Graduated Visitor Future postdoc

PARTON DISTRIBUTIONS WITHIN MESONS

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Motivation

- Understanding strong interactions is still being a challenge for physicists, even decades after the formulation of the fundamental theory of quarks and gluons, namely, **Quantum Chromodynamics** (**QCD**).

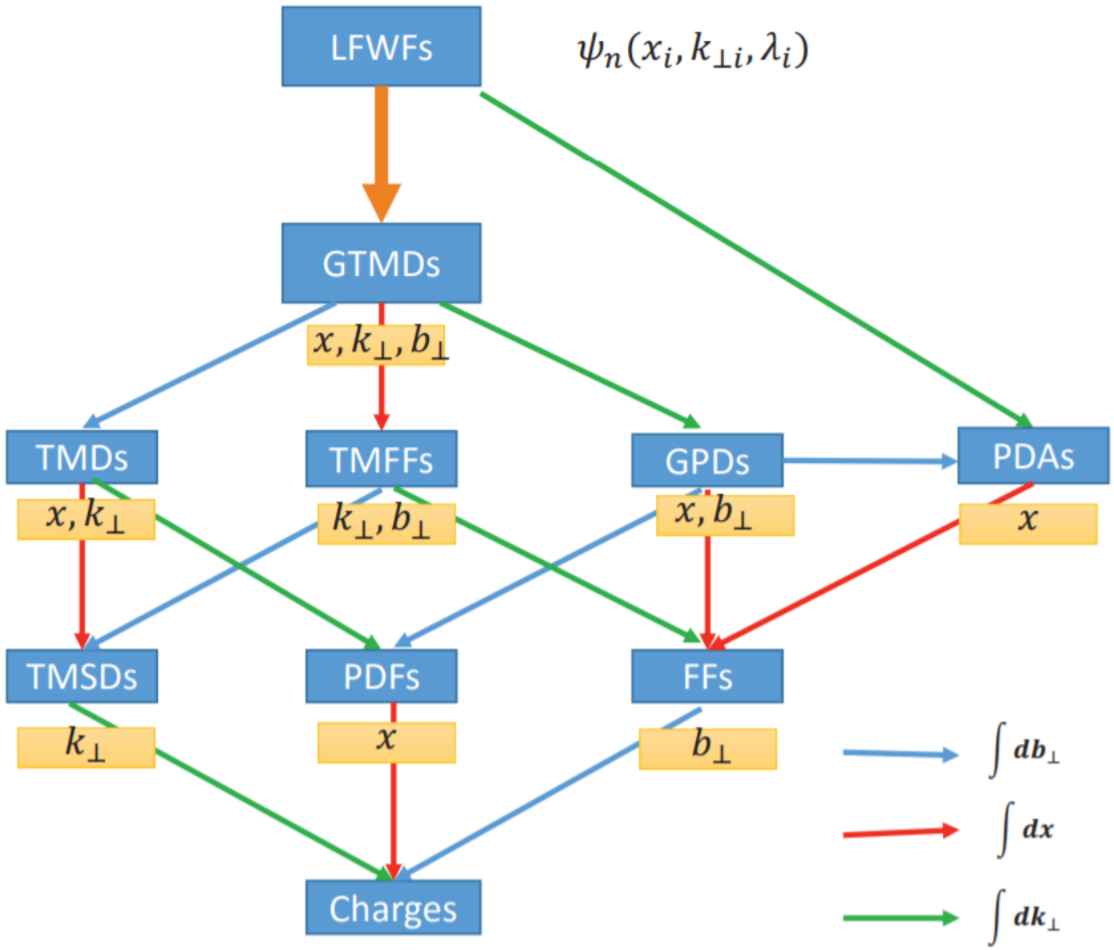
- QCD is characterized by two emergent phenomena: **confinement** and **dynamical chiral symmetry breaking** (**DCSB**), which have far reaching consequences in the hadron spectrum and their properties.

- Due to the non perturbative nature of QCD, unraveling the hadron structure, from the fundamental degrees of freedom, is an outstanding problem.

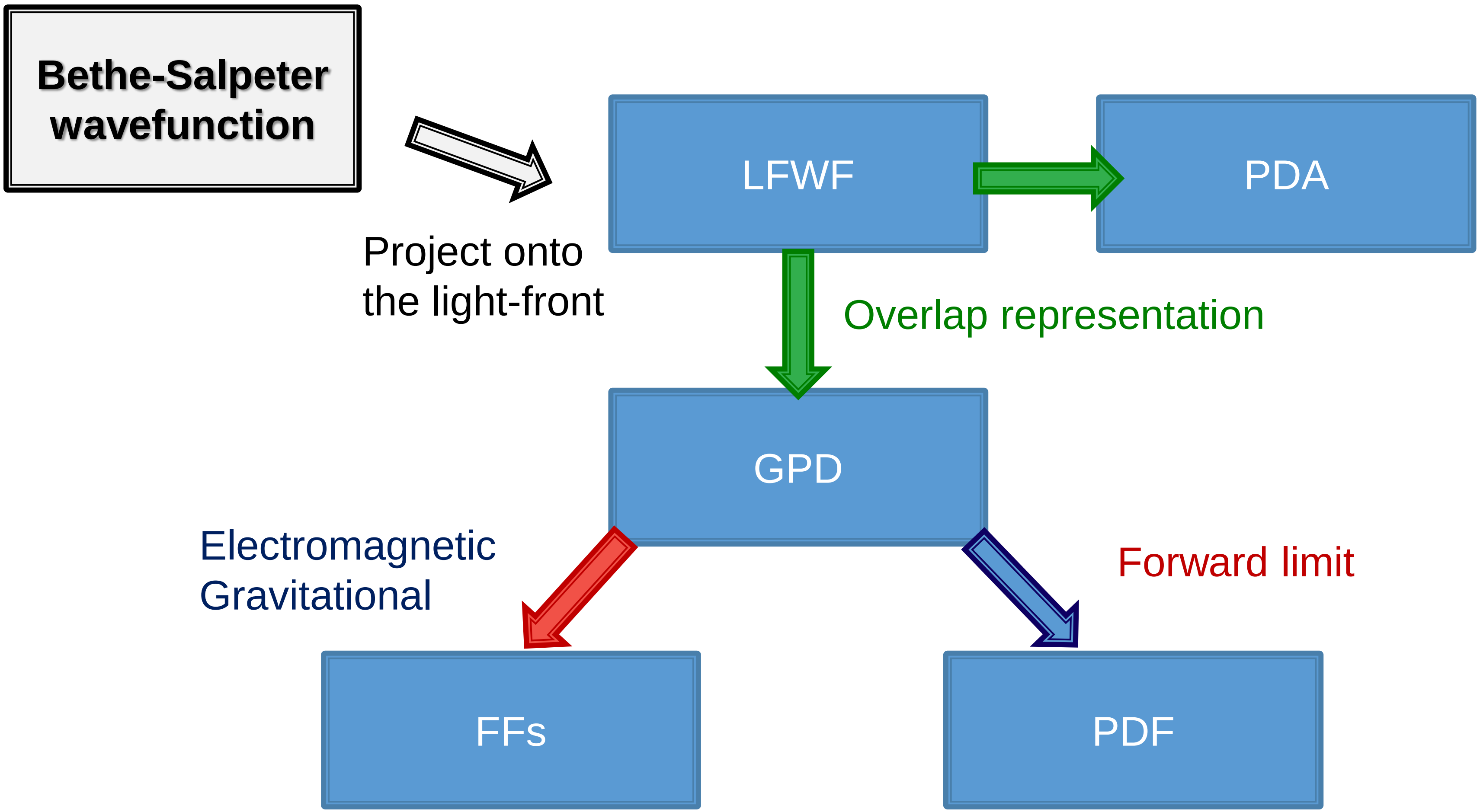
- I shall present an approach, based on **Dyson-Schwinger equations** (**DSEs**), to compute a choice of parton distributions within hadrons (pions and kaons).

Motivation

- Starting from the LFWFs there many kinds of parton distributions that can be obtained:



Our path…



Bethe-Salpeter wavefunction
Project onto the light-front
LFWF
PDA
Overlap representation
GPD
Electromagnetic
Gravitational
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PDF

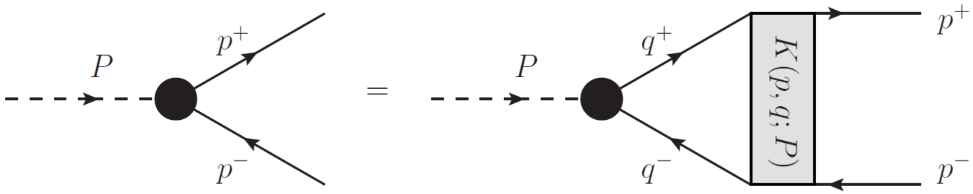
Bethe-Salpeter wave function

- The BS wave function is the sandwich of the BS amplitude and the quark-antiquark propagators:

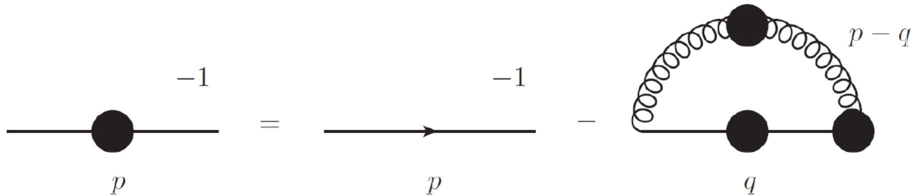
$$\chi_H(k_-^H; P_H) = S_q(k)\Gamma_H(k_-^H; P_H)S_{\bar{q}}(k - P_H) \ , \ k_-^H = k - P_H/2 \ .$$

$P_H^2 = -m_H^2$: **meson's mass**; Γ_H:**BS amplitude**; $S_{q(\bar{q})}$: **quark (antiquark) propagator**

- Tensor structure of Γ_H depends on the transformation properties of the meson. It obeys its corresponding BS equation:



- Analogously, $S_{q(\bar{q})}$ obey their corresponding DSE.



$$\chi_K(k_-^K;P_K) = S_u(k)\Gamma_K(k_-^K;P_K)S_s(k-P_K)$$

Bethe-Salpeter wave function

- Following **Phys.Rev. D97 (2018) no.9, 094014**, we employ a Nakanishi-like representation of Kaon BS wave function:

$$n_K\chi_K^{(2)}(k_-^K;P_K) = \underbrace{\mathcal{M}(k;P_K)}_{1}\int_{-1}^{1} d\omega\ \underbrace{\rho_K(\omega)}_{2}\underbrace{\mathcal{D}(k;P_K)}_{3}\ ,$$

1: Leading twist contribution to PDA (only γ_5 BSA):

$$\mathcal{M}(k;P_K) = -\gamma_5[\gamma\cdot P_K M_u + \gamma\cdot k(M_u - M_s) + \sigma_{\mu\nu}k_\mu P_{K\nu}]\ ,$$

2: Sprectral weight: To be chosen later.

3: Product of 3 quadratic forms in the denominator:

$$\mathcal{D}(k;P_K) = \Delta(k^2,M_u^2)\Delta((k-P_K)^2,M_s^2)\hat{\Delta}(k^2_{\omega-1},\Lambda_K^2)\ ,$$

$$\text{where: } \Delta(s,t) = [s+t]^{-1},\ \hat{\Delta}(s,t) = t\Delta(s,t)\ .$$

Bethe-Salpeter wave function

- Following **Phys.Rev. D97 (2018) no.9, 094014**, we employ a Nakanishi-like representation of Kaon BS wave function:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \mathcal{M}(k; P_K) \int_{-1}^{1} d\omega \; \rho_K(\omega) \mathcal{D}(k; P_K) \; ,$$

- Combining denominators (through Feynman parametrization) and rearranging the order of integration:

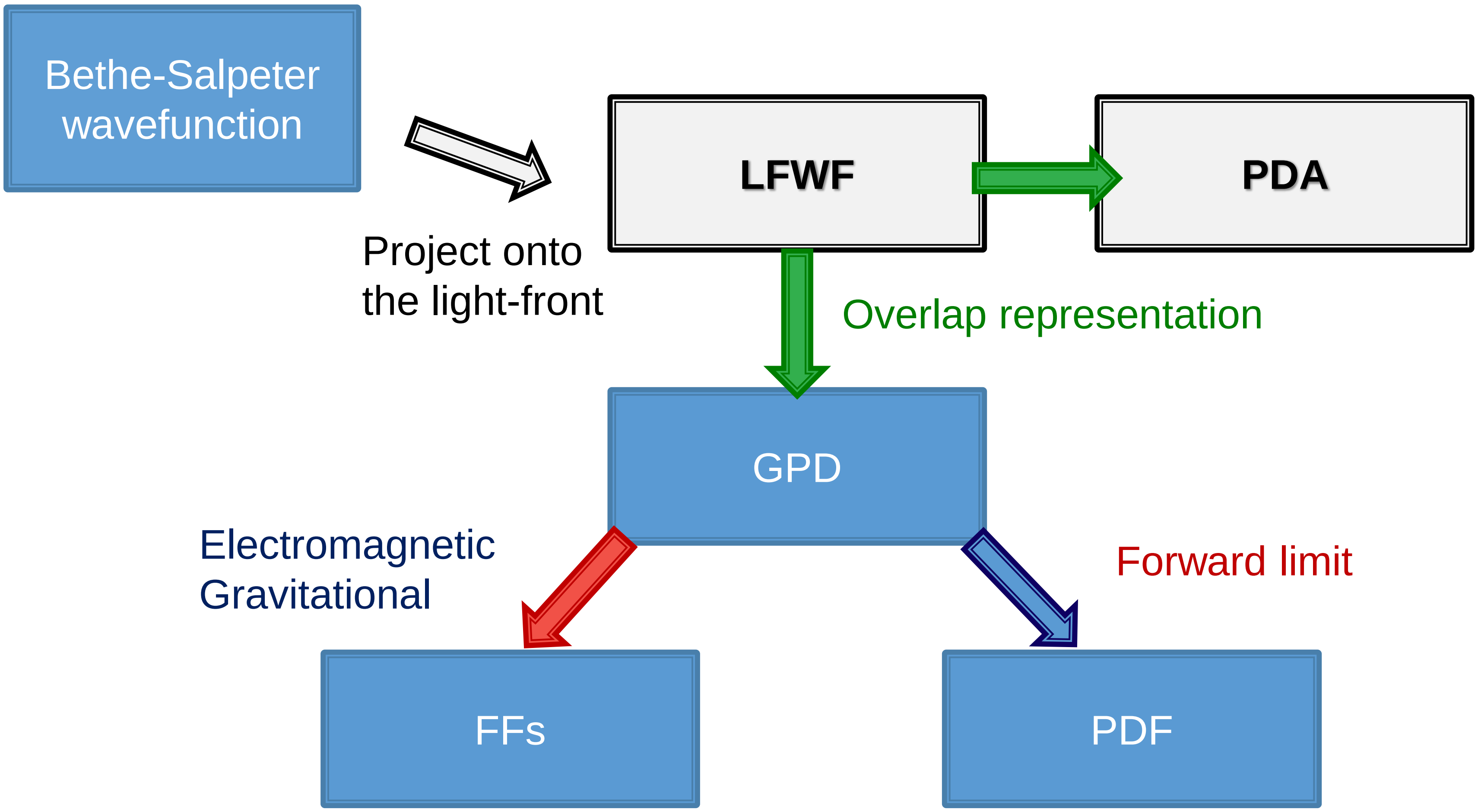
$$\chi_K^{(2)}(k_-^K; P_K) = \mathcal{M}(k; P_K) \int_0^1 d\alpha \; 2\chi_K(\alpha; \sigma^3(\alpha)) \; , \; \sigma = (k - \alpha P_K)^2 + \Omega_K^2 \; ,$$

where Ω_K^2 depends on the model and Feynman parameters, and:

$$\chi_K(\alpha; \sigma^3) = \left[\int_{-1}^{1-2\alpha} d\omega \int_{1+\frac{2\alpha}{\omega-1}}^{1} dv + \int_{1-2\alpha}^{1} d\omega \int_{\frac{\omega-1+2\alpha}{\omega+1}}^{1} dv \right] \frac{\rho_K(\omega)}{n_K} \frac{\Lambda_K^2}{\sigma^3} \; .$$

* Pion case is recovered when $M_s \to M_d$.

Our path…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LFWF: Pion and Kaon

- The pseudoscalar LFWF can be written:

$$f_K \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \mathrm{tr}_{CD} \int_{dk_{||}} \delta(n \cdot k - x n \cdot P_K) \gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K) \; .$$

- The moments of the distribution are given by:

$$< x^m >_{\psi_K^{\uparrow\downarrow}} = \int_0^1 dx x^m \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{1}{f_K n \cdot P} \int_{dk_{||}} \left[\frac{n \cdot k}{n \cdot P} \right]^m \gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K)$$

$$\int_0^1 d\alpha \alpha^m \left[\frac{12}{f_K} \mathcal{Y}_K(\alpha; \sigma^2) \right] \; , \quad \mathcal{Y}_K(\alpha; \sigma^2) = [M_u(1-\alpha) + M_s \alpha] \mathcal{X}(\alpha; \sigma_\perp^2) \; .$$

Uniqueness of Mellin moments →

$$\psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{12}{f_K} \mathcal{Y}_K(x; \sigma_\perp^2)$$

❖ Compactness of this result is a merit of the algebraic model.

LFWF: Pion and Kaon

- Notably, LFWF is determined from the Nakanishi weight:

$$\psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{12}{f_K}\mathcal{Y}_K(x; \sigma_\perp^2)\ , \quad \mathcal{Y}_K(\alpha; \sigma^2) = [M_u(1-\alpha) + M_s\alpha]\mathcal{X}_K(\alpha; \sigma_\perp^2)\ ,$$

$$\chi_K(\alpha; \sigma^3) = \left[\int_{-1}^{1-2\alpha} d\omega \int_{1+\frac{2\alpha}{\omega-1}}^{1} dv + \int_{1-2\alpha}^{1} d\omega \int_{\frac{\omega-1+2\alpha}{\omega+1}}^{1} dv\right] \frac{\rho_K(\omega)}{n_K}\frac{\Lambda_K^2}{\sigma^3}\ .$$

$$\Rightarrow \psi_K^{\uparrow\downarrow}(x, k_\perp^2) \sim \int d\omega\ \cdots \rho_K(\omega) \cdots$$

- Spectral density $\rho(\omega)$ could be obtained from solutions of the BS equation; for our modeling, a proper choice of is employed instead.

- **Medium term goal:** Obtain LFWF from realistic solutions of the Dyson-Schwinger and Bethe-Salpeter equations.
  - ❖ Compactness of this result is a merit of the algebraic model.

LFWF: Pion and Kaon

$$\varphi_K(x) = \frac{1}{16\pi^3}\int d^2k_\perp \psi_K^{\uparrow\downarrow}(x, k_\perp^2)$$

- Spectral density is chosen as:

$$u_G\rho_G(\omega) = \frac{1}{2b_0^G}\left[\operatorname{sech}^2\left(\frac{\omega-\omega_0^G}{2b_0^G}\right) + \operatorname{sech}^2\left(\frac{\omega+\omega_0^G}{2b_0^G}\right)\right][1+\omega\, v_G]\,,$$

Λ_π	b_0^π	w_0^π	v_π	Λ_K	b_0^K	w_0^K	v_K
M_u	0.1	0.73	0	$2\Lambda_\pi$	b_0^π	0.95	0.16

$$M_u = 0.31\ \text{GeV},\ M_s = 1.2M_u,\ m_\pi = 0.140\ \text{GeV},\ m_K = 0.49$$

- In order to produce empirical values of leptonic decay constants and broad and concave valence-quark DAs, such that:

$$\langle(2x-1)^2\rangle_{\varphi_\pi} := \int_0^1 dx\,(2x-1)^2\varphi_\pi(x) \approx 0.25\,, \qquad \frac{<1/x>_{AM}}{<1/x>_{CL}} \approx 1.15\,.$$

$$\langle 2x-1\rangle_{\varphi_K} \approx -0.04\,,\ \langle(2x-1)^2\rangle_{\varphi_K} \approx 0.25\,.$$

Phys.Rev. D97 (2018) no.9, 094014

‘Asymptotic’ model

Side note

- **Note**: If spectral density is chosen as: $\rho(\omega;\nu) \sim (1-\omega^2)^\nu$,

one obtains closed algebraic forms of PDAs and PDFs:

$$\phi(x;\nu) \sim [x(1-x)]^\nu \ , \quad q(x;\nu) \sim [x(1-x)]^{2\nu} \ .$$

- In particular, asymptotic PDA corresponds to ν=1.

Sketching the pion’s valence-quark generalised parton distribution

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[b]*CSSM, School of Chemistry and Physics University of Adelaide, Adelaide SA 5005, Australia*
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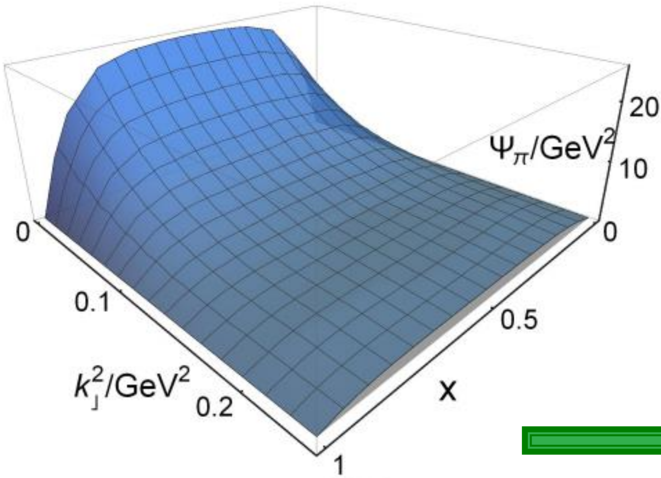
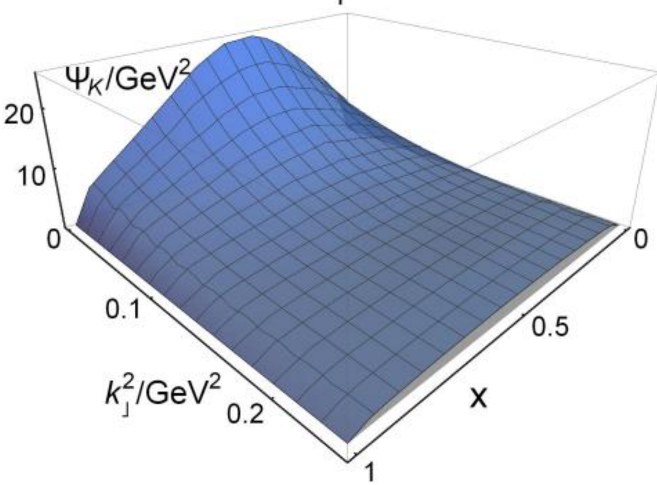
[d]*Departamento de Física Aplicada, Facultad de Ciencias Experimentales, Universidad de Huelva, Huelva E-21071, Spain*
[e]*Institute for Advanced Simulation, Forschungszentrum Jülich and JARA, D-52425 Jülich, Germany*

Phys.Lett. B741 (2015) 190-196

LFWF and PDA: Pion and Kaon





Phys.Rev. D97 (2018) no.9, 094014

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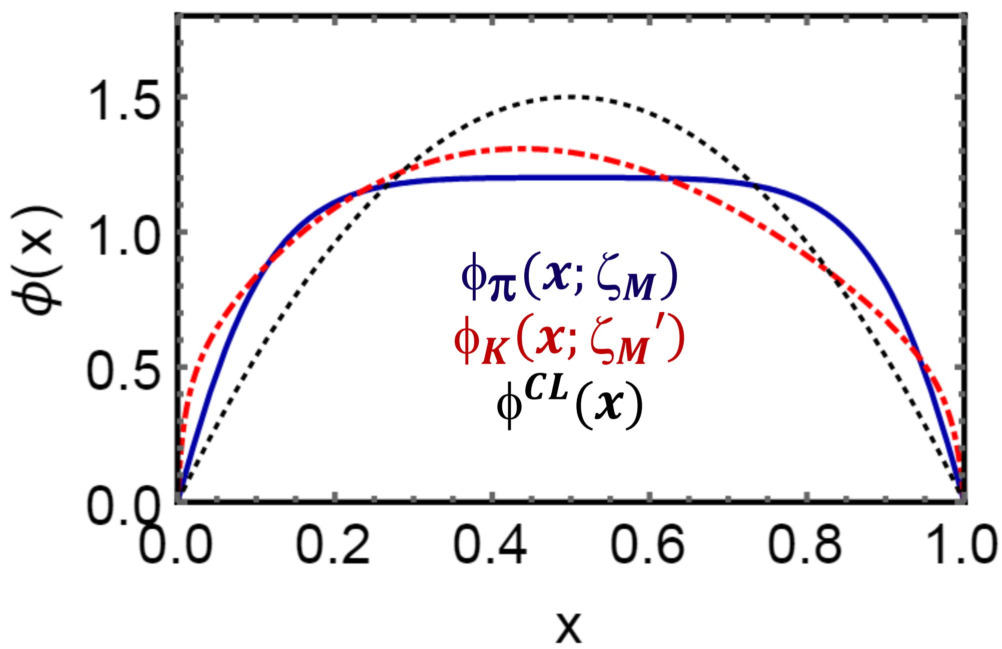
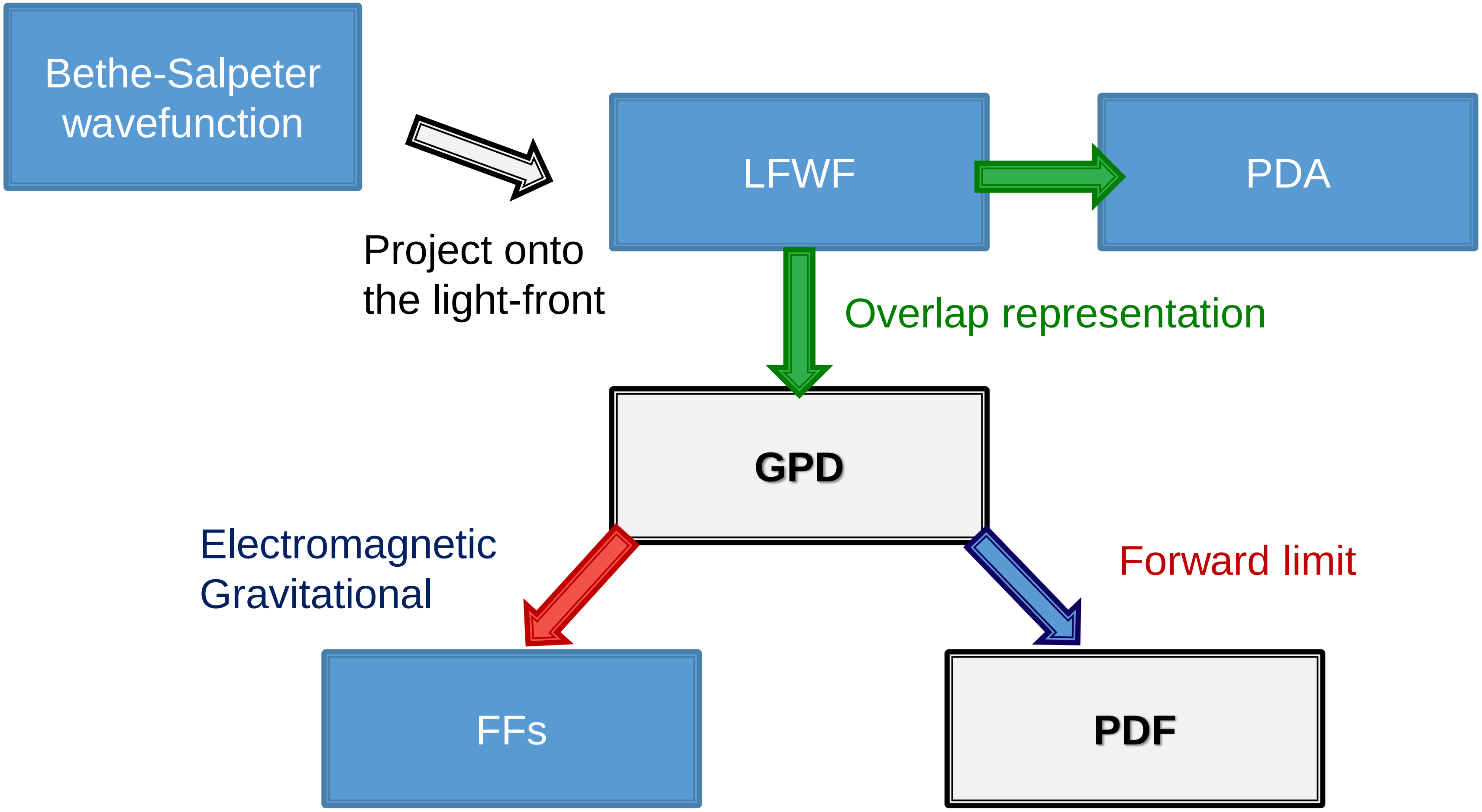


$\zeta_{M,M'}$: **Intrinsic** model **scales**

Our path…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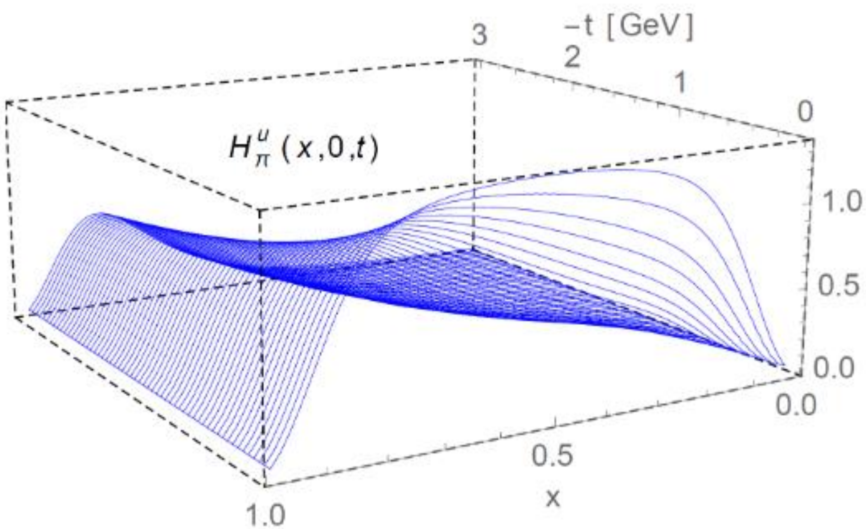
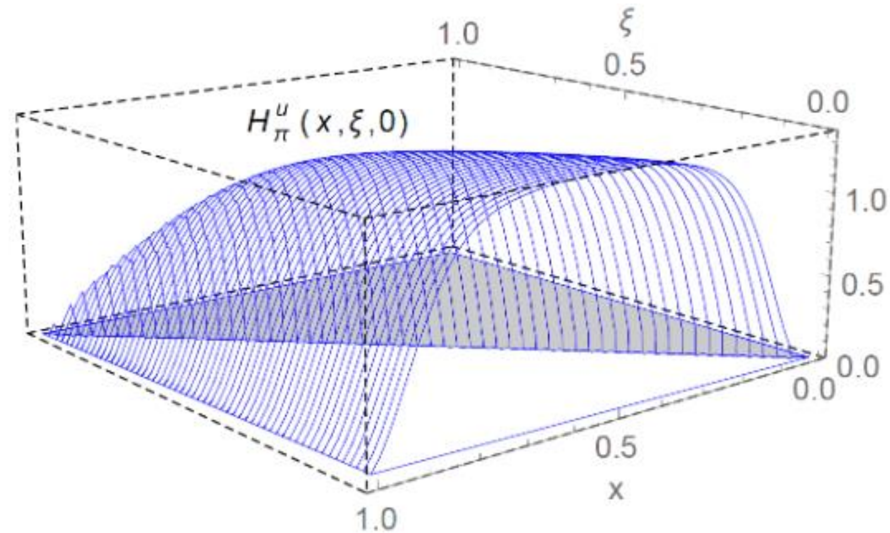
GPD: Overlap representation (Pion)

- A two-particle truncated expression for the Pion and Kaon **GPDs**, in the **DGLAP** kinematic **domain**, is obtained from the **overlap of the LFWF**:

$$H_M^q(x,\xi,t) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\,\pi^3}\Psi^*_{u\bar{f}}\left(\frac{x-\xi}{1-\xi},\mathbf{k}_\perp + \frac{1-x}{1-\xi}\frac{\Delta_\perp}{2}\right)\Psi_{u\bar{f}}\left(\frac{x+\xi}{1+\xi},\mathbf{k}_\perp - \frac{1-x}{1+\xi}\frac{\Delta_\perp}{2}\right)\,.$$





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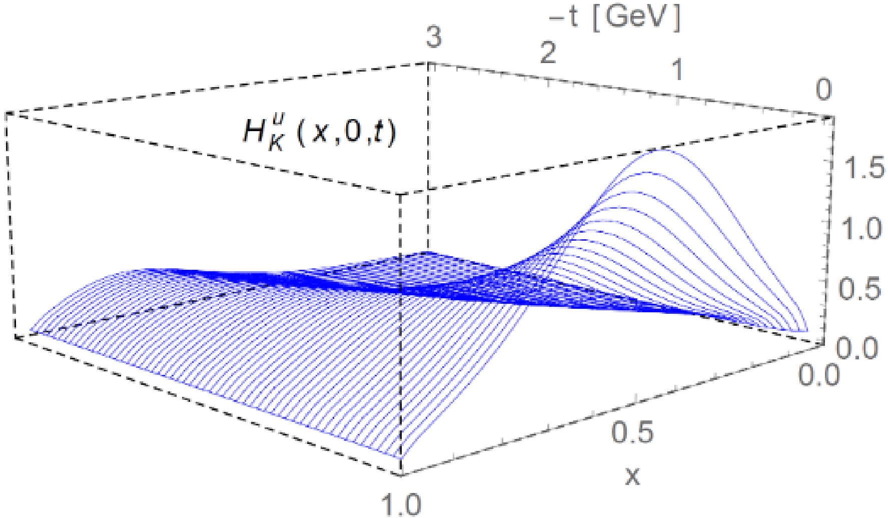
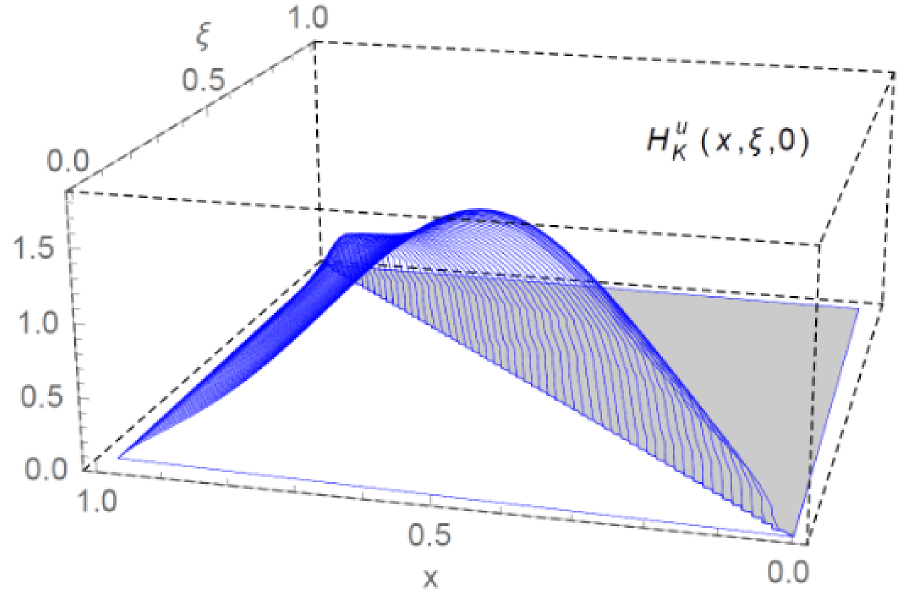
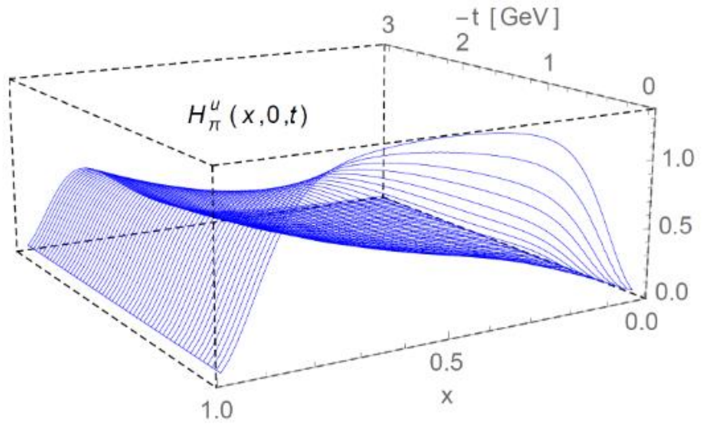
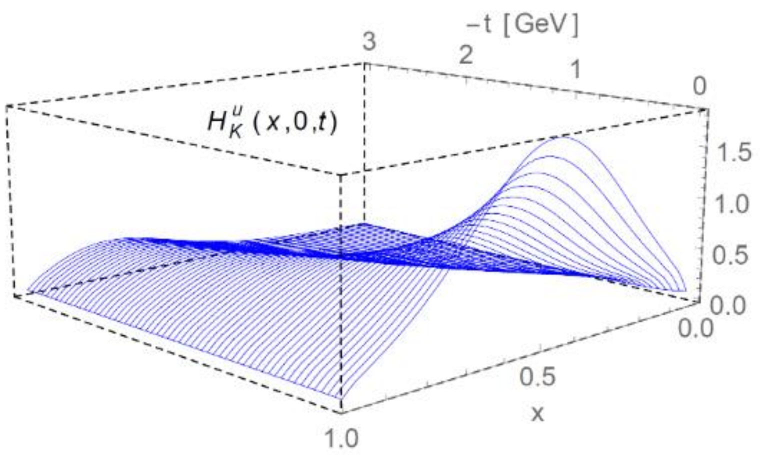


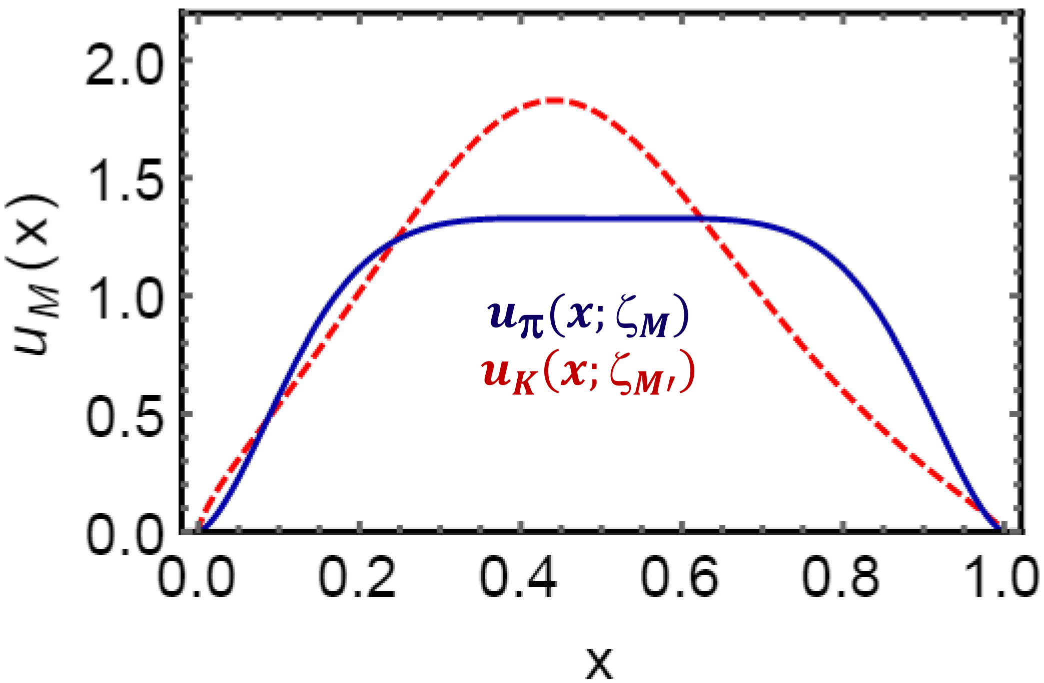


GPDs and PDFs: Pion and Kaon





$$H^u_M(x,0,0) = u_M(x)\ ,\ \ x > 0$$



$\zeta_{M,M'}$: **Intrinsic** model **scales**

Pion PDF

- To be able **to compare** the resulting pion **PDF** with experimental data, one **should obtain** the intrinsic **model scale**, **then** DGLAP **evolve to** the ζ=**5.2 GeV**.

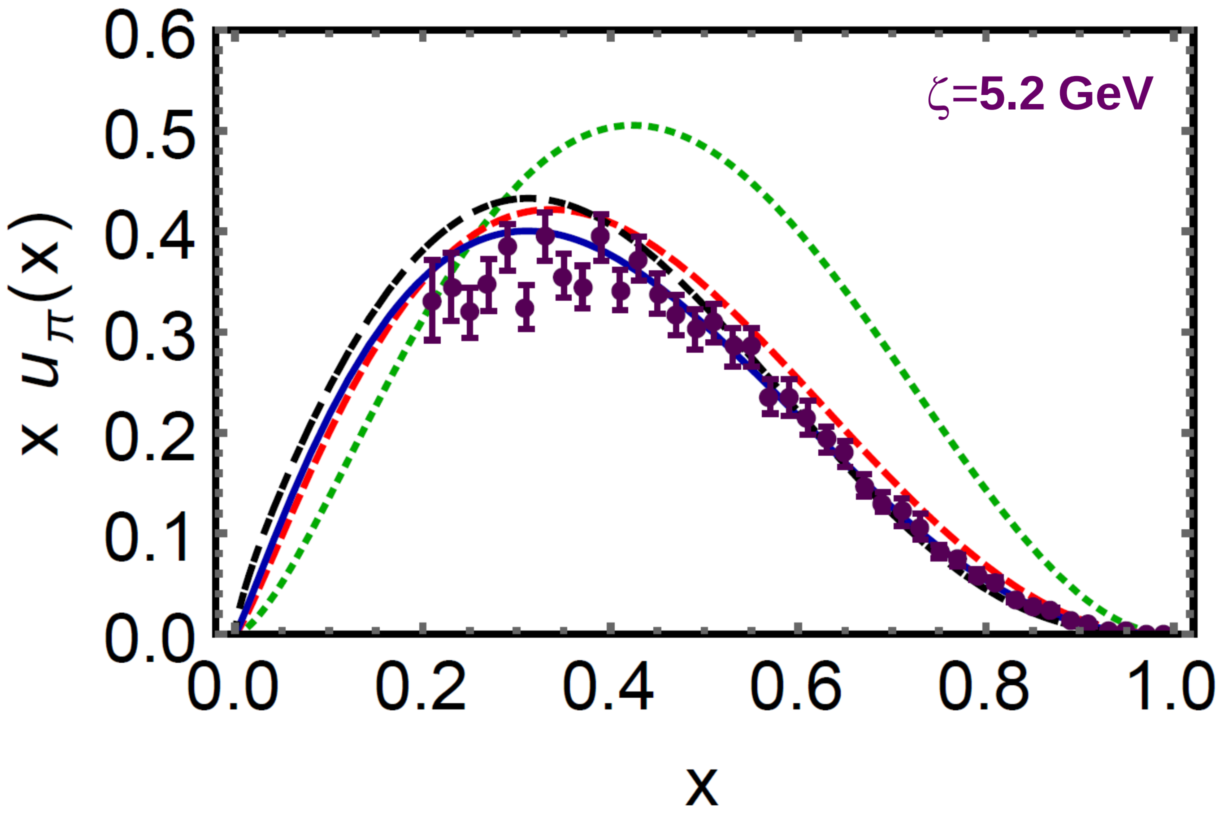
- The 1-loop DGLAP evolution equations:

$$< x^m_{\zeta_H} >^u_M = \int_0^1 dx\, x^m u_M(x)\ ,\ < x^m_{\zeta} >^u_M = < x^m_{\zeta_H} >^u_M \left[\frac{\alpha(\zeta^2)}{\alpha(\zeta_H^2)}\right]^{\gamma_0^m/\beta_0} .$$

- We guess the best initial scale: such that, for example, when the PDF is evolved to 2 GeV, one obtains the average of lattice moments (Brommet et al., Best et al., Detmold et al.):

	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$
average	0.26(8)	0.11(4)	0.058(27)

Pion PDF



$< x > = 0.24$

$< x > = 0.26$

$< x > = 0.33$

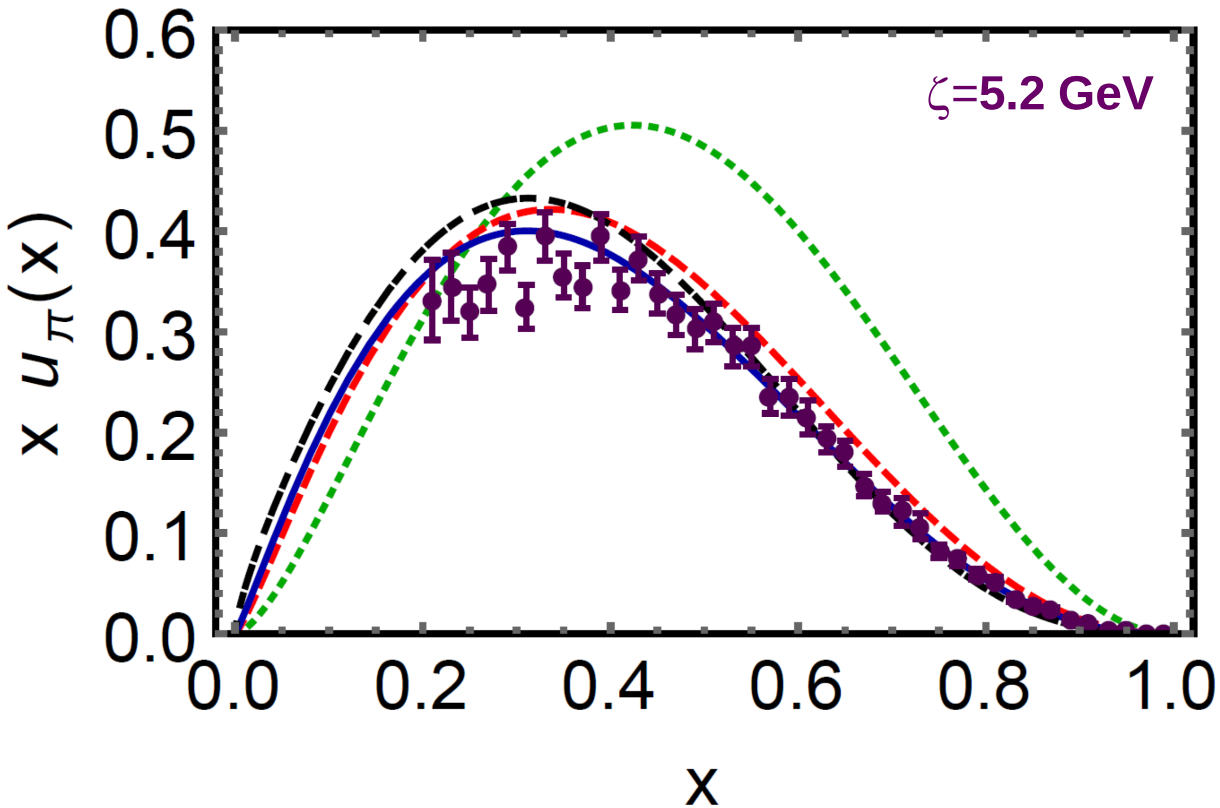
$< x > = 0.26$*

ζ_0 = 0.349 GeV: Obtained directly from the experimental data (π).

ζ_0 = 0.374 GeV: Obtained to best fit the lattice moments at 2 GeV (π).

ζ_0 = 0.510 GeV: Typical hadron scale. See for example: **Phys.Lett. B737 (2014) 23-29** and **Phys.Rev. D93 (2016) no.7, 074021***.

Pion PDF



$< x > = 0.24$

$< x > = 0.26$

$< x > = 0.33$

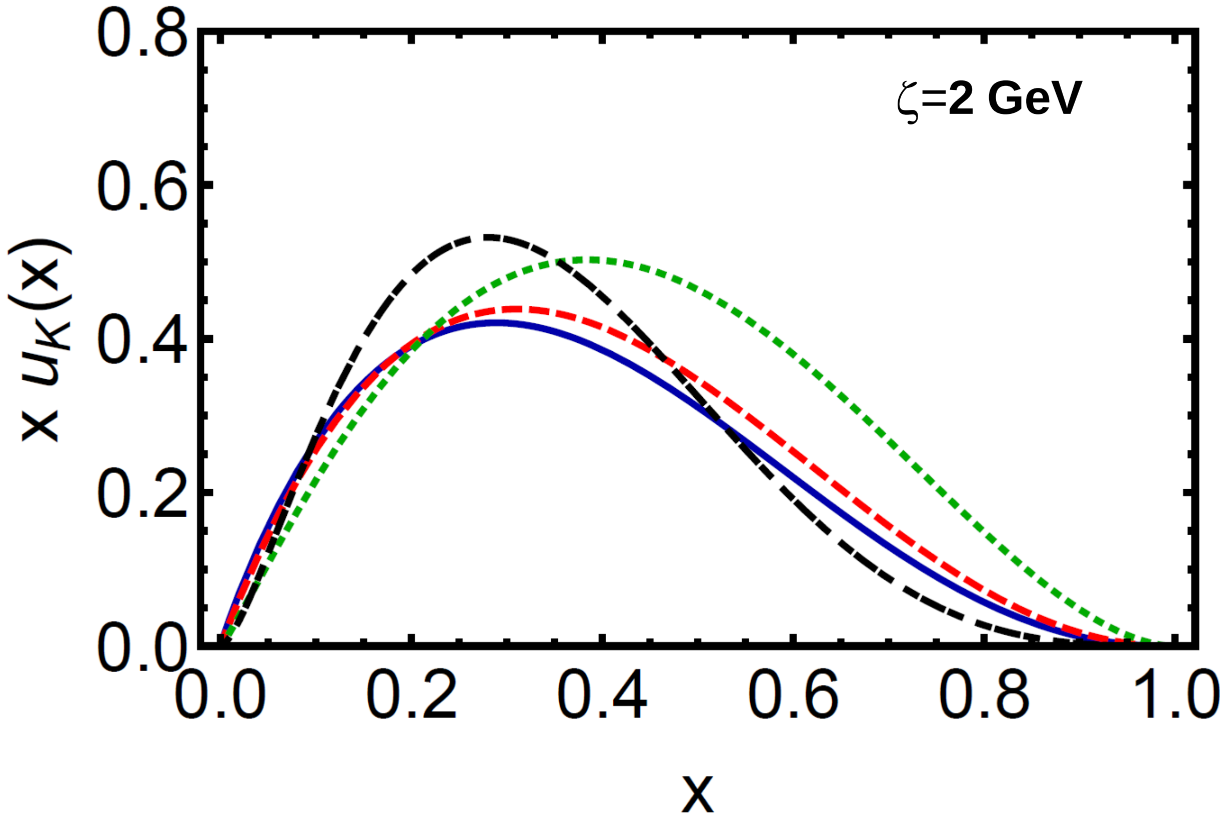
$< x > = 0.26^*$

ζ_0 = 0.349 GeV: Unsurprisingly, accurately matches data.

ζ_0 = 0.374 GeV: Good agreement with data and DSE prediction.

ζ_0 = 0.510 GeV: Leaves room to incorporate gluon content.

Kaon PDF



$< x > = 0.22$

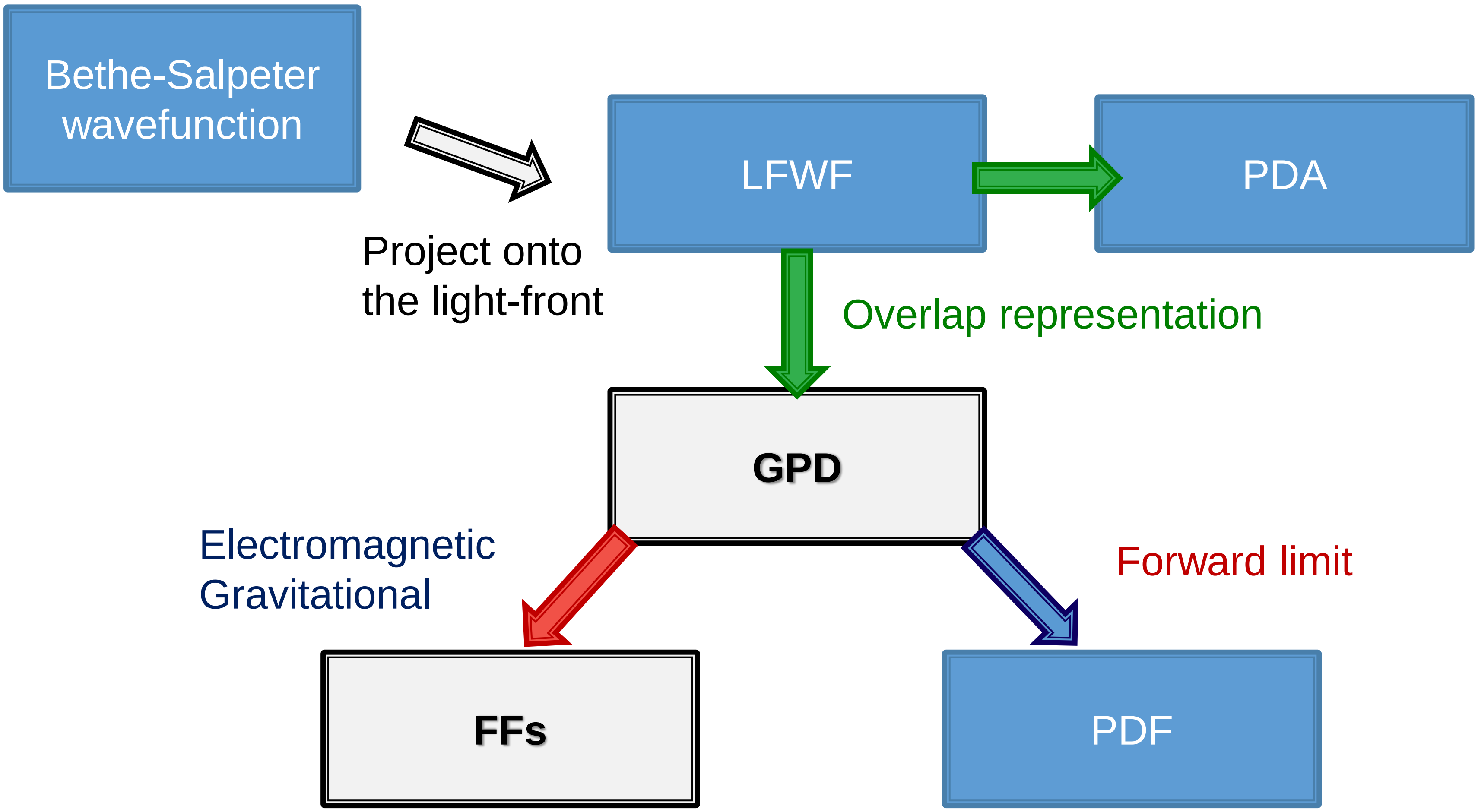
$< x > = 0.24$

$< x > = 0.29$

$< x > = 0.28$*

ζ_0 = 0.349 GeV: Obtained directly from the experimental data (π).
ζ_0 = 0.374 GeV: Obtained to best fit the lattice moments at 2 GeV (π).
ζ_0 = 0.510 GeV: Typical hadron scale. See for example: **Phys.Lett. B737 (2014) 23-29** and **Phys.Rev. D93 (2016) no.7, 074021***.

Our path…



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Elastic Electromagnetic FFs

- The **electromagnetic form factor**, associated to the quark flavor q in the meson M, can be computed as the GPD zero-th Mellin moment, such that:

$$F_M(\Delta^2) = e_u F_M^u(\Delta^2) + e_f F_M^f(\Delta^2) \ , \ F_M^q(-t = \Delta^2) = \int_{-1}^{1} dx \ H_M^q(x,\xi,t) \ .$$

Electric charges

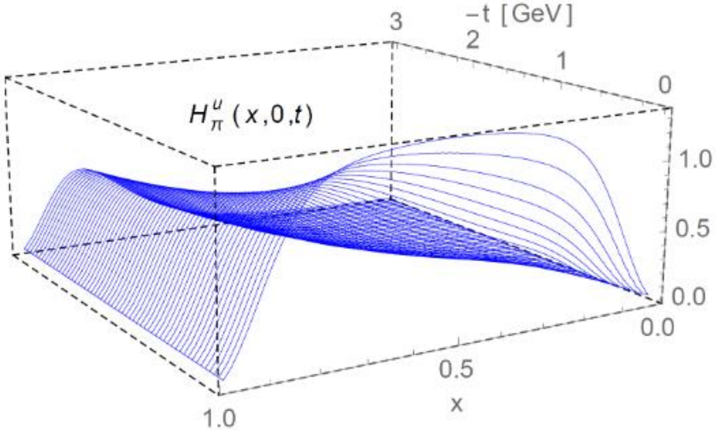
- The so-called **polynomiality** condition **guarantees** that the GPD zero-th order moment **is irrespective** of the value **of ξ**; hence we can simply take ξ=0.

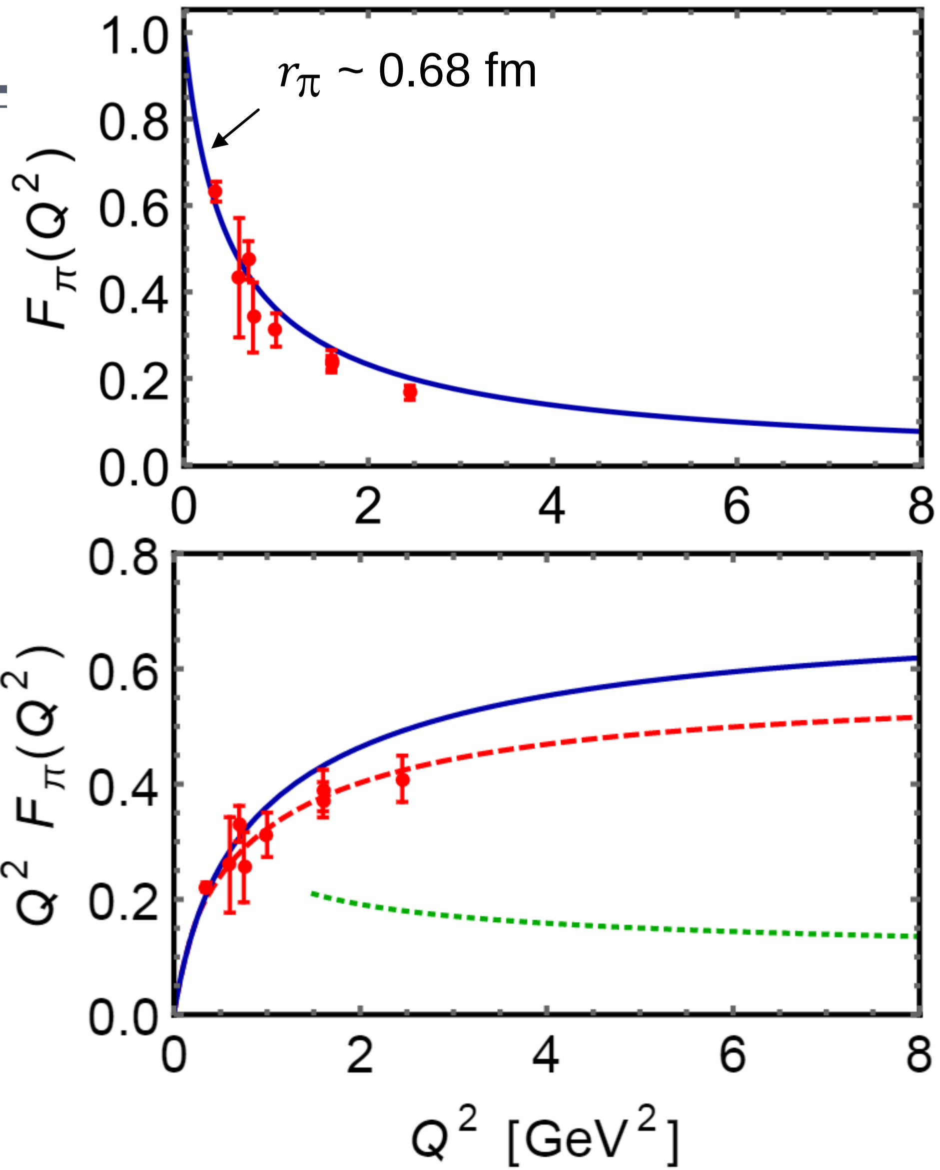
- Valence-quark GPD only takes non-zero values for - ξ < x < 1 (conversely, the antiquark GPD will be non-zero for -1 < x < ξ).

- For pion, charge conjugation + isospin symmetry ($m_u = m_d$) entail:

$$F_{\pi^+}^u(\Delta^2) = -F_{\pi^+}^d(\Delta^2) \Rightarrow F_{\pi^+}(\Delta^2) = \frac{2}{3} F_{\pi^+}^u(\Delta^2) - \frac{1}{3} F_{\pi^+}^d(\Delta^2) = F_{\pi^+}^u(\Delta^2) \ .$$

GPDs and EFF: Pion





Blue: Computed from GPD

Green: Computed from HS formula

Red: ‘*Evolved’* form factor

Hard-Scattering formula

Side note

- In studying hard exclusive processes, there are many instances in which one may appeal to factorization theorems (**Phys. Rev. D22, 2157 (1980)**).

- The amplitude involved can be written as a convolution of a hard-scattering kernel and the PDA. Such that:

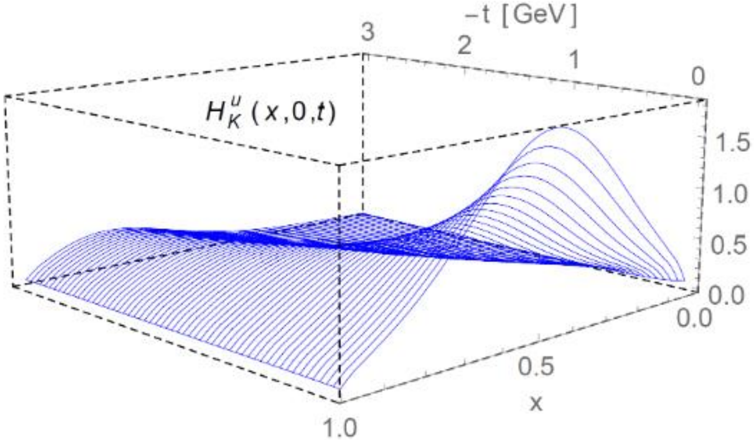
$$F_\pi(Q^2) = \int dx dy\, \phi_\pi^*(x;\zeta) T_{\gamma^*\pi\pi}(x,y,Q^2,\alpha_s(\zeta);\zeta)\phi_\pi(y;\zeta)$$

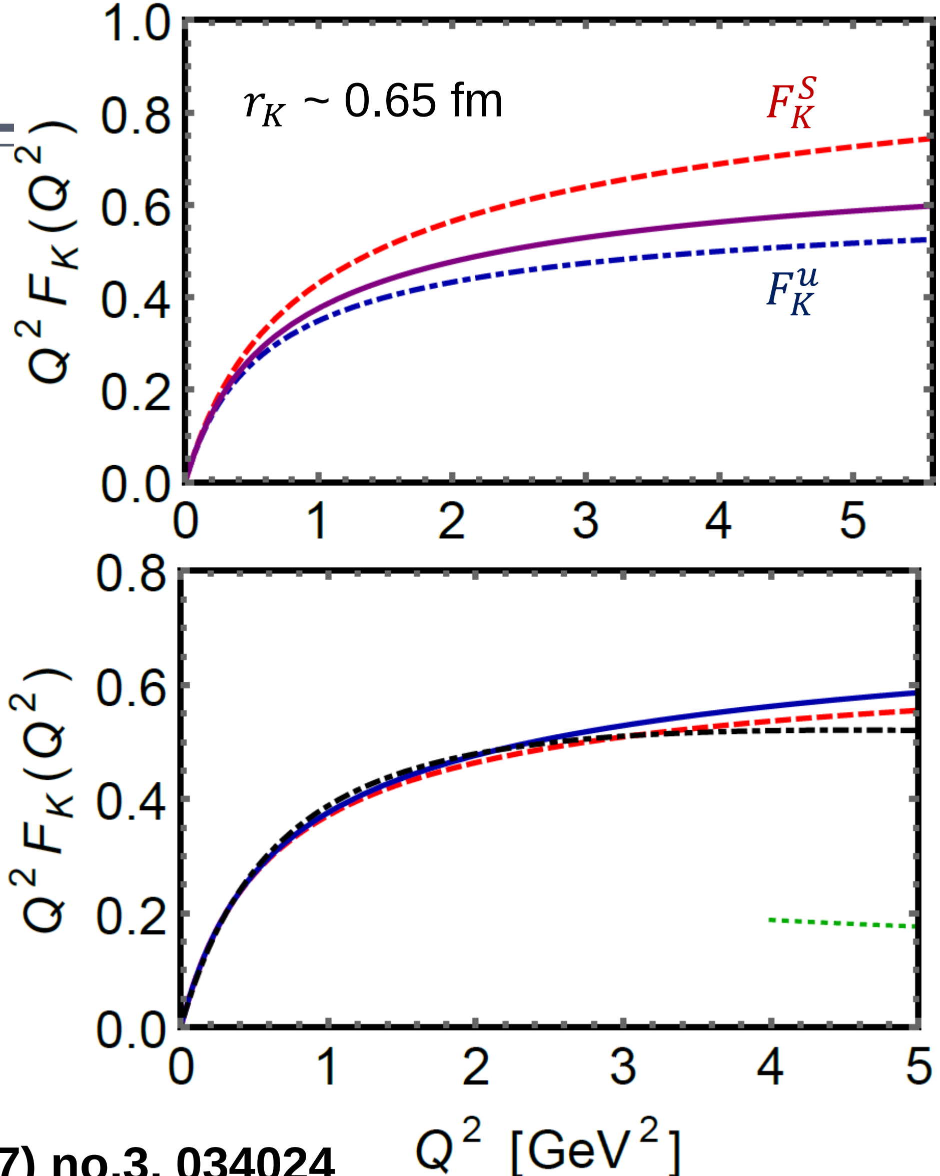
$$\exists Q_0 > \Lambda_{\rm QCD} \mid Q^2 F_\pi(Q^2) \overset{Q^2 > Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2 \left| \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\phi_\pi(x;\zeta^2 = Q^2)\right|^2$$

$$\phi_\pi(x;\zeta^2 = Q^2) \overset{\Lambda_{QCD}^2/Q^2 \simeq 0}{\approx} \phi^{cl}(x) = 6x(1-x) \ \Rightarrow Q^2 F_\pi(Q^2) \overset{\Lambda_{QCD}^2/Q^2 \simeq 0}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2$$

- In ref. **Phys. Rev. D93 (2016) no.7, 074017**, in the context of neutral pion electromagnetic transition form factor, we introduced a novel method to take into account scale evolution.

GPDs and EFF: Kaon





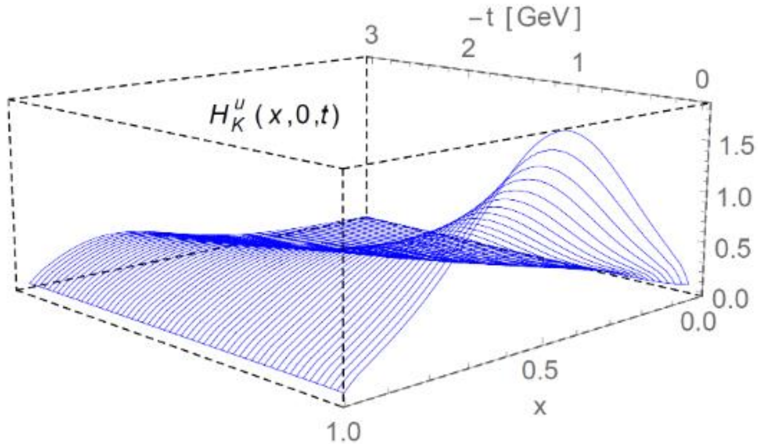
Blue: Computed from GPD

Green: Computed from HS formula

Red: '*Evolved'* form factor

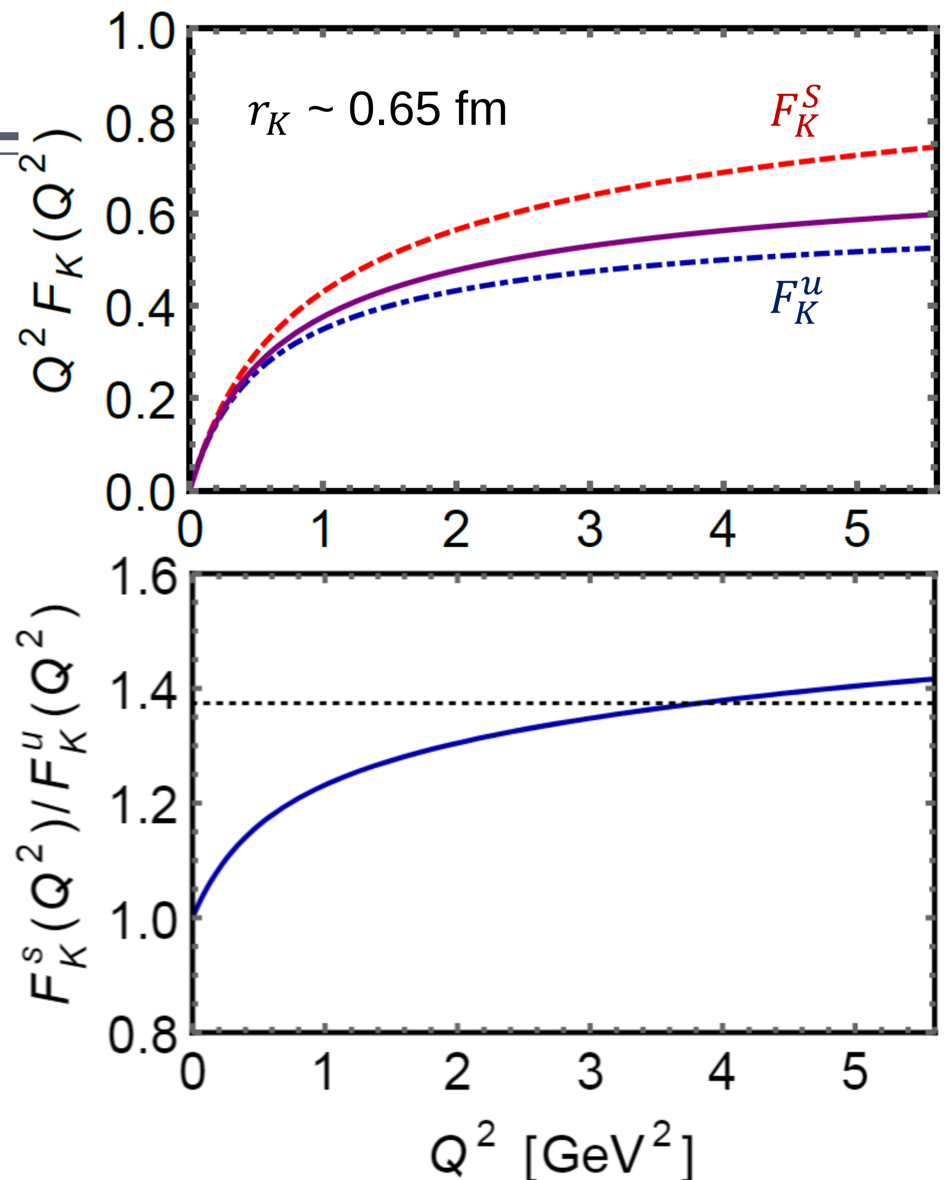
Black: DSE result, **Phys.Rev. D96 (2017) no.3, 034024**

GPDs and EFF: Kaon



$$\tilde{r}_K = \frac{F_K^s(Q^2)}{F_K^u(Q^2)} \overset{Q^2 \gg \Lambda_{QCD}^2}{\approx} \left(\frac{\omega_K^s}{\omega_K^u}\right)^2 \approx 1.37$$

$$\omega_\phi = \frac{1}{3}\int_0^1 \frac{\phi(x;\zeta)}{x}\,dx$$



If $\phi(x;\zeta)$ evolves, $\tilde{r}\to 1$ and flavor symmetry is restored.

Gravitational FFs

- Pion gravitational form factors are defined through*:

$$J_{\pi^+}(-t,\xi) \equiv \int_{-1}^{1} dx\; x H_{\pi^+}(x,\xi,t) = \Theta_2(t) - \Theta_1(t)\xi^2\ .$$

- Taking ξ=0 + isospin symmetric limit, one can readily compute:

$$\Theta_2(t) = \int_0^1 dx\; x[H^u_{\pi^+}(x,0,t) + H^d_{\pi^+}(x,0,t)] = \int_0^1 dx\; 2x H^u_{\pi^+}(x,0,t)\ .$$

- To obtain $\Theta_1(t)$, we need to take a non zero value of ξ; hence requiring the knowledge of the GPD in the ERBL region.

- Nevertheless, one can approximate $\Theta_1(t)$, by estimating the derivative of $J_{\pi^+}(-t,\xi)$ with respect to ξ^2 as:

$$D(\xi+\Delta/2) \equiv \frac{J(\xi+\Delta) - J(\xi)}{2(\xi+\Delta/2)\Delta}\ ,\ \Delta \to 0\ .$$

***Phys.Rev. D78 (2008) 094011.**

Gravitational FFs

$$D(\xi + \Delta/2) \equiv \frac{J(\xi+\Delta) - J(\xi)}{2(\xi+\Delta/2)\Delta}\ ,\ \Delta \to 0\ .$$

- Since we computed GPD only in DGLAP región, we restrict ourselves to the vicinity of $\xi \sim 0$, in which the derivative reduces to:

$$D(\Delta/2) \equiv \frac{J(\Delta) - J(0)}{\Delta^2}\ ,\ \Delta \to 0\ ,$$

- To get a clearer picture, let's split $J(-t, \xi)$ as follows:

$$J(-t,\xi) = \int_{-\xi}^{1} dx\ 2xH(x,\xi,t) = \left[\int_{-\xi}^{\xi} dx + \int_{\xi}^{1} dx\right] 2xH(x,\xi,t)$$

$$\Rightarrow J(-t,\xi) = J^{\mathrm{ERBL}}(-t,\xi) + J^{\mathrm{DGLAP}}(-t,\xi)\ ,$$

- Notice that, because of the polinomiality of the *complete* GPD:

$$J^{\mathrm{DGLAP}}(-t,\xi) = \Theta_2(t) - \xi^2\Theta_1(t)^{\mathrm{DGLAP}} + \sum_{i=1}^{\infty} c_i(t)\xi^{2+i}\ ,$$

$$J^{\mathrm{ERBL}}(-t,\xi) = -\xi^2\Theta_1(t)^{\mathrm{ERBL}} - \sum_{i=1}^{\infty} c_i(t)\xi^{2+i}$$

Gravitational FFs

- Thus, since so far we can only access DGLAP region:

$$J^{\mathrm{DGLAP}}(-t,\xi) = \Theta_2(t) - \xi^2\Theta_1(t)^{\mathrm{DGLAP}} + \sum_{i=1}^{\infty} c_i(t)\xi^{2+i}\,,$$

what I shall show as $\Theta_1(t)$, is in fact $\Theta_1(t)^{DGLAP}$ + some reminiscent dependence on ξ.

- One can argue that as long as we stay in the vicinity of ξ~0, DGLAP contribution dominates over the ERBL one (this is *a priori* unknown).

- The extensión to **ERBL region** is then **needed**. Taking advantage of the soft-pion theorem, one can conect PDA with $J(-t,\xi)^{ERBL}$ and thus with $\Theta_1(t)^{ERBL}$.

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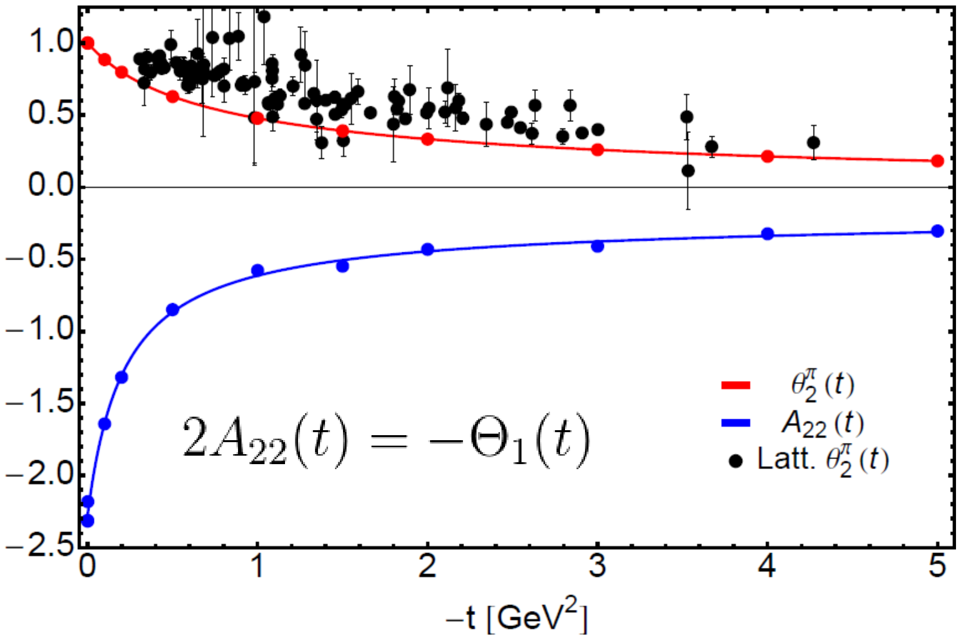
Gravitational FFs

- Besides being a pure valence quark GPD, there is a nice comparison of Θ_2 with lattice.

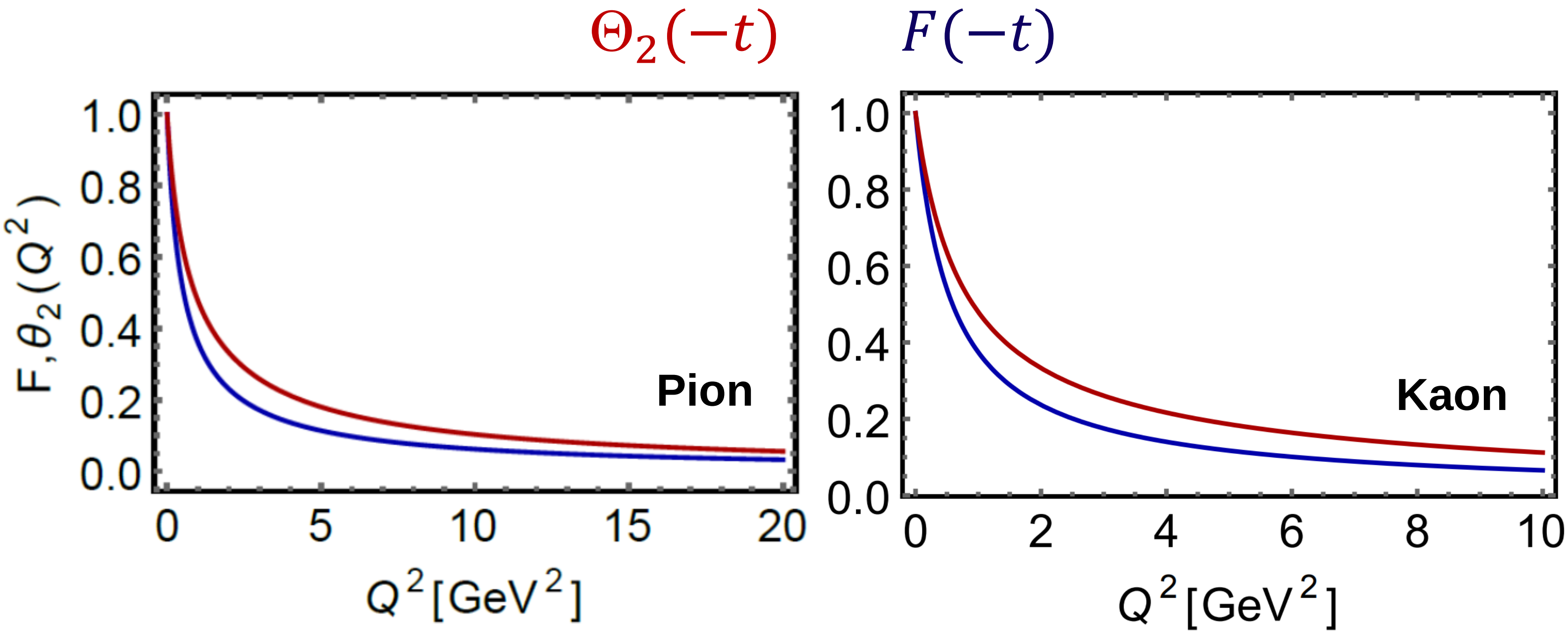
- Nonetheless, polinomiality of GPD is not fulfilled without the ERBL región. Such extension is necessary to provide a more reliable computation of Θ_1.



Latt.: D. Brommel, Ph.D. thesis, University of Regensburg, Regensburg, Germany (2007), DESY-THESIS-2007-023

Gravitational and electromagnetic FFs

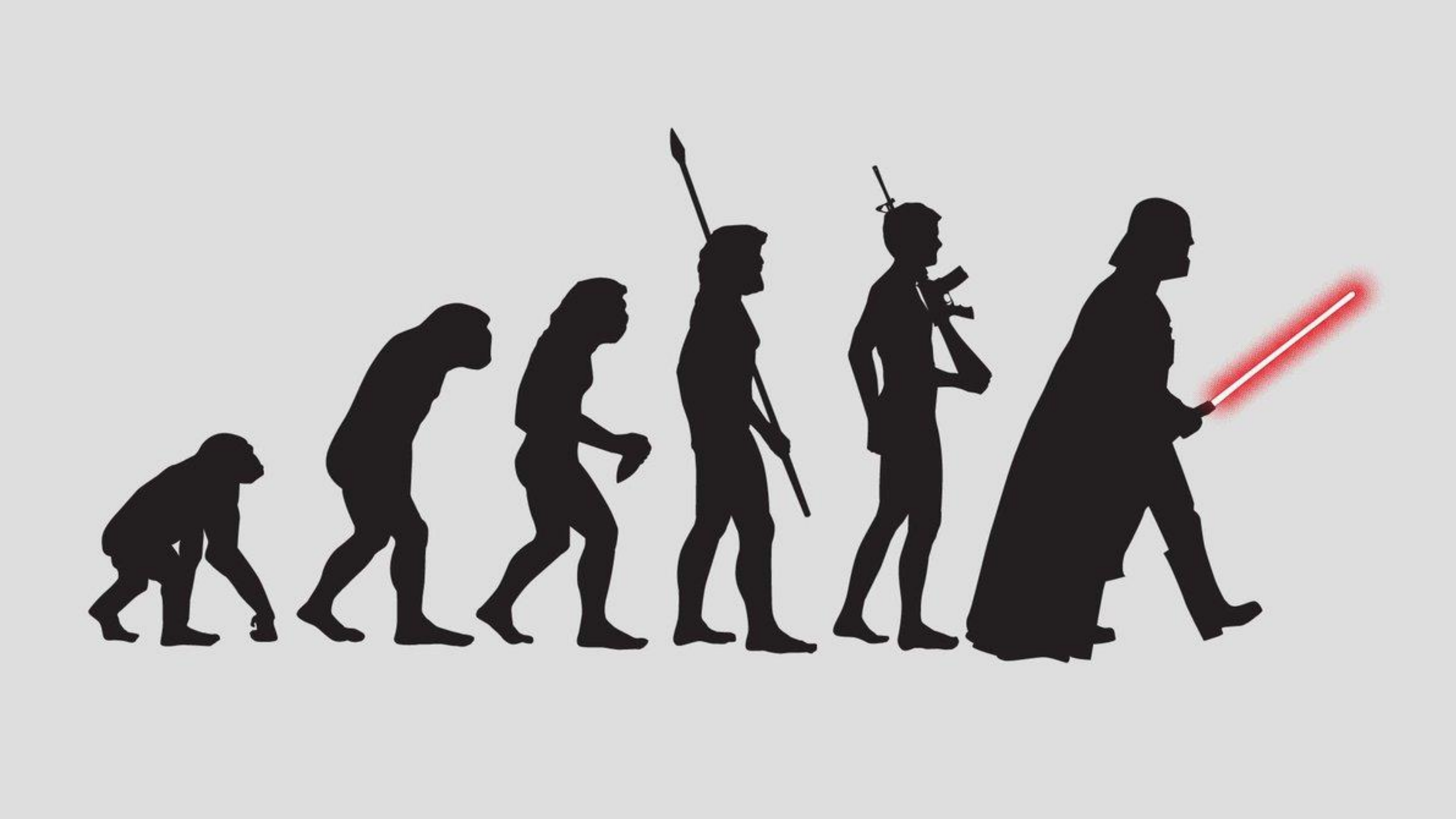
- Gravitational form factor $\Theta_2(t)$ harder than the electromagnetic one, but no obvious relation between the interaction radii.



$r_{\Theta_2}/r_F \approx 0.55/0.68 \approx 0.8$

$r_{\Theta_2}/r_F \approx 0.82$

(QCD) Evolution



PDA evolution

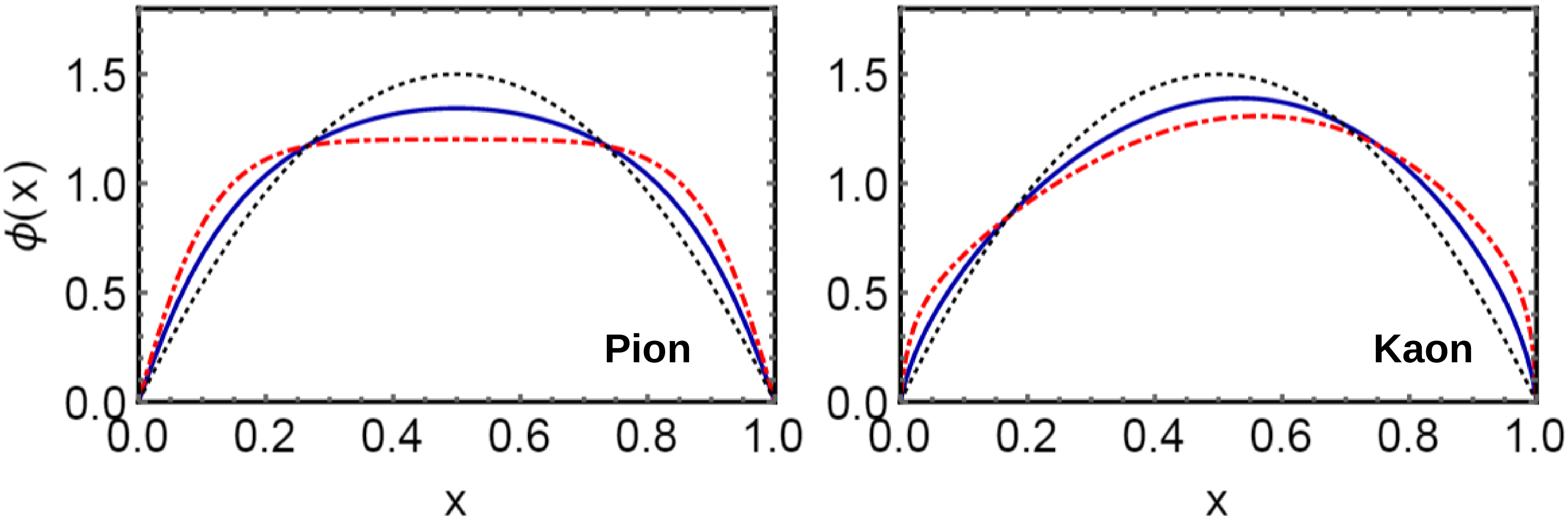
- We project **PDA** onto a 3/2-Gegenbauer polynomial basis. Such that it **evolves**, from an initial scale ζ_0 to a final scale ζ, **according to** the corresponding **ERBL equations**:

$$\phi(x;\zeta) = 6x(1-x)\left[1 + \sum_{n=1} a_n(\zeta) C_n^{3/2}(2x-1)\right] \ ,$$

$$a_n(\zeta) = a_n(\zeta_0)\left[\frac{\alpha(\zeta^2)}{\alpha(\zeta_0^2)}\right]^{\gamma_0^n/\beta_0} \ , \ \gamma_0^n = -\frac{4}{3}\left[3 + \frac{2}{(n+1)(n+2)} - 4\sum_{k=1}^{n+1}\frac{1}{k}\right] \ .$$

- Thus, any PDA at hadronic scale evolves logarithmically towards its conformal distribution, $\phi(x)=6x(1-x)$.
    - Quark mass and flavor become irrelevant. Broad PDA becomes narrower, skewed PDA becomes symmetric.

PDA evolution



$\phi_\pi(x, \zeta_0 = 0.51 \text{ GeV})$ $\phi_K(x, \zeta_0 = 0.51 \text{ GeV})$

$\phi_\pi(x, \xi = 2 \text{ GeV})$ $\phi_K(x, \xi = 2 \text{ GeV})$

$\phi_{CL}(x) = 6x(1-x)$

LFWF evolution

$$\phi(x) = \frac{1}{16\pi^3} \int d^2\vec{k}_\perp \psi^{\uparrow\downarrow}(x, k_\perp^2)$$

- We look for a way to evolve the LFWF.
- First, let's assume that the LFWF admits a similar Gegenbauer expansion. That is:

$$\psi(x, k_\perp^2; \zeta) = 6x(1-x) \left[\sum_{n=0} b_n(k_\perp^2; \zeta)\; C_n^{3/2}(2x-1) \right] ,$$

$$a_n(\zeta) \;=\; \frac{1}{16\pi^3} \int d^2\vec{k}_\perp\, b_n(k_\perp^2;\zeta)\ (\text{for } n \geq 1)\ ,\ \frac{1}{16\pi^3} \int d^2\vec{k}_\perp\, b_0(k_\perp^2;\zeta) = 1\ .$$

- 1-loop ERBL evolution of $a_n(\zeta)$ implies:

$$\frac{1}{a_n(\zeta)} \frac{d}{d\ln\zeta^2} a_n(\zeta) \;=\; \frac{\displaystyle\int d^2\vec{k}_\perp\ \frac{d}{d\ln\zeta^2} b_n(k_\perp^2;\zeta)}{\displaystyle\int d^2\vec{k}_\perp\ b_n(k_\perp^2;\zeta)} ,$$

$$\phi(x) = \frac{1}{16\pi^3} \int d^2\vec{k}_\perp \psi^{\uparrow\downarrow}(x, k_\perp^2)$$

LFWF evolution

- Now, if we take a factorization assumtion, we arrive at:

$$\frac{b_n(k_\perp^2;\zeta)}{b_n(k_\perp^2;\zeta_0)} = \frac{\widehat{b}_n(\zeta)}{\widehat{b}_n(\zeta_0)} = \left[\frac{\alpha(\zeta^2)}{\alpha(\zeta_0^2)}\right]^{\gamma_0^n/\beta_0} , \; b_n(k_\perp^2;\zeta) \equiv \widehat{b}_n(\zeta)\chi_n(k_\perp^2) \; .$$

- Suplemented by the condition $\chi_n(k_\perp^2) \equiv \chi(k_\perp^2)$, one gets $\widehat{b}_n(\zeta) \equiv a_n(\zeta)$.
- Such that, the followiong factorised form is obtained:

$$\psi(x, k_\perp^2; \zeta) \; \equiv \; \phi(x;\zeta) \; \chi(k_\perp^2)$$

→ **LFWF Evolves like PDA**

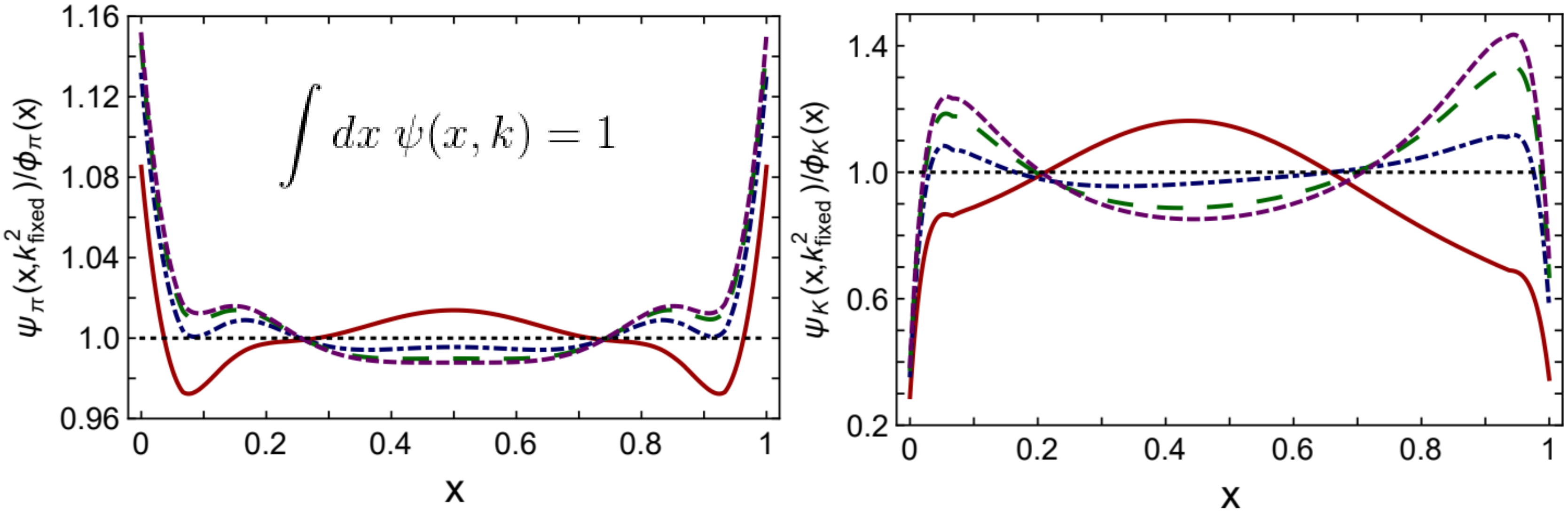
- Which is far from being a general result, but an useful approximation instead.

LFWF evolution

$$\psi(x, k_\perp^2; \zeta) \equiv \phi(x; \zeta)\, \chi(k_\perp^2)$$

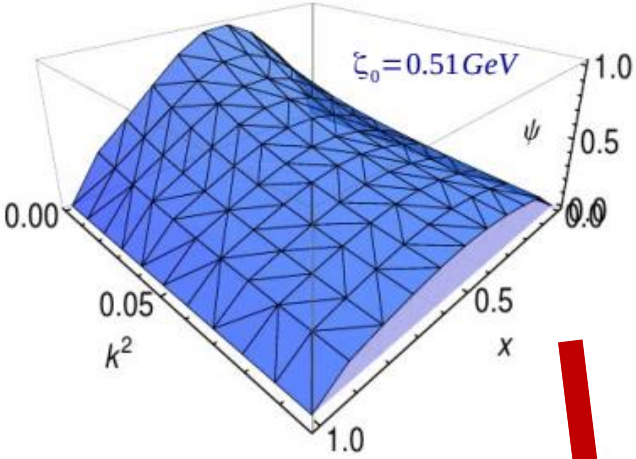
- A first validation of the factorized ansätz is addressed in **Phys.Rev. D97 (2018) no.9, 094014**:

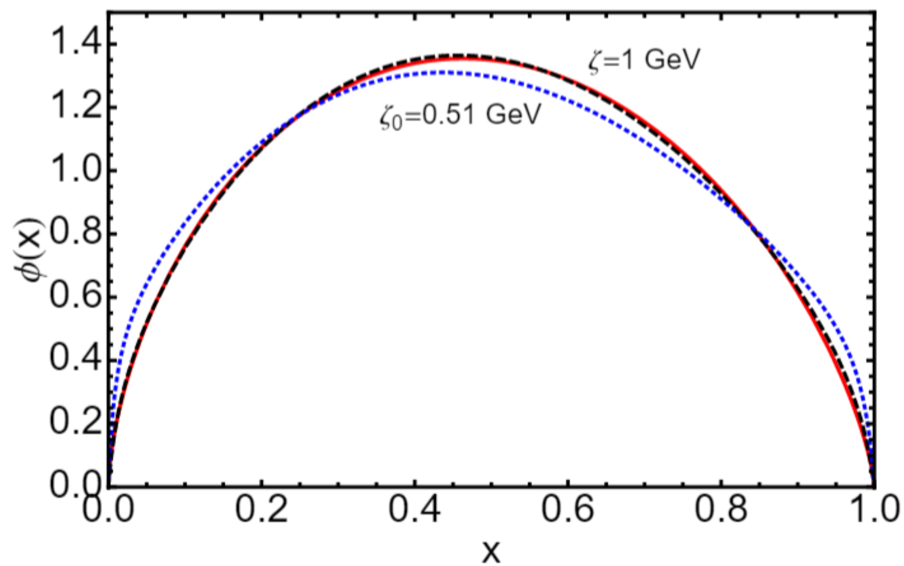
k^2=0, k^2=0.2 GeV, k^2=0.8 GeV, k^2=3.2 GeV

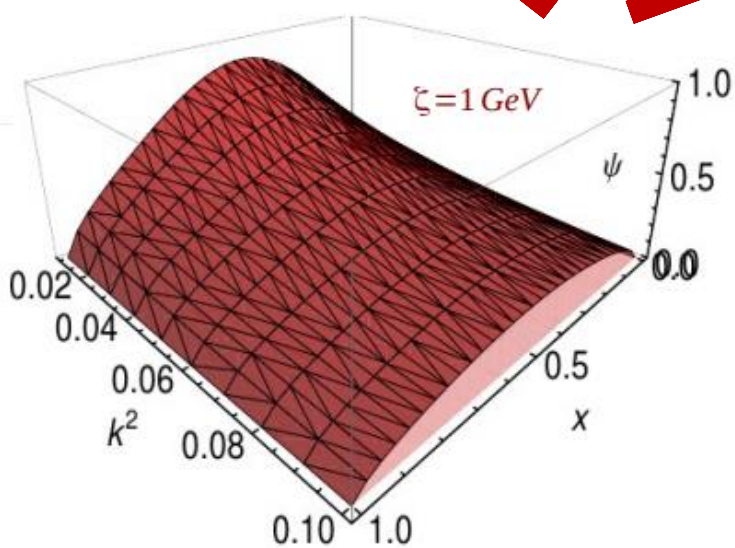


- If the factorized ansatz is a good approximation, then the plotted ratio must be 1. For the pion, it slightly deviates from 1; for the kaon, the deviation is much larger.

LFWF and PDA evolution





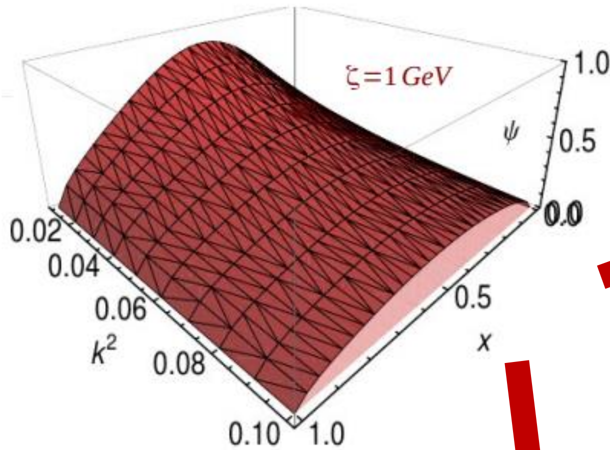


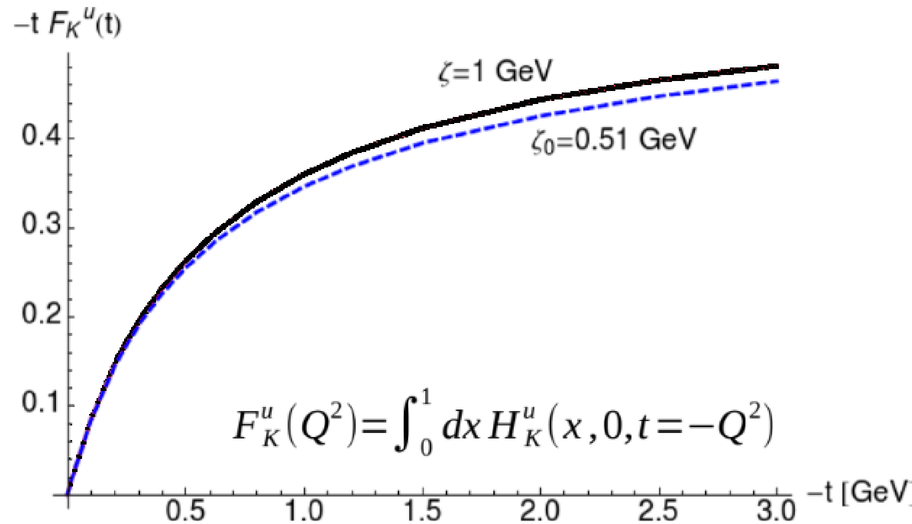
1) Compute LFWF and ERBL running of PDA

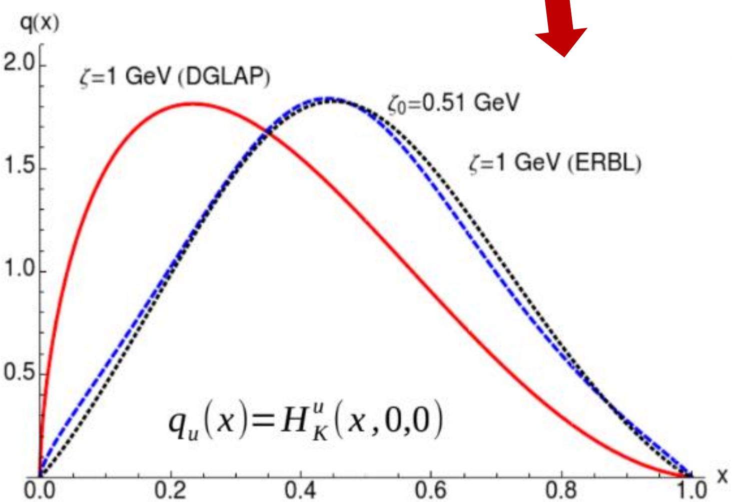
2) ERBL running of LFWF and compute PDA

Notably, **1)** and **2)** are **equivalent**. Factorization assumption and evolution seem reasonable.

LFWF and PDA evolution







1) Obtained from ERBL evolution of LFWF

2) Obtained from DGLAP evolution of GPD

Clearly, **1)** and **2)** are **not equivalent**. One must understand how ERBL and DGLAP regions are related.

GPDs, PDFs and EFFs

- Employing an QCD-based AM, we performed an exploratory study of:
  - **LFWF and PDAs.**
  - **Valence quark GPDs**: DGLAP region, in the overlap representation (see **Phys.Lett. B780 (2018) 287-293**).
  - **Valence quark PDFs**.
  - Elastic electromagnetic and gravitational **form factors**.

- **Qualitative results** of the obtained parton distributions are **consistent**. When available, we compare reasonably well with lQCD and experimental results.

- **Deeper understanding is needed** before attempting a full DSE based numerical computation.

GPDs, PDFs and EFFs

- Short, medium and long term **goals**:

  ✓ **Incorporate missing ingredients**: gluon content in kaon and pion (when studying PDFs), the rest of the Bethe-Salpeter amplitudes, etc.

  ✓ Improve our understanding how the parton distributions should **evolve with** their corresponding **evolution equations**. Connect ERBL and DGLAP regions.

  ✓ **Extension** of the GPD **to ERBL region** (see **Phys.Lett. B780 (2018) 287-293**, for example).

  ✓ **Realistic predictions**, based upon the **real solutions** of the quark propagator **DSEs** and **BSEs**.

❖ Reduce model dependence and provide **trustable predictions**.

Final remarks

- With several facilities at work all around the world, hadron physics is a very active field today: **it is the time to be interested in hadron physics**.

- Continuum QCD has evolved to the point where QCD connected predictions for elastic and transition form factors and parton distributions of **all types are within reach**:
  - **PDFs and GPDs:** Phys.Lett. B737 (2014) 23-29; Phys.Lett. B741 (2015) 190-196; Phys.Rev. D93 (2016) no.7, 074021
  - **PDAs and form factors:** Phys.Rev.Lett. 110 (2013) no.13, 132001; Phys.Rev.Lett. 111 (2013) no.14, 141802; Phys.Lett. B753 (2016) 330-335, Phys.Rev. D93 (2016) no.7, 074017; Phys.Lett. B783 (2018) 263-267; arXiv:1810.12313 [nucl-th].

- Lattice QCD and experiments provide crucial information to improve the theoretical predictions. Exist now an array of exciting predictions waiting for empirical validation.