Parton distribution amplitudes of s-wave and p-wave heavy quarkonia

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$\gamma^*\gamma \to \eta, \eta'$ transition form factors

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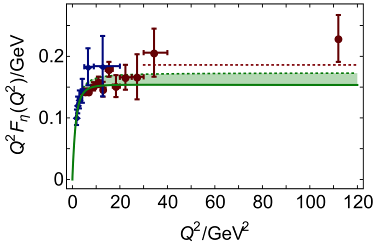
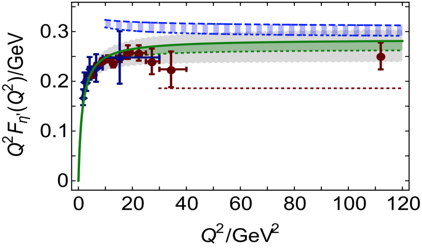
Abstract (arXiv)
Using a continuum approach to the hadron bound-state problem, we calculate $\gamma^*\gamma \to \eta, \eta'$ transition form factors on the entire domain of spacelike momenta, for comparison with existing experiments and in anticipation of new precision data from next-generation e^+e^- colliders. One novel feature is a model for the contribution to the Bethe-Salpeter kernel deriving from the non-Abelian anomaly, an element which is crucial for any computation of η, η' properties. The study also delivers predictions for the amplitudes that describe the light- and strange-quark distributions within the η, η'. Our results compare favourably with available data. Important to this at large-Q^2 is a sound understanding of QCD evolution, which has a visible impact on the η' in particular. Our analysis also provides some insights into the properties of η, η' mesons and associated observable manifestations of the non-Abelian anomaly.





- π and K polarised TMDs, Boer-Mulders function, in progress.

The quark structure of hadrons

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- Hadron structure functions:.
  - Form factors: the closest thing we have to a snapshot.
    - ★ e.g. $F(Q^2)$:
      momentum transfer Q.
  - The 1D picture of how quarks move within a hadron:
    - ★ PDFs and PDAs.
    - ★ e.g. $q(x)$:
      longitudinal momentum fraction x.
  - A multidimensional view of hadron structure:
    - ★ LFWFs, GPDs, TMDs etc..
    - ★ e.g. $\psi(x, k_\perp)$:
      longitudinal x and transverse momentum fraction $k_\perp$.
- FFs, PDFs, PDAs, LFWFs, GPDs, TMDs et cetera are related.

The quark structure of hadrons

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- LFWFs⟹Leading-twist PDAs

$$\phi(x) = \frac{1}{16\pi^3} \int dk_\perp^2 \, \psi^{\uparrow\downarrow}(x, k_\perp^2) \,. \quad (1)$$

- LFWFs⟹Unpolarised TMDs

$$f_1(x, k_\perp^2) = \frac{1}{16\pi^3} \sum_{\lambda_q, \lambda_{\bar{q}}} |\psi^{\lambda_q, \lambda_{\bar{q}}}(x, k_\perp^2)|^2$$

$$= \frac{1}{16\pi^3} (|\psi^{\uparrow\downarrow}|^2 + |\psi^{\downarrow\uparrow}|^2 + |\psi^{\uparrow\uparrow}|^2 + |\psi^{\downarrow\downarrow}|^2) \quad (2)$$

- Unpolarised TMDs⟹PDFs

$$q(x) = \int dk_\perp^2 \, f_1(x, k_\perp^2) \quad (3)$$

PDAs in theory

- Leading-twist PDAs

$$\phi(x) = \frac{1}{16\pi^3} \int dk_\perp^2 \psi^{\uparrow\downarrow}(x, k_\perp^2) \,. \tag{4}$$

- PDAs in theory
  - QCD sum rules:
    - ★ V.L. Chernyak and A.R. Zhitnitsky, Nucl. Phys. B 201, 492 (1982).
    - ★ P.Ball, and V.M. Braun, Phys.Rev. D54 (1996) 2182-2193, Nucl.Phys. B543 (1999) 201-238 .
    - ★ V.M. Braun, S.E. Derkachov, G.P. Korchemsky, and A.N. Manashov, Nucl.Phys. B553 (1999) 355-426.
    - ★ A.P. Bakulev, S.V. Mikhailov, and N.G. Stefanis, Phys.Lett. B508 (2001) 279-289, Phys.Lett. B590 (2004) 309-310.
  - Light-front QCD:
    - ★ G.P. Lepage and S.J. Brodsky, Phys. Lett. B 87, 359 (1979).
    - ★ S.J. Brodsky, and G.F. de Teramond, Phys.Rev.Lett. 96 (2006) 201601.
  - NJL model:
    - ★ E.R. Arriola, and W. Broniowski, Phys.Rev. D66 (2002) 094016.
  - Instanton model:
    - ★ A.E. Dorokhov, JETP Lett. 77 (2003) 63-67, Pisma Zh.Eksp.Teor.Fiz. 77 (2003) 68-72.
  - Lattice QCD:
    - ★ G. Martinelli, and C.T. Sachrajda, Phys.Lett. B190 (1987) 151-156, Phys.Lett. B217 (1989) 319-324.
    - ★ D. Daniel, R. Gupta, and D.G. Richards, Phys.Rev. D43 (1991) 3715-3724.
    - ★ V.M. Braun, M. Gockeler, R. Horsley, H. Perlt, D. Pleiter, P.E.L. Rakow, G. Schierholz, A. Schiller, W. Schroers, H. Stuben, and J.M. Zanotti, Phys.Rev. D74 (2006) 074501.
    - ★ UKQCD Collaboration, Phys.Lett. B641 (2006) 67-74.
  - Dyson-Schwinger Equations:
  - etc.

Partonic Structure of Heavy quarkonia

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- Particles: s-wave and p-wave Heavy quarkonia: $c\bar{c}$, $b\bar{b}$

  η_c, η_b, J/Ψ, Υ, h_c, h_b, χ_{c0}, χ_{b0}, χ_{c1}, χ_{b1}, χ_{c2}, χ_{b2}
  - Spectroscopy
  - Decay
  - Production
    - ★ Exclusive production in e^+e^- collisions
    - ★ $\sigma(e^+e^- \to J/\psi + H)$, where H is η_c, χ_{c0} or $\eta_c(2s)$
    - ★ Model the light-front wave functions of the quarkonia

- Physical quantities:
  - Leading-twist Parton distribution amplitudes (PDAs).

- The science questions:
  - $u\bar{u} \to b\bar{b}$: a picture from Goldstone mode to heavy-heavy systems.
  - s wave$\to$ p wave: difference between ground state and others.

Partonic Structure of Heavy quarkonia

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- S-wave: pseudoscalar and vector mesons
  - Light: π, K, ρ, ϕ
    and their radially-excited states
  - Heavy quarkonia:
    - ★ pseudoscalar: $^1S_0[(\eta_c, \eta_b), 0^{-+}]$ (S=0, L=0)
    - ★ vector: $^3S_1[(J/\Psi, \Upsilon), 1^{--}]$ (S=1, L=0)
- P-wave: scalar, axial-vector and tensor mesons
  - Light: σ, a_1, b_1, a_2 etc.
    need to go beyond rainbow-ladder
  - Heavy quarkonia:
    - ★ axial-vector: $^1P_1[(h_c, h_b), 1^{+-}]$ (S=0, L=1)
    - ★ scalar: $^3P_0[(\chi_{c0}, \chi_{b0}), 0^{++}]$ (S=1, L=1)
    - ★ axial-vector: $^3P_1[(\chi_{c1}, \chi_{b1}), 1^{++}]$ (S=1, L=1)
    - ★ tensor: $^3P_2[(\chi_{c2}, \chi_{b2}), 2^{++}]$ (S=1, L=1)

Leading twist

- Twist: $t = l - s$, l: the scaling dimension, s: spin projection.[1]
- ψ: l=3/2, ψ_+: s=1/2, ψ_-: s=-1/2.

$$\psi_+ = \frac{1}{2}\gamma_-\gamma_+\psi\,, \quad \psi_- = \frac{1}{2}\gamma_+\gamma_-\psi \tag{5}$$

- Operator $\overline{\psi}\gamma_\mu\psi$:

$$\begin{aligned} twist-2: &\quad \overline{\psi}_+\gamma_+\psi_+ \\ twist-3: &\quad \overline{\psi}_+\gamma_\perp\psi_- + \overline{\psi}_-\gamma_\perp\psi_+ \\ twist-4: &\quad \overline{\psi}_-\gamma_-\psi_- \end{aligned} \tag{6}$$

- Matrix elements:

$$\langle 0|\overline{\psi}(-z)\hat{O}\psi(z)|H(P)\rangle \tag{7}$$

  - $\hat{O}$: operator of twist $t = 2, 3, 4$.
- Twist-2 operator: $\overline{\psi}_+\hat{O}\psi_+$, and $\hat{O} \in \{\gamma_+, \gamma_+\gamma_5, \sigma_{+\perp}, \sigma_{+\perp}\gamma_5\}$.
  - s-wave:
    - ★ pseudoscalar: $\gamma_+\gamma_5$
    - ★ vector: γ_+, $\sigma_{+\perp}$
  - p-wave:
    - ★ scalar: γ_+
    - ★ axial-vector: $\gamma_+\gamma_5$, $\sigma_{+\perp}\gamma_5$
    - ★ tensor: γ_+, $\sigma_{+\perp}$

[1] V. Braun, G. Korchemsky and D. Mueller. The uses of conformal symmetry in QCD. Prog. Part. Nucl. Phys. 2003.

Operator properties

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- Matrix elements:

$$\langle 0|\overline{\psi}(-z)\hat{O}\psi(z)|H(P)\rangle \quad (8)$$

  - Twist-2 operator: $\overline{\psi}_+\hat{O}\psi_+$, and $\hat{O} \in \{\gamma_+,\gamma_+\gamma_5, \sigma_{+\perp},\sigma_{+\perp}\gamma_5\}$

- G-parity transform:

$$\begin{aligned}
\hat{G}\overline{u}(-z)\gamma_\mu d(z)\hat{G}^\dagger &= -\hat{C}\overline{d}(-z)\gamma_\mu u(z)\hat{C}^\dagger \\
&= \overline{u}(z)\gamma_\mu d(-z) \\
\hat{G}\overline{u}(-z)\gamma_\mu\gamma_5 d(z)\hat{G}^\dagger &= -\overline{u}(z)\gamma_\mu\gamma_5 d(-z) \\
\hat{G}\overline{u}(-z)\sigma_{\mu\nu} d(z)\hat{G}^\dagger &= \overline{u}(z)\sigma_{\mu\nu} d(-z) \\
\hat{G}\overline{u}(-z)\sigma_{\mu\nu}\gamma_5 d(z)\hat{G}^\dagger &= \overline{u}(z)\sigma_{\mu\nu}\gamma_5 d(-z)
\end{aligned} \quad (9)$$

$$\begin{aligned}
&\langle 0|\overline{u}(-z)\gamma_\mu\gamma_5 d(z)|H(P)\rangle \\
=&\langle 0|\hat{G}^\dagger \left(\hat{G}\overline{u}(-z)\gamma_\mu\gamma_5 d(z)\hat{G}^\dagger\right)\hat{G}|H(P)\rangle \\
=&\langle 0|\hat{G}^\dagger \left(-\overline{u}(z)\gamma_\mu\gamma_5 d(-z)\right)\hat{G}|H(P)\rangle
\end{aligned} \quad (10)$$

- π: $I^G = 1^- \rightarrow \hat{G}|\pi(P)\rangle = -|\pi(P)\rangle$

$$\begin{aligned}
&\langle 0|\overline{u}(-z)\gamma_\mu\gamma_5 d(z)|\pi(P)\rangle \\
=&\langle 0|\overline{u}(z)\gamma_\mu\gamma_5 d(-z)|\pi(P)\rangle \\
=&f_\pi P_\mu \int_0^1 dx e^{-i(2x-1)z\cdot P}\phi_\pi(x) \\
=&f_\pi P_\mu \int_0^1 dx e^{i(2x-1)*(-z)\cdot P}\phi_\pi(1-x)
\end{aligned} \quad (11)$$

- $\Longrightarrow \phi_\pi(x) = \phi_\pi(1-x)$

Operator properties

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- vector meson: ρ

$$\langle 0|\bar{u}(-z)\gamma_\mu d(z)|\rho(P)\rangle$$
$$=\langle 0|\left(\bar{u}(z)\gamma_\mu d(-z)\right)\hat{G}|\rho(P)\rangle$$
$$\langle 0|\bar{u}(-z)\sigma_{\mu\nu} d(z)|\rho(P)\rangle$$
$$=\langle 0|\left(\bar{u}(z)\sigma_{\mu\nu} d(-z)\right)\hat{G}|\rho(P)\rangle$$

- ρ: $I^G = 1^+$
$\rightarrow \hat{G}|\rho(P)\rangle = |\rho(P)\rangle$

- $\Longrightarrow \phi_\rho^{\parallel}(x) = \phi_\rho^{\parallel}(1-x)$
$\Longrightarrow \phi_\rho^{\perp}(x) = \phi_\rho^{\perp}(1-x)$

$$\begin{aligned}&\langle 0|\bar{u}(-z)\gamma_\mu d(z)|\rho(P)\rangle\\ =&\langle 0|\left(\bar{u}(z)\gamma_\mu d(-z)\right)|\rho(P)\rangle\\ =&P_\mu n\cdot\epsilon^\lambda f_\rho\int_0^1 dx e^{-i(2x-1)z\cdot P}\phi_\rho^{\parallel}(x)\\ =&P_\mu n\cdot\epsilon^\lambda f_\rho\int_0^1 dx e^{i(2x-1)*(-z)\cdot P}\phi_\rho^{\parallel}(1-x)\end{aligned} \tag{12}$$

$$\begin{aligned}&\langle 0|\bar{u}(-z)\gamma_\mu d(z)|\rho(P)\rangle\\ =&\langle 0|\left(\bar{u}(z)\gamma_\mu d(-z)\right)|\rho(P)\rangle\\ =&(\epsilon_\mu^\lambda P_\nu-\epsilon_\nu^\lambda P_\mu)f_\rho^{\perp}\int_0^1 dx e^{-i(2x-1)z\cdot P}\phi_\rho^{\perp}(x)\\ =&(\epsilon_\mu^\lambda P_\nu-\epsilon_\nu^\lambda P_\mu)f_\rho^{\perp}\int_0^1 dx e^{i(2x-1)*(-z)\cdot P}\phi_\rho^{\perp}(1-x)\end{aligned} \tag{13}$$

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- Twist-2 matrix element:
  - scalar meson:

$$\langle 0|\overline{\psi}(-z)\gamma_\mu\psi(z)|S_{0^{++}}(P)\rangle = -\langle 0|\overline{\psi}(z)\gamma_\mu\psi(-z)|S_{0^{++}}(P)\rangle \tag{14}$$

  - axial-vector meson:

$$\langle 0|\overline{\psi}(-z)\gamma_\mu\gamma_5\psi(z)|A_{1^{++}}(P)\rangle = \langle 0|\overline{\psi}(z)\gamma_\mu\gamma_5\psi(-z)|A_{1^{++}}(P)\rangle \tag{15}$$

$$\langle 0|\overline{\psi}(-z)\sigma_{\mu\nu}\gamma_5\psi(z)|A_{1^{++}}(P)\rangle = -\langle 0|\overline{\psi}(z)\sigma_{\mu\nu}\gamma_5\psi(-z)|A_{1^{++}}(P)\rangle \tag{16}$$

$$\langle 0|\overline{\psi}(-z)\gamma_\mu\gamma_5\psi(z)|A_{1^{+-}}(P)\rangle = -\langle 0|\overline{\psi}(z)\gamma_\mu\gamma_5\psi(-z)|A_{1^{+-}}(P)\rangle \tag{17}$$

$$\langle 0|\overline{\psi}(-z)\sigma_{\mu\nu}\gamma_5\psi(z)|A_{1^{+-}}(P)\rangle = \langle 0|\overline{\psi}(z)\sigma_{\mu\nu}\gamma_5\psi(-z)|A_{1^{+-}}(P)\rangle \tag{18}$$

  - tensor meson:

$$\langle 0|\overline{\psi}(-z)\gamma_\mu\psi(z)|T_{2^{++}}(P)\rangle = -\langle 0|\overline{\psi}(z)\gamma_\mu\psi(-z)|T_{2^{++}}(P)\rangle \tag{19}$$

$$\langle 0|\overline{\psi}(-z)\sigma_{\mu\nu}\psi(z)|T_{2^{++}}(P)\rangle = -\langle 0|\overline{\psi}(z)\sigma_{\mu\nu}\psi(-z)|T_{2^{++}}(P)\rangle \tag{20}$$

PDAs properties

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- PDAs:
  - scalar meson:

$$\varphi_S(x) = -\varphi_S(\bar{x}) \tag{21}$$

  - axial-vector meson:

$$\begin{aligned}
\varphi^{\parallel}_{A,1^{++}}(x) &= \varphi^{\parallel}_{A,1^{++}}(\bar{x}) \\
\varphi^{\perp}_{A,1^{++}}(x) &= -\varphi^{\perp}_{A,1^{++}}(\bar{x}) \\
\varphi^{\parallel}_{A,1^{+-}}(x) &= -\varphi^{\parallel}_{A,1^{+-}}(\bar{x}) \\
\varphi^{\perp}_{A,1^{+-}}(x) &= \varphi^{\perp}_{A,1^{+-}}(\bar{x})
\end{aligned} \tag{22}$$

  - tensor meson:

$$\begin{aligned}
\varphi^{\parallel}_{T}(x) &= -\varphi^{\parallel}_{T}(\bar{x}) \\
\varphi^{\perp}_{T}(x) &= -\varphi^{\perp}_{T}(\bar{x})
\end{aligned} \tag{23}$$

  - Antisymmetry PDAs: $x \Leftrightarrow \bar{x} = 1 - x$
  - $\varphi_S(x), \varphi^{\perp}_{A,1^{++}}(x), \varphi^{\parallel}_{A,1^{+-}}(x), \varphi^{\parallel}_{T}(x), \varphi^{\perp}_{T}(x)$

Leading twist PDAs of s-wave heavy quarkonia

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- Matrix elements:

$$\langle 0|\psi(-z)\gamma_5\gamma.n\psi(z)|\pi(P)\rangle = f_\pi n{\cdot}P\int_0^1 dx e^{-i(2x-1)z\cdot P}\phi(x)\,,$$
$$= tr_{CD}Z_2\int_{dq}^{\Lambda} e^{-iz\cdot q-iz\cdot(q-P)}\gamma_5\gamma.n\chi(q;P)\,. \tag{24}$$

- Projecting Bethe-Salpeter wave function onto the light front:

$$f_\pi\phi(x) = tr_{CD}Z_2\int_{dq}^{\Lambda}\delta(n{\cdot}q_+ - xn{\cdot}P)\gamma_5\gamma{\cdot}n\chi(q;P)\,, \tag{25}$$

$$f_V n{\cdot}P\phi_V^{\parallel}(x) = m_V tr_{CD}Z_2\int_{dq}^{\Lambda}\delta(n{\cdot}q_+ - xn{\cdot}P)\gamma{\cdot}nn_\lambda\chi_\lambda(q;P)\,, \tag{26}$$

$$f_V^{\perp}m_V^2\phi_V^{\perp}(x) = n{\cdot}Ptr_{CD}Z_T\int_{dq}^{\Lambda}\delta(n{\cdot}q_+ - xn{\cdot}P)\sigma_{\mu\lambda}P_\mu\chi_\lambda(q;P)\,, \tag{27}$$

  - n, light-like four-vector, $n^2 = 0$.
  - f_π,f_V, and $f_V^{\perp}$, decay constants.
  - $\chi(q;P)$, Bethe-Salpeter wave function, the solution of Bethe-Salpeter equation.

Methods computing PDAs with DSEs

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- Moments: $\langle x^m \rangle = \int_0^1 dx x^m \phi(x)$
  - Perturbation theory integral representations (PTIRs)[1]:
    - ★ Infinite number of Mellin moments.
    - ★ Combine denominators $\Rightarrow$ the integral over Feynman parameters.
    - ★ Represent the Bethe-Sapeter wave function with parameters.
  - "Brute-force" approach[2]:
    - ★ Limited number of Mellin moments.

- Spectral function: $\chi(q, P) = \int_{-1}^{1} dz \int_0^\infty d\gamma \frac{g(z,\gamma)}{(q^2 + zq\cdot P + \frac{1}{4}P^2 + M^2 + \gamma)^3}$
  - Maximum entropy method (MEM)[3]:
    - ★ Well-known method to solve the ill-posed inversion problem.
    - ★ Extract the weight function of Bethe-Salpeter wave function.

[1] L. Chang, I.C. Cloet, J.J. Cobos-Martinez, C.D. Roberts, S.M. Schmidt, and P.C. Tandy, Phys. Rev. Lett. 110, 132001 (2013),1301.0324.

[2] M. Ding, F. Gao, L. Chang, Y.X. Liu, and C.D. Roberts, Leading-twist parton distribution amplitudes of S-wave heavy-quarkonia, Phys.Lett. B753 (2016) 330-335.

[3] F. Gao, L. Chang, and Y.X. Liu, A novel algorithm for extracting the parton distribution amplitude from the Euclidean Bethe-Salpeter wave function. arXiv:1611.03560.

Leading twist PDAs of s-wave heavy quarkonia

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- Moments of PDAs:

$$\langle x^m \rangle = \int_0^1 dx x^m \phi_{\eta_c,\eta_b}(x) = \frac{1}{f_{\eta_c/\eta_b}} tr_{CD} Z_2 \int_{dq}^{\Lambda} \frac{(n\cdot q_+)^m}{(n\cdot P)^{m+1}} \gamma_5 \gamma\cdot n \chi_{\eta_c/\eta_b}(q;P)\,, \tag{28}$$

$$\langle x^m \rangle_{\parallel} = \int_0^1 dx x^m \phi^{\parallel}_{J/\psi,\Upsilon}(x) = \frac{m_V}{f_V} tr_{CD} Z_2 \int_{dq}^{\Lambda} \frac{[n\cdot q_+]^m}{[n\cdot P]^{m+2}} \gamma\cdot n n_\lambda \chi_\lambda(q;P)\,, \tag{29}$$

$$\langle x^m \rangle_{\perp} = \int_0^1 dx x^m \phi^{\perp}_{J/\psi,\Upsilon}(x) = \frac{1}{f_V^{\perp} m_V^2} tr_{CD} Z_T \int_{dq}^{\Lambda} \frac{[n\cdot q_+]^m}{[n\cdot P]^m} \sigma_{\mu\lambda} P_\mu \chi_\lambda(q;P)\,. \tag{30}$$

  - "Brute-force" approach.
  - Calculate directly, limited number of moments.
  - A factor $1/(1+q^2r^2)^{\frac{m}{2}}$ is introduced for $\langle x^m \rangle$, and each moment is a function of r, with reliable results extrapolated to $r^2 = 0$.
  - Reconstruct the PDAs from their moments.
- S: dressed-quark propagator; Γ: Bethe-Salpter amplitude.

$$\chi(k,P) = S(k_+)\Gamma(k,P)S(k_-)\,, \tag{31}$$

Bethe-Salpeter wavefucntion

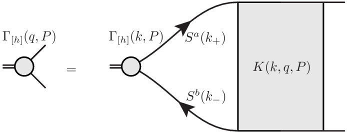
- S: dressed-quark propagator.

$$S(k) = Z(k^2,\zeta)/[i\gamma\cdot k + M(k^2,\zeta)]\,, \tag{32}$$

$$S^{-1}(k) = Z_2(i\gamma\cdot k + m^{bm}) + Z_2^2 \int_q^\Lambda g^2 D_{\mu\nu}(k-q)\frac{\lambda^a}{2} S(q)\Gamma_\nu^a(q,k)\,. \tag{33}$$

- Γ: Bethe-Salpeter amplitude.

$$[\Gamma(k,P)]_{tu} = Z_2^2 \int_q^\Lambda [S(q_+)\Gamma(q,P)S(q_-)]_{sr} K_{tu}^{rs}(q,k,P)\,. \tag{34}$$



Leading-twist PDAs of s-wave heavy-quarkonia

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- Quarkonia properties
- Current-quark masses were chosen in order to fit m_{η_c}, m_{η_b}.

	m_{η_c}	m_{η_b}	$m_{J/\Psi}$	m_{Υ}
ς_G^{RL}	2.98	9.39	3.26	9.52
ς_G^{DB}	2.98	9.39	3.07	9.46
expt.[1]	2.98	9.39	3.10	9.46

- Decay constants

	f_{η_c}	f_{η_b}	$f_{J/\Psi}$	$f^{\perp}_{J/\Psi}$	f_{Υ}	$f^{\perp}_{\Upsilon}$
ς_G^{RL}	0.389	0.597	0.410	0.337	0.552	0.489
ς_G^{DB}	0.262	0.543	0.255	0.213	0.471	0.421
expt.	0.238		0.294		0.506	
lQCD[2]	0.279	0.472	0.286		0.459	
DSE_{10}[3]	0.274	0.489	0.293		0.482	
CQM[4]	0.841	0.728	0.346		0.469	

[1] K. Olive et al. Particle data group collaboration. Chin. Phys. C, 2014.

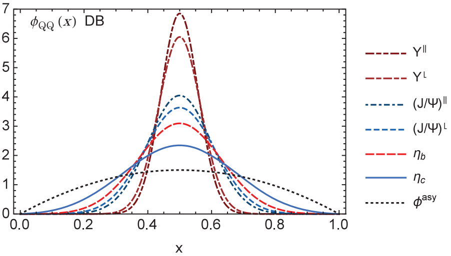
[2] C. McNeile et al. Heavy meson masses and decay constants from relativistic heavy quarks in full lattice QCD. Physical Review D, 2012.

[3] T.Nguyen et al. Soft and hard scale QCD dynamics in mesons. AIP Conf.Proc. 1361 (2011) 142-151.

[4] J.Segovia et al.$J^{PC} = 1^{--}$ hidden charm resonances. Physical Review D, 2008.

Leading-twist PDAs of s-wave heavy-quarkonia

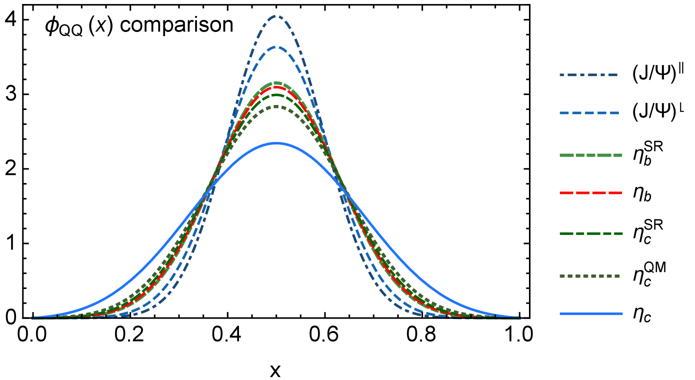
- Piecewise convex-concave-convex
- Deviate noticeably from $\phi_{NRQCD}(x) = \delta(x - 1/2)$
- Differences between pseudoscalar and vector meson PDAs, different vector-meson polarisations.
- $\Lambda_{QCD}/m_q(\zeta) \to 0$, $\phi(x) \to \delta(x - 1/2)$.



- Ordering of PDAs peak heights and widths: ($<_N$ means narrower than) $\phi^{asy} = 6x(1-x)$

$$\phi_{\Upsilon^{\parallel}} <_N \phi_{\Upsilon^{\perp}} <_N \phi_{J/\Psi^{\parallel}} <_N \phi_{J/\Psi^{\perp}} <_N \phi_{\eta_b} <_N \phi_{\eta_c} <_N \phi^{asy}(x)$$

Leading-twist PDAs of s-wave heavy-quarkonia



[1] H.-M. Choi, C.-R. Ji, Phys. Rev. D 76 (2007) 094010.

[2] V. V. Braguta, A. K. Likhoded, A. V. Luchinsky, Phys. Lett. B 646 (2007) 80-90.

[3] V. V. Braguta, Phys. Rev. D 75 (2007) 094016.

[4] T. Zhong, X.-G. Wu, T. Huang, Eur. Phys. J. C75 (2015) 45.

Leading-twist PDAs of s-wave heavy-quarkonia

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- PDA evolution with current-quark mass. Critical quark mass $m_q^c(\zeta)$, at which $\phi_{q\bar{q}}(x;\zeta) = \phi^{asy}(x)$

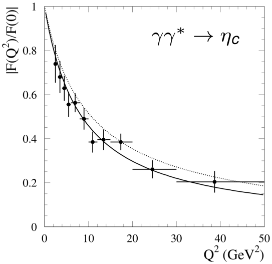
$$\begin{array}{cccc} & \phi_P & \phi_V^{\perp} & \phi_V^{\parallel} \\ m^c_{\varsigma_G^{DB}}(\zeta_2)/GeV & 0.15 & 0.13 & 0.12\,. \end{array} \quad (35)$$

- Fix point at $m_q^c(\zeta)$, quarkonium PDA is insensitive to changes under ERBL evolution
- Rms relative velocity of valence quark $\langle v^{2n}\rangle = (2n+1)\langle \xi^{2n}\rangle$

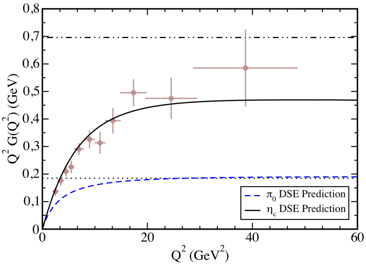
	η_c	η_b	$J/\Psi^{\perp}$	$J/\Psi^{\parallel}$	$\Upsilon^{\perp}$	$\Upsilon^{\parallel}$
$\langle v^2\rangle$	0.31	0.21	0.14	0.12	0.07	0.04
$\langle v^4\rangle$	0.16	0.07	0.03	0.02	0.01	0.00
$\langle v^6\rangle$	0.11	0.03	0.01	0.01	0.00	0.00

TFF of η_c

- Transition form factor $\gamma\gamma^* \to \eta_c$ in BaBar[1].



- TFF from DSEs[2].



- Asymptotic behaviour can be analyzed by PDAs:

$$\lim_{Q^2\to\infty} Q^2 G_{\eta_c}(Q^2) = 4\pi^2 \int_0^1 dx \frac{\frac{4}{9} f_{\eta_c} \phi_{\eta_c}(x)}{1-x} \tag{36}$$

[1] BaBar Collaboration. Measurement of the $\gamma\gamma^* \to \eta_c$ transition form factor. Phys.Rev. D81 (2010) 052010.

[2] K. Raya, M. Ding, A. Bashir, L. Chang and C.D. Roberts. Partonic structure of neutral pseudoscalars via two photon transition form factors, Phys.Rev. D95 (2017) 074014.

Leading-twist PDAs of s-wave heavy-quarkonia

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- Leading-twist PDAs of s-wave heavy-quarkonia: $\phi_P(x)$, $\phi_V^{\parallel}(x)$, $\phi_V^{\perp}(x)$.
  - Piecewise convex-concave-convex
  - Deviate noticeably from $\phi_{NRQCD}(x) = \delta(x - 1/2)$
  - Ordering of PDAs peak heights and widths: ($<_N$ means narrower than) $\phi^{asy} = 6x(1-x)$

$$\phi_{\Upsilon\parallel} <_N \phi_{\Upsilon\perp} <_N \phi_{J/\Psi\parallel} <_N \phi_{J/\Psi\perp} <_N \phi_{\eta_b} <_N \phi_{\eta_c} <_N \phi^{asy}(x)$$

  - Differences between pseudoscalar and vector meson PDAs, different vector-meson polarisations.
  - $\Lambda_{QCD}/m_q(\zeta) \to 0$, $\phi(x) \to \delta(x - 1/2)$.
  - Critical current quark mass $m_q^c(\zeta = 2GeV) = 0.15, 0.13, 0.12$GeV, $\phi(x) = \phi^{asy}(x)$.
  - $\langle v^{2n} \rangle$, $\langle v^4 \rangle$-corrections, pseudoscalar systems.
  - Predict $\gamma\gamma^* \to \eta_c$ transition form factor G_{η_c} with large Q^2.

PDAs of P-wave heavy quarkonia

- Twist-2 matrix element:
  - scalar meson [(χ_{c0}, χ_{b0}), 0^{++}]: γ_+

$$\langle 0|\psi(-z)\gamma_+\psi(z)|S_{0^{++}}(P)\rangle \tag{37}$$

  - axial-vector meson [(χ_{c1}, χ_{b1}), 1^{++}] , [(h_c, h_b), 1^{+-}]: $\gamma_+\gamma_5$, $\sigma_{+\perp}\gamma_5$

$$\begin{aligned} &\langle 0|\psi(-z)\gamma_+\gamma_5\psi(z)|A_{1^{++}}(P)\rangle \\ &\langle 0|\psi(-z)\sigma_{+\perp}\gamma_5\psi(z)|A_{1^{++}}(P)\rangle \\ &\langle 0|\psi(-z)\gamma_+\gamma_5\psi(z)|A_{1^{+-}}(P)\rangle \\ &\langle 0|\psi(-z)\sigma_{+\perp}\gamma_5\psi(z)|A_{1^{+-}}(P)\rangle \end{aligned} \tag{38}$$

  - tensor meson [(χ_{c2}, χ_{b2}), 2^{++}] : γ_+, $\sigma_{+\perp}$

$$\begin{aligned} &\langle 0|\psi(-z)\gamma_+\psi(z)|T_{2^{++}}(P)\rangle \\ &\langle 0|\psi(-z)\sigma_{+\perp}\psi(z)|T_{2^{++}}(P)\rangle \end{aligned} \tag{39}$$

PDAs of P-wave heavy quarkonia

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- scalar meson: $[(\chi_{c0}, \chi_{b0}), 0^{++}]$

$$f_S \varphi_S(x,\zeta) = tr_{CD} Z_2(\zeta,\Lambda) \int_{dq}^{\Lambda} \delta(n\cdot q_+ - x n\cdot P) \gamma\cdot n \chi_S(q;P)\,, \tag{40a}$$

- axial-vector meson: $[(\chi_{c1}, \chi_{b1}), 1^{++}]$, $[(h_c, h_b), 1^{+-}]$

$$f^{\parallel}_{A,1^{++}} \varphi^{\parallel}_{A,1^{++}}(x,\zeta) = tr_{CD} Z_2(\zeta,\Lambda) \int_{dq}^{\Lambda} \delta(n\cdot q_+ - x n\cdot P) \gamma_5 \gamma\cdot n n_\nu \chi^{1^{++}}_\nu(q;P)\,, \tag{41a}$$

$$f^{\perp}_{A,1^{++}} \varphi^{\perp}_{A,1^{++}}(x,\zeta) = tr_{CD} Z_T(\zeta,\Lambda) \int_{dq}^{\Lambda} \delta(n\cdot q_+ - x n\cdot P) \gamma_5 \sigma_{\mu\nu} n_\mu \chi^{1^{++}}_\nu(q;P)\,, \tag{41b}$$

$$f^{\parallel}_{A,1^{+-}} \varphi^{\parallel}_{A,1^{+-}}(x,\zeta) = tr_{CD} Z_2(\zeta,\Lambda) \int_{dq}^{\Lambda} \delta(n\cdot q_+ - x n\cdot P) \gamma_5 \gamma\cdot n n_\nu \chi^{1^{+-}}_\nu(q;P)\,, \tag{41c}$$

$$f^{\perp}_{A,1^{+-}} \varphi^{\perp}_{A,1^{+-}}(x,\zeta) = tr_{CD} Z_T(\zeta,\Lambda) \int_{dq}^{\Lambda} \delta(n\cdot q_+ - x n\cdot P) \gamma_5 \sigma_{\mu\nu} n_\mu \chi^{1^{+-}}_\nu(q;P)\,, \tag{41d}$$

- tensor meson: $[(\chi_{c2}, \chi_{b2}), 2^{++}]$

$$f^{\parallel}_T \varphi^{\parallel}_T(x,\zeta) = tr_{CD} Z_2(\zeta,\Lambda) \int_{dq}^{\Lambda} \delta(n\cdot q_+ - x n\cdot P) \gamma\cdot n n_\rho n_\sigma \chi_{\rho\sigma}(q;P)\,, \tag{42a}$$

$$f^{\perp}_T \varphi^{\perp}_T(x,\zeta) = tr_{CD} Z_T(\zeta,\Lambda) \int_{dq}^{\Lambda} \delta(n\cdot q_+ - x n\cdot P) \sigma_{\mu\rho} n_\rho n_\sigma \chi_{\mu\sigma}(q;P)\,. \tag{42b}$$

- the index ν of $\perp$-polarisation PDAs are only summed with the two $\perp$-polarisation directions.

Decay constants and normalisation

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- $f^{\parallel}_{A,1++}$, $f^{\perp}_{A,1+-}$: decay constants of axial-vector heavy quarkonia

$$3f^{\parallel}_{A,1++} m_{A,1++} = tr_{CD} Z_2 \int_{dq}^{\Lambda} \gamma_5 \gamma_\lambda \chi_\lambda(q;P)\,,$$

$$3f^{\perp}_{A,1+-} m^2_{A,1+-} = tr_{CD} Z_T \int_{dq}^{\Lambda} \gamma_5 \sigma_{\mu\lambda} P_\mu \chi_\lambda(q;P)\,.$$

- Define five quantities of mass dimension "one": f_S, $f^{\perp}_{A,1++}$, $f^{\parallel}_{A,1+-}$, $f^{\parallel}_T$, and $f^{\perp}_T$ to make odd PDAs dimensionless.
- PDAs normalised as:
  - even functions

$$\int_0^1 dx \varphi^{\parallel}_{A,1++}(x) = \int_0^1 dx \varphi^{\perp}_{A,1+-}(x) = 1\,. \tag{44}$$

  - odd functions

$$\begin{aligned} &\int_0^1 dx \xi (= 2x-1) \varphi_{S,0++}(x) \\ = &\int_0^1 dx \xi \varphi^{\perp}_{A,1++}(x) = \int_0^1 dx \xi \varphi^{\parallel}_{A,1+-}(x) \\ = &\int_0^1 dx \xi \varphi^{\parallel}_{T,2++}(x) = \int_0^1 dx \xi \varphi^{\perp}_{T,2++}(x) = 1\,. \end{aligned} \tag{45}$$

Asymptotic and heavy quark limit

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- asymptotic distribution:
  - All mass scale disappear in conformal limit: $P^2 = -M^2_{meson} \to 0$.
  - even functions

$$\varphi^{\parallel}_{A,1^{++}}(x) = \varphi^{\perp}_{A,1^{+-}}(x) = 6x(1-x) \tag{46}$$

  - odd functions

$$\begin{aligned}\varphi_{S,0^{++}}(x) = \varphi^{\perp}_{A,1^{++}}(x) = \varphi^{\parallel}_{A,1^{+-}}(x) = \varphi^{\parallel}_{T,2^{++}}(x) = \varphi^{\perp}_{T,2^{++}}(x)\\ = 30x(1-x)(2x-1)\end{aligned} \tag{47}$$

- heavy quark limit:
  - Meson are equivalent to two constituent quark: $M_{meson} = 2M_{constituent} \to \infty$.
  - even functions

$$\varphi^{\parallel}_{A,1^{++}}(x) = \varphi^{\perp}_{A,1^{+-}}(x) = \delta[x - 1/2]\,, \tag{48}$$

  - odd functions

$$\begin{aligned}\varphi_{S,0^{++}}(x) = \varphi^{\perp}_{A,1^{++}}(x) = \varphi^{\parallel}_{A,1^{+-}}(x) = \varphi^{\parallel}_{T,2^{++}}(x) = \varphi^{\perp}_{T,2^{++}}(x)\\ \propto \frac{1}{2h}\delta\left[x - (\frac{1}{2}+h)\right] - \frac{1}{2h}\delta\left[\bar{x} - (\frac{1}{2}+h)\right]\,.\end{aligned} \tag{49}$$

Leading-twist PDAs of P-wave heavy quarkonia

- Current-quark masses were chosen to fit the masses of ground state charmonium and bottomonium η_c and η_b.

	$m_{\chi_{c0}}$	$m_{\chi_{c1}}$	m_{h_c}	$m_{\chi_{c2}}$	$m_{\chi_{b0}}$	$m_{\chi_{b1}}$	m_{h_b}	$m_{\chi_{b2}}$
herein	3.333	3.45	3.443	3.575	9.82	9.88	9.87	9.937
expt.[1]	3.415	3.511	3.525	3.556	9.86	9.893	9.899	9.912

- Heavy quarkonia properties:

	$f^{\parallel}_{\chi_{c1}}$	$f^{\parallel}_{\chi_{b1}}$	$f^{\perp}_{h_c}$	$f^{\perp}_{h_b}$
herein	0.1779	0.157	0.1468	0.131
	$f^{\perp}_{\chi_{c1}}$	$f^{\perp}_{\chi_{b1}}$	$f^{\parallel}_{h_c}$	$f^{\parallel}_{h_b}$
herein	0.0414	0.0417	0.111	0.103

	$f_{\chi_{c0}}$	$f_{\chi_{b0}}$	$f^{\parallel}_{\chi_{c2}}$	$f^{\parallel}_{\chi_{b2}}$	$f^{\perp}_{\chi_{c2}}$	$f^{\perp}_{\chi_{b2}}$
herein	0.076	0.084	0.052	0.045	0.092	0.094

[1] K. Olive et al. Particle data group collaboration. Chin. Phys. C, 2014.

Leading-twist PDAs of P-wave heavy quarkonia

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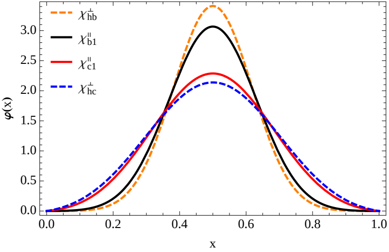
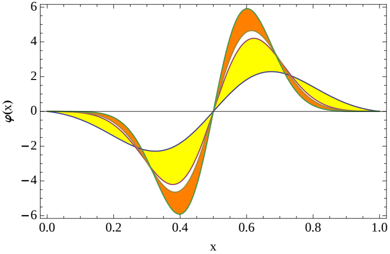
- even PDAs: $\varphi^{\parallel}_{1++}, \varphi^{\perp}_{1+-}$



- narrower than the asymptotic distribution $\varphi_{asy}(x) = 6x(1-x)$,
- broader than the distribution in heavy quark limit $\varphi(x) = \delta(x - 1/2)$,
- $b\bar{b}$ is narrower than $c\bar{c}$:

$$\varphi^{\perp}_{h_b} < N\varphi^{\parallel}_{\chi_{b1}} < N\varphi^{\parallel}_{\chi_{c1}} < N\varphi^{\perp}_{h_c},$$

- odd PDAs: $\varphi_S, \varphi^{\perp}_{1++}, \varphi^{\parallel}_{1+-}, \varphi^{\parallel}_T, \varphi^{\perp}_T$



- narrower than the asymptotic distribution $\varphi_{asy}(x) = 30x(1-x)(2x-1)$,
- broader than the distribution in heavy quark limit,
- charmonium: a wide band; bottomonium: a narrower band.

sensitivity of charmonium PDAs to the inner Lorentz structure is stronger than that of bottomonium systems.

Summary and outlook

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- **Summary:**
  - Twist-2 PDAs of s-wave heavy-quarkonia: η_c, η_b, J/Ψ, Υ.
    - ★ Piecewise convex-concave-convex even functions: $\phi_P(x)$, $\phi_V^{\parallel}(x)$, $\phi_V^{\perp}(x)$
    - ★ Deviate noticeably from $\phi_{NRQCD}(x) = \delta(x - 1/2)$
    - ★ Ordering of PDAs peak heights and widths: ($<_N$ means narrower than) $\phi^{asy} = 6x(1-x)$

$$\phi_{\Upsilon\parallel} <_N \phi_{\Upsilon\perp} <_N \phi_{J/\Psi\parallel} <_N \phi_{J/\Psi\perp} <_N \phi_{\eta_b} <_N \phi_{\eta_c} <_N \phi^{asy}(x)$$

    - ★ Differences between pseudoscalar and vector meson PDAs, different vector-meson polarisations.
    - ★ $\Lambda_{QCD}/m_q(\zeta) \to 0$, $\phi(x) \to \delta(x - 1/2)$.
    - ★ Critical current quark mass $m_q^c(\zeta = 2GeV) = 0.15, 0.13, 0.12$GeV, $\phi(x) = \phi^{asy}(x)$.
    - ★ $\langle v^{2n}\rangle$, $\langle v^4\rangle$-corrections, pseudoscalar systems.
    - ★ Predict $\gamma\gamma^* \to \eta_c$ transition form factor G_{η_c} with large Q^2.
  - Twist-2 PDAs of P-wave quarkonia: χ_{c1}, χ_{b1}, h_c, h_b, χ_{c0}, χ_{b0}, χ_{c2}, χ_{b2}.
    - ★ even PDAs: $\varphi_{1++}^{\parallel}$, $\varphi_{1+-}^{\perp}$.
    - ★ odd PDAs: $\varphi_{1++}^{\perp}$, $\varphi_{1+-}^{\parallel}$, φ_S, $\varphi_T^{\parallel}$, $\varphi_T^{\perp}$.
    - ★ narrower than the asymptotic distribution; broader than the distribution in heavy quark limit; $b\bar{b}$ is narrower than $c\bar{c}$: $\varphi_{h_b}^{\perp} <_N \varphi_{\chi_{b1}}^{\parallel} <_N \varphi_{\chi_{c1}}^{\parallel} <_N \varphi_{h_c}^{\perp}$.
    - ★ sensitivity of charmonium PDAs to the inner Lorentz structure is stronger than bottomonium.
- **Outlook:**
  - $\gamma\gamma^* \to \{\chi_{c0}, \chi_{c1}, \chi_{c2}, \chi_{b0}, \chi_{b1}, \chi_{b2}\}$ transition form factor with high Q^2.
  - $\sigma(e^+e^- \to J/\psi + H)$, where H is η_c, χ_{c0} or $\eta_c(2s)$.
  - higer twist PDAs, and PDFs, GPDs, TMDs, etc.
- **Thanks!**