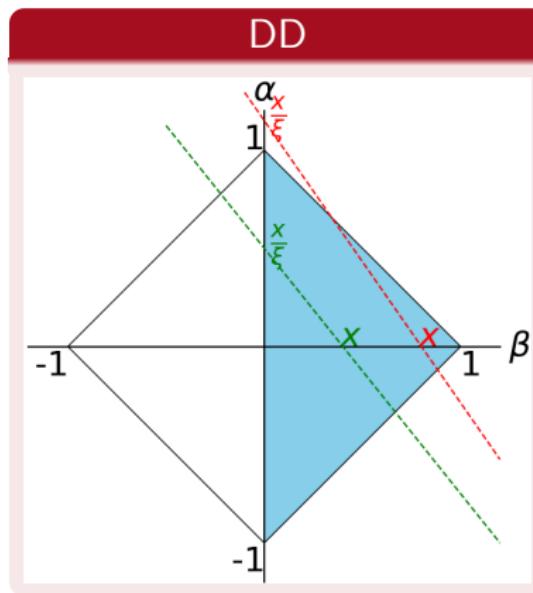
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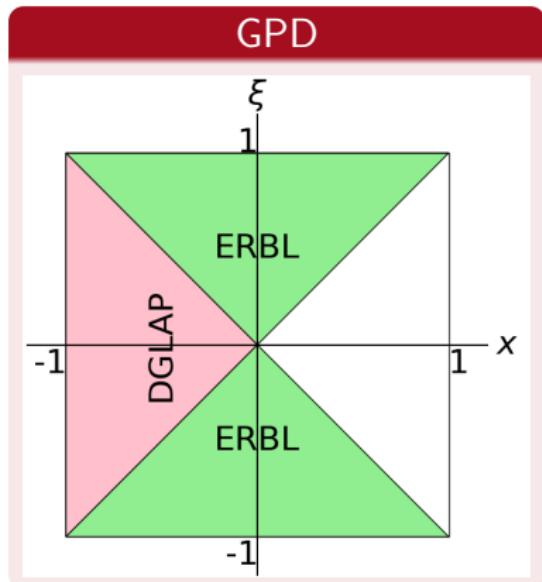
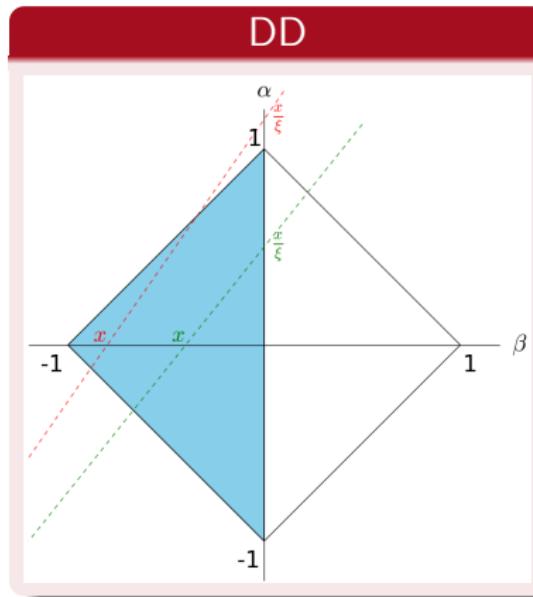
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How can we get a DD from a GPD in the DGLAP region?

- Restrict to quark GPDs ($\beta > 0$).
- Only ERBL region "sees" both $\beta > 0$ and $\beta < 0$.



Computation of the extension. Problem reduction.

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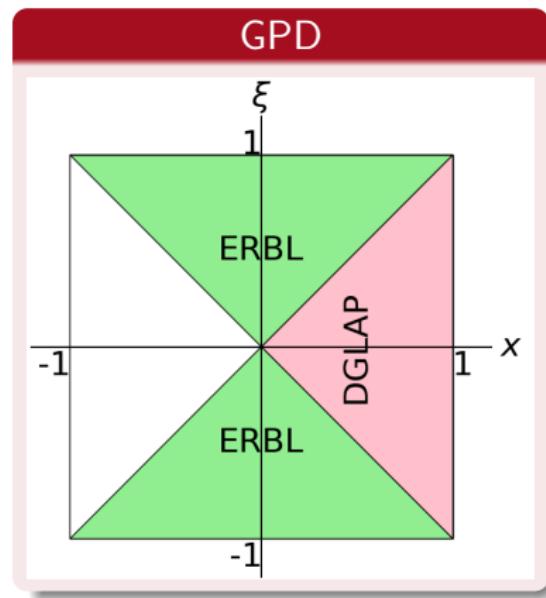
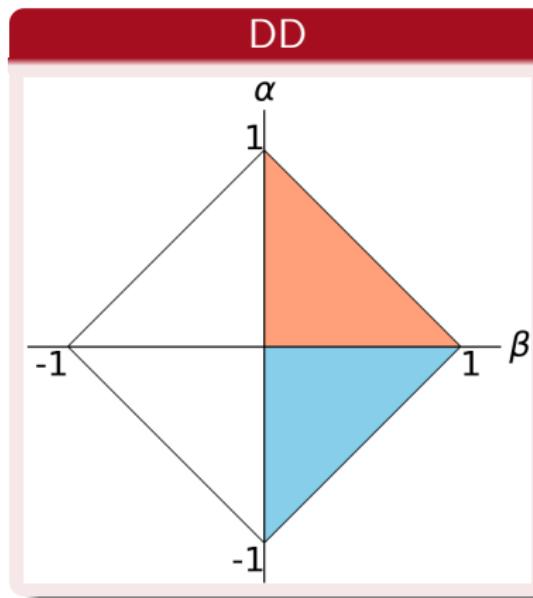
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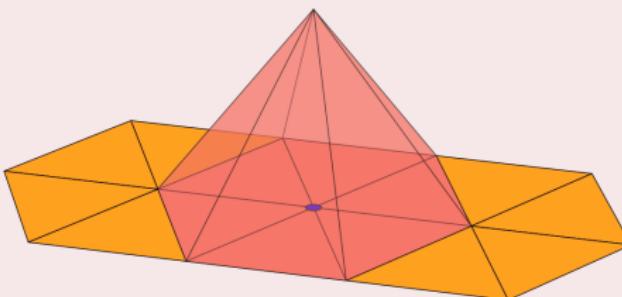
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How can we get a DD from a GPD in the DGLAP region?

- Restrict to quark GPDs ($\beta > 0$).
- Only ERBL region "sees" both $\beta > 0$ and $\beta < 0$.
- Use α -parity of the DD.



Example of a P1 basis function



- Discretize the DD on a mesh with $n \simeq 800$ triangular cells.
- Compute the Radon transform of a P1 basis function.
- Sample $m \simeq 4n$ (x, ξ) -lines intersecting the DD support.
- Solve a linear system $AX = B$ with A a sparse $m \times n$ matrix.
- Adopt an iterative regularization method: LSMR.

Fong and Saunders, arXiv:1006.0758

Examples - benchmarks (1/4).

Algebraic Bethe-Salpeter model.

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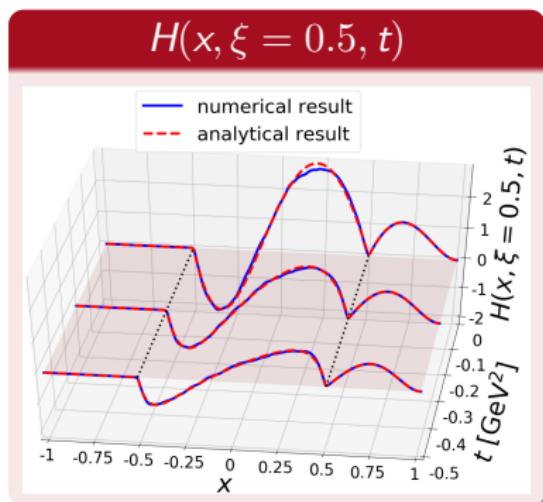
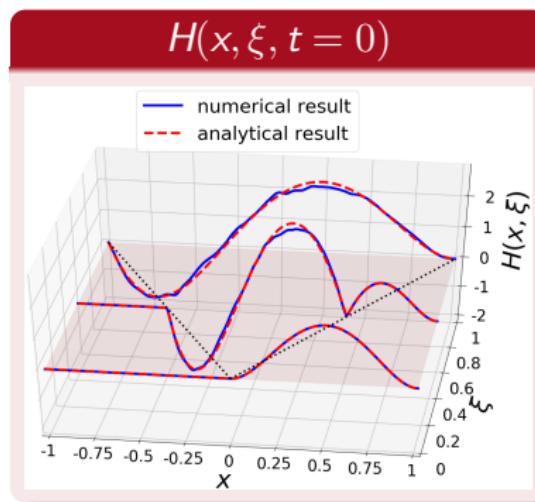
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$$\Psi_{I=0}(x, \mathbf{k}_\perp) = 8\sqrt{15}\pi \frac{M^3}{(\mathbf{k}_\perp^2 + M^2)^2} (1-x)x,$$

$$ik_\perp^j \Psi_{I=1}(x, \mathbf{k}_\perp) = 8\sqrt{15}\pi \frac{k_\perp^j M^2}{(\mathbf{k}_\perp^2 + M^2)^2} (1-x)x, \quad j = 1, 2$$



Examples - benchmarks (2/4).

Algebraic spectator model.

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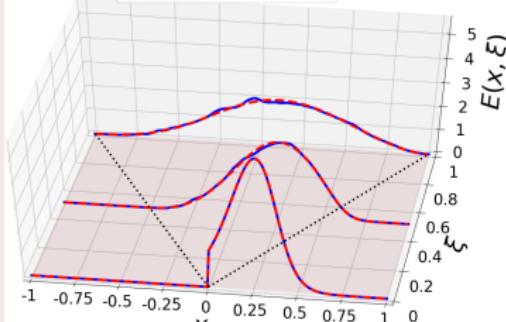
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$$\varphi(x, \mathbf{k}_\perp) = \frac{gM^{2p}}{\sqrt{1-x}} x^{-p} \left(M^2 - \frac{\mathbf{k}_\perp^2 + m^2}{x} - \frac{\mathbf{k}_\perp^2 + \lambda^2}{1-x} \right)^{-p-1}$$

Hwang and Müller, Phys. Lett. **B660**, 350 (2008)

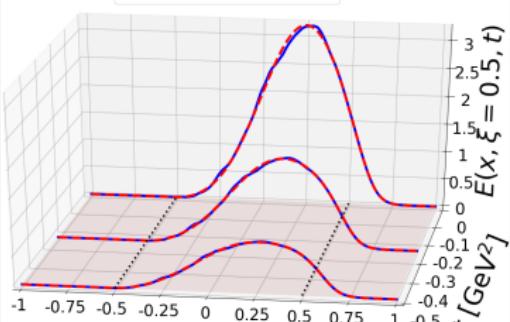
$H(x, \xi, t = 0)$

 numerical result
 analytical result



$H(x, \xi = 0.5, t)$

 numerical result
 analytical result



Chouika et al., Eur. Phys. J. **C77**, 906 (2017)

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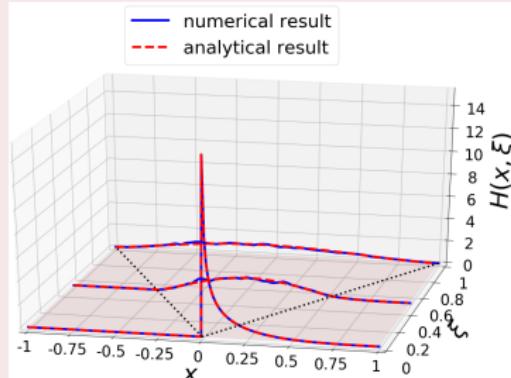
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Radyushkin DD Ansatz with phenomenological PDF:

$$q\text{Regge}(x) = \frac{35}{32} \frac{(1-x)^3}{\sqrt{x}} .$$

$H(x, \xi, t = 0)$



Chouika *et al.*, Eur. Phys. J. **C77**, 906 (2017)

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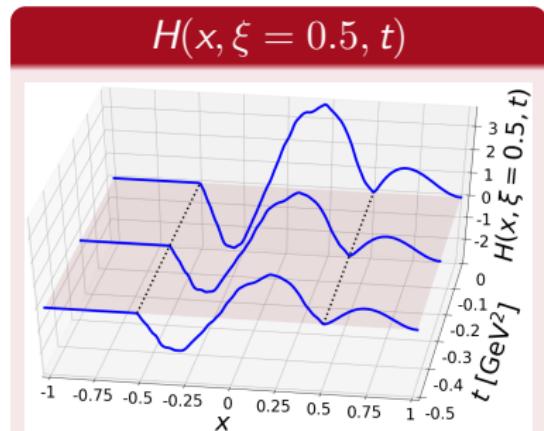
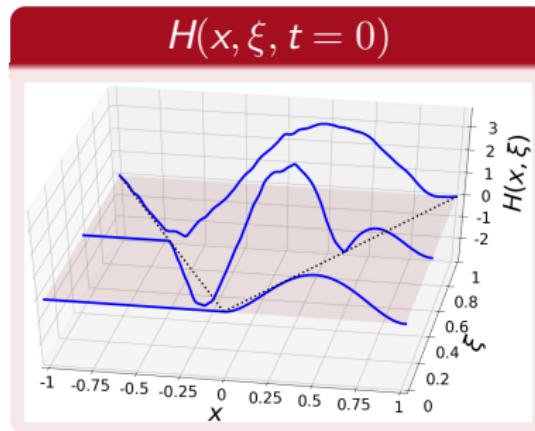
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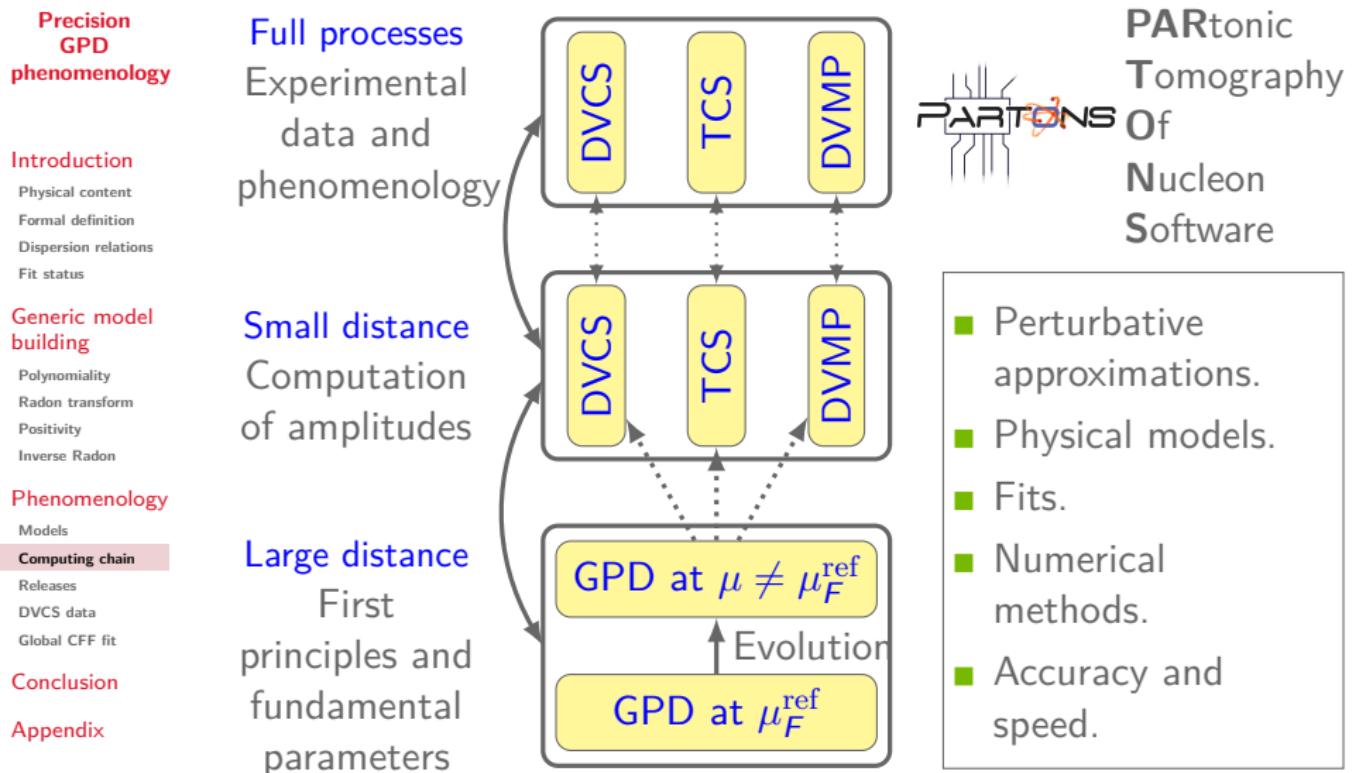
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$$\Psi(x, k_\perp^2) = \frac{4\sqrt{15}\pi}{M} \sqrt{x(1-x)} e^{-\frac{k_\perp^2}{4M^2(1-x)x}}.$$



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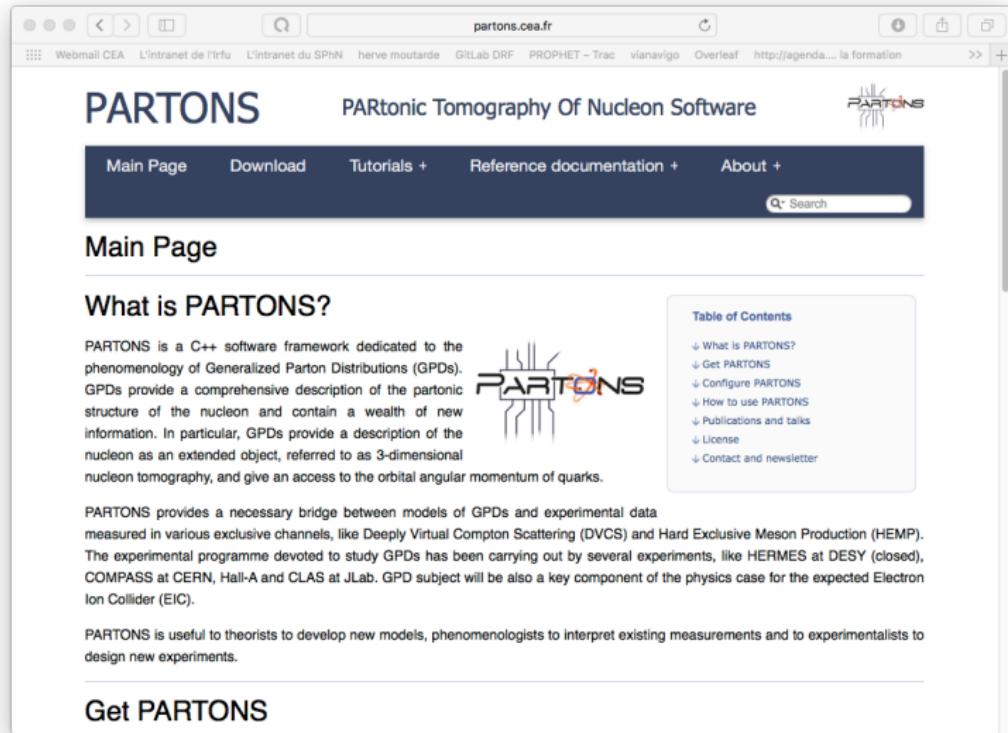
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The screenshot shows the main page of the PARTONS website at partons.cea.fr. The header includes the CEA Paris-Saclay logo and navigation links for Webmail CEA, L'intranet de l'Irfu, L'intranet du SPHIN, herve moutarde, GitLab DRF, PROPHET – Trac, vianavigo, Overleaf, and agenda. The page title is "PARTONS PARtonic Tomography Of Nucleon Software". Below the title are menu items: Main Page, Download, Tutorials +, Reference documentation +, and About +. A search bar is also present. The main content area features a large "Main Page" heading, followed by "What is PARTONS?", which describes it as a C++ software framework for GPD phenomenology. It mentions the Radon transform, positivity, and inverse Radon. To the right is a "Table of Contents" sidebar with links to "What is PARTONS?", "Get PARTONS", "Configure PARTONS", "How to use PARTONS", "Publications and talks", "License", and "Contact and newsletter". At the bottom, there is a "Get PARTONS" section and a citation for "Berthou et al., Eur. Phys. J. C78, 478 (2018)".

Berthou *et al.*, Eur. Phys. J. C78, 478 (2018)

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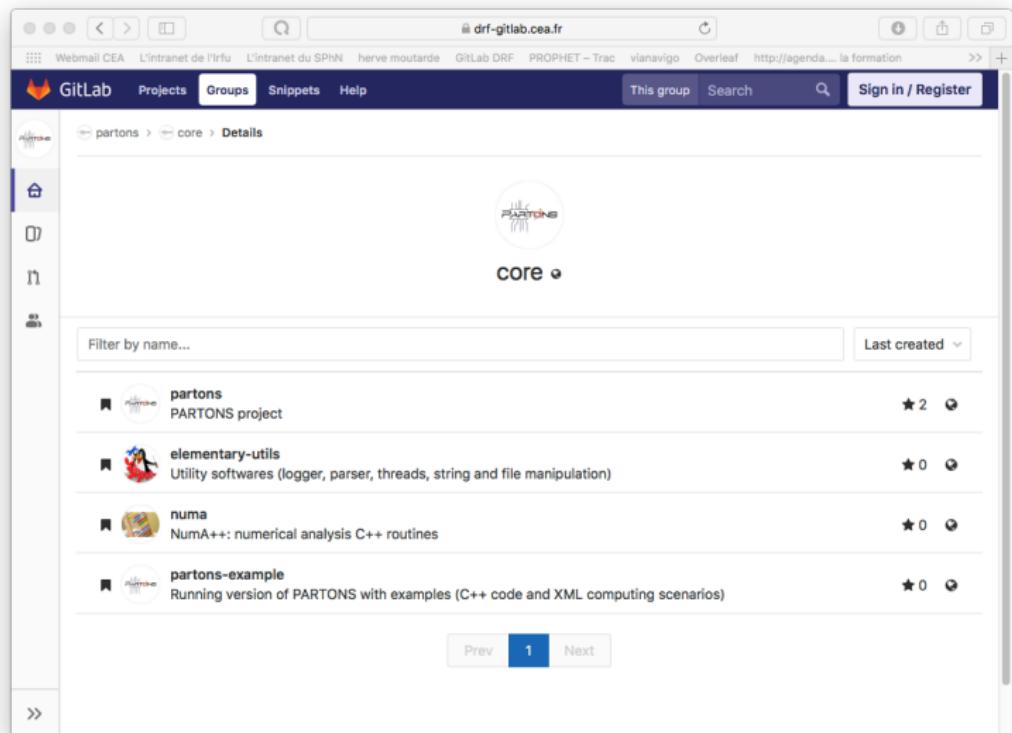
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The screenshot shows the CEA GitLab interface. The top navigation bar includes links for Webmail CEA, L'intranet de l'Irfu, L'intranet du SPHIN, herve moutarde, GitLab DRF, PROPHET – Trac, vianavigo, Overleaf, and http://agenda... la formation. The main header says "GitLab Projects Groups Snippets Help This group Search Sign in / Register". Below this, the breadcrumb navigation shows "partons > core > Details". A sidebar on the left has icons for Home, Groups, Projects, Snippets, Help, and a search/filter icon. The main content area displays a list of projects under the "core" group. Each project entry includes a thumbnail icon, the project name, a brief description, and a star rating. The "partons" project is listed first, followed by "elementary-utils", "numa", and "partons-example". At the bottom, there are navigation buttons for "Prev", "1", and "Next".

Project	Description	Rating
partons	PARTONS project	★ 2
elementary-utils	Utility softwares (logger, parser, threads, string and file manipulation)	★ 0
numa	NumA++: numerical analysis C++ routines	★ 0
partons-example	Running version of PARTONS with examples (C++ code and XML computing scenarios)	★ 0

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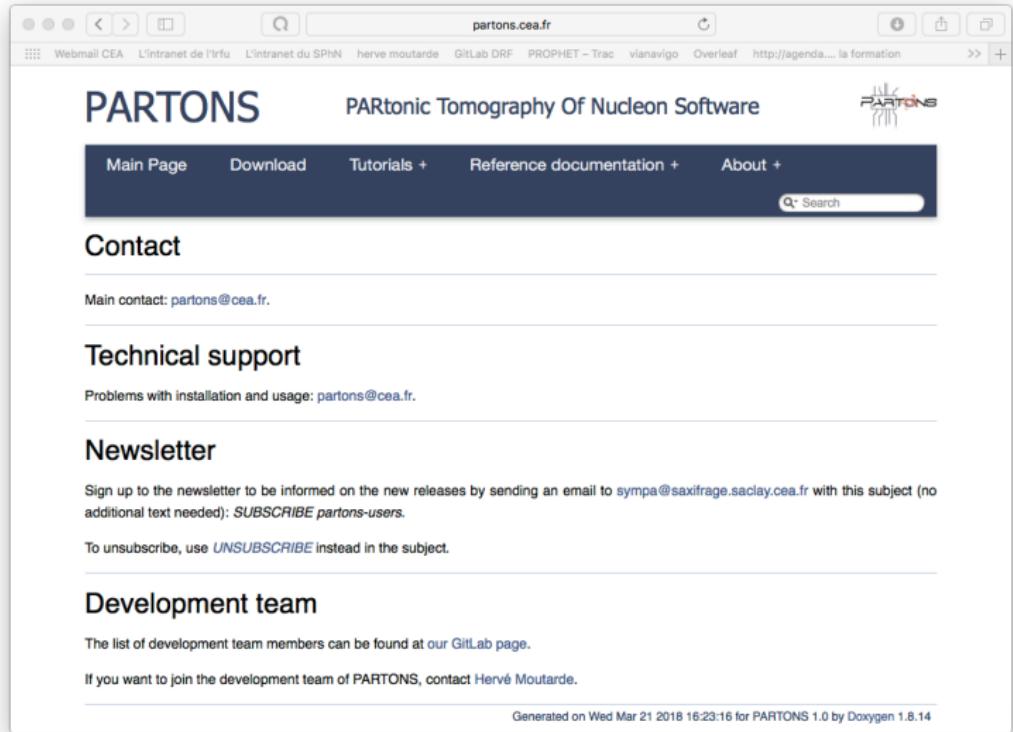
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The screenshot shows the 'Contact' section of the PARTONS website. The header includes the PARTONS logo and navigation links for Main Page, Download, Tutorials, Reference documentation, and About. A search bar is also present. The 'Contact' section contains links for Main contact (partons@cea.fr) and Technical support (partons@cea.fr). Below these, there's a section for Newsletter subscription with instructions to send an email to sympa@saxifrage.saclay.cea.fr with the subject 'SUBSCRIBE partons-users'. There's also a note about unsubscribing by sending 'UNSUBSCRIBE' instead. The Development team section links to the GitLab page for team members. The footer indicates the page was generated on March 21, 2018.

PARTONS PARtonic Tomography Of Nucleon Software

Main Page Download Tutorials + Reference documentation + About +

Contact

Main contact: partons@cea.fr.

Technical support

Problems with installation and usage: partons@cea.fr.

Newsletter

Sign up to the newsletter to be informed on the new releases by sending an email to sympa@saxifrage.saclay.cea.fr with this subject (no additional text needed): *SUBSCRIBE partons-users*.

To unsubscribe, use *UNSUBSCRIBE* instead in the subject.

Development team

The list of development team members can be found at our GitLab page.

If you want to join the development team of PARTONS, contact Hervé Moutarde.

Generated on Wed Mar 21 2018 16:23:16 for PARTONS 1.0 by Doxygen 1.8.14

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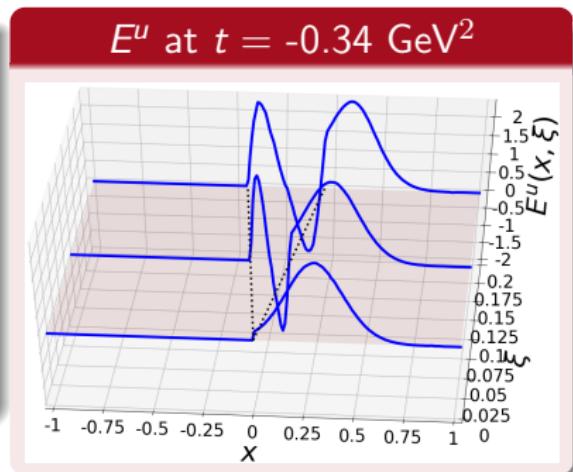
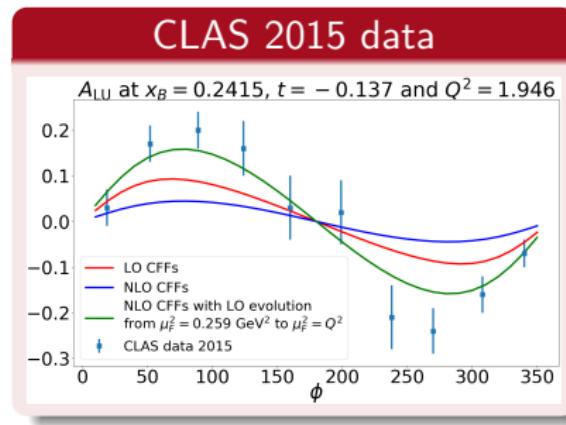
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- Only LO phenomenology achievable without extension to ERBL region.
- Computation of various DVCS observables in the valence region under different pQCD assumptions with PARTONS.

Chouika, PhD thesis (2018)

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- **Leading twist and leading order analysis.**
- Focus on the quark sector (intermediate to large x_B).
- Dispersion relations: CFF \mathcal{H} depends on **D-term** and **border function** $H(x, \xi = x)$.
- Tomography: model **skewing function** $H(x, x, t)/H(x, 0, t)$ consistently with perturbative QCD.
- Fit to PDFs and elastic form factors.
- Propagate uncertainties by **replica method**.

Moutarde *et al.*, Eur. Phys. J. **C78**, 890 (2018)

Selected DVCS measurements.

All existing sets except $d^4\sigma_{UU}^-$ from Hall A (2015-17) and HERA.

Precision GPD phenomenology

No.	Collab.	Year	Ref.	Observable	Kinematic dependence	No. of points used / all
1	HERMES	2001	[13]	A_{LU}^+	ϕ	10 / 10
2		2006	[114]	$A_C^{\cos i\phi}$	$i = 1$	4 / 4
3		2008	[115]	$A_C^{\cos i\phi}$	$i = 0, 1$	x_{Bj} 18 / 24
				$A_{UT,DVCS}^{\sin(\phi-\phi_S)\cos i\phi}$	$i = 0$	
				$A_{UT,I}^{\sin(\phi-\phi_S)\cos i\phi}$	$i = 0, 1$	
				$A_{UT,I}^{\cos(\phi-\phi_S)\sin i\phi}$	$i = 1$	
4		2009	[116]	$A_{LU,I}^{\sin i\phi}$	$i = 1, 2$	x_{Bj} 35 / 42
				$A_{LU,DVCS}^{\sin i\phi}$	$i = 1$	
				$A_C^{\cos i\phi}$	$i = 0, 1, 2, 3$	
5		2010	[117]	$A_{UL}^{\sin i\phi}$	$i = 1, 2, 3$	x_{Bj} 18 / 24
				$A_{LT}^{\cos i\phi}$	$i = 0, 1, 2$	
6		2011	[118]	$A_{LT,DVCS}^{\cos(\phi-\phi_S)\cos i\phi}$	$i = 0, 1$	x_{Bj} 24 / 32
				$A_{LT,DVCS}^{\sin(\phi-\phi_S)\sin i\phi}$	$i = 1$	
				$A_{LT,I}^{\cos(\phi-\phi_S)\cos i\phi}$	$i = 0, 1, 2$	
				$A_{LT,I}^{\sin(\phi-\phi_S)\sin i\phi}$	$i = 1, 2$	
7		2012	[119]	$A_{LU,I}^{\sin i\phi}$	$i = 1, 2$	x_{Bj} 35 / 42
				$A_{LU,DVCS}^{\sin i\phi}$	$i = 1$	
				$A_C^{\cos i\phi}$	$i = 0, 1, 2, 3$	
8	CLAS	2001	[14]	$A_{LU}^{-,\sin i\phi}$	$i = 1, 2$	— 0 / 2
9		2006	[120]	$A_{UL}^{-,\sin i\phi}$	$i = 1, 2$	— 2 / 2
10		2008	[121]	A_{LU}^-	ϕ	283 / 737
11		2009	[122]	A_{LU}^-	ϕ	22 / 33
12		2015	[123]	$A_{LU}^-, A_{UL}^-, A_{LL}^-$	ϕ	311 / 497
13		2015	[124]	$d^4\sigma_{UU}^-$	ϕ	1333 / 1933
14	Hall A	2015	[112]	$\Delta d^4\sigma_{LU}^-$	ϕ	228 / 228
15		2017	[113]	$\Delta d^4\sigma_{LU}^-$	ϕ	276 / 358
16	COMPASS	2018	[55]	b	—	1 / 1
					SUM:	2600 / 3970

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A selection of results.

2600 experimental points, 13 free parameters, $\chi^2/\text{dof} \simeq 0.91$.

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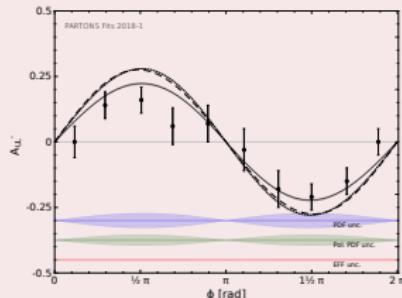
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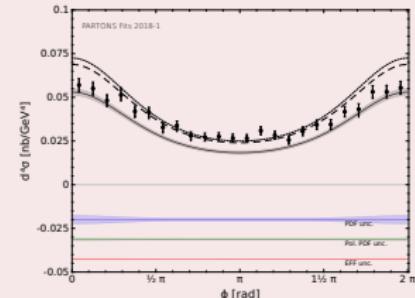
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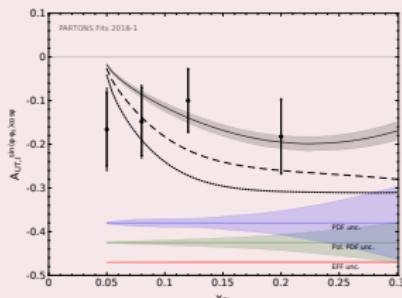
CLAS



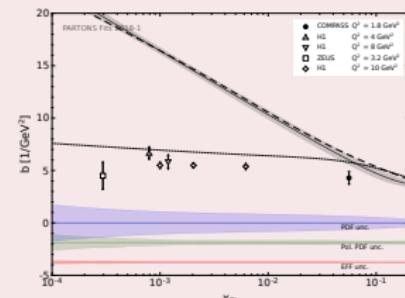
Hall A



HERMES



COMPASS



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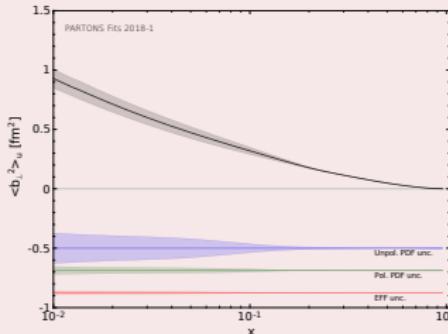
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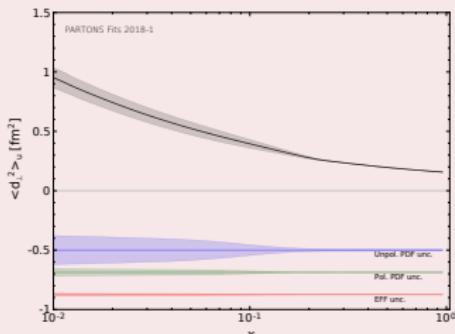
Aside on electromagnetic form factors

- Form factor: **Fourier transform** of charge density.
- The Fourier transform of a compactly-supported function is **analytic over the complex plane**.
- Form factors have a **branch cut** along the timelike axis.
- Hence the charge distribution ***cannot be bounded***.

Distance to center of mass



Distance to spectators



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- We can now build generic GPD models satisfying *a priori* all theoretical constraints.
- We now have tools to systematically relate these models to experimental data in multi-channel analysis.
- We now have an operating engine for global CFF fits.
- We revisit the mechanical properties of hadrons to assess how much we can learn from GPD extractions.

New studies become possible!

- Global GPD fits.
- Energy-momentum structure of hadrons.
- Quantitative impact of nonperturbative QCD ingredients on 3D hadron structure studies.
- GPD and TMD studies in a common framework.

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Proof (elements)

Let $\mathbf{x}_0 = (\mathfrak{b}, \mathfrak{a}) \in \mathbb{R}^2$, $s \in \mathbb{R}$ and $\delta > 0$ such that

$$\langle \mathbf{x}_0 | \omega_0 \rangle = \mathfrak{b} \cos \phi_0 + \mathfrak{a} \sin \phi_0 = s > s_0 + \delta.$$

Denote \mathcal{B} a ball containing \mathbf{x}_0 and the support of f , which is bounded by assumption.

We will show that $f = 0$ in a neighborhood of \mathbf{x}_0 in \mathcal{B} .

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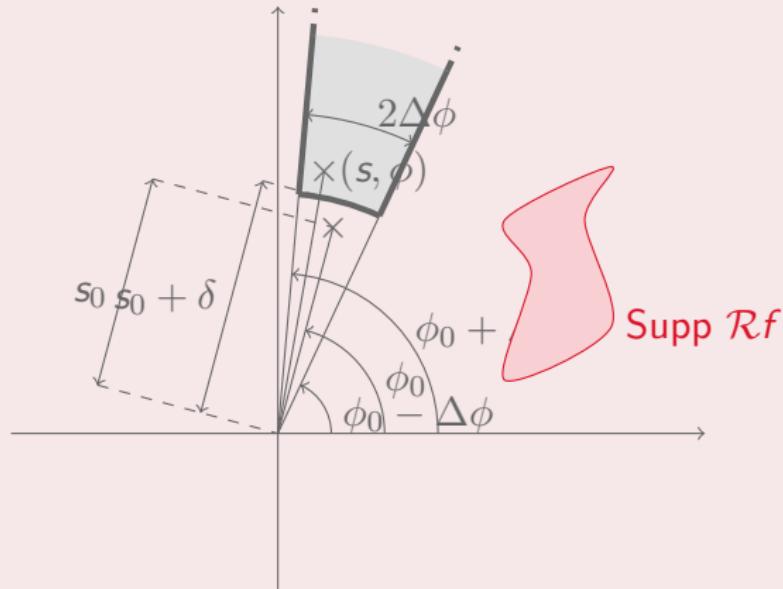
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Step 1

Identification of a neighborhood T of \mathbb{N}_0 s.t.:

$$\forall s > s_0 + \delta, \quad \forall \omega \in T, \quad \int_{\mathbb{R}^2} d\mathbb{N} \delta(s - \langle \mathbb{N} | \omega \rangle) f(\mathbb{N}) = 0.$$



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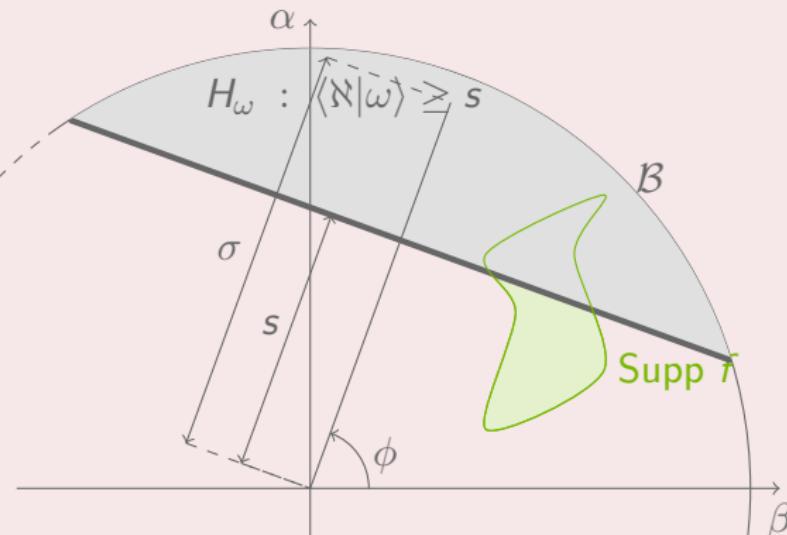
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Step 2

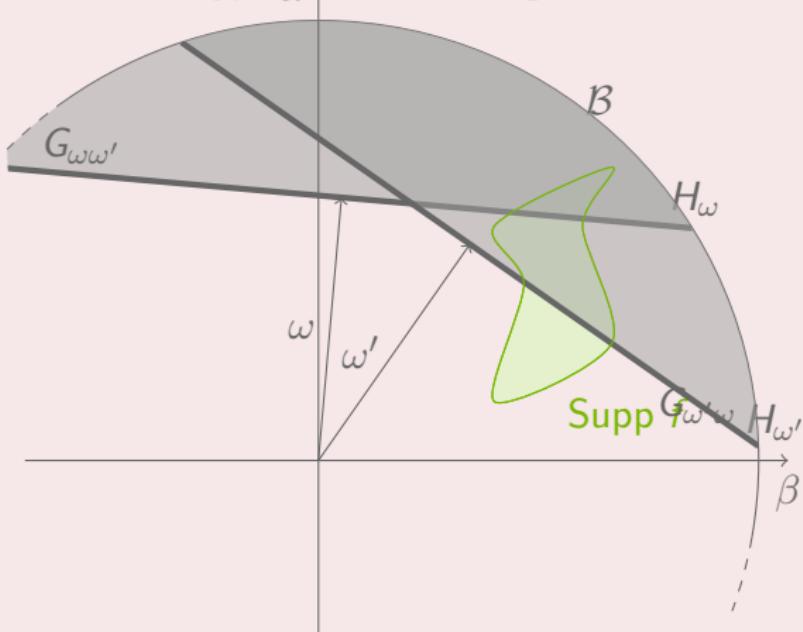
Prove by induction on the multi index $\mathbf{m} = (m, n)$ that, for all nonnegative integers m, n and $\omega \in T$:

$$\int_{[s_0 + \delta, +\infty[} ds \int_{\mathcal{B}} d\mathbb{N} \delta(s - \langle \mathbb{N} | \omega \rangle) \mathbb{N}^{\mathbf{m}} \langle \mathbb{N} | \omega \rangle^k f(\mathbb{N}) = 0 .$$



Step 2 (cont')

Induction step: infinitesimal change of the first cartesian coordinate of ω . G-type contributions go to 0.



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Step 3

Apply previous result with $k = 0$ and $\omega = \omega_0$, let δ goes to 0:

$$\text{for all } n, m \geq 0 \quad \int_{H_{\omega_0}} d\beta d\alpha \beta^m \alpha^n f(\beta, \alpha) = 0 .$$

Conclude by injectivity of the Fourier transform from $L^1(\mathbb{R}^2)$ into the set of continuous functions on \mathbb{R}^2 .

[◀ Back to uniqueness statement.](#)

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