Parton Distribution Amplitudes for heavy-light pseudoscalar mesons

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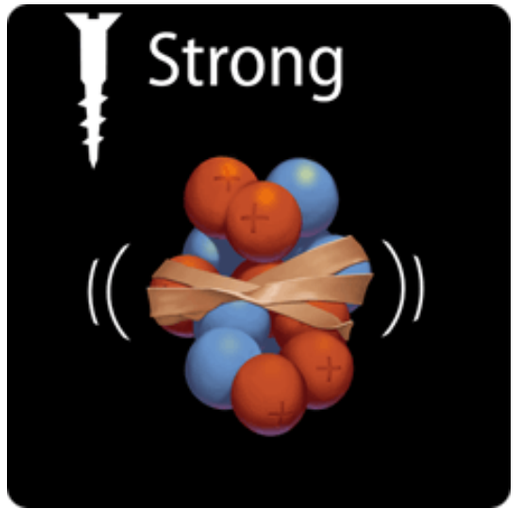
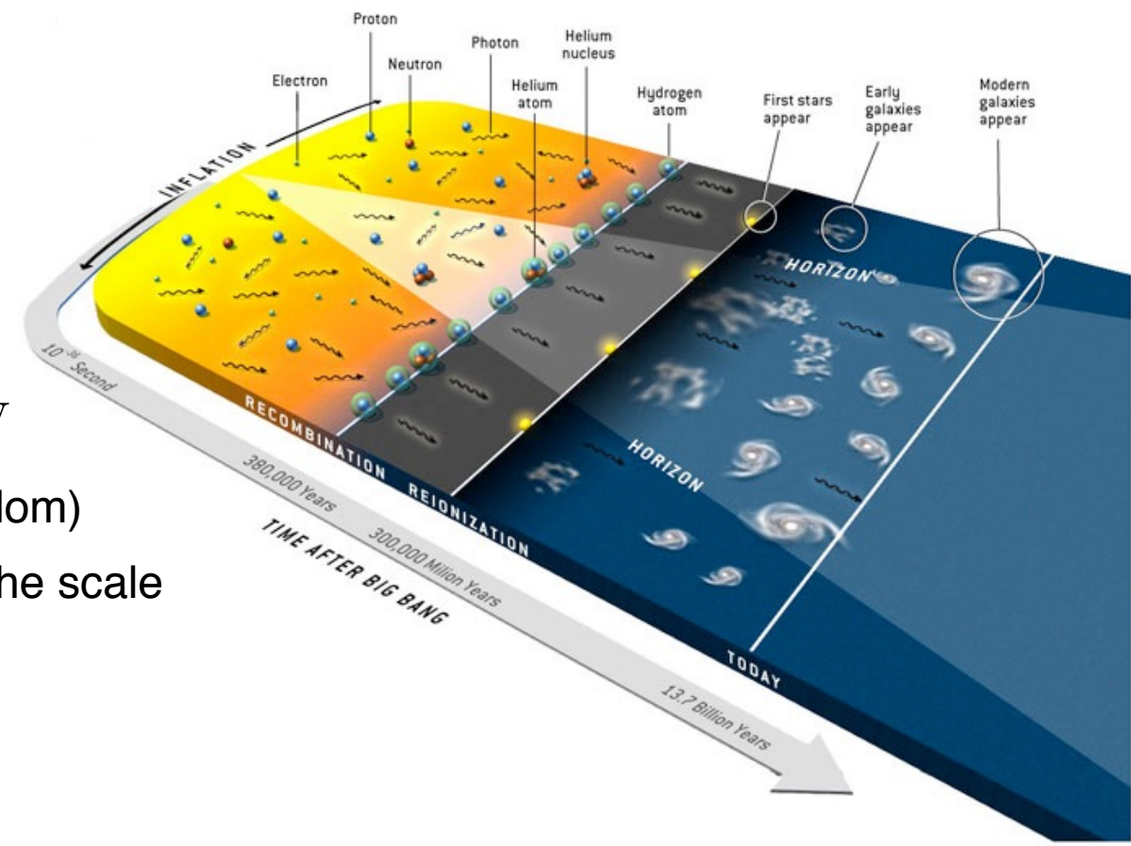
Quantum Chromo Dynamics

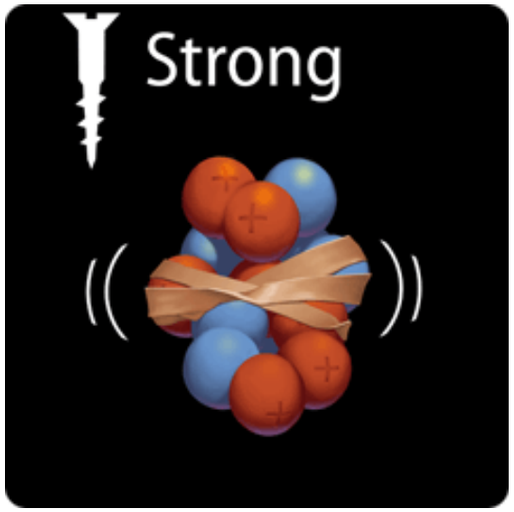


- **QCD started 10^{-6} secs. after the Big-Bang:**
  it is the glue that binds us all, and understanding its dynamics has profound implications
  - **Explain how massless gluons and light quarks are confined** and bind together to form hadrons…
  - **…thereby explain the origin of ~98% of the mass** in the visible Universe



- **QCD is *likely* a perfect theory**
  nothing needs to be added or changed
  - **Validated** over an incredible energy range: $0 \lesssim E \leq 8000\,\mathrm{GeV}$
  - **Unlikely to break down** at any energy scale (asymptotic freedom)
  - **No intrinsic parameters,** just need one observable to define the scale $\Lambda_{\mathrm{QCD}} \simeq 200 - 300\ \mathrm{MeV}$ QCD's "standard kilogram"
- **QCD is *a theory* not an effective theory**
- **However, it is innately nonperturbative**
  a priori no idea what such theory can produce

QCD: degrees of freedom



- **QCD basic degrees of freedom:**
  matter (quarks); gauge (gluons)

$$q_{\mathrm{f}}^{i} \begin{cases} \text{color} & i = 1,2,3 \\ \text{flavor} & \mathrm{f} = u,d,s,c,b,t \end{cases} \qquad A_{\mu}^{a} \begin{cases} \text{color} & a = 1,\ldots,8 \\ \text{spin} & \varepsilon_{\mu}^{\pm,0} \end{cases}$$

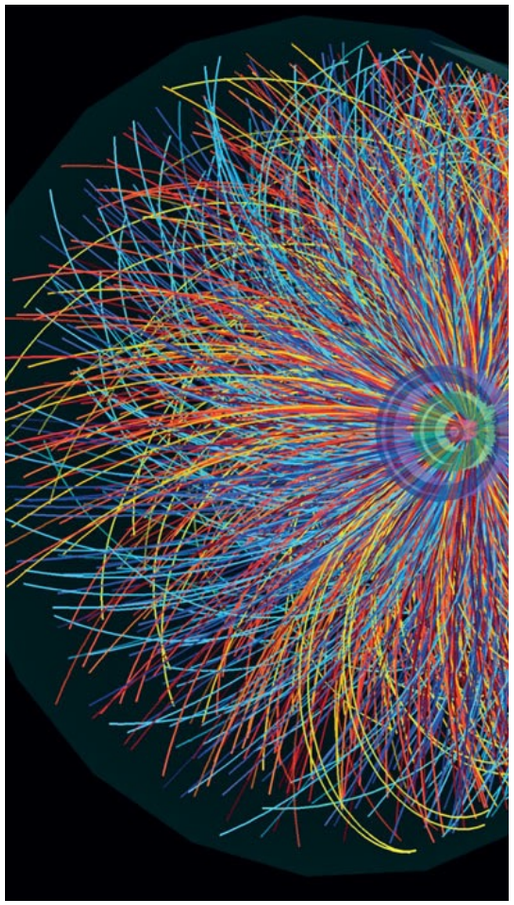
- **QCD action:**
  encodes all the dynamics

$$S_{\mathrm{QCD}} = \int \mathrm{d}^4x \left(\mathcal{L}_{\mathrm{I}} + \mathcal{L}_{\mathrm{GF+FPG}}\right) \qquad \mathcal{L}_{\mathrm{GF+FPG}} = s(\bar{c}^a \mathcal{F}^a - \xi/2\bar{c}^a b^a)$$

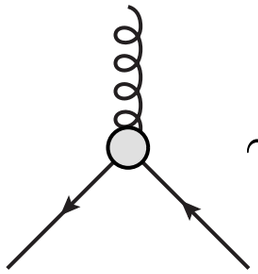
$$\mathcal{L}_{\mathrm{I}} = \bar{q}_{\mathrm{f}}^{i}(i\gamma^{\mu}D_{\mu} - m)_{ij}q_{\mathrm{f}}^{j} - \frac{1}{4}F_{\mu\nu}^{a}F_{a}^{\mu\nu}$$

- **Gauge fixing + FP ghost:**
  BRST exact, does not appear in spectrum

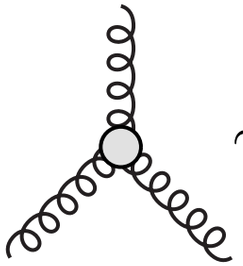
$$D_{\mu} = \partial_{\mu} - igA_{\mu}^{a}T^{a} \qquad F_{\mu\nu}^{a} = \partial_{\mu}A_{\nu}^{a} - \partial_{\nu}A_{\mu}^{a} + gf^{abc}A_{\mu}^{b}A_{\nu}^{c}$$



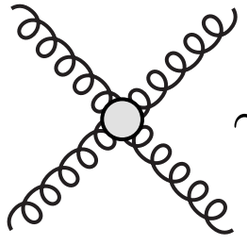
- **QCD (self-)interactions**
  dictate the theory's behavior



$\sim gA\bar{q}q$

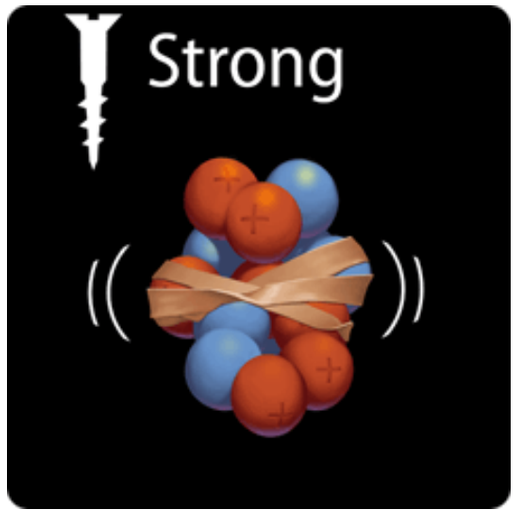


$\sim gA^3$

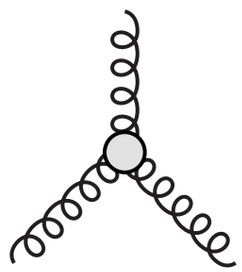


$\sim g^2A^4$

QCD: asymptotic freedom



- **Gluon self-interaction runs**
  implies QCD is non perturbative and quark/gluons are confined



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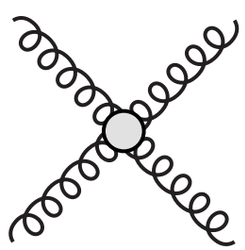

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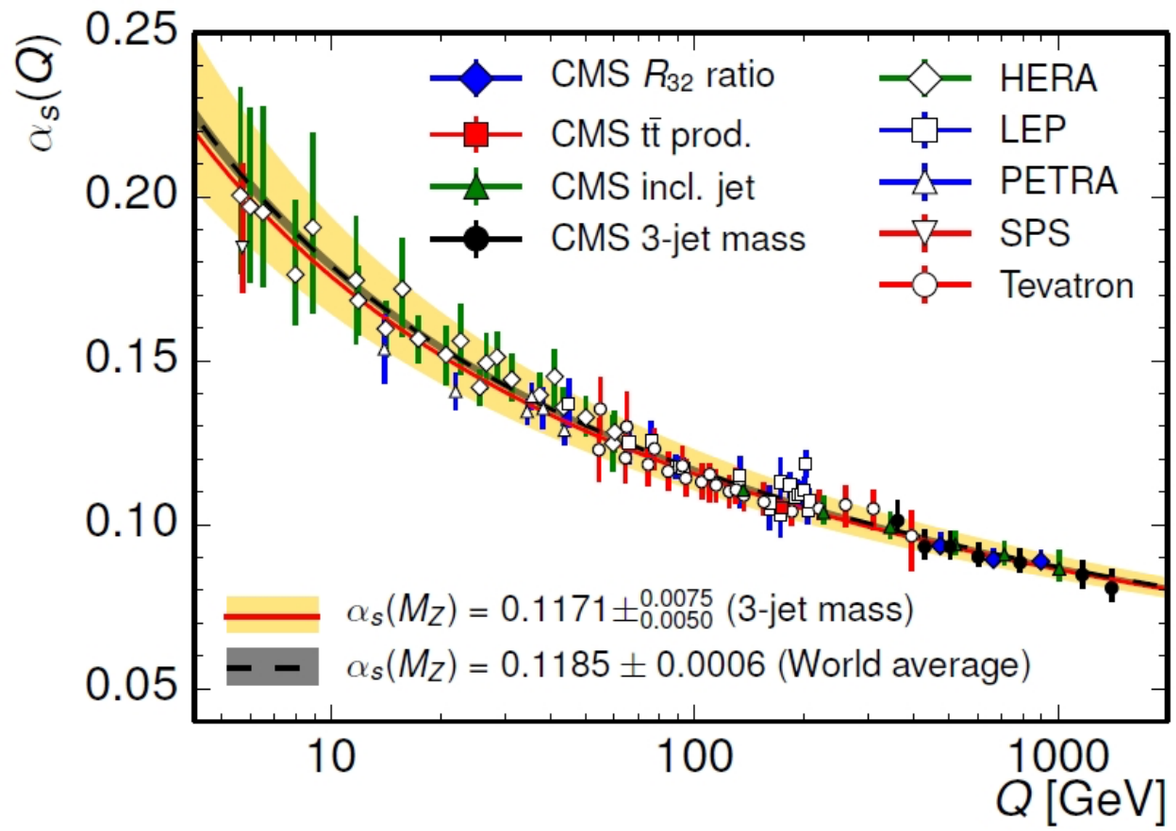
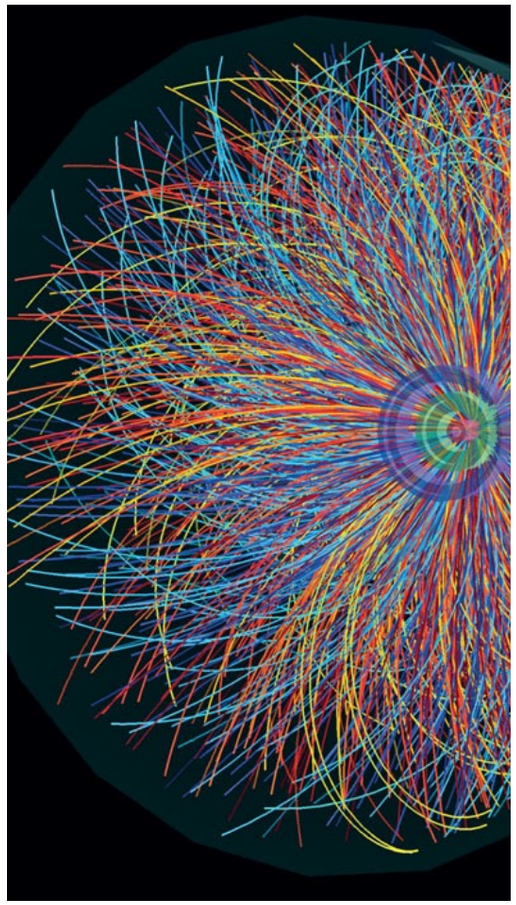


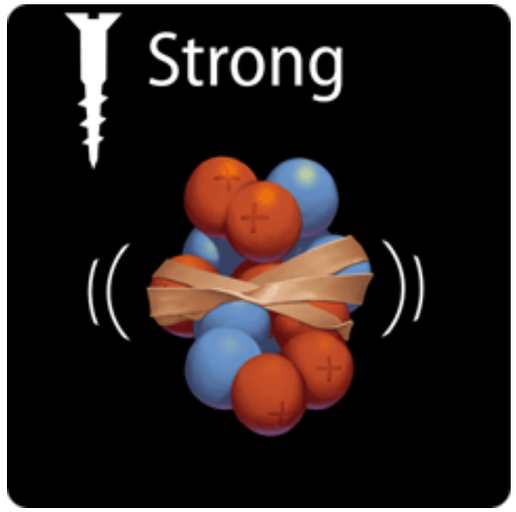


- **QCD interaction strength:**
  grows with separation between gluons and quarks

- **Typical scale of hadron physics**
  $r \sim 2 \text{ fm} \qquad \alpha \sim 0.5$

  - **Perturbation theory breaks down at this distance**
    QCD is entirely nonperturbative across almost the entire proton's volume

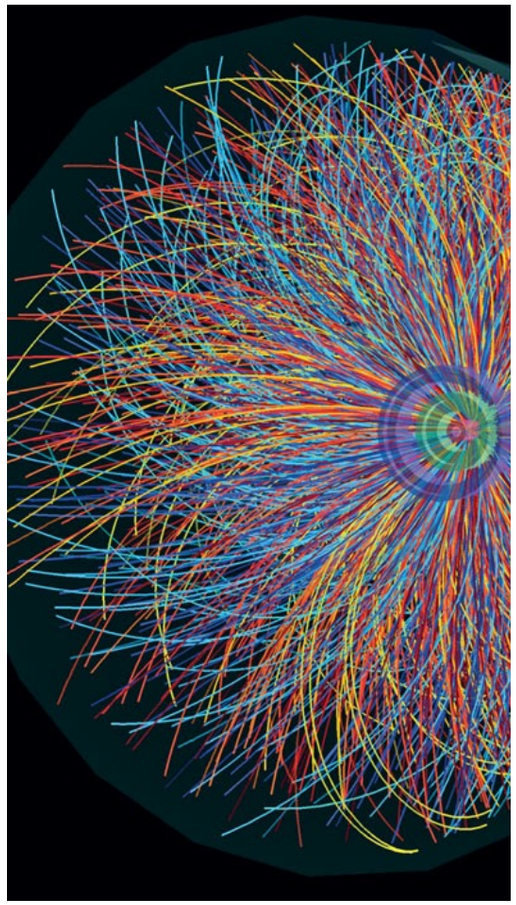
QCD: Schwinger-Dyson equations



- **Understanding the origin of mass in QCD**
  is quite likely inseparable from understanding confinement

- **One possible way of addressing this are Schwinger-Dyson eqs**
  which are quantum eom of Green's functions

- **SDEs: nonperturbative, covariant, IR/UV, light/heavy quarks; but:**
  infinite system of coupled integral equations
  - **Needs reliable truncation schemes**
    plus requires a gauge to be chosen (Landau)



- **Concentrate on SDE for 2-point functions** -1
  - **Three equations to be considered**
    quarks, gluons and ghosts

$$\mathcal{L}_{\mathrm{GF+FPG}} = s(\bar{c}^a \mathcal{F}^a - \xi/2 \bar{c}^a b^a)$$

  - **Gauge fixing + FP ghost:**
    BRST exact, does not appear in the spectrum

- **Capture two emergent phenomena**
  - Dynamical mass generation
  - Confinement (?)

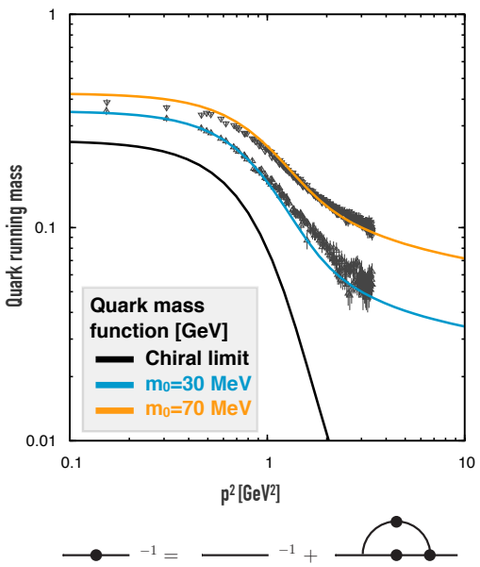
Intermezzo

Intermezzo

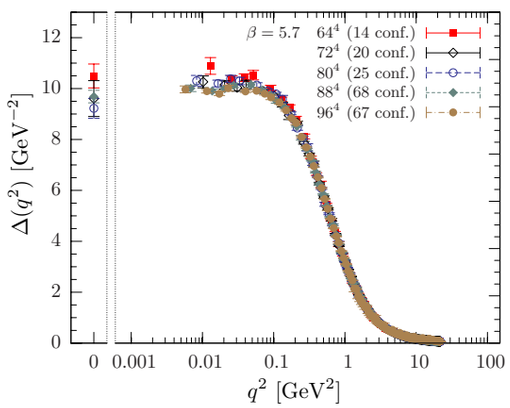
“This ‘telephone’ has too many shortcomings to be seriously considered as a means of communication. The device is inherently of no value to us.”
Western Union internal memo

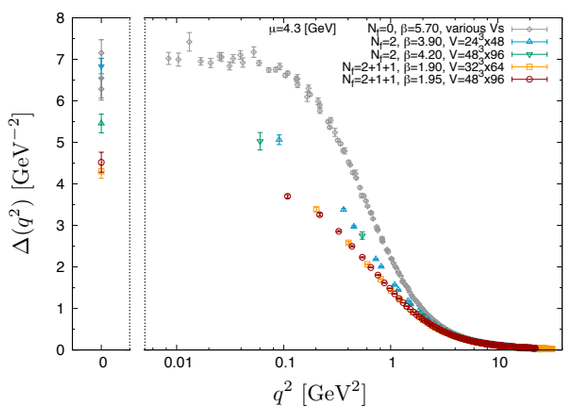
QCD in the IR

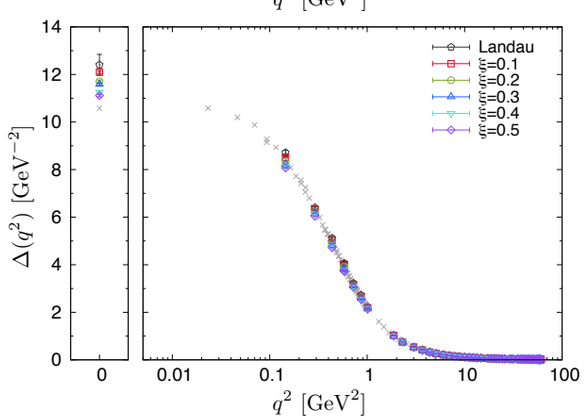
- **Dynamical χSB**
  in the matter sector

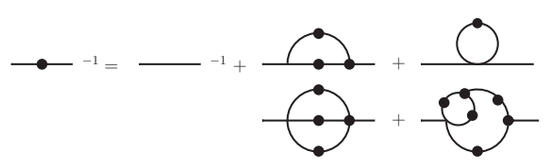


- **Saturating gluon propagator**
  dynamical gluon mass

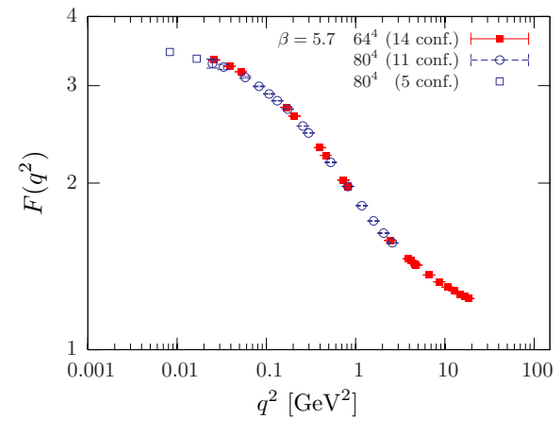


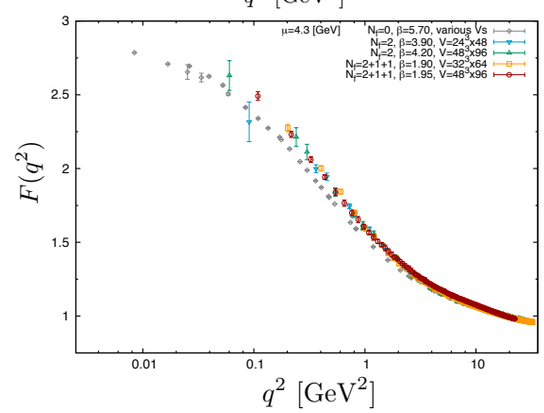




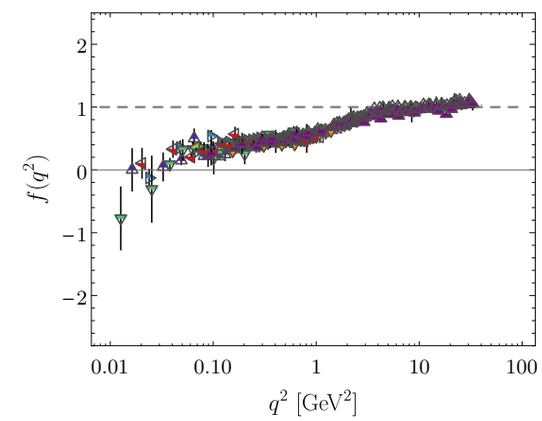


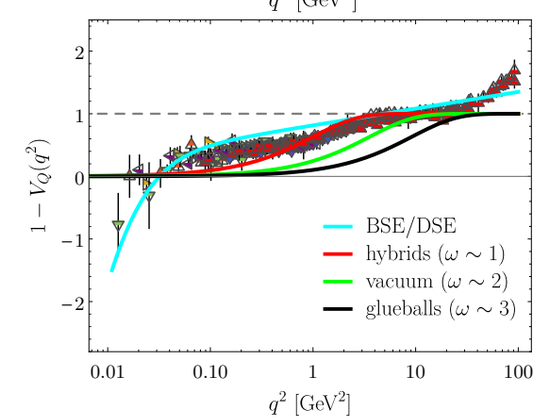
- **IR massless ghost**
  saturating dressing function

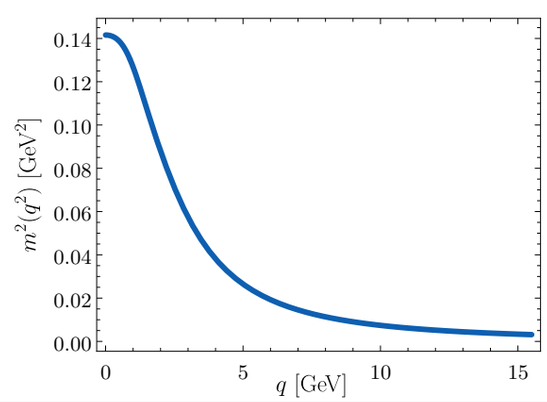




- **IR suppressed 3g vertex**
  due to ghost-gluon dynamics







PDAs within a DS framework

- **Meson leading twist PDA**
  projection of BS wave-function onto the light front
  Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, PRL 110 (2013)

$$f_{\rm PS}\varphi(x,\zeta) = Z_2(\zeta,\Lambda){\rm Tr}_{\rm CD}\int_{{\rm d}q}^{\Lambda}\delta(n\cdot q_+ - xn\cdot P)\gamma_5(\gamma\cdot n)\chi_{\rm PS}(q,P)$$

$f_{\rm PS}$ — **Meson's decay constant** (normalization of PDAs) $\int_0^1 {\rm d}x\,\phi(x,\zeta)=1$

$Z_2(\zeta,\Lambda)$ — **Quark wave-function renormalization** evaluated at ζ (2 GeV)

$\int_{{\rm d}q}^{\Lambda}$ — **Poincaré invariant regularization** of 4-dimensional integral

$\delta(n\cdot q_+ - xn\cdot P)$ — n **light-cone vector;** P **meson momentum**
$n\cdot P = -m_{\rm PS}$ $\quad P^2 = -m_{\rm PS}^2$

$\chi_{\rm PS}(q,P)$ — **BS wave-function:** $\chi_{\rm PS}(q,P) = S_{f_1}(q_+)\Gamma_{\rm PS}(q,P)S_{f_2}(q_-)$ $\quad q_+ = q+\eta P$, $q_- = q-(1-\eta)P$

$\Gamma_{\rm PS}(q,P)$ — **BS amplitude**
solution of the homogeneous BS equation

$$\Gamma_{\rm PS}^{\alpha\beta}(k,P) = \int_q \mathcal{K}^{\alpha\alpha';\beta\beta'}(q_\pm,k_\pm)[S_{f_1}(q_+)\Gamma_{\rm PS}^{\alpha\beta}(q,P)S_{f_2}(q_-)]_{\alpha'\beta'}$$

$S_{f_1}(q_+)$ — **Quark propagator**
solution of the gap equation

$$S_f^{-1}(k) = Z_2(-i\gamma\cdot k + m_f) + Z_1\int_{{\rm d}q}^{\Lambda} g^2 D^{\mu\nu}(k-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\Gamma_\nu(q,k)$$

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- **Not accessible**
  in Euclidean space

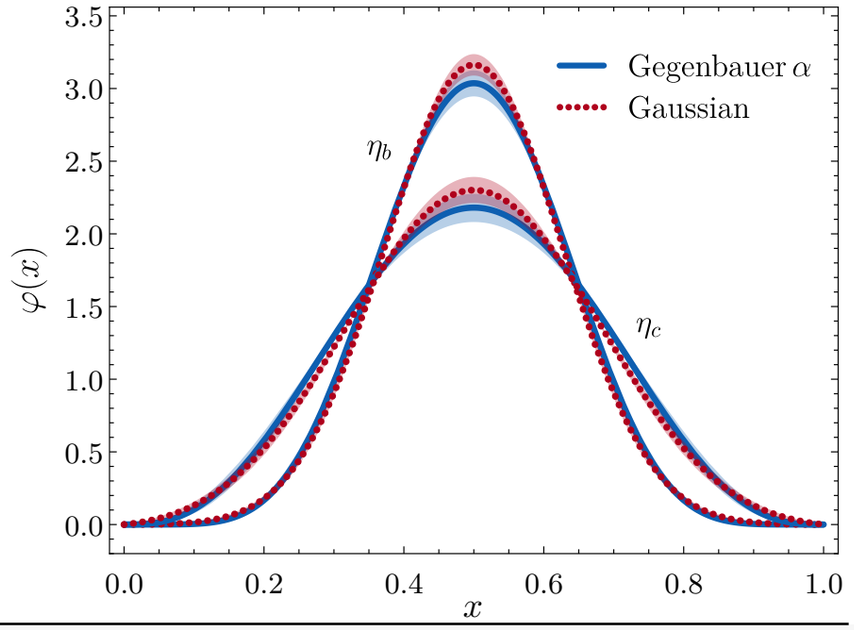
- **Mellin moments**
  used for reconstructing the PDA

$$\langle x^m\rangle = \frac{1}{f_{\rm PS}}Z_2(\zeta;\Lambda){\rm Tr}_{\rm CD}\int_{{\rm d}q}^{\Lambda}\frac{(n\cdot q_+)^m}{(n\cdot P)^{m+1}}\gamma_5(\gamma\cdot n)\,\chi_{\rm PS}(q;P)$$

- **Highly oscillatory integral**
  in Euclidean space
- **Requires damping factor**
  and extrapolation
  $1/(1+r^2k^2)^m; \qquad r^2\to 0$

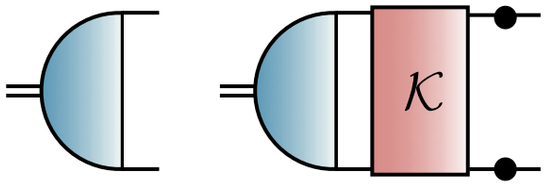
- **Two possible representations**
  for determining the full amplitude
  - **Gegenbauer**
    $\varphi(x) = \mathcal{N}x^\alpha(1-x)^\beta$
  - **Gaussian**
    $\varphi(x) = \mathcal{N}[1-(2x-1)^2]\,{\rm e}^{\mathfrak{a}^2(1-(2x-1)^2)+\mathfrak{b}^2(2x-1)}$

- **Main difference**
  end point behavior @ $x\to 1$
  - **Affects the inverse moment** $\int_0^1 {\rm d}x\,\phi(x)/(1-x)$
    large Q^2 behavior of form factors
  - **Heavy-heavy limit:** $\varphi(x)\to\delta(1-x/2)$
  - **Heavy-light limit:** $\varphi(x)\to\delta(1-x)$ (at least in principle, see later)



Propagators and BS amplitudes

- **Use RL truncation**
  for gap and BS equation



$$\Gamma_\nu = \gamma_\nu$$

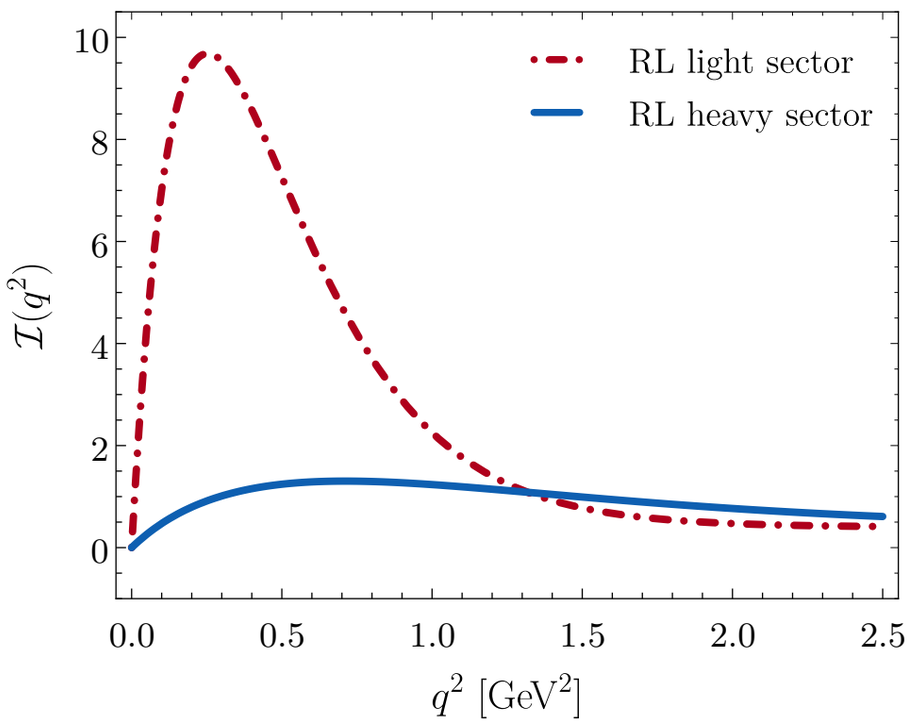
$$\mathcal{K}[S\Gamma_{\rm PS}S] = -P^{\mu\nu}\mathcal{G}\gamma_\mu[S\Gamma_{\rm PS}S]\gamma_\nu$$

- **For the interaction**
  we set
  Qin, Chang, Liu, Roberts and Wilson, PRC 84 (2011)

$$\mathcal{I}(k^2) = k^2\frac{\mathcal{G}_{\rm IR}(k^2)+\mathcal{G}_{\rm UV}(k^2)}{4\pi}$$

$$\mathcal{G}_{\rm IR}(k^2) = \frac{8\pi^2}{\omega^5}\varsigma^3 {\rm e}^{-k^2/\omega^2}$$

$$\mathcal{G}_{\rm UV}(k^2) = \frac{96\pi^2}{25}\frac{1-e^{-k^2/1[{\rm GeV}^2]}}{k^2\log[{\rm e}^2-1+(1+k^2/\Lambda^2)^2]}$$



- **Light quark sector** (GeV)
  $\varsigma = 0.8; \qquad \omega = 0.5$
- **Heavy quark sector** (GeV)
  $\varsigma = 0.6; \qquad \omega = 0.8$

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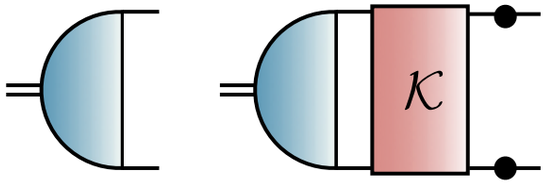

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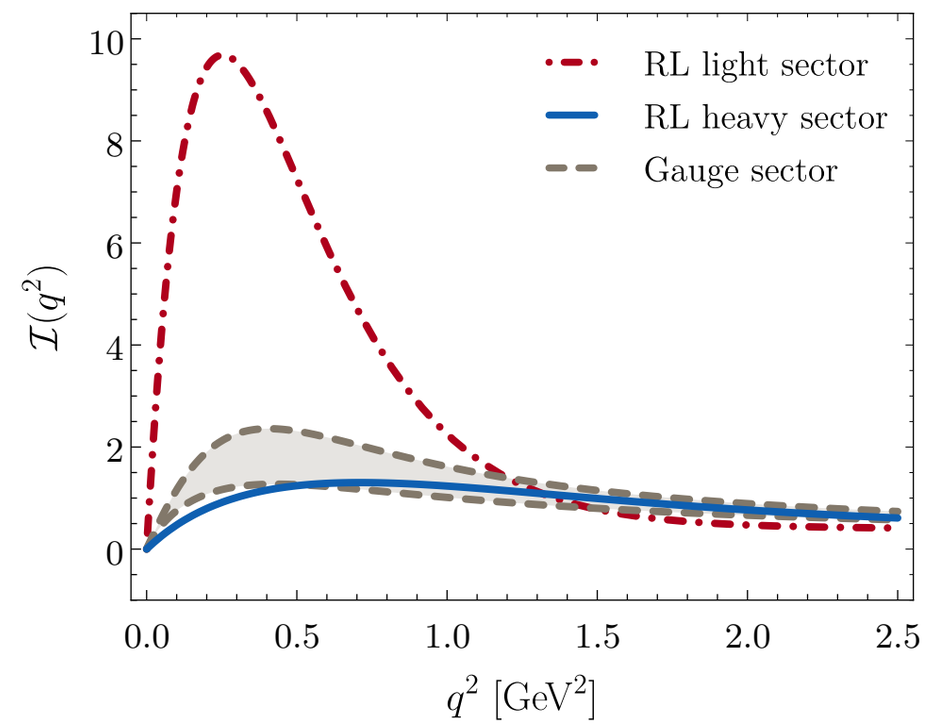
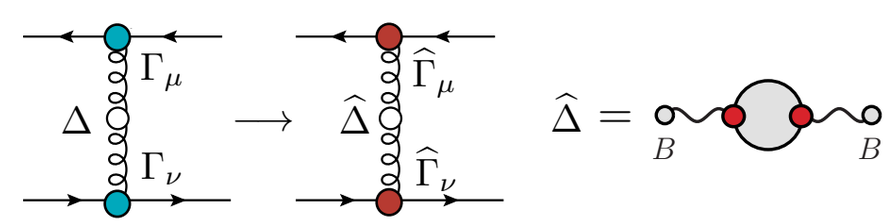
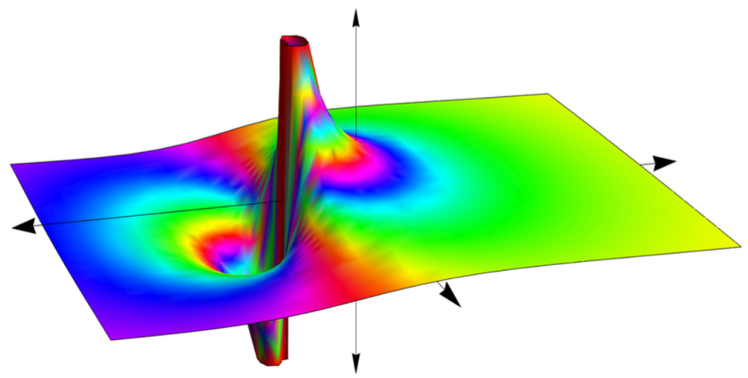


- **Heavy sector interaction**
  overlaps with gauge sector kernel determination
  DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)

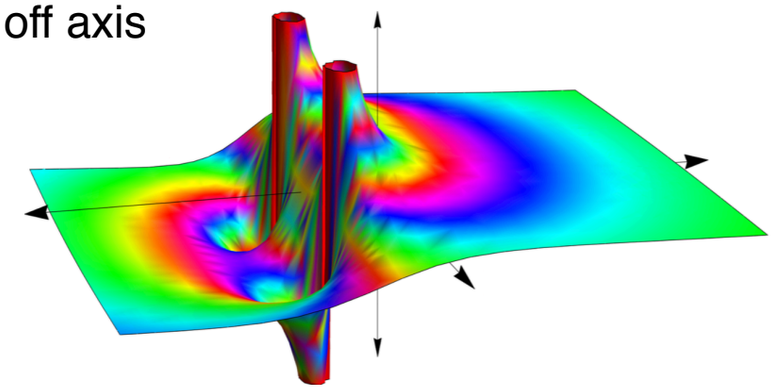


Propagators and BS amplitudes

- **SDE solutions**
  dramatic change in the analytic structure
  - **Normal particle**
    mass pole on the real axis

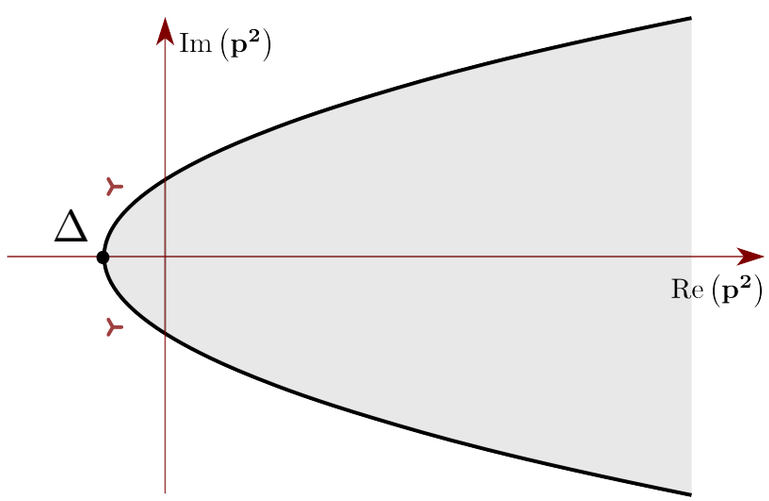


  - **SDE solutions**
    interactions move the mass pole off axis



  - **Complex conj. singularities**
    characterized by a dynamically generated mass scale

- **SDE singularities**
  impacts our ability to find BS solutions
  - **Quark propagators**
    need to be known on a parabolic region



  - **Cauchy th. not valid**
    if singularities are inside the contour
  - **Maximize parabola apices**
    probe bound states up to $M = \Delta_h + \Delta_l \;\; (\Delta_h \gg \Delta_l)$
  - **Use freedom to unbalance legs**
    in BS equation
    $q_+ = q + \eta P$
    $q_- = q - (1-\eta)P$ $\qquad \eta \in [\Delta_h/M, 1 - \Delta_l/M]$
  - **Still, if gap is too big**
    no BS solution exists anyway

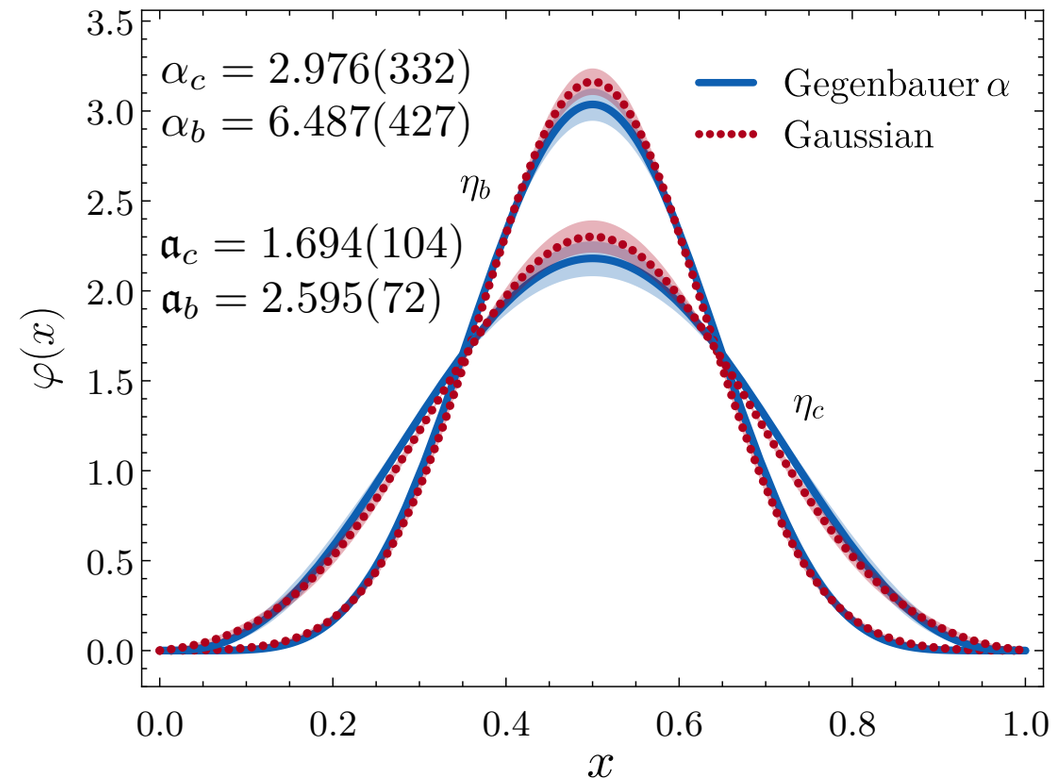
Heavy-light systems in RL

- **Choose current masses**
  for heavy quarks (in GeV): $m_c = 1.25,\ m_b = 4.35$

- **Fix interaction parameters**
  to heavy sector values: $\varsigma = 0.6;\ \omega = 0.8$

- **Evaluate properties of η mesons**

	Exp.				lQCD	
	m_i	f_i	m_i	f_i	m_i	f_i
η_c	2.98	0.272	2.98	0.238	2.98	0.279
η_b	9.38	0.501	9.39	/	9.39	0.472

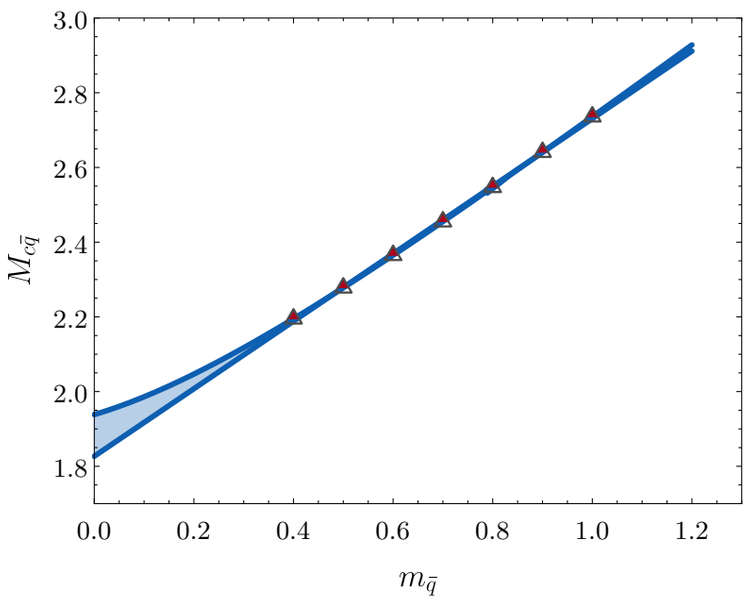


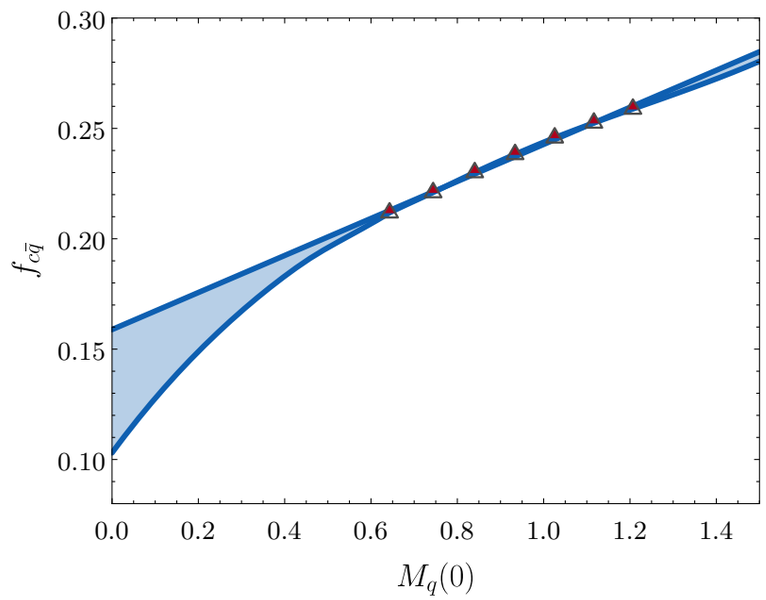
- **Construct an array of fictitious $h\bar{q}$ pseudoscalar mesons**
  down to the lowest m_q RL can accommodate
  - **Charm sector:** $m_q \sim 0.4$
  - **Bottom sector:** $m_q \sim 1.3$

- **Estimate PDAs parameters**
  use them to (SPM) extrapolate to lower m_q masses

Masses and decay constants

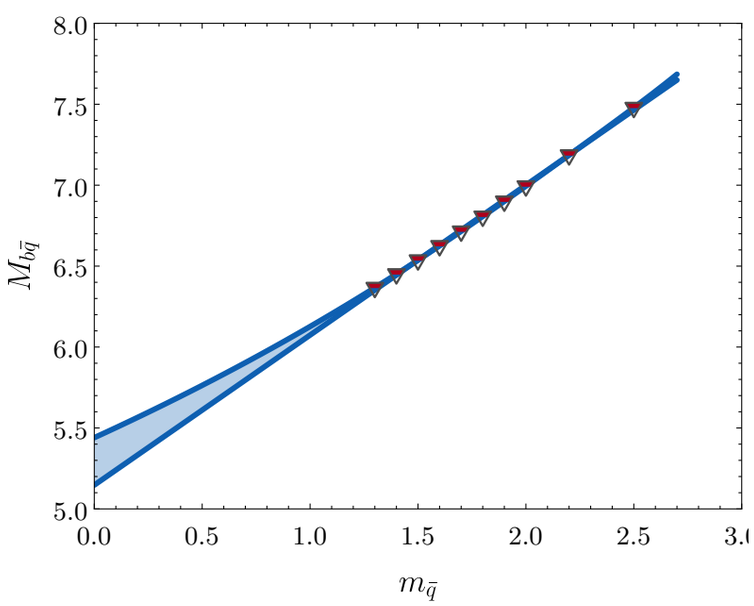
- **Charm sector**

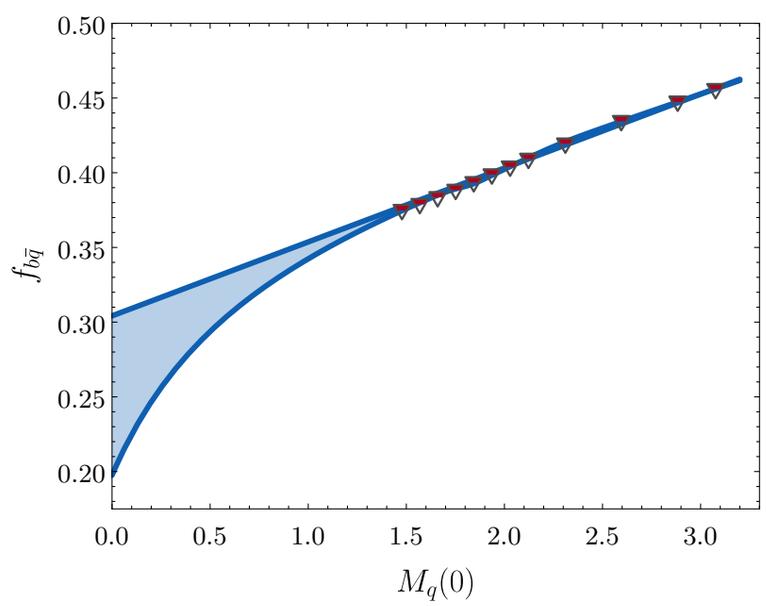




	This work		Exp.		lQCD	
	m_i	f_i	m_i	f_i	m_i	f_i
D	1.88(6)	0.200(6)	1.87	0.146	1.87	0.147
D_s	1.93(4)	0.218(1)	1.97	0.182/0.187	1.97	0.175

- **Bottom sector**

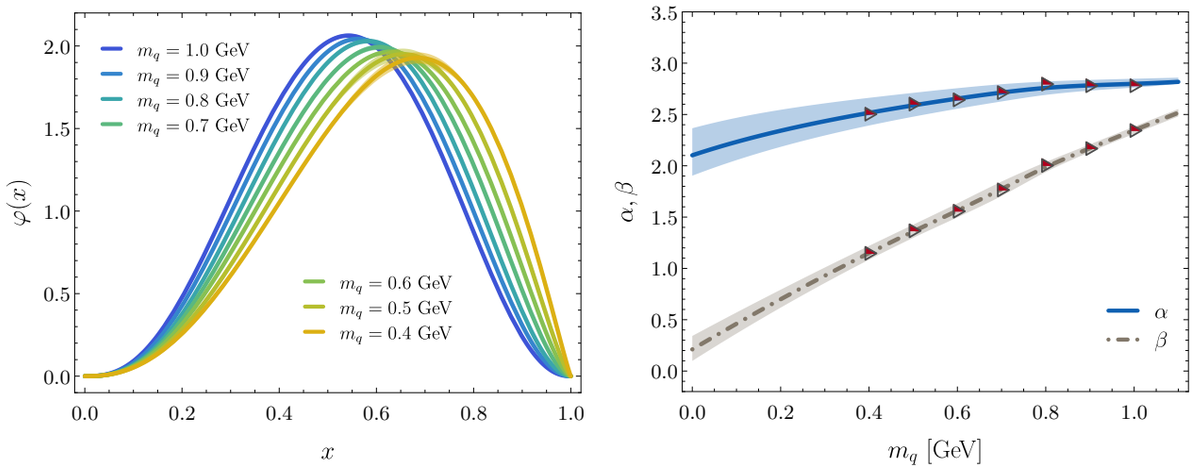


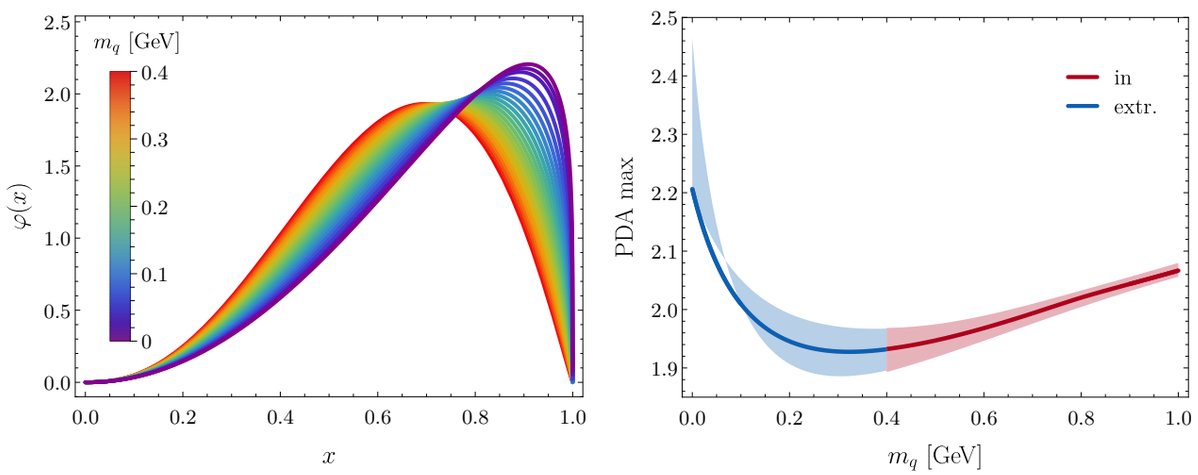


	This work		Exp.		lQCD	
	m_i	f_i	m_i	f_i	m_i	f_i
B	5.30(15)	0.313(17)	5.28	0.149	5.28	0.137
B_s	5.36(13)	0.328(11)	5.37	/	5.37	0.162
B_c	6.32(1)	0.373(2)	6.28	/	/	/

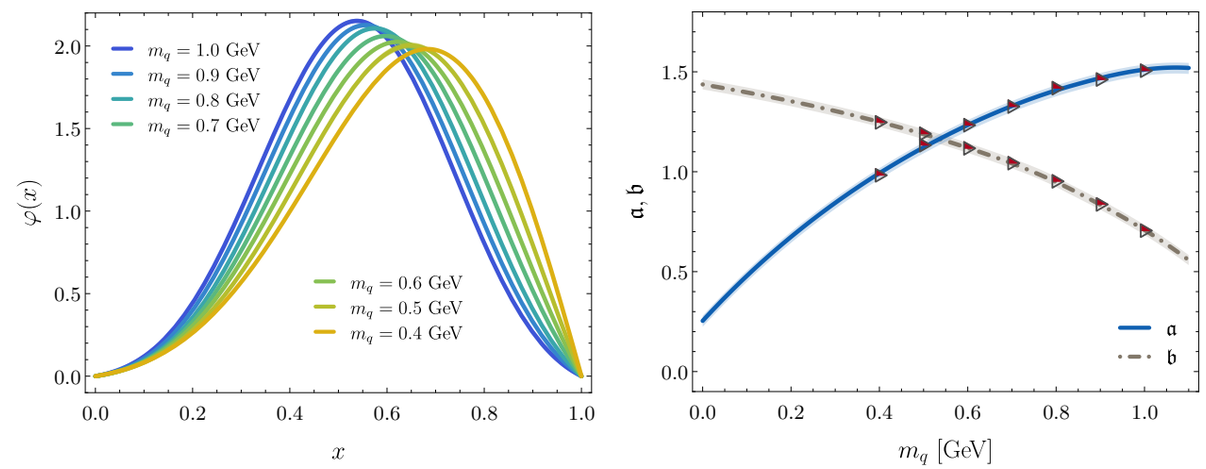
Charm sector

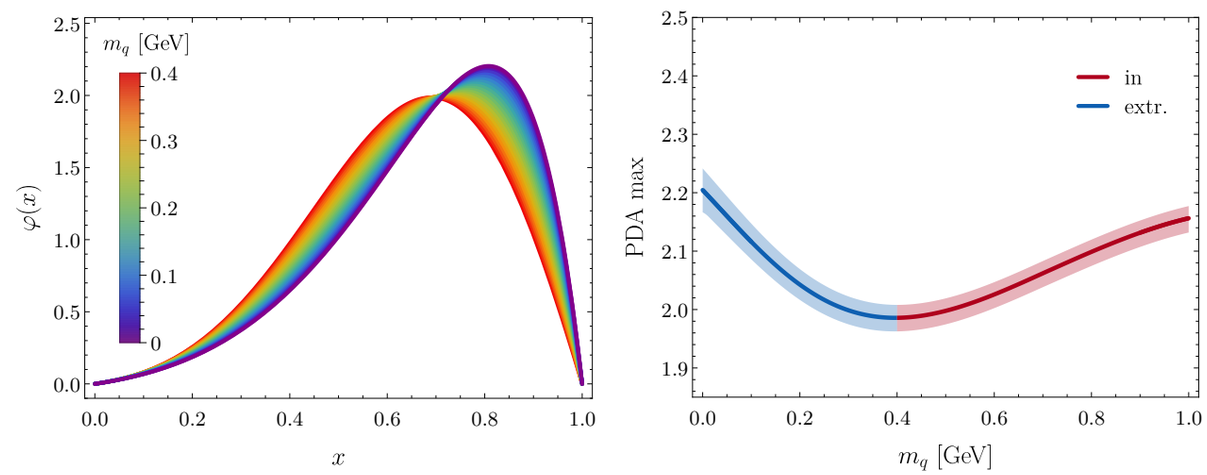
- **Gegenbauer**

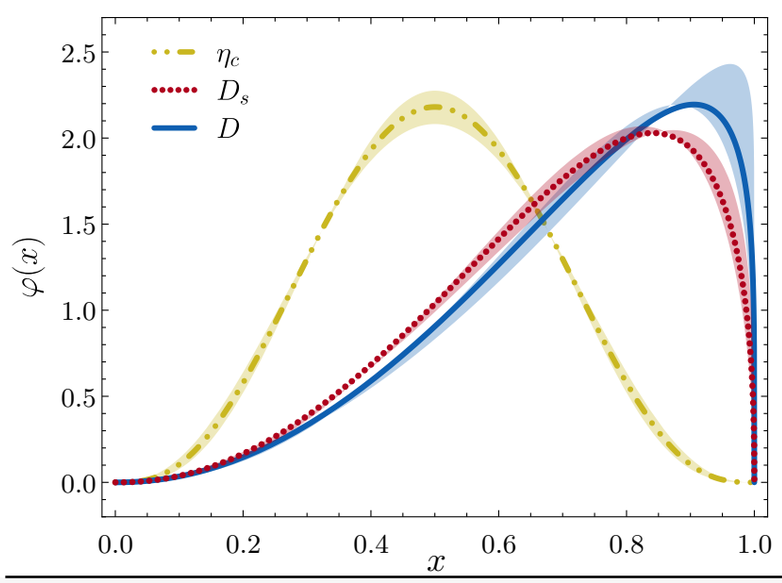




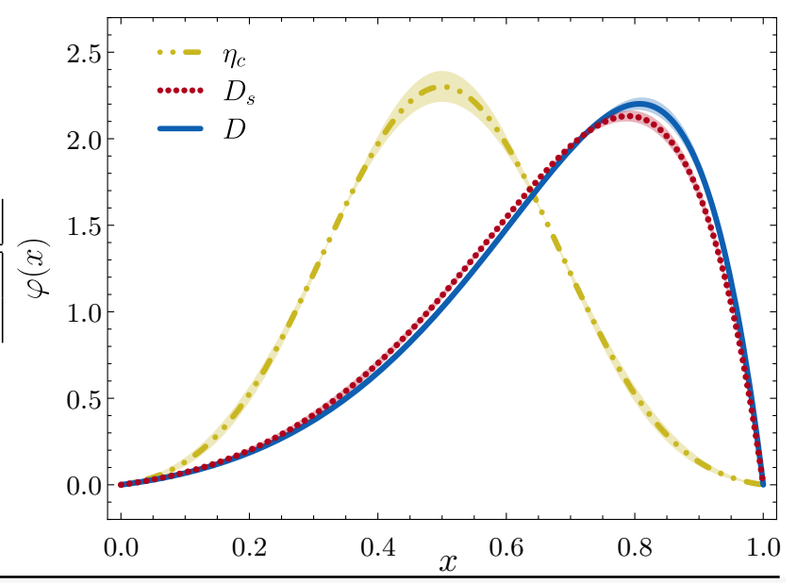
- **Gaussian**







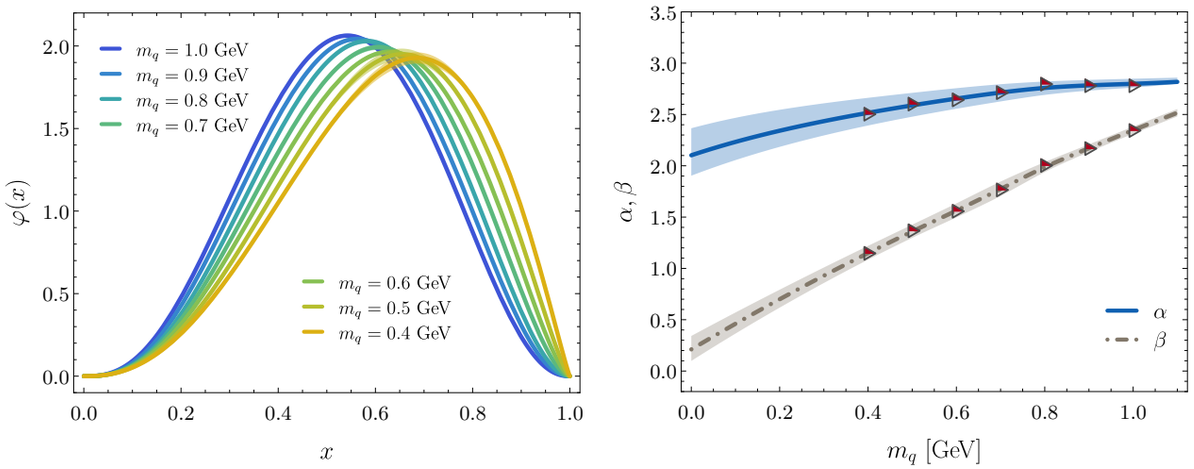
m_q	α	β	$\mathfrak{a}$	$\mathfrak{b}$
m_s	$2.212^{0.228}_{0.184}$	$0.418^{0.102}_{0.123}$	$0.442^{0.025}_{0.030}$	$1.404^{0.025}_{0.025}$
m_u	$2.107^{0.254}_{0.168}$	$0.220^{0.107}_{0.143}$	$0.262^{0.025}_{0.030}$	$1.436^{0.025}_{0.025}$

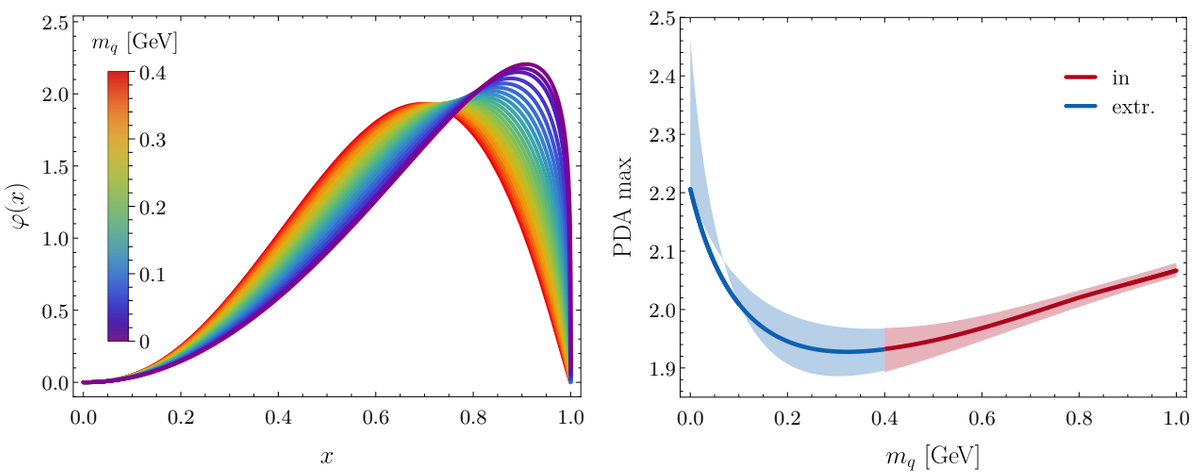


Charm sector

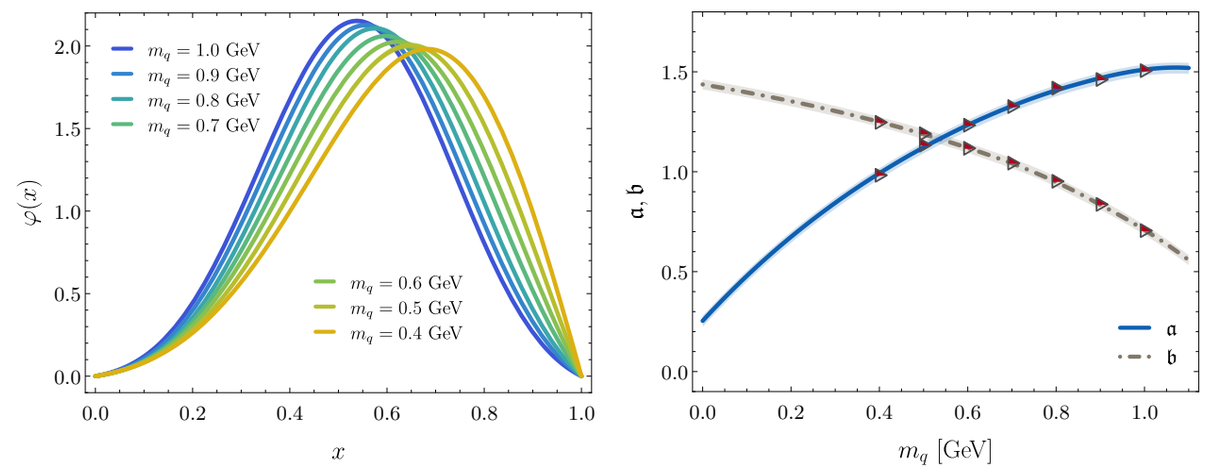
m_q	α	β	$\mathfrak{a}$	$\mathfrak{b}$
m_s	$2.212^{0.228}_{0.184}$	$0.418^{0.102}_{0.123}$	$0.442^{0.025}_{0.030}$	$1.404^{0.025}_{0.025}$
m_u	$2.107^{0.254}_{0.168}$	$0.220^{0.107}_{0.143}$	$0.262^{0.025}_{0.030}$	$1.436^{0.025}_{0.025}$

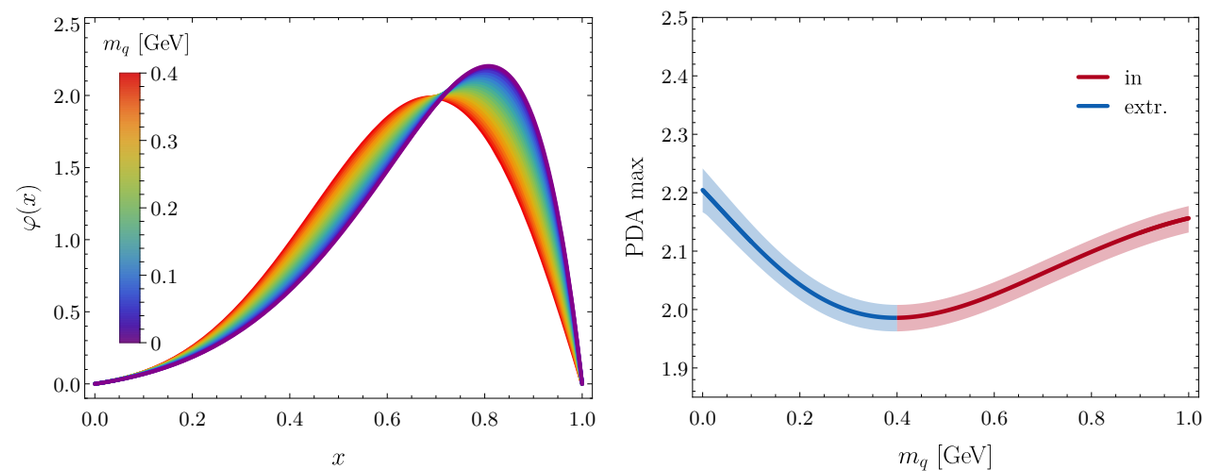
- **Gegenbauer**

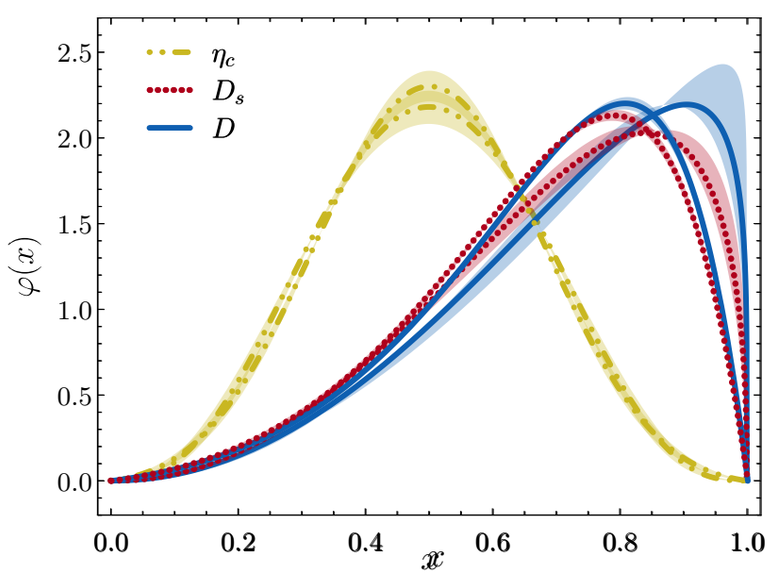




- **Gaussian**

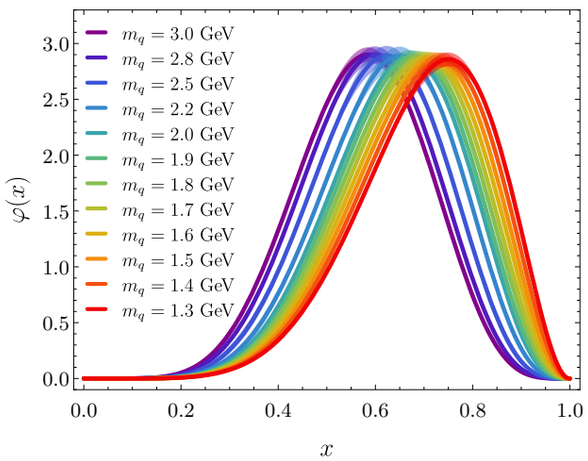


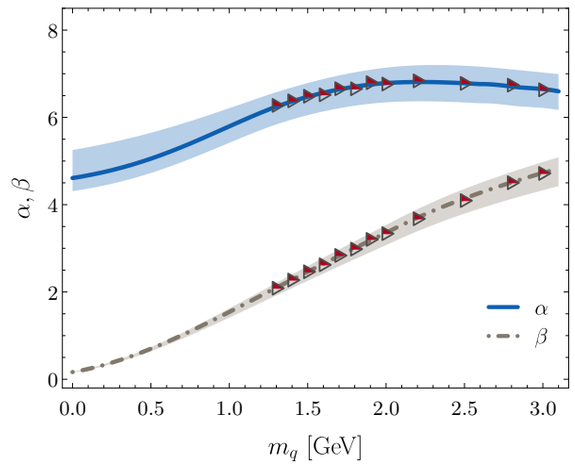


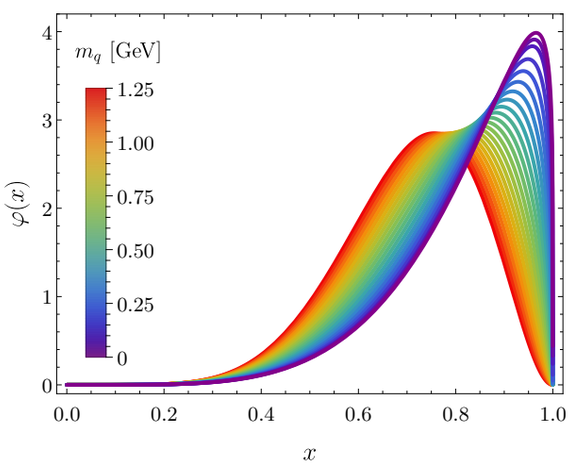


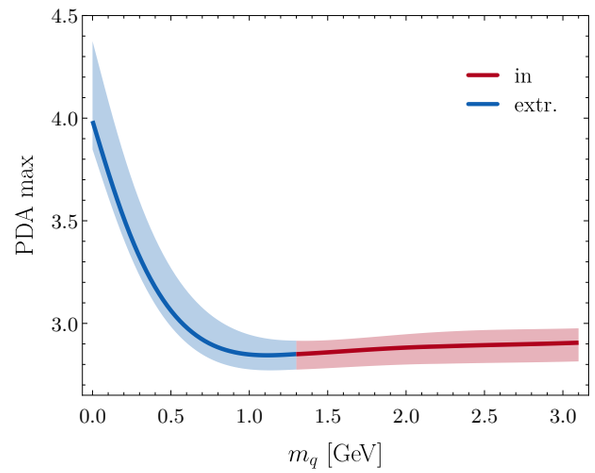
Bottom sector

- **Gegenbauer**

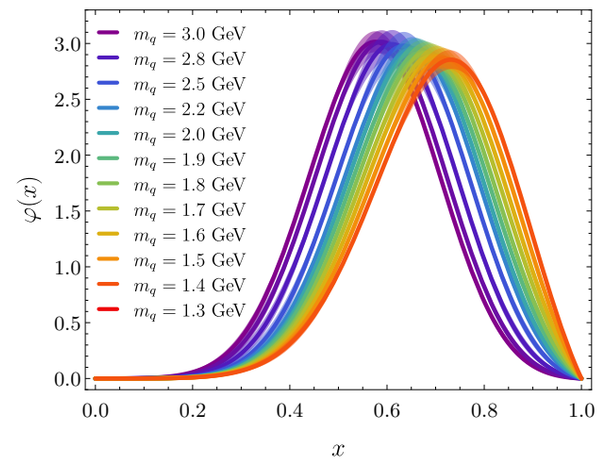


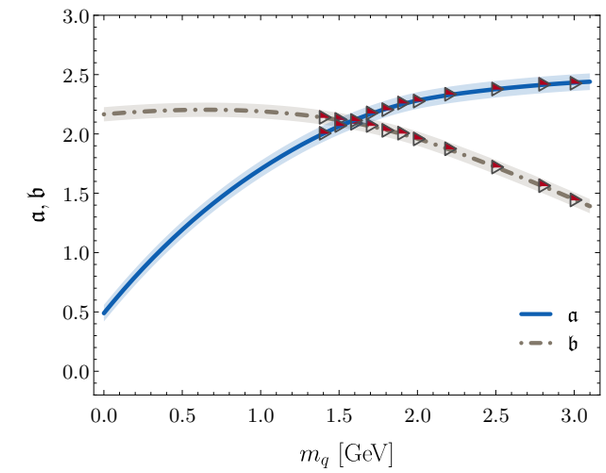


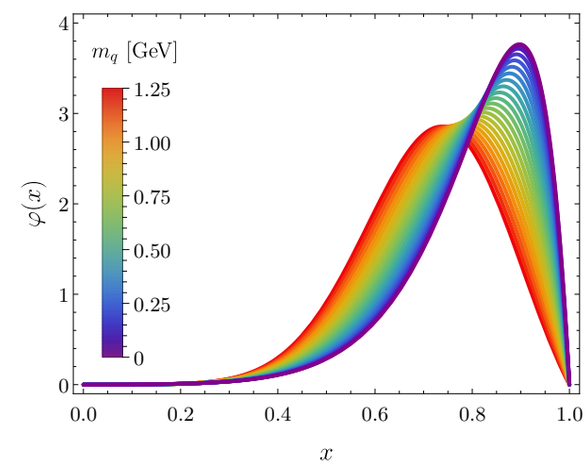


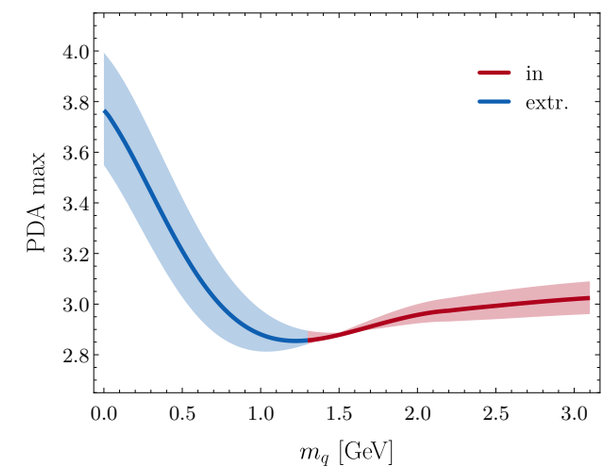


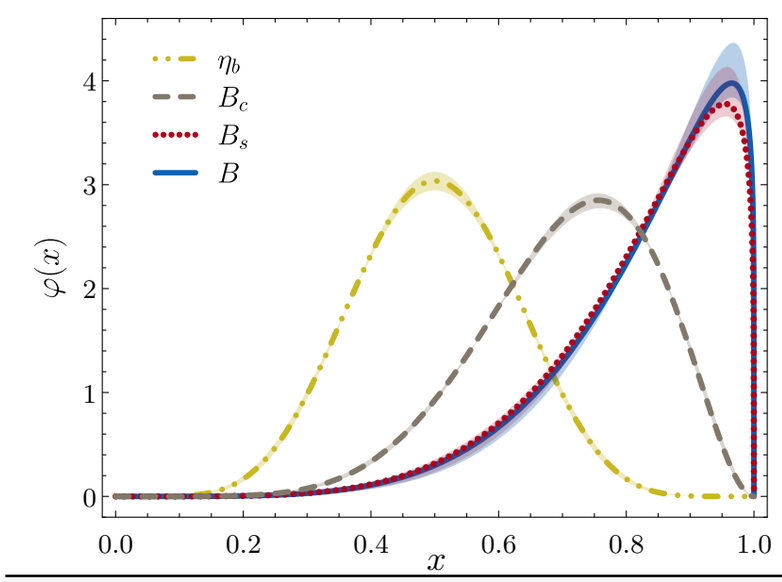
- **Gaussian**



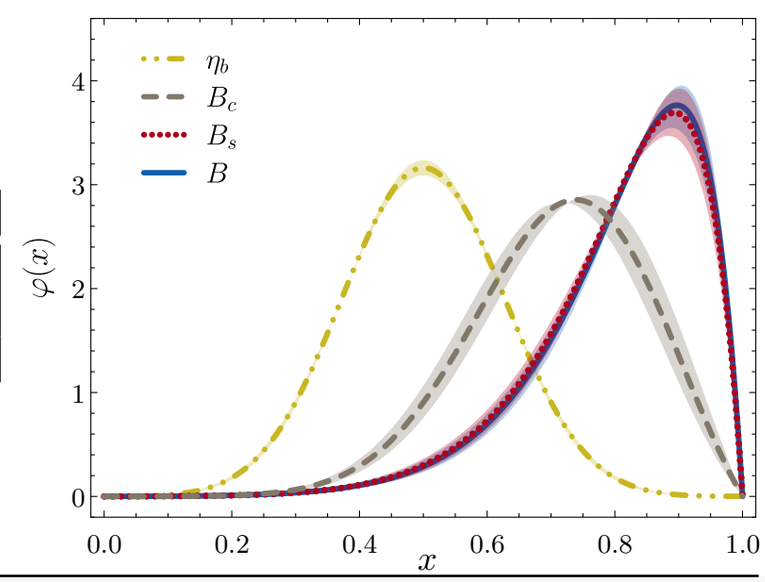








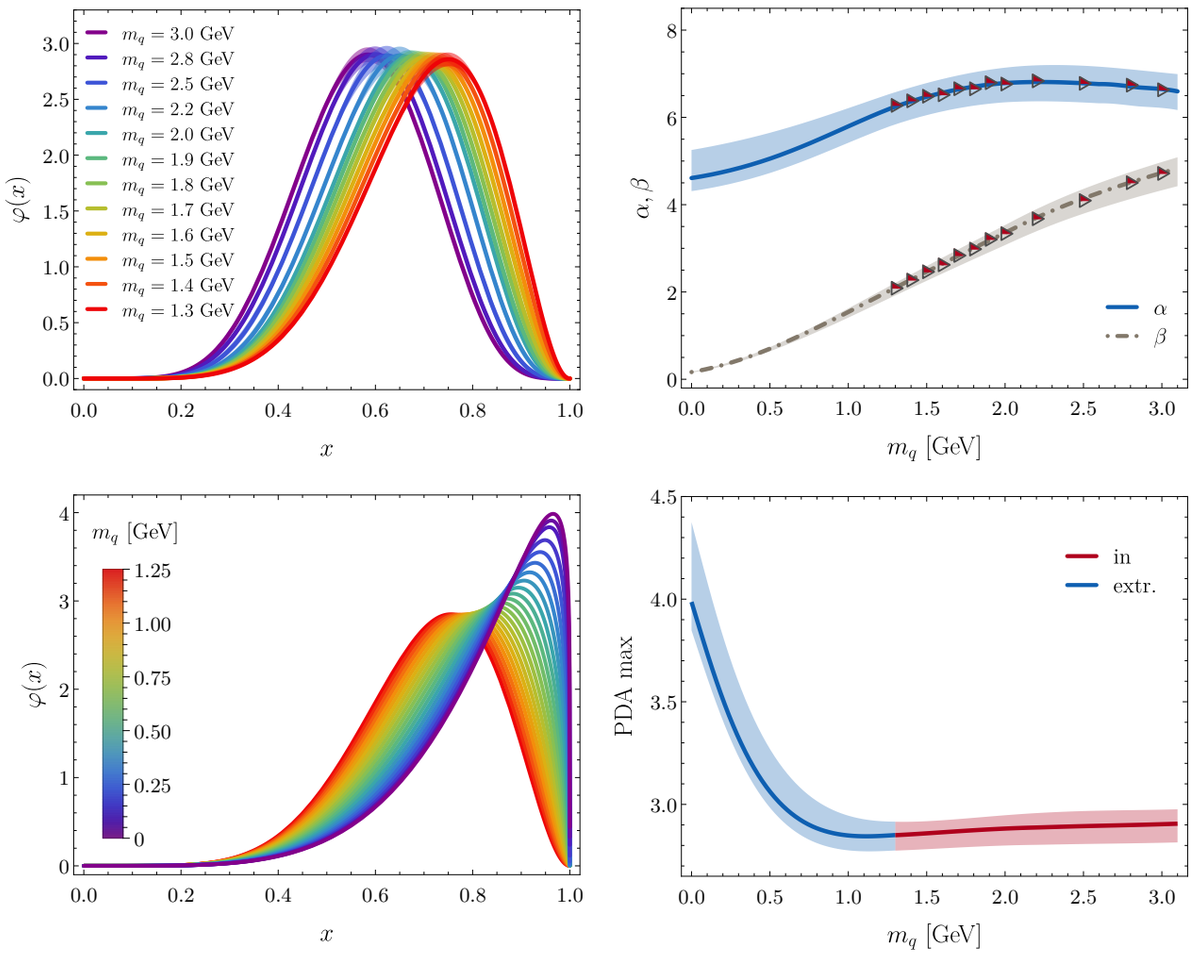
m_q	α	β	$\mathfrak{a}$	$\mathfrak{b}$
m_c	$6.179^{0.398}_{0.379}$	$1.994^{0.342}_{0.141}$	$1.901^{0.070}_{0.070}$	$2.163^{0.060}_{0.060}$
m_s	$4.660^{0.760}_{1.240}$	$0.230^{0.125}_{0.108}$	$0.621^{0.070}_{0.070}$	$2.174^{0.060}_{0.060}$
m_u	$4.612^{0.714}_{1.287}$	$0.144^{0.054}_{0.118}$	$0.495^{0.070}_{0.070}$	$2.166^{0.060}_{0.060}$



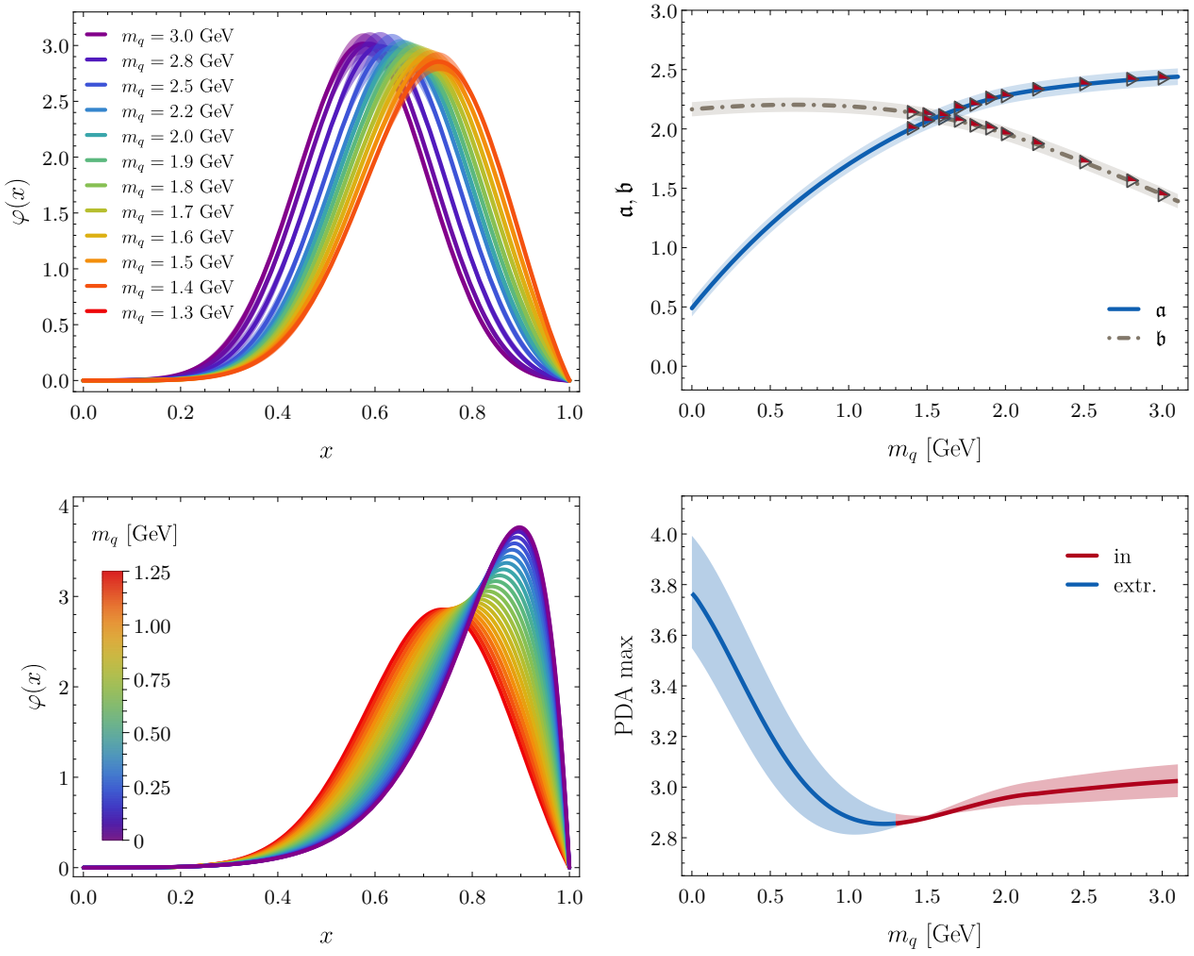
Bottom sector

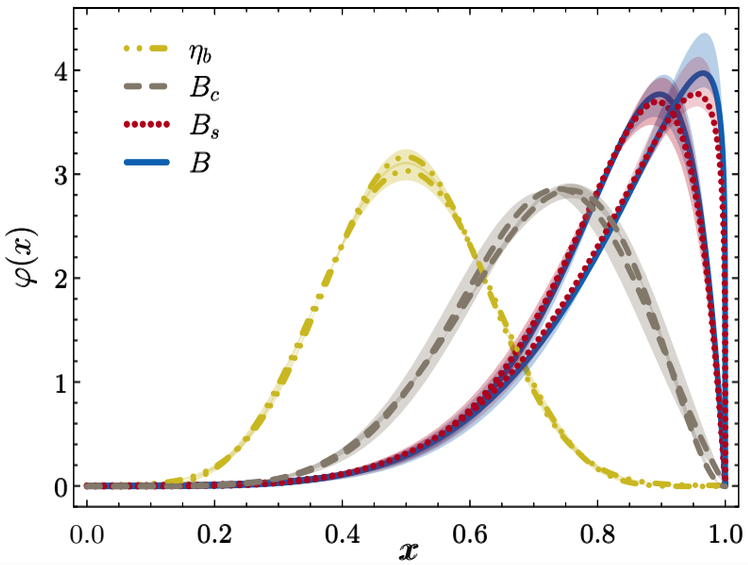
m_q	α	β	$\mathfrak{a}$	$\mathfrak{b}$
m_c	$6.179^{0.398}_{0.379}$	$1.994^{0.342}_{0.141}$	$1.901^{0.070}_{0.070}$	$2.163^{0.060}_{0.060}$
m_s	$4.660^{0.760}_{1.240}$	$0.230^{0.125}_{0.108}$	$0.621^{0.070}_{0.070}$	$2.174^{0.060}_{0.060}$
m_u	$4.612^{0.714}_{1.287}$	$0.144^{0.054}_{0.118}$	$0.495^{0.070}_{0.070}$	$2.166^{0.060}_{0.060}$

- **Gegenbauer**



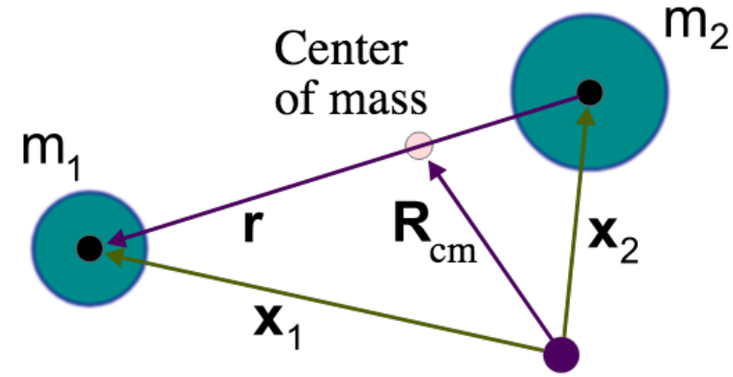
- **Gaussian**



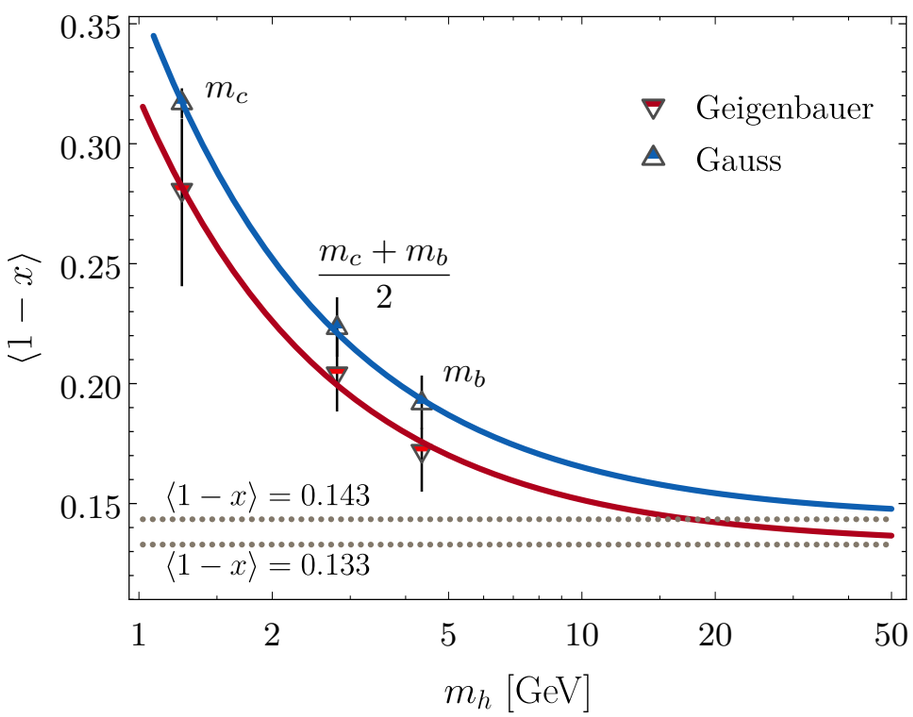


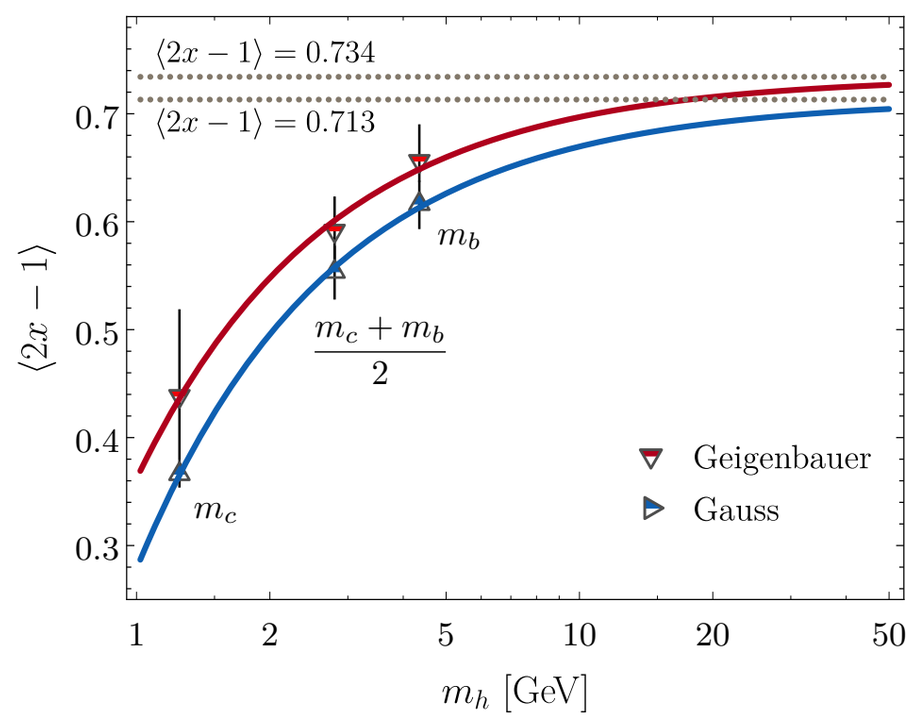
Infinite heavy mass limit

- **What happens when $m_h \to \infty$?**
  on general ground limiting moments saturate
  - **Two body problem**
    mass of the heavy-object controls overall translational motion of entire system; no effect on the relative motion of the light object
  - $\langle 2x-1 \rangle$ **moment**
    measure relative heavy/light momentum



- **Check with relevant moments**
  add extra point at intermediate heavy mass



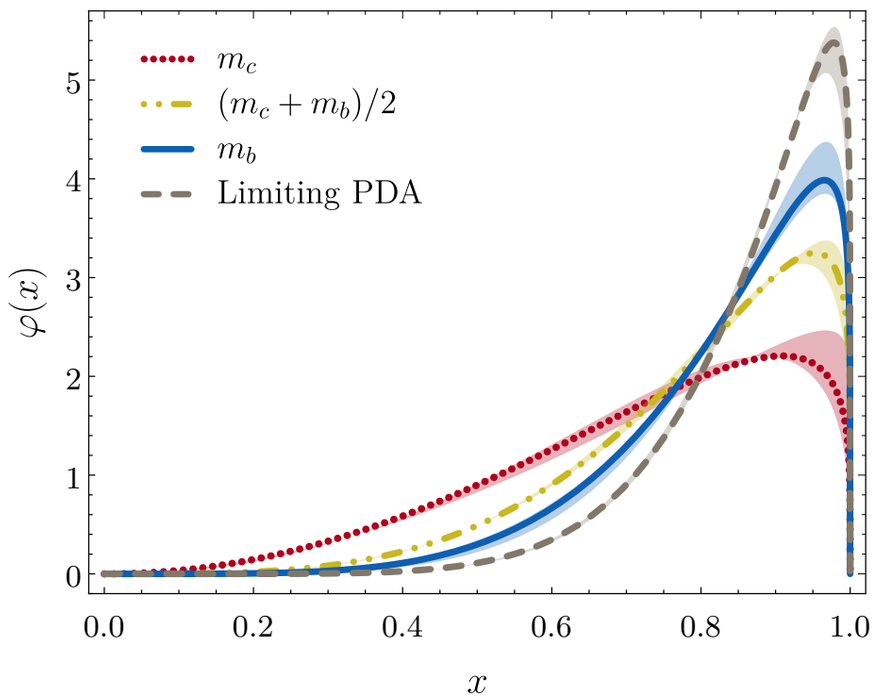


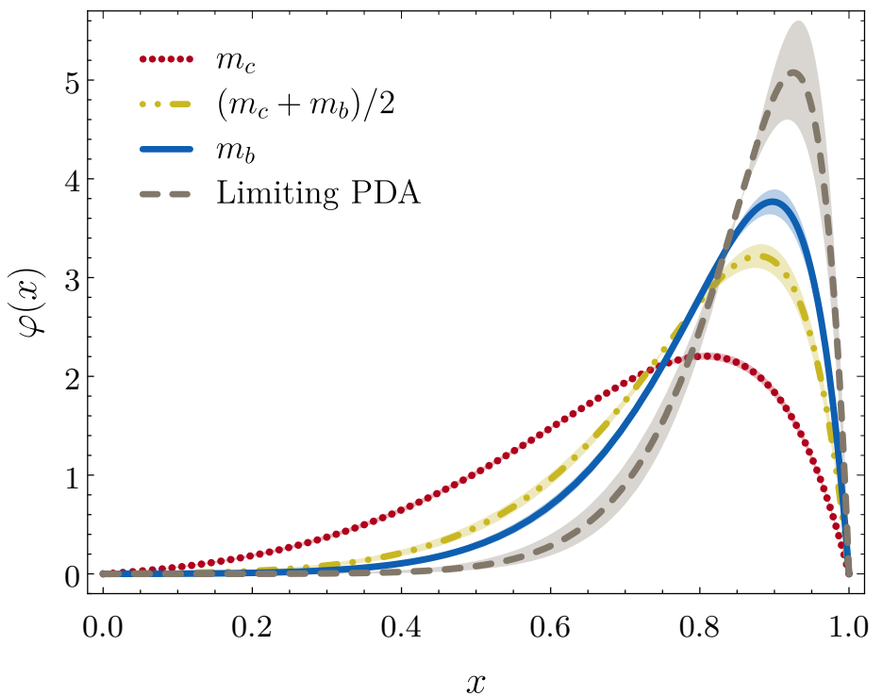
- **Fitting function**

$$a + \frac{b}{m_h}$$

Limiting PDA

- **Limiting PDA @** $m_h \to \infty$
  can be reconstructed from moments/parameters
  - **PDA stable**
    against fitting different moms/pars

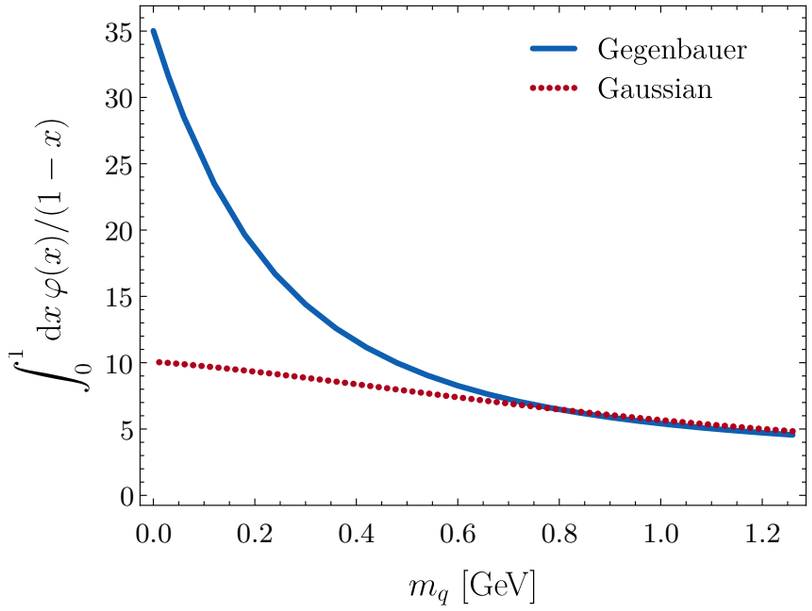




- **What does a limiting PDA imply?**

Gegenbauer or Gaussian?

- **Main difference**
  end point behavior @ $x \to 1$
  - **Affects the inverse moment** $\int_0^1 \mathrm{d}x\, \phi(x)/(1-x)$
    large Q^2 behavior of form factors
  - **Large factor expected**
    up to three times (bottom sector)

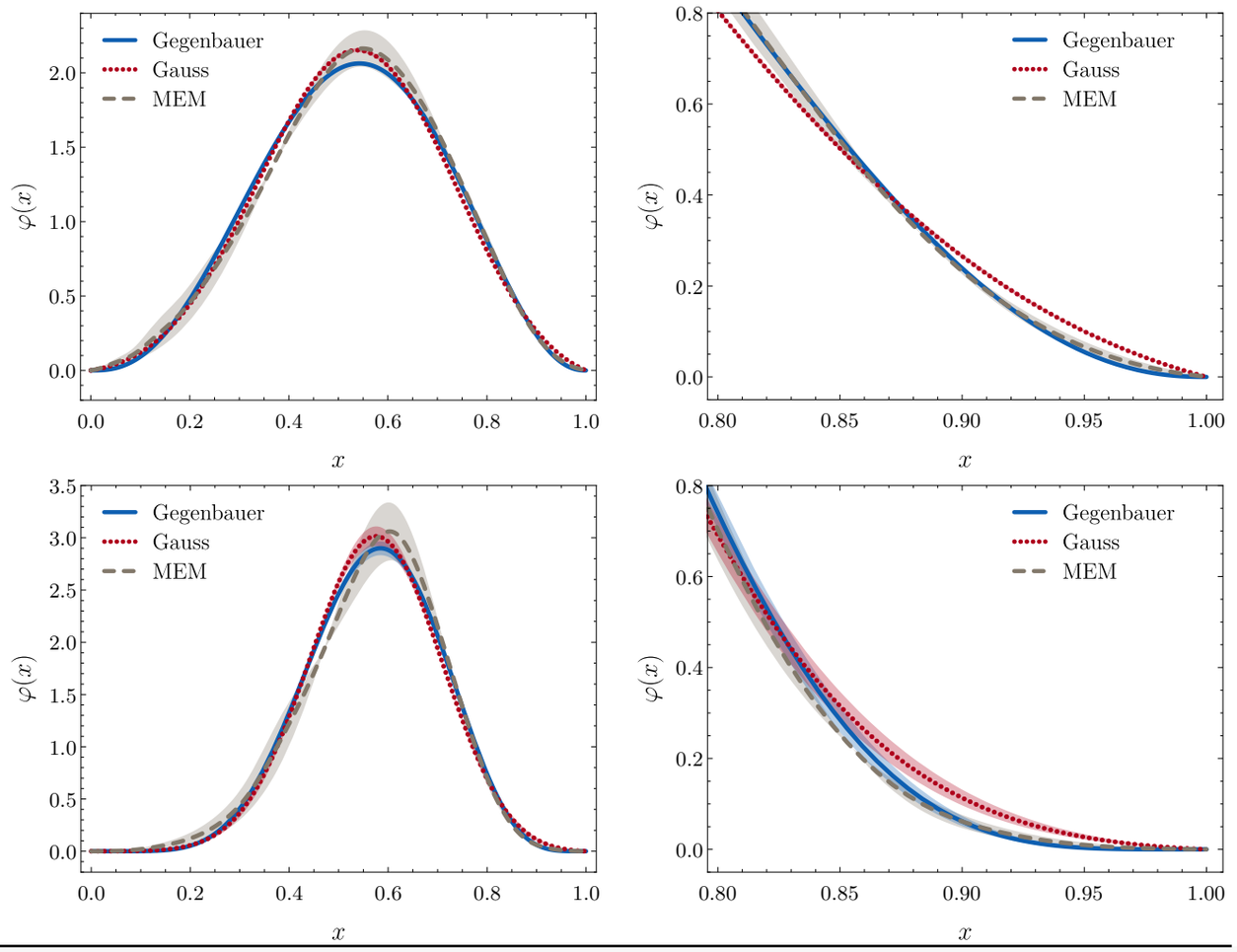


- **How to distinguish?**
  Use Bayesian determination
  Gao, Chang, Liu PLB 770 (2017)
  - **Model independent**
    determination of the PDA through

$$\Phi(k,P) = \int_{-1}^{1} \mathrm{d}z \int_0^\infty \mathrm{d}\gamma\, \frac{g(\gamma,z)}{\left(k^2 + zk\cdot P + \frac{1}{4}P^2 + \Lambda^2 + \gamma\right)^3}$$

$$\varphi(x) = \mathcal{N} \int_0^\infty \mathrm{d}\gamma\, \frac{g(\gamma, 1-2x)}{\gamma + \Lambda^2 - x(x-1)M^2}$$

  - **Use MEM**
    to extract g from numerical data of Φ
  - **Gegenbauer preferred**
    @ end point $x \to 1$



thank**you**